License Complementarity and Package Bidding: U.S. Spectrum Auctions*

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Abstract

U.S. spectrum licenses cover geographically distinct areas and often complement each other. A bidder seeking to acquire multiple licenses is then exposed to risks of winning only isolated patches. We investigate whether allowing bidders to bid for (predefined) packages of licenses alleviates the exposure problem and improves allocation efficiency. Using Auction 73 bidding data, we model the bidding process as an entry game with interdependent markets and evolving bidder belief. Bidders' decisions on bidding (and not bidding) provide bounds on licenses' stand-alone values and complementarity between licenses. With estimated bidder valuation, we conduct counterfactual analyses to show that the effects of package bidding on bidders' exposure risks depend on package format and package size. More importantly, package bidding increases auction revenue substantially, at the cost of reducing bidder surplus and increasing license allocation concentration.

Keywords: Spectrum Auctions, Complementarity, Package Bidding, Moment Inequalities

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1 Introduction

In the United States, firms that intend to provide cell phone services bid for spectrum licenses in simultaneous multi-round ascending auctions hosted by the Federal Communications Commission (FCC). Covering geographically distinct areas, these licenses often complement each other: a bidder may have higher willingness to pay if it can obtain two or more licenses together. To facilitate more efficient license allocation, in the mid 2000's, the FCC started to allow bidders to make a single bid for a group of licenses — a "package." For example, in Auction 73, conducted in early 2008, the FCC divided the country into different packages in Block C and designated the entire country as a single package in Block D.[1]

One key reason that the FCC started package bidding was to alleviate the "exposure problem" that often arose in previous spectrum auctions, which allowed only *à la carte* license bidding. Take, for example, a bidder with the highest willingness to pay for 20 MHz of spectrum covering the entire United States. As auction rounds and bidding prices increase, this bidder faces a tough choice. If the bidder continues to bid for all licenses, it is "exposed" to the risk that it will acquire only some of the licenses. Without complete nationwide coverage, the licenses won are not worth the high prices the bidder has bid. If the bidder decides to withdraw from bidding instead, it may fail to acquire licenses for which it actually has the highest value. In either case, the license allocation is inefficient.[2]

At first glance, package bidding does seem able to alleviate this exposure problem. If the FCC allows package bidding, it is all or nothing, so a bidder will no longer face the uncertainty of winning only a few licenses within a package. The exposure problem generated by a bidder seeking to win multiple *packages*, however, remains and could become exacerbated. More importantly, whether package bidding improves social surplus depends on the distribution of stand-alone values across bidders. For example, suppose that two bidders are bidding for two licenses, A and B, and the value of complementarity between the two licenses is \$5 million for both bidders. Suppose, also, that the stand-alone values of bidder 1 on license A and license B are \$10 million and \$2 million, respectively, while the stand-alone values of bidder 2 on license A and license B are \$2 million and \$10 million, just the opposite of bidder 1's values. If the FCC allows only *à la carte* bidding, then bidder 1 will win license A; bidder 2 will win license B; and the social surplus (equal to bidder surplus plus FCC revenue in this case) will be \$20 million. If the FCC auctions off licenses A and B in a single package, then

[1] Auction 73 auctions off the rights to operate the 700 MHz radio frequency band in the United States in five blocks: A, B, C, D, and E, each block covering different frequencies.

[2] Bulow, Levin, and Milgrom (2009) provide a detailed description of the exposure problem faced by a new entrant and point out that the source of this problem is the uncertainty about the final auction prices.

either bidder can win this package, and the social surplus is \$17 million, \$3 million lower than under *à la carte* bidding.

To design package bidding, therefore, the FCC needs information on two important empirical measures: one is the magnitude of complementarity across licenses, and the other is the distribution of stand-alone values across bidders. Estimating these bidder valuations is a challenging task, as the multi-round ascending auction is a dynamic game with heterogeneous bidders, heterogeneous licenses and correlated valuations. A bidder has almost infinite combinations of licenses to bid on under its constantly evolving belief about winning any set of licenses in each round of the auction. This curse of dimensionality problem makes it infeasible to calculate bidders' optimal strategy for choosing the licenses to bid on and how much to bid.[3]

This paper takes on this challenge. We use FCC Auction 73, the first auction in which the FCC designated packages to be auctioned off.[4] This auction had 1,099 licenses for sale, covering 698-793 MHz band (therefore, called "700 MHz Band"). The FCC divided these licenses into five blocks, each featuring a different market delineation and covering 6-22 MHz of spectrum. Between January 24 and March 18, 2008, 214 bidders participated in the auction, with 101 bidders successfully winning at least one license. These bidders were highly heterogeneous and included telecommunications giants such as AT&T and Verizon, regional carriers such as Cellular South, as well as small firms that qualified for the FCC's steep bidding credit. In total, Auction 73 raised \$18.958 billion for the FCC, selling 1,090 licenses. We focus on Block B, in which each license corresponded to the smallest FCC-designated coverage area, and we can assemble these licenses into packages of larger coverage areas in the counterfactual analyses.

The FCC has devised many rules to eliminate strategic bidding and to facilitate "straightforward bidding." The most notable rule in Auction 73 was the "limited disclosure" format, meaning that bidders did not observe competing bidders' identities in the bidding process. This format simplifies bidders' belief-formation process and strategy space because it reduces the dimensions of information that bidders can access. As a result of multiple FCC rules(details in Section 2), the vast majority of bidders bid exactly the FCC's designated minimum acceptable price in each round of the auction. Therefore, we model bidding decisions on the interdependent licenses in each auction round as an entry game with interdependent

[3]For example, Auction 73 had 1,099 licenses for sale. A bidder seeking a national footprint has $2^{1,099}$ combinations of potential strategies. In comparison, Cantillon and Pesendorfer (2006) have no more than three London bus routes in an auction, and Kim, Olivares, and Weintraub (2014) have about 30 units in each auction of Chilean school meals.

[4]Auction 51 was the first FCC auction that allowed bidders to assemble packages themselves.

markets. In this entry game, we estimate bidders' belief about final winning probabilities in every round of bidding using observed history by bidders and final winning outcomes. In each round, we assume that a bidder's bidding set is weakly preferable to any alternative feasible bidding set. Based on this assumption, we construct moment inequalities in the fashion of Pakes (2010) and Pakes, Porter, Ho, and Ishii (2015), using bidders' revealed preferences over different rounds on the same license to estimate complementarity. As bidding prices rise, bidders' provisional winning sets change; bidders' beliefs based on bidding history evolve; and bidders' decisions on bidding (and not bidding) inform us about the complementarity across licenses and licenses' stand-alone value to bidders.

Specifically, it is a bidder's change of bidding decisions that is most informative. When a bidder starts to bid on a license in the middle of the auction, the stand-alone value of this license has not changed, but the price for this license has become higher. The only reason that bidders start bidding must be that they now have a higher expected value of this license's contribution to the set of licenses the bidder perceives to win. This revealed preference characterizes the lower bound of the complementarity magnitude. Similarly, when a bidder stops bidding on a license, we obtain the upper bound of the complementarity magnitude. With complementarity identified from bidders' change in bidding behaviors on the same license, bidders' decisions on different licenses at a certain round show how license characteristics and bidder attributes affect their valuation of an individual license.

Our estimates of complementarity satisfy 85.7% of the bidder-license-round inequalities used in estimation. We estimate that the total complementarity of a nationwide license with 1 MHz's bandwidth is worth roughly \$0.87 billion. There is, however, substantial heterogeneity across different types of bidders in this valuation. The total complementarity in a 1 MHz nationwide license is worth \$1.36 billion to an average large bidder, \$0.28 billion to an average medium bidder, and \$0.91 billion to an average small bidder. The non-monotonic relationship between bidder size and bidder valuation of complementarity is a reflection of medium bidders' scattered bidding patterns. In bidders' stand-alone values, heterogeneity at the license level, including population, population density, and cell phone tower density in the area, plays a significant role, together with bidder-level heterogeneity.

With estimates on stand-alone values and complementarity, we perform counterfactual experiments to assess welfare trade-offs if the FCC adopts package bidding in Block B. We perform two alternative aggregations of Block B's small geographic areas. With these packages, the FCC can either exercise pure bundling (henceforth "pure package"), in which only package bidding is allowed, or mixed bundling (henceforth "mixed package"), in which both package bidding and *à la carte* bidding are allowed. Under each counterfactual policy, we evaluate welfare trade-offs by calculating the magnitude of the exposure problem, FCC revenue from selling off licenses, bidders' surplus from acquiring these licenses, and the final license allocation to different types of bidders.

We have three major findings. First, package bidding does alleviate the exposure problem to some extent, but this effect depends on package format and size. Our results show a nonlinear relationship between package size and the incidence of the exposure problem. For example, a bidder's "should have bid" regret increases substantially with the medium package size under the pure package policy. This is because, although package bidding eliminates the within-package exposure problem, it aggravates the between-package exposure problem. Bidders bidding on large packages face the risk of not winning all the packages they aims for. The incidence of the exposure problem depends on the balance of the within-package and between-package exposure problems. More importantly, we find that the mixed package is much more effective at reducing the exposure problem, as the bidders have more flexibility to choose bidding strategies. Decomposing a bidder's *ex-post* regret, we can see that the largest chunk of the exposure problem without package bidding comes from the *ex-post* regret due to large bidders' premature withdrawal. With mixed package bidding, a bidder's "should have bid" regret decreases dramatically, offsetting the small increase in "should have not bid" regret and, in turn, leading to smaller *ex-post* regret overall. The effect is especially prominent for large bidders, who bid more aggressively under mixed package bidding.

Second, package bidding could increase the total social welfare at the cost of redistributing bidder surplus to FCC revenue. The gain in social welfare is a result of a smaller exposure problem.[5] The redistribution effect is more prominent if the FCC bundles licenses into large packages. The driving force of this result is large bidders' more aggressive bidding behavior under large package bidding, which pushes up final bidding prices and allows the FCC to gain a substantial increase in revenue. Comparing different packages, a mixed package with medium-sized licenses strikes the best balance, with increased social surplus and a more balanced distribution of social surplus. These results confirm Cantillon and Pesendorfer (2006)'s findings: the welfare consequences of allowing package bidding are ambiguous and depend on the distributions of valuations and the size of the complementarity effect.

Lastly and very importantly, although bidders are generally worse off in package bidding, package bidding allocates many more licenses to large bidders at the expense of medium-sized and small bidders. Overall, package bidding creates a more concentrated market structure

[5]Under pure package with medium-sized licenses, the exposure problem gets exacerbated and, in turn, social welfare decreases.

(especially when packages cover large geographic areas). The implications of these results, as they concern the future competitive landscape in the cell phone market, are not included in our previous welfare assessment. One of the FCC's goals is to create and maintain a more llevel playing field for future cell phone providers (this goal is embodied by the steep bidding credit the FCC grants to small bidders) — package bidding actually works against this goal, which contradicts our previous result that assesses only short-term welfare trade-offs. Overall, due to these ambivalent welfare effects, we caution policy makers about the adoption of package bidding.

Package bidding is still in its explorative stage. Evidence that package bidding improves efficiency comes mostly from laboratory experiments – whether these conclusions can be extrapolated to real-world bidding is a subject of continuing controversy. Structure estimates of the valuation functions of FCC spectrum auctions are sparse, particularly because estimating complementarity across licenses is a challenging task.[6] Within this literature, we are closest to our predecessors Fox and Bajari (2013) and Yeo (2009); theirs are the only two papers allowing license complementarity in a structural estimation of FCC auctions.[7] Fox and Bajari (2013) estimate a matching game in which the equilibrium outcome of matches is pairwise stable (as in Fox (2010)), using bidding data from the final rounds of the 1995-1996 Broadband PCS C Block auction. This auction divided the continental U.S. into 480 small, geographically distinct licenses, and Fox and Bajari find that four large regional licenses would raise the allocative efficiency substantially. When they incorporate price information in their estimation, they find that the value of complementarity from a nationwide license is \$120 billion, which they claim as "absurdly high" (total bids for the C block amounted to only \$10 billion), and they caution readers about their result. Yeo (2009) also uses moment inequalities to estimate bidder valuations in Auction 66. Her goal is to evaluate markups of the winning bidders and to assess the level of competition in FCC spectrum auctions, which is very different from our intention. In Yeo's model, bidders believe that they will win all the licenses on which they bid in each round, and she obtains very wide, inconclusive interval estimates of license complementarity.

Most of the literature on FCC spectrum auctions investigates bidders' strategic behaviors.[8] Cramton and Schwartz (2002) find, in an early FCC spectrum auction, that a small

[6]Hong and Shum (2003) initialize the line of literature. They model bidding for each license as a single-unit auction and do not consider complementarity across licenses.

[7]Ausubel, Cramton, McAfee, and McMillan (1997) and Moreton and Spiller (1998) find strong reduced-form evidence on geographic license complementarity in U.S. spectrum auctions.

[8]There is also an extensive literature on collusion in auctions;Asker (2010), Conley and Decarolis (2016), Kawai and Nakabayashi (2015) are a few recent ones.

fraction of bidders tag the last three digits of their bids with the market number of a related license to signal to other bidders which licenses to bid or not bid on. This collusive bidding behavior results in significant FCC revenue loss. Doraszelski, Seim, Sinkinson, and Wang (2017) look into the case in which the FCC conducts a reverse auction for broadcast TV licenses: the FCC acquires spectrum from broadcast TV license holders and then repacks the acquired spectrum to mobile broadband spectrum. Their paper finds that multi-license holders of broadcast TV licenses strategically withhold some licenses to increase the price for their remaining licenses.[9] Our work has a different focus. As, since 2000s, the FCC has tightened its rules to regulate spectrum auctions and alleviate collusive concerns, bidders bid more straightforwardly, and we can recover bidders' valuation of licenses and the complementarity among licenses by characterizing bidder behaviors with simple rules. Our goal is use these model primitives to evaluate the effects of potential policies that could improve the allocative efficiency of spectrum auctions.

Our empirical strategy is inspired by Haile and Tamer (2003), who characterize an incomplete model and use only necessary conditions for identification and estimation. As Haile and Tamer (2003) point out, English auctions lack sufficient structure to yield a tractable theoretical model without significant abstractions. They construct bounds based on two simple assumptions: bidders bid no more than their valuations or let another bidder win at a price lower than their valuations. We follow this guiding principle: we do not invert the first-order conditions derived from a specific model to recover valuations from observed bids; instead, we use necessary conditions of an incomplete model to simplify the complex process of price bidding. We add to Haile and Tamer's contributions on two fronts: 1) we apply their insight to multiple-units auctions in which there is complementarity among these units; and 2) we allow the "revealed preferences" approach to reveal both bidder preferences and bidder belief. Our approach allows us to deal with both the computational and statistical curse of dimensionality that arises when the number of items increase in a combinatorial auction. Our work complements that of our predecessors (Cantillon and Pesendorfer (2006), Kim, Olivares, and Weintraub (2014)), who exploit bidders' first-order conditions in their profit-maximization problems in first-price combinatorial auctions. The most recent paper in this small but growing literature is by Gentry, Komarova, and Schiraldi (2018), who establish non-parametric identification of primitives in simultaneous first-price auctions with

[9] Strategic bidding is also a common theme in the literature on multi-unit auctions, which involve the sale of many related items, such as treasury, spectrum, electricity, and emission permits. Notable empirical examples along these lines include Wolfram (1997), Hortaçsu and Puller (2008), and Hortaçsu, Kastl, and Zhang (2018), among many others. Hortaçsu (2011) discusses empirical analysis of multi-unit auctions.

bidders' preferences over combinations and estimates cost synergies in Michigan highway procurement auctions.

The paper is organized as follows. In Sections 2 and 3, we present stylized facts in FCC spectrum auctions, and especially in Auction 73, to explain how they affect our modeling choices. In Section 4, we outline a model, formulate behavioral assumptions characterizing the solutions to the model, and describe how we construct moment inequalities from these behavioral assumptions. In Sections 5 and 6, we present estimation results, conduct counterfactuals, and discuss counterfactual results and implications.

2 FCC Spectrum Auctions

2.1 Spectrum Auctions Basics

Since 1994, the FCC has conducted auctions of licenses to transmit signals over specific bands of the electromagnetic spectrum. These auctions are open to any eligible company or individual that submits an application and upfront payment and that the FCC approves as a qualified bidder. In the 25 years since, FCC has conducted more than 100 auctions and raised hundreds of billions of dollars in revenue. With technology advancement that enables the use of different frequencies, the FCC has introduced new bandwidth to auctions four times, as the telephone industry transitioned from landlines to mobile devices and then to 3G and 4G data networks. Top auctions included Auction 35 (C and F Block Broadband Personal Communications Service), which garnered $16 billion in FCC revenue; Auction 66 (Advanced Wireless Services (AWS-1)), garnering $14 billion; Auction 73 (*apropos* of this study), $19 billion; and Auction 97 (Advanced Wireless Services (AWS-3)), $41 billion.[10] Major participants included incumbent cell phone carriers such as AT&T and Verizon; potential entrants into the wireless business such as the Dish Network; technology companies such as Google;[11] and even individual citizens. Most spectrum auctions, especially recent ones, have been conducted electronically, and results are accessible over Internet-based bidding system.[12] In a large auction, hundreds of firms bid for thousands of licenses in hundreds of rounds over a few weeks.

The FCC spectrum auctions adopt the format of a simultaneous multiple-round ascending auction. "Simultaneous" means that all licenses are up for bidding at the same time

[10]Personal Communication Services refers to the bandwidth from 1850 MHz to 1990 MHz, and Advanced Wireless Services refers to the range between 1710 MHz and 2180 MHz.

[11]For example, Google participated in Auction 73 in an effort to encourage open access of spectrum.

[12]Telephone bidding is also available to some qualified bidders.

throughout the entire auction. “Multiple-round” refers to discrete, successive rounds, the length of each round specified in advance by the FCC. “Ascending” means that in each round, the FCC adds a small increment to the winning bid (typically 10-20% of the provisional winning bid) to determine the acceptable minimum bid of a new round. There is no predetermined number of rounds. The auction will continue until a round occurs in which all bidder activities cease — no new bids, withdrawals of bids or use of proactive activity waivers (explained later in this section). This round becomes the closing round of the auction and determines the final outcome of winners and winning bids.

The FCC strictly controls information released before and after each round of bidding. Depending on information released after each round of bidding, there are two types of auctions: “full disclosure” and “limited disclosure.” In “full disclosure” auctions, after each round closes, the FCC releases information on bids placed by all bidders. In “limited disclosure” auctions, information release by the FCC is very restricted. In Auction 73, for example, the FCC withheld information until after the close of the auction concerning bidder identity, including, among other things, license selections, upfront payments and eligibility information (FCC (2008)). After each round closed, the FCC notified bidders only of the number of bids placed for a license in the round and the provisionally winning bids for this license. Bidding was strictly anonymous. Bidders were forbidden to share or discuss bidding strategies at any time. Before the adoption of the “limited information” format, the FCC’s flexible auction format and released information on bidding could be exploited by bidders, so they bid more strategically or even colluded with each other. This is why FCC has exercised tight control of information since mid-2000s.

Moreover, the FCC has auction rules to help reduce strategic bidding and bidding collusion. The most notable rules are the eligibility requirement and activity requirements (Bomberger (2007)). The FCC assigns each license a fixed number — “bidding units” — to measure bidder eligibility and activity. Before an auction starts, each bidder must submit an upfront payment that determines its bidding eligibility in the auction. A bidder’s eligibility is specified by the maximum number of bidding units on which the bidder’s upfront payment allows it to be active in a given round.[13] Bidder eligibility cannot be increased after the bidder submits its upfront payment. After bidding starts, the FCC measures a bidder’s activity by the sum of bidding units of the licenses on which the bidder places bids in the current round and licenses on which the bidder has provisionally winning bids from a previous round. Activity requirement dictates that a bidder must bid on a specified por-

[13]The upfront payment does not limit the dollar amount that a bidder may bid for any license.

tion of its maximum eligibility in a given round (80% and later 95% in Auction 73). If the activity requirement is not met, the FCC reduces the violating bidder's eligibility permanently, possibly preventing the bidder from further bidding in the auction.[14] In a real-time or continuous auction, bidders sometimes wait until the last minute to place their bids for various reasons. For example, eBay bidders often "snipe" the auctioned item in the final seconds of the auction. The FCC's eligibility and activity requirements prevent extremely slow bidding and sniping bids, and more generally, bidders' strategic use of postponing their bidding decision. In short, the FCC has implemented rules to ensure that participants bid actively throughout the auction.

2.2 License Complementarity and Package Bidding

Each license is associated with a geographically distinctive area —"Market Area" in FCC terminology. The FCC traditionally has five types of Market Areas. Going from small to large in geographic coverage, they are: Cellular Market Areas (CMAs), Basic Trading Areas (BTAs), Basic Economic Areas (BEAs), Major Trading Areas (MTAs), and Regional Economic Area Groupings (REAs). A typical CMA covers only three to four counties and a BEA roughly 15 counties, while an REA can cover hundreds of counties and span several states. In different splits, the United States are divided into 734 CMAs, 176 BEAs or 12 REAs. The FCC carefully balances geographic coverage associated with a license and its allotted bandwidth to avoid market power concentration and to induce entry into high-cost, sparsely populated rural areas.[15]

Spectrum licenses, especially the ones covering adjacent geographic areas, are complementary to each other. Complementarity exists when the value of the whole is greater than the sum of the parts. That is clearly the case in spectrum auctions because a typical cell phone carrier provides continuous coverage over at least a certain region. What is not clear, from both the FCC's and industry experts' perspectives, is the magnitude of such complementarity over a combination of licenses. A major justification of the *à la carte* auction format is to allow bidders to bid for a group of licenses that they would like to acquire

[14] A bidder has a limited number of "proactive" waivers to use to satisfy the activity requirement. In Auction 73, this number was three. The use of waivers preserves a bidder's current eligibility. The FCC will automatically apply a waiver for a bidder (instead of a bidder applying waivers proactively, which keeps the auction open) at the end of a round in which the bidder's activity is too low, unless the bidder has no waivers left or the bidder reduces eligibility voluntarily.

[15] Generally, there is a negative relationship between the size of the frequency bandwidth and that of the geographic coverage: licenses of narrower spectrum bandwidth is usually assigned a larger market area, and vice versa.

together. For example, if a small firm targets the entire Iowa market, it may want to bid for all Iowa licenses at a certain bandwidth in each round of bidding; if a national firm aims to fix bandwidth congestion in its current service areas, it may want to bid for more bandwidth in areas where the congestion problem is more severe. Different bidders have different plans for the acquired spectrum, and so it seems best to be left at the free disposal of bidders themselves.

Problems arise, however, with the free assembly on which a bidder needs to decide during the auction process. A typical bidder seeks to acquire multiple licenses that have a combined value higher than the sum of all stand-alone values due to complementarity. As bidders are never certain about the set of licenses they will win until the auction concludes, they are subject to the risks of either "overbidding" or "underbidding" *ex-post* regret. "Overbidding" regret happens when a bidder becomes the winner of isolated patches of licenses, which have a combined value lower than the winning bids. In contrast, "underbidding" regret happens when a bidder gives up bidding on a license for which it has higher value than a rival bidder's winning bid. Both types of *ex-post* regret constitute the "exposure problem," which is the result of license complementarity and bidder uncertainty about the auction allocation outcome.

One way to fix the problem is to allow bidders to submit bids on packages of licenses. The FCC first proposed a simple form of package bidding in 2000 (Auction 31), but it was not implemented until 2003 (Auction 51).[16] Following Auction 51, the FCC revised the package bidding format and implemented it in Block C, Auction 73. The most significant change from Auction 51 was that the packages were predefined by the FCC instead of being proposed by bidders.[17] For the 12 licenses in the C Block, FCC permitted *à la carte* bidding as well as package bidding.

The FCC and industry experts believe that package bidding, in general, should be an improvement over individual license bidding, especially when there is strong complementarity among licenses. As the revised FCC package bidding format endows the FCC with the power to set packages for bidding, understanding the magnitudes of license complementarity becomes a key policy input. The goal of this paper is to provide estimates of license complementarity to help the FCC make better decisions on how to divide license-coverage and set package format.

[16] Auction 51 is a small auction in which one bidder won five licenses (in one package) in three rounds.

[17] Bidder-defined package bidding becomes an intractable problem as the number of licenses and potential packages as well as the number of bidders increases.

3 Auction Set 73

Auction 73 began on January 24, 2008 and closed on March 18, 2008. The spectrum frequency to be auctioned off ("the 700 MHz band") had been occupied by television broadcasters and could then be used for flexible fixed, mobile, and broadcast. The FCC offered a total of 1,099 licenses in five blocks: 176 Basic Economic Area (BEA) licenses in the A and E Blocks; 734 Cellular Market Area (CMA) licenses in the B Block; 12 Regional Economic Area Grouping (REA) licenses (and three packages) in the C Block; and one nationwide license (NWA) in the D Block.[18]. After 261 rounds (in a matter of 38 days), 206 out of 214 qualified bidders placed at least one bid, and 101 of them won a total of 1,090 licenses.[19] Gross bids amounted to $19,120,378,000, from which the FCC actually received $18,957,582,150 (net bids, which is the amount the FCC receives after credits and discounts to bidders).

This was a "limited disclosure" auction, meaning that the FCC withheld all information on bidder identities until the auction concluded. In any round and for any license, bidders knew the characteristics of the license, the minimum acceptable bid by the FCC, the number of bids placed in all previous rounds, and the provisionally winning bid price. Bidders knew the provisionally winning licenses that they had in each round but they had no information aboout what licenses other bidders had provisionally won before the auction concluded.

In the following subsections, we report and analyze descriptive statistics of Auction 73. The statistics characterize an auction with highly heterogeneous licenses and equally, if not more so, heterogeneous bidders. More importantly, we highlight stylized facts that affect our model choices in subsequent sections.

3.1 Heterogeneous Licenses

Auction 73 consisted of five blocks of licenses. Table 1 reports summary statistics for different blocks of licenses and the top ten licenses (ranked from high to low in terms of winning bids). As shown in the table, these licenses varied substantially in bandwidth and a geographic area coverage, two most important characteristics of spectrum licenses. A license with wider bandwidth and a larger geographic area (usually corresponding to larger population) is more valuable. In these five blocks, the bandwidth went from 6 MHz (Block E) to 22 MHz (Block C), and the geographic coverage went from CMA (roughly three to four counties) to NWA

[18]Blocks A, B, C, and D provide paired spectrum, but Block E is unpaired. In paired spectrum, one frequency channel transmits in one direction (e.g., downstream), while a second frequency transmits in the opposite direction (e.g.; upstream). In contrast, unpaired spectrum provides a single frequency channel transmitting in both directions.

[19]The FCC held the remaining nine licenses and auctioned some off in later auctions.

Table 1: Licenses, Bids and Winning Bids

Block	# of Licenses	Market Area	Bandwidth (MHz)	Band*Pop (MHz*m)	Bid: # of Bidders	Bid: min ($)	Bid: max ($)	Win: # of Winners	Win: Sum Win ($)	Win: min ($)	Win: max ($)
A	176	BEA	12	309	80	7	580268	33	3961174	20	580268
B	734	CMA	12	194	200	1	892400	87	9143993	15	892400
C	15	REA	22	1280	26	13	4713823	4	4748319	550	1625930
D	1	NWA	10	2856	1	472042	472042	1	472042	472042	472042
E(unpaired)	176	BEA	6	154	50	3	224988	5	1266892	17	224988

License Name	Market Area	Bandwidth (MHz)	Band*Pop (MHz*m)	Bid: # of Bidders	Bid: min ($)	Bid: max ($)	Win: Win ($)
Mississippi Valley	REA	22	689	5	63932	1625930	1625930
Great Lakes	REA	22	1280	4	171433	1109715	1109715
Chicago, IL	CMA	12	97	6	38223	892400	892400
New York-Newark	CMA	12	194	9	59435	884703	884703
Central	REA	22	888	4	100035	723228	723228
LA-Riverside-Orange	BEA	12	216	6	33044	580268	580268
Northeast	REA	22	1101	3	324585	604624	502774
Los Angeles-Anaheim	CMA	12	187	13	26874	483981	483981
Nationwide	NWA	10	2856	1	472042	472042	472042
NYC-Long Is	BEA	12	309	4	83212	429356	429356

Note: 1) All monetary terms are in thousands of dollars. 2) Block C has 15 licenses: REA1-12, Pkg Atlantic, Pkg Pacific, Pkg 50 States, but there is no bidder for Pkg Pacific,and there is no winner for Pkg Atlantic, Pkg 50 States, REA009 and REA011.

(Nationwide Area: the first time that the FCC offered a license covering the entire United States). Block A licenses had the same geographic delineation as Block E licenses, but the doubled bandwidth in Block A drew more bidders and much higher bids than Block E. Although having much smaller geographic coverage than Block D, Block C licenses seemd to have been more valuable due to their 22 MHz of bandwidth, compared to Block D's mere six MHz.

Some blocks drew much more competition than others. Block B drew the highest number of bidders, while Block D attracted only one.[20] Block C is the most interesting block in this study, as the FCC auctioned off predefined packages in this block. Bidders could bid for 12 REAs licenses separately or for three packages separately: package 50 states (REA 1-8), package Atlantic (REA 9 and REA 11) and package Pacific (REA 10 and REA 12). Google seems to have been a strong contender in the beginning, but Verizon eventually became the biggest winner of Block C, winning regional licenses covering the continental United States and Hawaii (98% of U.S. population).[21]

3.2 Heterogeneous Bidders

We assign bidders into different quartiles based on the size of their upfront payment.[22] Table 2 reports the bidding behaviors and winnings of these quartiles, as well as the top ten winners. We can see that th top bidders in Auction 73 played the most important role in this auction. The top 25% bidders (53 in total) did the majority of the bidding and won 984 of the 1,099 offered licenses. Their gross bids were very close to the final winning gross bids in the entire auction.[23] AT&T won more licenses in terms of numbers, but Cellco (doing business as Verizon Wireless, referred to as Verizon henceforth) won more high-value licenses. Many of these bidders were industry veterans, including cell phone carriers (e.g., AT&T, Verizon, and Cellular South); cable companies (e.g., Cox); telecommunications firms providing Internet, TV and phone bundles (e.g., Frontier); and Internet giants (e.g., Google). One point to note is that many bidders and some winners did not offer cell phone services at the time of the

[20] Qualcomm was the only bidder in Block D. Qualcomm's provisionally winning bid ($472 million) for the Block D license, however, did not meet the FCC's reserve price ($1.3 billion).

[21] Google bid $1.037 billion in round 1 for package 50 states and increased its bids over time. In round 17, Google bid $4.713 billion for package 50 states. From round 27 to round 30, Verizon entered and placed eight different bids on REA 1 to REA 8 and won seven, spending $4.742 billion in total. Google gave up afterwards.

[22] Upfront payment determines the initial eligibility (maximum number of bidding units) of the bidders and is usually positively correlated with the size of the bidder.

[23] Gross bids reported in this table include Qualcomm's provisionally winning bid in Block D, which did not meet the FCC's reserve price and so is not counted in the FCC's reported final outcome.

auction. In other words, they became potential entrants into the industry after acquiring licenses.

Because of the distinct differences in bidding patterns, we separate bidders into three groups: large, medium and small. Large bidders included only AT&T and Verizon, which were incumbent cell phone carriers with the national footprint. Small bidders were bidders that qualified for the FCC's Designated Entity discount – a steep discount (15% and 25% in Auction 73) to encourage small or disadvantaged bidders to acquire spectrum licenses. Medium-sized bidders were those that were neither large nor small, including Mobility Spectrum LLC., Airwaves Inc., Qualcomm Inc., MetroPCS 700 MHz, Alltel Corporation, Frontier Wireless and many more. In Block B of Auction 73 (the auction block that we use for estimation and counterfactual analyses), there were 2 large, 86 medium and 112 small bidders.

Table 2: Bidders and Winners

		Bid			Win				
Firm Tier	Up.Pay ($)	# Lic.	min ($)	max ($)	# Lic.	Band*Pop (MHz*m)	Sum Win bids ($)	min ($)	max ($)
1st quartile	54212	31298	1	4713823	984	17531	19500000	17	1625930
2nd quartile	437	3148	4	19138	64	107	46955	15	8469
3rd quartile	105	1150	6	8055	24	36	26554	45	8055
4th quartile	30	822	1	2081	18	15	4430	20	793
		Bid			Win				
Firm Name	Up.Pay ($)	# Lic.	min ($)	max ($)	# Lic.	Band*Pop (MHz*m)	Sum Win bids ($)	min ($)	max ($)
AT&T	500000	6052	1	884703	227	2110	6636658	190	884703
Frontier	115253	2269	16	220188	168	1303	711871	51	62656
King Street	97000	3021	9	933360	152	487	400638	38	60918
Verizon Wireless	885000	3783	11	1625930	109	8508	9363160	107	1625930
CenturyTel	25000	2571	9	22151	69	212	148964	93	21928
Triad 700	57000	1296	15	80246	36	186	22694	46	3124
Cavalier	42000	1988	13	66872	35	322	61803	36	7811
Cellular South	29634	580	7	241365	24	180	191533	194	49201
Cox Wireless	36000	638	10	147893	22	248	304633	619	84119
David Miller	2250	384	10	4073	16	40	7812	32	4073

Note: 1) All monetary terms are in thousands of dollars. 2) Band*Pop represents frequency bandwidth times population associated with the license, measured by MHz times millions in population.

3.3 Stylized Facts of Bidder Behaviors

In this section, we establish four stylized facts of bidder behaviors that we observe in data. These stylized facts help us to set up the most parsimonious model to capture a very complicated bidding process.

1) Bidders Bid Minimum Acceptable Bids

In spectrum auctions, bidding is a discrete choice rather than a continuous choice. In each round of bidding, a bidder can choose from a limited number of possible bids in the (web-based) bidding system. The minimum of these bids is called the Minimum Acceptable Bid (MAB), which is determined by the FCC. The MAB of a license in a new round equals the provisionally winning bid (determined in the last round) plus an increment that may decrease over time.[24] At the beginning of the auction, the increment in the MAB is around 20% of the provisionally winning bid. At the end of the auction, the increment in the MAB is usually less than 10% of the provisionally winning bid. For Auction 73 Block C licenses, there were three MAB per license or one MAB per package in each round. For other licenses, only one MAB was available per license in each round. The FCC set this rule because it believed that bidders might use jump bids strategically to signal to other bidders if the FCC gave too many MAB options in each round.

From Auction 73's bidding data, we observe that over 99.8% of the bids equaled the MAB for an average round. Only nine bids succeeded the MAB, all of which occurred before round 17 in Block C. "Small Ventures USA, L.P." placed eight of these bids, and "Copper Valley Wireless, Inc." placed the other one.[25] After round 17, all bids were equal to the MAB. In other words, jump bids were extremely rare in Auction 73. When several bidders placed the same bid, a random number was used to determine the provisionally winning bids for the license.

2) Bidders Gradually Drop out of the Bidding Process

Auction 73 went through 261 rounds in total. As we observe from Table 3, the initial rounds were chaotic but the dust settled down as the auction progressed, as is typical of an FCC spectrum auction. In rounds 1 to 20, the bid-to-bidder ratio was over 100, but this ratio wen down to 50 in rounds 20 to 40 and then stabilized at 30 to 40 in later rounds. As shown in the "win" panel of Table 3, most winnings, including winnings of most high-value licenses, were determined in rounds 21 to 40. Auctions that drag on for over 100 rounds are, in fact, not for high-value licenses. In the last row of Table 3, the mean bids are only $1.2 million, and the mean winning bids are only $2.4 million, both the lowest among all six rows. Overall, bidders participated eagerly in the first rounds and gradually dropped out of

[24] If there are no bidders placing bids for a license, the FCC may slightly reduce the MAB of a license.

[25] Both of these bidders were medium-size bidders, with upfront payments of $700,000 and $528,000, respectively.

the race round by round. There is no evidence of bidders waiting strategically until the final rounds and coming in to snipe the licenses away from their competitors.

Table 3: Bidding Statistics Round by Round

Round	Bid						Win			
	# active bidders	# new bids	# active licenses	min ($)	mean ($)	max ($)	# licenses won	min ($)	mean ($)	max ($)
1 to 20	207	21382	1057	1	7111	4713823	86	42	38441	580268
21 to 40	125	6699	960	5	8981	1625930	535	18	27044	1625930
41 to 60	68	3041	325	4	3045	224988	92	32	7699	224988
61 to 80	57	1849	253	4	1827	146963	96	36	2984	62656
81 to 100	46	1434	177	4	1576	154999	91	55	4013	154999
> 100	48	2013	190	6	1255	81613	190	15	2412	81613

Note: All monetary terms are in thousands of dollars.

3) Bidders Bid Straightforwardly

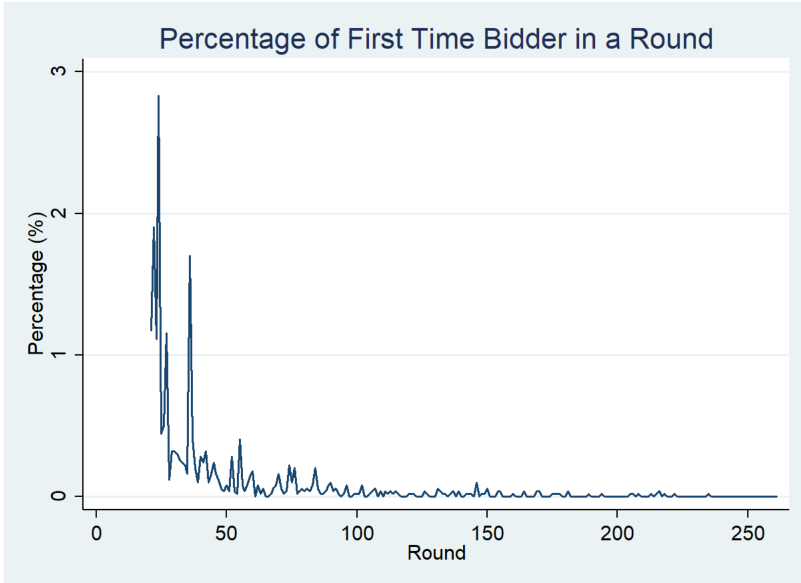
The bidding data suggest that bidders bid straightforwardly. Although Auction 73 lasted for 261 rounds, the average duration of bidding (last bid round - first bid round) was 22 rounds. The vast majority of bidders (88%) bid for fewer than 50 rounds and stopped bidding as MAB increased. In 19% of the bidder-license observations, bidders bid consecutively until they become provisional winners or until they give up.

We do not see patterns of bidders strategically delaying bidding in Auction 73. The total occurrences of "first-time-bidders" after round 20 accounted for only about 1,000 (bidder-license) observations. (Recall that there were more than 1,000 licenses and more than 200 bidders in this auction.) Figure 1 reports, for each round after round 20, the percentage of bidder-licenses in which a bidder bid on a license that it had not bid on before. Only about 3% of bidder-license observations after round 20 are first-time bids. This number reduces dramatically afterwards and becomes negligible after round 50. We suspect that these first-time bidders bid either: 1) to satisfy the FCC's eligibility requirement after they decided on stop bidding on some licenses; or 2) when some licenses became more valuable as the expected winning probability of adjacent licenses became larger as the auction progressed.

4) Bidders Win Licenses in Geographically Close Clusters

Figure 2 reports the final license allocations of Blocks A, B, C and E, a different color representing a different bidder. As shown in the figure, a bidder often wins a cluster of licenses that are located close together. This suggests complementarity across locations. Some bidders,

Figure 1: Percentage of First-Time Bids in Each Round



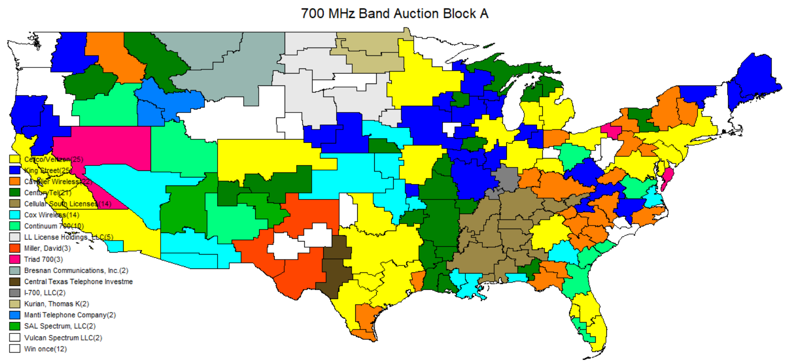
such as Verizon in Block C and Frontier in Block E, aim for nationwide complementarity, while others, such as Cellular South (covering the majority of Tennessee, Mississippi and Alabama in Block A) and King Street (doing business as US Cellular, covering quite a few of the eastern and midwestern states), aim for regional complementarity. Appendix 1 shows evidence that bidders bid on multiple licenses in the same round, further suggesting the existence of complementarity across different licenses.

4 Model

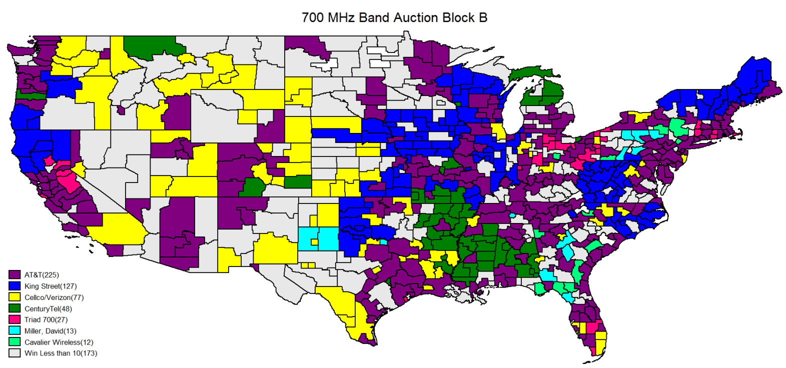
These stylized facts inform us in setting up a parsimonious model for estimation. First, as the vast majority of bidders bid at the MAB, we model an entry game instead of a "name a bid" pricing game. For any license at any given round, a bidder need only to decide whether or not to enter the bidding. Second, as rounds progress and bidders gradually drop out from licenses they have bid on, we believe that this is a process of bidders updating their beliefs in every round. Third, given the strong evidence that bidders bid straightforwardly, we assume that bidders consider every round (after the initial 20 rounds) the last round if the bidder takes no actions. That is, we minimize bidders' dynamic considerations and do not consider their "waiting strategically" problem. In fact, under the FCC's eligibility and activity requirement, "waiting" is almost a ruled-out option. Fourth and lastly, we allow any pair of licenses to be potentially complementary and bidders to bid for any combination of licenses, subject to

Figure 2: Final License Allocations of Block A, B, C and E

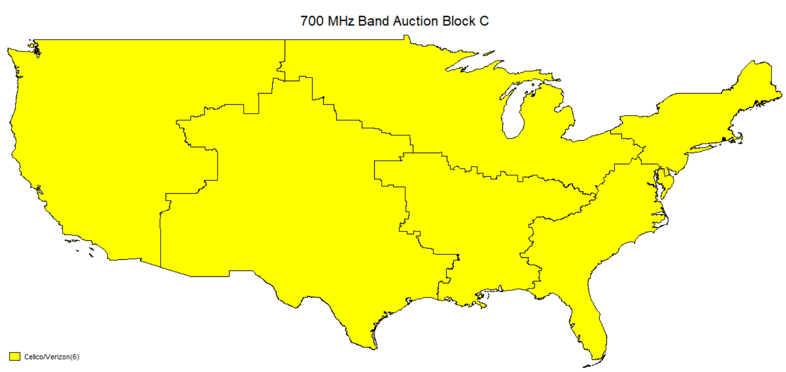
(a) Block A



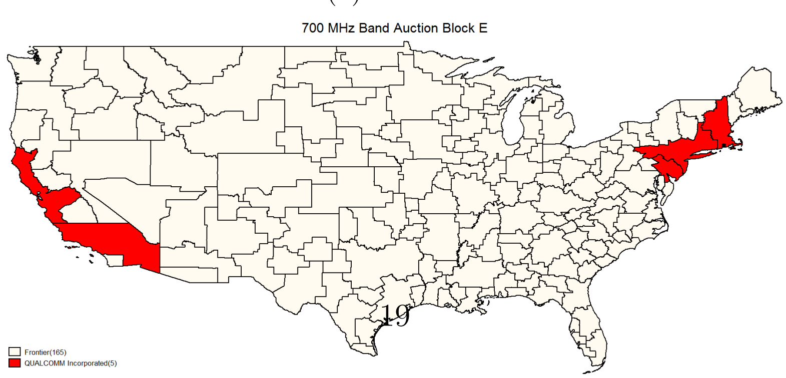
(b) Block B



(c) Block C



(d) Block E



their eligibility requirement. In summary, for each round of the auction, we have a bidder's static entry problem with interdependent markets and the bidder's evolving beliefs about the probability of winning.

In each round, bidders bid on a set of licenses to maximize their expected payoffs at the end of the auction, based on their updated beliefs about winning (instead of to maximize the present discounted value of the future profit stream, in the fashion of Ericson and Pakes (1995). This model choice is driven by the features of our setting, which differs from a typical dynamic game. The game is short — e.g., 38 days in Auction 73. In the auction, a bidder receives no flow payoff during the auction; all pay-offs are realized only at the end of the auction. Naturally, there are no random shocks in a bidder's utility function in each round. Bidders' valuation on any sets of licenses remain constant throughout the auction. What changes from round to round are bidders' evolving beliefs about the final outcome and license prices.

4.1 Model Setup and Notation

Notation and Timeline

There are N bidders (each bidder denoted by i) competing for L licenses (each license denoted by l). Rounds are denoted by t, with the last round denoted by T.[26] Each license covers multiple counties, with counties being the smallest geographic areas in our study. We denote counties by $c \in \mathbf{C}$ and the set of counties covered by license l by $\mathbf{C}_l$.

We now present notation according to the timeline of the game. Just before the auction, bidder i realizes its stand-alone values for all licenses and the complementarity between any two licenses. Let v_{il} denote bidder i's stand-alone value of license l. Let $\tau(l, l')$ denote the complementarity index between two licenses l and l'. The complementarity index is the same for all bidders, although its valuation may be different across different types of bidders. We will discuss the parametrization of v_{il} and $\tau(l, l')$ in Section 5.

At the beginning of a round t, bidder i observes a history h_{it}, which includes the prices and characteristics of all licenses, the number of other bidders in the previous rounds of all licenses, as well as the bidding history and the provisionally winning set of its own.[27] Let P_{lt} denote the MAB of license l in round t. The set of all MABs in round t is $\mathbf{P}_t = \{P_{lt}, l =$

[26] The ending round, T, is unknown at the beginning of and throughout the auction.

[27] A bidder's provisionally winning set includes the licenses on which the bidder bids and wins in the last round, as well as the licenses in the bidder's provisional winning set before last round that no other new bidders bid on in the last round.

$1, ..., L\}$. We denote the set of other bidder-license-round characteristics as $\mathbf{X}_{ilt}$.

Bidders form beliefs about the set of licenses that they can win given history h_{it}. Let $\mathbf{W}_{iT}$ denote the ultimate winning set of bidder i and $Pr(l \in \mathbf{W}_{iT}|h_{it}, \mathbf{B}_{it})$ denote bidder i's subjective belief about winning license l at the end of the auction. A bidder's bidding decisions $\mathbf{B}_{it}$, together with history h_{it}, affects its winning belief. Bidder i selects bidding set $\mathbf{B}_{it}$ to maximize its expected payoff. The FCC reveals the bidder's provisional winning set $\mathbf{W}_{it}$ at the end of each round and the bidder's final winning set $\mathbf{W}_{iT}$ at the end of the last round of the auction.

Information Set

A bidder's information set consists of both public and private information. Public information includes (1) the characteristics of all licenses; (2) the MABs of all licenses in the current and previous rounds; (3) the complementarity between any pair of licenses; and (4) the number of bidders on all licenses in all previous rounds. Private information includes (1) the bidder's stand-alone values for all licenses $\{v_{il}, \forall l\}$; (2) its own bidding history and, thus, the number of *other* bidders for any licenses in the previous rounds; and (3) its provisionally winning set in all previous rounds.

Restrictive Assumptions

We impose some restrictive assumptions to make our model tractable. First, we assume that a bidder's valuation of a set of licenses is independent of the identities of the winners of other licenses. Second, we assume that there is no complementarity or competition across blocks.[28] Third, we allow for only a pair-wise complementarity effect. Fourth, we ignore withdrawal possibilities in our model. Withdrawals are allowed in the auction with a steep penalty, so bidders rarely withdraw bids. Withdrawn bids consisted of less than 1% of the bidder-license-round combinations of Auction 73. Last, and most importantly, we assume away any unobservable heterogeneity at the license- or license-set level that are not absorbed by history h_{it}. That is, econometricians observe the same h_{it} as the bidders do.

[28] There may exist complementarity or competition effects across blocks. A bidder may want to acquire additional bandwidth, offered in a different block, of the same geographic market. A bidder may substitute licenses in one block with licenses of similar geographical coverage in another block.

4.2 Belief Formation

Due to the complementarity between licenses, a bidder's valuation of a license depends not only on the stand-alone value of the license, but also on the set of other licenses it wins. Moreover, bidders do not know exactly the set of licenses they will ultimately win *ex-ante*. Bidder i's ultimate winning set $\mathbf{W}_{iT}$ is revealed only after the auction concludes. A bidder forms beliefs about the probability of winning a set of licenses at the beginning of the auction and updates its beliefs round-by-round during the auction.

It is difficult to obtain the winning belief over all sets of licenses, as each bidder has 2^L possible ultimate winning sets. To simplify our analysis, we impose the following conditional independent assumption:

Assumption 1 ***Conditional Independence*** *Conditional on history h_{it} and bidding set $\boldsymbol{B}_{it}$, bidder i's subjective beliefs about winning two licenses are independent:*

$$Pr(l \in \boldsymbol{W}_{iT} \text{ and } l' \in \boldsymbol{W}_{iT}|h_{it}, \boldsymbol{B}_{it}) = Pr(l \in \boldsymbol{W}_{iT}|h_{it}, \boldsymbol{B}_{it}) \times Pr(l' \in \boldsymbol{W}_{iT}|h_{it}, \boldsymbol{B}_{it}) \quad (1)$$

This assumption means that the entire complementarity effect in bidders' beliefs is captured by history h_{it} and bidding set $\mathbf{B}_{it}$. For example, a bidder observes its provisional winning set and, in turn, knows the marginal contribution of license l to this provisional winning set. This bidder then chooses a bidding set $\mathbf{B}_{it}$ to realize the complementarity between license l and its provisional winning set. Conditional on the history and bidding set, the bidder's beliefs about winning two licenses are independent.

As we document in Section 3, bidders of Auction 73 bid rather straightforwardly; that is, there is no evidence of strategical delay in bidding. Bidders chose whether or not to bid as if they were facing a one-shot decision. Therefore, we make the following assumption to simplify a bidder's belief formation process:

Assumption 2 ***Final Round*** *At any round(after the initial rounds), bidder i believes the current round to be the final round of the auction.*

The above two assumptions allow us to specify bidder i's belief about winning license l in a tractable way. A bidder's belief is different in three cases: (1) license l is in bidder i's bidding set $\mathbf{B}_{it}$; (2) license l is not in bidder i's bidding set $\mathbf{B}_{it}$ but in bidder i's provisional winning set $\mathbf{W}_{it-1}$ in the last round; and (3) license l is not in bidder l's bidding set $\mathbf{B}_{it}$ or in the last round provisional winning set $\mathbf{W}_{it-1}$. Bidder i's belief about winning license l is specified as follows:

$$Pr(l \in \mathbf{W}_{iT}|h_{it}, \mathbf{B}_{it}) = \begin{cases} Pr(l \in \mathbf{W}_{iT}|h_{it}, l \in \mathbf{B}_{it}) & \text{if } l \in \mathbf{B}_{it} \\ Pr(l \in \mathbf{W}_{iT}|h_{it}, l \in \mathbf{W}_{it-1}\backslash\mathbf{B}_{it}) & \text{if } l \in \mathbf{W}_{it-1}\backslash\mathbf{B}_{it} \\ 0 & \text{if } l \notin (\mathbf{W}_{it-1} \bigcup \mathbf{B}_{it}). \end{cases} \tag{2}$$

In the above belief specification, a bidder's belief about winning license l is zero if the bidder is not a provisional winner of this license or does not bid on this license in the current round. This is the implication of our Assumption 2. For notation simplicity, we denote $Pr(l \in \mathbf{W}_{iT}|h_{it}, \mathbf{B}_{it})$ as $Pr(l|\mathbf{B}_{it})$ henceforth.[29]

4.3 Complementarity and The Expected Payoff of Bidding

This subsection discusses how the complementarity across licenses determines the marginal contribution of a license to a bidder's bidding set and then a bidder's expected value of its bidding set.

The Expected Value of Bidding Set $\mathbf{B}_{it}$

Let $EV_i(\mathbf{B}_{it})$ denote bidder i's expected value of all licenses if it bids on bidding set $\mathbf{B}_{it}$. Given its belief about winning license l ($Pr(l|\mathbf{B}_{it})$), the bidder's expected value of all licenses equals the sum of the expected stand-alone values v_{il} of all licenses and the expected complementarity effect among these licenses.

$$EV_i(\mathbf{B}_{it}) \;\; = \;\; \sum_l v_{il} \times Pr(l|\mathbf{B}_{it}) + \frac{1}{2}\beta_i \sum_l \sum_{l'} \tau(l, l') \times Pr(l|\mathbf{B}_{it}) \times Pr(l'|\mathbf{B}_{it}). \tag{3}$$

In the above equation, β_i denotes bidder i's valuation of a unit of complementarity index. This complementarity coefficient, β_i, is the key primitive parameter that we will estimate using this model. We assume that $\beta_i \geq 0$ and we allow different types of bidders to have different complementarity coefficients.

[29]The auction is not a first-order Markov process. Bidders' beliefs about winning licenses may depend on the entire history, which results in an enormous number of explanatory variables. We discuss how we reduce the dimensionality of explanatory variables when we estimate bidder beliefs in Section 5.

The Marginal Contribution of License l to Bidding Set $\mathbf{B}_{it}$

The marginal contribution of a license l to bidding set $\mathbf{B}_{it}$ is defined as the difference between the expected value of bidding $(\mathbf{B}_{it} \cup l)$ and bidding $(\mathbf{B}_{it} \backslash l)$.

$$\begin{aligned} \Delta EV_i(l, \mathbf{B}_{it}) &= EV_i(\mathbf{B}_{it} \cup l) - EV_i(\mathbf{B}_{it} \backslash l)) \\ &= [v_{il} + \beta_i \sum_{l'} \tau(l, l') \times Pr(l'|\mathbf{B}_{it})] \times Pr(l|\mathbf{B}_{it}). \end{aligned} \tag{4}$$

The Expected Payoff of Bidding Set $\mathbf{B}_{it}$

A bidder's expected payoff from bidding set $\mathbf{B}_{it}$ is the difference between its expected value of bidding set $\mathbf{B}_{it}$ and its expected payment.

$$\pi_i(\mathbf{B}_{it}, \mathbf{P}_t) = EV_i(\mathbf{B}_{it}) - \sum_l P_{lt} \times Pr(l|\mathbf{B}_{it}). \tag{5}$$

In the above equation, P_{lt} is the MAB of license l in round t.

A Bidder's Decision

In this model, a license's stand-alone value and the complementarity across licenses are constant over time. In round t, bidder i observes a history h_{it} and a vector of MABs $\mathbf{P}_t = \{P_{lt}, l = 1, ..., L\}$; forms a belief about the probability of winning any license l; and selects a set of licenses $\mathbf{B}_{it}$ to maximize its expected payoff $\pi_i(\mathbf{B}_{it}, \mathbf{P}_t)$. A bidder may want to bid on a license for which it has higher probability of winning rather than another license for which its valuation is higher, but the expected winning probability is lower.

4.4 Bayesian Nash Equilibrium

The equilibrium concept in this incomplete information game is Bayesian Nash Equilibrium. In each round t, bidder i chooses a set of licenses $\mathbf{B}_{it}$ to maximize its expected payoff. A strategy function of bidder i is denoted by $\sigma_i(\mathbf{v}_i, \tau_i(.), Pr(.))$, where $\mathbf{v}_i$ is the set of stand-alone values of all licenses; $\tau_i(.)$ is the set of complementarity indices between all pairs of licenses; and $Pr(.)$ is the bidder's belief about winning any set of licenses. The strategy function maps the private information $\mathbf{v}_i$, common state variables $\tau_i(.)$ and beliefs $Pr(.)$ into a set of binary choices. We define a Bayesian-Nash equilibrium as follows:

Definition 1 *A Bayesian-Nash Equilibrium consists of* $\sigma_i^*(\boldsymbol{v}_i, \tau_i(.), \boldsymbol{P}, Pr^*(.))$ *and* $Pr^*(.)$, *such that for each i, $\sigma_i^*(\boldsymbol{v}_i, \tau_i(.), \boldsymbol{P}, Pr^*(.)) = \boldsymbol{B}_{it}$ if and only if for any $\boldsymbol{B}'_{it}$,*

$$\pi_i(\boldsymbol{B}_{it}, \boldsymbol{P}_t) \geqslant \pi_i(\boldsymbol{B}'_{it}, \boldsymbol{P}_t); \tag{6}$$

and if all bidders use strategy $\sigma_i^*(\boldsymbol{v}_i, \tau_i(.), Pr^*(.))$, a bidder's probability of winning a license in the auction is the same as its belief $Pr^*(.)$.

According to Brouwer's Theorem, there is at least one Bayesian Nash Equilibrium.

The set of possible bidding sets is enormous. In our case, with N bidders and L licenses, in each round, a bidder may place 2^L different combinatorial bids, and the total number of possible bids for all bidders is $2^{L\times N}$. For a game with such a highly-dimensional state space, it is not possible to compute an equilibrium. Furthermore, there may be multiple equilibria. To estimate the model, we derive two behavior assumptions, which are necessary conditions of all Bayesian Nash equilibria in this game, and we exploit these behavioral assumptions to estimate the primitives of the model.

4.5 Behavior Assumptions

We adopt a "revealed preference" approach: at any point in time, a bidder's bidding set is weakly preferable to any alternative bidding set. Suppose that we observe that a bidder i bids for a set of licenses $\mathbf{B}_{it}$ in round t; then, it must be the case that its expected profit from bidding on $\mathbf{B}_{it}$ is weakly preferable to alternative action $\mathbf{B}'_{it}$. We have the following two situations:

(1) for any $l \in \mathbf{B}_{it}$, $\pi_i(\mathbf{B}_{it}, \mathbf{P}_t) \geqslant \pi_i(\mathbf{B}_{it} \backslash l, \mathbf{P}_t)$;

(2) for any $l \notin \mathbf{B}_{it}$ and $l \notin \mathbf{W}_{it-1}$, $\pi_i(\mathbf{B}_{it}, \mathbf{P}_t) \geqslant \pi_i(\mathbf{B}_{it} \cup l, \mathbf{P}_t)$ when l is under bidder i's eligibility constraint.

Using Equation (5), we derive the following two behavior assumptions, **BA1** and **BA2**, from the two inequalities above:

Behavior Assumption 1 ***(BA1)*** *If $l \in \boldsymbol{B}_{it}$,*

$$v_{il} + \beta_i \times \sum_{l'} \tau(l, l') \times Pr(l'|\boldsymbol{B}_{it}) \quad \geq \quad P_{lt}. \tag{7}$$

BA1 states that, if a bidder bids on license l, the expected marginal contribution of license l is higher than the MAB on this license. The marginal contribution of license l includes both the stand-alone value of license l and the increment in the expected complementarity between license l and all other licenses.

Behavior Assumption 2 ***(BA2)*** *If $l \notin \boldsymbol{B}_{it}$ and $l \notin \boldsymbol{W}_{it-1}$, but l is under bidder i's eligibility constraint, then*

$$v_{il} + \beta_i \times \sum_{l'} \tau(l, l') \times Pr(l'|\boldsymbol{B}_{it}) \leq P_{lt}. \tag{8}$$

BA2 states that, when a bidder does not bid on license l even though l is within its eligibility constraint, the marginal contribution of license l is lower than the MAB of license l in this round.

These two behavior assumptions can explain why a bidder starts to bid on a new license that it has not bid before, while the MAB monotonically increases, and also why a bidder eventually stops bidding. In early rounds, a bidder assigns low probabilities of winning licenses, so the expected complementarity of any license is small. When a bidder is more confident that it will win some licenses, it will start to bid on other licenses to gain complementarity. Eventually, it may find the price of a license too high and gives up bidding.

5 Identification, Estimation and Results

In this section, we first discuss the identification of the complementarity coefficient β_i and the stand-alone values of licenses. We then estimate the primitives of the model in the order of: 1) bidders' beliefs about winning probabilities; 2) the complementarity coefficient β_i; and 3) the stand-alone values of licenses as a function of license and bidder characteristics. We use data after the first 20 rounds in Block B of Auction 73 for estimation and counterfactual experiments. Block B has the smallest license coverage, so we can exercise different sizes of a license package.

We estimate the above primitives sequentially instead of simultaneously. The advantage of sequential estimation is that any possible misspecification of one primitive will not affect the estimation of another. For example, we do not need to impose any parametric specification assumptions on stand-alone values in the estimation of the complementarity effect. This way, the complementarity effect, the main focus of this study, is more robust to potential model misspecification. The disadvantage of sequential estimation is that the standard errors of estimates in the previous step may affect the standard errors of those in later steps.

5.1 Identification

Identification of the Complementarity Effect

We make use of changes in a bidder's bidding decisions on a license over different rounds to identify the complementarity effect. For bidder i on a license l, the stand-alone value of the license v_{il} and the complementarity between license l and any other license l' remains the same throughout the auction. The MAB P_{lt}, a bidder's belief about winning other licenses $Pr(l'|\mathbf{B}_{it}), \forall l' \neq l$, and, more importantly, a bidder's provisional winning set in each round change over time. The latter two elements determine the expected contribution of complementarity of a license to a bidder in each round. Changes in a bidder's bidding decisions on a given license result from changes in the MABs and the bidder's expected contribution of complementarity by the license. Since we observe the MABs in the data, we can exploit the bidder's bidding decisions on a license, given the prices over different rounds, to bound the complementarity effect.[30]

When we observe that a bidder starts bidding on a license it has not bid on in earlier rounds, this is because the increase in price is lower than the increase in the marginal contribution of the license. Since the stand-alone value of the license remains the same, this start-bidding decision at a given price generates a lower bound for the expected complementarity effect. When we observe that a bidder stops bidding on a license it has been bidding on, this is because the increase in license price is higher than the increase in the marginal contribution of the license. Similarly, the stop-bidding decision at a given price generates an upper bound for the expected complementarity effect. The upper and lower bounds of the complementarity effect are independent of the stand-alone value of licenses, and, therefore, we can separately identify the complementarity effect from the stand-alone value of licenses.

Identification of Stand-alone Values

We identify the stand-alone value of a license from a bidder's bidding decisions across different licenses. If a bidder chooses to bid on a license, the marginal contribution of this license must be greater than the license's MAB. The "revealed preference" in this action generates a lower bound for the stand-alone value (as the complementarity effect is already separately

[30] In our model, the marginal contribution of a license is shifted by each bidder's current provisional winning set, which is excluded from the utility of another license. This is in the spirit of using "variables that can be excluded a priori from the utility of one or more goods," as in Fox and Lazzati (2017) and Gentzkow (2007). In Gentzkow (2007), examples are "whether consumers have Internet access at work or a fast connection at home, which shift the utility of the online edition without affecting the utility of the print edition." Fox and Lazzati (2017) is more about non-parametric identification.

identified). On the contrary, if the bidder is eligible to bid on a license but chooses not, the "revealed preference" in this action generates an upper bound for the stand-alone value.

We will parametrize the stand-alone value of a license as a function of license- and bidder-level characteristics. As different licenses and bidders have different characteristics, a bidder's different bidding decisions across licenses in a given round identifies how these characteristics enter our parametrization. Basically, if a bidder bids on license l instead of on license l', the marginal contribution of license l towards a bidder's expected payoff is higher than that of license l'. As the complementarity effect is already identified from the previous step, the stand-alone values of different licenses can be separated from the marginal contributions of these licenses.

5.2 Estimation of Bidder Belief

5.2.1 Specification of Bidder Belief

This subsection discusses how we estimate a bidder's belief about winning a license. In round t, bidder i's belief about winning license l may depend on the entire history. To make estimation feasible, we use the following parametric specification, allowing bidder belief to depend only on history up to the last period, but our specification can easily accommodate more periods.[31] We estimate bidder belief for large, medium-sized and small bidders separately.

If bidder i places a bid in round t, or if bidder i does not bid on license l but it is a provisional winner of license l in round $t-1$, bidder i's belief about winning license l is

$$Pr(l|\mathbf{B}_{it}) = \Phi(\alpha_0 + fn(\sum_{l' \neq l} \tau(l', l)1[l' \in W_{i,t-1}])\alpha_1 + \mathbf{X}_{ilt}\alpha_2), \tag{9}$$

In equation (9), $\sum_{l' \neq l} \tau(l', l)1[l' \in W_{i,t-1}]$ is the contribution of license l to the complementarity effect in the provisional winning set of bidder i. We use a quadratic function $fn(.)$ to capture the nonlinear effects. The set of bidder-license-round characteristics X_{ilt} includes dummy variables indicating the number of other bidders that bid on license l plus 1 (there is always a provisional winner of license l in round $t-1$); population (measured as a fraction of U.S. population) times bandwidth; log population density; log tower density a bidder has in the license; [32] log upfront payment; and the number of rounds. When bidder i does not bid

[31] The large number of possible explanatory variables renders it infeasible to use non-parametric estimation.

[32] The tower density variable is defined as $\ln(\frac{Tower_{il}}{Area_l} + 1)$, where $Tower_{il}$ is the number of cell phone towers that bidder i has (by a certain date), and $Area_l$ is the fraction of license l's area in the United States. Appendix 2 explains how we construct the cell phone tower variable using the FCC cell phone tower registration database.

on license l but is a provisional winner of license l in round $t-1$, the set of bidder-license-round characteristics also includes the number of rounds in which bidder i is the provisional winner of license l. Lastly, $\Phi(.)$ is the normal density function.

Parameter α measures how different factors affect bidder i's belief about winning license l. In particular, α_1 measures how bidder i's belief about winning license l changes when the contribution of license l to the complementarity effect changes. A bidder may be more likely to win a license if it wins other licenses that have large complementarity with that license.

5.2.2 Estimates of Bidder Belief

Table 4 reports the marginal effect at covariates' mean for the estimates of equation (9).[33] The first three columns report the marginal effects if a bidder bids on a license, and the last three columns report the marginal effects if a bidder is a provisional winner of the license. Columns 1 and 4 are estimates for large bidders, columns 2 and 5 for medium-sized bidders, and columns 3 and 6 for small bidders.

Overall, the most notable pattern in Table 4 is the strong complementarity effect and the even stronger competition effect. The presence of competition lowers the probability of winning, while the presence of complementarity increases it. Other key license attributes often have contradictory effects across bidder types, suggesting that bidders select different licenses to bid on. Finally, if a bidder is still bidding or remains a provisional winner as the number of rounds increases, the bidder is more likely to win the license.

When the expected complementarity index between the focal license and the bidder's provisional winning set in the last round increases, a bidder is more likely to win this license. This relationship is often concave. For instance, if the complementarity index for a national bidder increases by 0.001 when the complementarity index is 0.005, a larger bidder's belief about winning increases by 0.3%.[34]

Small bidders are more sensitive to competitors. When there are one, two, three or more competitors in the last round, a small bidder's probability of winning the focal license decreases by 23.4%/34.1%/34.4% if it bids on the license, and by 12.3%/19.0%/15.8% if it is the provisional winner of this license. The competition effect for the large and medium-sized bidders, however, is much smaller and sometimes statistically insignificant.

Large bidders are more likely to win larger, densely-populated licenses, while small bidders are just the opposite. For instance, a 0.001 increase in population results in $0.001\ \times$

[33] Table A4 (in Appendix 5) reports estimates of Equation (9).

[34] Here is how we calculate the magnitude: based on Column 1 of Table 4, we have $(3.543 * 0.006 - 36.62 * 0.006^2) - (3.543 * 0.005 - 36.62 * 0.005^2) = 0.3\%$.

$0.883(estimates) \times 12(12MHzinBlockB) = 1\%$ increase in a large bidder's winning probability but a $0.001 \times 29.51 \times 12 = 35.4\%$ reduction in a small bidder's winning probability.

Table 4: Belief Estimation Results: Marginal Effect

	Bid			Provisional Winner		
	(1) Large	(2) Medium	(3) Small	(4) Large	(5) Medium	(6) Small
Complementarity	3.543***	27.97	119.6***	-0.566***	1.596***	20.80***
	(1.280)	(27.57)	(10.52)	(0.060)	(0.477)	(1.205)
Complem. Sq.	-36.62***	-6,192	-6,016***	0.058***	-6.305***	-819.5***
	(11.78)	(6,493)	(1,142)	(0.020)	(1.844)	(90.13)
$\#Competitor = 1$	0.043	0.276	-0.234***	-0.066***	-0.125***	-0.123***
	(0.052)	(0.213)	(0.090)	(0.005)	(0.011)	(0.008)
$\#Competitor = 2$	-0.016	0.128	-0.341***	-0.031**	-	-0.190***
	(0.056)	(0.218)	(0.093)	(0.013)		(0.029)
$\#Competitor \geq 3$	-	0.015	-0.344***	-0.098**	-0.041	-0.158***
		(0.250)	(0.109)	(0.040)	(0.078)	(0.049)
Pop*Bandwidth	0.883***	1.852*	-29.51***	1.088***	0.763	-0.017
	(0.322)	(0.991)	(2.591)	(0.112)	(0.496)	(0.376)
$\ln(\frac{Pop_c}{Area_c} + 1)$	-0.055***	-0.122***	-0.034**	0.003***	-0.024***	-0.037***
	(0.015)	(0.021)	(0.014)	(0.001)	(0.003)	(0.002)
$\ln(\frac{Tower_{ic}}{Area_c} + 1)$	0.009***	-0.010**	-	0.002***	-0.003***	-
	(0.002)	(0.005)		(0.167e-3)	(0.675e-3)	
ln Upfront Pay	-0.814***	-0.230e-3	0.010***	-0.126***	-0.007***	0.005***
	(0.014)	(0.007)	(0.003)	(0.002)	(0.001)	(0.416e-3)
Round	1.39e-4	1.61e-3***	4.21e-4***	1.29e-3***	6.88e-4***	2.72e-4***
	(4.51e-4)	(2.61e-4)	(1.45e-4)	(9.03e-05)	(3.79e-05)	(2.22e-05)
# Win Round				-2.67e-4***	1.70e-3***	1.44e-3***
				(9.08e-05)	(6.14e-05)	(3.17e-05)
# obs	1,706	1,779	3,664	229,654	246,933	329,880

Note: An observation is a bidder-license-round combination in Block B of Auction 73 (after the first 20 rounds). We include all bidding rounds and all standing-winning rounds for a bidder-license combination.

5.3 Estimation of Complementarity

5.3.1 Specification of the Complementarity Index

This subsection describes the construction of the complementarity index $\tau(l, l')$.

County Complementarity Index

As described in Section 2, different blocks have different geographic delineations that divide the U.S. and its territories into exclusive market areas. Geographic area definitions are BEAs in Block A and E, CMA in Block B, REA in Block C, and nationwide in block D. To make different blocks comparable, we use county, the greatest common divisor of CMA, BEA and REA, as the smallest geographic area in this study. Every license can be split into a number of whole counties, and every county is always entirely and exclusively included in a license.[35]

We follow the specification of the complementarity index in Fox and Bajari (2013). The complementarity index between any two counties c and c' with a bandwidth of $Bandwidth_{c,c'}$ is

$$\tau(c,c') = Bandwidth_{c,c'} \left[pop_c \frac{\frac{pop_c pop_{c'}}{dist^{\delta}_{c,c'}}}{\sum_{c'' \in \mathbf{C} \setminus c} \frac{pop_c pop_{c''}}{dist^{\delta}_{c,c''}}} + pop_{c'} \frac{\frac{pop_c pop_{c'}}{dist^{\delta}_{c,c'}}}{\sum_{c'' \in \mathbf{C} \setminus c'} \frac{pop_{c'} pop_{c''}}{dist^{\delta}_{c',c''}}} \right], \tag{10}$$

where pop_c is the fraction of the U.S. population in county c; $dist_{c,c'}$ is the distance between two counties c and c';[36] $\mathbf{C}$ is the set of all counties; and $Bandwidth$ is the bandwidth of the block. We set $\delta = 2$ such that our model is close to the "gravity model." A nice property of this index is that a nationwide license with bandwidth of 1 MHz has a total complementarity index of one.[37]

License Complementarity Index

Since each license (CMA, BEA or REA) exclusively contains one or more counties, we define the complementarity index between any two licenses l and l' as the sum of complementarity indices between the set of counties in l and the set of counties in l'.

$$\tau(l,l') = \sum_{c \in \mathbf{C}_l} \sum_{c' \in \mathbf{C}_{l'}} \tau(c,c'), \tag{11}$$

where $\mathbf{C}_l$ is the set of counties within license l.

[35]In contrast, Fox and Bajari (2013) use Basic Trading Area as the smallest geographic area in their study, which divides the continental United States into 480 mutually exclusive licenses.

[36]The distance between two counties is computed as the distance of the two county centers and is measured in kilometers. The minimum distance between two counties is set at ten kilometers to avoid small denominators.

[37]In Appendix 4, we compare different methods of constructing the complementarity index and find them to be similar in summary statistics.

Construction of Inequalities

We construct a set of inequalities based on **BA1** and **BA2**. For bidder i on license l, the set $\mathbf{BID}_{il}$ denotes the set of rounds in which bidder i bids on license l. According to **BA1**, if round $t \in \mathbf{BID}_{il}$, the marginal contribution of the license in round t is greater than the MAB of the license in round t.

$$v_{il} + \beta_i \sum_{l'} \tau(l,l') \times Pr(l'|\mathbf{B}_{it}) \quad \geq \quad P_{lt}. \tag{12}$$

In this inequality, $Pr(l'|\mathbf{B}_{it})$ is the "true" belief of the bidder, which is not observable to the econometrician. We replace this "true" belief with the estimated belief $\widehat{Pr}_t(l'|\mathbf{B}_{it})$. This way, we naturally introduce an error term ε_{ilt}, which is the difference between the expected complementarity contribution of a license under a bidder's "true" belief and that under its estimated belief.

$$\varepsilon_{ilt} = \beta_i(\sum_{l'} \tau(l,l') \times Pr(l'|\mathbf{B}_{it}) - \sum_{l'} \tau(l,l') \times \widehat{Pr}(l'|\mathbf{B}_{it})), \tag{13}$$

and inequality (12) becomes:

$$v_{il} + \beta_i SP(ilt) + \varepsilon_{ilt} \geq \quad P_{lt}, \tag{14}$$

where $SP(ilt) = \sum_l \tau(l,l') \times \widehat{Pr}(l'|\mathbf{B}_{it})$ is the expected complementarity contribution of a license under a bidder's estimated belief $\widehat{Pr}(l'|\mathbf{B}_{it})$.

Similarly, for bidder i on license l, the set $\mathbf{NOTBID}_{il}$ denotes the set of rounds in which bidder i does not bid on license l, while bidder i is not the provisional winner of license l and the license's assigned bidding units are under bidder i's unused eligibility. According to **BA2**, if $t' \in \mathbf{NOTBID}_{il}$

$$v_{il} + \beta_i \sum_{l'} \tau(l,l') \times Pr(l'|\mathbf{B}_{it}) \quad \leq \quad P_{lt'}. \tag{15}$$

We then replace this "true" belief with the estimated belief $\widehat{Pr}(l'|\mathbf{B}_{it})$ and introduce an error term $\varepsilon_{ilt'}$, which is the difference between the expected complementarity contribution of a license under a bidder's "true" belief and that under its estimated belief.

$$v_{il} + \beta_i SP(ilt') + \varepsilon_{ilt'} \quad \leq \quad P_{lt'}. \tag{16}$$

Once we have the two sets $\mathbf{BID}_{il}$ and $\mathbf{NOTBID}_{il}$, we match a round t in set $\mathbf{BID}_{il}$ with any round t' in set $\mathbf{NOTBID}_{il}$, sum up two inequalities and eliminate v_{il}:

$$\beta_i \left[SP(ilt) - SP(ilt')\right] + \varepsilon_{ilt} - \varepsilon_{ilt'} \quad \geq \quad P_{lt} - P_{lt'}. \tag{17}$$

Define $DS(l,t,t') = SP(ilt) - SP(ilt')$, where $DS(l,t,t')$ is the difference between the expected complementarity contribution of a license under a bidder's estimated belief in round t and that in round t'. We now construct conditional moment inequality:

$$E[\beta_i DS(l,t,t') - P_{lt} + P_{lt'} + \varepsilon_{ilt} - \varepsilon_{ilt'}|SP(ilt), SP(ilt'), Bid_{ilt}] \geq 0. \tag{18}$$

Assumption 3 ***Conditional Independence 2*** *Conditional on the expected complementarity and bidding decisions, errors generated by calculating the expected complementarity contribution of a license using a bidder's estimated belief have zero mean:*

$$E[\varepsilon_{ilt}|SP(ilt), Bid_{ilt}] = 0.$$

This assumption means that we do not allow for unobservable heterogeneity that bidders observe but that the econometrician does not observe. We can now rewrite the conditional inequality as:

$$E[\beta_i DS(l,t,t') - P_{lt} + P_{lt'}|DS(l,t,t'), Bid_{ilt}] \geq 0. \tag{19}$$

Following Andrews and Shi (2013), by properly selecting instruments we transform the conditional inequality in Equation 19 to unconditional inequality without losing identification power. For a non-negative instrument $z(DS(l,t,t'))$, we have

$$E\left[z(DS(l,t,t'))[\beta_i DS(l,t,t') - P_{lt} + P_{lt'}]\right] \geq 0. \tag{20}$$

Construction of Criterion Functions

Below, we discuss the construction of the criterion functions according to Equation (20). All inequalities belong to one of the following four categories:

Cat. 1: If $DS(l,t,t') > 0$ and $P_{lt} - P_{lt'} > 0$, $\beta_i \geq \frac{E[z(DS(l,t,t'))(P_{lt}-P_{lt'})]}{E[z(DS(l,t,t'))DS(l,t,t')]}$. This is a lower bound of the complementarity coefficient β_i. If bidder i does not bid on license l when the price is low (in round t') but starts to bid on the license when the price is high (in round

t), the increase in the expected complementarity contribution of this license must be higher than the increase in price, which generates a lower bound for complementarity.

Cat. 2: If $DS(l,t,t') < 0$ and $P_{lt} - P_{lt'} < 0$, $\beta_i \leq \frac{E[z(DS(l,t,t'))(P_{lt}-P_{lt'})]}{E[z(DS(l,t,t'))DS(l,t,t')]}$. This is an upper bound of the complementarity effect. If bidder i bids on on license l when the price is low (in round t) but stops bidding on this license when the price is high (in round t'), the increase in the expected complementarity contribution of this license must be lower than the increase in price, which generates an upper bound for complementarity.

Cat. 3: If $DS(l,t,t') > 0$ and $P_{lt} - P_{lt'} \leq 0$, $\beta_i \geq$ a non-positive number. However, it is uninformative because we assume that $\beta_i \geq 0$. So, inequalities in Cat 3 do not provide us with any new information.

Cat. 4: If $DS(l,t,t') < 0$ and $P_{lt} - P_{lt'} \geq 0$, $\beta_i \leq 0$. This contradicts our model because we assume that $\beta_i \geq 0$.

To estimate our model, we drop inequalities that either provide no information (Cat. 3) or contradict our model (Cat. 4) and use only the inequalities in Cat. 1 and Cat. 2 to estimate the complementarity effect. These two sets of inequalities identify the complementarity coefficient β_i.

We use the moment inequality approach to estimate the complementarity effect. We define the following criterion function following Chernozhukov, Hong, and Tamer (2007)

$$Q\left(\beta_i\right) = \sum_{z(DS(l,t,t'))} \min\{z(DS(l,t,t')) \times [\beta_i DS(l,t,t') - P_{lt} + P_{lt'}], 0\}^2, \tag{21}$$

where $\{z(DS(l,t,t'))\}$ is a set of instruments. Following Andrews and Shi (2013), we divide all inequalities to different groups. Instrument $z(.)$ is an indicator function whose value equals one if inequalities belong to a group, and zero otherwise. The selection of groups is based on the quantiles of $DS(.)$ values. For instance, when we construct instruments for inequalities in Cat. 1, we divide all inequalities into 4, 6, 8, 10, 12 and 14 groups. Suppose that we divide inequalities into 4 groups, the first group including all inequalities in the first quartile and the last group including the inequalities in the fourth quartile. The instrument $z(.)$ indicates whether an inequality belongs to a group. There will be a total of 108 instruments.[38]

The estimates of the complementarity effect minimize the criterion function. The true value of β_i should satisfy all inequalities if we specify the model correctly. If there are multiple (a set of) estimates of β_i that satisfy all inequalities (when $\beta_i = \hat{\beta}_i$, the value of the

[38]The total number of groups is $4+6+8+10+12+14 = 54$ for each of the two categories of inequalities.

Table 5: Estimates of the Complementarity Effect in Block B (Billion $)

	(1) Full Sample	(2) Large	(3) Medium	(4) Small
β	0.87	1.36	0.28	0.91
95% CI	[0.81 , 0.93]	[1.02 , 1.66]	[0.25 , 0.31]	[0.80 , 1.00]
# LBs	73,840	5,017	33,679	35,144
# Ubs	125,157	5,267	56,370	63,520

Note: The unit of observation is a bidder-license-round-pair. For bidder i, we include a bidding round and a not-bidding round as a pair for license l.

criterion function = 0), then we have a set estimate. If there is no estimate that satisfies all inequalities (there is no $\beta_i = \hat{\beta}_i$ such that the value of the criterion function = 0), we select a $\hat{\beta}_i$ that minimizes the criterion function as a point estimate.

5.3.2 Estimates of the Complementarity Effect

Table 5 reports the estimates of the complementarity effect. Column (1) reports empirical results for the full sample, Column (2) for large bidders, Column (3) for medium-sized bidders, and Column (4) for small bidders. In the estimation, for the set $\mathbf{NOTBID}_{il}$ (when bidder i does not bid on license l), we consider only licenses whose assigned bidding units are lower than the unused eligibility of the bidder. Appendix 6 reports results from a robustness check in which we add more restriction for the set $\mathbf{NOTBID}_{il}$: the bidding units of the license need to be higher than additional units required to maintain the eligibility level of the bidder (but still lower than the unused eligibility of the bidder). In this robustness check, we consider the possibility that a small license will not help the bidder to satisfy its activity requirement.

The estimates indicate that, when the complementarity index (of a nationwide license) increases by 1 MHz, the value from complementarity increases by $0.87 billion. The 95% confidence interval for this estimate is tight. The bandwidth in Block B is 12, and the total value of complementarity if a bidder bids on all licenses in Block B is 12 times $0.87 billion, or $10 billion. This is more than the actual total bidding in Block B (around $9.1 billion). Block B's final allocation is very fragmented, meaning that only limited complementarity is achieved.

The valuation of the complementarity effect has a non-monotonic relationship with bidder size. The value of complementarity in a nationwide license with bandwidth of 1 MHz is worth \$1.36 billion to an average large bidder, only \$0.28 billion to an average medium-sized bidder, and \$0.91 billion an average small bidder. A potential explanation for this pattern is that small bidders see future arbitrage opportunities — as large bidders may acquire small bidders' bandwidth (or even small bidders themselves) — and, therefore, align their willingness to pay with large bidders. Medium-sized bidders, however, bid according to their real need for spectrum. Medium-sized bidders' final win in Block B is very scattered (as shown in Figure 2), reflecting their low willingess to pay for complementarity. Appendix 7 shows that more concrete bidding patterns for the three types of bidders facing different expected complementary indices and suggests that medium-sized bidders' bidding decisions are much less affected by the gain in expected complementarity than are the decisions of the other two types of bidders.

5.4 Estimation of the Stand-alone Value of a License

5.4.1 Specification of the Stand-alone Value Function

After we obtain estimates of the complementarity effect, we estimate bidders' stand-alone values of licenses. We parametrize v_{il}, bidder i's stand-alone value of license l as a function of license characteristics and bidder characteristics:

$$\begin{aligned} v_{il} &= \theta_0 + \theta_1 ln(Pop_l) + \theta_2 \ln(\frac{Pop_l}{Area_l} + 1) + \theta_3 \ln(\frac{Tower_{il}}{Area_l} + 1) + \theta_4 \ln(UpPay_i) + \xi_{il} \\ &\equiv X^v_{il}\theta + \xi_{il}, \end{aligned} \tag{22}$$

where Pop_l is the fraction of license l's population in the United States; $Area_l$ is the fraction of license l's area in the United States; and $Tower_{il}$ is the number of bidder i's towers in license l. We use these three variables to construct $\frac{Pop_l}{Area_l}$, the population density of license l, and $\ln(\frac{Tower_{il}}{Area_l} + 1)$, the tower density of bidder i in license l, in the specification. We further include $\ln \mathrm{UpPay}_i$, the natural log of the upfront payment of bidder i measured in dollars. Again, we estimate equation (22) for large, medium-sized, and small bidders separately.

According to our discussion in the identification section, the identification of stand-alone values comes from a bidder's bidding decisions across different licenses. Denote π_{ilt} the payoff if bidder i bids on license l in round t.

$$\begin{aligned}\pi_{ilt} &= v_{il} + \beta_i \sum_{l'} \tau(l,l') \times Pr(l'|\mathbf{B}_{it}) - P_{lt} \\ &= X^v_{il}\theta + \beta_i \sum_{l'} \tau(l,l') \times Pr(l'|\mathbf{B}_{it}) - P_{lt} + \xi_{il}. \end{aligned} \tag{23}$$

Bidder i will bid on license l in round t if and only if $\pi_{ilt} \geq 0$.

To estimate parameter θ, we impose a parametric assumption on the distribution of ξ_{il}.

Assumption 4 ***Normal*** *ξ_{il} follows a normal distribution with mean zero conditional on X^v_{il}: $\xi_{il} \sim N(0, \sigma^2)$.*

With this assumption, we can write down the probability that bidder i will bid on license l at round t conditional on its information set:

$$\begin{aligned}\mathscr{P}(Bid_{ilt} &= 1|X^v_{il}, \tau(.,.), Pr(.), P_{lt};\theta) \\ &= \int_{\xi} 1[X^v_{il}\theta + \beta_i \sum_{l'} \tau(l,l') \times Pr(l'\mathbf{B}_{it}) - P_{lt} + \xi_{il} \geq 0]\frac{1}{\sigma}\phi(\frac{\xi_{il}}{\sigma})d\xi_{il} \\ &= \Phi(\frac{1}{\sigma}(X^v_{il}\theta + \beta_i \sum_{l'} \tau(l,l') \times Pr(l'|\mathbf{B}_{it}) - P_{lt})). \end{aligned} \tag{24}$$

In Equation (24), ϕ and Φ are, respectively, the standard normal p.d.f and c.d.f., and the vector $\{X^v_{il}, \tau(.,.), Pr(.), P_{lt}\}$ includes the corresponding covariates for every bidder, license and round in the sample. We allow σ, the standard deviation of ξ_{il}, to vary across different bidder types in estimation. Equation (24) shows that, after integrating over the unobserved heterogeneity ξ_{il}, we can write down the probability that bidder i will bid on license j in round t as a Probit model. We can write down the probability of bidder i not bidding on license l in round t in a similar fashion. The likelihood function is as follows:

$$\begin{aligned}\mathscr{L} = \sum_{l}\sum_{i,t} \frac{1}{\sum_{i,t} 1[Bid_{ilt}=1]} 1[Bid_{ilt}=1] ln(\mathscr{P}(Bid_{ilt}=1|X^v_{il}, \tau(.,.), Pr(.), P_{lt};\theta)) + \\ \sum_{l}\sum_{i,t} \frac{1}{\sum_{i,t} 1[Bid_{ilt}=0]} 1[Bid_{ilt}=0] ln(\mathscr{P}(Bid_{ilt}=0|X^v_{il}, \tau(.,.), Pr(.), P_{lt};\theta)). \end{aligned} \tag{25}$$

In Equation (25), we replace the true value of complementarity value β_i with its estimates $\hat{\beta}_i$ and $Pr(.)$ with its fitted value $\hat{P}r(.)$. We obtain estimates by maximizing the log-likelihood function. Because the identification of stand-alone values comes from cross-license variation, we weight the bidder-license-round observations such that one license has an effective bid observation and an effective not-bid observation. For instance, suppose that bidder A places five bids on license l, and bidder B places another four bids on license l throughout the entire bidding process. Each bid has $\frac{1}{9}$ weight in our estimation.

5.4.2 Estimates of Stand-alone Values

Table 6: Estimation of Stand-alone Values (in billion $)

	(1) Large	(2) Medium	(3) Small
$\ln(Pop_l)$ (θ_1)	0.114***	0.015***	-0.118***
	(0.014)	(0.003)	(0.006)
$\ln(\frac{Pop_l}{Area_l}+1)$ (θ_2)	-0.022	-0.007*	-0.045***
	(0.017)	(0.004)	(0.007)
$\ln(\frac{Tower_{il}}{Area_l}+1)$ (θ_3)	0.002	-0.002***	0.060***
	(0.003)	(0.001)	(0.007)
$\ln(UpPay_i)$ (θ_4)	0.692***	-0.008***	-0.069***
	(0.033)	(0.002)	(0.002)
Constant (θ_0)	-13.115***	0.246***	0.219***
	(0.693)	(0.039)	(0.071)
S.E. of ξ_{il} (σ)	0.380*	0.071	0.451
	(0.224)	(0.272)	(0.945)
# obs	29,553	45,542	95,151

Note: The unit of observation is a bidder-license-round combination. We include all bidding rounds and all not-bidding rounds (not-bidding but the license is under the bidder's eligibility constraint) of a bidder-license combination. We calculate standard errors using the Delta method.

Table 6 reports estimation results for bidders' stand-alone values. Columns (1), (2) and (3) are stand-alone value estimation results for large, medium-sized and small bidders respectively. Large and medium-sized bidders prefer large licenses, while small bidders prefer small licenses, suggesting that the valuation estimated here reflects budget constraints faced by small bidders. Coefficients for population density, tower density, and log upfront payment often have different signs across the three types of bidders, again suggesting that stand-alone values capture multiple differences among bidders, which choose very different licenses to bid for. Overall, heterogeneity at the license level, including bandwidth, population, population density, and cell phone tower density, plays a large role in bidders' stand-alone values.

5.5 Goodness of Fit

In this subsection, we compare bidders' predicted bidding decisions with their observed bidding decisions. To predict bidding decisions using our estimates, we first predict the stand-alone values of the licenses according to the estimates in Table 6. Then, for a bidder in a license-round, we compare the expected marginal contribution of the license (computed using Equation (5) and the complementarity estimates in Table 5) with the MAB. If the expected marginal contribution of the license is higher than the MAB, we predict that the bidder will bid on the license in this round. Otherwise, the bidder will not bid.

Table 7: Goodness of Fit

	Data				
	Not Bid		Bid		
Predicted	Frequency	Percentage	Frequency	Percentage	Total
Not Bid	142,279	83.6	3,310	1.9	145,589
Bid	21,009	12.3	3,648	2.1	24,657
Total	163,288	95.9	6,958	4.1	170,246

Table 7 shows how well our predicted bidding fits observed bidding. Our empirical analysis is based on a total of 170,246 (bidder-license-round) inequalities.[39] Our model predictions satisfy 85.7% = (83.6% + 2.1%) of these inequalities.

6 Counterfactual Analysis

In the counterfactual analysis, we compare the magnitude of the exposure problem, bidder surplus, FCC revenue and license allocation to the different types of bidders at the end of the auction under alternative package designs. As the example in our introduction illustrates, the welfare effect of package bidding depends on the distribution of bidder valuations and the size of license complementarity. In this section, we first describe our metrics used to measure the incidence of the exposure problem; next, we explain how we conduct simulations based on our estimates; and, lastly, we evaluate welfare trade-offs under alternative designs of package bidding.

[39]We have a total of 170,246 observations in Table 6

6.1 Two Types of the Exposure Problem

We characterize two types of the exposure problem: in any round for any bidder, if a license in the bidder's provisional winning set does not contribute value to its ultimate winning set, we call it the "overbidding" exposure problem; in any round for any bidder, if a license that could have contributed value to the ultimate winning set is not in the provisional winning set, we call it the "underbidding" exposure problem.

When a bidder bids on a license in a round, it must expect this bidding decision to be profitable. However, this bidder may not win all of the other licenses it expected to win and, in turn, the bidder may find it unprofitable to have this license at the end of the auction. In this case, the bidder may regret its bidding decision on this license, and, thus, we have the "overbidding" exposure problem. The magnitude of exposure from the set $\mathbf{W}_{it}$ on $\mathbf{W}_{iT}$, is

$$EP^1(\mathbf{W}_{it}, \mathbf{W}_{iT}) = -\sum_{l \in \mathbf{W}_{it}} \min\{v_{il} + \beta_i \sum_{l' \in (\mathbf{W}_{iT} \setminus l)} \tau(l, l') - P_l, 0\}, \tag{26}$$

where $v_{il} + \beta_i \times \sum_{l' \in (\mathbf{W}_{iT} \setminus l)} \tau(l, l')$ is the expected marginal contribution of license l towards the ultimate winning set $\mathbf{W}_{iT}$. If this expected marginal contribution is lower than the price, bidder i should not win license l and the difference between this contribution and the price is the magnitude of the "overbidding" exposure problem from this license. We then aggregate over the bidders' provisional winning set in this round to calculate the total "overbidding" exposure problem.

When a bidder does not bid on a license in a round, it must expect it to be unprofitable to do so. Again, the set of licenses that the bidder thinks it can win may not be the same as the ultimate winning set of the bidder. The bidder may regret that it did not bid on this license. This is the "underbidding" exposure problem. The magnitude of exposure from the set $\mathbf{W}_{it}$ on $\mathbf{W}_{iT}$, is

$$EP^2(\mathbf{W}_{it}, \mathbf{W}_{iT}) = \sum_{l \notin \mathbf{W}_{it}} \max\{v_{il} + \beta_i \sum_{l' \in (\mathbf{W}_{iT} \setminus l)} \tau(l, l') - P_l, 0\}. \tag{27}$$

If this expected marginal contribution of license l towards the ultimate winning set $\mathbf{W}_{iT}$ is higher than the price, bidder i regrets not having won license l. The difference between this expected marginal contribution and this price is the magnitude of the "underbidding" exposure problem. We then aggregate over the bidders' provisional winning set in this round to calculate the total "underbidding" exposure problem.

6.2 Benchmark Model

In this simulated auction, we assume away eligibility restrictions and activity requirements to simplify bidding decisions. These FCC rules are set to induce straightforward bidding in spectrum auctions, and our simulated bidding strategies already abide by these rules. We also do not intend to let budget constraints to affect our simulation results, as our model does not incorporate these constraints. Therefore, our results on different types of bidders are driven by the difference in their valuations, not by their budget constraints. To have a clean experiment, we restrict our analysis to the bidders that eventually win at least one license in Block B after 20 rounds, the block with the smallest geographic delineation. In total, we have 67 bidders (including 2 large bidders, 28 medium-sized bidders and 37 small bidders) bidding on 734 licenses.[40]

6.3 Benchmark Simulation Setup

Our simulation needs to use our estimated primitives in the model: the complementarity value between any two licenses (common to all bidders of the same type) and the stand-alone value of each license for each bidder. For any pair of licenses, we can calculate the complementarity index according to Equation 11. We the use the value of the complementarity effect as reported in the baseline estimation results in Table 5. Furthermore, if a bidder never bids on a license, we assume that the bidder has zero stand-alone value on this license and can not receive any complementarity value from it, which partially reflects the bidders' budget constraints.

Below, we explain how we simulate the stand-alone value of a license for a bid. For bidder i on license l, we obtain its fitted stand-alone value $X^v_{il}\hat{\theta}$ and estimates of the standard error of residual term ξ_{il} ($\hat{\sigma}$) from Table 6. Under **Assumption 4**, stand-alone value v_{it} follows a normal distribution with mean $X^v_{il}\hat{\theta}$ and standard deviation $\hat{\sigma}$. To limit extreme values and obtain stand-alone values consistent with real bidding patterns, we compute the upper and lower bounds of stand-alone values of a license for a bidder using bidding information.

First, we obtain a set of bidder-license-round specific lower bounds of a license's stand-alone values to a bidder from Equation (14), for all rounds when the bidder bids on this license. We select the maximum of all these bidder-license-round specific lower bounds as the lower bound of the license's stand-alone value to this bidder. Second, we obtain a set

[40] For our estimation, we use Block B data after 20 rounds. We restrict our analysis to winners after round 20 to save computational time. Winners for the majority of licenses (93%) are determined after the first 20 rounds. By restricting analysis to 67 bidders, we lose fewer than 500 bids (out of a total of over 7000 bids).

of bidder-license-round specific upper bounds of a license's stand-alone values to a bidder from Equation (16), for all rounds when a bidder does not bid on this license.[41] We select the minimum of all these bidder-license-round specific upper bounds as the license's upper bound of the stand-alone value to this bidder. Combining these two steps, we obtain a lower bound and an upper bound of each license's stand-alone value to each bidder.[42] Finally, we simulate a bidder's stand-alone value from a truncated normal distribution between the lower bound and upper bound of the license's stand-alone value to this bidder, using the fitted stand-alone value and estimates of the standard error of the residual term in Table 6.

Based on the value of complementarity between all pairs of licenses and the stand-alone values of all licenses for all bidders, we simulate the bidding decisions of the bidders in the auction.

6.4 Simulation Algorithm

We describe how we simulate the entire auction in this subsection. In any round t, bidder i observes its provisional winning set $\mathbf{W}_{it-1}$. The bidder forms an expectation of the probability of winning license l: $Pr(l|\mathbf{B}_{it})$, according to the belief formation functions in Equation (9). Bidder i determines its bidding set $\mathbf{B}_{it}$ to maximize its expected payoff in this round. Ideally, we want to compute the expected payoffs associated with all bidding strategies — that is, any possible set of licenses. For any bidder i, there are a total of 2^{734} possible bidding strategies in round t. It is computationally infeasible to evaluate the expected profit from all bidding strategies and find the optimal bidding set.

To solve this problem, we transform the profit-maximization problem into a search for the fixed points of the necessary conditions. This transformation simplifies our problem from a set with 2^{734} bidding possibilities to a set of fixed point of the necessary conditions, which has a much smaller dimension. Since all licenses are complements, if a bidder bids on any license, the marginal contribution of all other licenses towards the bidding set will increase. That is, the set of licenses displays a supermodularity property. We follow Jia (2008) and make use of the supermodularity property to compute the minimum and maximum bidding sets of the bidders in any round.

The intuition is that, if a bidder is willing to bid on a license even though it does not win any other licenses, the optimal bidding set must include this license. On the other hand, if a bidder is not willing to bid on a license even though it wins all other licenses, the optimal

[41] We assume the error terms in Equation (14) and Equation (16) to be zero in our simulations.

[42] When the lower bound is greater than the upper bound, we replace the upper bound with the lower bound.

bidding set must not include this license. So, we will compute the set of licenses on which the bidder should always bid (the minimum bidding set) and the set of licenses on which a bidder should potentially bid (all licenses minus the set of licenses that the bidder should not bid on, or the maximum bidding set). The bidder's optimal bidding set of the bidder should be in between the minimum and the maximum bidding set.

Below, we describe how we determine a bidder's minimum bidding set in round t. In any round t, the bidder's initial bidding set is an empty set, and we include licenses in this bidding set sequentially. In the first iteration, we compute the expected marginal contribution of all licenses towards the bidder's provisional winning set in round $t-1$. The bidding set in iteration 1 includes those licenses whose marginal contributions are higher than their current MABs. In the second iteration, we compute the expected marginal contributions of all licenses towards the provisional winning set in round $t-1$ and the bidding set in iteration 1. The bidding set in iteration 2 includes all licenses whose marginal contribution is higher than the current MABs. Similarly, in the $k-th$ iteration, we compute the expected marginal contributions of all licenses towards the provisional winning set in round $t-1$ and the bidding set in iteration $k-1$. The bidding set in iteration k includes all those licenses whose marginal contributions are higher than the current MABs. We iterate this sequence until the bidding set converges, and the converging set will be the minimum bidding set of the bidder in round t. This sequence will converge within 734 iterations.

We determine the maximum bidding set in round t using a similar method. The initial bidding set includes all licenses, and we delete licenses from this set sequentially. In any round t and iteration k, we compute the expected marginal contributions of all licenses towards the provisional winning set in round $t-1$ and the bidding set in iteration $k-1$. A bidder's bidding set in iteration k includes all licenses whose marginal contributions are higher than the current MABs.

Bidders submit their bids simultaneously. If there is only one bidder bidding on a license, that bidder will become the provisional winner of the license in this round. When there are multiple bidders bidding on a license, the winner will be randomly determined, and every bidder has an equal probability of winning. Once a bidder becomes the provisional winner, it will stop bidding on the license and will reconsider bidding on this license only after the round when another bidder makes a new bid on the license. The minimum acceptable price of a license increases by 10% if at least one bidder bids on this license in a round. The auction ends when no bidder places a new bid. Withdrawals are not allowed in this simulation. Appendix 8 describes our simulation procedure step by step.

6.5 Counterfactual Simulations

We simulate the benchmark model and four counterfactuals with different package designs 20 times. In each simulation, we redraw a set of the bidder's stand-alone values on all licenses.

In each simulation, we compute both the minimum and maximum bidding sets: a simulation in which all bidders' bidding sets are their minimum bidding sets and a simulation in which all bidders' bidding sets are their maximum bidding sets. We then take the average of different metrics across these simulations and report them in tables. The metrics include the magnitude of the exposure problem, bidder surplus, FCC revenue, and the final license allocations under different package designs.

Benchmark Simulation

In the benchmark simulation, we simulate the performance of 67 bidders bidding on 734 *à la carte* CMA licenses in block B. There is no package offered in the simulated auction.

Pure BEA and REA Packages

We conduct two counterfactual experiments in which we allow only predefined license packages. There are no *à la carte* CMA licenses offered in the simulated auction. In the first counterfactual experiment, we package all CMA licenses into 176 BEA packages.[43] In the second counterfactual experiment, we package all CMA licenses into into 6 REA packages.[44]

Mixed BEA and REA Packages

In the last two counterfactuals, we allow for mixed packages in the auction. Both *à la carte* CMA licenses and predefined (BEA and REA) packages of CMA licenses are for sale. A bidder can assemble *à la carte* CMA licenses and packages together. Suppose that a package contains three licenses: A, B and C. A bidder decides whether to bid on A, B and C separately or to bid on the entire package (let's call it package ABC), based on which yields higher expected payoffs. The FCC decides whether to sell the three licenses as a package or to sell licenses A, B, and C separately. The FCC first evaluates its revenue from selling package ABC and from selling *à la carte* licenses A, B, and C. If the revenue from package

[43]The BEA licenses defined by us in the simulated auction are not exactly the same as the BEA license defined by the FCC. This is because several BEA licenses may share one CMA license based on the FCC delineation. When this happens, we assign a CMA to the larger BEA.

[44]These 6 packages are the REA licenses 1 to 6 sold in block C, Auction 73, which cover the U.S. continent. REA licenses 7 to 12 cover Alaska, Hawaii, Guam and the Northern Mariana Islands, Puerto Rico and the U.S. Virgin Islands, American Samoa, and the Gulf of Mexico.

ABC is higher, the FCC will sell the three licenses as a package. Otherwise, the FCC will sell these licenses separately.

6.6 Simulation Results

6.6.1 The Exposure Problem

Table 8 (9) reports the magnitude of bidders' exposure problem at the end of the auction if all bidders bid on their minimum (maximum) bidding sets in all rounds of the auction. In each panel, the first three rows report, respectively, the magnitudes for large, medium-sized, and small bidders. The last row in each panel reports the summed value of the first three rows. The simulation results in the minimum bidding set and maximum bidding set are quite similar.

Table 8: Exposure Problem in the Last Round: Minimum Bidding Set (in million $)

		Minimum Bidding Set				
		Benchmark	Pure Package		Mixed Package	
		CMA	BEA	REA	BEA	REA
"Overbidding"	Large Bidders	69.66	116.86	157.90	232.17	181.18
	Medium Bidders	1.79	0.03	0.00	1.66	0.00
	Small Bidders	4.88	0.00	0.00	2.78	0.00
	Sum	76.33	116.89	157.90	236.60	181.18
"Underbidding"	Large Bidders	1278.45	2368.91	130.55	251.44	170.60
	Medium Bidders	31.70	444.95	0.00	3.31	0.00
	Small Bidders	52.31	487.20	0.00	5.31	0.00
	Sum	1362.46	3301.07	130.55	260.05	170.60
Total Exposure	Large Bidders	1348.11	2485.78	288.45	483.61	351.78
	Medium Bidders	33.49	444.98	0.00	4.97	0.00
	Small Bidders	57.19	487.20	0.00	8.08	0.00
	Sum	1438.79	3417.96	288.45	496.65	351.78

Package bidding may either alleviate or exacerbate the exposure problem. Package bidding produces two effects on the exposure problem: on the one hand, it eliminates the within-package exposure problem, because bidders will always win all the licenses within a package or none at all. On the other hand, it may increase the between-package exposure problem, because the packages are larger than individual licenses, and the risk of not obtaining several packages together is higher. Only a nationwide license produces no exposure

Table 9: Exposure Problem in the Last Round: Maximum Bidding Set (in million $)

		Benchmark	Maximum Bidding Set Pure Package		Mixed Package	
		CMA	BEA	REA	BEA	REA
"Overbidding"	Large Bidders	81.00	115.45	157.90	309.35	181.18
	Medium Bidders	1.73	0.03	0.00	1.67	0.00
	Small Bidders	5.29	0.00	0.00	3.27	0.00
	Sum	88.02	115.47	157.90	314.29	181.18
"Underbidding"	Large Bidders	1022.81	2336.31	130.55	237.27	170.60
	Medium Bidders	27.87	446.18	0.00	3.92	0.00
	Small Bidders	37.29	487.18	0.00	4.98	0.00
	Sum	1087.97	3269.67	130.55	246.17	170.60
Total Exposure	Large Bidders	1103.81	2451.76	288.45	546.62	351.78
	Medium Bidders	29.60	446.20	0.00	5.59	0.00
	Small Bidders	42.59	487.18	0.00	8.25	0.00
	Sum	1175.99	3385.15	288.45	560.45	351.78

problem. From reported results in Tables 8 and 9, we can see that the effects of package bidding on the exposure problem depend on package format and package size.

Tables 8 and 9, taken together, show that most package bidding regimes alleviate the exposure problem, especially the "underbidding" problem. The effect of the package format (pure package vs. mixed package) seems much larger than the effect of the package size (BEA vs. REA). Under the BEA pure package, in fact, the increase in the exposure problem is almost twofold, suggesting that the between-package exposure problem increases much more than the reduction in the within-package exposure problem. To deal with this problem, going for a larger package (REA) or a mixed package seems to work well: there is a significant reduction in the overall exposure problem under the REA pure package or the BEA mixed package. Under mixed bidding, however, the reduction in "underbidding" exposure risks comes at the cost of sizable increases in "overbidding" exposure risks, indicating more aggressive bidding behavior with a mixed package.

This more aggressive bidder behavior is more evident among large bidders. Different types of bidders bear different burdens when package policies change. Decomposing a bidder's ex-post regret, we can see that the largest chunk of the exposure problem under benchmark simulation comes from the *ex-post* regret stemming from large bidders' premature withdrawal. Package bidding, especially mixed package, changes this scenario drastically. Both tables show that large bidders have a higher "overbidding" exposure problem and a (much) lower "underbidding" exposure problem under a mixed package. This means that large bidders bid more aggressively: by the end of the auction, they will have won some licenses that they did not want to win (thus high "overbidding" regret) and will have lost few licenses that they wanted to win (thus low "underbidding" regret). The change in large bidders' bidding behavior is crucial in determining auction outcomes, as will be discussed in the next two subsections.

6.6.2 Bidder Surplus and FCC Revenue

Table 10 (11) reports the bidders' surplus (in the top panel) and FCC revenue (in the bottom panel) at the end of the auction if all bidders bid on their minimum (maximum) bidding sets in all rounds of the auction. In each panel, the first three rows report the values for large, medium-sized, and small bidders, respectively. The last row in each panel reports the summed value of the first three rows. The bottom row of the table sums up bidder surplus and FCC revenue to report the metric for social surplus under different simulations. Again, the simulation results in the minimum bidding set and maximum bidding set are quite similar.

Table 10: Bidders' Surplus and FCC Revenue: Minimum Bidding Set (in billion $)

		Minimum Bidding Set				
		Benchmark	Pure Package		Mixed Package	
		CMA	BEA	REA	BEA	REA
Bidder Surplus	Large Bidders	11.65	6.16	2.07	6.06	2.26
	Medium Bidders	0.04	0.00	0.00	0.00	0.00
	Small Bidders	0.12	0.04	0.00	0.00	0.00
	Sum 1	11.81	6.20	2.07	6.06	2.26
FCC Revenue	Large Bidders	7.84	12.30	19.28	15.30	19.30
	Medium Bidders	0.17	0.00	0.00	0.02	0.00
	Small Bidders	0.10	0.05	0.00	0.14	0.00
	Sum 2	8.12	12.40	19.28	15.40	19.30
Social Surplus	Sum 1 + Sum 2	19.93	18.60	21.36	21.46	21.56

The FCC revenue in the benchmark simulation ($8.12 to 8.18 billion) is close to the actual FCC revenue in Block B ($9.14 billion). Table 10 and Table 11 show that when the FCC exercises package bidding, the FCC has much to gain, but bidders have much to lose. All types of package bidding, except for BEA pure package, increase total social surplus to some

Table 11: Bidders' Surplus and FCC Revenue: Maximum Bidding Set (in billion $)

		Maximum Bidding Set Benchmark	Pure Package		Mixed Package	
		CMA	BEA	REA	BEA	REA
Bidder Surplus	Large Bidders	11.92	6.24	2.07	5.79	2.26
	Medium Bidders	0.03	0.00	0.00	0.00	0.00
	Small Bidders	0.19	0.04	0.00	0.00	0.00
	Sum 1	12.14	6.28	2.07	5.79	2.26
FCC Revenue	Large Bidders	7.88	12.30	19.28	15.60	19.30
	Medium Bidders	0.17	0.00	0.00	0.02	0.00
	Small Bidders	0.13	0.05	0.00	0.14	0.00
	Sum 2	8.18	12.30	19.28	15.70	19.30
Social Surplus	Sum 1 + Sum 2	20.32	18.58	21.36	21.49	21.56

extent, but the distribution between the FCC and bidders is quite uneven.[45] First, the FCC receives a huge gain in revenue under the package policy than under the benchmark. This is because large bidders bid much more aggressively under a mixed package, especially under the REA package. This aggressive bidding behavior pushes the final prices up, benefiting FCC revenue while hurting bidders, who bear the burden of high prices.

Second, putting bidders and the FCC together, a mixed package fares better than a pure package, as a mixed package leads to lower variances in social surplus. This is because a mixed package endows bidders with a much more flexible tool, allowing them to bid on *à la carte* licenses or packaged licenses, whichever yields a higher expected payoff.

Third, the FCC has much more to gain (and bidders have much to lose) when packages cover large geographic areas. This is consistent with the example in the introduction. When there is substantial heterogeneity in stand-alone values, a package that includes both licenses will wipe out the valuation heterogeneity because a package may contain a license that bidder 1 values a lot but that bidder 2 does not value, as well as another license that bidder 2 values a lot but that bidder 1 does not value. Therefore, package bidding does not award the individual license to the bidder who has the highest value for it. Bidders then compete in bidding for low-value packages, generating low bidder surplus.

Combining results on both bidder surplus and FCC revenue, we find that switching from *à la carte* bidding to mixed package increases social welfare substantially, but the FCC gains from the switch at the expense of bidder surplus. The best package policy seems to be the

[45]The BEA pure package has relatively low social surplus because bidders have the large exposure problem reported in Section 6.6.1.

BEA mixed package, which increases social welfare by more than $1 billion and results in a relatively more even split of social welfare.

6.6.3 Final License Allocation to Different Types of Bidders

Table 12 (13) reports final license allocation at the end of the auction if all bidders bid on the minimum (maximum) bidding set in all rounds. The five columns are the same as those in the last two tables. The first row reports the percentage of packages sold as bundles. The second to the fourth rows report the population-weighted market shares of large, medium-sized, and small bidders, respectively. The fifth row reports the Hirschman Herfindahl Index (HHI) based on population-weighted market shares. The sixth to eighth rows report the unweighted market shares of large, medium-sized, and small bidders, respectively. The last row reports the unweighted market shares.[46]

Table 12: License Allocation: Minimum Bidding Set

	Minimum Bidding Set				
	Benchmark	Pure Package		Mixed Package	
	CMA	BEA	REA	BEA	REA
Percentage of Packages	0.00	100.00	100.00	73.13	100.00
Market Share and HHI (Population Weighted)					
Market Share (Large)	87.86	97.91	100.00	96.21	100.00
Market Share (Medium)	5.14	0.12	0.00	0.83	0.00
Market Share (Small)	7.00	1.97	0.00	2.96	0.00
HHI (population)	6249	4788	6315	8090	5803
Market Share and HHI (Unweighted)					
Market Share (Large)	72.30	96.67	100.00	86.82	100.00
Market Share (Medium)	10.65	0.62	0.00	5.83	0.00
Market Share (Small)	17.06	2.72	0.00	7.35	0.00
HHI (licenses)	3839	5132	6500	6374	6056

Bidders do bid on packages under a mixed package policy. Under the BEA mixed package policy, roughly 73% of licenses are sold as packages. In contrast, under the REA mixed package policy, all licenses are sold as packages, although bidders can choose to bid on individual licenses. Large bidders win more licenses under both mixed and pure package regimes than under the benchmark, while medium-sized and small bidders lose almost all

[46] HHI based on population-weighted market shares is calculated as the sum of squared population shares won by each firm; HHI based on unweighted market shares is calculated as the sum of squared license shares won by each firm.

Table 13: License Allocation: Maximum Bidding Set

	Maximum Bidding Set				
	Benchmark	Pure Package		Mixed Package	
	CMA	BEA	REA	BEA	REA
Percentage of Packages	0.00	100.00	100.00	72.67	100.00
Market Share and HHI (Population Weighted)					
Market Share (Large)	87.27	97.91	100.00	96.22	100.00
Market Share (Medium)	4.30	0.12	0.00	0.81	0.00
Market Share (Small)	8.43	1.97	0.00	2.97	0.00
HHI (population)	6599	4846	6315	8369	5803
Market Share and HHI (Unweighted)					
Market Share (Large)	71.13	96.67	100.00	86.79	100.00
Market Share (Medium)	9.72	0.62	0.00	5.69	0.00
Market Share (Small)	19.15	2.72	0.00	7.52	0.00
HHI (licenses)	3957	5204	6500	6413	6056

market shares when the FCC switches to a package policy. This is because medium-sized and small bidders value complementarity much less than large bidders do, and, as a result, they are not able to compete on price with large bidders under package bidding. Switching from *à la carte* bidding to mixed package, a small number of large bidders win almost all licenses. The implications of these results, as they concern the future competitive landscape (and, in turn, consumer welfare) of the cell phone market, are not included in our short-run welfare assessment at the conclusion of the auction. A key policy goal of the FCC is to create and maintain a competitive marketplace for the telecommunications industries. This goal is embodied by the steep bidding credit the FCC offers to small bidders in spectrum auctions.[47] To achieve this goal, policies favoring small to medium-sized bidders are better than those favoring large, incumbent bidders.

7 Conclusion

In this paper, we evaluate the magnitude of license complementarity and, in turn, the welfare trade-offs of alternative package bidding policies in FCC spectrum auctions. The key to our approach is to construct moment inequalities derived from an incomplete model character-

[47] Another notable example of this goal is the 1996 Telecommunications Act, which aimed to promote competitive entry into the local telephone market. The act eliminated a state's authority to erect legal entry barriers to local telephone markets. The act also required incumbents to offer free interconnections and to lease their network to any new entrant(Fan and Xiao (2015)).

izing bidder behaviors. We find that heterogeneity among bidders plays a large role in their valuation of license complementarity: a large bidder, such as AT&T, values a 1 MHz nationwide license at almost four times more than a medium-sized bidder does. Under a mixed package bidding policy, these large bidders gain more freedom because they can assemble *à la carte* licenses and packages to bid on with less concern about winning only isolated patches of licenses. As a result, they bid more aggressively and wins more licenses, giving the FCC a large revenue boost, while effectively driving medium-sized and small bidders out of license allocation. We recommend that policy makers exercise caution when considering package bidding as a policy remedy. Although having the benefit of reducing bidders' *ex post* regret in most cases, package bidding may favor license allocation to more-powerful players in the telecommunications market, which hampers the development of a more level field for all types of competitors.

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Appendix

1) Bidders Bid on Multiple Licenses in the Same Round

Table A1 reports the number of licenses that a bidder bid on in a given round of all five blocks in Auction 73. We restrict our sample to a sub-sample that starts from round 21 and contains only bidders who placed at least one bid in a round in the license's associated block. There are 5,358 such bidder-block-round observations. As shown in the table, bidders often placed multiple bids in the same round. A bidder bid on a maximum of 197 licenses in the same round. This is clear evidence of bidders' expected complementarity over multiple licenses.

Table A1: A Bidder Places Multiple Bids in a Round

# Licenses	Frequency	Percentage
1	3,219	59.78
2	922	17.12
3 to 5	745	13.83
6 to 10	275	5.11
11 to 20	119	2.21
21 to 197	105	1.95
Total	5,385	100

2) Constructing Cell Phone Tower Variable

The FCC maintains a cell phone tower registration database that records all tower ownership or usage information.[48] We use two data sets from this database: the EN data set and the RA data set. The EN data set records the owner of a tower, whereas the RA data set matches each tower to a county. We count the number of towers a firm has in a county after we merge these two data sets. Lastly, we manually match firm names in the tower registration database with the identity of the bidders in the auction data to obtain the number of towers a bidder has in a county.

A drawback of the data is that ownership changes over time, but the FCC does not keep historical records database. So, what we use in this paper is the tower ownership structure in late 2016, when we started the project. As Auction 73 happened in 2008, there may be changes in the number of towers that a bidder has in a county during the eight-year gap.

[48]The website is `http://wireless.fcc.gov/uls/`.

These changes include: change of ownership, new towers, merger between bidders, etc. Still, these data are the best data we could get to measure bidders' stock of cell phone towers in different counties. We think that this variable is a combination of towers that had been built before Auction 73 and towers that would be built after Auction 73, which is a good measure of bidder heterogeneity.

3) Summary Statistics of Variables Used for Estimation

Table A2: Summary Statistics for Estimation

Panel I: Table (4), Belief Estimation, # obs: 813,616					
Variable	Mean	Median	Std	Min	Max
Complementarity	0.006	0.000	0.038	0	1.41
Complementarity Sq	0.002	0.000	0.037	0	1.99
$\# \, Competitor = 1$	0.775	1.000	0.418	0	1
$\# \, Competitor = 2$	0.004	0.000	0.059	0	1
$\# \, Competitor \geq 3$	0.001	0.000	0.025	0	1
Pop*Bandwidth	0.023	0.008	0.062	0.00	0.65
$\ln(\frac{Pop_c}{Area_c} + 1)$	1.061	0.824	0.835	0.01	4.72
$\ln(\frac{Tower_{ic}}{Area_c} + 1)$	1.433	0.000	3.115	0	10.89
ln Upfront Pay	17.130	17.859	2.875	7.90	20.60
Round	141.000	141.000	69.570	21	261
# Win Round	21.450	0.000	52.209	0	256
Panel II: Table (5), Complementarity Effect Estimation, # obs: 198,997					
	Mean	Median	Std	Min	Max
Price (no bid)	1555728	315750	10600000	4575	937000000
Complementarity Index (no bid)	0.001	0.000	0.003	0.000	0.202
Price (bid)	1151258	277000	8880105	4875	804000000
Complementarity Index (bid)	0.001	0.000	0.003	0.000	0.155
Panel III: Table (6), Stand-alone Value Estimation, # obs: 170,246					
	Mean	Median	Std	Min	Max
Bid this round	0.041	0.000	0.198	0.000	1.000
$\ln(Pop_l)$	-7.584	-7.506	0.996	-12.061	-2.914
$\ln(\frac{Pop_l}{Area_l} + 1)$	0.785	0.625	0.615	0.005	3.917
$\ln(\frac{Tower_{il}}{Area_l} + 1)$	0.904	0.000	2.583	0.000	10.892
$\ln(UpPay_i)$	17.737	17.553	1.669	7.901	20.601
Expected Comple. - price (in Billion $)	-0.003	0.000	0.022	-0.973	0.139

4) Comparison of Different Complementarity Indices

In this paper, we do not incorporate the measure of travel complementarity, as in Fox and Bajari (2013) and Yeo (2009), for the following reasons: first, their measures of license complementarity are at the BTA or CMA level, but our measure is at the county level. If we construct measures of travel complementarity at the county level, there may be substantial bias in the measure of complementarity because there will be high complementarity between counties with an airport, but no complementarity in the counties with no airport even though they are close to an airport. Second, we deem travel complementarity not as important as distance complementarity. This argument is consistent with the empirical findings in Fox and Bajari (2013)'s empirical results, where the two travel complementarity parameters are not significant. Third, Fox and Bajari (2013) have shown that the three complementarity measures (one distance complementarity index and two travel complementarity indices) are highly correlated.

Table A3: Summary of Complementarity by Market Type

		County-level		CMA-level		CMA-level 2	
Market Type		mean	s.d	mean	s.d	mean	s.d
CMA	Within	0.415	2.526	-	-	-	-
	Between	0.003	0.047	0.004	0.067	0.004	0.130
BEA	Within	3.018	7.904	1.999	4.683	4.913	9.140
	Between	0.032	0.232	0.054	0.376	0.033	0.492
REA	Within	136.054	17.143	123.562	39.999	156.386	40.416
	Between	12.245	10.220	16.102	11.705	2.792	3.943
National license	Benchmark	1000		1000		1000	

Note: Complementarity listed in the table is complementarity indices $\times 1000$.

Table A3 compares three measures of market complementarity. All of them are based on the geographic complementarity function in Fox and Bajari (2013) (with $\delta = 2$). Measure 1 is measured at the county level. This is the one we discussed in the main text and used as our empirical input. Measures 2 and 3 are at the CMA level. The difference between measures 2 and 3 is that measure 3 follows Yeo (2009) and treats the distance between CMA l and l' as the minimum distance between two counties within CMA l and CMA l' (These two counties must belong to the same state). These summary statistics show that all three measures are very close to each other.

5) Belief Estimation Result

Table A4 reports estimates of Equation (9). This table is structured the same way as Table 4.

Table A4: Belief Estimation Result

	Bid			Provisional Winner		
	(1) Large	(2) Medium	(3) Small	(4) Large	(5) Medium	(6) Small
Complementarity	15.91***	78.99	329.8***	-10.27***	15.23***	170.5***
	(5.765)	(77.91)	(30.46)	(1.097)	(4.555)	(9.932)
Comple. Sq	-164.5***	-17,490	-16,590***	1.052***	-60.16***	-6,718***
	(53.05)	(18,348)	(3,184)	(0.369)	(17.62)	(740.2)
# $Competitor = 1$	0.194	0.781	-0.647***	-1.192***	-1.194***	-1.007***
	(0.231)	(0.603)	(0.249)	(0.099)	(0.110)	(0.070)
# $Competitor = 2$	-0.071	0.362	-0.940***	-0.555**	-	-1.560***
	(0.253)	(0.615)	(0.259)	(0.228)		(0.236)
# $Competitor \geq 3$	-	0.0414	-0.949***	-1.786**	-0.394	-1.298***
		(0.706)	(0.300)	(0.722)	(0.748)	(0.404)
Pop*Bandwidth	3.968***	5.232*	-81.360***	19.75***	7.283	-0.141
	(1.449)	(2.808)	(7.491)	(2.035)	(4.737)	(3.085)
$\ln(\frac{Pop_c}{Area_c} + 1)$	-0.248***	-0.344***	-0.094**	0.060***	-0.231***	-0.300***
	(0.065)	(0.061)	(0.038)	(0.021)	(0.030)	(0.015)
$\ln(\frac{Tower_{ic}}{Area_c} + 1)$	0.042***	-0.028**	-	0.029***	-0.0278***	-
	(0.010)	(0.013)		(0.003)	(0.006)	
ln(Upfront Pay)	-3.656***	-0.001	0.027***	-2.285***	-0.071***	0.042***
	(0.146)	(0.019)	(0.010)	(0.047)	(0.008)	(0.003)
Round/10	0.006	0.046***	0.012***	0.235***	0.066***	0.022***
	(0.020)	(0.008)	(0.004)	(0.016)	(0.004)	(0.002)
# Win Round				-0.005***	0.017***	0.012***
				(0.002)	(0.001)	(0.259e-3)
Constant	73.88***	-1.165*	0.305	46.15***	1.246***	-0.192***
	(3.008)	(0.658)	(0.287)	(0.968)	(0.121)	(0.056)
# obs	1,706	1,779	3,664	229,654	246,933	329,880

6) Robustness to Table 5

We imposed two behavior assumptions on bidders: **BA1** and **BA2**. **BA1** is straightforward to establish: if a bidder bids on a license, then: 1) the marginal contribution of the license to the set of licenses that the bidder believes it will win is higher than the MAB on the license;

Table A5: Estimates of Complementarity Effect (in million \$)

	(1) Full Sample	(2) Large	(3) Medium	(4) Small
β	14.4	12.8	9.0	14.5
95% CI	[12.1 , 19.0]	[6.6 , 21.1]	[6.1 , 12.9]	[10.1 , 17.3]
# LBs	1057	127	504	426
# Ubs	2053	31	675	1347

Note: The unit of observation is a bidder-license-round-pair. For bidder i, we include a bidding round and not-bidding round as a pair for license l.

and 2) the license is under the bidder's eligibility constraint. **BA2**, however, warrants extra discussion. The actions of "not bid" or "stop bidding" have more possibilities than we allow under the "straightforward" bidder assumption. There are three potential reasons that a bidder chooses not to bid on a license: 1) the marginal contribution of the license is lower than the MAB; 2) the bidding units of the license are higher than the remaining eligibility of the bidder; 3) the bidding units of the license, although lower than the bidder's remaining eligibility, are too low to satisfy the FCC's activity requirement, so the bidder may want to bid on other licenses instead to maintain its eligibility. Our baseline estimation (results shown in Table 5) incorporate both reason (1) and reason (2), but does not consider reason (3). In this robustness check, we add one more restriction: the bidding units of the license need to be higher than the additional units required to maintain the eligibility level of the bidder (but still lower than the unused eligibility of the bidder). Note that this restriction substantially reduces the number of observations used in the estimation, which is probably why we have much wider confidence intervals in Table A5.

7) Expected Complementarity and Bidding Patterns

Table A6 presents the mean and median value of a license's marginal contribution in the expected complementarity index (as defined by Equation 5) when a bidder bids on the license and when a bidder does not bid on the license.

Large bidders have the highest difference between the expected complementarity index when they bid and the expected complementarity index when they do not bid, followed by small bidders. The value for medium-sized bidders is close to zero. These bidding patterns indicate that large bidders are more likely to bid when the gain in expected complementarity

Table A6: A License's Marginal Contribution to the Expected Complementarity Index

(a) When a bidder bids on a license					
	Mean	Median	s.d	min	max
Large	0.0130	0.0066	0.0217	0.0002	0.2857
Medium	0.0007	0.0003	0.0010	0.0000	0.0162
Small	0.0012	0.0005	0.0022	0.0000	0.0694
(b) When a bidder does not bid on a license					
	Mean	Median	s.d	min	max
Large	0.0074	0.0041	0.0141	0.0000	0.2878
Medium	0.0007	0.0004	0.0011	0.0000	0.0109
Small	0.0008	0.0002	0.0017	0.0000	0.0849
(c) Difference between (a) and (b)					
	Mean	Median			
Large	0.0056	0.0025			
Medium	-0.0001	-0.0001			
Small	0.0003	0.0003			

is higher. However, medium-sized bidders' bidding decisions are much less affected by the gain in expected complementarity. This is consistent with our estimates of β_i: large bidders have the highest complementarity, small bidders the second highest, and medium-sized bidders the lowest.

8) Counterfactual Simulation Procedure

This section describes the procedure we use to conduct one iteration of our counterfactual simulation.

Begin: outerloop

Step 1. For each bidder, take random draws of all licenses' stand-alone values from the truncated normal distribution (Details discussed in Section 6.3).

Step 2. We start with round $t = 1$. If $t = 1$, predict the probability of each bidder ultimately winning each license if the license is in its bidding set (there is no provisional winning set). If $t > 1$, calculate the marginal contribution of each license to the complementarity index of its provisional winning set in the last round, and predict the probability of each bidder ultimately winning each license if the license is its bidding set or provisional winning set according to Equation (9).

Step 3. **Begin: Innerloop**: Compute the minimum bidding sets of each bidder. Here, we make use of the supermodularity property in this game and iterate to obtain a bidding set. In any iteration k, when we compute the minimum bidding set of a bidder, we compute the marginal contribution of all licenses not in the current bidding set towards the bidding set in iteration $k-1$ plus its provisional winning set in round $t-1$. We then move all licenses with positive marginal contribution to the current bidding set. We iterate until no licenses outside the bidding set make a positive contribution to the current bidding set. We compute the maximum bidding sets of each bidder similarly(Details discussed in Section 6.4). **End: Innerloop**

Step 4. At the end of round t, each license's provisional winner is determined. When there is only one bid on a license, this bidder becomes the provisional winner of the license. When there are multiple bids on a license, we take a random draw to decide the winner, and all bidders on this license have an equal probability of becoming the provisional winner.

Step 5. Price increases by 10% for round $t+1$ when there is a new bid on this license in round t.

Update $t = t+1$ *and iterate Steps 2-5 until no one places a new bid.*

Step 6. Auction ends if no one places a new bid.

End: outerloop