Theory for Extending Single-Product Production Function Estimation to Multi-Product Settings *

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Abstract

We introduce a new methodology for estimating multi-product production functions. It embeds the seminal contributions of Diewert (1973) and Lau (1976) in a semi-parametric econometric framework following Olley and Pakes (1996). We address the simultaneity of inputs and outputs while allowing for and estimating one unobserved technical efficiency term for each firm-product, each one of which may be freely correlated with inputs and outputs. We show how to translate the structural parameters into the reduced form parameters that give the elasticity of each output with respect to each input. For each output the sum of these input coefficients is the returns to scale for that output. We show how to use these estimates to recover estimates of firm-product marginal costs by extending the Hall (1988) single-product result to our multi-product setting. The main advantage of our framework is that it does not require multi-product production to be a collection of single-product production functions, which rules out the possibility that outputs are substitutes or complements with one another. Our empirical results using panel multi-production production data are largely consistent with our theoretical restrictions and strongly reject the single-product production approximation to multi-product production.

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1 Introduction

A well-known anecdotal fact is that most firms that produce one good also produce at least one or more other similar goods. This fact has been confirmed in recent micro-level production data across several countries.[1] Prior to these micro-level data being available, most micro-level data sets on production only included total revenue generated by all of a firm's products, in addition to measurements on categories of input expenditures like the wage bill, spending on capital or intermediate inputs. In these new data - in addition to the same input measurements - product-level quantities and revenues are reported separately for each product. In this paper we show how this kind of data can be used to improve upon previous estimation of production functions and the implied estimates of marginal costs and markups.

We introduce a new methodology for estimating multi-product production functions. It embeds the seminal contributions of Diewert (1973) and Lau (1976) in a semi parametric econometric framework following Olley and Pakes (1996) and the ensuing literature. The intuition behind the approach is straightforward. The standard single product production function gives the maximal output for any combination of inputs (e.g. labor, capital, and intermediate inputs). A multi-product production function extends the single product setting by giving the maximal output achievable of any one good holding inputs levels *and* the levels of other output goods produced constant.

We estimate one output equation for each output by relating that output to aggregate inputs and the other individual output levels. This approach allows for the possibility that any one output can be a substitute or a complement with any other output. We extend results from Petropoulos (2001) and Ackerberg, Benkard, Berry, and Pakes (2007) to address the simultaneity of inputs and outputs. Our approach allows for and estimates one unobserved technical efficiency term for each firm-product, each one of which may be freely correlated with inputs and outputs.

We show how to translate the structural parameters into the reduced form parameters that give the elasticity of each output with respect to each input. For each output the sum of these input coefficients is the returns to scale for that output which will generally vary by output. We then derive several testable conditions that must hold for the structural estimates to be consistent with multi-product production.

Consistent with multi-product production theory, in the Belgian data all but five of the forty-eight input coefficients are positive, and thirty eight of these forty-three positive input coefficient estimates are strongly statistically significant. The coefficient on "other good output" is always negative and highly significant suggesting quantities are

[1] See e.g. Bernard, Redding and Schott (2010, 2011) and Mayer, Melitz and Ottaviano (2014, 2018).

substitutes for one another holding inputs constant. The significance in every case of the twelve estimated quantity coefficients strongly rejects the use of single-product production as an approximation to multi-product production.

When we translate the structural coefficients into their reduced-form counterparts elasticities are almost all positive and result in returns to scale range from 0.88 to 1.247 for ten of twelve quantities with five quantities having returns to scale almost identical to 1. We find a very high correlation of technical efficiency across products within a firm suggesting an unobserved managerial ability that can be applied across the production of different products. We also find that firms are about 40% more technically efficient at producing their highest revenue product. Finally when we examine similar multi-product data from France we find very similar results across all margins.

Hall (1986, 1988) shows in the case of single-product production, minimization of the variable cost function yields a relationship between the markup and the elasticities of output with respect to an input and observed input expenditures. We derive the multivariate analog where we express marginal costs as a function of output-input elasticities, individual input expenditures, and output quantities. Deloecker and Warzynski (2012) assume single-product production and recover firm-specific markups by applying Hall's insights to standard micro-level production data where only total revenue is recorded. Using this new micro-level data on individual output quantities and revenues we show how one can allow for and estimate one unobserved marginal cost term for each firm-product using the multi-product variable cost minimization problem.

Using Indian manufacturing data, De Loecker et al (2016) is the first paper to tackle these questions since the availability of this kind of data. They impose two key assumptions in their representation of multi-product production. *Assumption A1* maintains that all production is single-product production, thereby ruling out the possibility that outputs might be substitutes or complements in production. They estimate production function parameters using only the observations on single-product firms. *Assumption A4* maintains that it is possible to partition inputs across these different single-product production functions. They then show that cost minimization - under *A1 and A4* - provides for a rule for partitioning inputs across the different single-product production technologies, and they use this rule for identifying multi-product technical efficiency residuals at multi-product firms.

Several follow-up papers using multi-product data extend the De Loecker et al (2016) methodology on important economic dimensions. Valmari (2016) extends the cost minimization conditions to profit maximization conditions by adding a demand side to the model. Gong and Sickles (2018) show how to allow for different production functions for multi-product firms versus single-product firms in a setting where stochastic frontier analysis is the maintained production model. Orr (2019) provides alternatives to assumptions *A1 and A4* and Itoga (2019) follows Orr's extension. None of these papers allows any one output to be a possible substitute or complement to any other output, but DLGKP and the papers that followed do provide several restrictions that may be useful for improving the precision of estimates in our setting.[2]

The rest of the paper is structured as follows. In Sections 2 and 3 we cover the theory and identification. Section 4 discusses estimation of marginal costs. Section 5 discusses estimation and Section 6 describes the data. Section 7 has the results and Section 8 concludes.

2 Multi-Product Production

Using Diewert (1973) and Lau (1976) we review the theoretical conditions under which single- and multi-product production functions exist and their testable implications.

2.1 Single Product Firms

The primitive of production analysis is the firm's production possibilities set T. In the single-product setting T lives in the non-negative orthant of R^{1+N} and contains all values of the single output q that can be produced by using N inputs $x = (x_1, x_2, \ldots, x_N)$, so if $(\tilde{q}_1, \tilde{x}) \in T$, then $\tilde{q}_1$ is producible given $\tilde{x}$, The single-product production function $F(x)$ - the production frontier - is defined as:

$$q^* = F(x) \equiv max\{q \,|\, (q, x) \in T\}.$$

$F(x)$ admits some well-known testable properties. If inputs are *freely disposable* then an output level achieved with the vector of inputs x' can always be achieved with a vector of inputs x'' where $x'' \geq x'$. This implies the production function is weakly increasing in inputs (Diewert (1973)). The production function $F(x)$ should also be concave in the freely variable inputs holding fixed inputs constant and it should be quasi-concave in the fixed inputs holding the freely variable inputs constant (Lau (1976)).

2.2 Multi-Product Firms

With M outputs and N inputs the firm's production possibilities set T lives on the non-negative orthant of R^{M+N}. It contains all of the combinations of M non-negative

[2]There is also an interest among practitioners in extending the Directional Distance Function to allow for multi-product production (see e.g. Fare, Martins-Filho, and Vardanyan (2009) or Kuosmanen and Johnson (2019).)

outputs $q = (q_1, q_2, \ldots, q_M)$ that can be produced by using N non-negative inputs $x = (x_1, x_2, \ldots, x_N)$ so if $(\tilde{q}, \tilde{x}) \in T$ then $\tilde{q} = (\tilde{q_1}, \ldots, \tilde{q_J})$ is achievable using $\tilde{x} = (\tilde{x_1}, \ldots, \tilde{x_N})$. For good j produced by the firm let the output production of other goods be denoted by q_{-j}. For any (q_{-j}, x), if $max\{q_j \,|\, (q_j, q_{-j}, x) \in T$ is finite, then Diewert (1973) defines the *transformation function* as

$$q_j^* = F_j(q_{-j}, x) \equiv max\{q_j \,|\, (q_j, q_{-j}, x) \in T\}.$$

If no positive output of q_j is possible given (q_{-j}, x) then he assigns

$$F_j(q_{-j}, x) = -\infty.$$

We develop the properties of $F_j(q_{-j}, x)$ under a mix of assumptions from Diewert (1973) and Lau (1976).

We follow Lau (1976) and divide outputs and inputs (q_{-j}, x) into those that are variable v in the short-run and those that are not, denoted by K. Alternatively, we could do all of our analysis conditional on q_{-j}, with (v, K) partitioning only the variable from the fixed inputs. We sometimes abuse notation by expressing (q_{-j}, x) as (v, K) and by writing $F_j(v, K)$.

We assume the production possibilities set T satisfies the following four *Conditions P*:

(i) P.1 T is a non-empty subset of the non-negative orthant of R^{M+N}

(ii) P.2 T is closed and bounded,

(iii) P.3 If $(q, x_k, x_{-k}) \in T$ then $(q, x'_k, x_{-k}) \in T \; \forall x'_k \geq x_k$.

(iv) P.4 The sets $T^K = \{v \,|\, (v, K) \in T\}$ are convex for every K; the sets $T^v = \{K \,|\, (v, K) \in T\}$ are convex in K for every v.

Conditions P.1 and P.2 are weak regularity conditions on T that require the production set to be non-empty, closed, and bounded. Condition P.3 is a free disposal condition on inputs; if you can produce q_j given (q_{-j}, x), then you can produce q_j with any $x^{'} \geq x$. Diewert (1973) uses these free disposal conditions to prove that output is weakly increasing in any input holding all other inputs and outputs constant. Diewert (1973) then shows if we add the condition that T is convex, there exists a well-defined production function that is concave in the inputs, ensuring decreasing marginal rates of substitution among inputs. The standard concavity tests in a single product setting can be directly extended to test whether the multi-product theory holds.

Convexity on T rules out the possibility of increasing returns to scale. Condition P.4 extends the convexity on T assumption from Diewert (1973) to the disjoint biconvexity assumption of Lau (1976). Under disjoint biconvexity we have convexity of the freely variable inputs and outputs v holding fixed inputs and outputs K constant, and convexity in the fixed variables K holding the freely variable v constant. This setup allows for the possibility of overall increasing returns to scale - non-convexities in T - while maintaining decreasing marginal rates of substitution between elements in v and similarly for the elements in K. The analysis can be done unconditionally or conditional on outputs q_{-j}.

The following theorem formalizes the above claims.

Theorem 2.1 (The Transformation Function) *Under P.1-P.4 the function $F_j(q_{-j}, x)$ is an extended real-valued function defined for each $(q_{-j}, x) \geq (0_{M-1}, 0_N)$ and is non-negative on the set where it is finite. $F_j(q_{-j}, x)$ is non-decreasing in x holding q_{-j} constant, $F_j(v, K)$ is concave in v for all K, and $F_j(v, K)$ and quasi-concave in K for all v.*

Proof: See Appendix A.

The empirical implication of disjoint convexity is as follows. Convexity in the elements of v (conditional on any K) results in a production function that is concave in v holding K constant. For the elements in K convexity in K given v results in the production function being quasi-concave in K given v. As before these tests can be conducted unconditionally or conditional on outputs q_{-j}.

2.3 "Unobserved" Inputs

Historically, in the single-product production literature it is common to allow for a component of the error to affect output and be observed by the firm when it is making its input decisions (Griliches and Mairesse (1995)). This factor is an "unobserved" technical efficiency term that is unobserved to the researcher and is allowed to be freely correlated with input choices. In our setting with multiple products we want to allow for one possible "unobserved" technical efficiency term for each output produced, with the entire vector of these unobserved shocks denoted

$$\omega = (\omega_1, \omega_2, \ldots, \omega_M).$$

In this section we briefly outline how to incorporate these factors into our theory framework. The main result is all of the components of the theorem continue to hold with the caveat now that everything is conditional on ω.

We extend the production possibilities set to the case where - in addition to containing observed M outputs q and observed N inputs x - we now add the "unobserved" M inputs

ω, so $(q, x, \omega) \in R^{M+N+M}$. $(q, x, \omega) \in T$ if (e.g.) the vector of outputs q can be produced with observed and unobserved inputs x and ω respectively. We define the "Generalized Transformation Function" as the Transformation Function that has all of ω as arguments in production. Define

$$q*_j = F_j(q_{-j}, x, , \omega) = max(q_j | q_j, q_{-j}, x, \omega) \in T$$

and let it equal $-\infty$ if there is no non-negative q_j such that $(q_j, q_{-j}, x, \omega) \in T$.

Now we assume the production possibilities set T satisfies the following four *Conditions P'*:

(i) P'.1 T is a non-empty subset of the non-negative orthant of R^{M+N}

(ii) P'.2 T is closed and bounded,

(iii) P'.3 If $(q, x_k, x_{-k}, \omega) \in T$ then $(q, x'_k, x_{-k}, \omega) \in T$ $\forall x'_k \geq x_k$.

(iv) P'.4 The sets $T^K = \{v \,|\, (v, K, \omega) \in T\}$ are convex for every K given ω; the sets $T^v = \{K \,|\, (v, K, \omega) \in T\}$ are convex in K for every v given ω.

All of the results for the transformation function hold but now they are conditional on ω.

Theorem 2.2 (The Generalized Transformation Function) *Under P'.1-P'.4 the function $F_j(q_{-j}, x, \omega)$ is an extended real-valued function defined for each $(q_{-j}, x) \geq (0_{M-1}, 0_N)$ and is non-negative on the set where it is finite. $F_j(q_{-j}, x, \omega)$ is non-decreasing in x holding q_{-j} and ω constant. Given ω, $F_j(v, K, , \omega)$ is concave in v for all K quasi-concave in K for all v.*

Proof: See Appendix A.

Convexity in the elements of v conditional on any K and ω results in a production function that is concave in v holding K and ω constant. For the elements in K convexity in K given v and ω results in the production function being quasi-concave in K given v and ω. All previously discussed tests are available in this setting after conditioning on ω.

3 Identification of Production Function Parameters

We use a simultaneous equations system suggested by Theorem 2.2 to develop several results. We then illustrate in the two-product setting.

3.1 General Setup

Using Cobb-Douglass specifications for each of the goods with all variables in logs we write the system of M production equations as:

$$q_{jt} = \beta_0^j + \beta_l^j l_t + \beta_k^j k_t + \beta_m^j m_t + \gamma_{-j}^j \prime q_{-jt} + \omega_{jt} \qquad j = 1 \cdots M \tag{1}$$

where (l_t, k_t, m_t) denote total labor, capital, and materials use in the production of all goods.[3] Input production parameters vary by good j and are given by $\beta^j = (\beta_l^j, \beta_k^j, \beta_m^j)$. q_{-jt} denotes the $M-1$ column vector of all other outputs excluding q_j and γ_{-j}^j denotes the $M-1$ row vector of output elasticities for all other products excluding j. ω_{jt} is the unobserved input for good j and we suppress it throughout this section for transparency. From Theorem 2.2 we have the first testable implication.

Lemma 3.1 *The multi-product production function is only well-defined when $\beta^j > 0 \ \forall j$.*

For transparency we suppress time subscripts and we write this system in matrix form by first moving outputs to left hand side yielding

$$q_j - \gamma_{-j}^j \prime q_{-j} = \beta_0^j + \beta_l^j l + \beta_k^j k + \beta_m^j m \qquad j = 1 \cdots M. \tag{2}$$

We then express the system as

$$\Gamma Q = \beta X \tag{3}$$

with $Q = (q_1, \ldots, q_M)'$ the $Mx1$ vector of quantities, Γ the MxM matrix of associated quantity parameters with the diagonals normalized to one, $X = (1, l, k, m)'$ and β the associated $Mx4$ matrix of stacked rows $(\beta_0^j, \beta_l^j, \beta_k^j, \beta_m^j)$ $j = 1, \ldots, M$.

For the reduced form of the multi-product production function not only must Γ^{-1} exist but it also must be that the determinant of Γ is positive. The positive determinant ensures that the non-diagonal elements are sufficiently small relative to the diagonal element of 1.

Lemma 3.2 *A necessary and sufficient condition for the existence of the reduced form multi-product production function is the determinant of Γ must be positive.*

With Γ invertible we can solve for Q directly as

$$Q = \Gamma^{-1} \beta X \tag{4}$$

These reduced form equations characterize how changing any one or any collection of inputs affects the levels of each of the different outputs. Each production output has its own measure of returns-to-scale that can be computed from $\Gamma^{-1} \beta$.

[3] This discussion generalizes immediately to a trans-log system of equations which Diewert (1973) advocates. We allow for a quadratic in quantities in our robustness section.

Lemma 3.3 *Returns to scale for the jth output are given by the sum of the last three elements of the jth row of $\Gamma^{-1}\beta$.*

If multi-product production can be characterized as a collection of single-product production functions, as maintained e.g. in Foster, Haltiwanger, and Syverson (2008) and De Loecker, Goldberg, Khandelwal, Pacvnik (2016)), it follows directly that all of the off-diagonal elements of Γ must equal zero.

Lemma 3.4 *A necessary condition for single-product production functions to exist in a multi-product setting is $\gamma_{ij} = 0 \;\; \forall i \neq j, \;\; i, j = 1, \ldots, M$.*

Significance of any subset of these γ_{ij}'s or any function of them rejects the single-product approximation. We explore all of these testable implications below in our data.

3.2 Illustration in a 2-product Setting

We illustrate using Dhyne, Petrin and Warzynski (2014), who look at the bread and cakes industry in Belgium, where most firms that produce one also produce the other. We suppress ω's in this section. Let q_{Bt} and q_{Ct} denote output quantities of bread and cakes and let the structural production function for bread and for cakes be given by the system of equations

$$q_{Bt} = \beta_0 + \beta_l^b l_t + \beta_k^b k_t + \beta_m^b m_t + \gamma_C q_{Ct} \tag{5}$$

$$q_{Ct} = \beta_0 + \beta_l^c l_t + \beta_k^c k_t + \beta_m^c m_t + \gamma_B q_{Bt} \tag{6}$$

Lemma 3.1 says the production parameters $(\beta_l^b, \beta_k^b, \beta_m^b, \beta_l^c, \beta_k^c, \beta_m^c)$ must be positive to be consistent with multi-product production. If we condition on outputs, and if labor and materials are the flexible inputs and capital is the fixed input then either estimated equation should be concave in l and m given k and quasi-concave in k given l and m. These are the standard single-product conditions extended to each of the multi-product equations.

Suppressing time subscripts and moving quantities to the left hand side we have in matrix form

$$\begin{bmatrix} q_B \\ q_C \end{bmatrix} \quad \begin{bmatrix} 1 & -\gamma_C \\ -\gamma_B & 1 \end{bmatrix} = \left(\begin{bmatrix} \beta_0^b & \beta_l^b & \beta_k^b & \beta_m^b \\ \beta_0^c & \beta_l^c & \beta_k^c & \beta_m^c \end{bmatrix} \begin{bmatrix} 1 \\ l \\ k \\ m \end{bmatrix} \right).$$

From Lemma 3.2 solving for the reduced form for quantities requires invertibility of this Γ matrix which leads to the testable condition that $\gamma_B * \gamma_C < 1$.

Inverting out Γ to solve for the reduced form we have

$$\begin{bmatrix} q_B \\ q_C \end{bmatrix} = \frac{1}{1-\gamma_C\gamma_B}\left(\begin{bmatrix} \beta_0^b + \gamma_B\beta_0^c & \beta_l^b + \gamma_B\beta_l^c & \beta_k^b + \gamma_B\beta_k^c & \beta_m^b + \gamma_B\beta_m^c \\ \beta_0^c + \gamma_C\beta_0^l & \beta_l^c + \gamma_C\beta_l^b & \beta_k^c + \gamma_C\beta_k^b & \beta_m^c + \gamma_C\beta_m^b \end{bmatrix}\begin{bmatrix} 1 \\ l \\ k \\ m \end{bmatrix}\right).$$

A one percent increase in labor leads to an increase in bread quantities of

$$\frac{1}{1-\gamma_C\gamma_B} * (\beta_l^b + \gamma_B\beta_l^c)$$

percent and cake quantities of

$$\frac{1}{1-\gamma_C\gamma_B} * (\beta_l^c + \gamma_C\beta_l^b)$$

percent. Lemma 3.3 has returns to scale for bread production equal to

$$\frac{1}{1-\gamma_C\gamma_B}(\beta_l^b + \gamma_B\beta_l^c + \beta_k^b + \gamma_B\beta_k^c + \beta_m^b + \gamma_B\beta_m^c),$$

and similarly for cake. Lemma 3.4 says treating bread and cakes separately as single product production functions requires $(\gamma_B, \gamma_C) = 0$. We now turn to estimation of marginal costs.

4 Identification of Marginal Costs

Hall (1986, 1988) shows in the case of single-product production cost minimization identifies the markup as a function of the observed elasticities of revenue with respect to an input and the observed firm expenditures on that input. We derive the multivariate analog using the variable cost function to show how to express marginal costs as a function of output-input elasticities, individual input expenditures, and output quantities. We show how/when we can identify one unobserved marginal cost term for each firm-product. For the rest of the paper we use x to exclusively denote the N_1 freely variable inputs and K to exclusively denote the remaining $N - N_1$ fixed inputs.

In order to use cost minimization to invert out marginal costs the reduced form production functions must exist so we can express each of the M outputs only as a function of inputs:

$$q_m = q_m(x, K, \omega) \quad m = 1, \ldots, M.$$

Letting q_m^* denote the desired output for each good m. Minimization of the variable cost function is given by

$$Min_x \;\; P * x \;\; s.t. \;\; q_m(x, K, \omega) > q_m^* \;\; m = 1 \ldots M$$

where $P = [P_1 \cdots P_N]'$ denote the input prices for inputs 1 through N. This minimization yields the following first order conditions for the N_1 freely variable inputs x:

$$P_i = \sum_{m=1}^{M} \frac{\partial q_m(x, K, \omega)'}{\partial x_i} \lambda_m \quad i = 1, \ldots, N_1,$$

and $\lambda = [\lambda_1 ... \lambda_M]'$ denote marginal costs - the lagrange multipliers - for outputs one through M.

As we show below if $N_1 > M$ we are overidentified, if $N_1 = M$ we are exactly identified, and if $N_1 < M$ we are underidentified. To simplify discussion we analyze the case where we are just identified; if we were overidentified it would add a set of $N_1 - M$ additional restrictions to the just identified case. In matrix notation we have

$$P = [\frac{\partial q(x, K, \omega)'}{\partial x}]\lambda$$

with $[\frac{\partial q(x,K,\omega)'}{\partial x}]$ the MxM $(N_1 x N_1)$ matrix of partial derivatives. If $[\frac{\partial q(x,K,\omega)'}{\partial x}]$ is invertible - which is readily checked for any set of point estimates - we have

$$[\frac{\partial q(x, K, \omega)'}{\partial x}]^{-1} P = \lambda$$

In the single-product case it simplifies down to

$$\lambda = \frac{P_i x_i}{q * \epsilon_i},$$

which is the result from Hall (1986, 1988). Multiplying this formula through by $\frac{1}{p_q}$, where p_q denotes the price of output we have the output elasticity divided by the markup is equal to the ratio of input expenditure to revenue:

$$\frac{\epsilon_i}{\mu} = \frac{P_i x_i}{p_q q}$$

where $\mu = \frac{p_o}{\lambda}$. This is the approach proposed in De Loecker and Warzynski (2012) to invert out markups in standard plant-level data where only input expenditures and revenue are observed, and where one has an estimate of the elasticity of output with respect to input i. *The difference between these last two expressions illustrates the value of observing quantities of outputs (or, alternatively, individual prices of outputs); without them, one can estimate markups using observed revenue shares and estimated elasticities, but one cannot separate price from marginal cost.*

4.1 Illustration in a 2-product Setting

We focus on the two output case to illustrate. From cost minimization in the two-product setting there are two first-order conditions given as

$$P = \begin{bmatrix} \frac{\partial q_1}{\partial x_1} & \frac{\partial q_2}{\partial x_1} \\ \frac{\partial q_1}{\partial x_2} & \frac{\partial q_2}{\partial x_2} \end{bmatrix} \lambda$$

where $P = [P_1 \; P_2]'$ denote the input prices for inputs one and two and $\lambda = [\lambda_1 \; \lambda_2]'$ denote marginal costs for outputs one and two. Inverting the matrix and premultiplying prices by this inverse yields marginal costs:

$$\frac{1}{det} \begin{bmatrix} \frac{\partial q_2}{\partial x_2} & -\frac{\partial q_2}{\partial x_1} \\ -\frac{\partial q_1}{\partial x_2} & \frac{\partial q_1}{\partial x_1} \end{bmatrix} P = \lambda$$

where the determinant is given as

$$det = \frac{\partial q_1}{\partial x_1}\frac{\partial q_2}{\partial x_2} - \frac{\partial q_2}{\partial x_1}\frac{\partial q_1}{\partial x_2}.$$

Using the relationship that

$$\frac{\partial q_i}{\partial x_j} = \frac{\partial q_i}{\partial x_j}\frac{x_j}{q_i}\frac{q_i}{x_j} = \epsilon_{ij}\frac{q_i}{x_j},$$

and solving for marginal costs yields:

$$\lambda_1 = \frac{P_1 x_1 * \epsilon_{22} - P_2 x_2 * \epsilon_{21}}{(\epsilon_{11}\epsilon_{22} - \epsilon_{12}\epsilon_{21}) * q_1}$$

and

$$\lambda_2 = \frac{P_2 x_2 * \epsilon_{11} - P_1 x_1 * \epsilon_{12}}{(\epsilon_{11}\epsilon_{22} - \epsilon_{12}\epsilon_{21}) * q_2}.$$

The marginal costs (λ_1, λ_2) are a function only of $(\epsilon_{11}, \epsilon_{12}, \epsilon_{21}, \epsilon_{22})$, input expenditures, and output quantities. We now turn to estimation of the structural production function parameters.

5 Estimation

We review the standard proxy approach in the single-product production setup and then turn to our multi-product extension.

5.1 Single-product production setting

We have for q_t:

$$q_t = \beta_l l_t + \beta_k k_t + \beta_m m_t + \omega_t + \epsilon_t \tag{7}$$

where we have replaced the shock with its two components, i.e. $\varepsilon_t = \omega_t + \eta_t$. ϵ_t is assumed to be i.i.d. error upon which the firm does not act (like measurement error or specification error). ω_t is the technical efficiency shock, a state variable observed by the firm but unobserved to the econometrician. ω_t is assumed to be first-order Markov and is the source of the simultaneity problem as firm observe their shock before choosing their freely variable inputs l_t and m_t. k_t also responds to ω_t but with a lag as investments made in period $t-1$ come online in period t. This assumption allows k_t to be correlated with expected value of ω_t given ω_{t-1}. as ω_{t-1} - denoted $E[\omega_t|\omega_{t-1}]$ - but maintains that the innovation in the productivity shock $\xi_t = \omega_t - E[\omega_t|\omega_{t-1}]$ is unknown at the time the investment decision was made in $t-1$ and is therefore uncorrelated with current k_t.

The control function approaches of OP and LP both provide weak conditions under which there exists a proxy variable $h_t(k_t, \omega_t)$ that is a function of both state variables and that is monotonic in ω_t given k_t. The variables may include either investment (OP) or materials, fuels, electricity, or services (LP) (e.g.). Given the monotonocity there exists some function $g(\cdot)$,

$$\omega_t = g(k_t, h_t)$$

allowing ω_t to be written as a function of k_t and h_t.[4] For estimation Wooldridge (2009) uses a single index restriction to approximate unobserved productivity, writing

$$\omega_t = g(k_t, h_t) = \mathbf{c}(k_t, h_t)'\beta_\omega$$

where $\mathbf{c}(k_t, h_t)$ is a known vector function of (k_t, h_t) chosen by researchers with parameter vector β_ω to be estimated. The conditional expectation $E[\omega_t|\omega_{t-1}]$ can then be written as

$$E[\omega_t|\omega_{t-1}] = f(\mathbf{c}(k_{t-1}, h_{t-1})'\beta_\omega)$$

for some unknown function $f(\cdot)$, which Wooldridge (2009) approximates using a polynomial.

Replacing ω_t with its expectation and innovation, the estimating equation becomes

$$q_t = \beta_l l_t + \beta_k k_t + \beta_m m_t + E[\omega_t|\omega_{,t-1}] + \xi_t + \epsilon_t \tag{8}$$

[4] Kim, Petrin, and Song (2016) extend Hu and Schennach (2008) to allow for measurement error in all of the variables in the proxy function.

For expositional transparency we use only the first-order approximation term for $f(\cdot)$, which yields our error term

$$[\xi_t + \epsilon_t](\theta) = q_t - \beta_l l_t - \beta_k k_t - \beta_m m_t - \mathbf{c}(h_{t-1}, k_{t-1})'\beta_\omega \tag{9}$$

with the parameters to $\beta = (\beta_l, \beta_k, \beta_m, \beta_\omega)$.

We formulate the moment condition using materials m_t as the proxy but any other available proxy cited above could also be used here. The only change would be the set of conditioning variables. When m_t is the proxy a sufficient set of conditioning variables given as (e.g.) $x_t = (k_t, k_{t-1}, m_{t-1}, m_{t-2}, l_{t-1})$. Let θ_0 denote the true parameter value. Wooldridge shows that the conditional moment restriction

$$s(x_t;\theta) \quad \equiv \quad E[[\xi_t + \epsilon_t](\theta)|x_t] \text{ and } s(x_t;\theta_0) = 0$$

is sufficient for identification of β in the single product case (up to a rank condition on the instruments).[5] In equation (12) a function of m_{t-1} and k_{t-1} conditions out $E[\omega_t|\omega_{t-1}]$. ξ_t is not correlated with k_t, so k_t can serve as an instrument for itself. Lagged labor l_{t-1} and twice lagged materials m_{t-2} serve as instruments for l_t and m_t.

5.2 Multi-product production setting

In the multi-product case we have a system of M_t output equations:

$$q_{jt} = \beta_0^j + \beta_l^j l_t + \beta_k^j k_t + \beta_m^j m_t + \gamma_{-j}^j{\prime} q_{-jt} + \omega_{jt} + \eta_{jt} \qquad j = 1 \cdots M \tag{10}$$

We denote the vector of technical efficiency shocks as $\omega_t = (\omega_{1t}, \omega_{2t}, \ldots, \omega_{M_t})$ and assume $E[\omega_t|\omega_{t-1}] = \omega_{t-1}$. Choices of inputs will now generally be based not only on ω_{jt} but also on all of the other technical efficiency shocks ω_{-jt}. This frustrates the "inverting out" of ω_t that allows one to express ω_t as a function of k_t and a single proxy h_t as is done in the single product case.

We extend suggestions from Petropoulos (2001) and Ackerberg, Benkard, Berry, and Pakes (2007) to allow for these multiple unobserved technical efficiency shocks. Suppose we observe (at least) one proxy variable for every technical efficiency shock. Let $\mathbf{h}_t = (h_{1t}, \ldots, h_{Lt})$ denote the $1XL$ vector of available proxies. Each of these variables will generally be a function of k_t and $(\omega_{1t}, \omega_{2t}, \ldots, \omega_{M_t})$ and we write the vector of proxies as $\mathbf{h}_t(k_t, \omega_t)$. Conditional on k_t if $\mathbf{h}_t(k_t, \omega_t)$ is one-to-one and onto in ω_t then we can invert the proxy variables to get the $1XL$ vector of functions $\omega_t = \mathbf{g}(k_t, \mathbf{h}_t)$; in the next

[5] The Wooldridge formulation is robust to the Ackerberg, Caves, and Frazer (2015) criticism of OP/LP.

subsection we provide a simple condition that ensures this invertibility. Included in this vector of functions is

$$\omega_{jt} = g_j(k_t, \mathbf{h}_t), \qquad j = 1 \cdots M$$

which then motivates including a function of (k_t, h_t) in the estimation to control for ω_{jt}.[6]

The rest of the estimation proceeds in a manner similar to the single-product case. We use the same single index restriction to approximate unobserved productivity, so we have

$$\omega_{jt} = g_j(k_t, \mathbf{h}_t) = \mathbf{c}_j(k_t, \mathbf{h}_t)'\beta_{\omega_j}$$

where $\mathbf{c}_j(k_t, \mathbf{h}_t)$ is a known vector function of $(k_t, \mathbf{h}_t)$ chosen by researchers. $E[\omega_{jt}|\omega_{t-1}]$ is now given as

$$E[\omega_{jt}|\omega_{t-1}] = f_j(\mathbf{c}_j(k_{t-1}, \mathbf{h}_{t-1})'\beta_{\omega_j})$$

for some unknown function $f_j(\cdot)$. Again we use only the first-order approximation term for $f_j(\cdot)$ to keep exposition to a minimum.

Re-expressing in terms of firm's expectations we have

$$q_{jt} = \beta_l^j l_t + \beta_k^j k_t + \beta_m^j m_t + \gamma_{-j}^j{}' q_{-jt} + E[\omega_{jt}|\omega_{t-1}] + \xi_{jt} + \epsilon_{jt} \tag{11}$$

with $\xi_{jt} = \omega_{jt} - E[\omega_{jt}|\omega_{t-1}]$. The error is

$$[\xi_{jt} + \epsilon_{jt}](\theta) = q_{jt} - \beta_l^j l_t - \beta_k^j k_t - \beta_m^j m_t - \gamma_{-j}^j{}' q_{-jt} - \mathbf{c}_j(k_{t-1}, \mathbf{h}_{t-1})'\beta_{\omega_j}$$

with the new parameters γ_{-j}^j added to $\beta^j = (\beta_l^j, \beta_k^j, \beta_m^j, \gamma_{-j}^j, \beta_{\omega_j})$.

An additional key difference from the single product case is the need for instruments for q_{-jt}, which might either be lagged values of q_{-jt} or inputs lagged even further back. Let the set of conditioning variables be given as (e.g.) $x_{jt} = (q_{-j,t-1}, k_t, k_{t-1}, \mathbf{h}_{t-1}, m_{t-1}, l_{t-1})$.[7] Let θ_0 denote the true parameter value. The conditional moment restriction

$$s(x_{jt};\theta) \;\;\equiv\;\; E[[\xi_{jt} + \epsilon_{jt}](\theta)|x_{jt}] \text{ and } s(x_{jt};\theta_0) = 0$$

continues to be sufficient for identification of β as long as a rank condition holds.

5.3 Multivariate Control Functions

It is straightforward to use cost minimization and the implicit function theorem to prove $\mathbf{h}_t(k_t, \omega_t)$ is a bijection under a full rank condition on $\sum_{m=1}^{M} \frac{\lambda_m \partial q_m^2}{\partial x \partial \omega'}$. From the first-order conditions of cost minimization we have

$$P_i = \sum_{m=1}^{M} \frac{\partial q_m(x, K, \omega)'}{\partial x_i} \lambda_m \quad i = 1, \ldots, N$$

[6]The extension of Kim, Petrin, and Song (2016) of Hu and Schennach (2008) to allow for measurement error in the single-product production setting extends directly to the multi-product setup.

[7]If h_t contains m_t (l_t) then one would add m_{t-2} (l_{t-2}) to the conditioning set.

which has - when evaluated at the optimal x - a second derivative matrix given by

$$\sum_{m=1}^{M} \lambda_m \frac{\partial^2 q_m(x, K, \omega)}{\partial x \, \partial x'}$$

which is symmetric and positive definite and thus invertible. Using the implicit function theorem we have the following expression when evaluated at the optimal x:

$$\sum_{m=1}^{M} \lambda_m \frac{\partial^2 q_m(x, K, \omega)}{\partial x \, \partial x'} * \frac{\partial x}{\partial \omega'} + \sum_{m=1}^{M} \lambda_m \frac{\partial^2 q_m(x, K, \omega)}{\partial \omega \ \partial x'} = 0.$$

Given the invertibility of $\sum_{m=1}^{M} \lambda_m \frac{\partial^2 q_m(x,K,\omega)}{\partial x \, \partial x'}$ we can solve for $\frac{\partial x}{\partial \omega'}$ as

$$\frac{\partial x}{\partial \omega'} = \Big(\sum_{m=1}^{M} \lambda_m \frac{\partial^2 q_m(x, K, \omega)}{\partial x \, \partial x'}\Big)^{-1} * \sum_{m=1}^{M} \lambda_m \frac{\partial^2 q_m(x, K, \omega)}{\partial \omega \ \partial x'}$$

Thus $\mathbf{h}_t(k_t, \omega_t)$ is a bijection under a full rank condition on $\sum_{m=1}^{M} \lambda_m \frac{\partial^2 q_m(x,K,\omega)}{\partial x \partial \omega'}$. The condition requires that each change in ω results in a matrix of changes in the sum of the derivatives across the outputs that is not perfectly collinear. Analysis of a simple case with Cobb-Douglass production functions did not suggest any obvious reason why this full rank condition should not hold.

5.4 Illustration in a 2-product Setting

In the case of two-product production we have an equation for good 1

$$q_{1t} = \beta_l^1 l_t + \beta_k^1 k_t + \beta_m^1 m_t + \gamma^1 q_{2t} + \omega_{1t} + \epsilon_{1t} \tag{12}$$

and an equation for good 2

$$q_{2t} = \beta_l^2 l_t + \beta_k^2 k_t + \beta_m^2 m_t + \gamma^2 q_{1t} + \omega_{2t} + \epsilon_{2t}. \tag{13}$$

We use as our two proxies investment and materials, and we write these input demands as $i_t(k_t, \omega_{1t}, \omega_{2t})$ and $m_t = m(k_t, \omega_{1t}, \omega_{2t})$. If the bivariate function (i_t, m_t) is one-to-one and onto with $(\omega_{1t}, \omega_{2t})$ then this bivariate bijection can be inverted and there exist functions $g_1(\cdot)$ and $g_2(\cdot)$ such that $\omega_{1t} = g_1(k_t, i_t, m_t)$ and $\omega_{2t} = g_2(k_t, i_t, m_t)$. For either j we approximate

$$\omega_j = g_j(k_t, i_t, m_t) = \mathbf{c}_j(k_t, i_t, m_t)'\beta_{\omega_j}$$

where $\mathbf{c_j}(k_t, i_t, m_t)$ is a known vector function of (k_t, i_t, m_t) chosen by researchers. The nonparametric conditional mean function for either j is given as

$$E[\omega_{jt}|\omega_{t-1}] = f_j(\mathbf{c_j}(k_{t-1}, i_{t-1}, m_{t-1})'\beta_{\omega_j}) \quad j = 1, 2$$

for some unknown functions $f_1(\cdot)$ and $f_2(\cdot)$. The error now becomes

$$[\xi_{jt} + \epsilon_{jt}](\theta) = q_{jt} - \beta_l^j l_t - \beta_k^j k_t - \beta_m^j m_t - \gamma_{-j}^j q_{-jt} - f_j(\mathbf{c_j}(k_{t-1}, i_{t-1}, m_{t-1})'\beta_\omega) \quad j = 1, 2.$$

Let the set of conditioning variables be given as (e.g.) $x_{jt} = (q_{-j,t-1}, k_t, k_{t-1}, i_{t-1}, m_{t-1}, m_{t-2})$. Let θ_0 denote the true parameter value. The conditional moment restrictions for each equation are given as

$$s(x_{jt};\theta) \equiv E[[\xi_{jt} + \epsilon_{jt}](\theta)|x_{jt}] \text{ and } s(x_{jt};\theta_0) = 0 \quad j = 1, 2.$$

We now turn to our multi-product data.

6 Data

6.1 The Belgian PRODCOM survey

Statistical offices around the world are running production surveys through which they collect precise information about the products made by firms that are intended for use in industrial statistics. These datasets cover a large subset of mostly manufacturing firms and typically contain both values and quantities for each good produced by firms.

In this paper, we use the firm-product level production data based on a production survey (PRODCOM) collected by Statistics Belgium.[8]

The survey is designed to cover at least 90% of production value in each NACE 4-digit industry by including all Belgium firms with a minimum of 10 employees or total revenue above 2.5 million Euros.[9] The sampled firms are required to disclose monthly product-specific revenues and quantities sold of all products at the PRODCOM 8 digit level (e.g. 11.05.10.00 for "Beer made from malt", 23.51.11.00 for "Cement clinkers" in the PRODCOM 2008-2017 classification).

Our analysis covers the entire period through which the data is available, 1996-2017. This creates two difficulties: in 2008, PRODCOM both significantly reduced its sample size to administrative costs and changed its classification system (the first 4 digits of a PRODCOM code refer to a NACE 4 digit sector and the NACE classification has been revised in 2008 implying a complete redefinition of the PRODCOM codes). In addition to that major revision, PRODCOM codes are marginally revised on a yearly basis. We therefore use annual concordance tables provided by Eurostat to follow the

[8]See http://statbel.fgov.be/fr/statistiques/collecte_donnees/enquetes/prodcom/ and http://statbel.fgov.be/nl/statistieken/gegevensinzameling/enquetes/prodcom/ for more details in French and Dutch, or Eurostat in English (http://ec.europa.eu/eurostat/web/prodcom). This dataset was previously used in Bernard et al., 2019 and Amiti et al., forthcoming.

[9]NACE is a French acronym for the European Statistical Classification of Economic Activities.

specific products over our sample period and use only those products with no confusion regarding the concordance (one to many and many to one)-

In our empirical analysis, we perform several cleaning procedures to avoid outliers. First, we only keep firms that have their principal business activities in manufacturing as classified by NACE. First, for each 4-digit industry we compute the median ratios of total revenue over employment, capital over employment, total revenue over materials and wage bill over labor (average wage), and we exclude those observations more than five times the interquartile range below or above the median. Second, we only keep firm-product observations where the share of the product's revenue in the firm's total revenue is at least 5%. Third, we use the Value Added Tax revenue data that provides us with a separate check against the revenue numbers firms report to PRODCOM. Comparing the tax administrative data revenue numbers with the revenue numbers reported in the PRODCOM data, we find that between 85% and 90% of firms report similar values for both. We exclude firms if they do not report a total value of production to PRODCOM that is at least 90% of the revenue they report to the tax authorities.

As will become clearer in the next subsection, we aggregate monthly revenues and quantities to the quarterly level and calculate the associated quarterly unit price. This is done in order to use the same time dimension than the other datasets that we need for our analysis.

Table 1: Average share of a firm's revenue derived by its individual products, 1996 to 2017

Product ranking within a firm determined by its share of the firm's total revenue.

	Number of products produced by the firm at the Prodcom 8-digit level						
	1	2	3	4	5	More than 5	N
Product rank							
1	100	78.1	69.9	64.8	60.0	50.5	
2		21.9	22.9	23.2	22.5	21.7	
3			7.2	9.0	10.6	11.6	
4				3.0	5.0	6.5	
5					1.9	3.8	
6+						5.9	
Share of manufacturing output	24.7	17.8	11.4	9.4	3.7	33.0	100
# observations	37,284	34,068	22,875	18,324	12,380	79,199	204,130

Note: For any product rank i each column j reports the average share (in %) of the i-th product in total output for firms producing j products.

Table 1 shows the average revenue share of products in firms' portfolios when they are producing a different number of products at two levels of aggregation (8-digit and 2-digit

PRODCOM). We observe 204,130 firm-product yearly observations between 1996 and 2017. As has been noted in other product-level data sets, the majority of firms produce multiple products.[10] At the 8-digit level of disaggregation, multi-product firms are responsible for 75.3% of total value of manufacturing output. Most firms produce between one and five products and these firms account for 67% of the value of manufacturing output. For firms producing two goods the core good accounts for 78.1% of revenue. Similarly for firms producing three goods 69.9% of revenue comes from the core product. Even for firms producing six or more goods the core good is responsible for 50.5% of revenue..

To test the pure Diewert-Lau framework, our analysis requires the identification of firms producing the same subset of products. For this purpose, we identified a few specific industries where firms producing two goods were the most commonly observed form of production. We identified six 2-product environment (combos) that fit to our requirements[11]: bread and cake; marble and other building stones; doors of plastic and doors of metal; structures of iron, steel and aluminium, and doors of metal; windows of wood, and joinery and carpentry of wood; and bricks, and prefabricated structures of cement.

Table 2 shows the product portfolio description for those 6 environments. The main message that this table conveys is that, for these 6 economic environments that we identified, the most observed form of production is when firms produce these two exact products associated to a combination, or at least this type of production pattern is a common form of production. The more obvious example is bread and cake: out of 9,621 observations, firms producing bread produce 2-products in 8,064 cases; out of these 8,064 two-product firms, 7,855 also produce cake. As we go down the list, the number of observations becomes lower and the share of single product firms also goes up.[12]

6.2 Firm Input Measurements

Quarterly measurements of firms inputs from 1997 to 2016 are obtained from the VAT fiscal declarations of firm revenue, the National Social Security database, and the Central Balance Sheet Office database. For tax liability purposes, Belgian firms have to report in their VAT fiscal declarations both their sales revenues and their purchases. Purchases are reported into three separate categories : material inputs and services directly used for production, other inputs and services used for supporting activities, and acquisition of capital goods. Using this information, we construct quarterly measures for both types of

[10] See e.g. Bernard et. al (2010) or Goldberg et. al (2010).

[11] See Appendix B for the full product description

[12] For one of our combinations, we realize that firms producing PRODCOM products within a 6-digit code were providing different unit of measurements. We therefore used the most common unit provided for code 222314.

Table 2: Number of observations by product and product scope, selected combinations of products

	# obs.	# obs. w/ 1 product	# obs. w/ 2 products	# obs. w/ 3 products	More than 3 products		# obs.	# obs. (w/ same unit)
107111	9,621	721	8,064	542	294	combo	7,855	7,855
107112	10,020	761	8,155	578	526	107111-107112		
237011	2,050	386	1,178	417	69	combo	1,132	1,132
237012	2,922	953	1,394	542	33	237011-237012		
222314	4,539	1,345	2,421	614	159	combo	1,766	1,065
251210	6,780	2,757	3,000	831	192	222314-251210		
251123	13,892	8,895	3,292	1,094	611	combo	892	892
251210	6,780	2,757	3,000	831	192	251123-251210		
162311	4,602	1,875	1,169	904	654	combo	595	595
162319	3,580	1,181	1,045	422	932	162311-162319		
236111	3,048	1,413	752	593	290	combo	464	464
236112	5,163	3,008	1,298	678	179	236111-236112		

intermediate inputs and for investments. For measures of firm employment, we use data from the National Social Security declarations, where firms report on a quarterly basis their level of employment and their total wage bill. To construct a quarterly measure of capital we start with data from the Central Balance Sheet Office, which records annual measures of firm assets for all Belgian firms. For the first year a firm is in our data, we take the total fixed assets as reported in the annual account as their starting capital stock. We then use standard perpetual inventory methods to build out a capital stock for each firm-quarter.[13]

7 Results

7.1 6-digit analysis

Table 3 shows the coefficients of our generalized transformation function for the 6 selected combinations of two goods. The top panel shows the results when the log of quantity of the first good (bread in the first example) is considered as left hand side variable and regressed on aggregate firm-level inputs and the log of quantity of the second good (cake in the first column). The bottom panel shows a similar regressions when log of quantity of the second good (cake in column 1') is regressed on inputs and the log of quantity of the first good (bread in column 1').[14]

The first row of each panel shows the coefficient of the log of production of the other good conditional on input use. The coefficient is always negative and highly significant suggesting quantities are substitutes for one another holding inputs constant. The significance in every case of the twelve estimated quantity coefficients strongly rejects the use of single-product production as an approximation to multi-product production.

Consistent with multi-product production theory, all but five of the forty-eight input

[13]In order to build the capital stock, we assume a constant depreciation rate of 8% per year for all firms. Real capital stock is computed using the quarterly deflator of fixed capital gross accumulation. The initial capital stock in $t = t_0$, where period t_0 represents the 4th quarter of the first year of observation of the firm, is given by

$$K_{t_0} = \frac{Total\ fixed\ assets_{first\ year\ of\ observation}}{P_{K;t_0}}$$

The capital stock in the subsequent periods is given by

$$K_t = (1 - 0.0194)\, K_{t-1} + \frac{I_t}{P_{K;t}}$$

We assume that the new investment is not readily available for production and that it takes one year from the time of investment for a new unit of capital to be fully operational.

[14]See Dhyne, Petrin and Warzynski (2016) for a joint estimation of production function, demand function and cost function for the Belgian bread and cake industry.

coefficients are positive, and thirty eight of these forty-three positive input coefficient estimates are strongly statistically significant. These findings are based on relatively small sample sizes for each industry which, except for Bread and Cakes, ranges from between 255 to 996 observations. The last row at the bottom of Table 3 shows that there is a very high correlation of technical efficiency terms within-firm, perhaps related to unmeasured managerial ability that can be translated to the various products that the firm produces.

We translate the structural coefficients into their reduced-form counterparts in Table 4 to investigate individual elasticities of output for each input and calculate overall returns to scale for each quantity. Overall the reduced form elasticities look reasonable as they are almost all positive and result in returns to scale 0.88 to 1.247 for ten of twelve quantities with five quantities having returns to scale almost identical to 1. Two of the twelve reduced form capital coefficient estimates have point estimates that are negative - although not statistically significant - arising from the fact that the estimated coefficient on capital from the structural equation is probably too low (and statistically insignificant). Not surprising is the returns to scale estimates that seem most unrealistic, like those in column 4' and 5', arise because of one or more failures of the theoretical properties of structural production function estimates.

Table 3: Multi-product production function estimates at 6-digit Prodcom level, Belgian data

Dependent variable q_{ijt} is log of the quantity sold in physical units at the 6-digit product level of good j by firm i at time t

	(1) 107111-107112	(2) 237011-237012	(3) 222314-251210	(4) 251123-251210	(5) 162311-162319	(6) 236111-236112
$q_{(-j)}$	-0.374***	-0.629***	-0.769***	-0.417***	-0.303***	-0.334***
	(0.010)	(0.048)	(0.032)	(0.058)	(0.041)	(0.038)
l	0.405***	0.442***	0.364***	0.267***	0.650***	-0.071
	(0.017)	(0.066)	(0.068)	(0.074)	(0.043)	(0.091)
k	0.101***	0.163***	0.070***	0.286***	0.606***	0.722***
	(0.009)	(0.050)	(0.022)	(0.032)	(0.075)	(0.064)
$m1$	0.602***	0.979***	1.216***	0.571***	0.404***	0.655***
	(0.016)	(0.060)	(0.062)	(0.085)	(0.058)	(0.049)
$m2$	0.305***	0.042	0.350***	0.219***	-0.240***	0.108
	(0.012)	(0.066)	(0.036)	(0.041)	(0.054)	(0.084)
	(1') 107112-107111	(2') 237012-237011	(3') 251210-222314	(4') 251210-251123	(5') 162319-162311	(6') 236112-236111
$q_{(-j)}$	-0.555***	-0.390***	-0.720***	-0.687***	-1.307***	-0.963***
	(0.015)	(0.033)	(0.030)	(0.102)	(0.161)	(0.108)
l	0.547***	0.276***	0.365***	-0.037	0.741***	0.342**
	(0.019)	(0.055)	(0.066)	(0.098)	(0.146)	(0.141)
k	0.145***	0.399***	0.005	0.068	0.841***	1.027***
	(0.010)	(0.037)	(0.022)	(0.054)	(0.185)	(0.108)
$m1$	0.721***	0.955***	1.187***	1.185***	1.077***	0.682***
	(0.019)	(0.040)	(0.060)	(0.061)	(0.095)	(0.104)
$m2$	0.163***	-0.234***	0.276***	0.108*	-0.459***	0.142
	(0.015)	(0.052)	(0.037)	(0.057)	(0.110)	(0.131)
	Correlation between ω_1 and ω_2					
	0.81	0.89	0.92	0.84	0.78	0.95
# obs.	7,262	996	895	596	255	360

Note: Each column reports the estimated coefficients using a modified variant of the GMM Wooldrige estimator. Explanatory variables are in logs and include firm-level labor, the standard real indices for materials and for capital - i.e. the dollar value of each - and the physical quantity of the other good produced by the firm. We include the product's price as an additional control. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 4: Elasticities of Output with Respect to Inputs and Returns to Scale Implied by the Strutural Estimates

	(1) 107111-107112	(2) 237011-237012	(3) 222314-251210	(4) 251123-251210	(5) 162311-162319	(6) 236111-236112
l	0.253	0.356	0.187	0.396	0.704	*-0.273*
	(0.020)	(0.066)	(0.086)	(0.100)	(0.073)	(0.135)
k	0.059	*-0.116*	0.149	0.361	0.582	0.558
	(0.009)	(0.057)	(0.037)	(0.038)	(0.228)	(0.101)
$m1$	0.419	0.502	0.679	0.107	0.129	0.630
	(0.018)	(0.049)	(0.069)	(0.070)	(0.122)	(0.061)
$m2$	0.308	0.250	0.309	0.244	-0.168	0.089
	(0.012)	(0.075)	(0.045)	(0.048)	(0.121)	(0.162)
RTS	1.039	0.992	1.324	1.108	1.247	1.005
	(1') 107112-107111	(2') 237012-237011	(3') 251210-222314	(4') 251210-251123	(5') 162319-162311	(6') 236112-236111
l	0.407	0.137	0.230	*-0.309*	*-0.179*	0.605
	(0.019)	(0.061)	(0.074)	(0.103)	(0.179)	(0.188)
k	0.112	0.443	*-0.102*	*-0.180*	0.080	0.489
	(0.009)	(0.046)	(0.037)	(0.060)	(0.595)	(0.131)
$m1$	0.488	0.759	0.698	1.112	0.909	0.075
	(0.020)	(0.038)	(0.062)	(0.060)	(0.269)	(0.101)
$m2$	-0.008	-0.331	0.054	-0.059	-0.239	0.056
	(0.014)	(0.060)	(0.040)	(0.097)	(0.359)	(0.190)
RTS	0.999	1.009	0.880	0.564	0.570	1.225
# obs.	7,262	996	895	596	255	360

Note: Each column reports the reduced form coefficients. s.e. are obtained by bootstrap, using 100 random replications.

Table 5: Regression of Technical Efficiency ω (TFPQ) on CORE Product Indicator

	ω
CORE	0.410***
	(0.025)
Product dummies	YES
Time dummies	YES
R^2	0.69
# obs.	26,032

* p<0.10, ** p<0.05, *** p<0.01.

The recent international trade literature has focussed on whether a firm's core product - the one that represents the largest share of the revenue that the firm generates - is positively correlated with its technical efficiency. Most theory models have the core competent products produced most efficiently. We regress our measure of technical efficiency (TFPQ) on an indicator variable equal to one if the product generates more than 50% of the total sales of the firm. Results are shown in Table 5 where we pool all of our productivity estimates across industries obtained from the estimated values from Table 3. After controlling for product and time fixed effects we find that firms are approximately 40% more technically efficient at producing their core relative to non-core products.

7.2 4-digit analysis

We next try to adopt several aggregation strategies to consider different product markets and possibly increase our sample size. Our first approach aggregates physical output within a 4-digit PRODCOM code for firms operating in two 4-digit environments. We estimate this framework for a subset of firms in the furniture industry. Table 6 shows the results are largely consistent with the theory as the estimated matrix $\hat{\Gamma}$ is positive definite for all product pairs and almost all of the input coefficients are positive.

7.3 Robustness with French data

We replicate the analysis using a sample of French firms (see Smeets and Warzynski (2019) for more information about the dataset). Data are collected annually for the period 2009-2017, and we only use one variable for material, but the rest of the analysis is otherwise similar. Table 7 shows the results at the 6-digit level and Table 8 at the 4-digit level. The number of observations is a bit lower because of the relatively shorter panel and the annual dimension of the data, but results again are largely consistent with our theory results.

Table 6: Multi-product production function estimates at 4-digit Prodcom level, Belgian data

Dependent variable q_{ijt} is log of the quantity sold in physical units at the 4-digit product level of good j by firm i at time t

	(1)	(2)	(1')	(2')
	3100-3109	3102-3109	3109-3100	3109-3102
$q_{(-j)}$	-0.261***	-0.262***	-0.732***	-0.797***
	(0.021)	(0.027)	(0.061)	(0.075)
l	0.416***	0.652***	0.638***	1.693**
	(0.049)	(0.117)	(0.088)	(0.167)
k	0.156	0.152	0.860***	-0.134
	(0.200)	(0.262)	(0.352)	(0.407)
m	0.910***	0.447***	0.631***	0.125
	(0.048)	(0.090)	(0.103)	(0.144)
# obs.	1,205	885	1,205	885

Note: Each column reports the estimated coefficients using a modified variant of the GMM Wooldrige estimator. Explanatory variables are in logs and include firm-level labor, the standard real indices for materials and for capital - i.e. the dollar value of each - and the physical quantity of the other good produced by the firm We include the product's price as an additional control. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 7: Multi-product production function estimates at 6-digit Prodcom level, French data

Dependent variable q_{ijt} is log of the quantity sold in physical units at the 6-digit product level of good j by firm i at time t

	(1)	(2)	(3)	(4)	(5)	(6)
	181212-181219	162311-162319	310912-310913	237011-237012	310210-310913	251123-251210
$q_{(-j)}$	-0.652***	-0.301***	-0.074	-0.561**	-0.212***	-0.464***
	(0.110)	(0.049)	(0.048)	(0.241)	(0.047)	(0.079)
l	0.340***	0.714***	0.311***	0.633***	0.235***	0.312***
	(0.077)	(0.117)	(0.106)	(0.190)	(0.084)	(0.154)
k	0.073**	0.142**	0.045	0.052	0.298***	0.334***
	(0.036)	(0.052)	(0.052)	(0.062)	(0.056)	(0.084)
m	1.200***	0.464***	0.652***	0.856***	0.696***	0.838***
	(0.094)	(0.114)	(0.093)	(0.231)	(0.071)	(0.115)
	(1')	(2')	(3')	(4')	(5')	(6')
	181219-181212	162319-162311	310913-310912	237012-237011	310913-310210	251210-251123
$q_{(-j)}$	-0.541***	-0.601***	-0.224*	-0.410***	-0.532***	-0.399***
	(0.081)	(0.087)	(0.086)	(0.087)	(0.130)	(0.071)
l	0.334***	0.585***	0.011	0.547***	0.076	0.066
	(0.074)	(0.172)	(0.169)	(0.191)	(0.161)	(0.147)
k	-0.009	0.186**	0.118	0.113	0.145**	0.230***
	(0.034)	(0.074)	(0.077)	(0.083)	(0.074)	(0.070)
m	1.092***	0.513***	0.986***	0.818***	1.148***	1.098***
	(0.075)	(0.150)	(0.141)	(0.170)	(0.122)	(0.123)
# obs.	569	349	380	359	334	312

Note: Each column reports the estimated coefficients using a modified variant of the GMM Wooldrige estimator. Explanatory variables are in logs and include firm-level labor, the standard real indices for materials and for capital - i.e. the dollar value of each - and the physical quantity of the other good produced by the firm We include the product's price as an additional control. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 8: Multi-product production function estimates at 4-digit Prodcom level, French data

Dependent variable q_{ijt} is log of the quantity sold in physical units at the 4-digit product level of good j by firm i at time t

	(1)	(2)	(1')	(2')
	3102-3109	3101-3109	3109-3102	3109-3101
$q_{(-j)}$	-0.153***	-0.297***	-0.239***	-0.492***
	(0.024)	(0.043)	(0.061)	(0.094)
l	0.244***	0.285**	0.350***	0.393**
	(0.059)	(0.129)	(0.076)	(0.157)
k	0.192***	0.160**	0.112*	0.322***
	(0.033)	(0.063)	(0.045)	(0.101)
m	0.750***	0.726***	0.702***	1.021***
	(0.049)	(0.146)	(0.084)	(0.183)
# obs.	1,078	342	1,079	342

Note: Each column reports the estimated coefficients using a modified variant of the GMM Wooldrige estimator. Explanatory variables are in logs and include firm-level labor, the standard real indices for materials and for capital - i.e. the dollar value of each - and the physical quantity of the other good produced by the firm We include the product's price as an additional control. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

7.4 Marginal costs and markups

We derive estimates of marginal costs using the theory from Section 4. We apply the formulas on our subsample of bread and cake producers where the number of observations is the largest and report the results in Table 9. For a loaf of bread we find the interquartile range of marginal costs of between 0.85 and 1.96 Euros. For cakes we find an interquartile range of 2.46 and 7.58 Euros. While these levels do not seem unreasonable, when we compare them to the interquartile range of unit prices for bread 1.14 to 1.71 and cake of 3.43 and 5.12 Euros, we can see that marginal costs for some products may be upwardly biased. In particular the right tail of the distribution of marginal costs for cakes appears to be inflated as markups fall below one.

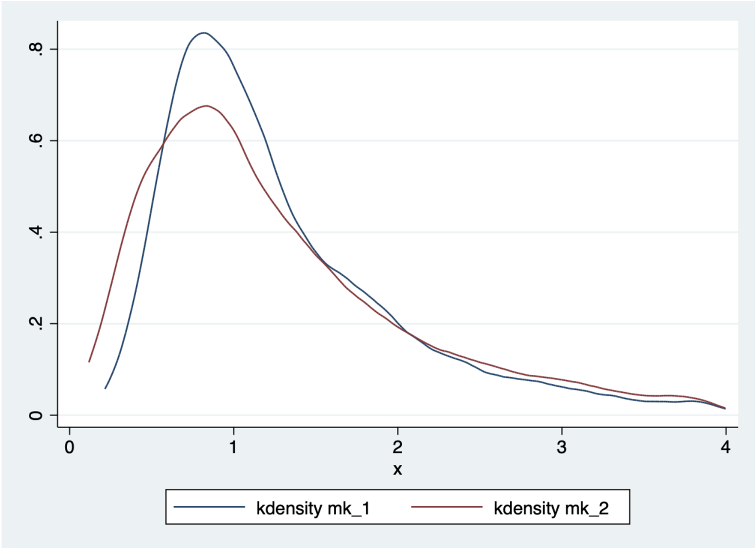
Table 9: Summary statistics on marginal costs estimates and prices. Bread and cake producers, Belgium

	Marginal cost		**Price**	
	Bread	Cake	Bread	Cake
mean	1.504	5.61	1.474	5.032
10%	0.576	1.579	1.103	3.15
25%	0.855	2.465	1.149	3.431
50%	1.301	4.158	1.325	3.802
75%	1.962	7.582	1.715	5.127
90%	2.704	11.923	1.971	8.894
std dev	0.869	4.313	0.422	3.109

We then use our marginal cost estimates to compute markups. The estimates that we obtain are sensible, with an average markup of 1.13 for bread and 1.37 for cake. The distribution of markups for bread and cake is shown in Figure 1. We observe for both products a concentration around one, and a fat tail on the right of 1. Again there is evidence that for some goods the estimates of marginal costs are inflated.

As a next exercise, we then correlate our marginal costs and markup measures to the productivity for both bread and cake. Results in Table 10 show that markups are positively correlated with productivity for both products. These results are in line with previous research using product level information and estimation markups and productivity (see e.g. Foster, Haltiwanger and Syverson, 2008 and De Loecker et al., 2016).

Figure 1: Distribution of markups for bread (mk_1) and cake (mk_2)



8 Conclusion

We introduce a new methodology for estimating multi-product production functions. It embeds the seminal contributions of Diewert (1973) and Lau (1976) in a semi-parametric econometric framework following Olley and Pakes (1996). We address the simultaneity of inputs and outputs while allowing for and estimating one unobserved technical efficiency term for each firm-product, each one of which may be freely correlated with inputs and outputs. We show how to translate the structural parameters into the reduced form parameters that give the elasticity of each output with respect to each input. For each output the sum of these input coefficients is the returns to scale for that output. We show how to use these estimates to recover estimates of firm-product marginal costs by extending the Hall (1988) single-product result to our multi-product setting. The main advantage of our framework is that it does not require multi-product production to be a collection of single-product production functions, which rules out the possibility that outputs are substitutes or complements with one another. Our empirical results using panel multi-production production data are largely consistent with our theoretical restrictions and strongly reject the single-product production approximation to multi-product production.

Table 10: Relationship between prices, marginal costs, markups and TFPQ (ω)

	Bread		
	logp	*logMC*	Markup
$TFPQ$	-0.299***	-0.789***	0.563***
	(0.005)	(0.011)	(0.021)
R^2	0.42	0.53	0.18
# obs.	4,569	4,569	4,569
	Cake		
	logp	*logMC*	Markup
$TFPQ$	-0.512***	-0.694***	0.152***
	(0.007)	(0.012)	(0.017)
Adj. R^2	0.52	0.42	0.04
# obs.	4,569	4,569	4,569

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Appendix

The first result is from Diewert (1973) and the last two results are from Lau (1976). The results below when we classify outputs and inputs into flexible (a ν) ones and fixed ones (a K). The result extends directly to the case where we hold other outputs q_{-j} constant and in which case (ν, K) contains only flexible and fixed inputs.

Proof of Theorem 3.1

Under P.1-P.4 $F_j(q_{-j}, x)$ is

(1) non-decreasing in x

Let $(q_{-g}, x) \geq (0_{M-1}, 0_N)$ and suppose $q_g^* = F(q_{-g}, x)$ is finite. Then $(q_g^*, q_{-g}, x') \in T \;\; \forall x' \geq x$ by free disposal. But $F(q_{-g}, x') \geq q_q^* = F(q_{-g}, x)$.

Under P.1-P.4 $F_j(v, K)$ is

(2a) concave in v ∀ K

Suppose $q_j^* = F(v_j, K) \;\; j = 1, 2$. Then $(q_j^*, v_j, K) \in T \;\; j = 1, 2$.

By convexity $(\lambda\, q_1^* + (1-\lambda)q_2^*, \lambda\, v_1 + (1-\lambda)v_2, K) \in T \;\; 0 < \lambda < 1$.

$$\begin{aligned} Then\ q^* &= F(\lambda\, v_1 + (1-\lambda)v_2, K) \\ &= max(q \,|\, (q, \lambda\, v_1 + (1-\lambda)v_2, K) \in T) \\ &\geq \lambda\, q_1^* + (1-\lambda)q_2^* \\ &= \lambda\, F(v_1, K) + (1-\lambda)F(v_2, K) \end{aligned}$$

(2b) quasi-concave in K ∀ v

Suppose $q_j^* \equiv F(v, K_j) \; for\, j = 1, 2$. Then $(q_j^*, v, K_j) \in T \; for\, j = 1, 2$. Let $\tilde{q} = min(q_1^*, q_2^*)$. Then $(\tilde{q}, v, K_j) \in T \; for\, j = 1, 2$. Then convexity of T in K $\forall v$ implies $(\tilde{q}, v, \lambda K_1 + (1-\lambda)K_2) \in T \;\; for 0 < \lambda < 1$ With $K_\lambda \equiv \lambda K_1 + (1-\lambda)K_2$ we have

$$q_\lambda = F(v, K_\lambda) \geq \tilde{q} = min(F(v, K_1)\, ,\, F(v, K_2).$$

Appendix B: choice of 6-digit combinations

Specification 1

10.71.11 Fresh bread
10.71.12 Fresh pastry goods and cakes

Specification 2

23.70.11 Marble, travertine, alabaster, worked, and articles thereof (except setts, curbstones, flagstones, tiles, cubes and similar articles); artificially coloured granules, chippings and powder of marble, travertine and alabaster
23.70.12 Other worked ornamental or building stone and articles thereof; other artificially coloured granules and powder of natural stone; articles of agglomerated slate

Specification 3

22.23.14 Doors, windows and frames and thresholds for doors; shutters, blinds and similar articles and parts thereof, of plastics
25.12.10 Doors, windows and their frames and thresholds for doors, of metal

Specification 4

25.11.23 Other structures and parts of structures, plates, rods, angles, shapes and the like, of iron, steel or aluminium
25.12.10 Doors, windows and their frames and thresholds for doors, of metal

Specification 5

16.23.11 Windows, French windows and their frames, doors and their frames and thresholds, of wood
16.23.19 Builders' joinery and carpentry, of wood, n.e.c.

Specification 6

23.61.11 Tiles, flagstones, bricks and similar articles, of cement, concrete or artificial stone
23.61.12 Prefabricated structural components for building or civil engineering, of cement, concrete or artificial stone