New-Keynesian Trade: Understanding the Employment and Welfare Effects of Sector-Level Shocks*

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There is a growing empirical consensus suggesting that sector-specific productivity increases in a foreign country can have important unemployment and nonemployment effects across the different regions of a domestic economy. Such employment changes cannot be explained by the workhorse quantitative trade model since it assumes full employment and a perfectly inelastic labor supply curve. In this paper we show how adding downward nominal wage rigidity and home employment allows the quantitative trade model to generate changes in unemployment and nonemployment that match those uncovered by the empirical literature studying the "China Shock." We also compare the associated welfare effects predicted by this model with those in the model without unemployment. We find that the China Shock leads to welfare increases in most states of the U.S., including many that experience unemployment during the transition. On average across U.S. states, nominal rigidities reduce the gains from the China Shock from 36 to 30 basis points.

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1 Introduction

In their influential paper, Autor, Dorn, and Hanson (2013) (henceforth ADH) show that US commuting zones more exposed to the "China shock" suffered significant increases in unemployment and exit from the labor force relative to less exposed regions. In contrast, the standard quantitative trade model assumes full employment and a perfectly inelastic labor supply curve (see e.g., Costinot and Rodriguez-Clare (2014)), implying that all the adjustment takes place through wages rather than employment (see Kim and Vogel (2018)). In this paper we show how adding downward nominal wage rigidity (DNWR) and home employment allows the quantitative trade model to generate changes in unemployment and nonemployment that match those uncovered by ADH during a transition period. We can then also investigate the welfare effects of the China shock on the augmented model with DNWR. The proposed model can also be used to study the labor market and welfare consequences of other shocks such as trade liberalization or the joining of preferential trade agreements.

We start from the standard gravity model of trade with multiple sectors and an input-output structure. We further assume that there are multiple regions inside the United States, that there is no labor mobility across regions or countries, and that labor supply is upward sloping because workers have the option to engage in home production. The way in which the China shock affects employment here is clear: economies with positive net exports in sectors experiencing the strongest productivity increases in China (i.e., more exposed to the China shock) suffer a worsening of their terms of trade, and this leads to a decline in the real wage and employment as some workers exit the labor force to engage in home production. Even with a very large elasticity of substitution between home production and work, however, the model is not able to replicate the strong declines in employment in the U.S. regions most exposed to the China shock documented by ADH, and of course the model cannot generate any changes in unemployment.

We then add DNWR as in Schmitt-Grohe and Uribe (2016) so that the wage in each region can fall by no more than $100(1-\delta)$ percent each year. The implications of DNWR depend on our assumptions regarding monetary policy and the exchange rate system. We assume that all countries except the United States have flexible exchange rates vis-á-vis the dollar, and that the world nominal GDP in dollars grows at a constant and exogenous rate which we set equal to zero without loss of generality. A negative terms-of-trade shock implies a contraction in labor demand, and if δ is low enough then this leads to a temporary increase in unemployment that subsequently falls as nominal wages can adjust downwards. If home production is available to workers, then DNWR will lead to even bigger declines in employment as more workers prefer to exit the labor force rather than face the possibility of unemployment.

We quantify these effects using the "exact-hat algebra" approach to counterfactual analysis popularized by Dekle, Eaton, and Kortum (2007) and extended to a dynamic context by Caliendo, Dvorkin, and Parro (2019). This methodology ensures that our model perfectly matches the sector-level input-output and trade data at the beginning of the period of analysis (year 2000), and makes the calibration more transparent, as we only need to calibrate the trade elasticity, the elasticity of labor supply (κ), and the parameter governing the importance of DNWR (δ).

Our quantitative analysis requires data for sector-level input-output flows as well as bilateral trade flows across all pairs of U.S. regions, China, and the rest of the world. We leverage minimal and transparent assumptions to build such a dataset by combining four primary data sources. We use the World Input-Output Database (WIOD) to obtain country-level bilateral trade flows for each year. We also use the 2002 Commodity Flow Survey (CFS) to construct bilateral trade flows in manufacturing between U.S. states for 2000-2007. We then combine these sources with the 2008 bilateral trade flows between U.S. states and other countries from the U.S. Census to construct their counterparts for year 2000. Finally, we use the Regional Economic Accounts of the Bureau of Economic Analysis (BEA) to obtain state-level production and consumption in services, which we combine with an estimated gravity model for trade in services to infer bilateral trade in services across U.S. states and between states and the other countries. The resulting dataset contains 13 sectors (12 of them in manufacturing and a catch-all services sector) for 50 U.S. states plus 36 additional countries and an aggregate rest of the world during the years 2000 to 2007, the period used in ADH.

Coming to the three key parameters that we need, we pick the trade elasticity from the trade literature, and we calibrate κ and δ so that the model matches two key moments from ADH, namely the way in which more exposed commuting zones experience increases in unemployment rates and decreases in labor-force participation rates. As is common in the literature, we think of the China shock as a productivity improvement that varies across sectors in China, and we follow Caliendo, Dvorkin, and Parro (2019) in calibrating these sector-level productivity shocks so that the model-implied changes in US imports from China match those in the data when projected on the increase in imports from China by other countries similar to the U.S. (in the spirit of ADH). We do this each year so that we can trace out the dynamic response of the economy to the China shock as it unfolded over the period of analysis.

Our calibration leads to a value of δ that is in the ballpark of the value used by Schmitt-Grohe and Uribe (2016) and a value of κ that is high relative to estimates in the labor literature. This is not surprising given that this is the only way in which our model is able to generate the large effects of the China shock on labor-force participation.

The calibrated model generates a significant temporary decline in employment in the regions most exposed to the China shock. For example, we see that Indiana experienced a decline in employment of up to 3.5% over the years 2001-2007, but then went back to a level of employment above the one before the shock. This is the typical dynamic response we see for the most exposed states, and it arises from the combination of three forces. First, the China shock itself is not constant but grows in strength starting in 2001, peaking in 2003, and then becoming negligible by 2008. Second, a shock that requires a decline in the nominal wage to maintain full employment will increase unemployment in the short run under a DNWR, but then this unemployment will erode quickly as the nominal wage can fall around 2% each year. Third, the China shock leads to increases in the real wage for almost all regions, including most of the ones for which full employment would require a decline in the nominal wage.[1] Since the real wage governs labor supply, and since there is no unemployment in the long run, this implies an increase in employment after the economy fully adjusts to the China shock. For the U.S. as a whole, the calibrated model implies that the China shock is responsible for around 0.8 percent-

[1]This implies that the states most exposed to the China shock tend to experience both an increase in the real wage and an increase in unemployment. This may seem paradoxical, but it is a natural consequence of a shock that implies both an improvement in the terms of trade and a decline in the export price index. To see this more clearly, consider a small open economy and imagine that the price index of its exports falls while the price index of its imports falls even more. Since the terms of trade have improved, the real wage and employment would both increase in the absence of nominal frictions. However, the fact that the price index of its exports has fallen requires the nominal wage to also decline, and if this is higher than $1-\delta$, then there would be temporary unemployment.

age points of unemployment in the U.S. in 2004 (this is around a 15% of total unemployment in the U.S. that year, which was 5.4%).

Finally, we study the implications of our model for the welfare effects of the China shock, and we compare our results to those that occur when we remove the DNWR (i.e., when setting $\delta = 0$). We compute welfare as the present discounted value of the utility flow in the future, with a discount rate of 0.97 and a utility flow given by the average real wage across all households in an economy (employed, unemployed, and in home production). Remarkably, welfare increases in almost all regions, including many that experience unemployment during the transition. For the U.S. as a whole, although the China shock remains good for welfare in the presence of nominal frictions, those benefits are smaller with these frictions, specifically DNWR reduces the average U.S. welfare gain from 0.36% to 0.30%. To see how DNWR matters for welfare in some of the most affected states, consider again Indiana. If we compute the welfare effect under the same China shock and same parameters except that we switch off DNWR by setting $\delta = 0$, we see that welfare increases by 3 basis points in Indiana, rather than decreasing by 7 basis points as in the model with DNWR.

Our paper follows in the footsteps of a large literature that analyzes the impacts of trade shocks on different regions or countries. Papers like ADH, Caliendo, Dvorkin, and Parro (2019), Galle, Rodriguez-Clare, and Yi (2017), and Adao, Arkolakis, and Esposito (2019) focus on the effect of the China shock on commuting zones or states in the U.S., but using models without unemployment. There is a large literature exploring the effect of trade on unemployment in models with search and matching frictions, see e.g. Davidson and Matusz (2004), Helpman et al. (2010), Hasan et al. (2012) and Heid and Larch (2016). More recently, Kim and Vogel (2018) introduce search and matching frictions and a labor-leisure choice into a multi-sector trade model where each commuting zone is treated as a small-open economy affected by the China shock. They study how this model can match the ADH findings for the effect of the China shock on income per capita decomposed into the effect on wages, labor supply, and unemployment. We instead focus on DNWR as the friction that generates unemployment, and emphasize the employment and welfare implications of the China shock in a model that allows for intermediate goods and the general-equilibrium implications across U.S. states and between those and the rest of the world.[2]

More closely related to our paper is Eaton, Kortum, and Neiman (2013), which studies the extent to which unmodeled cross-country relative wage rigidities can explain the increases in unemployment and decreases in GDP observed in countries undergoing sudden stops. Relative to this paper, our contribution is to show how DNWR can lead to such relative wage rigidities, to extend the analysis to terms-of-trade shocks in a multi-sector model, and to quantify the effect of the China shock on unemployment and nonemployment across U.S. states over the 2000 - 2007 period.

On the side of open-economy macroeconomics, classic contributions like Clarida, Gali, and Gertler (2002) or various papers by Gali and Monacelli (2005, 2008, 2016) have introduced nominal rigidities in models with trade, Schmitt-Grohe and Uribe (2016) uses a downward nominal wage rigidity to study the effects of trade shocks on a small open economy, Choudhri, Faruqee, and Tokarick (2011) studies the implications of nominal rigidities for the gains from trade in a two-country model, and Nakamura and Steinsson (2014), Beraja, Hurst, and Ospina (2016), or Chodorow-Reich and Wieland (2017) deal with multiple heterogenous regions in a model with nominal rigidities. None of these papers connect to actual sector-level trade flows and hence cannot be used for quantitative analysis for something like the China shock.

The rest of the paper proceeds as follows: Section 2 introduces the general framework that incorporates a rich trade structure with dynamic aspects and nominal rigidities. After introducing the model, this section also discusses equilibrium and exact hat algebra. Section 3 describes the data, the calibration of the China shock, the exposure measure that we will use, and the calibration of parameters δ and κ. Section 4 describes our main results and Section 5 concludes.

[2] Pessoa (2016) allows for search and matching in a fully dynamic multi-sector model and explores the effects on workers originally employed in sectors differently exposed to the China shock. The paper does not explore the aggregate effects on employment and unemployment, or how such effects matter for welfare relative to a model without unemployment.

2 A Quantitative Trade Model with Nominal Rigidities

We present a multi-sector quantitative trade model with an input-output structure as in Caliendo and Parro (2015), but extended to allow for multiple periods, an upward sloping labor supply, and downward nominal wage rigidity. We assume that the United States is composed of multiple regions. Since our intention here is to focus on the role of nominal rigidities in affecting employment, we assume that there is no labor mobility across those regions, which is in any case a reasonable assumption given our focus on the short to medium term.

2.1 Basic Assumptions

Our presentation of the consumption, production, and trade sides of the model will be brief, since this is well known. There are M regions in the U.S., plus $I - M$ regions outside of the U.S. (for a total of I regions). There are S sectors in the economy (indexed by s or k). In each region (indexed by i or j) and each period, a representative consumer devotes all income to expenditure $P_{j,t}C_{j,t}$, where $C_{j,t}$ and $P_{j,t}$ are aggregate consumption and the price index in region j in period t, respectively. Aggregate consumption is a Cobb-Douglas aggregate of consumption across the S different sectors with expenditure shares $\alpha_{j,s}$. As in a multi-sector Armington trade model, consumption in each sector is a CES aggregate of the consumption of the good of each of the I regions, with elasticity of substitution $\sigma_s > 1$ in sector s.

Each region produces good k with a Cobb-Douglas production function, using labor (with share $\phi_{j,k}$) and intermediates inputs from all sectors (with the share of intermediate inputs coming from sector s denoted by $\phi_{j,sk}$), with $\phi_{j,k} + \sum_s \phi_{j,sk} = 1$. Under perfect competition, given iceberg trade costs $\tau_{ij,k,t} \geq 1$, assuming that intermediates are aggregated in the same way as consumption goods, and letting $W_{i,t}$ denote the wage in region i at time t, the price in country j of good k produced by i at time t is $\tau_{ij,k,t} A_{i,k,t}^{-1} W_{i,t}^{\phi_{i,k}} \prod_s P_{i,s,t}^{\phi_{i,sk}}$, where $P_{i,s,t}$ is the price index of sector s in country i at time t and is given by z

$$P_{j,k,t}^{1-\sigma_k} = \sum_{i=1}^{I} \left(\tau_{ij,k,t} A_{i,k,t}^{-1} W_{i,t}^{\phi_{i,k}} \prod_{s=1}^{S} P_{i,s,t}^{\phi_{i,sk}} \right)^{1-\sigma_k} . \tag{1}$$

For future purposes, note also that

$$P_{i,t} = \prod_{s=1}^{S} P_{i,s,t}^{\alpha_{i,s}}. \tag{2}$$

In Appendix B we provide more details on this model.

2.2 Labor Supply and Downward Nominal Wage Rigidity

We denote the total population of region i with $\overline{L}_i$ (we assume this doesn't vary with time because of the short time ranges we will deal with). Agents can either stay at home or look for work in the market sector. We assume that home production has a utility flow of μ_i (also does not vary with time), while the market option offers an expected real income of $\omega_{i,t}$. We denote the number of agents that look for work in the market sector with $\ell_{i,t}$. There are independent draws for the individual's preference to stay home or work that come from a Frechet distribution with shape parameter κ. The share of people looking for work in the market is then

$$\pi_{i,t} \equiv \ell_{i,t}/\overline{L}_i = \omega_{i,t}^{\kappa}/(\mu_i^{\kappa} + \omega_{i,t}^{\kappa}), \tag{3}$$

while ex-ante instantaneous utility (before the shock is realized) is

$$u_{i,t} \propto \left(\mu_i^{\kappa} + \omega_{i,t}^{\kappa}\right)^{1/\kappa}.$$

We denote the number of agents that are actually employed in region i at time t by $L_{i,t}$. In the standard trade model, labor market clearing requires that the sum of labor used across sectors in a region be equal to labor supply, $L_{i,t} \equiv \sum_{s=1}^{S} L_{i,s,t} = \ell_{i,t}$. We depart from the standard model and instead follow Schmitt-Grohe and Uribe (2016) by assuming that there is downward nominal wage rigidity (DNWR), which might lead to an employment level that is strictly below labor supply,

$$L_{i,t} \leq \ell_{i,t}. \tag{4}$$

All prices and wages up to now are expressed in U.S. dollars. In contrast, the downward nominal wage rigidity of a region is in terms of its local currency unit. Letting $W_{i,t}^{LCU}$ denote the wage of region i at time t in local currency units, the DNWR takes the following form:

$$W_{i,t}^{LCU} \geq \delta W_{i,t-1}^{LCU}, \qquad \delta \geq 0.$$

Denote the exchange rate between the local currency unit of region i and the local currency unit of region 1 (which is the U.S. dollar) in period t with $E_{i,t}$ (this is given in dollars per local currency units of region i). This implies that $W_{i,t} = W_{i,t}^{LCU} E_{i,t}$, and hence the DNWR in dollars entails

$$W_{i,t} \;\; \geq \;\; \frac{E_{i,t}}{E_{i,t-1}} \delta W_{i,t-1}.$$

Since all regions within the U.S. share the dollar as their local currency unit, then $E_{i,t} = 1$ and $W_{i,t}^{LCU} = W_{i,t} \; \forall \; i \leq M$. This means that the DNWR in states of the U.S. takes the familiar form $W_{i,t} \geq \delta W_{i,t-1}$. For the $I - M$ regions outside of the U.S., the LCU is not the dollar and so the behavior of the exchange rate will affect how the DNWR affects the real economy. We assume that the exchange rate against the dollar is fully flexible in all countries outside the U.S. This implies that those countries have no DNWR in terms of dollars. The DNWR in dollars can then be simply captured by

$$W_{i,t} \geq \delta_i W_{i,t-1}, \qquad \delta_i \geq 0, \tag{5}$$

with

$$\delta_i = \delta \quad \forall \; i \leq M \qquad \text{and} \qquad \delta_i = 0 \quad \forall \; i > M.$$

Besides equations (4) and (5), we additionally have the complementary slackness condition:

$$(\ell_{i,t} - L_{i,t})(W_{i,t} - \delta_i W_{i,t-1}) = 0. \tag{6}$$

Since we know that people in the market sector get the real wage of $W_{i,t}/P_{i,t}$ with probability

$L_{i,t}/\ell_{i,t}$ we can express the real income from working in the market sector as

$$\omega_{i,t} = \frac{W_{i,t}L_{i,t}}{P_{i,t}\ell_{i,t}}. \tag{7}$$

2.3 Nominal Anchor

So far we have introduced nominal elements to the model (i.e. the DNWR), but we haven't introduced a nominal anchor that constraints or determines nominal quantities and prevents nominal wages from rising so much in each period as too make the DNWR always non-binding. The idea here is that each country has a central bank that is not willing to allow inflation to be too high, because inflation is costly (for reasons left out of the model). In traditional macro models this is usually implemented via a Taylor rule, where the nominal interest rate reacts to inflation in order to keep price growth in check. We instead use a nominal anchor that captures the same idea in a way that naturally lends itself to quantitative implementation.

In particular, we assume that world nominal GDP in dollars grows at a constant rate across years,

$$\sum_{i=1}^{I} W_{i,t}L_{i,t} = \gamma \sum_{i=1}^{I} W_{i,t-1}L_{i,t-1}. \tag{8}$$

This says that world aggregate demand in dollars grows at a gross rate of γ. Although this might seem far from realistic, it nonetheless has some desirable properties: it can lead to unemployment even in the context of two countries that have a single region each; it can be seen as capturing a fixed level of aggregate demand in the context of a global liquidity trap; it can motivate "currency wars" since countries might want to manipulate their exchange rate to bring aggregate demand to their home country; and it will lead to particularly nice properties for the solution algorithm that we will use. This rule can also be seen as indicating that one of the countries (the U.S. in our specification) has a nominal GDP targeting rule where the target is for world GDP instead of just for U.S. GDP.

2.4 Equilibrium

Letting $R_{i,s,t}$ denote total revenues in sector s of country i, noting that the demand of industry k of country j of intermediates from sector s is $\phi_{j,sk} R_{j,k,t}$, and allowing for exogenous deficits as in Dekle, Eaton, and Kortum (2007), the market clearing condition for sector s in country i can be written as

$$R_{i,s,t} = \sum_{j=1}^{I} \lambda_{ij,s,t} \left(\alpha_{j,s} \left(W_{j,t} L_{j,t} + D_{j,t} \right) + \sum_{k=1}^{S} \phi_{j,sk} R_{j,k,t} \right), \tag{9}$$

where

$$\lambda_{ij,k,t} \equiv \frac{(\tau_{ij,k,t} A_{i,k,t}^{-1} W_{i,t}^{\phi_{i,k}} \prod_{s=1}^{S} P_{i,s,t}^{\phi_{i,sk}})^{1-\sigma_k}}{\sum_{r=1}^{I} (\tau_{rj,k,t} A_{r,k,t}^{-1} W_{r,t}^{\phi_{r,k}} \prod_{s=1}^{S} P_{r,s,t}^{\phi_{r,sk}})^{1-\sigma_k}}, \tag{10}$$

are sector-k trade shares in period t and $D_{j,t}$ are transfers received by region j, with $\sum_j D_{j,t} = 0$. In turn, labor market clearing in each region requires that

$$W_{i,t} L_{i,t} = \sum_{s=1}^{S} \phi_{i,s} R_{i,s,t}. \tag{11}$$

Given last-period wages $\{W_{i,t-1}\}$ and last period employment $\{L_{i,t-1}\}$, the period t equilibrium is a set of wages $\{W_{i,t}\}$, employment $\{L_{i,t}\}$, trade shares $\{\lambda_{ij,s,t}\}$, country and sector-country prices indices $\{P_{i,t}\}$ and $\{P_{i,s,t}\}$, and revenues $\{R_{i,s,t}\}$ such that equations (1) - (11) hold, with $\delta_i = \delta$ for $i \leq M$ (U.S. regions) and $\delta_i = 0$ for all $i > M$ (other countries).

2.5 Discussion

Consider a shock that requires the relative wage of some region i to fall to maintain full employment in that region. This could be for example a negative productivity shock, an increase in productivity abroad, or a decline in transfers to the region. If δ is low enough or the exchange rate can depreciate (e.g., $\delta_i = 0$) then wages can adjust downwards in the required magnitude without causing unemployment, while if γ is high enough then again there would be no unemployment, since no downward adjustment is needed in the wage. However, there are combinations of parameters δ_i and γ that will lead to unemployment after the shock, although there would then be a decline in unemployment towards zero as the DNWR and the nominal anchor allow for adjustment year after year.

We clarify that having multiple regions is not critical for the shock to lead to unemployment given the particular form of our nominal anchor. To see this, imagine that the U.S. was composed of a single region and consider a shock as above for that region. If γ was high enough then the adjustment could take place without unemployment in the U.S. since wages in dollars in the rest of the world could increase enough to generate the necessary relative wage adjustment. However, if γ is low and δ is high, this full adjustment would not be possible and there would be (temporary) unemployment in the U.S.

With multiple regions in the U.S. we could also have unemployment after a shock with the more natural nominal anchor rule that simply imposed that

$$\sum_{i=1}^{M} W_{i,t} L_{i,t} = \gamma \sum_{i=1}^{M} W_{i,t-1} L_{i,t-1}.$$

Imagine that there was a shock that affected only one of the regions in the U.S., requiring that the wage in that region fall relative to the wages of the other regions. With a low enough γ and a high enough δ, this cannot take place, and so there would be (temporary) unemployment after the shock.

2.6 Hat Algebra

Our goal is to use a calibrated version of the model above to compute the welfare effects of a trade shock or the closing of a country's trade deficit. We want to do this using actual data for U.S. states as well as outside countries, but without having to calibrate technology levels and iceberg trade costs along the transition and without requiring data on nominal wages or available labor (since this would require taking a stance on what efficiency units we are measuring things in). To do so, we follow the exact hat algebra methodology of Dekle, Eaton, and Kortum (2007) and the extension of that methodology to dynamic settings proposed in Caliendo, Dvorkin, and Parro (2019). Our counterfactual exercises then only require data on nominal GDP, $Y_{i,t} \equiv W_{i,t} L_{i,t}$, trade deficits, $D_{i,t}$, revenues, $R_{i,s,t}$, the fraction of workers in the market sector $\pi_{i,t}$, and trade shares $\lambda_{ij,s,t}$ at time zero, $t = t_0$, whatever shocks we are interested in, and the model's parameters, namely δ, γ, κ, $\{\alpha_s\}$, $\{\phi_{i,s}\}$, and $\{\phi_{i,sk}\}$.

We use the variable $\hat{x}_t$ to denote x_t/x_{t-1} for any variable x. To express the equilibrium system in hats and only leave it in terms of observable data in period zero (when we assume the economy was in a steady state where every country had full employment) we follow a process described in Appendix C. There we show that the equilibrium system in hats is given by:

$$\hat{R}_{i,s,t}R_{i,s,t-1} = \sum_{j=1}^{I} \hat{\lambda}_{ij,s,t}\lambda_{ij,s,t-1}\left(\alpha_{j,s}\left(\hat{W}_{j,t}\hat{L}_{j,t}Y_{j,t-1} + \hat{D}_{j,t}D_{j,t-1}\right) + \sum_{k=1}^{S}\phi_{j,sk}\hat{R}_{j,k,t}R_{j,k,t-1}\right) \quad \forall\, i, \forall\, s$$

$$\hat{\lambda}_{ij,s,t} = \frac{(\hat{\tau}_{ij,s,t}\hat{A}_{i,s,t}^{-1}\hat{W}_{i,t}^{\phi_{i,s}}\prod_{k=1}^{S}\hat{P}_{i,k,t}^{\phi_{i,ks}})^{1-\sigma_s}}{\sum_{r=1}^{I}\lambda_{rj,s,t-1}(\hat{\tau}_{rj,s,t}\hat{A}_{r,s,t}^{-1}\hat{W}_{r,t}^{\phi_{r,s}}\prod_{k=1}^{S}\hat{P}_{r,k,t}^{\phi_{r,ks}})^{1-\sigma_s}} \quad \forall\, i, \forall\, s$$

$$\hat{P}_{i,s,t}^{1-\sigma_s} = \sum_{j=1}^{I}\lambda_{ji,s,t-1}\left(\hat{\tau}_{ji,s,t}\hat{A}_{j,s,t}^{-1}\hat{W}_{j,t}^{\phi_{j,s}}\prod_{k=1}^{S}\hat{P}_{j,k,t}^{\phi_{j,ks}}\right)^{1-\sigma_s} \quad \forall\, i, \forall\, s$$

$$\hat{P}_{i,t} = \prod_{s=1}^{S}\hat{P}_{i,s,t}^{\alpha_{i,s}}, \quad \forall\, i$$

$$\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1} = \sum_{s=1}^{S}\phi_{i,s}\hat{R}_{i,s,t}R_{i,s,t-1} \quad \forall\, i$$

$$\prod_{q=1}^{t}\hat{L}_{i,q} \le \prod_{q=1}^{t}\hat{\ell}_{i,q}, \hat{W}_{i,t} \ge \delta_i, CS \quad \forall\, i$$

$$\hat{\ell}_{i,t} = \frac{(\hat{W}_{i,t}\hat{L}_{i,t}/(\hat{P}_{i,t}\hat{\ell}_{i,t}))^{\kappa}}{1-\pi_{i,t-1}+\pi_{i,t-1}(\hat{W}_{i,t}\hat{L}_{i,t}/(\hat{P}_{i,t}\hat{\ell}_{i,t}))^{\kappa}}, \quad \forall\, i$$

$$\gamma\sum_{i=1}^{I}Y_{i,t-1} = \sum_{i=1}^{I}\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1}. \quad \text{single}$$

For each period t this is a system of equations which we can use to solve for the quantities that we care about ($\hat{R}_{i,s,t}$, $\hat{\lambda}_{ij,s,t}$, and $\hat{P}_{i,s,t}$ for all i and s, and $\hat{P}_{i,t}$, $\hat{W}_{i,t}$, $\hat{L}_{i,t}$, and $\hat{\ell}_{i,t}$ for all i) given the objects that we already know from the previous period ($Y_{i,t-1}$, $\lambda_{ij,s,t-1}$, $D_{i,t-1}$, $R_{i,s,t-1}$, $\pi_{i,t-1}$, $\{\hat{\ell}_{i,q}\}_{q=1}^{t-1}$ and $\{\hat{L}_{i,q}\}_{q=1}^{t-1}$ for all i, j, s) and the time t shocks ($\hat{A}_{i,s,t}$, $\hat{D}_{i,t}$ and $\hat{\tau}_{ij,s,t}$ for all i, j, s). Thus, starting at $t = 1$ we can solve this system with information on $Y_{i,0}$, $\lambda_{ij,s,0}$, $D_{i,0}$, $R_{i,s,0}$ and $\pi_{i,0}$ for all i, j, s (assuming that we leave a steady state where $L_{i,0} = \ell_{i,0}$) and the shocks ($\hat{A}_{i,s,1}$, $\hat{D}_{i,1}$ and $\hat{\tau}_{ij,s,1}$ for all i, j, s) and obtain $\hat{W}_{i,1}$ and $\hat{L}_{i,1}$ for all i, from these we can also obtain $Y_{i,1}$, $\lambda_{ij,s,1}$, $D_{i,1}$,

$R_{i,s,1}$, $\pi_{i,1}$ and $\ell_{i,1}$ for all i, j, s. Then we can move forward to period 2 and solve for $\hat{W}_{i,2}$ and $\hat{L}_{i,2}$ for all i. We can keep doing this process to solve the system forward while requiring only period zero information and the shocks hitting the economy.

Besides being interested in employment effects, we are also interested in the welfare effects of the China shock. With our current setup we can express the change in instantaneous utility as

$$\hat{u}_{i,t} = \left(1 - \pi_{i,t-1} + \pi_{i,t-1}(\hat{W}_{i,t}\hat{L}_{i,t}/(\hat{P}_{i,t}\hat{\ell}_{i,t}))^{\kappa}\right)^{1/\kappa}.$$

We will combine this expression for instantaneous utility with a standard lifetime utility function which is time-separable with discount factor β.

3 Data and Calibration

3.1 Data Description

We use trade and production data for 50 U.S. states, 36 additional countries, and an aggregate rest of the World region, for a total of 87 regions. We consider 13 sectors, of which, the first 12 are manufacturing sectors classified according to the North American Industry Classification System (NAICS). The last sector puts together all service sectors. We provide a brief description of the data here and relegate the details to Appendix A.

The international country-level data for all 13 sectors comes from the World Input-Output Database (WIOD). We follow Caliendo, Dvorkin, and Parro (2019) to construct the bilateral manufacturing trade flows between U.S. states for 2000 by combining WIOD and the 2002 Commodity Flow Survey (CFS). To do this, we first compute the bilateral expenditure shares across regions and sectors from the 2002 CFS. Next, we assign the total U.S. domestic sales from WIOD according to the bilateral shares calculated in the first step. This way, the bilateral trade flows matrix for the 50 U.S. states would match the total U.S. domestic sales from WIOD in each sector.

We use U.S. Census data to compute the sector-level bilateral trade flows between U.S. states and other countries for the 12 manufacturing sectors. While the U.S. Census data on sector-by-state-by-country bilateral trade starts in 2008, we use this year to project our bilateral trade matrix for 2000. In particular, we assume that the importance of each state in the total exports (imports) to (from) other countries in each sector remains constant at the 2008 levels. We provide more details on this in Appendix A.2.

Finally, we use information on region-level production and consumption in services together with bilateral distances to construct a matrix of bilateral trade flows in services consistent with a gravity structure. The details of this procedure are explained in Appendix A.1.4.

The trade elasticity σ_s is assumed to be constant across sectors and to take the value of 6, consistent with the trade literature. For inter-temporal comparisons, when computing welfare, we use a discount factor β of 0.97 (at the annual level).

3.2 Calibration of the China Shock

We calibrate Chinese productivity changes to match the predicted changes in import values from China to the U.S., as in Caliendo, Dvorkin, and Parro (2019) or Galle, Rodriguez-Clare, and Yi (2017). By "predicted" we mean that we do not use the actual changes in import values from China to the U.S., but the predicted values based on the changes in import values from China to the other high income countries used in ADH.[3]

We need to calibrate $\hat{A}_{China,s,t}$ for $s = 1, ..., 12$ and $t = 2001, ..., 2007$; these are 84 parameters. Calibrating these 84 parameters individually would add too much noise. Instead, we decompose the total productivity shock in sector s and time t into a component coming from a sectoral productivity increase that is constant from 2000 to 2007 and a component coming from a time productivity increase that is constant across sectors, i.e. $\hat{A}_{China,s,t} = \hat{A}^1_{China,s}\hat{A}^2_{China,t}$. This means we have to estimate only 19 parameters instead of 84.[4] We choose these 19 parameters in order to match 2 types of targets.

The first type of target we have relates to total changes in imports from China to the U.S. between 2000 and 2007 in a given sector. As mentioned above, we don't want to use the actual

[3] To be precise, we use the subset of the ADH countries that are available in the 2013 version of the WIOD, namely Australia, Germany, Denmark, Spain, Finland, and Japan. New Zealand and Switzerland are included in the "other country" category of ADH but are not included in WIOD.

[4] Actually the 19 parameters condense to 18 parameters, since the multiplicative nature of our two components means that the level is not well identified (for example if all the $\hat{A}^1_{China,s}$ were multiplied by k and all the $\hat{A}^2_{China,t}$ were divided by k, all the $\hat{A}_{China,s,t}$ would stay the same). We solve this by setting $\hat{A}^2_{China,2003} = 1$.

changes in imports in the U.S., we want to use the predicted values instead. We obtain these predicted values from the following regression:

$$\Delta M_{US,C,s}^{2007-2000} = \beta \Delta M_{OC,C,s}^{2007-2000} + \varepsilon_s,$$

where $\Delta M_{US,C,s}^{2007-2000}$ is the change in imports from China to the U.S. between 2000 and 2007 in sector s, $\Delta M_{OC,C,s}^{2007-2000}$ is the change in imports from China to the other high-income countries between 2000 and 2007 in sector s, and β is the coefficient of interest. This is a regression with 12 data points, but it has a very high R^2.[5] We denote the predicted values from this regression by $\Delta \hat{M}_{US,C,s}^{2007-2000}$.

The second type of target we have relates to total changes in imports from China to the U.S. across two given years for all manufacturing sectors. Once again, we don't want to use actual changes, so we obtain the predicted changes from the following regression:

$$\Delta M_{US,C,t} = a + b \Delta M_{OC,C,t} + \varepsilon_t,$$

where $\Delta M_{US,C,t}$ is the change in imports from China to the U.S. between year $t-1$ and year t in all manufacturing sectors, $\Delta M_{OC,C,t}$ is the change in imports from China to the other high-income countries between year $t-1$ and year t in all manufacturing sectors, and b is the coefficient of interest.[6] This is a regression with just seven data points but, it still has a high R^2. We denote the predicted values from this regression by $\Delta \hat{M}_{US,C,t}$.

We pick the $\hat{A}^1_{China,s}$ and $\hat{A}^2_{China,t}$ such that the total productivity changes in China $\hat{A}_{China,s,t} = \hat{A}^1_{China,s} \hat{A}^2_{China,t}$ deliver changes in imports in our model that simultaneously match the 12 values of $\Delta \hat{M}_{US,C,s}^{2007-2000}$ and the 7 values of $\Delta \hat{M}_{US,C,t}$.[7]

[5]We do not include a constant in this regression because it can lead to certain sectors needing to import a (gross) negative amount from China, which is impossible.

[6]In this regression including a constant a doesn't introduce any complications.

[7]Since the sum of all the $\Delta \hat{M}_{US,C,s}^{2007-2000}$ has to equal the sum of all the $\Delta \hat{M}_{US,C,t}$, this means that instead of 19 targets we only really have 18 targets, which corresponds well with footnote 4.

3.3 Exposure to China

We use a model-consistent measure of exposure to China that is analogous to the one proposed by ADH. We define:

$$\text{Exposure}_i \equiv \sum_{s=1}^{S} \frac{VA_{i,s,2000}}{VA_{i,2000}} \frac{\Delta_{2000}^{2007} M_{C-Other,s}}{Y_{US,s,2000}},$$

where $VA_{i,s,2000}$ is the value added of state i in sector s in year 2000. This value can be taken directly from the U.S. Bureau of Economic Analysis (BEA). $Y_{US,s,2000}$ is total U.S. production in sector s in year 2000, which is taken directly from the WIOD Database. Finally, $\Delta_{2000}^{2007} M_{C-Other,s}$ is the change in imports from China to other high-income countries from 2000 to 2007 in sector s. This is taken directly from WIOD as well.[8]

The main difference between our empirical exposure measure and the one in ADH is the use of value added shares instead of employment shares as weights. However, this is internally consistent with our model for two reasons. The first one is that since labor is the only factor of production, the value added is equal to the contribution of labor. This means that $VA_{i,s,2000} = W_{i,s,2000} L_{i,s,2000}$. Second, since workers are mobile across sectors we have that $W_{i,s,2000} = W_{i,2000}\ \forall s$. These points together imply that $\frac{L_{i,s,2000}}{L_{i,s,2000}} = \frac{VA_{i,s,2000}}{VA_{i,s,2000}}$. Moreover, we renormalize our exposure measure to have the same mean as the measure in ADH for comparability purposes.

3.4 Calibration of DNWR and Labor Supply Elasticity

Parameters δ (governing the amount of downward nominal wage rigidity), κ (governing labor supply choice), and γ (governing the nominal anchor), will be important in our model. Parameters γ and δ however, are somewhat redundant, since what matters is their relative value. Hence, we will assume that γ is 1, and put the burden of adjustment on δ, following a procedure similar to the one in Schmitt-Grohe and Uribe (2016).

We want to choose δ and κ simultaneously to match the empirical estimates obtained in ADH regarding the effects of exposure to China on both unemployment and labor force par-

[8] Here we focus again on the other six high-income countries available in WIOD.

ticipation.[9] ADH find a 0.221 increase in unemployment and a 0.553 decrease in labor force participation (during the 2000-2007 period) for each additional $1000 of exposure to China. Intuitively, we can imagine that δ governs the amount of unemployment generated by exposure to China for a given κ. Hence, by increasing δ we can increase the effect of exposure to China on unemployment. Similarly, we can imagine that κ governs the fall in the labor force generated by exposure to China for a given δ. Hence, by increasing κ we can increase the effect of exposure to China on labor force participation.[10]

By following this calibration process we obtain the values $\delta = 0.982$ and $\kappa = 7.2$. Our estimate for δ falls squarely in the range advocated by Schmitt-Grohe and Uribe (2016) who obtain an annual δ of 0.984 (after "normalizing" γ to one like we do).[11] This estimate implies that wages can fall up to 1.8% annually (or 0.45% quarterly) without generating unemployment.

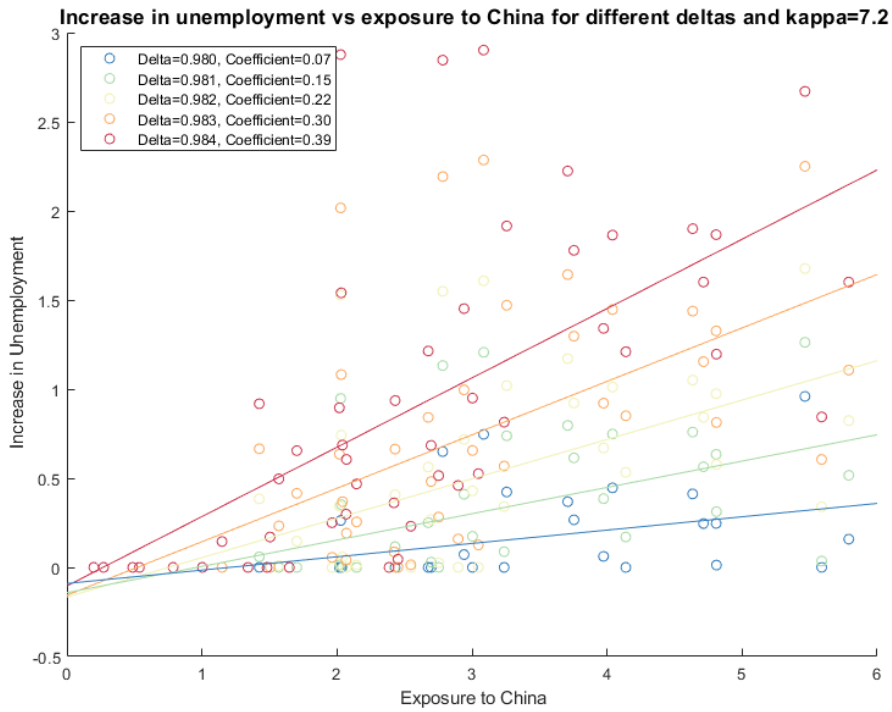
In Figure 1 we provide some illustration of how the identification of δ and κ works. Panel (a) of that figure shows a scatterplot of the increase in unemployment against the exposure to China for the calibrated level of $\kappa = 7.2$ and for different levels of δ. We can see that a higher δ leads to a higher slope in the regression of unemployment on exposure to China (the coefficient is reported in the legend for convenience). For the calibrated parameter value of $\delta = 0.982$ the coefficient obtained in the regression is 0.22, which is the target that we obtained from ADH. Similarly, panel (b) of Figure 1 shows a scatterplot of the decrease in labor force participation against the exposure to China for the calibrated level of $\delta = 0.982$ and for different levels of κ. We can see that bigger κ's lead to a bigger slope in the regression of labor force participation on exposure to China (the coefficients are also reported in the legend). For the calibrated parameter value of $\kappa = 7.2$ the coefficient obtained in the regression is 0.55, which is the target that we obtained from ADH.

Table 1 reproduces some of the empirical estimates in ADH (in particular the ones displayed in their table 5, panel B, for all education levels) and compares them with the counterpart of these estimates coming from the model. Columns (3) and (4) just illustrate the fact

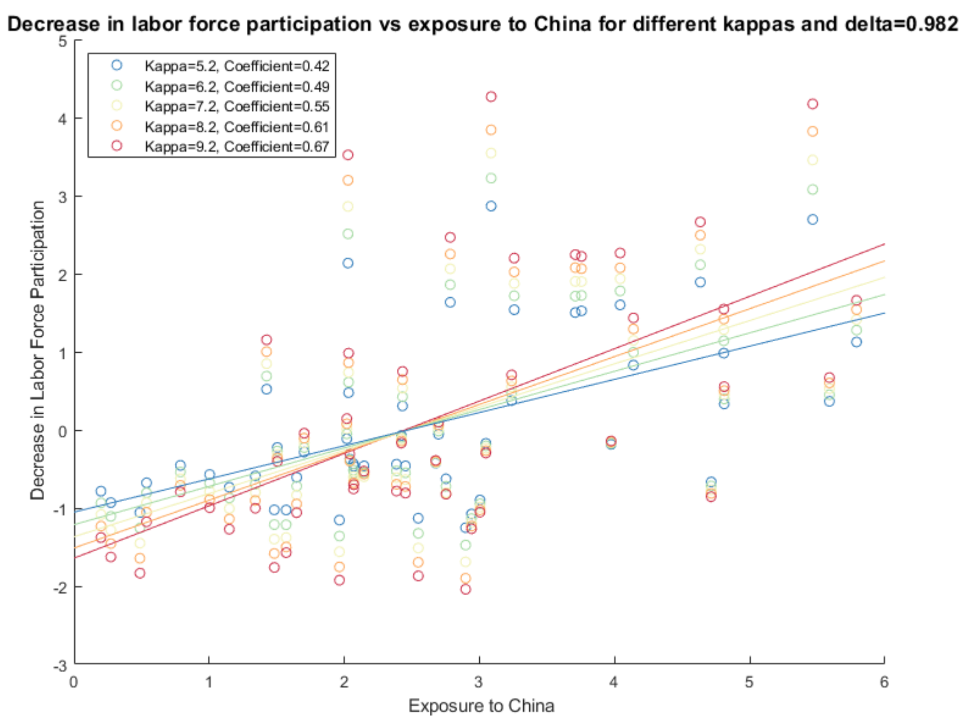
[9] The way we measure unemployment and labor force participation, which are the target variables in our identification, is as averages of the 2006-2008 values. We do this in order to be consistent with ADH.

[10] This intuition only works to a first approximation, since in reality both δ and κ affect both coefficients. But it is undeniable that δ has a stronger impact on the unemployment effect while κ has a stronger impact on the labor force participation effect.

[11] Schmitt-Grohe and Uribe (2016) obtain a quarterly value of δ of 0.996, which would correspond to an annual value of 0.984. However, they end up using a delta of 0.96 in their paper to be conservative.

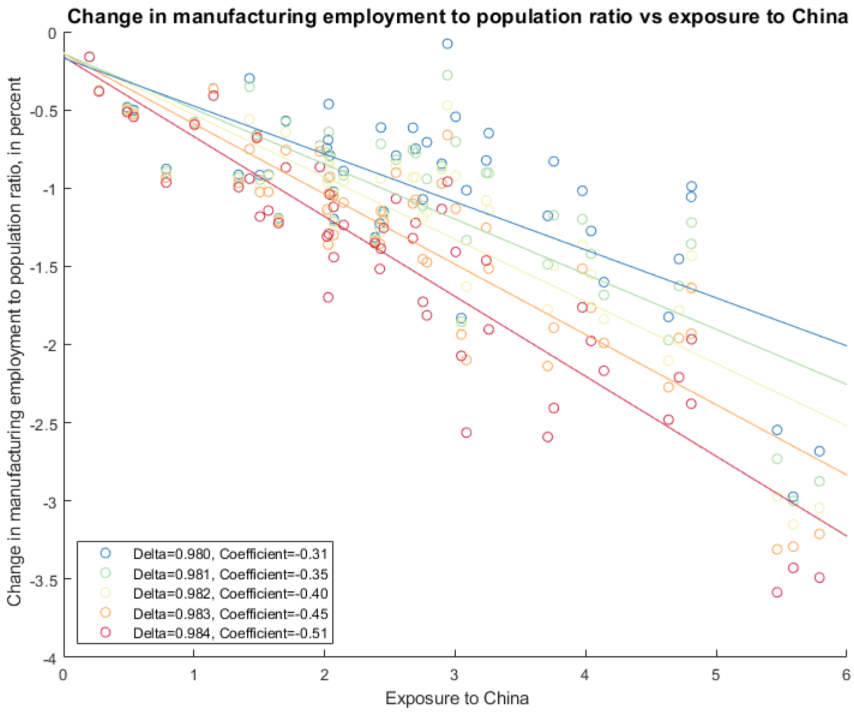


(a) Identification of δ

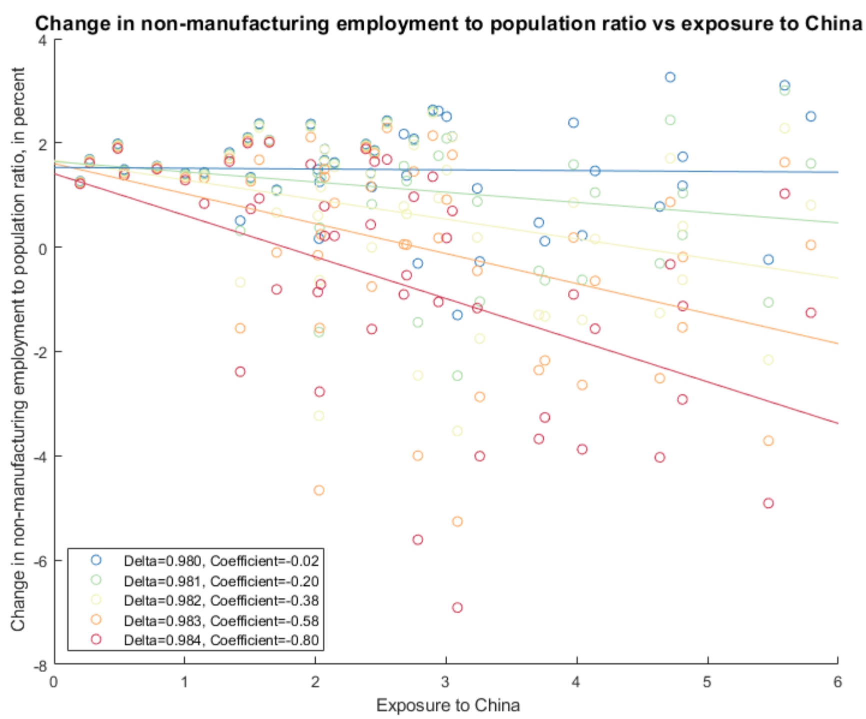


(b) Identification of κ

Figure 1: Illustration of the Identification



(a) Manufacturing Employment



(b) Non-Manufacturing Employment

Figure 2: Sectoral Employment vs Exposure to China

Table 1: Exposure to China and Employment Effects

	Mfg. emp. (1)	Non-mfg. emp. (2)	Unemployment (3)	Not in L. Force (4)
ADH empirical results	-0.596	-0.178	0.221	0.553
RUV model results	-0.397	-0.377	0.221	0.553

Notes: This table compares the employment effects obtained in ADH (table 5, panel b, all education levels) with the ones obtained in our model. The employment effects represent the 2007-2000 change in a given employment category regressed on exposure to China. The exposure measure is very similar in both papers and has the same mean (2.63 thousands of dollars of imports from china per worker). The 2007-2000 changes are converted into decadal changes by multiplying by 10/7.

that we have targeted these moments in ADH and so we obtain the same coefficients as they do. However, ADH also obtain the effects of exposure to China on manufacturing and non-manufacturing employment (columns 1 and 2), which are moments that we didn't target in our calibration. So, in principle our model could feature any numbers in these columns, including numbers with different signs (e.g. a fall in manufacturing employment but an increase in non-manufacturing employment). The results from the model are reassuring, even for these non-targeted moments we see that the model does pretty well, producing results that are relatively similar to the ones in ADH.

We illustrate some of the things mentioned in the previous paragraph in Figure 2. Panel (a) shows the relationship between the 2000-2007 change in the manufacturing employment to population ratio (in percent) and exposure to China for several different values of parameter δ and a value of $\kappa = 7.2$. We can see that manufacturing employment always falls more in regions that are more exposed to China, and the slope of this relationship becomes steeper with δ. Panel (b) of Figure 2 shows non-manufacturing employment instead, and it displays a different pattern. Here the slope also becomes steeper with increases in δ, but for low values of δ (when the nominal rigidity is almost non-binding), non-manufacturing employment doesn't react much to exposure to China. For even lower values of δ, non-manufacturing employment increases with exposure to China. An increase in non-manufacturing employment in regions more exposed to China does not match the patterns presented in ADH, and this points to the fact that the DNWR plays an important role in getting the model to be consistent with the well-identified empirical effects.

4 China Shock and Unemployment

Now that we have introduced the general model with rigidities, as well as our data and calibration, we will apply the model to study the China shock and the effects that this shock had across different states of the U.S. With our model we can obtain the reaction of wages, employment, unemployment, labor force participation, and real wages for all the 87 regions included in our model. The effects of exposure to China on employment across U.S. states for our calibrated parameters are shown in Figure 1.

In Figure 3 we plot three variables related to the labor market for the U.S. on average. The leftmost panel plots the cumulative changes in employment over population (this is given as a ratio so that 1 means no change and 1.01 means a cumulative increase of 1%). This variable starts at 1 in the first period (which corresponds to 2000), falls up to 2.2% in 2004 and subsequently recovers to end roughly 1% higher. In the middle panel we plot the time path for the cumulative changes (in ratios) in labor force participation over population. This variable falls up to roughly 1.5% in 2004 before recovering to end up roughly 1% higher. Finally, in the rightmost panel we plot the time path for the cumulative changes (in ratios) in employment over labor force participation. This variable falls up to 0.8% (corresponding to an unemployment of 0.8%) in 2004 before recovering to its original value. Notice that all unemployment generated by the China shock eventually disappears, since the DNWR allows for more adjustment to occur each year and eventually the shock stops hitting the economy.[12]

Now we turn to welfare. In Figure 4 we provide a scatter plot of the percentage change in welfare across US states against exposure to China. The figure illustrates the fact that states that are more exposed to China see a smaller welfare increase. This is due to the fact that more exposed states see a bigger fall in their terms of trade, more unemployment, and a bigger fall in labor force participation.

However, it is important to notice that in our model 42 states gain from the China shock and only 8 states have a welfare decrease. The state that gains the most from the China shock is California, which sees a welfare increase of 67 basis points, while the state that suffers the most

[12]We view this as a positive aspect of the model, since believing the China shock had permanent unemployment effects is hard to square with the historically low level of unemployment that we are observing currently.

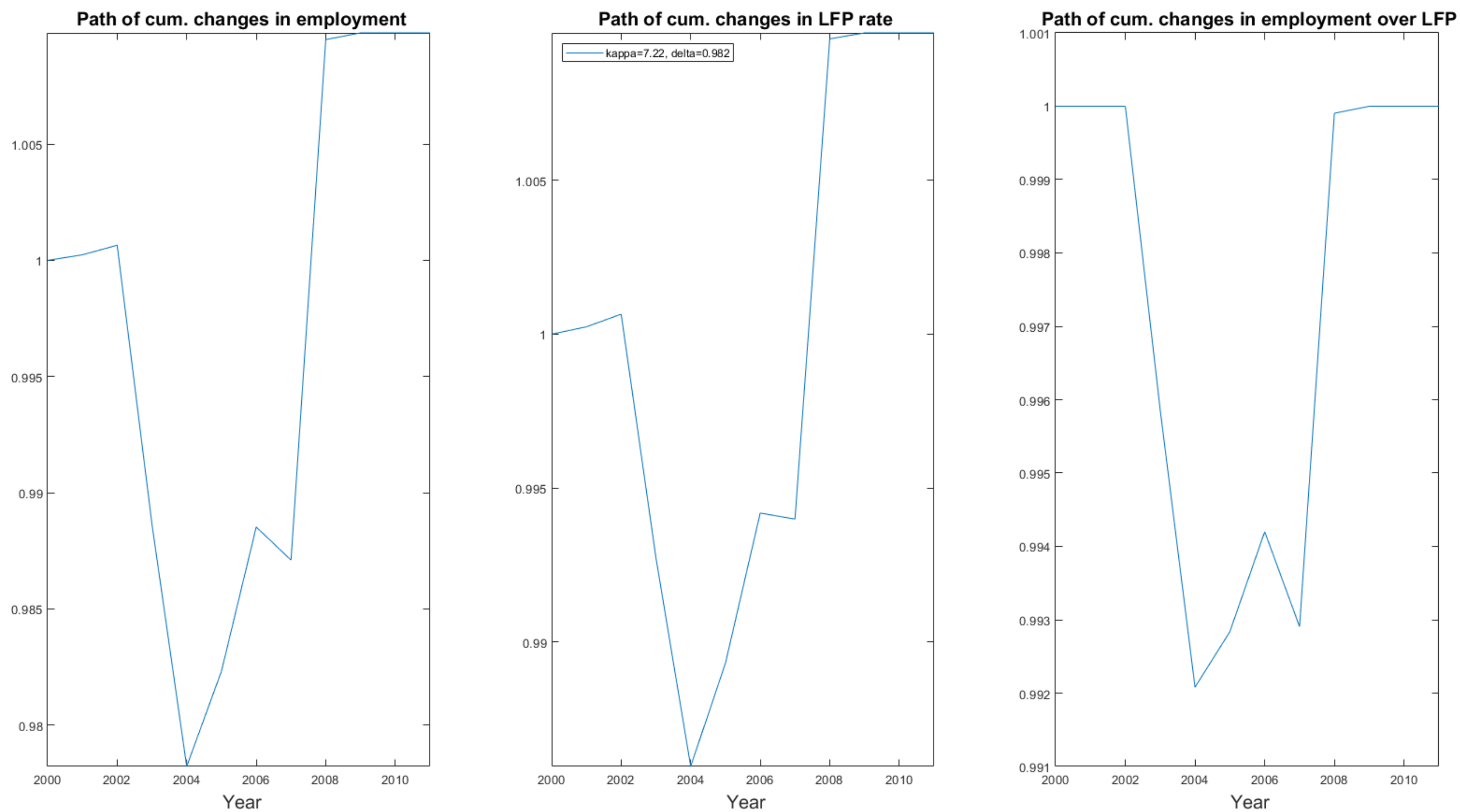


Figure 3: Time paths of different employment related variables for the U.S. as a whole

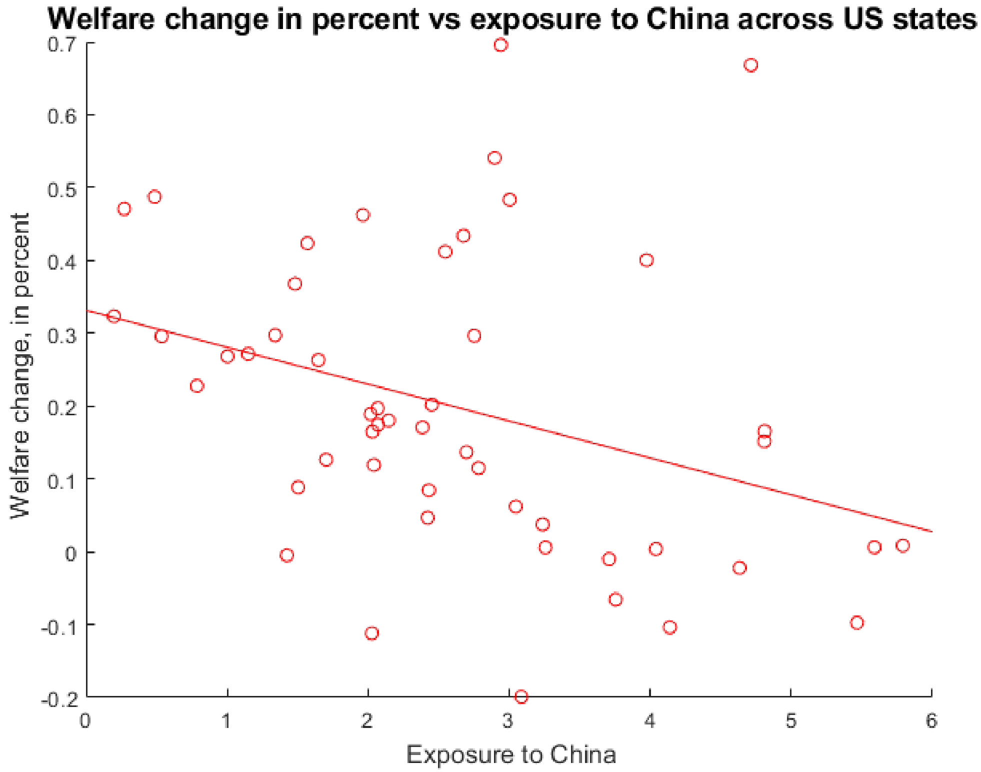


Figure 4: Welfare against Chinese exposure for calibrated parameters

is Iowa which suffers a welfare decrease of 20 basis points. We can also plot a map of welfare across states, to get a sense of which states gained or lost more from the China shock. This is done in Figure 5.

On average, the U.S. sees a welfare increase of 30 basis points if we weight states by their population and of 20 basis points if we don't weight. The fact that the U.S. as a whole gains from the China shock is true even though we match the employment effects captured in ADH, which typically have been interpreted as implying that the China shock had negative overall (welfare) effects. The reason why most states gain is that they also consume the goods where China had a productivity increase, so they see a fall in their consumer prices which pushes welfare up. This positive effect on welfare is counteracted by the unemployment generated by the China shock, which affects more exposed states disproportionately.

So far we have been discussing the welfare effects in our model with DNWR. But we can also explore what happens if we shut down the nominal rigidity ($\delta = 0$) and leave all other parameters unchanged. In this case the China shock is more beneficial for the U.S. as a

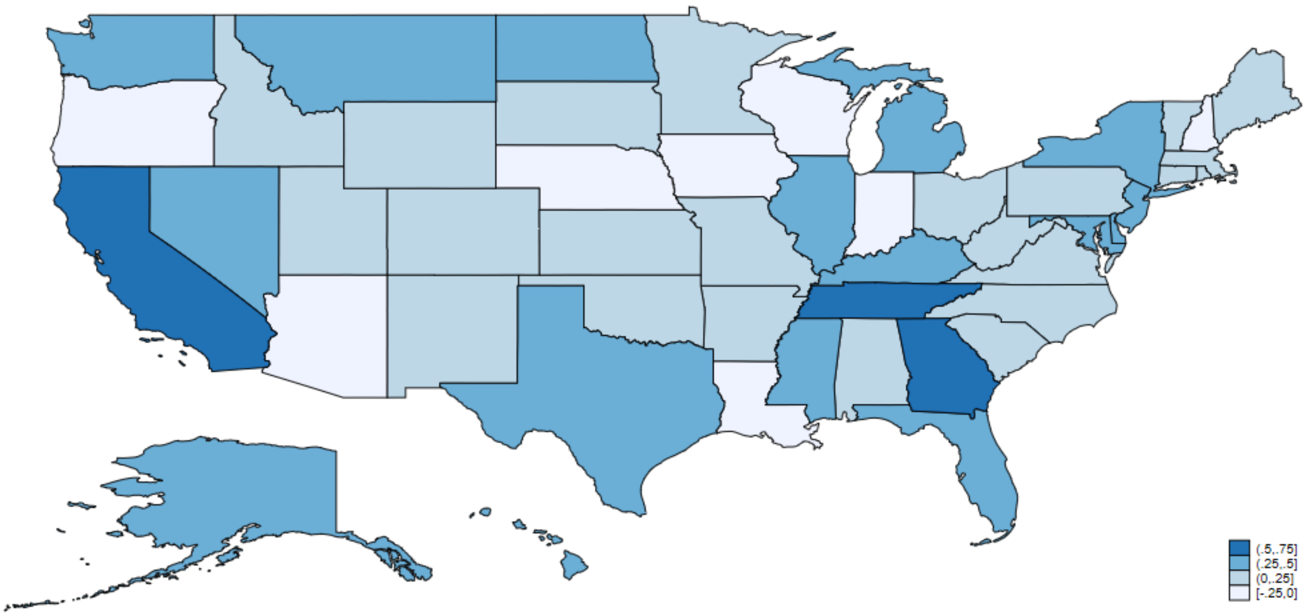


Figure 5: Welfare map across U.S. states for calibrated parameters

whole. The average weighted welfare increase is 36 basis points, while the unweighted one is 25 basis points. This calculation uses our discount factor of $\beta = 0.97$. For different assumptions about the discount factor, the fraction of welfare gains that are "eliminated" when DNWR is incorporated varies. This is shown in table 2. For $\beta = 0.97$, 15 to 25% of the gains are eliminated by the DNWR, for $\beta = 0.95$ this range jumps to 25 to 35%, and for a very high value of $\beta = 0.9$ the range becomes 55 to 75%. It is also worth mentioning that the range of welfare gains that we obtain in our model without rigidities is in the ballpark of other measures of the welfare gain from the China shock obtained from theoretical international trade models that have studied this topic.

5 Conclusion

In this paper we build a Neo-Keynesian model of trade to capture the fact that unemployment can emerge after trade shocks due to nominal rigidities. Our model combines the richness in the trade structure of international trade models (several regions and sectors) with the dynamic structure and nominal rigidities of open economy macro models. The nominal rigidity, which is a downwardly rigid nominal wage, can generate unemployment if nominal demand

Table 2: Welfare gains from the China shock across different discount factors

	Weighted			Unweighted		
	$\delta = 0$	calibrated δ	% decrease	$\delta = 0$	calibrated δ	% decrease
β	(1)	(2)	(3)	(4)	(5)	(6)
0.99	0.3948	0.3718	5.8257	0.2769	0.2562	7.4756
0.98	0.3790	0.3352	11.5567	0.2657	0.2263	14.8288
0.97	0.3638	0.3008	17.3172	0.2550	0.1982	22.2745
0.96	0.3491	0.2686	23.0593	0.2446	0.1720	29.6811
0.95	0.3350	0.2386	28.7761	0.2346	0.1476	37.0844
0.94	0.3214	0.2106	34.4742	0.2250	0.1250	44.4444
0.93	0.3083	0.1846	40.1233	0.2157	0.1040	51.7849
0.92	0.2957	0.1604	45.7558	0.2069	0.0845	59.1590
0.91	0.2836	0.1379	51.3752	0.1983	0.0665	66.4650
0.90	0.2720	0.1171	56.9485	0.1901	0.0499	73.7507

Notes: This table displays the average welfare gains from the China shock, for the U.S. as a whole, across different values of the discount factor β. The panel "Weighted" refers to a weighted average across the U.S. using state income as weights, while the panel "Unweighted" refers to a simple unweighted average across states. Columns (1) and (4) display the gains in percent when the DNWR is inactive ($\delta = 0$). Columns (2) and (5) display the gains in percent for our calibrated δ value of 0.982. Finally, columns (3) and (6) display the percentage decrease in the welfare gain when going from $\delta = 0$ to the calibrated δ.

is not growing sufficiently fast, captured in the model by having nominal demand grow at a constant rate.

We apply this model to quantify the effects of the China shock across regions of the United States, with a realistic calibration, but several simplifying assumptions. We find that the China shock is responsible of up to 0.8 percentage points of U.S. unemployment, but this can go as high as 2.7 percentage points for the states affected the most. Regarding welfare we find that on aggregate the welfare increase in the U.S. with nominal rigidities, 30 basis points, is about 83% of the one that would occur without rigidities (36 basis points). Importantly, the effect is still positive (even though it could be negative if nominal rigidities were even higher) and we can disaggregate it across regions to show which states suffered the most from the China shock through high unemployment.

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Appendix

A Data Construction

In this appendix, we provide details on the construction of the data we briefly described in Section 3.1. We divide this appendix into two parts. In the first part, we discuss how we combine different data sources to compute an internally consistent bilateral trade-flow matrix for all sectors for the years after 2008 (which is the period when Census data is available). In the second part, we discuss how we use the previous step to construct a bilateral trade-flows matrix for 2000. We explain this last step in more detail in Appendix A.2. Throughout this section we refer to the sector k exports of region i to region j as $X_{ij,k}$.

A.1 Construction of Bilateral Trade Flows Between Regions

The final objective is to construct a bilateral trade-flows matrix with entries $X_{ij,k}$. For now, let us focus on the years after 2008 (when the state-by-sector trade data from Census is available). We construct the matrix of $X_{ij,k}$ in four steps explained below.

A.1.1 Step 1: Bilateral Trade between Countries

In the first step we focus on the case where both i and j are countries. Thus, we simply take $X_{ij,k} = X_{ij,k}^{WIOD}$, where $X_{ij,k}^{WIOD}$ are the bilateral trade flows that come directly from the WIOD database without any further calculations.

A.1.2 Step 2: Manufacturing Trade between U.S. States and Countries

For the second step, we combine Census and WIOD data to calculate the trade flows between each of the 50 US states and the other 37 country regions. We scale state-level imports and exports data from Census to match the U.S. totals in WIOD. More precisely, the exports (imports) of state i to (from) country j in manufacturing sector k is computed as a proportion of WIOD's US export (imports) to (from) country j in sector k. This proportion is computed using Census data as the exports (imports) of state i to (from) j in sector k relative to the total

US exports (imports) to (from) j in sector k.

Mathematically, let $X_{ij,k}^{census}$ be the bilateral trade flows between regions i and j, in sector k, according to the Census database. Define the share of exports of US State i in sector k as:

$$y_{ij,k}^{census} \equiv \frac{X_{ij,k}^{census}}{\sum_{h \in US} X_{hj,k}^{census}}.$$

Define also the share of imports of State j in sector k as:

$$e_{ij,k}^{census} \equiv \frac{X_{ij,k}^{census}}{\sum_{l \in US} X_{il,k}^{census}},$$

then $\forall k = 1, ..., 12$:

$$X_{ij,k} = \begin{cases} e_{ij,k}^{census} X_{i\,US,k}^{WIOD} & \forall i \notin US,\ \forall j \in US \\ y_{ij,k}^{census} X_{US\,j,k}^{WIOD} & \forall i \in US,\ \forall j \notin US \end{cases}.$$

A.1.3 Step 3: Manufacturing Trade among U.S. States

In the third step we focus on manufacturing bilateral trade between U.S. States. For this, we combine WIOD Data for the total trade of the USA with itself, and the closest Commodity Flow Survey (CFS) for each year. Similarly to the previous explanation, the export of state i to state j in manufacturing sector k is computed as a share of WIOD's US trade with itself in sector k, where the share is what state i exports to j in sector k represent in sector k total trade according to CFS.

Mathematically, define $X_{ij,k}^{CFS}$ as the bilateral trade flows between state i and state j, in manufacturing sector k, according to the CFS. We first construct:

$$x_{ij,k}^{CFS} \equiv \frac{X_{ij,k}^{CFS}}{\sum_h \sum_l X_{hl,k}^{CFS}} \quad \forall (i \in US,\ \&\ j \in US)\ \forall k = 1, ..., 12,$$

then $X_{ij,k} = x_{ij,k}^{CFS} X_{US,US,k}^{WIOD}\ \forall (i \in US,\ \&\ j \in US), \quad \forall k = 1, ..., 12.$

A.1.4 Step 4: Trade in Services

We compute bilateral trade flows for services $k = 13$ using a Gravity Structure that matches WIOD totals for trade in services between countries (including USA).

As inputs we need total expenditures in services for each region (X_i), as well as total production in services (Y_i). For the case of countries we take this directly from WIOD. For the case of U.S. states we take these variables from the Regional Economic Accounts of the Bureau of Economic Analysis. We scale the state-level services GDP and expenditures such that they aggregate to the USA totals in WIOD. Finally, we also require to compute bilateral distances. We describe how to construct these distances below. Since this step focuses only on the services sector ($k = 13$), we remove the subscript k for a more compact notation.

Theory. Start with the gravity equation:

$$X_{ij} = \left(\frac{w_i \tau_{ij}}{P_j}\right)^{-\varepsilon} X_j,$$

where $P_j^{-\varepsilon} = \sum_i \left(w_i \tau_{ij}\right)^{-\varepsilon}$. But $\sum_j X_{ij} = Y_i$ and hence $\sum_j \left(\frac{w_i \tau_{ij}}{P_j}\right)^{-\varepsilon} X_j = Y_i$. This implies $w_i^{-\varepsilon} \Pi_i^{-\varepsilon} = Y_i$, where $\Pi_i^{-\varepsilon} = \sum_j \tau_{ij}^{-\varepsilon} P_j^{\varepsilon} X_j$. Hence we have:

$$X_{ij} = \frac{\tau_{ij}^{-\varepsilon}}{\left(\Pi_i P_j\right)^{-\varepsilon}} Y_i X_j = \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} \tilde{P}_j^{-1} Y_i X_j,$$

where $\tilde{P}_j \equiv P_j^{-\varepsilon}$ and $\tilde{\Pi}_i \equiv \Pi_i^{-\varepsilon}$, and $\tilde{\tau}_{ij} \equiv \tau_{ij}^{-\varepsilon}$. Imagine that we have X_{ij} for some country pairs. In particular, imagine that for $i \in S_j$ we know X_{ij}. Then:

$$\begin{aligned}
\tilde{P}_j &= \sum_{i \notin S_j} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} Y_i + \sum_{i \in S_j} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} Y_i \\
&= \sum_{i \notin S_j} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} Y_i + \sum_{i \in S_j} X_{ij} \tilde{\Pi}_i \tilde{P}_j Y_i^{-1} X_j^{-1} \tilde{\Pi}_i^{-1} Y_i \\
&= \sum_{i \notin S_j} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} Y_i + \sum_{i \in S_j} X_{ij} \tilde{P}_j X_j^{-1}.
\end{aligned}$$

Let $S_i^* \equiv \{j | i \in S_j\}$, then:

$$\tilde{\Pi}_i = \sum_{j \notin S_i^*} \tilde{\tau}_{ij} \tilde{P}_j^{-1} X_j + \sum_{j \in S_i^*} \tilde{\tau}_{ij} \tilde{P}_j^{-1} X_j = \sum_{j \notin S_i^*} \tilde{\tau}_{ij} \tilde{P}_j^{-1} X_j + \sum_{j \in S_i^*} X_{ij} \tilde{\Pi}_i Y_i^{-1},$$

so the system is now:

$$\begin{aligned} \tilde{P}_j &= \sum_{i \notin S_j} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} Y_i + \sum_{i \in S_j} X_{ij} \tilde{P}_j X_j^{-1} \\ \tilde{\Pi}_i &= \sum_{j \notin S_i^*} \tilde{\tau}_{ij} \tilde{P}_j^{-1} X_j + \sum_{j \in S_i^*} X_{ij} \tilde{\Pi}_i Y_i^{-1}. \end{aligned}$$

Letting

$$X_{S_j,j} \equiv \sum_{i \in S_j} X_{ij}, \quad \lambda_j \equiv 1 - X_{S_j,j}/X_j, \quad X_{i,S_i^*} = \sum_{j \in S_i^*} X_{ij}, \quad \lambda_i^* \equiv 1 - X_{i,S_i^*}/Y_i,$$

then:

$$\begin{aligned} \lambda_j \tilde{P}_j &= \sum_{i \notin S_j} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} Y_i \\ \lambda_i^* \tilde{\Pi}_i &= \sum_{j \notin S_i^*} \tilde{\tau}_{ij} \tilde{P}_j^{-1} X_j. \end{aligned}$$

The full system is:

$$\begin{aligned} \tilde{P}_j &= \sum_i \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} Y_i \quad j \in US \\ \tilde{\Pi}_i &= \sum_j \tilde{\tau}_{ij} \tilde{P}_j^{-1} X_j \quad i \in US \\ \lambda_j \tilde{P}_j &= \sum_{i \notin S_j} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} Y_i \quad j \notin US \\ \lambda_i^* \tilde{\Pi}_i &= \sum_{j \notin S_i^*} \tilde{\tau}_{ij} \tilde{P}_j^{-1} X_j \quad i \notin US \end{aligned}$$

Define $P_s = (\tilde{P}_1, ..., \tilde{P}_{50})'$ for the states, $P_c = (\tilde{P}_{51}, ..., \tilde{P}_{87})'$ for the countries. Similarly for Π_s, Π_c,λ_c, and λ_c^*. Define $\lambda = (1_{1\times 50}, \lambda_c, 1_{1\times 37}, \lambda_c^*)'$. Define $S = (P_s, P_c, \Pi_s, \Pi_c)'$, and with some abuse of notation $S^{-1} = \left(P_s^{-1}, P_c^{-1}, \Pi_s^{-1}, \Pi_c^{-1}\right)'$. Define $(\mathcal{T}Y)_{ss}$, $(\mathcal{T}Y)_{sc}$, $(\mathcal{T}Y)_{cs}$,$(\mathcal{T}X)_{ss}$,

$(\mathcal{T}X)_{sc}$, $(\mathcal{T}X)_{cs}$ as follows:

$$\boldsymbol{(\mathcal{T}Y)_{ss}} = \begin{pmatrix} \tilde{\tau}_{s_1 s_1} Y_{s_1} & \cdots & \tilde{\tau}_{s_k s_1} Y_{s_k} \\ \vdots & \ddots & \vdots \\ \tilde{\tau}_{s_1 s_k} Y_{s_1} & \cdots & \tilde{\tau}_{s_k s_k} Y_{s_k} \end{pmatrix},$$

and the others are defined analogously. The full system can be written as:

$$\lambda \circ S = \begin{pmatrix} 0 & 0 & (\mathcal{T}Y)_{ss} & (\mathcal{T}Y)_{sc} \\ 0 & 0 & (\mathcal{T}Y)_{cs} & 0 \\ (\mathcal{T}X)_{ss} & (\mathcal{T}X)_{sc} & 0 & 0 \\ (\mathcal{T}X)_{cs} & 0 & 0 & 0 \end{pmatrix} \cdot S^{-1},$$

or as:

$$\lambda \circ S = B \cdot S^{-1},$$

where $\circ$ is the element-by-element product and B is the big matrix. Given $\{X_j\}$ and $\{Y_i\}$ and $\{\tilde{\tau}_{ij}\}$ (more on the computation of $\tilde{\tau}_{ij}$ below) , we can get $\{\tilde{P}_j\}$ and $\{\tilde{\Pi}_i\}$. Then we compute $\{X_{ij}\}$ from

$$X_{ij} = \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} \tilde{P}_j^{-1} Y_i X_j.$$

Computation of the bilateral resistance $\tilde{\tau}_{ij}$. To solve the gravity system, we must first compute $\tilde{\tau}_{ij}\ \forall i,\ j$. We proceed by assuming the following functional form:

$$\tilde{\tau}_{ij} = \beta_0^{\iota_{ij}} dist_{ij}^{\beta_1} \exp\left(\xi_{ij}\right),$$

where ι_{ij} is an indicator variable equal to 1 if $i = j$, and ξ_{ij} is an idiosyncratic error term. β_1 captures the standard distance elasticity and β_0 captures the additional resistance of trading with others versus with oneself.

To calculate $dist_{ij}$, we follow the same procedure used in the GeoDist dataset of CEPII to calculate international (and intranational) bilateral trade distances. The idea is to calculate the distance between two countries based on bilateral distances between the largest cities of those two countries, those inter-city distances being weighted by the share of the city in the overall country's population (Head and Mayer, 2002).

We use population for 2010 and coordinates data for all US counties, and all cities around the world with more than 300,000 inhabitants. For those countries with less than two cities of this size, we take the largest cities. Coordinates are important to calculate the physical bilateral distances in kms between each county r in state i and county s in state j (d_{rs} $\forall r \in i$, $s \in j$ and $\forall i, j = 1, ..., 50$), and define $dist\,(ij)$ as:

$$dist\,(ij) = \left(\sum_{r\,\in\,i} \sum_{s\,\in\,j} \left(\frac{pop_r}{pop_i} \right) \left(\frac{pop_s}{pop_j} \right) d_{rs}^{\theta} \right)^{1/\theta}, \tag{12}$$

where pop_h is the population of country/state h. We set $\theta = -1$.

Given our definition of $\tilde{\tau}_{ij}$ we can write the gravity equation in the following way.

$$X_{ij} = \beta_0^{\iota_{ij}} dist_{ij}^{\beta_1} \exp\left(\xi_{ij}\right) \tilde{\Pi}_i^{-1} \tilde{P}_j^{-1} Y_i X_j.$$

Taking logs we can write the previous equation as:

$$\ln X_{ij} = \delta_i^o + \delta_j^d + \tilde{\beta}_0 \iota_{ij} + \beta_1 \ln dist_{ij} + \xi_{ij}, \tag{13}$$

where $\tilde{\beta}_0 = \ln \beta_0$ and the δs are fixed effects. We first estimate the equation above using a 2000-2011 panel of bilateral trade flows in services between countries from WIOD. We present our OLS estimation results for two versions of this equation in Table 3. Column (1) controls for year, origin, and destination fixed effects. Column (2) controls for origin-by-year and destination-by-year fixed effects. Both specification find quantitatively similar results. We take from this exercise a value of the effect of the own-country dummy ($\hat{\tilde{\beta}}_0$) equal to 6.45 and a distance elasticity $\hat{\beta}_1$ equal to -0.7. We take these estimates and compute our estimate of the bilateral resistance term as $\hat{\tilde{\tau}}_{ij} = exp(\hat{\tilde{\beta}}_0 \iota_{ij} + \hat{\beta}_1 \ln dist_{ij})$.

Table 3: Estimation of Own-Country Dummy and Distance Elasticity

Dep. Var.: $\ln X_{ij,t}$	(1)	(2)
ι_{ij}	6.476^{***}	6.453^{***}
	(0.098)	(0.100)
$\ln dist_{ij}$	-0.698^{***}	-0.707^{***}
	(0.020)	(0.019)
Year	Yes	No
Orig.	Yes	No
Dest.	Yes	No
Year×Orig.	No	Yes
Year×Dest.	No	Yes
Observations	17,141	17,141
Adjusted R^2	0.77	0.78

Notes: This table displays the OLS estimates of specifications analogous to the one in equation (13). The outcome variable $\ln X_{ij,t}$ is the log exports in services of country i sent to country j. The own-country dummy ι_{ij} is defined as an indicator function equal to one whenever country i is the same as country j. Finally, $\ln dist_{ij}$ is the log distance between country i and country j. This variable is computed according to equation (12). Robust standard errors are presented in parenthesis. *** denotes statistical significance at the 1%.

A.2 Projection of Bilateral Trade Flows between Regions to year 2000

The Bilateral Trade Flows between regions for 2000 cannot be computed the same way as it was done for 2008, since the Census dataset used begins in 2008. Therefore, the computation method is adapted to take into account this issue; values that used Census data will be calculated using the 2008 trade flows matrix and the 2000 WIOD dataset.

First, the part of the method regarding the calculation of trade flows between countries, and trade between states remains the same, with the exception of changing the year of interest of the WIOD and CFS databases used to 2000 (the nearest CFS dataset is 2002). Furthermore, an adjustment is made in the computation of trade between states in sector 13; the same occurs with the trade flows between states and countries. Those amounts will be calculated using a proportionality method that assumes the share of specific trade flow in total US exports or imports to or from a country remains mostly constant in time, an assumption that is later checked and discussed.

Specifically, the trade amounts between states regarding services (state i exports to state j) are computed as a share of 2000 WIOD's US trade with itself in sector 13, where the share is what state i exports to state j represent in total US sector 13-bilateral trade in 2008 trade matrix. Similarly, state i exports to country j in sector k are calculated as a share of 2000 WIOD's US exports to j in sector k, where the share is what state i 2008 exports to country j represent in the total US sector k-exports to j in 2008 trade matrix. Analogously, state j imports from country i in sector k are calculated as a share of 2000 WIOD's US imports from i in sector k, where the share is what state j 2008 imports from country i represent in the total US sector k-imports from i in 2008 trade matrix. These modifications to the initial method allows obtaining the 2000 trade matrix between regions. Hereby, "Base" will refer to the 2008 Trade Matrix.

A.2.1 Trade between U.S. States and countries

Define the share of exports of US State i in sector k, going to country j as:

$$y_{ij,k}^{Base} \equiv \frac{X_{ij,k}^{Base}}{\sum_{h \in US} X_{hj,k}^{Base}} \quad \forall i \in US\,,\, j \notin US.$$

Similarly, only for $j \in US$ define the share of imports of State j in sector k, coming from country i as:

$$e_{ij,k}^{Base} \equiv \frac{X_{ij,k}^{Base}}{\sum_{l \in US} X_{il,k}^{Base}} \quad \forall i \notin US\,,\, j \in US.$$

Define $\forall k k = 1, ..., 13$:

$$X_{ij,k} = \begin{cases} e_{ij,k}^{Base} X_{i\,US,k}^{WIOD} & \forall i \notin US,\ \forall j \in US \\ y_{ij,k}^{Base} X_{US\,j,k}^{WIOD} & \forall i \in US,\ \forall j \notin US \end{cases} \quad .$$

A.2.2 Trade between U.S. States

First construct:

$$x_{ij,13} \equiv \frac{X_{ij,13}^{Base}}{\sum_h \sum_l X_{hl,13}^{Base}} \quad \forall (i \in US, \ \& \ j \in US),$$

and then define:

$$X_{ij,13} = x_{ij,13} \cdot X_{US,US,13}^{WIOD} \quad \forall (i \in US, \ \& \ j \in US).$$

B Input-Output Loop

Here we elaborate on the way the Input-Output loop works. There are I regions and S sectors, and to produce output in each region and sector firms need to combine labor with all the sectoral aggregates (the version of them available in that region). Specifically, the technology to produce the differentiated good of industry s in region i at time t is

$$Y_{i,s,t} = \left(\phi_{i,s}^{-\phi_{i,s}} \prod_{k=1}^{S} \phi_{i,ks}^{-\phi_{i,ks}} \right) A_{i,s,t} L_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^{S} M_{i,ks,t}^{\phi_{i,ks}},$$

where $M_{i,ks,t}$ is the quantity of the composite good of industry k used in region i to produce in sector s at time t, $\phi_{i,s}$ is the labor share in region i, sector s, $\phi_{i,ks}$ is the share of inputs that sector s uses from sector k in region i, and $1 - \phi_{i,s} = \sum_{k=1}^{S} \phi_{i,ks}$. The resource constraint for the composite good produced in region j, sector k, at time t is

$$M_{j,k,t} = C_{j,k,t} + \sum_{s=1}^{S} M_{j,ks,t}.$$

In turn, the resource constraint for good s produced by region i at time t is

$$Y_{i,s,t} = \sum_{j=1}^{I} \tau_{ij,s,t} Y_{ij,s,t}.$$

The composite in sector k is produced according to

$$M_{j,k,t} = \left(\sum_{i=1}^{I} Y_{ij,k,t}^{\frac{\sigma_k - 1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k - 1}}.$$

Total labor is defined as

$$L_{i,t} \equiv \sum_{s=1}^{S} L_{i,s,t}.$$

Now let's move to the equations in terms of the prices and values. Let's start with prices. Let $P_{i,s,t}$ be the price of $M_{i,s,t}$, $p_{ij,s,t}$ be the price of $Y_{i,s,t}$ in j at time t, and $W_{i,t}$ be the nominal wage in region i at time t. We know that

$$\begin{aligned} p_{ii,s,t} &= A_{i,s,t}^{-1} W_{i,t}^{\phi_{i,s}} \prod_{k=1}^{S} P_{i,k,t}^{\phi_{i,ks}}, \\ p_{ij,s,t} &= \tau_{ij,s,t} p_{ii,s,t}, \\ P_{j,s,t} &= \left(\sum_{i=1}^{I} p_{ij,s,t}^{1-\sigma_s} \right)^{1/(1-\sigma_s)}, \end{aligned}$$

Combining the last three equations we obtain:

$$P_{j,s,t}^{1-\sigma_s} = \sum_{i=1}^{I} \left(\tau_{ij,s,t} A_{i,s,t}^{-1} W_{i,t}^{\phi_{i,s}} \prod_{k=1}^{S} P_{i,k,t}^{\phi_{i,ks}} \right)^{1-\sigma_s},$$

which, for each time period t, is a system of $I \times S$ equations in $I \times S$ unknowns that can be used to solve for the $P_{j,s,t}$'s given the trade costs ($\tau_{ij,s,t}$'s), technologies ($A_{i,s,t}$'s), wages ($W_{i,t}$'s), labor shares ($\phi_{i,s}$'s) and input output coefficients ($\phi_{i,ks}$), note that we don't allow the labor shares and input output coefficients to vary with time.

This system of $I \times S$ equations in $I \times S$ unknowns is well behaved and can be solved using contraction mapping techniques, where you start with a guess for the $I \times S$ prices (denoted $PI_{j,s,t}$), and obtain a new guess (denoted $PE_{j,s,t}$) as follows:

$$PE_{j,s,t} = \left(\sum_{i=1}^{I} \left(\tau_{ij,s,t} A_{i,s,t}^{-1} W_{i,t}^{\phi_{i,s}} \prod_{k=1}^{S} PI_{i,k,t}^{\phi_{i,ks}} \right)^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}}$$

We iterate until the difference between PE and PI is very small and this provides a solution to the system. This is a similar method to the one followed in Caliendo and Parro (2015).

Getting back to the description of the setup of the model, the price of final output in region j at time t is given by

$$P_{j,t} \;=\; \prod_{s=1}^{S} P_{j,s,t}^{\alpha_{j,s}}.$$

Now let's move on to resource constraints in value. Multiplying the resource constraint for $M_{j,k,t}$ by $P_{j,k,t}$ we get

$$Z_{j,k,t} = P_{j,k,t}C_{j,k,t} + \sum_{s=1}^{S} P_{j,k,t}M_{j,ks,t},$$

where $Z_{j,k,t} \equiv P_{j,k,t}M_{j,k,t}$ denotes the total expenditure of region j in industry k at time t. Let $\lambda_{ij,k,t}$ be the share of that expenditure spent on imports from i,

$$\lambda_{ij,k,t} \equiv \frac{p_{ij,k,t}Y_{ij,k,t}}{Z_{j,k,t}}.$$

We know that

$$\lambda_{ij,k,t} = \frac{p_{ij,k,t}^{1-\sigma_k}}{\sum_l p_{lj,k,t}^{1-\sigma_k}} = \frac{p_{ij,k,t}^{1-\sigma_k}}{P_{j,k,t}^{1-\sigma_k}} = \frac{\left(\tau_{ij,k,t}A_{i,k,t}^{-1}W_{i,k,t}^{\phi_{i,k}}\prod_{s=1}^{S}P_{i,s,t}^{\phi_{i,sk}}\right)^{1-\sigma_k}}{\sum_{r=1}^{I}\left(\tau_{rj,k,t}A_{r,k,t}^{-1}W_{r,k,t}^{\phi_{r,k}}\prod_{s=1}^{S}P_{r,s,t}^{\phi_{r,sk}}\right)^{1-\sigma_k}}.$$

Let $R_{i,k,t} = p_{ii,k,t}Y_{i,k,t}$ represent the sales of good k by region i at time t. Multiplying the resource constraint for $Y_{i,k,t}$ above by $p_{ii,k,t}$ we get

$$p_{ii,k,t}Y_{i,k,t} = \sum_{j=1}^{I} \tau_{ij,k,t}p_{ii,k,t}Y_{ij,k,t},$$

and hence

$$R_{i,k,t} = \sum_{j=1}^{I} \lambda_{ij,k,t}Z_{j,k,t}.$$

Plugging in from the resource constraint above for $Z_{j,k,t}$ we then have

$$R_{i,k,t} = \sum_{j=1}^{I} \lambda_{ij,k,t} \left(P_{j,k,t} C_{j,k,t} + \sum_{s} P_{j,k,t} M_{j,ks,t} \right).$$

Finally, note that

$$P_{j,k,t} M_{j,ks,t} = \phi_{j,ks} R_{j,s,t},$$

and

$$P_{j,k,t} C_{j,k,t} = \alpha_{j,k} \left(W_{j,t} L_{j,t} + D_{j,t} \right),$$

hence

$$R_{i,k,t} = \sum_{j=1}^{I} \lambda_{ij,k,t} \left(\alpha_{j,k} \left(W_{j,t} L_{j,t} + D_{j,t} \right) + \sum_{s} \phi_{j,ks} R_{j,s,t} \right).$$

For each time period t, this is a linear system of $I \times S$ equations in $I \times S$ unknowns that can be used to solve for the $R_{i,k,t}$'s given the trade shares ($\lambda_{ij,k,t}$'s), Cobb-Douglas shares ($\alpha_{j,k}$'s), wages ($W_{j,t}$'s), labor quantities ($L_{j,t}$'s), deficits ($D_{j,t}$'s), and input output coefficients ($\phi_{j,ks}$). Since this is a linear system in the R's, it is relatively easy to solve. Of this total production ($R_{i,k,t}$), we know that a fraction $\phi_{i,k}$ is payed to labor, so we can write:

$$\begin{aligned} W_{i,t} L_{i,k,t} &= \phi_{i,k} R_{i,k,t} \\ W_{i,t} L_{i,t} &= \sum_{k=1}^{S} \phi_{i,k} R_{i,k,t}, \end{aligned}$$

which will be needed as an equilibrium condition.

Summarizing, the equilibrium system in each period t is described by the following equations:

$$R_{i,s,t} = \sum_{j=1}^{I} \lambda_{ij,s,t} \left(\alpha_{j,s} \left(W_{j,t} L_{j,t} + D_{j,t} \right) + \sum_{k} \phi_{j,sk} R_{j,k,t} \right) \qquad \forall\, i, \forall\, s$$

$$\lambda_{ij,s,t} = \frac{(\tau_{ij,s,t} A_{i,s,t}^{-1} W_{i,t}^{\phi_{i,s}} \prod_k P_{i,k,t}^{\phi_{i,ks}})^{1-\sigma_s}}{\sum_{r=1}^{I} (\tau_{rj,s,t} A_{r,s,t}^{-1} W_{r,t}^{\phi_{r,s}} \prod_k P_{r,k,t}^{\phi_{r,ks}})^{1-\sigma_s}} \qquad \forall\, i, \forall\, s$$

$$P_{i,s,t}^{1-\sigma_s} = \sum_{j=1}^{I} \left(\tau_{ji,s,t} A_{j,s,t}^{-1} W_{j,t}^{\phi_{j,s}} \prod_{k=1}^{S} P_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s} \qquad \forall\, i, \forall\, s$$

$$
\begin{aligned}
P_{i,t} &= \prod_{s=1}^{S} P_{i,s,t}^{\alpha_{i,s}} && \forall\, i \\
W_{i,t}L_{i,t} &= \sum_{s=1}^{S} \phi_{i,s} R_{i,s,t} && \forall\, i \\
L_{i,t} &\leq \ell_{i,t}, W_{i,t} \geq \delta_i W_{i,t-1}, CS && \forall\, i \\
\ell_{i,t} &= \frac{(W_{i,t}L_{i,t}/(P_{i,t}\ell_{i,t}))^{\kappa}}{\mu_i^{\kappa} + (W_{i,t}L_{i,t}/(P_{i,t}\ell_{i,t}))^{\kappa}} \overline{L}_i && \forall\, i \\
\sum_{i=1}^{I} W_{i,t}L_{i,t} &= \gamma \sum_{i=1}^{I} W_{i,t-1}L_{i,t-1}. && \text{single}
\end{aligned}
$$

C Hat Algebra

The equations defining the trade shares

$$
\lambda_{ij,k,t} = \frac{(\tau_{ij,k,t} A_{i,k,t}^{-1} W_{i,t}^{\phi_{i,k}} \prod_s P_{i,s,t}^{\phi_{i,sk}})^{1-\sigma_k}}{\sum_{r=1}^{I} (\tau_{rj,k,t} A_{r,k,t}^{-1} W_{r,t}^{\phi_{r,k}} \prod_s P_{r,s,t}^{\phi_{r,sk}})^{1-\sigma_k}},
$$

by multiplying, dividing, and using the definition of the $\lambda's$, can easily be express in hats as:

$$
\hat{\lambda}_{ij,k,t} = \frac{(\hat{\tau}_{ij,k,t} \hat{A}_{i,k,t}^{-1} \hat{W}_{i,t}^{\phi_{i,k}} \prod_s \hat{P}_{i,s,t}^{\phi_{i,sk}})^{1-\sigma_k}}{\sum_{r=1}^{I} \lambda_{rj,k,t-1} (\hat{\tau}_{rj,k,t} \hat{A}_{r,k,t}^{-1} \hat{W}_{r,t}^{\phi_{r,k}} \prod_s \hat{P}_{r,s,t}^{\phi_{r,sk}})^{1-\sigma_k}}.
$$

Given the definition of the trade shares, the equations for prices:

$$
P_{i,s,t}^{1-\sigma_s} = \sum_{j=1}^{I} \left(\tau_{ji,s,t} A_{j,s,t}^{-1} W_{j,t}^{\phi_{j,s}} \prod_{k=1}^{S} P_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s},
$$

cannot be simplified much, so we simply express it in terms of hats as:

$$
\hat{P}_{i,s,t}^{1-\sigma_s} = \sum_{j=1}^{I} \lambda_{ji,s,t-1} \left(\hat{\tau}_{ji,s,t} \hat{A}_{j,s,t}^{-1} \hat{W}_{j,t}^{\phi_{j,s}} \prod_{k=1}^{S} \hat{P}_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s}.
$$

The system for revenues:

$$R_{i,s,t} \;=\; \sum_{j=1}^{I} \lambda_{ij,s,t} \left(\alpha_{j,s}(W_{j,t}L_{j,t} + D_{j,t}) + \sum_{k=1}^{S} \phi_{j,sk} R_{j,k,t} \right),$$

can be expressed as:

$$\hat{R}_{i,s,t} R_{i,s,t-1} \;=\; \sum_{j=1}^{I} \hat{\lambda}_{ij,s,t} \lambda_{ij,s,t-1} \left(\alpha_{j,s}(\hat{W}_{j,t}\hat{L}_{j,t}Y_{j,t-1} + \hat{D}_{j,t}D_{j,t-1}) + \sum_{k=1}^{S} \phi_{j,sk} \hat{R}_{j,k,t} R_{j,k,t-1} \right).$$

The labor market clearing condition:

$$W_{i,t}L_{i,t} \;=\; \sum_{s=1}^{S} \phi_{i,s} R_{i,s,t},$$

also cannot be simplified much, so we leave it as:

$$\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1} \;=\; \sum_{s=1}^{S} \phi_{i,s} \hat{R}_{i,s,t} R_{i,s,t-1}.$$

Similarly, the equation:

$$\sum_{i=1}^{I} W_{i,t}L_{i,t} = \gamma \sum_{i=1}^{I} Y_{i,t-1},$$

can be expressed as:

$$\sum_{i=1}^{I} \hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1} = \gamma \sum_{i=1}^{I} Y_{i,t-1},$$

Now we turn to the equations in the downward nominal wage rigidity bloc. The equation $W_{i,t} \geq \delta_i W_{i,t-1}$ is easily turn into hats as:

$$\hat{W}_{i,t} \geq \delta_i,$$

while the equations for labor can be reworked as follows:

$$\begin{aligned} L_{i,t} &\leq \ell_{i,t} \\ \frac{L_{i,t}}{L_{i,t-1}} \frac{L_{i,t-1}}{L_{i,t-2}} \cdots \frac{L_{i,1}}{L_{i,0}} L_{i,0} &\leq \frac{\ell_{i,t}}{\ell_{i,t-1}} \frac{\ell_{i,t-1}}{\ell_{i,t-2}} \cdots \frac{\ell_{i,1}}{\ell_{i,0}} \ell_{i,0} \end{aligned}$$

$$
\begin{aligned}
\hat{L}_{i,t}\hat{L}_{i,t-1}\ldots\hat{L}_{i,1}L_{i,0} &\leq \hat{\ell}_{i,t}\hat{\ell}_{i,t-1}\ldots\hat{\ell}_{i,1}\ell_{i,0} \\
L_{i,0}\prod_{q=1}^{t}\hat{L}_{i,q} &\leq \ell_{i,0}\prod_{q=1}^{t}\hat{\ell}_{i,q}.
\end{aligned}
$$

Since we assume that in period zero the economy was in a steady state where every region had full employment we know that $L_{i,0} = \ell_{i,0}$ so we finally obtain:

$$
\prod_{q=1}^{t}\hat{L}_{i,q} \leq \prod_{q=1}^{t}\hat{\ell}_{i,q}.
$$

Next we turn the labor force participation equation into hats starting from the definition of $\pi_{i,t}$:

$$
\begin{aligned}
\frac{\pi_{i,t}}{\pi_{i,t-1}} &= \left(\frac{\omega_{i,t}^{\kappa}}{\mu^{\kappa}+\omega_{i,t}^{\kappa}}\right)\Big/\left(\frac{\omega_{i,t-1}^{\kappa}}{\mu^{\kappa}+\omega_{i,t-1}^{\kappa}}\right) \\
\hat{\pi}_{i,t} &= \frac{\hat{\omega}_{i,t}^{\kappa}}{\frac{\mu^{\kappa}}{\mu^{\kappa}+\omega_{i,t-1}^{\kappa}}+\frac{\omega_{i,t-1}^{\kappa}}{\mu^{\kappa}+\omega_{i,t-1}^{\kappa}}\frac{\omega_{i,t}^{\kappa}}{\omega_{i,t-1}^{\kappa}}} \\
&= \frac{\hat{\omega}_{i,t}^{\kappa}}{1-\pi_{i,t-1}+\pi_{i,t-1}\hat{\omega}_{i,t}^{\kappa}}.
\end{aligned}
$$

Noticing that $\hat{\pi}_{i,t} = \hat{\ell}_{i,t}$ (because $\overline{L}_i$ is fixed across time) and using the definition of $\omega_{i,t}$ in equation (7) we obtain the following hat equation:

$$
\hat{\ell}_{i,t} = \frac{(\hat{W}_{i,t}\hat{L}_{i,t}/(\hat{P}_{i,t}\hat{\ell}_{i,t}))^{\kappa}}{1-\pi_{i,t-1}+\pi_{i,t-1}(\hat{W}_{i,t}\hat{L}_{i,t}/(\hat{P}_{i,t}\hat{\ell}_{i,t}))^{\kappa}}.
$$

With this we have all the hat equations that we need. Recall that in our setup we can express instantaneous utility as:

$$
u_{i,t} \propto \left(\mu^{\kappa}+\omega_{i,t}^{\kappa}\right)^{1/\kappa} = (\omega_{i,t}^{\kappa}/\pi_{i,t})^{1/\kappa} = \omega_{i,t}\pi_{i,t}^{-1/\kappa}.
$$

The change in utility can then be expressed as:

$$
\hat{u}_{i,t} = \hat{\omega}_{i,t}\hat{\pi}_{i,t}^{-1/\kappa} = \hat{\omega}_{i,t}\left(\frac{\hat{\omega}_{i,t}^{\kappa}}{1-\pi_{i,t-1}+\pi_{i,t-1}\hat{\omega}_{i,t}^{\kappa}}\right)^{-1/\kappa} = \left(1-\pi_{i,t-1}+\pi_{i,t-1}\hat{\omega}_{i,t}^{\kappa}\right)^{1/\kappa}.
$$

D Modifying the Data to Fit a New Set of Alphas

From our data on bilateral trade flows, labor shares, and input-output coefficients we can back out a set of Cobb-Douglas parameters which are the $\alpha_{i,s}$'s. There are certain situations where we might want to change these α's. One reason that we might want to do this is because the original α's implied by our data might be slightly negative, which is not ideal. Another reason might be that we want to equalize the α's between all regions of the United States or all regions of the World (as done in Caliendo, Dvorkin, and Parro (2019)).

Imagine that we know the new set of alphas that we want to obtain, and we want to recover the data (bilateral trade flows) that is compatible with these new alphas. The equilibrium system to obtain the data for the new alphas is the following (notice that we are basically applying the system without the DNWR and with $\gamma = 1$):

$$
\begin{aligned}
\hat{R}_{i,s,t}R_{i,s,t-1} &= \sum_{j=1}^{I} \hat{\lambda}_{ij,s,t}\lambda_{ij,s,t-1}\left(\alpha'_{j,s}(\hat{W}_{j,t}Y_{j,t-1} + D_{j,t-1}) + \sum_{k=1}^{S} \phi_{j,sk}\hat{R}_{j,k,t}R_{j,k,t-1}\right) \\
\hat{\lambda}_{ij,k,t} &= \frac{\left(\hat{W}_{i,t}^{\phi_{i,k}} \prod_{s=1}^{S} \hat{P}_{i,s,t}^{\phi_{i,sk}}\right)^{1-\sigma_k}}{\sum_{r=1}^{I} \lambda_{rj,k,t-1}\left(\hat{W}_{r,t}^{\phi_{r,k}} \prod_{s=1}^{S} \hat{P}_{r,s,t}^{\phi_{r,sk}}\right)^{1-\sigma_k}} \\
\hat{P}_{j,k,t}^{1-\sigma_k} &= \sum_{i=1}^{I} \lambda_{ij,k,t-1}\left(\hat{W}_{i,t}^{\phi_{i,k}} \prod_{s=1}^{S} \hat{P}_{i,s,t}^{\phi_{i,sk}}\right)^{1-\sigma_k} \\
\hat{W}_{i,t}Y_{i,t-1} &= \sum_{s=1}^{S} \phi_{i,s}\hat{R}_{i,s,t}R_{i,s,t-1} \\
\sum_{i=1}^{I} \hat{W}_{i,t}Y_{i,t-1} &= \sum_{i=1}^{I} Y_{i,t-1}.
\end{aligned}
$$

In this system the R_{t-1}, λ_{t-1}, α', Y_{t-1}, D_{t-1}'s are all data, and the $\hat{W}$, $\hat{P}$, $\hat{R}$ and $\hat{\lambda}$'s are the outcomes. From these outcomes we can construct the new bilateral trade flow matrix.