Monetary Policy, Redistribution, and Risk Premia*

Rohan Kekre† Moritz Lenel‡

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Abstract

We study the transmission of monetary policy through risk premia in a heterogeneous agent New Keynesian environment. Heterogeneity in households' marginal propensity to take risk (MPR) summarizes differences in risk aversion, constraints, rules of thumb, background risk, and beliefs relevant for portfolio choice on the margin. An unexpected reduction in the nominal interest rate redistributes to households with high MPRs, lowering risk premia and amplifying the stimulus to investment. Quantitatively, this mechanism rationalizes the role of news about future excess returns in driving the stock market response to monetary policy shocks.

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†University of Chicago Booth School of Business. Email: rohan.kekre@chicagobooth.edu.

‡Princeton University. Email: lenel@princeton.edu.

1 Introduction

A growing literature finds that expansionary monetary policy lowers risk premia. This has been established for the equity premium in stock markets, the term premium in nominal bonds, and the external finance premium on risky corporate debt.[1] The basic New Keynesian framework as described by Woodford (2003) and Gali (2008) does not capture this aspect of monetary policy transmission. As noted by Kaplan and Violante (2018), this is equally true for an emerging body of heterogeneous agent New Keynesian models whose focus on heterogeneous marginal propensities to consume has substantially enriched the implications for aggregate consumption but less so for asset prices and therefore investment.

This paper demonstrates that a New Keynesian model with heterogeneous households differing instead in risk-bearing capacity can quantitatively rationalize the observed effects of policy on risk premia, amplifying the transmission to the real economy through investment. An expansionary monetary policy shock lowers the risk premium if it redistributes to households with a high marginal propensity to take risk (MPR), defined as the marginal propensity to save in capital relative to save overall. With underlying heterogeneity in risk aversion, portfolio constraints, rules of thumb, background risk, or beliefs, high MPR households borrow in the bond market from low MPR households to hold leveraged positions in capital. By generating unexpected inflation, raising profit income relative to labor income, and raising the price of capital, an expansionary monetary policy shock endogenously redistributes to high MPR households and thus lowers the market price of risk. In a calibration matching portfolio heterogeneity in the U.S. economy, this mechanism rationalizes the role of news about lower future excess returns in driving the empirically observed increase in the stock market. The stimulus to investment is amplified relative to a counterfactual representative agent economy without heterogeneity in monetary policy exposures and MPRs.

Our baseline environment enriches an otherwise standard New Keynesian model with Epstein and Zin (1991) preferences and heterogeneity in risk aversion. Households consume, supply labor subject to adjustment costs in nominal wages, and choose a portfolio of nominal bonds and claims on capital. Production is subject to aggregate TFP shocks. Monetary policy follows a standard Taylor (1993) rule. Heterogeneity in risk aversion endogenously generates heterogeneity in MPRs and exposures to a monetary policy shock. Epstein-Zin preferences allow us to flexibly model this heterogeneity as distinct from households' intertemporal elasticities of substitution. We begin by analytically characterizing the effects of a monetary policy shock in a simple two-period version of this environment, providing an organizing framework for the quantitative analysis of the infinite horizon which follows.

[1]See Bernanke and Kuttner (2005), Hanson and Stein (2015), and Gertler and Karadi (2015), respectively.

An expansionary monetary policy shock lowers the risk premium by endogenously redistributing wealth to households with a high marginal propensity to save in capital relative to save overall — that is, a high MPR. Redistribution to high MPR households lowers the risk premium because of market clearing in asset markets: to the extent households on aggregate wish to increase their portfolio share in capital, its expected return must fall relative to that on bonds. An expansionary monetary policy shock redistributes across households by revaluing their initial balance sheets: it deflates nominal debt, raises the profits earned using capital, and raises the price of capital. More risk tolerant households hold leveraged positions in capital and have a higher MPR. Hence, an expansionary monetary policy shock will lower the risk premium by endogenously redistributing to these households.

These insights are robust to alternative sources of heterogeneity beyond risk aversion. We consider a richer environment in which households may also face portfolio constraints or follow rules-of-thumb as in models of limited participation or the financial accelerator; households may be subject to idiosyncratic background risk as in the large literatures on uninsured labor or entrepreneurial income risk; and households may have subjective beliefs regarding the value of capital. In this general environment, the MPR continues to summarize the relevant cross-sectional heterogeneity to evaluate the risk premium effects of redistributive shocks. Moreover, because each of these forms of heterogeneity imply that households holding more levered positions in capital will be the ones with a high MPR, they continue to imply that expansionary monetary policy will lower the risk premium through redistribution.

The reduction in the risk premium matters for the real economy depending on the endogenous reaction of the monetary feedback rule. Absent nominal rigidity, the investment effect of a shock lowering the risk premium depends critically on households' intertemporal elasticities of substitution, which in turn control the relative strength of income and substitution effects and thus the equilibrium response of the safe real interest rate to the shock.[2] With nominal rigidity, the response of the real interest rate also critically depends on the monetary policy rule. In our environment, this logic applies to the risk premium effects of the primitive monetary shock itself. Provided that the central bank's feedback rule limits the rise in the real interest rate following this shock, the decline in the risk premium will be reflected in lower required returns to capital, amplifying the stimulus to investment.

Accounting for the risk premium effects of monetary policy is important given empirical evidence implying that it may be a key component of the transmission mechanism. We refresh this point from Bernanke and Kuttner (2005) using the structural vector autoregression instrumental variables (SVAR-IV) approach in Gertler and Karadi (2015). Using monthly

[2]Intuitively, even if the risk premium falls, it could either be that the expected return to capital falls or rises (and thus investment rises or falls), in the latter case because the safe real interest rate rises sufficiently.

data from July 1979 through June 2012, we run a six-variable VAR including the excess return on the S&P 500 and the real return on a T-bill. Using Fed Funds surprises on FOMC days as an instrument, we find that an unexpected loosening of monetary policy resulting in a roughly 25*bp* reduction in the 1-year Treasury bond leads to an unexpected increase in excess returns of 2*pp*. Using a Campbell and Shiller (1988) decomposition, 1.1*pp* (55%) of this increase is driven by lower future excess returns, posing a strong challenge to existing New Keynesian frameworks where virtually all of the effect on the stock market operates instead through higher dividends or lower risk-free rates.

Extending the model to the infinite horizon, we investigate whether a calibration to the U.S. economy is capable of rationalizing these facts. We parameterize the model to match the cross-sectional heterogeneity in wealth, labor income, and financial portfolios in the Survey of Consumer Finances, together disciplining the model-implied exposures to a monetary policy shock and MPRs. We use global solution methods to solve the model and capture the non-linear model dynamics with aggregate risk.[3] To make the computational burden tractable, we summarize the cross-sectional heterogeneity into three groups of households: two groups corresponding to the small fraction with high wealth relative to labor income, but differing in their risk tolerance and thus portfolio share in capital, and one group corresponding to the large fraction holding little wealth relative to labor income.

In response to an expansionary monetary policy shock, we find that the redistribution across households with heterogeneous MPRs can quantitatively explain the risk premium effects of monetary policy found in the data. Using the same Campbell-Shiller decomposition on model impulse responses as we used on the data, 47% of the excess return on equity in our baseline parameterization arises from news about lower future excess returns, compared to 55% in the data and 2% in a counterfactual representative agent economy. Consistent with the analytical results, the effect on the risk premium is amplified in parameterizations with a more persistent shock and thus larger debt deflation; higher stickiness and thus a larger increase in profit income relative to labor income; or higher investment adjustment costs and thus a larger increase in the price of capital.

The reduction in the risk premium through redistribution in turn amplifies the effect of policy on investment. We consider a monetary shock which generates the same (roughly 25*bp*) decline in the 1-year nominal bond yield as in our empirical analysis.[4] Relative to

[3]Our model does not provide a novel way to rationalize the level of the equity premium. We add a rare disaster to match this moment, following Barro (2006), but other approaches include adding long-run risk (Bansal and Yaron (2004)), habits (Campbell and Cochrane (1999)), idiosyncratic risk (Gomes and Michaelides (2008)), or limited participation and heterogeneous intertemporal elasticities of substitution (Guvenen (2009)). To obtain an accurate solution especially given the disaster, we use a global method.

[4]While we do not include the 1-year bond in our set of traded assets, we can compute its hypothetical yield by assuming that, in each state, the highest-valuation household prices the bond.

the representative agent economy, our baseline parameterization increases the peak response of investment to this shock from 1.5*pp* to 1.7*pp*. A countervailing effect on consumption means that the effect on output is more modest. Consistent with the analytical results, the amplification of the investment response is even stronger when the monetary policy rule features a less aggressive tightening in response to the primitive monetary shock.

Related literature Our paper contributes to the rapidly growing literature on heterogeneous agent New Keynesian models by studying the transmission of monetary policy through risk premia. Our characterization of the redistributive effects of monetary policy relates especially to the work of Doepke and Schneider (2006), Auclert (2018), Kaplan, Moll, and Violante (2018), and Luetticke (2018). We build on Doepke and Schneider (2006) in our measurement of household portfolios, informing the heterogeneity in exposures to a monetary policy shock. Relative to Auclert (2018), we demonstrate that it is the covariance of these exposures with MPRs rather than MPCs which matters for policy transmission through risk premia rather than risk-free rates. And relative to Kaplan et al. (2018) and Luetticke (2018), we demonstrate that this framework can match the stock market reaction to monetary policy shocks when assets differ in their riskiness rather than in their liquidity.

In doing so, we bring to the HANK literature many established insights from heterogeneous agent and intermediary-based asset pricing. The wealth distribution is a crucial determinant of the market price of risk as in other models with heterogeneous risk aversion (e.g., Garleanu and Panageas (2015)), segmented markets (e.g., He and Krishnamurthy (2013)), rules-of-thumb (e.g., Chien, Cole, and Lustig (2012)), background risk (e.g., Heaton and Lucas (1996)), or heterogeneous beliefs (e.g., Geanakoplos (2009)). We build on this literature by focusing on the changes in wealth induced by a monetary policy shock in a production economy with nominal rigidities. In studying this question we follow Alvarez, Atkeson, and Kehoe (2009) and Drechsler, Savov, and Schnabl (2018), who study the effects of monetary policy on risk premia in an exchange economy with segmented markets and in a model of banking, respectively.[5] We instead study these effects operating through the balance sheet revaluation of heterogeneous agents in a conventional New Keynesian setting.

Indeed, our paper most directly builds on prior work focused on risk premia in New Keynesian economies. We clarify the sense in which Bernanke, Gertler, and Gilchrist (1999) served as a seminal HANK model focused on heterogeneity in MPRs rather than MPCs.[6]

[5]More recently, in complementary work Bhandari, Evans, and Golosov (2019) construct a segmented markets model in the spirit of Alvarez, Lucas, and Weber (2001) and Alvarez et al. (2009) in which monetary policy also has effects on risk premia.

[6]In Bernanke et al. (1999), households can only trade bonds while entrepreneurs can trade bonds and capital. In equilibrium, households have a zero MPR while entrepreneurs have a positive MPR. Changes in the distribution of net worth across these agents thus affects credit spreads and economic activity.

As we demonstrate, however, heterogeneity in MPRs need not rely on market segmentation, justifying its relevance even in markets which may not be intermediated by specialists. In relating movements in the risk premium to the real economy, we make use of the insight in Caballero and Farhi (2018), Caballero and Simsek (2018), and Ilut and Schneider (2014) that an increase in the risk premium will induce a recession if the safe interest rate does not sufficiently fall in response.[7] We build especially on the first two papers, as well as Brunnermeier and Sannikov (2012, 2016), in emphasizing the effects of heterogeneity in asset valuations on risk premia. Relative to these papers, we explore the importance of such heterogeneity for monetary transmission in a calibration to the U.S. economy.

Outline The remainder of the paper is structured as follows. In section 2 we characterize our main insights in a two-period environment, characterizing the mechanisms through which a monetary easing will endogenously redistribute to high MPR households in a wide variety of settings. This provides an organizing framework for our quantitative analysis in the infinite horizon in section 3. Calibrated to the U.S. economy, the redistribution toward high MPR households rationalizes the empirical evidence on the effect of a monetary policy easing on the equity premium and amplifies the stimulus through investment.

2 Analytical insights in a two-period environment

We first characterize our main conceptual insights in a two-period environment allowing us to obtain simple analytical results. An expansionary monetary policy shock lowers the risk premium on capital if it redistributes to households with relatively high MPRs. Heterogeneity in risk aversion induces a joint distribution of MPRs and monetary policy exposures such that an expansionary shock lowers the risk premium. A similar result obtains with heterogeneity in portfolio constraints, rules-of-thumb, background risk, or beliefs. The transmission of monetary policy through investment is amplified given the decline in the risk premium provided that the monetary feedback rule limits any rise in the real interest rate.

2.1 Baseline environment and equilibrium

There are two periods, 0 and 1. While we later relax a number of the specific features of this environment to demonstrate the generality of our results, this baseline environment is the

[7]While these authors make this point in the case of a time-varying price of risk (as in our model), a similar result obtains in the case of New Keynesian models with a time-varying quantity of risk as in Basu and Bundick (2017) and Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez (2015) and in a neoclassical setting with money as in DiTella (2018).

one we will extend to the infinite horizon and study quantitatively in section 3 of the paper.

Households There is a unit measure of households indexed by $i \in [0,1]$, each comprising a continuum of members $j \in [0,1]$ supplying a differentiated variety of labor. There is full consumption insurance within each household. Household i has Epstein-Zin preferences over consumption in each period $\{c_0^i, c_1^i\}$ and labor supply $\{\ell_0^i(j)\}_{j=0}^1$

$$v_0^i = \left((1-\beta^i) \left(c_0^i \Phi^i \left(\int_0^1 \ell_0^i(j) dj \right) \right)^{1-1/\psi^i} + \beta^i \left(\mathbb{E}_0 \left[(c_1^i)^{1-\gamma^i} \right] \right)^{\frac{1-1/\psi^i}{1-\gamma^i}} \right)^{\frac{1}{1-1/\psi^i}} \tag{1}$$

with discount factor β^i, intertemporal elasticity of substitution ψ^i, relative risk aversion γ^i, and (dis)utility of labor given by the function $\Phi^i(\cdot)$. We assume for simplicity that households exogenously supply one unit of non-differentiated labor in period 1, though of course this assumption will be relaxed in the infinite horizon environment we study in the next section.

In addition to consuming and supplying labor, the household chooses its position in a nominal bond B_0^i and in capital k_0^i subject to the resource constraints

$$\begin{aligned} P_0 c_0^i + B_0^i + Q_0 k_0^i \leq\ & (1-\tau) \int_0^1 W_0(j) \ell_0^i(j) dj - \int_0^1 AC_0^W(j) dj\ + \\ & (1+i_{-1}) B_{-1}^i + (\Pi_0 + (1-\delta_0) Q_0) k_{-1}^i + T_0^i, \qquad (2) \\ P_1 c_1^i \leq\ & W_1 + (1+i_0) B_0^i + \Pi_1 k_0^i. \qquad (3) \end{aligned}$$

B_{-1}^i and k_{-1}^i are its endowments in these same assets. In terms of the economy's nominal unit of account ("dollars"),[8] the consumption good trades at P_t dollars at t, labor services for member j earn an after-tax wage $(1-\tau)W_0(j)$ dollars in period 0 and W_1 dollars in period 1, one dollar in bonds purchased at t yields $1+i_t$ dollars at $t+1$, and one unit of capital purchased for Q_t dollars at t yields a dividend Π_{t+1} plus non-depreciated value of capital $(1-\delta_{t+1})Q_{t+1}$ at $t+1$. We assume that capital fully depreciates after its use in period 1 ($\delta_1 = 1$), when the economy ends. Following Rotemberg (1982), in period 0 the household pays a quadratic cost of setting its wage for member j

$$AC_0^W(j) = \frac{\chi^W}{2} W_0 \ell_0 \left(\frac{W_0(j)}{W_{-1}} - 1 \right)^2$$

given some reference wage W_{-1} and the aggregate wage bill $W_0 \ell_0$ defined below. We assume this is a cost paid to the government, rebated back to households through the i-specific

[8] Following Woodford (2003), we model the economy at the cashless limit.

government transfers T_0^i.

Supply-side A union representing each labor variety j across households chooses $W_0(j), \ell_0(j)$ to maximize the social welfare of union members subject to the allocation rule

$$\ell_0^i(j) = \ell^i(\ell_0(j)) := \int_0^1 \ell^i(\ell_0(j))di = \ell_0(j) \tag{4}$$

and Pareto weights $\{\mu^i\}$.[9] A representative labor packer purchases varieties supplied by each union and combines them to produce a CES aggregate with elasticity of substitution ϵ and which it can sell at price W_0, earning profits

$$W_0 \left[\int_0^1 \ell_0(j)^{(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)} - \int_0^1 W_0(j)\ell_0(j)dj. \tag{5}$$

The representative producer hires ℓ_0 units of the labor aggregate in period 0 and ℓ_1 units of labor directly from households in period 1, and combines these with k_{t-1} units of capital rented from households each period t to produce the final good with TFP z_t. In period 0 the producer also uses $\left(\frac{k_0}{k_{-1}}\right)^{\chi^x} x_0$ units of the consumption good to produce x_0 new capital goods, where χ^x indexes the degree of adjustment costs and here we assume the representative producer takes k_0 as given. Taken together, the producer earns profits

$$\Pi_0 k_{-1} = P_0 z_0 \ell_0^{1-\alpha} k_{-1}^{\alpha} - W_0\ell_0 + Q_0 x_0 - P_0 \left(\frac{k_0}{k_{-1}}\right)^{\chi^x} x_0, \tag{6}$$

$$\Pi_1 k_0 = P_1 z_1 \ell_1^{1-\alpha} k_0^{\alpha} - W_1 \ell_1. \tag{7}$$

Future TFP is uncertain in period 0, following

$$\log z_1 \sim N\left(\log \bar{z} - \frac{1}{2}\sigma^2, \sigma^2\right). \tag{8}$$

Policy The government sets $\tau = -\frac{1}{\epsilon-1}$ to undo the effects of monopolistic competition in the labor market. The government sets lump-sum transfers T_0^i according to

$$T_0^i = \int_0^1 AC_0^W(j)dj + \tau \int_0^1 W_0(j)\ell_0^i(j)dj, \tag{9}$$

[9] We demonstrate the robustness of our analytical results to the case of individually-supplied labor by each household in appendix B. In our quantitative analysis, we find the computation of the equilibrium more robust with a union because it reduces the dimension of the fixed point to be solved. Hence, to provide analytical insights closer to the model studied quantitatively, we assume the same union structure here.

reflecting the rebating of wage adjustment costs and the payroll tax. Finally, the government sets monetary policy $\{i_0, P_1\}$ by committing to a fixed $P_1 = \bar{P}_1$, eliminating inflation risk in the nominal bond, and setting i_0 according to the feedback rule

$$1 + i_0 = (1 + \bar{i}) \left(\frac{P_0}{P_{-1}} \right)^{\phi} m_0 \tag{10}$$

consistent with a standard Taylor rule with reference price P_{-1}, where m_0 is the monetary shock of interest. Note that in this two-period setting, the equilibrium can be locally unique even when $\phi \leq 1$, including the useful benchmark case where $\phi = -1$ and hence the real interest rate between periods 0 and 1

$$1 + r_1 \equiv (1 + i_0) \frac{P_0}{P_1} = \frac{(1 + \bar{i}) P_{-1}}{\bar{P}_1} m_0,$$

so a shock to the nominal rate translates one-for-one into a shock to the real rate.[10]

Market clearing Market clearing in goods each period is

$$\int_0^1 c_0^i di + \left(\frac{k_0}{k_{-1}} \right)^{\chi^x} x_0 = z_0 \ell_0^{1-\alpha} k_{-1}^{\alpha}, \tag{11}$$

$$\int_0^1 c_1^i di = z_1 \ell_1^{1-\alpha} k_0^{\alpha}, \tag{12}$$

in labor is

$$\left[\int_0^1 \ell_0(j)^{(\epsilon-1)/\epsilon} dj \right]^{\epsilon/(\epsilon-1)} = \ell_0, \tag{13}$$

$$\int_0^1 \ell_1^i di = \ell_1, \tag{14}$$

in the capital rental market is

$$\int_0^1 k_{t-1}^i di = k_{t-1}, \quad t \in \{0, 1\}, \tag{15}$$

in the capital claims market is

$$(1 - \delta_0) \int_0^1 k_{-1}^i di + x_0 = \int_0^1 k_0^i di, \tag{16}$$

[10] As in the infinite horizon model, between periods t and $t+1$ we denote i_t the nominal interest rate known in period t and r_{t+1} the realized real interest rate depending on the price level in period $t+1$.

and in bonds is

$$\int_0^1 B_{t-1}^i di = 0. \tag{17}$$

Equilibrium Given the state variables $\{W_{-1}, P_{-1}, \{B_{-1}^i, k_{-1}^i\}, i_{-1}, z_0, m_0\}$ and stochastic process for z_1 in (8), the definition of equilibrium is then standard:

Definition 1. *An equilibrium is a set of prices and policies such that: (i) each household i chooses $\{c_0^i, B_0^i, k_0^i, c_1^i\}$ to maximize (1) subject to (2)-(3), (ii) each union j chooses $\{W_0(j), \ell_0(j)\}$ to maximize the social welfare of its members subject to the allocation rule (4), (iii) the labor packer chooses $\{\ell_0(j)\}$ to maximize profits (5), (iv) the representative producer chooses $\{\ell_0, x_0\}$ and ℓ_1 to maximize profits (6)-(7), (v) the government sets $\{T_0^i\}$ according to (9) and $\{i_0, P_1\}$ according to $P_1 = \bar{P}_1$ and (10), and (vi) the goods, labor, capital, and bond markets clear according to (11)-(17).*

Since labor varieties and unions j are symmetric, $\ell_0(j) = \ell_0$ and we drop j going forward.

We will analytically study this economy around the point with zero aggregate risk and $m_0 = \bar{m}_0$: $\{\sigma = 0, z_1 = \bar{z}_1, m_0 = \bar{m}_0\}$. For any variable n, we denote $\bar{n}$ to be its value at the point of approximation, and $\hat{n}$ its log/level deviation from this point (except for σ, which is a perturbation parameter but will not be denoted as $\hat{\sigma}$). For expositional simplicity we do not treat z_0 as a perturbation parameter of interest, but we do so in appendix B. Like monetary policy shocks, TFP shocks redistribute across households, generating state-dependence in the risk premium and affecting the transmission of TFP shocks to economic activity.

2.2 Limiting portfolios and MPRs

To understand the effects of monetary policy shocks through risk premia, it will prove useful to first understand the determinants of households' portfolios in equilibrium as well as their marginal portfolio choices given an additional unit of income.

To do so, it is helpful to re-write households' micro-level optimization problem as

$$\begin{aligned} \max \left((1-\beta^i) \left(c_0^i \Phi^i(\ell_0^i) \right)^{1-1/\psi^i} + \beta^i \left(\mathbb{E}_0 \left[(c_1^i)^{1-\gamma^i} \right] \right)^{\frac{1-1/\psi^i}{1-\gamma^i}} \right)^{\frac{1}{1-1/\psi^i}} \quad s.t. \\ c_0^i + b_0^i + q_0 k_0^i = y_0^i(w_0 \ell_0^i, P_0, \pi_0, q_0), \\ c_1^i = w_1 + (1+r_1) b_0^i + \pi_1 k_0^i, \end{aligned} \tag{18}$$

where we have denominated in lower-case the real analogs to the nominal variables introduced earlier, we have made use of the definition of the real interest rate

$$1 + r_1 \equiv (1 + i_0)\frac{P_0}{P_1},$$

and we have collected households' income in period 0 — which they take as exogenous along with $\{\ell_0^i, q_0, w_1, r_1, \pi_1\}$ — as a function of non-predetermined variables

$$y_0^i(w_0\ell_0^i, P_0, \pi_0, q_0) \equiv w_0\ell_0^i + \frac{1}{P_0}(1 + i_{-1})B_{-1}^i + (\pi_0 + (1 - \delta_0)q_0)k_{-1}^i.$$

Defining the real savings of household i chosen in period 0 as

$$a_0^i \equiv b_0^i + q_0 k_0^i,$$

its equilibrium portfolio share in capital is given by $\frac{q_0 k_0^i}{a_0^i}$ and its policy functions imply the marginal propensities to consume, save in bonds, save in capital, and save overall

$$\left\{\frac{\partial c_0^i}{\partial y_0^i}, \frac{\partial b_0^i}{\partial y_0^i}, \frac{\partial k_0^i}{\partial y_0^i}, \frac{\partial a_0^i}{\partial y_0^i}\right\},$$

where these partial derivatives hold fixed all prices and other variables which the household takes as given in (18): $\ell_0^i = \ell^i(\ell_0)$, which enters its utility, as well as q_0, r_1, and the probability distributions over w_1 and π_1. We then define a useful summary of the household's portfolio choice on the margin:

Definition 2. *Household i's* marginal propensity to take risk (MPR) *is given by*

$$mpr_0^i \equiv \frac{q_0 \frac{\partial k_0^i}{\partial y_0^i}}{\frac{\partial a_0^i}{\partial y_0^i}}.$$

The MPR summarizes the degree to which the household allocates the marginal dollar to capital instead of the bond. As we clarify in an extension where monetary policy allows P_1 to be stochastic in appendix B, it need not be that the nominal bond is riskless in real terms. Nonetheless, we give the MPR its name because under any realistic calibration the payoff on capital is more risky than on bonds.

We can better understand the structural determinants of households' portfolios and MPRs by taking their limits as aggregate risk falls to zero. In doing so, we apply techniques developed by Devereux and Sutherland (2011) in the context of open-economy macroeconomics to the present heterogeneous agent New Keynesian environment and our particular statistics of interest. These authors take the difference across countries of a second-order approximation to optimal portfolio choice for each country, and then make use of the method of undetermined coefficients, to characterize cross-country portfolio shares as aggregate risk falls to zero. Analogous steps can be used to characterize households' portfolio shares in the present environment. Moreover, a second-order approximation to the *partial derivatives* of the first-order conditions of (18) with respect to y_0^i can be used to characterize the *marginal* portfolio responses to income as aggregate risk falls to zero. These steps lead to the first result of the paper, the proof of which (along with all other proofs of results in the paper) is in appendix A:

Proposition 1. *At the limit of zero aggregate risk, household i's portfolio share in capital is*

$$\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} = \left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i} \right) \frac{\gamma}{\gamma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i},$$

and its MPR is

$$\overline{mpr}_0^i = \frac{\gamma}{\gamma^i},$$

where

$$\gamma = \left[\int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \frac{1}{\gamma^i} di \right]^{-1} \tag{19}$$

is the harmonic average of risk aversion weighted by households' period 1 consumption.

Intuitively, a household's portfolio share in capital and MPR is higher the less risk averse it is relative to the appropriately-defined average household in the economy. Even though we are asking how the individual household allocates wealth both in equilibrium and in response to a marginal dollar of income, the risk aversion of all other households is relevant because this controls the prices facing the household in general equilibrium.

Proposition 1 further clarifies two useful points regarding the MPR. First, it captures a dimension of heterogeneity in principle orthogonal to the marginal propensities to consume and save which have been emphasized in prior work: while in the limit of zero aggregate risk the latter are fully determined by households' attitudes towards consumption across *dates* (discount factors and intertemporal elasticities of substitution), MPRs are governed by attitudes towards consumption across *states* (relative risk aversion). Second, the MPR may be quite distinct from observed portfolios because it captures portfolio allocation on the margin. Indeed, a household's portfolio share in capital depends not only on risk aversion but its motive to hedge labor income also subject to TFP shocks, captured by $\frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i}$ in (1). While this hedging motive matters for equilibrium portfolios, it is irrelevant on the margin.

2.3 Monetary policy, redistribution, and risk premia

The joint distribution of household portfolios and MPRs determines the effect of a monetary policy shock on the expected excess returns on capital, to which we now turn.

Let $1 + r_1^k$ denote the gross real returns on capital

$$1 + r_1^k \equiv \frac{\Pi_1}{Q_0}\frac{P_0}{P_1} = \frac{\pi_1}{q_0}.$$

Combining household i's first-order conditions with respect to bonds and capital yields

$$\mathbb{E}_0\left[\left(c_1^i\right)^{-\gamma^i}\left(r_1^k - r_1\right)\right] = 0.$$

Approximating this condition up to third order in the perturbation parameters, and using market clearing in bonds and capital, we obtain:

Proposition 2. *Up to third order in the perturbation parameters $\{\sigma, \hat{z}_1, \hat{m}_0\}$, the risk premium on capital is*

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \zeta_{m_0}\hat{m}_0\sigma^2 + o(||\cdot||^4), \tag{20}$$

where γ was defined in (19) and

$$\zeta_{m_0} = \gamma\int_0^1 \bar{\xi}_{m_0}^i\left(\overline{mpr}_0 - \overline{mpr}_0^i\right)di, \tag{21}$$

where $\bar{\xi}_{m_0}^i \equiv \overline{\frac{d\left[c_1^i/\int_0^1 c_1^{i'}di'\right]}{dm_0}}$ is the effect of a monetary shock on household i's consumption share in period 1, and $\overline{mpr}_0 \equiv \int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'}di'}\overline{mpr}_0^i di = 1$ is the weighted average MPR in (1).

The coefficient of interest is ζ_{m_0}, summarizing the effect of a monetary policy shock on the risk premium. As is evident, in this simple two-period model a monetary policy shock affects the risk premium only through redistribution.[11] If monetary policy does not redistribute ($\bar{\xi}_{m_0}^i = 0$ for all i) or households have identical MPRs ($\overline{mpr}_0^i = \overline{mpr}_0 = 1$ for all i), the shock has no effect on the risk premium. Away from this case, redistributing wealth to households with relatively high MPRs will lower the risk premium. Intuitively, such redistribution raises the relative demand for capital versus bonds, lowering the required excess returns to clear asset markets.

[11] This is no longer the case in the infinite horizon where, for instance, monetary policy affects the future path of real interest rates and thus the intertemporal hedging demand for capital.

The relevant measure of redistribution toward household i is the (endogenous) change in its future consumption share

$$\bar{\xi}^i_{m_0} \equiv \overline{\frac{d\left[c^i_1/\int_0^1 c^{i'}_1 di'\right]}{dm_0}},$$
$$= \frac{1}{\int_0^1 \bar{c}^{i'}_1 di'}\left[\overline{\frac{dc^i_1}{dm_0}} - \frac{\bar{c}^i_1}{\int_0^1 \bar{c}^{i'}_1 di'}\int_0^1 \overline{\frac{dc^{i'}_1}{dm_0}} di'\right]. \tag{22}$$

Using standard tools from price theory, we can decompose each household's change in future consumption as follows:

Proposition 3. *A household's change in future consumption in response to a monetary policy shock is given by*

$$\overline{\frac{dc^i_1}{dm_0}} = (1+\bar{r}_1)\underbrace{\overline{\frac{\partial a^i_0}{\partial y^i_0}}}_{MPS}\Bigg[\underbrace{-\frac{(1+i_{-1})B^i_{-1}}{\bar{P}_0}\frac{1}{\bar{P}_0}\overline{\frac{dP_0}{dm_0}} + k^i_{-1}\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right)}_{\text{balance sheet revaluation}}$$
$$+\underbrace{\left(\overline{\frac{dw_0\ell^i_0}{dm_0}} + \frac{1}{1+\bar{r}_1}\overline{\frac{dw_1}{dm_0}}\right)}_{\text{change in non-traded income}} + \underbrace{\bar{a}^i_0\frac{1}{1+\bar{r}_1}\overline{\frac{d(1+r_1)}{dm_0}}}_{\text{income effect}}$$
$$+\underbrace{\psi^i c^i_0\frac{1}{1+\bar{r}_1}\overline{\frac{d(1+r_1)}{dm_0}} + \left(\psi^i - 1\right)\bar{w}_0\left(1-\bar{\tau}^{\ell^i_0}\right)\overline{\frac{d\ell^i_0}{dm_0}}}_{\text{substitution effects}}\Bigg]$$

given the labor wedge for household i $\bar{\tau}^{\ell^i_0} \equiv 1 - \frac{-\bar{c}^i_0\Phi^{i'}(\bar{\ell}^i_0)/\Phi^i(\bar{\ell}^i_0)}{(1-\alpha)\bar{z}_0(\bar{\ell}_0)^{-\alpha}k^{\alpha}_{-1}}$.

The resulting redistribution summarized in (22) reflects heterogeneity in the marginal propensity to save; heterogeneity in changes in wealth; and heterogeneity in substitution effects. We focus on the second here.[12]

First, a cut in the nominal interest rate will trigger a revaluation of household balance sheets. If it generates unexpected inflation ($\overline{\frac{dP_0}{dm_0}} < 0$), it will redistribute toward net debtors in the nominal bond. If it raises short-run profits ($\overline{\frac{d\pi_0}{dm_0}} < 0$) or raises asset prices capitalizing the future stream of profits ($\overline{\frac{dq_0}{dm_0}} < 0$), it will redistribute toward owners of capital. Second, a cut in the nominal interest rate will change the net present value of non-traded labor income. If it lowers the real wage in the short run ($\overline{\frac{dw_0}{dm_0}} > 0$), the standard effect with sticky

[12] The role of the marginal propensity to save and substitution effect due to a change in the real interest rate is straightforward. The substitution effect due to a change in $\bar{\ell}^i_0$ reflects the non-separability of labor from consumption in period 0 when $\psi^i \neq 1$.

nominal wages rather than prices, it will redistribute to households supplying less labor. If it raises the quantity of labor demanded ($\overline{\frac{d\ell_0}{dm_0}} < 0$), it will redistribute to households whose labor demand is especially sensitive to the aggregate. Third, if a cut in the nominal interest rate lowers the equilibrium real interest rate ($\overline{\frac{d(1+r_0)}{dm_0}} > 0$), it will redistribute wealth away from net savers through a Slutsky income effect. These heterogeneous exposures to a monetary shock have been previously exposited in the literature on HANK models, as by Auclert (2018).[13] Our analysis demonstrates that it is their covariance with MPRs rather than MPCs which matters for transmission through risk-premia rather than risk-free rates.

A sufficient condition for the aggregate effects of a monetary policy shock to take the signs conjectured in the previous paragraph is that the degree of nominal rigidity χ^W is sufficiently high.[14] In the useful benchmark case where households differ in risk aversion, households' endowments entering period 0 are consistent with their chosen portfolios that period,[15] and households are otherwise fully symmetric, Propositions 1-3 then imply that an expansionary monetary policy shock will lower the risk premium through these channels.

By Proposition 1, more risk tolerant households will hold levered positions in capital, and will further allocate more of the marginal dollar to capital (have a higher MPR). By Proposition 3 and the above discussion, a monetary expansion will redistribute to these households through the balance sheet revaluation channel, devaluing their debt burden,

[13] We can re-arrange the terms in Proposition 3 to obtain a decomposition consistent with Auclert (2018). Since $\bar{q}_0 = \frac{\bar{\pi}_1}{1+\bar{r}_1}$, we have that

$$\overline{\frac{dq_0}{dm_0}} = \frac{1}{1+\bar{r}_1}\overline{\frac{d\pi_1}{dm_0}} - \frac{\bar{q}_0}{1+\bar{r}_1}\overline{\frac{d(1+\bar{r}_1)}{dm_0}}.$$

It follows that we can equivalently write the sum of the balance sheet revaluation, change in non-traded income, and income effect as

$$\left[-\frac{(1+i_{-1})B^i_{-1}}{\bar{P}_0}\frac{1}{\bar{P}_0}\overline{\frac{dP_0}{dm_0}}\right] + \left[\left(\bar{b}^i_0 + \bar{q}_0(\bar{k}^i_0 - (1-\delta_0)k^i_{-1})\right)\frac{1}{1+\bar{r}_1}\overline{\frac{d(1+r_1)}{dm_0}}\right] +$$
$$\left[k^i_{-1}\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\frac{1}{1+\bar{r}_1}\overline{\frac{d\pi_1}{dm_0}}\right) + \left(\overline{\frac{dw_0\ell^i_0}{dm_0}} + \frac{1}{1+\bar{r}_1}\overline{\frac{dw_1}{dm_0}}\right)\right].$$

The first bracketed term corresponds to the Fisher channel, the second bracketed term to the interest rate exposure channel, and the third bracketed term to the earnings heterogeneity channel in Auclert (2018). In our decomposition, we find it convenient to explicitly account for the effect on the price of capital to aid the interpretation of our quantitative simulations in the next section.

[14] Consider the effects of an increase in the nominal price level $\bar{P}_0$ induced by the monetary policy shock on aggregate labor $\bar{\ell}_0$. First, by lowering the real wage, the increase in $\bar{P}_0$ will stimulate $\bar{\ell}_0$ through the standard Keynesian labor demand channel. Second, by raising the real interest rate, putting downward pressure on investment and thus (through the resource constraint) upward pressure on consumption, the increase in $\bar{P}_0$ will lower $\bar{\ell}_0$ through the wealth effect on labor supply. A higher degree of nominal rigidity lowers the importance of labor supply relative to labor demand in determining equilibrium in the labor market.

[15] In the infinite horizon, this is indeed the case when approximating around the deterministic steady-state.

raising their profits earned on capital, and raising the re-sale value of their capital. When households are symmetric in their claims on labor income, total wealth, and intertemporal elasticities of substitution, the balance sheet revaluation channel will be the only one which redistributes across households. Taken together, Proposition 2 implies that the risk premium will fall. This is formalized in the following result, where we further assume zero wage inflation at the point of approximation ($W_{-1} = \bar{W}_0$) only to simplify the exposition of the proof.

Proposition 4. *Suppose that at the point of approximation households' initial endowments in bonds and capital are identical to their choices in period 0 ($B^i_{-1} = \bar{P}_0 \bar{b}^i_0$, $k^i_{-1} = \bar{k}^i_0$) and there is zero wage inflation ($W_{-1} = \bar{W}_0$). Suppose households differ in risk aversion $\{\gamma^i\}$ but their total wealth and all other preference parameters, technologies, and endowments are fully symmetric. Then for χ^W sufficiently big, $\zeta_{m_0} > 0$ and hence a cut in the nominal interest rate lowers the risk premium on capital.*

This analytical benchmark is useful because, as we later show, redistribution through balance sheet revaluation indeed drives the risk premium effects of monetary policy in our quantitative analysis in section 3.

2.4 Generalizations to other sources of heterogeneity

The preceding results do not rely on heterogeneity in preferences alone. We demonstrate in this section that they generalize to environments with heterogeneity in portfolio constraints, rules-of-thumb, background risk, or beliefs. Importantly, across these cases the MPR remains the relevant statistic to evaluate the effects of redistribution on risk premia. The joint distribution of exposures and MPRs induced by these sources of heterogeneity continue to imply that an expansionary monetary policy shock will lower the risk premium.

Binding constraints or rules-of-thumb Suppose a measure of households in the set C are not at an interior optimum in their portfolio choice because of the additional constraint

$$q_0 k^i_0 = \omega^i_0 a^i_0,$$

reflecting either a binding leverage constraint or a rule-of-thumb in their portfolio allocation. When $\omega^i_0 = 0$ in particular, this means the household cannot trade capital, as in models of limited participation. In this setting we then generalize Propositions 1-2 to:

Corollary 1. *With binding constraints or rules-of-thumb, households' limiting portfolios and MPRs are*

$$\frac{\bar{q}_0\bar{k}_0^i}{\bar{a}_0^i} = \begin{cases} \omega_0^i & \text{for } i \in C, \\ \left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i}\right)\frac{\gamma}{\gamma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i} & \text{for } i \notin C, \end{cases}$$

$$\overline{mpr}_0^i = \begin{cases} \omega_0^i & \text{for } i \in C, \\ \frac{\gamma}{\gamma^i} & \text{for } i \notin C, \end{cases}$$

where

$$\gamma = \left(\int_{i\notin C} \frac{\bar{c}_1^i}{\int_{i'\notin C}\bar{c}_1^{i'}di'}\frac{1}{\gamma^i}di\right)^{-1}\left(1 - \frac{\int_{i\notin C}(1+\bar{r}_1)\bar{b}_0^i di}{\int_{i\notin C}\bar{c}_1^i di}\right).$$

Up to third order in $\{\sigma, \hat{z}_1, \hat{m}_0\}$*, we obtain (20) with* γ *defined above and*

$$\zeta_{m_0} = \left(\int_{i\notin C} \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'}di'}\frac{1}{\gamma^i}di\right)^{-1}\int_0^1 \bar{\xi}_{m_0}^i\left(\overline{mpr}_0 - \overline{mpr}_0^i\right)di,$$

where $\bar{\xi}_{m_0}^i \equiv \overline{\frac{d\left[(1+r_0)a_0^i/\int_0^1 c_1^{i'}di'\right]}{dm_0}}$ *for* $i \in C$*,* $\bar{\xi}_{m_0}^i \equiv \overline{\frac{d\left[c_1^i/\int_0^1 c_1^{i'}di'\right]}{dm_0}}$ *for* $i \notin C$*, and*
$\overline{mpr}_0 \equiv \int_{i\in C}\frac{(1+\bar{r}_1)\bar{a}_0^i}{\int_{i'\in C}(1+\bar{r}_1)\bar{a}_0^{i'}di' + \int_{i'\notin C}\bar{c}_1^{i'}di'}\overline{mpr}_0^i di + \int_{i\notin C}\frac{\bar{c}_1^i}{\int_{i'\in C}(1+\bar{r}_1)\bar{a}_0^{i'}di' + \int_{i'\notin C}\bar{c}_1^{i'}di'}\overline{mpr}_0^i di = 1.$

The risk premium γ now depends not only on the weighted average risk aversion of unconstrained households, but also on the leverage which these households must take in aggregate to hold the economy's capital stock after accounting for the positions of constrained households. For this reason, the effect of a monetary policy shock on the risk premium ζ_{m_0} depends on the MPRs of constrained households. For instance, if wealth transfers to households who cannot trade capital and thus have $\overline{mpr}_0^i = 0$, in equilibrium the remaining households must be induced to hold a more levered position in capital and thus the risk premium must rise. This is consistent with prior asset pricing models with segmented markets or rules-of-thumb such as Guvenen (2009), Chien et al. (2012), and He and Krishnamurthy (2013) as well as macro models of the financial accelerator such as Bernanke et al. (1999).

Background risk Suppose households are subject to idiosyncratic risk beyond the aggregate risk already described: their labor productivity and quality of capital together are subject to a shock ϵ_1^i, where both are modeled as changes in the efficiency units of each factor in the production function. ϵ_1^i is *iid* across households, independent of the aggregate TFP shock z_1, and follows

$$\log \epsilon_1^i \sim N\left(-\frac{1}{2}\eta^i\sigma^2, \eta^i\sigma^2\right).$$

We then generalize Propositions 1-2 to:[16]

Corollary 2. *With background risk, households' limiting portfolios and MPRs are*

$$\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} = \left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i}\right)\frac{\gamma}{\gamma^i(1+\eta^i)} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i},$$
$$\overline{mpr}_0^i = \frac{\gamma}{\gamma^i(1+\eta^i)}.$$

where

$$\gamma = \left(\int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \frac{1}{\gamma^i(1+\eta^i)} di\right)^{-1}.$$

Up to third order in $\{\sigma, \hat{z}_1, \hat{m}_0\}$, we obtain (20) with γ defined above and

$$\zeta_{m_0} = \gamma \int_0^1 \bar{\xi}_{m_0}^i \left(\overline{mpr}_0 - \overline{mpr}_0^i\right) di,$$

where $\bar{\xi}_{m_0}^i \equiv \overline{\frac{d\left[c_1^i / \int_{i' \notin C} c_1^{i'} di'\right]}{dm_0}}$ and $\overline{mpr}_0 \equiv \int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \overline{mpr}_0^i di = 1$.

This environment captures features of the large literatures in macroeconomics and asset pricing with uninsurable labor income risk and/or entrepreneurial income risk. Corollary 2 is consistent with the empirical finding that households with greater background risk η^i uncorrelated with the stock market hold a lower portfolio share in stocks (Heaton and Lucas (2000)). It also implies that households with different amounts of background risk will have different MPRs. Redistribution across these households in turn will induce changes in the risk premium owing to their trades in asset markets.

Subjective beliefs Suppose households differ in their beliefs regarding TFP. In particular, household i believes

$$\log z_1 \sim N\left(\log \bar{z}_1 - \frac{1}{2}\varsigma^i \sigma^2, \varsigma^i \sigma^2\right)$$

even though the objective (true) probability distribution remains described by (8). We may label households with $\varsigma^i > 1$ "pessimists" and households with $\varsigma^i < 1$ "optimists". We then generalize Propositions 1-2 to:

[16] We continue to define $1 + r_1^k \equiv \frac{\pi_1}{q_0}$, a claim on capital aggregating over the idiosyncratic risk, even though each household i faces the set of asset returns $\{1 + r_0, 1 + r_1^{k,i} \equiv \frac{\pi_1 \epsilon_1^i}{q_0}\}$.

Corollary 3. *With subjective beliefs, households' limiting portfolios and MPRs are*

$$\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} = \left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i} \right) \frac{\gamma}{\gamma^i \varsigma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i},$$
$$\overline{mpr}_0^i = \frac{\gamma}{\gamma^i \varsigma^i}.$$

where

$$\gamma = \left(\int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \frac{1}{\gamma^i \varsigma^i} di \right)^{-1}.$$

Up to third order in $\{\sigma, \hat{z}_1, \hat{m}_0\}$, *we obtain (20) with* γ *defined above and*

$$\zeta_{m_0} = \gamma \int_0^1 \bar{\xi}_{m_0}^i \left(\overline{mpr}_0 - \overline{mpr}_0^i \right) di,$$

where $\bar{\xi}_{m_0}^i \equiv \overline{\frac{d\left[c_1^i / \int c_1^{i'} di'\right]}{dm_0}}$ *and* $\overline{mpr}_0 \equiv \int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \overline{mpr}_0^i di = 1$.

Consistent with the empirical literature on belief disagreements, as in the work of Giglio, Maggiori, Stroebel, and Utkus (2019) and Meeuwis, Parker, Schoar, and Simester (2019), beliefs will be reflected in individual portfolio choice and marginal portfolio responses to shocks. Consistent with the theoretical literature exploring the consequences of such disagreements, as in the work of Geanakoplos (2009), Simsek (2013), and Caballero and Simsek (2018), these individual responses can then have important consequences for the macroeconomy through their effect on asset prices.

Robustness of the effects of monetary policy In each of the above cases, the additional dimension of heterogeneity continues to imply that households with a high portfolio share in capital also have a high MPR. It follows that this remains true in an environment where households can vary along *all* of these dimensions. Proposition 3 summarizing households' exposure to a monetary policy shock remains unchanged. Hence, an expansionary monetary policy shock will redistribute to high MPR households as in the case with heterogeneity in risk aversion alone, lowering the risk premium and generalizing Proposition 4:

Proposition 5. *Suppose that at the point of approximation households' initial endowments in bonds and capital are identical to their choices in period 0 (*$B_{-1}^i = \bar{P}_0 \bar{b}_0^i$, $k_{-1}^i = \bar{k}_0^i$*) and there is zero wage inflation (*$W_{-1} = \bar{W}_0$*). Suppose households differ in risk aversion* $\{\gamma^i\}$*, being constrained and (among those that are) constraints* $\{\omega^i\}$*, the degree of background risk* $\{\eta^i\}$*, and beliefs* $\{\varsigma^i\}$*; but their total wealth and all other preference parameters, technologies, and endowments are fully symmetric. Then for* χ^W *sufficiently big,* $\zeta_{m_0} > 0$ *and hence a cut* *in the nominal interest rate lowers the risk premium on capital.*

In this sense, while our quantitative analysis focuses on differences in risk aversion, we expect that our insights are robust to these other potential sources of heterogeneity generating the same distribution of MPRs and exposures to a monetary policy shock.

2.5 Monetary policy, redistribution, and investment

Before turning to our quantitative analysis in the infinite horizon, we conclude our analysis of this environment by asking how the risk premium effects of monetary policy matter for the transmission of policy to the real economy. We focus in particular on aggregate investment.

In terms of the model's perturbation parameters, new capital is given by[17]

$$\hat{k}_0 = \underbrace{\delta^{k_0}_{m_0}\hat{m}_0 + \tfrac{1}{2}\delta^{k_0}_{m_0^2}\hat{m}_0^2 + \tfrac{1}{6}\delta^{k_0}_{m_0^3}\hat{m}_0^3}_{\substack{\text{effects absent}\\ \text{aggregate risk}}} + \frac{1}{2}\delta^{k_0}_{\sigma^2}\sigma^2 + \underbrace{\tfrac{1}{2}\delta^{k_0}_{m_0\sigma^2}\hat{m}_0\sigma^2}_{\substack{\text{effect with}\\ \text{aggregate risk}}} + o(||\cdot||^4) \tag{23}$$

for some coefficients $\delta^{k_0}_{\cdot}$. The first three terms reflect the effects of a monetary policy shock on investment in environments even absent aggregate risk, and they are well understood. They reflect the standard channels through intertemporal substitution as well as heterogeneity in the marginal propensities to consume versus save. We instead focus on the additional effects of a monetary policy shock in environments with aggregate risk, summarized in the term $\delta^{k_0}_{m_0\sigma^2}$. Equivalently, we seek to understand the investment responses which accompany a change in the risk premium summarized by ζ_{m_0} in the previous results.

Since the return on capital is

$$1 + r^k_1 = \frac{\pi_1}{q_0} = \frac{\alpha z_1 k_0^{\alpha-1}}{(k_0/k_{-1})^{\chi^x}},$$

we have that the expected return on capital is

$$\mathbb{E}_0\hat{r}^k_1 = -\left(1 - \alpha + \chi^x\right)\hat{k}_0.$$

That is, the required return on capital falls in the amount of new capital both because the marginal product of capital falls and the price of capital rises, the latter reflecting optimal investment among producers. Now consider the case where monetary policy redistributes to high MPR households and thus lowers the risk premium. Then to evaluate the effects

[17] Formally, we work with new capital k_0 rather than investment x_0 only for expositional simplicity: $1+r^k_1$ is exactly log-linear in k_0 but not x_0, recalling that $k_0 = (1-\delta_0)k_{-1} + x_0$. However, the results presented here extend naturally from k_0 to x_0.

on investment we must determine whether the required return on capital falls (and thus investment rises) or the safe real interest rate simply rises (and thus investment may remain unchanged or even fall).

Absent nominal rigidity, the equilibrium response of the real interest rate depends crucially on households' intertemporal elasticity of substitution. This reflects the presence of offsetting income and substitution effects in response to the shock. As described in Gourio (2012) and other papers in the literature on time-varying risk premia, in the case of a unitary intertemporal elasticity of substitution the real interest rate varies by exactly the amount to keep investment unchanged. This can explain what Cochrane (2017) calls the "macro-finance separation" in joint analyses of asset pricing and business cycles such as Tallarini Jr. (2000).

With nominal rigidity, Caballero and Farhi (2018) and Caballero and Simsek (2018) point out that the monetary policy rule also determines the extent to which the real interest rate fluctuates. These authors demonstrate, for instance, that a decrease in the risk premium will induce a boom if the nominal rate remains at the zero lower bound. In our environment, these insights apply to the risk premium effect of a primitive monetary policy shock itself. In the useful benchmark in which monetary policy targets a constant real interest rate subject to monetary policy shocks ($\phi = -1$), we obtain:

Proposition 6. *If monetary policy follows the rule (10) with $\phi = -1$, then in (23)*

$$\frac{1}{2}\delta^{k_0}_{m_0\sigma^2} = -\frac{1}{1-\alpha+\chi^x}\zeta_{m_0}.$$

Hence, if monetary policy lowers the risk premium by redistributing to high MPR households, as under the conditions in Proposition 5, it will amplify the stimulus to investment.

3 Quantitative relevance in the infinite horizon

We now evaluate the quantitative relevance of these insights in an extended infinite horizon setting. We first revisit the empirical evidence on the equity premium response to monetary policy shocks which poses a strong challenge to existing New Keynesian frameworks in which the equity premium barely moves. We then calibrate our model to match standard "macro" moments as well as novel "micro" moments from the Survey of Consumer Finances which discipline the cross-sectional heterogeneity in MPRs and exposures to monetary policy. In response to an unexpected monetary easing in our model economy, wealth endogenously redistributes to relatively high MPR households, rationalizing the equity premium response found in the data and contributing to the stimulus in real activity through investment.

3.1 Empirical effects of monetary policy shocks in U.S. data

The effects of an unexpected shock to monetary policy have been the subject of a large literature in empirical macroeconomics. In response to an unexpected loosening, this literature finds that the price level rises and production expands, consistent with workhorse New Keynesian models. But, as found in Bernanke and Kuttner (2005) and a number of subsequent papers using asset pricing data, the evidence further suggests that risk premia fall, posing a challenge to workhorse models where risk premia barely move.[18]

We refresh the findings in Bernanke and Kuttner (2005) using the structural vector autoregression instrumental variables (SVAR-IV) approach in Gertler and Karadi (2015). Using monthly data from July 1979 through June 2012, we first run a six-variable, six-lag VAR using the 1-year Treasury yield, CPI, industrial production, S&P 500 return relative to the 1-month T-bill, 1-month T-bill relative to the change in CPI, and smoothed dividend-price ratio on the S&P 500.[19,20] Over January 1991 through June 2012 we then instrument the residuals in the 1-year Treasury yield (the *monetary policy indicator*) with an external instrument: Fed Futures surprises on FOMC days aggregated to the month level from Gertler and Karadi (2015). The identification assumptions are that the exogenous variation in the monetary policy indicator in the VAR are due to the structural monetary shock and that the instrument is correlated with this structural shock but not the five others in the VAR. Under these assumptions, a first-stage regression of the monetary policy residual on the surprise, followed by a second-stage regression of all other residuals on the predicted residual, can be used to identify the effects of a monetary policy shock on all variables in the VAR.

We first demonstrate the validity of the first stage in Table 1. We consider 5 possible measures of policy surprises constructed in Gertler and Karadi (2015): using the current Fed Funds futures contract; the 3-month ahead Fed Funds futures contract; the 2-quarters ahead 3-month Eurodollar contract; the 3-quarters ahead 3-month Eurodollar contract; or the 4-quarters ahead 3-month Eurodollar contract. In all cases, the 1-year Treasury yield rises given a positive surprise, as would be expected. The effects are statistically significant at conventional levels, and for the first two instruments the F statistics above 10 exceed the threshold of a strong instrument recommended by Stock, Wright, and Yogo (2002). Since shocks in our model will be to the current nominal interest rate, to maximize comparability

[18]This effect on risk premia may co-exist with the revelation of information due to the shock, a channel studied by Nakamura and Steinsson (2018) and others. The analysis of Jarocinski and Karadi (forthcoming) implies that by confounding "pure" monetary policy shocks with such information shocks in our analysis, our estimates may understate the increase in the stock market following a pure monetary easing.

[19]The series for the 1-year Treasury yield, CPI, and industrial production are taken from the dataset provided by Gertler and Karadi (2015). The remaining series are from CRSP.

[20]The smoothed dividend-price ratio is computed as the 3-month moving average of dividends paid in a month divided by the price of the stock at the end of the month, value-weighted over stocks in the S&P 500.

	1-year Treasury yield					
Current Fed Funds	0.85					0.25
	(0.22)					(0.43)
Expected Fed Funds, 3 mos ahead		1.15				1.26
		(0.28)				(0.46)
Expected 3-mo ED rate, 2 qtrs ahead			0.84			1.72
			(0.31)			(1.38)
Expected 3-mo ED rate, 3 qtrs ahead				0.68		-4.90
				(0.30)		(1.82)
Expected 3-mo ED rate, 4 qtrs ahead					0.70	3.04
					(0.31)	(1.05)
Observations	258	258	258	258	258	258
Adj R^2	0.05	0.06	0.03	0.02	0.02	0.07
F-statistic	14.46	16.27	7.59	5.24	5.00	6.39

Table 1: effects of monetary policy instruments on first-stage residuals of VAR

Notes: heteroskedasticity-robust standard errors given in parenthesis.

we focus on the current Fed Funds futures contract as our baseline instrument.

We then plot the impulse responses to a negative monetary policy shock using this instrument in Figure 1. Since the structural monetary policy shock is not observed, its magnitude should be interpreted through the lens of the 22*bp* decrease in the 1-year yield on impact. Consistent with the wider literature, industrial production and the price level rise, and the real interest rate falls. Excess returns rise by 2*pp* in the first month but fall to be small and negative in the months which follow. This is consistent with a decline in the equity premium, also suggested by the fall in the dividend/price ratio.

Following Bernanke and Kuttner (2005), we can more formally decompose this 2*pp* excess return on the stock market into the contribution from news about higher dividend growth, lower real risk-free discount rates, and lower future excess returns using a Campbell-Shiller decomposition:

$$\begin{aligned}(\text{excess return})_t - \mathbb{E}_{t-1}[(\text{excess return})_t] &= (\mathbb{E}_t - \mathbb{E}_{t-1})\sum_{j=0}\kappa^j\Delta(\text{dividends})_{t+j} \\ - (\mathbb{E}_t - \mathbb{E}_{t-1})\sum_{j=0}\kappa^j(\text{real rate})_{t+j} &- (\mathbb{E}_t - \mathbb{E}_{t-1})\sum_{j=1}\kappa^j(\text{excess return})_{t+j},\end{aligned} \tag{24}$$

where $\kappa = \frac{1}{1+\frac{d}{p}}$ and $\frac{d}{p}$ is the steady-state dividend yield. Using the SVAR-IV to compute the revised expectations in real rates and excess returns given the monetary shock, we obtain the decomposition in Table 2.[21] 1.1*pp* (55%) of the increase in the stock market is due to

[21]Following Bernanke and Kuttner (2005), our VAR enables us to compute $(\text{excess return})_t -$

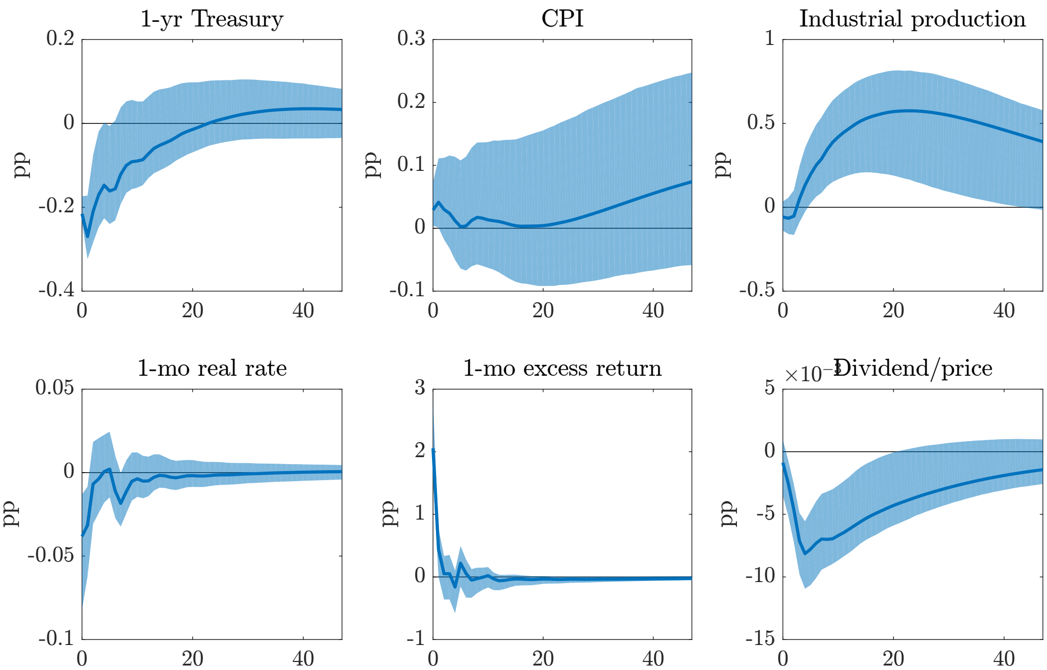

Figure 1: effects of 1 SD monetary shock given current Fed Funds instrument

Notes: 95% confidence interval at each horizon is computed using the wild bootstrap (to account for uncertainty in the coefficients of the VAR) with 10,000 iterations, following Mertens and Ravn (2013) and Gertler and Karadi (2015).

news about lower future excess returns, $0.7pp$ (35%) is due to news about higher dividend growth, and only $0.2pp$ (10%) is due to news about lower risk-free rates. Hence, the SVAR-IV approach validates the message from Bernanke and Kuttner (2005): monetary policy shocks primarily change excess returns through their effects on risk premia.

The important role of the risk premium in explaining the change in excess returns is robust to details of the estimation approach. In appendix C.1 we modify the estimation approach along a number of dimensions. First, we change the number of lags used in the VAR, ranging from 4 months to 8 months. Second, we change the sample periods over which the VAR and/or first-stage is estimated. Third, we add variables to the VAR, such as the Gilchrist and Zakrajsek (2012) excess bond premium and other credit spreads used in Gertler and Karadi (2015) as well as the term spread and other variables known to predict excess stock market returns used in Bernanke and Kuttner (2005). Fourth, we change the instrument used for the monetary policy shock, using the three-month ahead Fed Funds futures contract

$\mathbb{E}_{t-1}[(\text{excess return})_t]$, $(\mathbb{E}_t - \mathbb{E}_{t-1})\sum_{j=0}\rho^j(\text{real rate})_{t+j}$, and $(\mathbb{E}_t - \mathbb{E}_{t-1})\sum_{j=1}\rho^j(\text{excess return})_{t+j}$, and we assign to dividend growth the residual implied by (24). The alternative of directly including dividend growth on the S&P 500 in the VAR is made complicated by their strong seasonality in the data.

Current excess return	2.06
	[1.60,2.83]
Dividend growth news	0.72
	[-0.35, 1.78]
Real rate news	-0.21
	[-0.54, 0.13]
Future excess return news	-1.12
	[-2.46, -0.15]

Table 2: effects of 1 SD monetary shock on current excess returns and components

Notes: decomposition in (24) uses $\kappa = 0.9962$ following Campbell and Ammer (1993), and 95% confidence interval in brackets is computed using the wild bootstrap (to account for uncertainty in the coefficients of the VAR) with 10,000 iterations, following Mertens and Ravn (2013) and Gertler and Karadi (2015).

rather than the current contract. Across these cases we confirm the message of the baseline estimates above: in response to a monetary policy shock which reduces the 1-year Treasury yield by 17-23*bp*, excess returns on impact rise by 1.6-3.2*pp*, and news about future excess returns explains 35%-75% of this increase.

Unlike a local projection, the use of a VAR enables us to generate the long-horizon forecasts needed to implement the decomposition in (24). As noted by Stock and Watson (2018), we can test the assumption of invertibility implicit in the SVAR-IV approach both by assessing whether lagged values of the instrument have forecasting power when included in the VAR and by comparing the estimated impulse responses to those obtained using a local projection with instrumental variables (LP-IV). We demonstrate in appendix C.1 that both of these tests fail to reject the null hypothesis that invertibility in our application is satisfied. We further provide a visual comparison of the impulse responses over 4 years using the SVAR-IV and LP-IV approaches. Our estimates using the SVAR-IV lie within the (quite large) confidence intervals obtained using the LP-IV at virtually every horizon.

3.2 Infinite horizon environment

We now investigate whether redistribution can quantitatively rationalize the stock market response to a monetary policy shock and, if so, its implications for policy transmission to the real economy. We first extend the model to the infinite horizon, building on the environment from section 2.1. We describe the necessary changes when moving to the infinite horizon here and more fully describe the environment and our solution approach in appendix D.

3.2.1 Household preferences

Household i now maximizes a generalization of (1)

$$v_t^i = \left((1-\beta) \left(c_t^i \Phi \left(\int_0^1 \ell_t^i(j) dj \right) \right)^{1-1/\psi} + \beta \mathbb{E}_t \left[\left(v_{t+1}^i \right)^{1-\gamma^i} \right]^{\frac{1-1/\psi}{1-\gamma^i}} \right)^{\frac{1}{1-1/\psi}}, \tag{25}$$

with endogenous labor supply each period. We assume for simplicity that β, ψ, and $\Phi(\cdot)$ are now identical across types. We further assume for simplicity that the labor allocation rule and Pareto weights are symmetric: $\ell^i(\ell_t) = \ell_t$ and $\mu^i = 1$. We parameterize the disutility of labor as in Shimer (2010),

$$\Phi(\ell_t^i) = \left(1 + (1/\psi - 1)\, \bar{\theta} \frac{(\ell_t^i)^{1+1/\theta}}{1+1/\theta} \right)^{\frac{1/\psi}{1-1/\psi}}, \tag{26}$$

where θ controls the Frisch elasticity of labor supply and $\bar{\theta}$ controls the disutility of labor.

3.2.2 Aggregate productivity

In the infinite horizon we need to specify the dynamics of aggregate productivity. In line with previous work studying asset prices in production economies,[22] we assume that productivity z_t follows a unit root process

$$\log(z_t) = \log(z_{t-1}) + \sigma \epsilon_t^z + \varphi_t, \tag{27}$$

where ϵ_t^z is an *iid* shock from a standard Normal distribution and φ_t is a rare disaster equal to zero with probability $1-p$ and $\underline{\varphi} < 0$ with probability p. We introduce the disaster to help match the level of the equity premium in our calibration. We further assume that the occurrence of the disaster destroys capital and reduces the reference wage in households' wage adjustment costs in proportion to the decline in productivity. The first assumption implies that aggregate output is

$$y_t \equiv (z_t \ell_t)^{1-\alpha} \left(k_{t-1} \exp(\varphi_t) \right)^{\alpha}, \tag{28}$$

where productivity is now labor-augmenting and thus consistent with balanced growth.

[22]See, for example, Tallarini Jr. (2000) or Barro (2006).

3.2.3 Monetary and fiscal policy

Finally, in the infinite horizon monetary policy generalizes (10) and follows a standard Taylor (1993) rule as

$$1 + i_t = (1 + \bar{i}) \left(\frac{P_t}{P_{t-1}} \right)^{\phi} m_t, \tag{29}$$

where policy shocks follow an AR(1) process

$$\log m_t = \rho \log m_{t-1} + \varsigma \epsilon_t^m \tag{30}$$

where ϵ_t^m is an *iid* shock from a standard Normal distribution.

Fiscal policy is characterized by $\tau = -\frac{1}{\epsilon - 1}$ and household-specific lump-sum taxes

$$T_t^i = \int_0^1 AC_t^W(j) dj + \tau \int_0^1 W_t(j) \ell_t^i(j) dj + P_t tr_t^i. \tag{31}$$

Relative to (9) in the two-period environment, (31) adds another component of transfers

$$tr_t^i = \omega_t^i \big[(\Pi_t + (1 - \delta) Q_t) k_{t-1} \exp(\varphi_t) \\ - (1 + i_{t-1}) B_{t-1}^i - (\Pi_t + (1 - \delta) Q_t) k_{t-1}^i \exp(\varphi_t) \big] \tag{32}$$

where $\omega_t^i = \omega^i$ for all households except a positive measure, for whom $\omega_t^i = \omega_t$ ensures that $\int_0^1 tr_t^i di = 0$. These transfers give us an additional degree of freedom to match the wealth distribution of households in the data: in the extreme case where $\omega_t^i = 1$ for all i, households have identical financial wealth inclusive of transfers each period. We assume that households anticipate all future transfers except their own, which they assume to be zero.

3.2.4 Equilibrium and model solution

The optimization problems of households and unions are naturally extended to incorporate the dynamics of the infinite horizon, and the optimization problem of producers are unchanged. The definition of equilibrium then generalizes Definition 1.

We solve the model globally using numerical methods. While the perturbation approach used in the two-period environment remains feasible in the infinite horizon, we turn to this solution approach for two reasons. First, we find that household portfolios and MPRs solved analytically at the deterministic steady-state have non-trivial differences from their values at the stochastic steady-state solved globally.[23,24] Second, a perturbation solution is ill-suited to handle the addition of a disaster to our exogenous driving forces.

Given this global solution approach, we now limit the degree of heterogeneity across households to make the computational burden tractable. We divide the continuum of households into a finite number of groups within which households are perfectly symmetric. We choose three groups in our baseline parameterization, denoted $i \in \{a, b, c\}$ and where we now understand the index i to refer to groups and the representative household of each group. The fraction of households belonging to group i is denoted λ^i, where $\sum_i \lambda^i = 1$.

We solve a stationary transformation of the economy obtained by dividing all real variables except labor by z_t and nominal variables by $P_t z_t$. As is shown in appendix D, in the transformed economy we obtain a recursive representation of the equilibrium in which the aggregate state in period t is given by the monetary policy shock m_t, scaled aggregate capital $k_{t-1}/\exp(\sigma\epsilon_t^z)$, scaled prior period's real wage $w_{t-1}/\exp(\sigma\epsilon_t^z)$, and wealth shares $\{s_t^i\}$ of $I-1$ groups, where

$$s_t^i \equiv \lambda^i \frac{(1+i_{t-1})B_{t-1}^i + (\Pi_t + (1-\delta)Q_t)k_{t-1}^i \exp(\varphi_t) + tr_t^i}{(\Pi_t + (1-\delta)Q_t)k_{t-1}\exp(\varphi_t)}. \tag{33}$$

Productivity shocks inclusive of disasters only govern the transition across states, but do not separately enter the state space itself.

We solve the model over a large grid of the aggregate states, making sure that the solution is robust to larger grid sizes and boundaries. When forming expectations over prices and policies, we use quadrature and linear interpolation over aggregate states, but (for households' value functions) interpolate using cubic splines over individual wealth. The stochastic equilibrium is determined through backward iteration, while dampening the updating of

[23]We wish to emphasize, however, that the qualitative insights of section 2 continue to be relevant in the infinite horizon. For instance, setting $p = 0$ so that there is no disaster, we can prove the following analog of Proposition 2 in this infinite horizon environment around the deterministic steady-state (denominated without time subscripts):

$$\mathbb{E}_0\hat{r}_1^k - \mathbb{E}_0\hat{r}_1 = \Gamma\sigma^2 + \zeta_{m_0}\hat{m}_0\sigma^2 + o(||\cdot||^4),$$

where

$$\zeta_{m_0} = \tilde{\zeta}_{m_0} + \gamma\int_0^1 \bar{\xi}_{m_0}^i\left(\overline{mpr} - \overline{mpr}^i\right)di$$

and $\tilde{\zeta}_{m_0}$ is the effect of a monetary shock on the risk premium in a counterfactual economy with $\gamma^i = \gamma$ for all i, given $\gamma = \left(\int_0^1 \frac{\bar{c}^i}{\int_0^1 \bar{c}^{i'}di'}\frac{1}{\gamma^i}di\right)^{-1}$, $\bar{\xi}_{m_0}^i = \overline{\frac{d\left[c_1^i/\int_0^1 c_1^{i'}di'\right]}{dm_0}}$, $\overline{mpr} = \int_0^1 \frac{\bar{c}^i}{\int_0^1 \bar{c}^{i'}di'}\overline{mpr}^i di$, and $\overline{mpr}^i \equiv \overline{\frac{q_0\frac{\partial k_0^i}{dy_0^i}}{\frac{\partial a_0^i}{dy_0^i}}}$. Hence, beyond the risk premium effect of monetary policy in an otherwise identical economy with homogenous risk aversion, it will lower the risk premium if it redistributes to households with relatively high MPRs.

[24]We use the term "stochastic steady-state" to refer to the set of prices and policies to which the economy converges when agents fully anticipate the stochastic properties of productivity, monetary policy, and disasters, but no such shocks are ever realized.

the price of capital and individuals' expectations over the dynamics of the aggregate states. The code is written in Fortran and parallelized using OpenMP, so that convergence can be achieved in less than twenty minutes on a modern computing system with eight cores. The computational algorithm is further described in appendix D.

3.3 Parameterization and the stochastic steady-state

We now parameterize the model to match micro moments informing the heterogeneity across groups as well as macro moments regarding the business cycle and asset prices.

3.3.1 Micro moments: the distribution of wealth, labor income, and portfolios

We seek to replicate patterns in the distribution of wealth, labor income, and financial portfolios in U.S. data, giving us confidence in the model-implied distribution of MPRs and exposures to a monetary policy shock. We work with the 2016 Survey of Consumer Finances (SCF) and proceed in three steps.

First, we decompose each household's nominal wealth (A^i) into claims on the economy's capital stock (Qk^i, in positive net supply) and bonds (B^i, in zero net supply accounting for the government and rest of the world).[25,26] In the same spirit as Doepke and Schneider (2006), the key step in doing this is to account for the implicit leverage households have on capital through the leverage of firms and of financial intermediaries. In particular, if household i owns \$1 in equity in a firm which has net leverage

$$\frac{\text{assets net of bonds}}{\text{equity}} = lev^{firm},$$

then we assign the household

$$\{Qk^i = lev^{firm}, B^i = 1 - lev^{firm}\}.$$

If household i owns \$1 in equity in an intermediary which has net leverage

$$\frac{\text{assets net of bonds}}{\text{equity}} = lev^{inter}$$

and the intermediary's assets net of bonds are equity claims on the above firm, then we

[25] While all observations are as of 2016, we drop time subscripts anticipating that we will calibrate the model's stochastic steady-state to match these moments.

[26] Consistent with the traded assets in our model, we do not distinguish between assets in zero net supply but having different duration. Extending the model to feature a richer set of traded assets would be valuable.

<table>
<tr><td colspan="2"></td><td colspan="2">$\frac{A^i}{W\ell^i}$</td></tr>
<tr><td colspan="2"></td><td>$\geq p60$</td><td>$< p60$</td></tr>
<tr><td rowspan="2">$\frac{Qk^i}{A^i}$</td><td>$\geq p99.5$</td><td>Group a
Share households: 0.2%
Share $\sum_i W\ell^i$: 0.0%
Share $\sum_i A^i$: 0.2%
Median $\frac{Qk^i}{A^i}$: 6.6</td><td rowspan="2">Group c
Share households: 60.3%
Share $\sum_i W\ell^i$: 77.3%
Share $\sum_i A^i$: 11.3%
Median $\frac{Qk^i}{A^i}$: 0.3</td></tr>
<tr><td>$< p99.5$</td><td>Group b
Share households: 39.5%
Share $\sum_i W\ell^i$: 22.7%
Share $\sum_i A^i$: 88.5%
Median $\frac{Qk^i}{A^i}$: 0.7</td></tr>
</table>

Table 3: heterogeneity in wealth to labor income and the capital portfolio share

Notes: observations are weighted by SCF sample weights.

assign the household

$$\{Qk^i = lev^{inter}lev^{firm}, B^i = 1 - lev^{inter}lev^{firm}\}.$$

We use the Financial Accounts of the United States as well as analyses of hedge funds and private equity to inform these leverage assumptions. We outline the specific assumptions and present the resulting aggregate decomposition of household net worth in appendix C.2. We focus on measures of $\{A^i, Qk^i, B^i\}$ excluding assets and liabilities associated with households' primary residence and vehicles.[27]

Second, we stratify households by their wealth to labor income $\{\frac{A^i}{W\ell^i}\}$ and capital portfolio share $\{\frac{Qk^i}{A^i}\}$, defining subsamples mapping to our three groups.[28] We sort households on these variables based on Proposition 1, which demonstrated that the capital portfolio share is informative about households' risk aversion and thus MPR only after properly accounting for their non-traded exposure to aggregate risk through labor income. Group a corresponds to households with high wealth to labor income and a high capital portfolio share, group b corresponds to households with high wealth to labor income but a low capital portfolio share, and group c corresponds to households with low wealth to labor income. We define

[27] As suggested by the large literatures on housing and consumer durables, households' choices to accumulate these assets and associated liabilities reflect factors not well captured by our parsimonious framework. For this reason, we exclude them from our calculation of cross-sectional moments.

[28] For each household we measure labor income as total wage and salary income for the previous calendar year as reported in the SCF summary extract.

	$1\{hbus^i = 1\}$	$1\{age^i > 54, lf^i = 0\}$
$1\{i = a\}$	0.45	0.26
	(0.15)	(0.12)
$1\{i = b\}$	0.12	0.52
	(0.01)	(0.01)
Observations	6,227	6,227
Adj R^2	0.03	0.34

Table 4: indicators for private business wealth or being retired on group indicators

Notes: observations are weighted by SCF sample weights and standard errors adjust for imputation and sampling variability following Pence (2015). Each specification includes a constant term (not shown), capturing the baseline probability of holding private business wealth or being retired among households in group c.

"high" wealth to labor income as households in the top 40% of households ordered by this measure, and a "high" capital portfolio share as households in the top 0.5% of households ordered by this measure.

Third, we summarize the labor income, wealth, and financial portfolios of these three groups which we seek to match, provided in Table 3. Group a households constitute 0.2% of households, earn a negligible fraction of labor income, hold 0.2% of wealth, and have a median capital portfolio share of 6.6. Group b households constitute 39.5% of households, earn 22.7% of labor income, hold 88.5% of wealth, and have a median capital portfolio share of 0.7. Finally, group c households constitute 60.3% of households, earn 77.3% of labor income, hold 11.3% of wealth, and have a median capital portfolio share of 0.3. To better understand the nature of households in each group, in Table 4 we first project an indicator for the household holding private business assets on households' group indicator. We find that households in group a are especially more likely to hold private business assets. We then project an indicator for the household head being older than 54 and out of the labor force, together capturing a retired household head, on households' group indicator. We find that households in group b are especially more likely to be retired.

3.3.2 Macro moments: business cycle dynamics and asset prices

We also calibrate the model to match standard macro moments regarding the business cycle and asset prices. In terms of the business cycle, we seek to match the volatilities of the growth rates of consumption, investment and hours worked. We use NIPA data on consumption of non-durables and services, investment in durables and capital, and hours worked, together with the time series of the working age population provided by the BLS, to estimate quarterly per capita growth rates in those series over the sample period Q1 1983 to Q1 2018. In terms of asset prices, we seek to match the average real interest rate and equity premium. We estimate the annualized average real interest rate and equity premium over July 1979 - June 2012 using the data from CRSP described in Section 3.1. We will compare the equity premium to that implied by our model assuming that an equity claim (with return r^e) is a levered claim to capital with a debt to equity ratio of 1.5.

3.3.3 Parameterization

A model period corresponds to one quarter. After setting a subset of parameters in accordance with the literature, we calibrate the remaining parameters to be consistent with the macro and micro moments described above. We note that all stochastic properties of the model are estimated using a simulation where no disasters are realized in sample.[29]

Externally set parameters A subset of model parameters summarized in Table 5 are set externally in accordance with the literature. Among the model's preference parameters, we set the IES below but close to one, namely $\psi = 0.95$. This parameter also governs the substitutability between consumption and labor and Shimer (2010) argues such a parameterization is in line with standard models of time allocation which predict that the marginal utility of consumption is higher when households work more. The Frisch elasticity of labor supply is set to $\theta = 0.75$, consistent with the micro evidence in Chetty, Guren, Manoli, and Weber (2011). The three types have measure $\lambda^a = 0.2\%$, $\lambda^b = 22.7\%$ and $\lambda^c = 77.1\%$, as determined in our analysis of the SCF micro data in Table 3.

On the production side, we choose $\alpha = 0.33$ for the capital share of production and a quarterly depreciation rate of 2.5%, standard values in the literature. The disaster probability is set to $p = 0.5\%$, which follows Barro (2006) and implies that a disaster shock is expected to occur every 50 years. The depth of the disaster is set to $\underline{\varphi} = -10\%$, lower than Barro (2006) but more consistent with the long-run effects of a disaster estimated by Nakamura, Steinsson, Barro, and Ursua (2013) after accounting for the recovery after an initial disaster. We choose an elasticity of substitution across worker varieties $\epsilon = 10$ and Rotemberg wage adjustment costs of $\chi^W = 200$, which together imply a Calvo (1983)-equivalent frequency of wage adjustment around 4 quarters, consistent with the evidence in Grigsby, Hurst, and Yildirmaz (2019). The Taylor coefficient on inflation is set to $\phi = 1.5$ and monetary policy shocks have a standard deviation of $\varsigma = 0.25\%/4$ with zero persistence. Finally, we assume that the wage markup is perfectly offset by $\tau = -\frac{1}{\epsilon-1} = -\frac{1}{9}$.

[29] We make this choice to maximize comparability between the model and data, since our data is all from the post-World War II period.

	Description	Value	Notes
ψ	IES	0.95	Shimer (2010)
θ	Frisch elasticity	0.75	Chetty et al. (2011)
λ^a	measure of a households	0.2%	SCF
λ^b	measure of b households	22.7%	SCF
α	1 - labor share	0.33	
δ	depreciation rate	2.5%	
ϵ	elast. of subs. across workers	10	
τ	undoes wage markup	-0.11	
χ^W	Rotemberg wage adj costs	200	$\approx \mathbb{P}(\text{adjust}) = 4$ qtrs
ϕ	Taylor coeff. on inflation	1.5	Taylor (1993)
η	std. dev. MP shock	0.25%/4	
ρ	persistence MP shock	0	
p	disaster probability	0.5%	Barro (2006)
$\underline{\varphi}$	disaster shock	-10%	

Table 5: externally set parameters

Calibrated parameters We calibrate the remaining parameters to target the macro and micro moments described above. While there is no one-to-one mapping between individual model parameters and those moments, Table 6 reports in each line a parameter choice, as well as a moment in model and data that this parameter is closely linked to.

The standard deviation of the productivity shock σ is set to 0.6%, which is a key determinant of the model's ability to match quarterly consumption growth volatility of 0.5%. The scale of the capital adjustment cost is set to $\chi^x = 7$ to dampen the volatility of investment growth in order to match the data. The discount factor is one key determinant of the expected real interest rate. Due to the precautionary savings motive, $\beta = 0.99$ is high enough to match the low annualized real rate observed in the data. Households' risk aversion parameters, $\gamma^a = 0.5$, $\gamma^b = 30$, and $\gamma^c = 27.5$, are drivers of both the average risk premium in the economy and households' portfolio choices. The lump-sum wealth transfers across households are chosen to approximate the measured wealth shares of the three groups; the reported values tr^i/a^i denote the size of the transfer rule relative to households' respective wealth in the stochastic steady-state. Finally we set the disutility of labor to $\bar{\theta} = 0.83$, which targets a level of labor $\ell = 1.0$ and only serves as a normalization.

	Description	Value	Moment	Target	Model
σ	std. dev. prod.	0.6%	$\sigma(\Delta \log c)$	0.5%	0.5%
ϕ^x	capital adj cost	7	$\sigma(\Delta \log x)$	1.7%	1.6%
β	discount factor	0.99	$\mathbb{E}r_{+1}$	1.4%	1.4%
γ^b	RRA b	30	$\mathbb{E}\left[r^e_{+1} - r_{+1}\right]$	7.1%	6.9%
γ^a	RRA a	0.5	k^a/a^a	6.6	11.0
γ^c	RRA c	27.5	k^c/a^c	0.3	0.1
tr^a/a^a	transfer to a	-0.3%	$a^a/\sum_i \lambda^i a^i$	0.2%	4.0%
tr^c/a^c	transfer to c	0.0002%	$a^c/\sum_i \lambda^i c^i$	11.3%	17.2%
$\bar{\theta}$	ℓ disutility	0.83	ℓ	1	1

Table 6: targeted moments and calibrated parameters

Notes: targeted business cycle moments are from Q1/83-Q1/18 NIPA and targeted asset pricing moments are from 7/79-6/12 data underlying the VAR. The equity premium in the model is calculated assuming a debt/equity ratio of 1.5 on a stock market claim. The stochastic properties of the model are estimated over a sample with no disaster realizations.

3.3.4 Untargeted moments

Table 7 reports the values of several untargeted moments and their empirical counterparts where available. In terms of macro moments, the model closely matches the quarterly volatilities of output growth and employment growth. It undershoots the volatility of expected real interest rates and, especially, the volatility of expected excess returns. This implies that the time-variation in expected returns operating through productivity shocks is limited in the present calibration, recalling that productivity shocks and monetary shocks are the only realized shocks when simulating the model (there are no disaster realizations) and monetary policy shocks are fully transitory and have small standard deviation.[30]

In terms of micro moments, the model generates heterogeneity in quarterly MPRs at the stochastic steady-state consistent with Proposition 1 in the analytical results. Group a households are the most risk tolerant and have the highest MPR, borrowing \$10 for every \$1 of the marginal dollar in net worth to invest in capital. Group b and c households have relatively higher levels of risk aversion and correspondingly lower MPRs.

Quasi-experimental evidence is consistent with the positive covariance between MPRs and capital portfolio shares in our calibration. Using data on Norwegian lottery winners, the estimates of Fagereng, Holm, and Natvik (2019b) imply that the marginal propensity to

[30]In appendix E we simulate the impulse responses following a productivity shock, demonstrating that the endogenous redistribution induced by a productivity shock toward high MPR households puts downward pressure on the risk premium, as in the case of a monetary policy shock. However, as Table 7 makes clear, this channel alone is quantitatively not strong enough to match the observed time-variation in excess returns.

Moment (ann.)	Data	Model
$\sigma(\Delta \log y)$	0.6%	0.6%
$\sigma(\Delta \log \ell)$	0.7%	0.7%
$\sigma(\mathbb{E}r_{+1})$	0.8%	0.1%
$\sigma\left(\mathbb{E}\left[r^e_{+1} - r_{+1}\right]\right)$	5.4%	0.1%
mpr^a		11.0
mpr^b		0.9
mpr^c		0.4

Table 7: untargeted macro and micro moments

Notes: business cycle moments are from Q1/83-Q1/18 NIPA and asset pricing moments are from 7/79-6/12 data underlying the VAR. The stochastic properties of the model are estimated over a sample with no disaster realizations.

save in risky assets relative to save overall rises with liquid assets.[31] Using data on Swedish lottery winners, Briggs, Cesarini, Lindqvist, and Ostling (2015) find that lottery winners reduce their portfolio share in risky assets by more if they are low wealth.[32] These findings are consistent with a positive covariance between MPRs and the portfolio share in risky assets since the latter is rising in wealth.[33] Using U.S. tax data, Hoopes, Langetieg, Nagel, Reck, Slemrod, and Stuart (2017) find that households with high taxable income, dividend income, and private business income are more likely to sell stocks following periods of market tumult. While this may reflect heterogeneous responses to prices rather than changes in income, it remains consistent with the positive covariance between MPRs and risky portfolio shares.

The evidence is more mixed on the magnitude of MPRs. The above studies using lottery winners report average marginal propensities to save in risky assets relative to save overall of 0.05-0.15,[34] below those in our calibration even after accounting for reasonable estimates of the leverage of firms and intermediaries in which households invest.[35] However, using

[31] In Table 8 of their paper, the authors report that the marginal propensity to save in stocks, bonds, and mutual funds rises from 0.021 to 0.068, and the marginal propensity to save in these assets, deposits, or repay debt rises from 0.435 to 0.672, from the first to fourth quartile in liquid assets. The ratio thus rises from $0.021/0.435 = 0.05$ to $0.068/0.672 = 0.10$.

[32] See Figure 3 in their paper.

[33] The positive relationship between the portfolio share in risky assets and wealth is pervasive in the literature, documented in the Scandinavian context by, for instance, Calvet, Campbell, and Sodini (2007).

[34] See footnote 31 for estimates from Fagereng et al. (2019b). In Table B.8 of Briggs et al. (2015), the authors report a marginal propensity to save in risky assets of 0.085 and marginal propensity to save in these assets, safe assets, bank accounts, or repay debt of 0.58, implying a ratio of 0.15.

[35] Following section 3.3.1, we must account for firms' and intermediaries' net leverage to translate households' equity claims into capital claims and thus the MPR. Applying net leverage of 2, reflecting the various measures of leverage estimated in appendix C.2, yields MPRs implied by these estimates of 0.1-0.3.

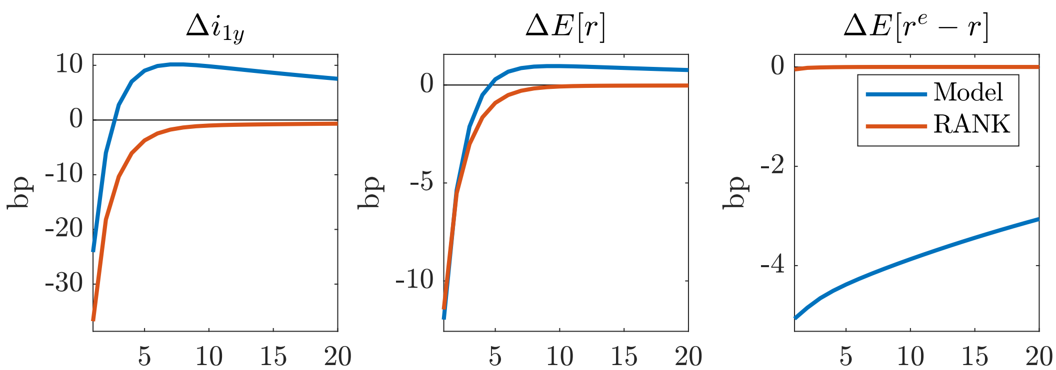

Figure 2: expected returns after negative monetary policy shock

Notes: all series are plotted as quarterly (non-annualized) deviations from the stochastic steady-state, except for the 1-year nominal bond yield Δi_{1y}. *bp* denotes basis points (0.01%).

the Panel Study of Income Dynamics, Brunnermeier and Nagel (2008) document significant inertia in financial portfolios, with a negative change in the risky share after receiving one dollar of cash or deposits but an increase in the risky share after receiving one dollar of unexpected returns on risky assets.[36] As lottery winnings are paid out as cash or riskless deposits, they may understate households' MPRs in response to dividends or capital gains, more relevant for the balance sheet revaluation emphasized in this paper. Among recipients of private business income, they may particularly understate the MPR because investment in private businesses is not included in the definition of (traded) risky assets. Accumulating further evidence on MPRs and refining the framework developed in this paper to match these moments, much as the literature has been able to do for MPCs, would be valuable.

3.4 Impulse responses to a monetary policy shock

We now simulate the effects of a negative shock to the nominal interest rate.

3.4.1 Model versus RANK

We first compare in Figures 2, 3, and 4 the impulse responses of the model with heterogeneity to a counterfactual representative agent New Keynesian (RANK) economy. In the latter, we set $\gamma^i = 11$ across all groups, equal to the harmonic mean of risk aversion in the model weighted by the consumption share of each group at the stochastic steady-state, and transfers $tr^i = 0$ for all groups. The difference between these two impulse responses thus isolates the redistributive effects of monetary policy in our setting.

[36]These results are consistent with the more recent work of Fagereng, Holm, Moll, and Natvik (2019a) using Norwegian administrative data finding evidence that households "save by holding" on to nearly 100% of assets experiencing capital gains.

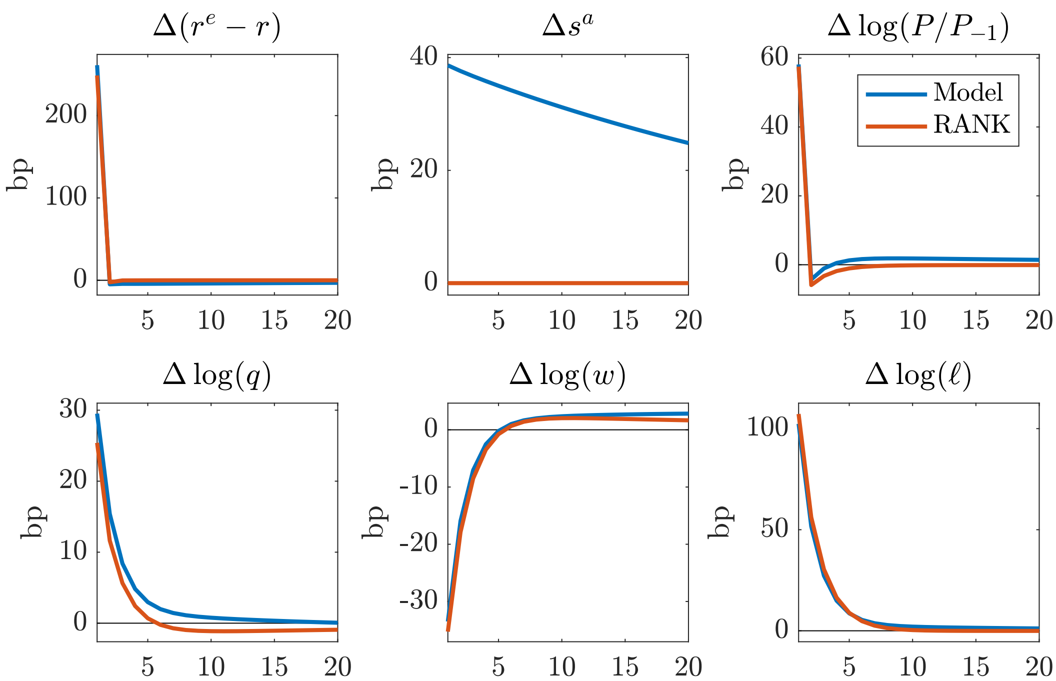

Figure 3: redistribution after negative monetary policy shock

Notes: all series are plotted as quarterly (non-annualized) deviations from the stochastic steady-state. *bp* denotes basis points (0.01%).

Figure 2 summarizes the effect of the monetary policy shock on key expected returns of interest. The first panel reports the change in the yield on a 1-year nominal bond due to the shock. We obtain this yield by computing, in each state, the price that each household would be willing to pay for the asset. We then set the price to that of the highest-valuation household. Even though this asset is not traded in our model, we price it so that we can calibrate the magnitude of the primitive shock to the nominal interest rate ϵ_0^m to be consistent with the 22*bp* reduction in the Treasury yield estimated in Figure 1. The second and third panels depict the resulting change in the expected real interest rate and the expected excess returns on capital. The decline in the former reflects the monetary non-neutrality in our setting, while the decline in the latter demonstrates that the risk premium declines substantially in our model relative to the representative agent case.

Figure 3 demonstrates that redistribution drives the decline in the risk premium in our model. The first panel of the first row demonstrates that realized excess returns on capital are substantially positive on impact, followed by small negative returns in the quarters which follow — consistent with the decline in the expected excess returns in the previous panel, and exactly the pattern estimated in the data in Figure 1. Through the lens of Proposition 3, the substantially positive excess returns on impact endogenously redistribute to the high

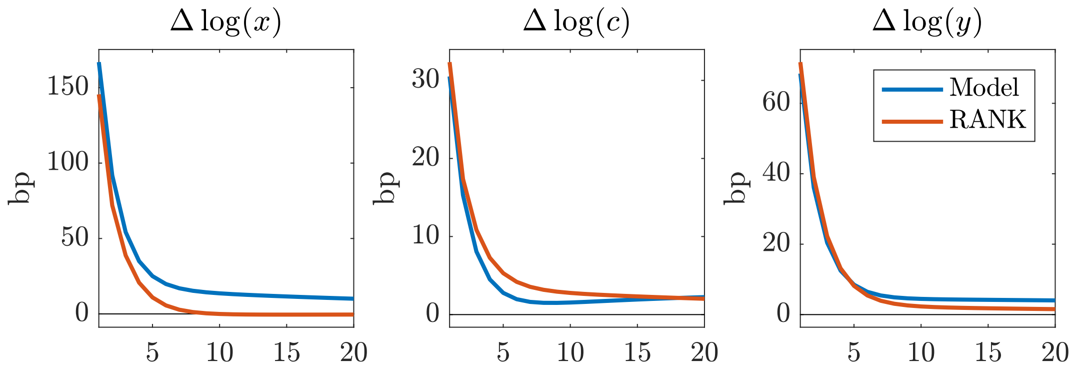

Figure 4: quantities after negative monetary policy shock

Notes: all series are plotted as quarterly (non-annualized) deviations from the stochastic steady-state. *bp* denotes basis points (0.01%).

MPR a households who hold levered claims on capital.[37] Indeed, the second panel in this row demonstrates that the financial wealth share of a households persistently rises after the shock. In part this results from unexpected inflation which lowers the realized real interest rate, shown in the third panel. In part it also results from an increase in the price of capital, shown in the first panel of the second row. This reflects the increase in profits in the short-run because there are lower real wages and higher employment in this sticky wage environment, shown in the second and third panels of this row.

Figure 4 examines the consequences of this redistribution for the transmission of the policy shock to the real economy. Comparing the investment response in the model to the representative agent case, the stimulus to investment on impact is amplified from 1.5*pp* to 1.7*pp*. Investment remains persistently higher in the following periods, leading to an amplification of capital accumulation relative to the representative agent case. The amplification of investment in our model is countervailed by a mitigation of the stimulus through consumption, so that the overall stimulus to output is little changed.

Quantitatively, the price and quantity effects of the monetary policy shock are consistent with the empirical estimates even though these were not targeted in the calibration. First, the impact effect on the stock market of 2.4*pp* is comparable to the 2*pp* increase estimated in Figure 1. Second and crucially, a Campbell-Shiller decomposition on the model impulse responses matches the role of news about lower future excess returns in driving this increase in the stock market in the data. We summarize this decomposition in Table 8. The performance of our model contrasts starkly with the counterfactual representative agent economy, where essentially none of the transmission to the stock market operates though news about future

[37] This redistribution occurs both from b and c households. On impact, we find that the financial wealth share of the b households falls by 13*pp* and the financial wealth share of the c households falls by 26*pp*.

% Excess return	Data	Model	RANK
Δ Dividends	35%	27%	56%
−Real rates	10%	26%	42%
−Excess returns	55%	47%	2%

Table 8: Campbell-Shiller decomposition of excess returns after monetary shock

Notes: estimates from data correspond to Table 2. Comparable estimates obtained in the model assuming a debt/equity ratio of 1.5 on a stock market claim.

excess returns. Third, the peak stimulus to output in the model of $0.7pp$ is comparable to the peak stimulus to industrial production estimated in Figure 1, giving us confidence in the model's predictions for transmission to the real economy.[38]

The difference between the model and representative agent case indeed is almost fully accounted for by the balance sheet revaluation characterized in our analytical results and described above. Figure 5 plots the differences between the model and RANK impulse responses for a subset of key variables of interest. It then compares this difference to the impulse response in the model following a one-time transfer of wealth across households exactly equal to the initial change in wealth shares in our model induced by the monetary policy shock.[39] As is evident, the difference in impulse responses is almost fully accounted for by the balance sheet revaluation induced by the monetary policy shock.

3.4.2 Inspecting the mechanism through sensitivity analysis

We seek to further understand the sources and implications of the redistribution in the model by varying a set of key parameters. Each column of Table 9 corresponds to a different parameterization, where in each column only a single parameter is changed from our baseline parameterization in the first column. The first three rows reports the change in inflation, the price of capital, and the realized excess return on equity in period 0 relative to the economy's stochastic steady-state in period -1. These three variables are central to understand the sources of redistribution in response to a monetary policy shock. The last three rows then report the differential responses in the wealth share of group a households, the expected return on equity, and investment between that parameterization and the RANK benchmark.

The second column reports the results for an economy in which monetary policy shocks

[38]The shape of the quantity responses differs from the hump-shapes typically estimated in the data, however. Adding features such as investment adjustment costs (as opposed to the present capital adjustment costs) could improve the model in this dimension, following Christiano, Eichenbaum, and Evans (2005).

[39]Following Figure 1 and footnote 37, we increase the wealth share of a households by $39bp$, reduce the wealth share of b households by $13bp$, and reduce the wealth share of c households by $26bp$ in period 0.

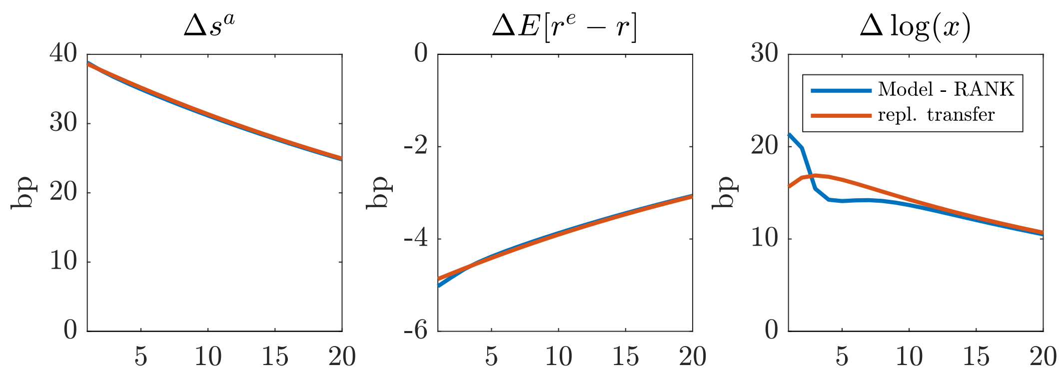

Figure 5: replicating wealth transfer of negative monetary policy shock

Notes: all series are plotted as quarterly (non-annualized) deviations. *bp* denotes basis points (0.01%).

are persistent, setting $\rho = 0.3$, demonstrating the importance of redistribution through debt deflation. In that case, a monetary policy shock induces a stronger response of the inflation rate relative to the baseline, as can be seen in row 1. Through this debt deflation channel, the realized excess return on capital on impact rises in row 3 and the change in the wealth share of the levered household group a increases in row 4. In line with proposition 2, the larger redistribution to high MPR households amplifies the decline in the risk premium in row 5. Correspondingly, the monetary policy shock induces a larger investment response through redistribution, seen in row 6.

An increase in the capital adjustment cost to $\chi^x = 12$, as reported in the third column, amplifies the redistribution through asset prices. In that case a monetary policy shock induces a larger effect on the price of capital and therefore increases the unexpected return on capital, as reported in rows 2 and 3, respectively. Redistribution is slightly larger and the risk premium declines more than in the baseline case in rows 4 and 5. These effects are not especially large because of the countervailing effect of a smaller inflation response in row 1: the higher adjustment cost limits quantity responses in the capital market, evident in row 6, in turn dampening the response in the labor market.

The fourth column eliminates nominal wage rigidity by setting $\chi^W = 0$, demonstrating the role of changes in profit income in inducing redistribution across households. When wage rigidity is zero, the decline in the real wage and the stimulus to employment is mitigated. It follows that the short-run increase in profits is mitigated. This not only reduces the increase in the current dividend but it also reduces the increase in the price of capital which capitalizes future dividends, evident in row 2. The decline in the risk premium is thus mitigated in row 5, even though the redistribution through debt deflation is in fact amplified in row 1.

Finally, the last column reports the results when monetary policy is less responsive to changes in the inflation rate by setting $\phi = 1.3$, demonstrating the role of the monetary

Row		Baseline	$\rho = 0.3$	$\chi^x = 12$	$\chi^W = 0$	$\phi = 1.3$
1	$\Delta \log(P/P_{-1})$	$58bp$	$79bp$	$54bp$	$71bp$	$65bp$
2	$\Delta \log(q)$	$30bp$	$34bp$	$37bp$	$3bp$	$33bp$
3	$\Delta(r^e - r)$	$89bp$	$115bp$	$92bp$	$75bp$	$100bp$
4	$\Delta^2 s^a$	$39bp$	$50bp$	$40bp$	$33bp$	$42bp$
5	$\Delta^2 E[r^e - r]$	$-5bp$	$-6bp$	$-5bp$	$-4bp$	$-5bp$
6	$\Delta^2 \log(x)$	$21bp$	$25bp$	$17bp$	$20bp$	$23bp$

Table 9: impact effects of negative monetary policy shock across parameterizations

Notes: for each parameterization, rows 1–3 report the change in a particular variable on impact of the monetary policy shock relative to the stochastic steady-state. Rows 4–6 report the difference in that change relative to the RANK benchmark. All rows report quarterly (non-annualized) changes, and *bp* denotes basis points (0.01%).

feedback rule in mediating the transmission from risk premia to the real economy. A less responsive Taylor rule dampens the extent to which risk premia movements are absorbed by changes in the risk-free rate. Consistent with the discussion of Proposition 6, this will lead to a stronger investment response. Comparing the response of investment in row 6 to the baseline parameterization, we see that a less responsive Taylor rule leads to a slightly stronger investment response versus the baseline, despite a similar change in equity premium.

4 Conclusion

In this paper we revisit the monetary transmission mechanism in a New Keynesian environment with heterogeneous propensities to bear risk. An expansionary monetary policy shock lowers the risk premium if it redistributes to households with high MPRs. Heterogeneity in risk aversion, portfolio constraints, rules of thumb, background risk, or beliefs induce a joint distribution of monetary policy exposures and MPRs such that an expansionary shock redistributes in this way. In a calibration matching micro-level heterogeneity in the U.S. economy, this mechanism quantitatively rationalizes the stock market effects of monetary policy which have eluded existing frameworks and amplifies its transmission through investment.

References

Alvarez, F., A. Atkeson, and P. J. Kehoe (2009): "Time-varying risk, interest rates, and exchange rates in general equilibrium," *Review of Economic Studies*, 76, 851–878.

Alvarez, F., R. E. Lucas, and W. E. Weber (2001): "Interest rates and inflation," *American Economic Review Papers & Proceedings*, 91, 219–225.

Ang, A., S. Gorovyy, and G. B. van Inwegen (2011): "Hedge fund leverage," *Journal of Financial Economics*, 102, 102–126.

Auclert, A. (2018): "Monetary policy and the redistribution channel," *Working paper*.

Axelson, U., T. Jenkinson, P. Stromberg, and M. S. Weisbach (2013): "Borrow cheap, buy high? The determinants of leverage and pricing in buyouts," *Journal of Finance*, 68, 2223–2267.

Bansal, R. and A. Yaron (2004): "Risks for the long run: a potential resolution of asset pricing puzzles," *Journal of Finance*, 59, 1481–1509.

Barro, R. J. (2006): "Rare disasters and asset markets in the twentieth century," *Quarterly Journal of Economics*, 121, 823–866.

Basu, S. and B. Bundick (2017): "Uncertainty shocks in a model of effective demand," *Econometrica*, 85, 937–958.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): *The financial accelerator in a quantitative business cycle framework*, Elsevier, vol. 1 of *Handbook of Macroeconomics*, 1341–1393.

Bernanke, B. S. and K. N. Kuttner (2005): "What explains the stock market's reaction to Federal Reserve policy?" *Journal of Finance*, 60, 1221–1257.

Bhandari, A., D. Evans, and M. Golosov (2019): "Monetary policy with segemented markets," *Working paper*.

Briggs, J., D. Cesarini, E. Lindqvist, and R. Ostling (2015): "Wealth, portfolio allocations, and risk preference," *Working paper*.

Brunnermeier, M. and Y. Sannikov (2012): "Redistributive monetary policy," *Proceedings of the Symposium at Jackson Hole*.

Brunnermeier, M. K. and S. Nagel (2008): "Do wealth fluctuations generate time-varying risk aversion? Micro-evidence on individuals' asset allocation," *American Economic Review*, 98, 713–736.

Brunnermeier, M. K. and Y. Sannikov (2016): "The I theory of money," *Working paper*.

Caballero, R. and E. Farhi (2018): "The safety trap," *Review of Economic Studies*, 85, 223–274.

Caballero, R. and A. Simsek (2018): "A risk-centric model of demand recessions and monetary policy," *Working paper*.

Calvet, L. E., J. Y. Campbell, and P. Sodini (2007): "Down or out: assessing the welfare costs of household investment mistakes," *Journal of Political Economy*, 115, 707–747.

Calvo, G. A. (1983): "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 12, 383–398.

Campbell, J. Y. and J. Ammer (1993): "What moves the stock and bond markets? A variance decomposition for long-term asset returns," *Journal of Finance*, 48, 3–37.

Campbell, J. Y. and J. H. Cochrane (1999): "By force of habit: a consumption-based explanation of aggregate stock market behavior," *Journal of Political Economy*, 107, 205–251.

Campbell, J. Y. and R. J. Shiller (1988): "The dividend-price ratio and expectations of future dividends and discount factors," *Review of Financial Studies*, 1, 195–228.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011): "Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins," *American Economic Review*, 101, 471–475.

Chien, Y., H. Cole, and H. Lustig (2012): "Is the volatility of the market price of risk due to intermittent portfolio rebalancing," *American Economic Review*, 106, 2859–2896.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal rigidities and the dynamic effects of a shock to monetary policy," *Journal of Political Economy*, 113, 1–45.

Cochrane, J. H. (2017): "Macro-Finance," *Review of Finance*, 21, 945–985.

Devereux, M. B. and A. Sutherland (2011): "Country portfolios in open economy macro-models," *Journal of the European Economic Association*, 9, 337–369.

DiTella, S. (2018): "A neoclassical theory of liquidity traps," *Working paper*.

Doepke, M. and M. Schneider (2006): "Inflation and the redistribution of nominal wealth," *Journal of Political Economy*, 114, 1069–1097.

Drechsler, I., A. Savov, and P. Schnabl (2018): "A model of monetary policy and risk premia," *Journal of Finance*, 63, 317–373.

Epstein, L. G. and S. E. Zin (1991): "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: an empirical analysis," *Econometrica*, 99, 263–286.

Fagereng, A., M. B. Holm, B. Moll, and G. Natvik (2019a): "Saving behavior across the wealth distribution: the importance of capital gains," *Working paper.*

Fagereng, A., M. B. Holm, and G. J. Natvik (2019b): "MPC heterogeneity and household balance sheets," *Working paper.*

Fernandez-Villaverde, J., P. Guerron-Quintana, K. Kuester, and J. Rubio-Ramirez (2015): "Fiscal volatility shocks and economic activity," *American Economic Review*, 105, 3352–3384.

Forni, M. and L. Gambetti (2014): "Sufficient information in structural VARs," *Journal of Monetary Economics*, 66, 124–136.

Gali, J. (2008): *Monetary policy, inflation, and the business cycle*, Princeton University Press.

Garleanu, N. and S. Panageas (2015): "Young, Old, Conservative, and Bold. The implications of finite lives and heterogeneity for asset pricing," *Journal of Political Economy*, 123, 670–685.

Geanakoplos, J. (2009): "The leverage cycle," *NBER Macroeconomics Annual*, 24, 1–66.

Gertler, M. and P. Karadi (2015): "Monetary policy surprises, credit costs, and economic activity," *American Economic Journal: Macroeconomics*, 7, 44–76.

Giglio, S., M. Maggiori, J. Stroebel, and S. Utkus (2019): "Five facts about beliefs and portfolios," *Working paper.*

Gilchrist, S. and E. Zakrajsek (2012): "Credit spreads and business cycle fluctuations," *American Economic Review*, 102, 1692–1720.

Gomes, F. and A. Michaelides (2008): "Asset pricing with limited risk sharing and heterogeneous agents," *Review of Financial Studies*, 21, 415–448.

Gourio, F. (2012): "Disaster Risk and Business Cycles," *American Economic Review*, 102, 2734–2766.

Grigsby, J., E. Hurst, and A. Yildirmaz (2019): "Aggregate nominal wage adjustments: new evidence from administrative payroll data," *Working paper.*

Guvenen, F. (2009): "A parsimonious macroeconomic model for asset pricing," *Econometrica*, 77, 1711–1750.

Hanson, S. G. and J. C. Stein (2015): "Monetary policy and long-term real rates," *Journal of Financial Economics*, 115, 429–448.

He, Z. and A. Krishnamurthy (2013): "Intermediary Asset Pricing," *American Economic Review*, 2, 732–770.

Heaton, J. and D. Lucas (2000): "Portfolio choice and asset prices: the importance of entrepreneurial risk," *Journal of Finance*, 55, 1163–1198.

Heaton, J. and D. J. Lucas (1996): "Evaluating the effects of incomplete markets on risk sharing and asset pricing," *Journal of Political Economy*, 104, 443–487.

Hoopes, J., P. Langetieg, S. Nagel, D. Reck, J. Slemrod, and B. Stuart (2017): "Who sold during the crash of 2008-09? Evidence from tax-return data on daily sales of stock," *Working paper.*

Ilut, C. L. and M. Schneider (2014): "Ambiguous business cycles," *American Economic Review*, 104, 2368–2399.

Jarocinski, M. and P. Karadi (forthcoming): "Deconstructing monetary policy surprises – the role of information shocks," *AEJ: Macroeconomics.*

Kaplan, G., B. Moll, and G. L. Violante (2018): "Monetary policy according to HANK," *American Economic Review*, 108, 697–743.

Kaplan, G. and G. L. Violante (2018): "Microeconomic heterogeneity and macroeconomic shocks," *Journal of Economic Perspectives.*

Luetticke, R. (2018): "Transmission of monetary policy with heterogeneity in household portfolios," *Working paper.*

McKinsey (2018): "The rise and rise of private markets," .

Meeuwis, M., J. A. Parker, A. Schoar, and D. I. Simester (2019): "Belief disagreement and portfolio choice," *Working paper.*

Mertens, K. and M. O. Ravn (2013): "The dynamic effects of personal and corporate income tax changes in the United States," *American Economic Review*, 103, 1212–1247.

Nakamura, E. and J. Steinsson (2018): "High frequency identification of monetary non-neutrality: the information effect," *Quarterly Journal of Economics*, 133, 1283–1330.

Nakamura, E., J. Steinsson, R. Barro, and J. Ursua (2013): "Crises and recoveries in an empirical model of consumption disasters," *American Economic Journal: Macroeconomics*, 5, 35–74.

Pence, K. (2015): "SCFCOMBO: Stata module to estimate errors using the Survey of Consumer Finances," Tech. rep., Boston College Department of Economics.

Ramey, V. A. (2016): *Macroeconomic shocks and their propagation*, Elsevier, vol. 2A of *Handbook of Macroeconomics*, 71–162.

Rotemberg, J. J. (1982): "Sticky prices in the United States," *Journal of Political Economy*, 90, 1187–1211.

Shimer, R. (2010): *Labor markets and business cycles*, Princeton University Press.

Simsek, A. (2013): "Belief disagreements and collateral constraints," *Econometrica*, 81, 1–53.

Stock, J. H. and M. W. Watson (2018): "Identification and estimation of dynamic causal effects in macroeconomics using external instruments," *The Economic Journal*, 128, 917–948.

Stock, J. H., J. H. Wright, and M. Yogo (2002): "A survey of weak instruments and weak identification in generalized method of moments," *Journal of Business and Economic Statistics*, 20, 518–529.

Tallarini Jr., T. D. (2000): "Risk-sensitive business cycles," *Journal of Monetary Economics*, 45, 507–532.

Taylor, J. B. (1993): "Discretion Versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.

Woodford, M. (2003): *Interest and prices: foundations of a theory of monetary policy*, Princeton University Press.

Appendix For Online Publication

A Proofs

In this appendix we prove the results stated in section 2.

Proposition 1

Proof. We first characterize households' portfolio share in capital in the limit of zero aggregate risk. Optimal portfolio choice is

$$\mathbb{E}_0 (c_1^i)^{-\gamma^i} (r_1^k - r_1) = 0.$$

Up to first-order, optimal portfolio choice yields

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 = o(|| \cdot ||^2).$$

It follows that given the first-order expansion in terms of the states

$$\hat{r}_1^k = \hat{z}_1 + \delta_{m_0}^{r_1^k} \hat{m}_0 + o(|| \cdot ||^2),$$
$$\hat{r}_1 = \delta_{m_0}^{r_1} \hat{m}_0 + o(|| \cdot ||^2),$$

with coefficients $\delta_{\cdot}^{\cdot}$, we can conclude

$$\delta_{m_0}^{r_1^k} = \delta_{m_0}^{r_1}$$

by the method of undetermined coefficients.

Up to second-order, optimal portfolio choice yields

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\mathbb{E}_0 (\hat{r}_1^k)^2 - \frac{1}{2}\hat{r}_1^2 = \gamma^i \mathbb{E}_0 \hat{c}_1^i \left(\hat{r}_1^k - \hat{r}_1\right) + o(|| \cdot ||^3).$$

Using the above first-order approximations of $\hat{r}_1^k$ and $\hat{r}_1$ in terms of the underlying states, and the first-order approximation of $\hat{c}_1^i$

$$\hat{c}_1^i = \delta_{z_1}^{c_1^i} \hat{z}_1 + \delta_{m_0}^{c_1^i} \hat{m}_0 + o(|| \cdot ||^2),$$

it follows that optimal portfolio choice implies

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i \delta_{z_1}^{c_1^i} \sigma^2 + o(|| \cdot ||^3). \tag{34}$$

Anticipating the result in Proposition 2 that

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + o(||\cdot||^3),$$

it follows that

$$\delta_{z_1}^{c_1^i} = \frac{\gamma}{\gamma^i}. \tag{35}$$

Approximating up to first order the period 1 resource constraint and equilibrium wages and profits

$$w_1 = (1-\alpha) z_1 k_0^\alpha,$$
$$\pi_1 = \alpha z_1 k_0^{\alpha-1},$$

the method of undetermined coefficients implies

$$\delta_{z_1}^{c_1^i} = \frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i}.$$

Substituting in (35) and re-arranging, we can conclude that

$$\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} = \frac{\bar{\pi}_1 \bar{k}_0^i}{(1+\bar{r}_1)\bar{a}_0^i} = \frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i}\frac{\gamma}{\gamma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i},$$

where the first equality uses $\bar{q}_0 = \frac{\bar{\pi}_1}{1+\bar{r}_1}$ absent aggregate risk.

We now characterize households' marginal responses to a unit of income in the limit of zero aggregate risk. Differentiating households' optimal portfolio choice condition yields

$$0 = \mathbb{E}_0^i m_{0,1}^i \frac{\gamma^i}{c_1^i}\left(r_1^k - r_1\right)\frac{\partial c_1^i}{\partial y_0^i}$$

where the household's stochastic discount factor between periods 0 and 1 is

$$m_{0,1}^i \equiv \frac{\beta^i}{1-\beta^i}(c_0^i)^{\frac{1}{\psi^i}}\Phi^i(l_0^i)^{1-\frac{1}{\psi^i}}(ce_0^i)^{\gamma^i - \frac{1}{\psi^i}}(c_1^i)^{-\gamma^i}.$$

Differentiating households' period 1 resource constraint yields

$$\frac{\partial c_1^i}{\partial y_0^i} = (1+r_1)\frac{\partial b_0^i}{\partial y_0^i} + \pi_1 \frac{\partial k_0^i}{\partial y_0^i}.$$

Combining the previous two equations yields

$$0 = \mathbb{E}_0 m_{0,1}^i \frac{\gamma^i}{c_1^i}(r_1^k - r_1)\left((1+r_1)\frac{\partial b_0^i}{\partial y_0^i} + \pi_1 \frac{\partial k_0^i}{\partial y_0^i}\right). \tag{36}$$

A second-order approximation then implies

$$\begin{aligned} 0 = &\left((1+\bar{r}_1)\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}\right)\left(\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2\right) \\ &- \left((1+\bar{r}_1)\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}\right)\frac{\gamma^i+1}{\bar{c}_1^i}\left(\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right)\sigma^2 + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}\sigma^2 + o(||\cdot||^3). \end{aligned} \tag{37}$$

Again anticipating the result in Proposition 2 that

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + o(||\cdot||^3)$$

and the above result that

$$\frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i} = \frac{\gamma}{\gamma^i},$$

it follows from (37) that

$$\bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}} = \frac{\gamma}{\gamma^i}\overline{\frac{\partial a_0^i}{\partial y_0^i}}$$

using $\bar{q}_0 = \frac{\bar{\pi}_1}{1+\bar{r}_0}$ and $\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}} = \overline{\frac{\partial a_0^i}{\partial y_0^i}}$. The expression for $\overline{mpr}_0^i \equiv \frac{\bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}}}{\overline{\frac{\partial a_0^i}{\partial y_0^i}}}$ then follows. □

Proposition 2

Proof. We first derive the result up to second order. Multiplying both sides of (34) by $\frac{\bar{c}_1^i}{\gamma^i}$, integrating over all households i, and making use of the market clearing conditions which imply that

$$\int_0^1 \bar{c}_1^i di = \int_0^1 \left(\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right) di,$$

we obtain

$$\mathbb{E}_0 \hat{r}_1(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \left(\frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'}\frac{1}{\gamma^i}\right)^{-1}\sigma^2 + o(||\cdot||^3), \tag{38}$$

defining γ as in the claim.

We now derive the result up to third order. The third-order approximation of optimal portfolio choice for household i is

$$\begin{aligned}\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\mathbb{E}_0(\hat{r}_1^k)^2 - \frac{1}{2}\hat{r}_1^2 \\ = \gamma^i\mathbb{E}_0\hat{c}_1^i\left(\hat{r}_1^k - \hat{r}_1\right) - \frac{1}{2}\left(\gamma^i\right)^2\mathbb{E}_0\left(\hat{c}_1^i\right)^2\left(\hat{r}_1^k - \hat{r}_1\right) \\ + \frac{1}{2}\gamma^i\mathbb{E}_0\hat{c}_1^i\left((\hat{r}_1^k)^2 - \hat{r}_1^2\right) - \frac{1}{6}\left(\mathbb{E}_0(\hat{r}_1^k)^3 - \hat{r}_1^3\right) + o(||\cdot||^4). \quad (39)\end{aligned}$$

A second-order expansion of $\hat{r}_1^k$ and $\hat{r}_1$ in terms of the underlying states yields

$$\begin{aligned}\hat{r}_1^k &= \hat{z}_1 + \delta^{r_0}_{m_0}\hat{m}_0 + \frac{1}{2}\delta^{r_0}_{m_0^2}\hat{m}_0^2 + \left(-\frac{1}{2} + \gamma + \frac{1}{2}\delta^{r_0}_{\sigma^2}\right)\sigma^2, \\ \hat{r}_0 &= \delta^{r_0}_{m_0}\hat{m}_0 + \frac{1}{2}\delta^{r_0}_{m_0^2}\hat{m}_0^2 + \frac{1}{2}\delta^{r_0}_{\sigma^2}\sigma^2\end{aligned}$$

where we have already made use of the fact that, by the method of undetermined coefficients, (38) implies

$$\begin{aligned}\frac{1}{2}\delta^{r_1^k}_{m_0^2} &= \frac{1}{2}\delta^{r_1}_{m_0^2}, \\ \frac{1}{2}\delta^{r_1^k}_{\sigma^2} - \frac{1}{2}\delta^{r_1}_{\sigma^2} + \frac{1}{2} &= \gamma.\end{aligned}$$

A second-order expansion of $\hat{c}_1^i$ in terms of the underlying states yields

$$\hat{c}_1^i = \delta^{c_1^i}_{m_0}\hat{m}_0 + \delta^{c_1^i}_{z_1}\hat{z}_1 + \frac{1}{2}\delta^{c_1^i}_{m_0^2}\hat{m}_0^2 + \delta^{c_1^i}_{m_0 z_1}\hat{m}_0\hat{z}_1 + \frac{1}{2}\delta^{c_1^i}_{z_1^2}\hat{z}_1^2 + \frac{1}{2}\delta^{c_1^i}_{\sigma^2}\sigma^2 + o(||\cdot||^3).$$

Substituting these into (39) and collecting terms, we obtain

$$\begin{aligned}\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \\ \gamma^i\delta^{c_1^i}_{z_1}\sigma^2 + \left[\gamma^i\left(\delta^{c_1^i}_{m_0}\gamma + \delta^{c_1^i}_{m_0 z_1}\right) - \left(\gamma^i\right)^2\delta^{c_1^i}_{m_0}\delta^{c_1^i}_{z_1} + \gamma^i\delta^{c_1^i}_{z_1}\delta^{r_1}_{m_0} - \gamma\delta^{r_1}_{m_0}\right]\hat{m}_0\sigma^2 + o(||\cdot||^4).\end{aligned}$$

Making use of (35) substantially simplifies this to

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i\delta^{c_1^i}_{z_1}\sigma^2 + \gamma^i\delta^{c_1^i}_{m_0 z_1}\hat{m}_0\sigma^2 + o(||\cdot||^4). \quad (40)$$

Again multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i}$, integrating over all households i, and making use of the market clearing conditions, we obtain

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \frac{\gamma}{\int_0^1 \bar{c}_1^i di} \left(\int_0^1 \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di \right) \hat{m}_0 \sigma^2 + o(|| \cdot ||^4).$$

Then, taking a second-order approximation of the period 1 resource constraint and equilibrium wages and profits, the method of undetermined coefficients implies

$$\bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} + \bar{c}_1^i \delta_{m_0}^{c_1^i} \delta_{z_1}^{c_1^i} = \alpha \bar{w}_1 \delta_{m_0}^{k_0} + \bar{\pi}_1 \delta_{m_0}^{k_0^i} - (1-\alpha)\bar{\pi}_1 \bar{k}_0^i \delta_{m_0}^{k_0}. \tag{41}$$

It follows that

$$\int_0^1 \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di = -\int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} \delta_{z_1}^{c_1^i} di + \bar{\pi}_1 \int_0^1 \delta_{m_0}^{k_0^i} di,$$

using

$$\int_0^1 \alpha \bar{w}_1 \delta_{m_0}^{k_0} di = \alpha \bar{w}_1 \delta_{m_0}^{k_0} = \alpha(1-\alpha)\bar{z}_1 \bar{k}_0^\alpha \delta_{m_0}^{k_0},$$

$$\int_0^1 (1-\alpha)\bar{\pi}_1 \bar{k}_0^i \delta_{m_0}^{k_0} di = (1-\alpha)\bar{\pi}_1 \bar{k}_0 \delta_{m_0}^{k_0} = \alpha(1-\alpha)\bar{z}_1 \bar{k}_0^\alpha \delta_{m_0}^{k_0}$$

implied by market clearing and the definition of equilibrium wages and profits.[40] Since a first-order approximation to capital claims market clearing implies

$$\int_0^1 \delta_{m_0}^{k_0^i} di = \bar{k}_0 \delta_{m_0}^{k_0},$$

it further follows that

$$\int_0^1 \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di = -\int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} \delta_{z_1}^{c_1^i} di + \bar{\pi}_1 \bar{k}_0 \delta_{m_0}^{k_0}.$$

Moreover, since a first-order approximation to goods market clearing implies

$$\int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} di = \alpha \bar{z}_1 \bar{k}_0^\alpha \delta_{m_0}^{k_0}$$

[40]Note that we linearize rather than log-linearize with respect to $\{k_0^i, b_0^i, a_0^i\}$ since in principle these may be negative.

and $\bar{\pi}_1 = \alpha \bar{z}_1 \bar{k}_0^{\alpha-1}$, it further follows that

$$\begin{aligned}
\int_0^1 \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di &= -\int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} \delta_{z_1}^{c_1^i} di + \int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} di, \\
&= \int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \delta_{z_1}^{c_1^i}\right) di, \\
&= \int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \frac{\gamma}{\gamma^i}\right) di
\end{aligned}$$

where the final line uses (35). Hence, we can conclude

$$\begin{aligned}
\zeta_{m_0} &= \frac{\gamma}{\int_0^1 \bar{c}_1^i di} \left(\int_0^1 \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di\right), \\
&= \frac{\gamma}{\int_0^1 \bar{c}_1^i di} \int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \frac{\gamma}{\gamma^i}\right) di.
\end{aligned}$$

Recall from Proposition 1 that $\overline{mpr}_0^i \equiv \frac{\gamma}{\gamma^i}$. Since the definition of (19) implies

$$\int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \frac{\gamma}{\gamma^i}\right) di = \int_0^1 \left(\bar{c}_1^i \delta_{m_0}^{c_1^i} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \int_0^1 \bar{c}_1^{i'} \delta_{m_0}^{c_1^{i'}} di'\right) \left(1 - \frac{\gamma}{\gamma^i}\right) di$$

and

$$1 = \int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \overline{mpr}_0^i di \equiv \overline{mpr}_0,$$

and we further have

$$\begin{aligned}
\overline{\frac{d\left[c_1^i / \int_0^1 c_1^{i'}\right]}{dm_0}} &= \frac{1}{\int_0^1 \bar{c}_1^{i'} di'} \left(\overline{\frac{dc_1^i}{dm_0}} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \int_0^1 \overline{\frac{dc_1^{i'}}{dm_0}} di'\right), \\
&= \frac{1}{\int_0^1 \bar{c}_1^{i'} di'} \left(\bar{c}_1^i \delta_{m_0}^{c_1^i} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \int_0^1 \bar{c}_1^{i'} \delta_{m_0}^{c_1^{i'}} di'\right),
\end{aligned}$$

we obtain the expression for ζ_{m_0} given in the claim. □

Proposition 3

Proof. Assuming that $\frac{dc_1^i}{dm_0}$ is continuous in σ, it is equivalent to characterize $\frac{dc_1^i}{dm_0}$ and then evaluate its limit at the deterministic steady-state ($\sigma = 0$) or simply compute $\frac{d\bar{c}_1^i}{d\bar{m}_0}$ at this limit. It is expositionally simpler to do the latter, so we do that here.

Re-consider households' micro-level optimization problem (18) given $\sigma = 0$:

$$\max \left((1-\beta^i) \left(\bar{c}_0^i \Phi^i(\bar{\ell}_0^i) \right)^{1-\frac{1}{\psi^i}} + \beta^i \left(\bar{c}_1^i \right)^{1-\frac{1}{\psi^i}} \right)^{\frac{1}{1-\frac{1}{\psi^i}}} \quad s.t.$$
$$\bar{c}_0^i + \bar{a}_0^i = \bar{y}_0^i(\bar{w}_0\bar{\ell}_0^i, \bar{P}_0, \bar{\pi}_0, \bar{q}_0),$$
$$\bar{c}_1^i = \bar{w}_1 + (1+\bar{r}_1)\bar{a}_0^i,$$

defining policy functions

$$\bar{c}_1^i(\bar{y}_0^i(\bar{w}_0\bar{\ell}_0^i, \bar{P}_0, \bar{\pi}_0, \bar{q}_0), \bar{\ell}_0^i, 1+\bar{r}_1, \bar{w}_1),$$

where recall that

$$\bar{y}_0^i(\bar{w}_0\bar{\ell}_0^i, \bar{P}_0, \bar{\pi}_0, \bar{q}_0) = \bar{w}_0\bar{\ell}_0^i + \frac{1}{\bar{P}_0}(1+i_{-1})B_{-1}^i + (\bar{\pi}_0 + (1-\delta_0)\bar{q}_0)k_{-1}^i.$$

It follows that

$$\frac{d\bar{c}_1^i}{d\bar{m}_0} = \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}\left[-\frac{1}{\bar{P}_0}B_{-1}^i \frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} + k_{-1}^i\left(\frac{d\bar{\pi}_0}{d\bar{m}_0} + (1-\delta_0)\frac{d\bar{q}_0}{d\bar{m}_0} \right) + \frac{d\bar{w}_0\bar{\ell}_0^i}{dm_0} \right]$$
$$+ \frac{\partial \bar{c}_1^i}{\partial \bar{\ell}_0^i}\frac{d\bar{\ell}_0^i}{d\bar{m}_0} + \frac{\partial \bar{c}_1^i}{\partial(1+\bar{r}_1)}\frac{d\bar{r}_1}{d\bar{m}_0} + \frac{\partial \bar{c}_1^i}{\partial \bar{w}_1}\frac{d\bar{w}_1}{d\bar{m}_0}, \quad (42)$$

where each of the partial derivatives is evaluated with respect to the policy function above. We now characterize each of these partial derivatives in turn.

First note that it is clearly the case that

$$\frac{\partial \bar{c}_1^i}{\partial \bar{w}_1} = \frac{1}{1+\bar{r}_1}\frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}.$$

Then define the expenditure minimization problem dual to the utility maximization problem above

$$\min \bar{c}_0^{i,h} + \bar{a}_0^{i,h} \quad s.t.$$
$$\left((1-\beta^i) \left(\bar{c}_0^{i,h} \Phi^i(\bar{\ell}_0^{i,h}) \right)^{1-\frac{1}{\psi^i}} + \beta^i \left(\bar{c}_1^{i,h} \right)^{1-\frac{1}{\psi^i}} \right)^{\frac{1}{1-\frac{1}{\psi^i}}} \geq \bar{u}^i,$$
$$\bar{c}_1^{i,h} = \bar{w}_1 + (1+\bar{r}_1)\bar{a}_0^{i,h},$$

where we use h superscripts to denote compensated (Hicksian) policies. Letting

$$\bar{e}_0^i(\bar{u}^i, \bar{\ell}_0^i, 1+\bar{r}_1, \bar{w}_1)$$

denote the level of period 0 expenditure solving this problem, duality implies

$$\bar{c}_1^i(\bar{e}_0^i(\bar{u}^i, \bar{\ell}_0^i, 1+\bar{r}_1, \bar{w}_1), \bar{\ell}_0^i, 1+\bar{r}_1, \bar{w}_1) = \bar{c}_1^{i,h}(\bar{u}^i, \bar{\ell}_0^i, 1+\bar{r}_1, \bar{w}_1).$$

This leads to Slutsky identities

$$\begin{aligned}
\frac{\partial \bar{c}_1^i}{\partial \bar{\ell}_0^i} &= \frac{\partial \bar{c}_1^{i,h}}{\partial \bar{\ell}_0^i} - \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}\frac{\partial \bar{e}_0^i}{\partial \bar{\ell}_0^i}, \\
\frac{\partial \bar{c}_1^i}{\partial (1+\bar{r}_1)} &= \frac{\partial \bar{c}_1^{i,h}}{\partial (1+\bar{r}_1)} - \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}\frac{\partial \bar{e}_0^i}{\partial (1+\bar{r}_1)}.
\end{aligned}$$

By the Envelope Theorem,

$$\begin{aligned}
\frac{\partial \bar{e}_0^i}{\partial \bar{\ell}_0^i} &= -\bar{c}_0^{i,h}\frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}, \\
\frac{\partial \bar{e}_0^i}{\partial (1+\bar{r}_1)} &= -\frac{1}{1+\bar{r}_1}\bar{a}_0^{i,h},
\end{aligned}$$

so that we may further write the above identities as

$$\begin{aligned}
\frac{\partial \bar{c}_1^i}{\partial \bar{\ell}_0^i} &= \frac{\partial \bar{c}_1^{i,h}}{\partial \bar{\ell}_0^i} + \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}\left(\bar{c}_0^{i,h}\frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}\right), \\
\frac{\partial \bar{c}_1^i}{\partial (1+\bar{r}_1)} &= \frac{\partial \bar{c}_1^{i,h}}{\partial (1+\bar{r}_1)} + \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}\frac{1}{1+\bar{r}_1}\bar{a}_0^{i,h}.
\end{aligned}$$

Substituting the above results into (42), using $\bar{c}_0^{i,h} = \bar{c}_0^i$ and $\bar{a}_0^{i,h} = \bar{a}_0^i$ implied by duality, and collecting terms, we obtain

$$\begin{aligned}
\frac{d\bar{c}_1^i}{d\bar{m}_0} = \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}\Bigg[&-\frac{1}{\bar{P}_0}B_{-1}^i\frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} + k_{-1}^i\left(\frac{d\bar{\pi}_0}{d\bar{m}_0} + (1-\delta_0)\frac{d\bar{q}_0}{d\bar{m}_0}\right) + \frac{d\bar{w}_0\bar{\ell}_0^i}{d\bar{m}_0} + \frac{1}{1+\bar{r}_1}\frac{d\bar{w}_1}{d\bar{m}_0} \\
&+ \frac{1}{1+\bar{r}_1}\bar{a}_0^i\frac{d\bar{r}_1}{d\bar{m}_0} + \bar{c}_0^i\frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}\frac{d\bar{\ell}_0^i}{d\bar{m}_0}\Bigg] + \frac{\partial \bar{c}_1^{i,h}}{\partial \bar{l}_0^i}\frac{d\bar{l}_0^i}{d\bar{m}_0} + \frac{\partial \bar{c}_1^{i,h}}{\partial (1+\bar{r}_1)}\frac{d\bar{r}_1}{d\bar{m}_0} \quad (43)
\end{aligned}$$

We next characterize the compensated derivatives $\frac{\partial \bar{c}_1^{i,h}}{\partial \bar{l}_0^i}$ and $\frac{\partial \bar{c}_1^{i,h}}{\partial (1+\bar{r}_1)}$. The compensated policies solve the system

$$(1-\beta^i)(\Phi^i(\bar{l}_0^i))^{1-\frac{1}{\psi^i}}(\bar{c}_0^{i,h})^{-\frac{1}{\psi^i}} = \beta^i(\bar{c}_1^{i,h})^{-\frac{1}{\psi^i}}(1+\bar{r}_1),$$

$$\left((1-\beta^i)\left(\bar{c}_0^{i,h}\Phi^i(\bar{\ell}_0^{i,h})\right)^{1-\frac{1}{\psi^i}} + \beta^i(\bar{c}_1^{i,h})^{1-\frac{1}{\psi^i}}\right)^{\frac{1}{1-\frac{1}{\psi^i}}} = \bar{u}^i,$$

$$\bar{c}_1^{i,h} = \bar{w}_1 + (1+\bar{r}_1)\bar{a}_0^{i,h}.$$

Straightforward differentiation of this system yields

$$\frac{\partial \bar{c}_1^{i,h}}{\partial \bar{l}_0^i} = \frac{-\frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}}{\frac{1}{\psi^i}\frac{1}{\bar{c}_0^{i,h}}\frac{1}{1+\bar{r}_1} + \frac{1}{\psi^i}\frac{1}{\bar{c}_1^{i,h}}},$$

$$\frac{\partial \bar{c}_1^{i,h}}{\partial \bar{r}_0} = \frac{\frac{1}{1+\bar{r}_1}}{\frac{1}{\psi^i}\frac{1}{\bar{c}_0^{i,h}}\frac{1}{1+\bar{r}_1} + \frac{1}{\psi^i}\frac{1}{\bar{c}_1^{i,h}}}.$$

Differentiating the system defining uncompensated policies

$$(1-\beta^i)(\Phi^i(\bar{l}_0^i))^{1-\frac{1}{\psi^i}}(\bar{c}_0^{i,h})^{-\frac{1}{\psi^i}} = \beta^i(\bar{c}_1^{i,h})^{-\frac{1}{\psi^i}}(1+\bar{r}_1),$$

$$\bar{c}_0^i + \bar{a}_0^i = \bar{y}_0^i$$

$$\bar{c}_1^i = \bar{w}_1 + (1+\bar{r}_1)\bar{a}_0^i,$$

implies that

$$\frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i} = \frac{\frac{1}{\psi^i}\frac{1}{\bar{c}_0^i}}{\frac{1}{\psi^i}\frac{1}{\bar{c}_0^i}\frac{1}{1+\bar{r}_1} + \frac{1}{\psi^i}\frac{1}{\bar{c}_1^i}}.$$

Hence, making use of duality ($\bar{c}_0^i = \bar{c}_0^{i,h}$ and so on), we can more succinctly write

$$\frac{\partial \bar{c}_1^{i,h}}{\partial \bar{l}_0^i} = \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}\left(-\psi^i \bar{c}_0^i \frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}\right),$$

$$\frac{\partial \bar{c}_1^{i,h}}{\partial \bar{r}_0} = \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}\left(\psi^i \bar{c}_0^i \frac{1}{1+\bar{r}_1}\right).$$

Combining the prior results and using

$$\frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i} = (1+\bar{r}_0)\frac{\partial \bar{a}_0^i}{\partial \bar{y}_0^i}$$

and the definition of the static labor wedge in this environment

$$\bar{\tau}^{\ell_0^i} = 1 - \frac{-\bar{c}_0^i \Phi^{i\prime}(\bar{l}_0^i)/\Phi^i(\bar{l}_0^i)}{\bar{w}_0}$$

yields the stated result in the claim. □

Proposition 4

Proof. Combining (22) with Proposition 3 and using

$$\begin{aligned}
\bar{c}_1^i &= \bar{c}_1, \\
\overline{\frac{\partial a_0^i}{\partial y_0^i}} &= \overline{\frac{\partial a_0}{\partial y_0}}, \\
\psi^i &= \psi, \\
\bar{\tau}^{\ell_0^i} &= \bar{\tau}^{\ell_0}, \\
\bar{\ell}_0^i &= \bar{\ell}_0, \\
\overline{\frac{d\ell_0^i}{dm_0}} &= \overline{\frac{d\ell_0}{dm_0}},
\end{aligned}$$

as assumed in the claim, we obtain

$$\bar{\xi}_{m_0}^i = \frac{1}{\bar{c}_1}(1+\bar{r}_1)\overline{\frac{\partial a_0}{\partial y_0}}\left[-\frac{(1+i_{-1})B_{-1}^i}{P_0}\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}} + (k_{-1}^i - k_{-1})\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right)\right].$$

By Proposition 1 and the assumptions in the claim,

$$\begin{aligned}
k_{-1}^i = \bar{k}_0^i &= \bar{a}_0\left[\frac{\bar{c}_1}{(1+\bar{r}_1)\bar{a}_0}\frac{\gamma}{\gamma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0}\right], \\
&= \frac{\bar{k}_0}{\alpha}\left(\frac{\gamma}{\gamma^i} - 1\right) + \bar{k}_0, \\
B_{-1}^i = \bar{P}_0\bar{b}_0^i &= \bar{P}_0\left[\bar{a}_0 - \bar{k}_0^i\right], \\
&= -\bar{P}_0\frac{\bar{k}_0}{\alpha}\left(\frac{\gamma}{\gamma^i} - 1\right),
\end{aligned}$$

where

$$\gamma = \left[\int_0^1 \frac{1}{\gamma^i} di\right]^{-1}$$

and we use $\bar{q}_0 = 1$ following the assumption that $k_{-1} = \bar{k}_0$. It follows that

$$-\frac{(1+i_{-1})B^i_{-1}}{P_0} = (1+i_{-1})\frac{\bar{k}_0}{\alpha}\left(\frac{\gamma}{\gamma^i} - 1\right),$$
$$k^i_{-1} - k_{-1} = \frac{\bar{k}_0}{\alpha}\left(\frac{\gamma}{\gamma^i} - 1\right).$$

Hence,

$$\begin{aligned}\zeta_{m_0} &= \gamma \int_0^1 \bar{\xi}^i_{m_0}(\overline{mpr}_0 - \overline{mpr}^i_0)di,\\ &= \gamma \int_0^1 \bar{\xi}^i_{m_0}\left(1 - \frac{\gamma}{\gamma^i}\right)di,\\ &\propto -(1+i_{-1})\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}} - \overline{\frac{d\pi_0}{dm_0}} - (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\end{aligned} \tag{44}$$

where the second line uses Proposition 1 and the third line uses the above results.

To sign (44), we now compute $\{\frac{dP_0}{dm_0}, \frac{d\pi_0}{dm_0}, \frac{dq_0}{dm_0}\}$ at the limit of $\sigma = 0$. Assuming these derivatives are continuous in σ, their values at the limit of $\sigma = 0$ will be equal to $\{\overline{\frac{dP_0}{dm_0}}, \overline{\frac{d\pi_0}{dm_0}}, \overline{\frac{dq_0}{dm_0}}\}$. The limiting Euler equation

$$\left(\bar{c}^i_0\right)^{-\frac{1}{\psi}}\Phi(\bar{\ell}_0)^{1-\frac{1}{\psi}} = \beta(1+\bar{r}_1)\left(\bar{c}^i_1\right)^{-\frac{1}{\psi}}$$

implies

$$-\frac{1}{\psi}\frac{1}{\bar{c}_0}\frac{d\bar{c}^i_0}{d\bar{m}_0} + \left(1 - \frac{1}{\psi}\right)\epsilon^{\Phi}_{\bar{\ell}_0}\frac{1}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} = \frac{1}{1+\bar{r}_1}\frac{d(1+\bar{r}_1)}{d\bar{m}_0} - \frac{1}{\psi}\frac{1}{\bar{c}_1}\frac{d\bar{c}^i_1}{d\bar{m}_0}, \tag{45}$$

where we write the elasticity of $\Phi(\ell_0)$ with respect to ℓ_0 evaluated at $\bar{\ell}_0$

$$\epsilon^{\Phi}_{\bar{\ell}_0} \equiv \frac{\Phi'(\bar{\ell}_0)\bar{\ell}_0}{\Phi(\bar{\ell}_0)}.$$

The union's limiting labor supply condition

$$\int_0^1 (\bar{v}^i_0)^{\frac{1}{\psi}}(\bar{c}^i_0)^{-\frac{1}{\psi}}\Phi(\bar{\ell}_0)^{1-\frac{1}{\psi}}\left[\frac{\bar{W}_0}{\bar{P}_0} + \bar{c}^i_0\frac{\Phi'(\bar{\ell}_0)}{\Phi(\bar{\ell}_0)} + \frac{\bar{W}_0}{\bar{P}_0}\frac{\chi^W}{\epsilon}\frac{\bar{W}_0}{W_{-1}}\left(\frac{\bar{W}_0}{W_{-1}} - 1\right)\right]di = 0$$

implies

$$\frac{1}{\bar{W}_0}\frac{d\bar{W}_0}{d\bar{m}_0} - \frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} - \frac{1}{\bar{c}_0}\left(\int_0^1 \frac{d\bar{c}^i_0}{d\bar{m}_0}di\right) - \left(\epsilon^{-\Phi'}_{\bar{\ell}_0} - \epsilon^{\Phi}_{\bar{\ell}_0}\right)\frac{1}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} + \frac{\chi^W}{\epsilon}\frac{1}{\bar{W}_0}\frac{d\bar{W}_0}{d\bar{m}_0} = 0. \tag{46}$$

where we have used the symmetry across households and $W_{-1} = \bar{W}_0$ at the point of approximation, and further defined the elasticity of the marginal disutility of labor

$$\epsilon_{\bar{\ell}_0}^{-\Phi'} \equiv \frac{-\Phi''(\bar{\ell}_0)\bar{\ell}_0}{-\Phi'(\bar{\ell}_0)}.$$

The limiting labor demand condition in period 0

$$\frac{\bar{W}_0}{\bar{P}_0} = (1-\alpha)z_0\bar{\ell}_0^{-\alpha}k_{-1}^{\alpha},$$

implies

$$\frac{1}{\bar{W}_0}\frac{d\bar{W}_0}{d\bar{m}_0} - \frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} = -\alpha\frac{1}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0}. \tag{47}$$

The limiting optimal investment condition

$$\bar{q}_0 = \left(\frac{\bar{k}_0}{k_{-1}}\right)^{\chi^x}$$

implies

$$\frac{d\bar{q}_0}{d\bar{m}_0} = \chi^x\frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0} \tag{48}$$

where we have used $k_{-1} = \bar{k}_0$ at the point of approximation. The limiting goods market clearing condition in period 0

$$\int_0^1 \bar{c}_0^i di + \bar{q}_0\left(\bar{k}_0 - (1-\delta_0)k_{-1}\right) = z_0\bar{\ell}_0^{1-\alpha}k_{-1}^{\alpha}$$

implies

$$\int_0^1 \frac{d\bar{c}_0^i}{d\bar{m}_0}di + \frac{d\bar{k}_0}{d\bar{m}_0} + \delta\bar{k}_0\frac{d\bar{q}_0}{d\bar{m}_0} = (1-\alpha)z_0\bar{\ell}_0^{-\alpha}k_{-1}^{\alpha}\frac{d\bar{\ell}_0}{d\bar{m}_0}. \tag{49}$$

where we use $\bar{q}_0 = 1$ and $\bar{k}_0 = k_{-1}$ at the point of approximation. The limiting goods market clearing condition in period 1

$$\int_0^1 \bar{c}_1^i di = \bar{z}_1\bar{k}_0^{\alpha}$$

implies

$$\int_0^1 \frac{d\bar{c}_1^i}{d\bar{m}_0}di = \alpha\bar{z}_1\bar{k}_0^{\alpha-1}\frac{d\bar{k}_0}{d\bar{m}_0}. \tag{50}$$

The limiting definition of the returns

$$1 + \bar{r}_1 = \frac{\alpha\bar{z}_1\bar{k}_0^{\alpha-1}}{\bar{q}_0}$$

implies

$$\frac{1}{1+\bar{r}_1}\frac{d(1+\bar{r}_1)}{d\bar{m}_0} = (\alpha-1)\frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0} - \frac{d\bar{q}_0}{d\bar{m}_0} \tag{51}$$

where we again use $\bar{q}_0 = 1$ at the point of approximation. Finally, the limiting Fisher equation together with the monetary policy rules (10) and $P_1 = \bar{P}_1$

$$1+\bar{r}_1 = \frac{(1+\bar{i})}{(P_{-1})^{\phi}}\frac{(\bar{P}_0)^{1+\phi}}{\bar{P}_1}\bar{m}_0$$

implies

$$\frac{1}{1+\bar{r}_1}\frac{d(1+\bar{r}_1)}{d\bar{m}_0} = (1+\phi)\frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} + \frac{1}{\bar{m}_0}. \tag{52}$$

Combining (48), (51), and (52) yields

$$\frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0} = -\frac{1}{1-\alpha+\chi^x}\left((1+\phi)\frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} + \frac{1}{\bar{m}_0}\right). \tag{53}$$

Combining (45), (48), (49), (50), and (52) yields

$$\frac{1}{\bar{c}_0}\left(z_0\bar{\ell}_0^{-\alpha}k_{-1}^{\alpha}(1-\alpha)\frac{d\bar{\ell}_0}{d\bar{m}_0} - (\delta_0\chi^x+1)\frac{d\bar{k}_0}{d\bar{m}_0}\right) + (1-\psi)\epsilon^{\Phi}_{\bar{\ell}_0}\frac{1}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} = \\ -\psi\left((1+\phi)\frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} + \frac{1}{\bar{m}_0}\right) + \alpha\frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0}.$$

Since by assumption $W_{-1} = \bar{W}_0$, it follows from the union's optimal labor supply and the representative producer's optimal labor demand that each household's labor wedge is zero at the point of approximation:

$$\bar{\tau}_0^{\ell_0^i} = \bar{\tau}_0^{\ell_0} = 1 - \frac{-\bar{c}_0\Phi'(\bar{\ell}_0)/\Phi(\bar{\ell}_0)}{(1-\alpha)z_0\bar{\ell}_0^{-\alpha}k_{-1}^{\alpha}} = 0.$$

Hence we can further simplify the above as

$$-\psi\epsilon^{\Phi}_{\bar{\ell}_0}\frac{1}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} = -\psi\left((1+\phi)\frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} + \frac{1}{\bar{m}_0}\right) + \left(\alpha + (\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}\right)\frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0}. \tag{54}$$

Combining (46), (47), (48), and (49) yields

$$\frac{\frac{\chi^W}{\epsilon}}{1+\frac{\chi^W}{\epsilon}}\frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} = \left(\alpha + \frac{1}{1+\frac{\chi^W}{\epsilon}}\left(\epsilon^{-\Phi'}_{\bar{\ell}_0} - 2\epsilon^{\Phi}_{\bar{\ell}_0}\right)\right)\frac{1}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} - \frac{1}{1+\frac{\chi^W}{\epsilon}}(\delta_0\chi^x+1)\frac{1}{\bar{c}_0}\frac{d\bar{k}_0}{d\bar{m}_0}, \tag{55}$$

where we have again used the result that each household's labor wedge is zero at the point of approximation. Then (53)-(55) are 3 equations in the 3 unknowns $\{\frac{d\bar{\ell}_0}{d\bar{m}_0}, \frac{d\bar{k}_0}{d\bar{m}_0}, \frac{d\bar{P}_0}{d\bar{m}_0}\}$. Solving this system yields

$$\frac{\bar{m}_0}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} = -\frac{\left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}\right)\frac{1}{1-\frac{\epsilon}{\chi^W}(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}\frac{1+\phi}{1-\alpha+\chi^x}}}{-\psi\epsilon^{\Phi}_{\bar{\ell}_0} + \left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{k_0}{c_0}}{1-\alpha+\chi^x}\right)(1+\phi)\left(\frac{\alpha\left(1+\frac{\epsilon}{\chi^W}\right)+\frac{\epsilon}{\chi^W}\left(\epsilon^{-\Phi'}_{\bar{\ell}_0}-2\epsilon^{\Phi}_{\bar{\ell}_0}\right)}{1-\frac{\epsilon}{\chi^W}(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}\frac{1+\phi}{1-\alpha+\chi^x}}\right)},$$

$$\frac{\bar{m}_0}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} = \frac{1}{1-\frac{\epsilon}{\chi^W}\left(\delta_0\chi^x+1\right)\frac{\bar{k}_0}{\bar{c}_0}\frac{1+\phi}{1-\alpha+\chi^x}}\times$$

$$\left[-\frac{\left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}\right)\frac{\alpha\left(1+\frac{\epsilon}{\chi^W}\right)+\frac{\epsilon}{\chi^W}\left(\epsilon^{-\Phi'}_{\bar{\ell}_0}-2\epsilon^{\Phi}_{\bar{\ell}_0}\right)}{1-\frac{\epsilon}{\chi^W}(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}\frac{1+\phi}{1-\alpha+\chi^x}}}{-\psi\epsilon^{\Phi}_{\bar{\ell}_0} + \left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}\right)(1+\phi)\left(\frac{\alpha\left(1+\frac{\epsilon}{\chi^W}\right)+\frac{\epsilon}{\chi^W}\left(\epsilon^{-\Phi'}_{\bar{\ell}_0}-2\epsilon^{\Phi}_{\bar{\ell}_0}\right)}{1-\frac{\epsilon}{\chi^W}(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}\frac{1+\phi}{1-\alpha+\chi^x}}\right)} + \right.$$

$$\left.\frac{\epsilon}{\chi^W}\left(\delta_0\chi^x+1\right)\frac{\bar{k}_0}{\bar{c}_0}\frac{1}{1-\alpha+\chi^x}\right],$$

and then $\frac{\bar{m}_0}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0}$ implied by (53). For χ^W sufficiently large, each will be negative. Indeed, in the limit $\chi^W \to \infty$, these imply

$$\frac{\bar{m}_0}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} = -\frac{\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}}{-\psi\epsilon^{\Phi}_{\bar{\ell}_0} + \left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}\right)(1+\phi)\alpha} < 0,$$

$$\frac{\bar{m}_0}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} = -\alpha\frac{\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}}{-\psi\epsilon^{\Phi}_{\bar{\ell}_0} + \left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}\right)(1+\phi)\alpha} < 0,$$

$$\frac{\bar{m}_0}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0} = -\frac{1}{1-\alpha+\chi^x}\frac{-\psi\epsilon^{\Phi}_{\bar{\ell}_0}}{-\psi\epsilon^{\Phi}_{\bar{\ell}_0} + \left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}\right)(1+\phi)\alpha} < 0.$$

Since

$$\frac{d\bar{\pi}_0}{d\bar{m}_0} = (1-\alpha)\frac{\bar{\pi}_0}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} \propto \frac{d\bar{\ell}_0}{d\bar{m}_0}$$

and

$$\frac{d\bar{q}_0}{d\bar{m}_0} = \chi^x\frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0} \propto \frac{d\bar{k}_0}{d\bar{m}_0},$$

it follows from (44) that $\zeta_{m_0} > 0$. $\square$

Corollary 1

Proof. First consider the case of a household i facing a binding leverage constraint or rule-of-thumb ($i \in C$). If the household maintains

$$q_0 k_0^i = \omega_0^i a_0^i$$

in response to a marginal change in income, clearly

$$q_0 \frac{\partial k_0^i}{\partial y_0^i} = \omega_0^i \frac{\partial a_0^i}{\partial y_0^i}$$

and so

$$mpr_0^i \equiv \frac{q_0 \frac{\partial k_0^i}{\partial y_0^i}}{\frac{\partial a_0^i}{\partial y_0^i}} = \omega_0^i.$$

Provided the household remains constrained in the limit of zero aggregate risk, it follows that

$$\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} = \omega_0^i,$$
$$\overline{mpr}_0^i = \omega_0^i.$$

Now consider a household i at an interior optimum in portfolio choice ($i \notin C$). Optimal portfolio choice remains

$$\mathbb{E}_0 (c_1^i)^{-\gamma^i} (r_1^k - r_1) = 0.$$

As in the proof of Propositions 1 and 2, we successively consider higher-order approximations and repeatedly make use of the method of undetermined coefficients and market clearing.

The first- and second-order approximations imply (34) as in the proof of Proposition 1. As before,

$$\delta_{z_1}^{c_1^i} = \frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i}.$$

Multiplying both sides of (34) by $\frac{\bar{c}_1^i}{\gamma^i}$ but now integrating *only* over households $i' \notin C$ and dividing by $\int_{i \notin C} \left[\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right] di$ yields

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\mathbb{E}_0 \sigma^2 = \gamma \sigma^2 + o(||\cdot||^3)$$

for γ as defined in the claim, noting that

$$\frac{\int_{i\notin C}\left[\bar{w}_1+\bar{\pi}_1\bar{k}_0^i\right]di}{\int_{i\notin C}\bar{c}_1^i di}=1-\frac{\int_{i\notin C}(1+\bar{r}_1)\bar{b}_1^i di}{\int_{i\notin C}\bar{c}_1^i di}.$$

Moreover, by (34) we obtain (35) for $i\notin C$. It follows then that, as in the proof of Proposition 1, we obtain

$$\begin{aligned}\frac{\bar{q}_0\bar{k}_0^i}{\bar{a}_0^i}&=\left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i}\right)\frac{\gamma}{\gamma^i}-\frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i},\\ \overline{mpr}_0^i&=\frac{\gamma}{\gamma^i}\end{aligned}$$

for $i\notin C$.

A third-order approximation implies (39) as in the proof of Proposition 2. Using the same steps outlined therein yields (40). Multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i}$ but now again integrating only over households $i\notin C$ and dividing by $\int_{i\notin C}\left[\bar{w}_1+\bar{\pi}_1\bar{k}_0^i\right]di$ yields

$$\mathbb{E}_0\hat{r}_1^k-\hat{r}_0+\frac{1}{2}\sigma^2=\gamma\sigma^2+\frac{\gamma}{\int_{i\notin C}\left[\bar{w}_1+\bar{\pi}_1\bar{k}_0^i\right]di}\left(\int_{i\notin C}\bar{c}_1^i\delta_{m_0z_1}^{c_1^i}di\right)\hat{m}_0\sigma^2+o(||\cdot||^4).$$

By the period 1 resource constraint and equilibrium wages and profits, we again obtain (41). Integrating again only over households $i\notin C$ yields

$$\begin{aligned}\int_{i\notin C}\bar{c}_1^i\delta_{m_0z_1}^{c_1^i}di&=-\int_{i\notin C}\bar{c}_1^i\delta_{m_0}^{c_1^i}\delta_{z_1}^{c_1^i}di+\int_{i\notin C}\left[\alpha\bar{w}_1\delta_{m_0}^{k_0}+\bar{\pi}_1\delta_{m_0}^{k_0^i}-(1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{m_0}^{k_0}\right]di,\\
&=\int_{i\notin C}\bar{c}_1^i\delta_{m_0}^{c_1^i}\left(1-\delta_{z_1}^{c_1^i}\right)di+\int_{i\notin C}\left[\alpha\bar{w}_1\delta_{m_0}^{k_0}+\bar{\pi}_1\delta_{m_0}^{k_0^i}-(1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{m_0}^{k_0}-\bar{c}_1^i\delta_{m_0}^{c_1^i}\right]di,\\
&=\int_{i\notin C}\bar{c}_1^i\delta_{m_0}^{c_1^i}\left(1-\delta_{z_1}^{c_1^i}\right)di+\\
&\quad\int_{i\notin C}\Big[\alpha\bar{w}_1\delta_{m_0}^{k_0}+\bar{\pi}_1\delta_{m_0}^{k_0^i}-(1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{m_0}^{k_0}-\\
&\qquad\left(\alpha\bar{w}_1\delta_{m_0}^{k_0}+(1+\bar{r}_1)\delta_{m_0}^{b_0^i}+\bar{b}_0^i\delta_{m_0}^{r_1}+\bar{\pi}_1\delta_{m_0}^{k_0^i}-(1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{m_0}^{k_0}\right)\Big]di,\\
&=\int_{i\notin C}\bar{c}_1^i\delta_{m_0}^{c_1^i}\left(1-\delta_{z_1}^{c_1^i}\right)di-\int_{i\notin C}\left[(1+\bar{r}_1)\delta_{m_0}^{b_0^i}+\bar{b}_0^i\delta_{m_0}^{r_1}\right]di,\\
&=\int_{i\notin C}\bar{c}_1^i\delta_{m_0}^{c_1^i}\left(1-\delta_{z_1}^{c_1^i}\right)di+\int_{i\in C}\left[(1+\bar{r}_1)\delta_{m_0}^{b_0^i}+\bar{b}_0^i\delta_{m_0}^{r_1}\right]di,\\
&=\int_{i\notin C}\bar{c}_1^i\delta_{m_0}^{c_1^i}\left(1-\delta_{z_1}^{c_1^i}\right)di+\int_{i\in C}\left[(1+\bar{r}_1)\delta_{m_0}^{a_0^i}+\bar{a}_0^i\delta_{m_0}^{r_1}\right]\left(1-\omega_0^i\right)di,\end{aligned}$$

where the third equality substitutes in for $\bar{c}_1^i\delta_{m_0}^{c_1^i}$ implied by the period 1 resource constraint and equilibrium wages and profits; the fifth equality uses bond market clearing $\int_0^1 b_0^i di = 0$ both at the point of approximation and up to first order; and the final equality uses $b_0^i = (1-\omega_0^i)a_0^i$ both at the point of approximation and up to first order among constrained households. Using (35) and the expression for γ as defined in the claim, then note that

$$\int_{i\notin C} \bar{c}_1^i \delta_{m_0}^{c_1^i}\left(1-\delta_{z_1}^{c_1^i}\right)di = \int_{i\notin C}\left(\bar{c}_1^i\delta_{m_0}^{c_1^i} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'}di'}\int_0^1 \bar{c}_1^{i'}\delta_{m_0}^{c_1^{i'}}di'\right)\left(1-\frac{\gamma}{\gamma^i}\right) + \left(\int_0^1 \bar{c}_1^{i'}\delta_{m_0}^{c_1^{i'}}di'\right)\left(\frac{\int_{i'\notin C}\bar{c}_1^{i'}di'}{\int_0^1 \bar{c}_1^{i'}di'} - \frac{\int_{i'\notin C}\left[\bar{w}_1 + \bar{\pi}_1\bar{k}_0^{i'}\right]di'}{\int_0^1 \bar{c}_1^{i'}di'}\right).$$

Furthermore,

$$\int_{i\in C}\left[(1+\bar{r}_1)\delta_{m_0}^{a_0^i} + \bar{a}_0^i\delta_{m_0}^{r_1}\right]\left(1-\omega_0^i\right)di = \int_{i\in C}\left[(1+\bar{r}_1)\delta_{m_0}^{a_0^i} + \bar{a}_0^i\delta_{m_0}^{r_1} - \frac{(1+\bar{r}_1)\bar{a}_0^i}{\int_0^1 \bar{c}_1^{i'}di'}\int_0^1 \bar{c}_1^{i'}\delta_{m_0}^{c_1^{i'}}di'\right]\left(1-\omega_0^i\right) + \left(\int_0^1 \bar{c}_1^{i'}\delta_{m_0}^{c_1^{i'}}di'\right)\left(\frac{(1+\bar{r}_1)\int_{i'\in C}\bar{a}_0^{i'}(1-\omega_0^{i'})di'}{\int_0^1 \bar{c}_1^{i'}di'}\right).$$

Since bond market clearing implies

$$\frac{\int_{i'\notin C}\bar{c}_1^{i'}di'}{\int_0^1 \bar{c}_1^{i'}di'} - \frac{\int_{i'\notin C}\left[\bar{w}_1 + \bar{\pi}_1\bar{k}_0^{i'}\right]di'}{\int_0^1 \bar{c}_1^{i'}di'} + \frac{(1+\bar{r}_1)\int_{i'\in C}\bar{a}_0^{i'}(1-\omega_0^{i'})di'}{\int_0^1 \bar{c}_1^{i'}di'} = 0,$$

it follows that

$$\int_{i\notin C} \bar{c}_1^i \delta_{m_0}^{c_1^i}\left(1-\delta_{z_1}^{c_1^i}\right)di + \int_{i\in C}\left[(1+\bar{r}_1)\delta_{m_0}^{a_0^i} + \bar{a}_0^i\delta_{m_0}^{r_1}\right]\left(1-\omega_0^i\right)di = \int_{i\notin C}\left(\bar{c}_1^i\delta_{m_0}^{c_1^i} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'}di'}\int_0^1 \bar{c}_1^{i'}\delta_{m_0}^{c_1^{i'}}di'\right)\left(1-\frac{\gamma}{\gamma^i}\right) + \int_{i\in C}\left[(1+\bar{r}_1)\delta_{m_0}^{a_0^i} + \bar{a}_0^i\delta_{m_0}^{r_1} - \frac{(1+\bar{r}_1)\bar{a}_0^i}{\int_0^1 \bar{c}_1^{i'}di'}\int_0^1 \bar{c}_1^{i'}\delta_{m_0}^{c_1^{i'}}di'\right]\left(1-\omega_0^i\right).$$

Furthermore note that using the definition of γ given in the claim and bond market clearing,

$$1 = \int_{i\notin C}\frac{\bar{c}_1^i}{\int_{i'\notin C}\bar{c}_1^{i'}di' + \int_{i'\in C}(1+\bar{r}_1)\bar{a}_0^{i'}di'}\overline{mpr}_0^i di + \int_{i\in C}\frac{(1+\bar{r}_1)\bar{a}_0^i}{\int_{i'\notin C}\bar{c}_1^{i'}di' + \int_{i'\in C}(1+\bar{r}_1)\bar{a}_0^{i'}di'}\overline{mpr}_0^i di = \overline{mpr}_0.$$

Finally, since

$$\overline{\frac{d\left[c_1^i/\int_0^1 c_1^{i'}\right]}{dm_0}} = \frac{1}{\int_0^1 \bar{c}_1^{i'} di'}\left(\overline{\frac{dc_1^i}{dm_0}} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'}\int_0^1 \overline{\frac{dc_1^{i'}}{dm_0}} di'\right),$$
$$= \frac{1}{\int_0^1 \bar{c}_1^{i'} di'}\left(\bar{c}_1^i \delta_{m_0}^{c_1^i} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'}\int_0^1 \bar{c}_1^{i'} \delta_{m_0}^{c_1^{i'}} di'\right),$$

and

$$\overline{\frac{d\left[(1+r_1)a_0^i/\int_0^1 c_1^{i'}\right]}{dm_0}} = \frac{1}{\int_0^1 \bar{c}_1^{i'} di'}\left(\overline{\frac{d(1+r_1)a_0^i}{dm_0}} - \frac{(1+r_1)a_0^i}{\int_0^1 \bar{c}_1^{i'} di'}\int_0^1 \overline{\frac{dc_1^{i'}}{dm_0}} di'\right),$$
$$= \frac{1}{\int_0^1 \bar{c}_1^{i'} di'}\left((1+\bar{r}_1)\delta_{m_0}^{a_0^i} + \bar{a}_0^i \delta_{m_0}^{r_1} - \frac{(1+\bar{r}_1)\bar{a}_0^i}{\int_0^1 \bar{c}_1^{i'} di'}\int_0^1 \bar{c}_1^{i'} \delta_{m_0}^{c_1^{i'}} di'\right),$$

we can combine all of the previous results to write

$$\frac{\gamma}{\int_{i\notin C}\left[\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right] di}\left(\int_{i\notin C} \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di\right) = \gamma \frac{\int_0^1 \bar{c}_1^i di}{\int_{i\notin C}\left[\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right] di}\int_0^1 \bar{\xi}_{m_0}^i \left(\overline{mpr}_0 - \overline{mpr}_0^i\right) di$$

for $\bar{\xi}_{m_0}^i = \overline{\frac{d\left[c_1^i/\int_0^1 c_1^{i'}\right]}{dm_0}}$ for $i \notin C$ and $\bar{\xi}_{m_0}^i = \overline{\frac{d\left[(1+r_1)a_0^i/\int_0^1 c_1^{i'}\right]}{dm_0}}$ for $i \in C$. Again noting that

$$\frac{\int_0^1 \bar{c}_1^i di}{\int_{i\notin C}\left[\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right] di} = \left(1 - \frac{\int_{i\notin C}(1+\bar{r}_1)\bar{b}_0^i di}{\int_{i\notin C} \bar{c}_1^i di}\right)^{-1} \frac{\int_0^1 \bar{c}_1^i di}{\int_{i\notin C} \bar{c}_1^i di}.$$

yields the expression for ζ_{m_0} given in the claim. □

Corollary 2

Proof. The period 1 consumption of each household i is now

$$c_1^i = w_1 \epsilon_1^i + (1+r_1)b_0^i + \pi_1 \epsilon_1^i k_0^i,$$

where the real wage and real profits per unit of capital remain

$$w_1 = (1-\alpha) z_1 k_0^\alpha,$$
$$\pi_1 = \alpha z_1 k_0^{\alpha-1}$$

since, by the law of large numbers, the aggregate efficiency units of labor supplied remains 1 and aggregate capital among households of type i remains k_0^i.[41] Define the capital return facing each household i

$$1 + r_1^{k,i} \equiv \frac{\pi_1 \epsilon_1^i}{q_0},$$

distinct from the return on capital aggregating over idiosyncratic risk

$$1 + r_1^k \equiv \frac{\pi_1}{q_0}.$$

Then household i's optimal portfolio choice is now

$$\mathbb{E}_0 \left(c_1^i\right)^{-\gamma^i} \left(r_1^{k,i} - r_1\right) = 0.$$

Using approximations up to first and second order as in the proof of Proposition 1 yields the analog to (34) in this environment,

$$\mathbb{E}_0 \hat{r}_1^{k,i} - \hat{r}_1 + \frac{1}{2}(1+\eta^i)\sigma^2 = \gamma^i \left(\delta_{z_1}^{c_1^i} + \eta^i \delta_{\epsilon_1^i}^{c_1^i}\right) \sigma^2 + o(|| \cdot ||^3). \tag{56}$$

Given the definitions of the idiosyncratic and aggregate capital returns,

$$\hat{r}_1^{k,i} = \hat{\epsilon}_1^i + \hat{r}_1^k.$$

By assumption,

$$\mathbb{E}_0 \hat{\epsilon}_1^i = -\frac{1}{2}\eta^i \sigma^2.$$

It follows that

$$\mathbb{E}_0 \hat{r}_1^{k,i} = -\frac{1}{2}\eta^i \sigma^2 + \mathbb{E}_0 \hat{r}_1^k,$$

so that (56) implies for the aggregate capital claim

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i \left(\delta_{z_1}^{c_1^i} + \eta^i \delta_{\epsilon_1^i}^{c_1^i}\right) \sigma^2 + o(|| \cdot ||^3). \tag{57}$$

By the period 1 resource constraint and equilibrium wages and profits,

$$\delta_{z_1}^{c_1^i} = \frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i}$$

[41] Recall that we are assuming a double continuum of households now, where the continuum of households of type i are each subject to a distinct shock ϵ_1^i which is *iid* within and across i.

as in the baseline environment and

$$\delta^{c_1^i}_{\epsilon_1^i} = \frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i} = \delta^{c_1^i}_{z_1}.$$

Hence, (57) implies

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i \left(1 + \eta^i\right) \delta^{c_1^i}_{z_1} \sigma^2 + o(||\cdot||^3). \tag{58}$$

Multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i(1+\eta^i)}$, integrating over all households i, and making use of the market clearing conditions, we obtain

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma \sigma^2 + o(||\cdot||^3)$$

for γ as defined in the claim. Furthermore, it follows from (58) that we generalize (35) to

$$\delta^{c_1^i}_{z_1} = \frac{\gamma}{\gamma^i(1+\eta^i)},$$

which implies

$$\frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i} = \frac{\gamma}{\gamma^i(1+\eta^i)}$$

and thus the expression for $\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i}$ given in the claim.

Differentiating each household's optimality conditions and resource constraints generalizes (36) to

$$0 = \mathbb{E}_0 m_{0,1}^i \frac{\gamma^i}{c_1^i} \left(r_1^{k,i} - r_1\right) \left((1+r_1)\frac{\partial b_0^i}{\partial y_0^i} + \pi_1 \epsilon_1^i \frac{\partial k_0^i}{\partial y_0^i}\right).$$

A second-order approximation then generalizes (37) to

$$\begin{aligned} 0 = &\left((1+\bar{r}_1)\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}\right)\left(\mathbb{E}_0 \hat{r}_1^{k,i} - \hat{r}_1 + \frac{1}{2}(1+\eta^i)\sigma^2\right) \\ &- \left((1+\bar{r}_1)\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}\right) \frac{\gamma^i + 1}{\bar{c}_1^i} \left(\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right)(1+\eta^i)\sigma^2 + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}(1+\eta^i)\sigma^2 + o(||\cdot||^3). \end{aligned}$$

Using the above results, this implies

$$\bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}} = \frac{\gamma}{\gamma^i(1+\eta^i)} \overline{\frac{\partial a_0^i}{\partial y_0^i}},$$

from which the expression for $\overline{mpr}_0^i$ in the claim follows.

Finally, optimal portfolio choice up to third order, the above results, and steps analogous to those used in the proof of Proposition 2 yields the analog of (40)

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i\left(\delta_{z_1}^{c_1^i} + \delta_{\epsilon_1^i}^{c_1^i}\eta^i\right)\sigma^2 + \gamma^i\left(\delta_{m_0 z_1}^{c_1^i} + \delta_{m_0\epsilon_1^i}^{c_1^i}\eta^i\right)\hat{m}_0\sigma^2 + o(||\cdot||^4). \tag{59}$$

A second order expansion of the period 1 resource constraint implies

$$\begin{aligned}\bar{c}_1^i\delta_{m_0 z_1}^{c_1^i} + \bar{c}_1^i\delta_{m_0}^{c_1^i}\delta_{z_1}^{c_1^i} &= \alpha\bar{w}_1\delta_{m_0}^{k_0} + \bar{\pi}_1\delta_{m_0}^{k_0^i} - (1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{m_0}^{k_0},\\ &= \bar{c}_1^i\delta_{m_0\epsilon_1^i}^{c_1^i} + \bar{c}_1^i\delta_{m_0}^{c_1^i}\delta_{\epsilon_1^i}^{c_1^i},\end{aligned}$$

from which we can conclude

$$\delta_{m_0 z_1}^{c_1^i} = \delta_{m_0\epsilon_1^i}^{c_1^i}$$

since $\delta_{\epsilon_1^i}^{c_1^i} = \delta_{z_1}^{c_1^i}$ as argued above. It follows from (59) that

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i\left(1+\eta^i\right)\delta_{z_1}^{c_1^i}\sigma^2 + \gamma^i\left(1+\eta^i\right)\delta_{m_0 z_1}^{c_1^i}\hat{m}_0\sigma^2 + o(||\cdot||^4).$$

Then multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i(1+\eta^i)}$, integrating over all households i, and making use of the market clearing conditions, we obtain

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \frac{\gamma}{\int_0^1 \bar{c}_1^i di}\left(\int_0^1 \bar{c}_1^i\delta_{m_0 z_1}^{c_1^i} di\right)\hat{m}_0\sigma^2 + o(||\cdot||^4).$$

Then following similar steps as in the proof of Proposition 2, using

$$\delta_{z_1}^{c_1^i} = \frac{\gamma}{\gamma^i(1+\eta^i)} = \overline{mpr}_0^i$$

implied by the above results, yields the expression for ζ_{m_0} given in the claim. □

Corollary 3

Proof. Denote with $\mathbb{E}_0^i$ the expectation under household i's subjective beliefs, and $\mathbb{E}_0$ that under the objective (true) probability distribution. Household i's optimal portfolio choice is then characterized by

$$\mathbb{E}_0^i\left(c_1^i\right)^{-\gamma^i}\left(r_1^k - r_1\right) = 0.$$

Using approximations up to first and second order as in the proof of Proposition 1 yields the analog to (34) in this environment,

$$\mathbb{E}_0^i \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\varsigma^i \sigma^2 = \gamma^i \delta_{z_1}^{c_1^i} \varsigma^i \sigma^2 + o(||\cdot||^3). \tag{60}$$

By the definition of returns,

$$\hat{r}_1^k = \hat{z}_1 + (\alpha - 1)\hat{k}_0 - \hat{q}_0$$

where there is no uncertainty over $\hat{k}_0$ or $\hat{q}_0$ as of period 0. Hence,

$$\mathbb{E}_0^i \hat{r}_1^k + \frac{1}{2}\varsigma^i \sigma^2 = \mathbb{E}_0 \hat{r}_1^k + \frac{1}{2}\sigma^2.$$

Hence, (60) implies

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i \delta_{z_1}^{c_1^i} \varsigma^i \sigma^2 + o(||\cdot||^3). \tag{61}$$

By the period 1 resource constraint and equilibrium wages and profits,

$$\delta_{z_1}^{c_1^i} = \frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i}$$

as in the baseline environment. Multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i \varsigma^i}$, integrating over all households i, and making use of the market clearing conditions, we obtain

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma \sigma^2 + o(||\cdot||^3)$$

for γ as defined in the claim. Furthermore, it follows from (61) that we generalize (35) to

$$\delta_{z_1}^{c_1^i} = \frac{\gamma}{\gamma^i \varsigma^i},$$

which implies

$$\frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i} = \frac{\gamma}{\gamma^i \varsigma^i}$$

and thus the expression for $\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i}$ given in the claim.

Differentiating each household's optimality conditions and resource constraints generalizes (36) to

$$0 = \mathbb{E}_0^i m_{0,1}^i \frac{\gamma^i}{c_1^i} \left(r_1^k - r_1 \right) \left((1 + r_1) \frac{\partial b_0^i}{\partial y_0^i} + \pi_1 \frac{\partial k_0^i}{\partial y_0^i} \right).$$

A second-order approximation then generalizes (37) to

$$0 = \left((1+\bar{r}_1)\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1\overline{\frac{\partial k_0^i}{\partial y_0^i}}\right)\left(\mathbb{E}_0^i \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\varsigma^i\sigma^2\right)$$
$$- \left((1+\bar{r}_1)\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1\overline{\frac{\partial k_0^i}{\partial y_0^i}}\right)\frac{\gamma^i+1}{\bar{c}_1^i}\left(\bar{w}_1 + \bar{\pi}_1\bar{k}_0^i\right)\varsigma^i\sigma^2 + \bar{\pi}_1\overline{\frac{\partial k_0^i}{\partial y_0^i}}\varsigma^i\sigma^2 + o(||\cdot||^3).$$

Using the above results, this implies

$$\bar{q}_0\overline{\frac{\partial k_0^i}{\partial y_0^i}} = \frac{\gamma}{\gamma^i\varsigma^i}\overline{\frac{\partial a_0^i}{\partial y_0^i}},$$

from which the expression for $\overline{mpr}_0^i$ in the claim follows.

Finally, optimal portfolio choice up to third order, the above results, and steps analogous to those used in the proof of Proposition 2 yields the analog of (40)

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i\delta_{z_1}^{c_1^i}\varsigma^i\sigma^2 + \gamma^i\delta_{m_0z_1}^{c_1^i}\varsigma^i\hat{m}_0\sigma^2 + o(||\cdot||^4). \tag{62}$$

Multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i\varsigma^i}$, integrating over all households i, and making use of the market clearing conditions, we obtain

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \frac{\gamma}{\int_0^1 \bar{c}_1^i di}\left(\int_0^1 \bar{c}_1^i\delta_{m_0z_1}^{c_1^i}di\right)\hat{m}_0\sigma^2 + o(||\cdot||^4).$$

Then following similar steps as in the proof of Proposition 2, using

$$\delta_{z_1}^{c_1^i} = \frac{\gamma}{\gamma^i\varsigma^i} = \overline{mpr}_0^i$$

implied by the above results, yields the expression for ζ_{m_0} given in the claim. □

Proposition 5

Proof. We first note that, in the general environment featuring portfolio constraints / rules-of-thumb, background risk, and subjective beliefs regarding aggregate TFP, households' limiting portfolios and MPRs are

$$\frac{\bar{q}_0\bar{k}_0^i}{\bar{a}_0^i} = \begin{cases} \omega_0^i \ \text{ for } \ i \in C, \\ \left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i}\right)\frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i} \ \text{ for } \ i \notin C, \end{cases} \tag{63}$$

$$\overline{mpr}_0^i = \begin{cases} \omega_0^i \ \text{ for } \ i \in C, \\ \frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} \ \text{ for } \ i \notin C, \end{cases} \tag{64}$$

where

$$\gamma = \left(\int_{i\notin C} \frac{\bar{c}_1^i}{\int_{i'\notin C}\bar{c}_1^{i'}di'}\frac{1}{\gamma^i(1+\eta^i)\varsigma^i}di\right)^{-1}\left(1 - \frac{\int_{i\notin C}(1+\bar{r}_1)\bar{b}_0^i di}{\int_{i\notin C}\bar{c}_1^i di}\right). \tag{65}$$

Up to third order in $\{\sigma, \hat{z}_1, \hat{m}_0\}$, we obtain (20) with γ as in (65) and

$$\zeta_{m_0} = \left(\int_{i\notin C} \frac{\bar{c}_1^i}{\int_{i'\notin C}\bar{c}_1^{i'}di'}\frac{1}{\gamma^i(1+\eta^i)\varsigma^i}di\right)^{-1}\int_0^1 \bar{\xi}_{m_0}^i\left(\overline{mpr}_0 - \overline{mpr}_0^i\right)di, \tag{66}$$

where $\bar{\xi}_{m_0}^i \equiv \overline{\frac{d\left[(1+r_0)a_0^i/\int_0^1 c_1^{i'}di'\right]}{dm_0}}$ for $i \in C$ and $\bar{\xi}_{m_0}^i \equiv \overline{\frac{d\left[c_1^i/\int_0^1 c_1^{i'}di'\right]}{dm_0}}$ for $i \notin C$, and $\overline{mpr}_0 \equiv \int_{i\in C}\frac{(1+\bar{r}_1)\bar{a}_0^i}{\int_{i'\in C}(1+\bar{r}_1)\bar{a}_0^{i'}di'+\int_{i'\notin C}\bar{c}_1^{i'}di'}\overline{mpr}_0^i di + \int_{i\notin C}\frac{\bar{c}_1^i}{\int_{i'\in C}(1+\bar{r}_1)\bar{a}_0^{i'}di'+\int_{i'\notin C}\bar{c}_1^{i'}di'}\overline{mpr}_0^i di = 1$. The proof of these results combines the proofs in Corollaries 1-3 and we do not repeat it here.

A household's limiting change in future consumption in response to a monetary policy shock $\overline{\frac{dc_1^i}{dm_0}}$ remains characterized by Proposition 3. Given the limiting period 1 budget constraint

$$\bar{c}_1^i = \bar{w}_1 + (1+\bar{r}_1)\bar{a}_0^i, \tag{67}$$

a household's limiting change in $(1+r_1)a_0^i$ in response to a monetary policy shock is characterized by

$$\overline{\frac{d(1+r_1)a_0^i}{dm_0}} = \overline{\frac{dc_1^i}{dm_0}} - \overline{\frac{dw_1}{dm_0}}. \tag{68}$$

Then, under the assumptions that households are identical except for $\{\gamma^i, \omega_0^i, \eta^i, \varsigma^i\}$ and whether or not they are constrained, it follows that for unconstrained households ($i \notin C$)

$$\bar{\xi}_{m_0}^i = \frac{1}{\bar{c}_1}(1+\bar{r}_1)\overline{\frac{\partial a_0}{\partial y_0}}\left[-\frac{(1+i_{-1})}{P_0}B_{-1}^i\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}} + (k_{-1}^i - k_{-1})\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right)\right].$$

as in the baseline case. By (63)

$$
\begin{aligned}
k_{-1}^i = \bar{k}_0^i &= \bar{a}_0 \left[\frac{\bar{c}_1}{(1+\bar{r}_1)\bar{a}_0} \frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0} \right], \\
&= \frac{\bar{k}_0}{\alpha} \left(\frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} - 1 \right) + \bar{k}_0, \\
B_{-1}^i = \bar{P}_0 \bar{b}_0^i &= \bar{P}_0 \left[\bar{a}_0 - \bar{k}_0^i \right], \\
&= -\bar{P}_0 \frac{\bar{k}_0}{\alpha} \left(\frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} - 1 \right),
\end{aligned}
$$

where we use $\bar{q}_0 = 1$ and, by (65),

$$
\gamma = \left[\int_{i \notin C} \frac{1}{\gamma^i(1+\eta^i)\varsigma^i} di \right]^{-1} \frac{\int_{i \notin C} \left[\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i \right] di}{\bar{c}_1}.
$$

It follows that

$$
\begin{aligned}
-\frac{(1+i_{-1})B_{-1}^i}{P_0} &= (1+i_{-1}) \frac{\bar{k}_0}{\alpha} \left(\frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} - 1 \right), \\
k_{-1}^i - k_{-1} &= \frac{\bar{k}_0}{\alpha} \left(\frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} - 1 \right).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
&\left(\int_{i \notin C} \frac{\bar{c}_1^i}{\int_{i' \notin C} \bar{c}_1^{i'} di'} \frac{1}{\gamma^i(1+\eta^i)\varsigma^i} di \right)^{-1} \int_{i \notin C} \bar{\xi}_{m_0}^i (\overline{mpr}_0 - \overline{mpr}_0^i) di, \\
&= \left[\int_{i \notin C} \frac{1}{\gamma^i(1+\eta^i)\varsigma^i} di \right]^{-1} \int_{i \notin C} \bar{\xi}_{m_0}^i \left(1 - \frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} \right) di, \\
&= \left(\int_{i \notin C} \frac{1}{\gamma^i(1+\eta^i)\varsigma^i} di \right)^{-1} \left[-\frac{1}{\bar{c}_1}(1+\bar{r}_1) \overline{\frac{\partial a_0}{\partial y_0}} \frac{\bar{k}_0}{\alpha} \int_{i \notin C} \left(1 - \frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} \right)^2 di \right] \times \\
&\quad \left[(1+i_{-1})\bar{q}_0 \frac{1}{P_0} \overline{\frac{dP_0}{dm_0}} + \overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0) \overline{\frac{dq_0}{dm_0}} \right]. \qquad (69)
\end{aligned}
$$

where the second equality uses (64) and the third equality uses the above results.

For constrained households ($i \in C$),

$$
\begin{aligned}
\bar{\xi}^i_{m_0} &= \frac{1}{\bar{c}_1}\left[\overline{\frac{d(1+r_1)a^i_0}{dm_0}} - \frac{(1+\bar{r}_1)\bar{a}_0}{\bar{c}_1}\int_0^1 \overline{\frac{dc^i_1}{dm_0}}di\right], \\
&= \frac{1}{\bar{c}_1}\left[\overline{\frac{d(1+r_1)a^i_0}{dm_0}} - \int_0^1 \overline{\frac{dc^i_1}{dm_0}}di\right] + \frac{1}{\bar{c}_1}\frac{\bar{w}_1}{\bar{c}_1}\int_0^1 \overline{\frac{dc^i_1}{dm_0}}di, \\
&= \frac{1}{\bar{c}_1}(1+\bar{r}_1)\overline{\frac{\partial a_0}{\partial y_0}}\left[-\frac{(1+i_{-1})}{P_0}B^i_{-1}\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}} + (k^i_{-1}-k_{-1})\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right)\right] - \\
&\quad \frac{1}{\bar{c}_1}\overline{\frac{dw_1}{dm_0}} + \frac{1}{\bar{c}_1}\frac{\bar{w}_1}{\bar{c}_1}\int_0^1 \overline{\frac{dc^i_1}{dm_0}}di, \\
&= \frac{1}{\bar{c}_1}(1+\bar{r}_1)\overline{\frac{\partial a_0}{\partial y_0}}\left[-\frac{(1+i_{-1})}{P_0}B^i_{-1}\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}} + (k^i_{-1}-k_{-1})\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right)\right] + \\
&\quad \frac{\bar{w}_1}{\bar{c}_1}\left[\frac{1}{\bar{c}_1}\int_0^1 \overline{\frac{dc^i_1}{dm_0}}di - \frac{1}{\bar{w}_1}\overline{\frac{dw_1}{dm_0}}\right],
\end{aligned}
$$

where the second equality uses (67) and the third equality uses (68) as well as Proposition 3. By (63)

$$
\begin{aligned}
k^i_{-1} = \bar{k}^i_0 &= \frac{\bar{a}_0\omega^i_0}{\bar{q}_0}, \\
&= \bar{k}_0(\omega^i_0 - 1) + \bar{k}_0, \\
B^i_{-1} = \bar{P}_0\bar{b}^i_0 &= \bar{P}_0\bar{a}_0(1-\omega^i_0), \\
&= -\bar{P}_0\bar{k}_0\left(\omega^i_0 - 1\right).
\end{aligned}
$$

It follows that

$$
\begin{aligned}
-\frac{(1+i_{-1})B^i_{-1}}{\bar{P}_0} &= (1+i_{-1})\bar{k}_0\left(\omega^i_0 - 1\right), \\
k^i_{-1} - k_{-1} &= \bar{k}_0\left(\omega^i_0 - 1\right).
\end{aligned}
$$

Hence,

$$\left(\int_{i\notin C}\frac{\bar{c}_1^i}{\int_{i'\notin C}\bar{c}_1^{i'}di'}\frac{1}{\gamma^i(1+\eta^i)\varsigma^i}di\right)^{-1}\int_{i\in C}\bar{\xi}^i_{m_0}(\overline{mpr}_0-\overline{mpr}^i_0)di,$$
$$=\left(\int_{i\notin C}\frac{1}{\gamma^i(1+\eta^i)\varsigma^i}di\right)^{-1}\int_{i\in C}\bar{\xi}^i_{m_0}\left(1-\omega_0^i\right)di,$$
$$=\left(\int_{i\notin C}\frac{1}{\gamma^i(1+\eta^i)\varsigma^i}di\right)^{-1}\times$$
$$\left(\left[-\frac{1}{\bar{c}_1}(1+\bar{r}_1)\overline{\frac{\partial a_0}{\partial y_0}}\bar{k}_0\int_{i\in C}\left(1-\omega_0^i\right)^2di\right]\left[(1+i_{-1})\bar{q}_0\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}}+\overline{\frac{d\pi_0}{dm_0}}+(1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right]+\right.$$
$$\left.\left(\int_{i\in C}(1-\omega_0^i)di\right)\frac{\bar{w}_1}{\bar{c}_1}\left[\frac{1}{\bar{c}_1}\int_0^1\overline{\frac{dc_1^i}{dm_0}}di-\frac{1}{\bar{w}_1}\overline{\frac{dw_1}{dm_0}}\right]\right). \tag{70}$$

Now note that the characterization of $\overline{\frac{dP_0}{dm_0}}$, $\overline{\frac{d\pi_0}{dm_0}}$, and $\overline{\frac{dq_0}{dm_0}}$ is unchanged from the proof of Proposition 4. Furthermore, since limiting goods market clearing in period 1

$$\int_0^1\bar{c}_1^i di=\bar{z}_1\bar{k}_0^\alpha$$

implies

$$\frac{1}{\bar{c}_1}\int_0^1\frac{d\bar{c}_1^i}{d\bar{m}_0}di=\alpha\frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0}$$

while limiting labor demand in period 1

$$\bar{w}_1=(1-\alpha)\bar{z}_1\bar{k}_0^\alpha$$

implies

$$\frac{1}{\bar{w}_1}\frac{d\bar{w}_1}{d\bar{m}_0}=\alpha\frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0},$$

we have that

$$\left[\frac{1}{\bar{c}_1}\int_0^1\overline{\frac{dc_1^i}{dm_0}}di-\frac{1}{\bar{w}_1}\overline{\frac{dw_1}{dm_0}}\right]=0.$$

Hence, combining (69) and (70) in (66) implies

$$
\begin{aligned}
\zeta_{m_0} &= \left(\int_{i \notin C} \frac{\bar{c}_1^i}{\int_{i' \notin C} \bar{c}_1^{i'} di'} \frac{1}{\gamma^i (1+\eta^i) \varsigma^i} di \right)^{-1} \int_0^1 \bar{\xi}_{m_0}^i (1 - \overline{mpr}_0^i) di \\
&\propto -(1+i_{-1}) \frac{1}{P_0} \overline{\frac{dP_0}{dm_0}} - \overline{\frac{d\pi_0}{dm_0}} - (1-\delta_0) \overline{\frac{dq_0}{dm_0}}, \\
&> 0
\end{aligned}
$$

for χ^W sufficiently large. □

Proposition 6

Proof. Recall that the monetary policy rule (10) and $P_1 = \bar{P}_1$ implies a real interest rate

$$
1 + r_1 = \frac{(1+\bar{i})}{P_{-1}^{\phi}} \frac{(P_0)^{1+\phi}}{\bar{P}_1} m_0,
$$

which then implies the exact log-linear relationship

$$
\hat{r}_1 = (1+\phi) \hat{P}_0 + \hat{m}_0.
$$

When $\phi = -1$, it follows that

$$
\hat{r}_1 = \hat{m}_0.
$$

Given the expansion in state variables

$$
\hat{r}_1^k = \delta_{m_0}^{r_1} \hat{m}_0 + \hat{z}_1 + \frac{1}{2} \delta_{\sigma^2}^{r_1^k} \sigma^2 + \frac{1}{2} \delta_{z_1^2}^{r_1^k} z_1^2 + \frac{1}{2} \delta_{m_0 \sigma^2}^{r_1^k} \hat{m}_0 \sigma^2 + \frac{1}{2} \delta_{m_0 z_1^2}^{r_1^k} \hat{m}_0 z_1^2 + o(|| \cdot ||^4),
$$

it follows from Proposition 2 that

$$
\frac{1}{2} \delta_{m_0 \sigma^2}^{r_1^k} + \frac{1}{2} \delta_{m_0 z_1^2}^{r_1^k} = \zeta_{m_0}.
$$

Now by the definition of the return on capital, we have the exact log-linear relationship

$$
\hat{r}_1^k = \hat{z}_1 - (1 - \alpha + \chi^x) \hat{k}_0
$$

as derived in the main text. It follows by the method of undetermined coefficients

$$\frac{1}{2}\delta^{r_1^k}_{m_0\sigma^2} = -\left(1-\alpha+\chi^x\right)\frac{1}{2}\delta^{k_0}_{m_0\sigma^2},$$
$$\frac{1}{2}\delta^{r_1^k}_{m_0 z_1^2} = 0.$$

Hence, the above results imply

$$\frac{1}{2}\delta^{r_1^k}_{m_0\sigma^2} = -\frac{1}{1-\alpha+\chi^x}\zeta_{m_0},$$

proving the claim. □

B Additional analytical results

In this appendix we provide supplementary analytical results accompanying section 2. We exclude proofs of these supplemental results for brevity, but they are available on request.

B.1 Individually supplied labor

We first demonstrate the robustness of our analytical results to individually-supplied labor rather than the union set-up assumed in the main text.

B.1.1 Modified environment and equilibrium

We dispense with the index j and assume households directly supply distinct varieties of labor to the market at wages $\{W_0^i\}$. Household preferences thus can be written

$$v_0^i = \left(\left(1-\beta^i\right)\left(c_0^i\Phi^i\left(\ell_0^i\right)\right)^{1-1/\psi^i} + \beta^i\left(\mathbb{E}_0\left[\left(c_1^i\right)^{1-\gamma^i}\right]\right)^{\frac{1-1/\psi^i}{1-\gamma^i}}\right)^{\frac{1}{1-1/\psi^i}}$$

and the resource constraints become

$$\begin{aligned} P_0c_0^i + B_0^i + Q_0k_0^i \leq\ & (1-\tau)W_0^i\ell_0^i - AC_0^{W,i}\ + \\ & (1+i_{-1})B_{-1}^i + (\Pi_0 + (1-\delta_0)Q_0)k_{-1}^i + T_0^i, \\ P_1c_1^i \leq\ & W_1 + (1+i_0)B_0^i + \Pi_1 k_0^i \end{aligned}$$

with adjustment costs

$$AC_0^{W,i} = \frac{\chi^W}{2}W_0\ell_0\left(\frac{W_0^i}{W_{-1}}-1\right)^2.$$

The labor packer directly hires labor from households and combines it using the CES aggregator, earning profits

$$W_0 \left[\int_0^1 (\ell_0^i)^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)} - \int_0^1 W_0^i \ell_0^i di.$$

The notation in the government transfer condition (9) and labor market clearing condition (13) must be changed, and the equilibrium in Definition 1 is otherwise the same.

In equilibrium households will generically supply different amounts of labor and earn different wages solving

$$\frac{W_0^i}{P_0} + c_0^i \frac{\Phi^{i'}(\ell_0^i)}{\Phi^i(\ell_0^i)} + \frac{W_0}{P_0} \frac{\chi^W}{\epsilon} \frac{\ell_0}{\ell_0^i} \frac{W_0^i}{W_{-1}} \left(\frac{W_0^i}{W_{-1}} - 1 \right) = 0,$$
$$\ell_0^i = \left(\frac{W_0^i}{W_0} \right)^{-\epsilon} \ell_0,$$

conditional on their choice of consumption c_0^i and the aggregates $\{W_0, P_0, \ell_0\}$. This contrasts with labor supply in the baseline economy, where households earn identical wages and the representative union's labor supply condition is

$$\int_0^1 \mu^i \left(v_0^i\right)^{\frac{1}{\psi^i}} \left(c_0^i\right)^{-\frac{1}{\psi^i}} \Phi^i(\ell^i(\ell_0^i))^{1-\frac{1}{\psi^i}} \ell^{i'}(\ell_0) \Bigg[\frac{W_0}{P_0} \left(\frac{\ell^i(\ell_0)}{\ell_0 \ell^{i'}(\ell_0)} \frac{1}{1-\epsilon} - \frac{\epsilon}{1-\epsilon} \right) + c_0^i \frac{\Phi^{i'}(\ell^i(\ell_0))}{\Phi^i(\ell^i(\ell_0))}$$
$$+ \frac{W_0}{P_0} \frac{1}{\ell^{i'}(\ell_0)} \frac{\chi^W}{\epsilon} \frac{W_0}{W_{-1}} \left(\frac{W_0}{W_{-1}} - 1 \right) \Bigg] di = 0.$$

given the allocation rule $\ell^i(\cdot)$ defined in (4) and Pareto weights $\{\mu^i\}$.

B.1.2 Robustness of results

Now each household is characterized by a marginal propensity to work $\frac{\partial \ell_0^i}{\partial y_0^i}$ and marginal propensity to set its wage $\frac{\partial w_0^i}{\partial y_0^i}$ in addition to its marginal propensities to consume, save in bonds, save in capital, and save overall. Nonetheless, Proposition 1 remains unchanged.

The characterization of the risk premium up to third order in Proposition 2 is unchanged. However, the monetary policy exposures $\bar{\xi}_{m_0}^i$ now reflect households' alternative adjustment on the supply-side. In particular, Proposition 3 must be adjusted to reflect the fact that each household is no longer a price-taker in the labor market. However, when households are identical except for risk aversion and their portfolio shares, these changes are irrelevant for $\bar{\xi}_{m_0}^i$; that is, balance sheet revaluation remains the only source of redistribution. This is formalized in the following result:

Proposition 7. *Consider the case with individually-supplied labor by each household but assume the conditions of Proposition 4 hold. Then*

$$\bar{\xi}^i_{m_0} = \frac{1}{\bar{c}_1}(1+\bar{r}_1)\overline{\frac{\partial a_0}{\partial y_0}}\left[-\frac{(1+i_{-1})B^i_{-1}}{P_0}\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}} + (k^i_{-1} - k_{-1})\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right)\right]$$

where $\bar{c}_1$ is the identical level of consumption across households and $\overline{\frac{\partial a_0}{\partial y_0}}$ the identical marginal propensity to save of households at the point of approximation.

It immediately follows that Proposition 4 remains unchanged. The generalizations to other forms of heterogeneity in Corollaries 1-3 are unchanged, and thus Proposition 5 is unchanged as well. Finally, the effect of a monetary policy shock on capital accumulation operating through the change in the risk premium in Proposition 6 is unchanged.

B.2 Inflation risk in the nominal bond

We next demonstrate the robustness of our analytical results to inflation risk in the nominal bond.

B.2.1 Modified environment and equilibrium

We generalize the baseline environment described in section 2.1 so that the monetary authority lets the future price level vary with TFP

$$P_1 = \bar{P}_1(z_1)^\iota.$$

The baseline environment featured $\iota = 0$. It is further straightforward to add another source of risk in P_1 corresponding to a distinct monetary policy shock, but we do not do that for expositional parsimony. Beyond this change to monetary policy, the definition of equilibrium in Definition 1 is otherwise unchanged.

In the baseline economy the realized real interest rate was given by

$$1 + r_1 \equiv (1+i_0)\frac{P_0}{P_1} = \frac{1+\bar{i}}{P^\phi_{-1}}\frac{P_0^{1+\phi}}{\bar{P}_1}m_0$$

and thus was known with certainty as of period 0. In contrast in the present economy the realized real interest rate given by

$$1 + r_1 \equiv (1+i_0)\frac{P_0}{P_1} = \frac{1+\bar{i}}{P^\phi_{-1}}\frac{P_0^{1+\phi}}{\bar{P}_1(z_1)^\iota}m_0$$

is uncertain as of period 1.

B.2.2 Robustness of results

Proposition 1 must be adjusted because households' limiting portfolios and MPRs are affected by the presence of inflation risk:

Proposition 8. *With inflation risk in the nominal bond,*

$$\frac{\bar{q}_0\bar{k}_0^i}{\bar{a}_0^i} = \frac{1}{1+\iota}\left[\left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i}\right)\frac{\gamma}{\gamma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i} + \iota\right],$$

$$\overline{mpr}_0^i = \frac{1}{1+\iota}\left[\frac{\gamma}{\gamma^i} + \iota\right],$$

where γ remains characterized by (19).

Expected excess returns up to third order are also modified:

Proposition 9. *Up to third order in the perturbation parameters $\{\sigma, \hat{z}_1, \hat{m}_0\}$,*

$$\mathbb{E}_0\hat{r}_1^k - \mathbb{E}_0\hat{r}_1 + \frac{1}{2}\left(1-\iota^2\right)\sigma^2 = \gamma\left(1+\iota\right)\sigma^2 + \zeta_{m_0}\hat{m}_0\left(1+\iota\right)^2\sigma^2 + o(||\cdot||^4),$$

where γ is defined in (19) and ζ_{m_0} is defined in (21).

Intuitively, as $\iota \to -1$, the real payoff to the nominal bond perfectly replicates that of capital, eliminating any excess returns on capital and the effect of a monetary policy shock on those expected excess returns. Aside from an appropriate re-scaling of each term, however, it remains the case that the distribution of monetary policy exposures and MPRs determines the effect of a monetary shock on the risk premium.

Proposition 3 characterizing the change in households' consumption in response to a monetary policy shock is unchanged. It follows that the balance sheet revaluation underlying Proposition 4 remains unchanged. The generalizations to other forms of heterogeneity in Corollaries 1-3 must be modified like the above results, but Proposition 5 remains unchanged. Finally, the effect of a monetary policy shock on capital accumulation operating through the change in the risk premium in Proposition 6 is unchanged.

B.3 Effect of TFP shocks

We next demonstrate that our analytical insights extend beyond monetary policy shocks to any shock which redistributes across households in period 0 or (in expectation) period 1. Here we focus on a TFP shock in period 0, corresponding to our quantitative analysis of productivity shocks in appendix E. Formally, we treat $\hat{z}_0$ as another perturbation parameter of interest. For expositional simplicity, we assume $\hat{m}_0 = 0$, though it is straightforward to consider both TFP and monetary shocks since they simply enter additively.

We first obtain the analog of Proposition 2 for a TFP shock:

Proposition 10. *Up to third order in the perturbation parameters $\{\sigma, \hat{z}_1, \hat{z}_0\}$,*

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \zeta_{z_0}\hat{z}_0\sigma^2 + o(||\cdot||^4),$$

where γ was defined in (19) and

$$\zeta_{z_0} = \gamma \int_0^1 \bar{\xi}_{z_0}^i \left(\overline{mpr}_0 - \overline{mpr}_0^i\right) di,$$

where $\bar{\xi}_{z_0}^i \equiv \overline{\frac{d\left[c_1^i / \int_0^1 c_1^{i'} di'\right]}{dz_0}}$ is the effect of a TFP shock on household i's consumption share in period 1 and $\overline{mpr}_0 \equiv \int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \overline{mpr}_0^i = 1$ is the weighted average MPR in (1).

As is evident, ζ_{z_0} parallels ζ_{m_0} for a monetary shock. In this simple two-period environment, a TFP shock affects the risk premium only through redistribution. If a positive TFP shock redistributes wealth to households with high MPRs, it will lower the risk premium.

The change in consumption relevant to evaluate the redistributive effects of a TFP shock is analogous to Proposition 3:

Proposition 11. *A household's change in future consumption in response to a TFP shock is given by*

$$\begin{aligned}\overline{\frac{dc_1^i}{dz_0}} = (1+\bar{r}_1) \underbrace{\overline{\frac{\partial a_0^i}{\partial y_0^i}}}_{MPS} \Bigg[& \underbrace{- \frac{(1+i_{-1})B_{-1}^i}{\bar{P}_0} \frac{1}{\bar{P}_0} \overline{\frac{dP_0}{dz_0}} + k_{-1}\left(\overline{\frac{d\pi_0}{dz_0}} + (1-\delta_0)\overline{\frac{dq_0}{dz_0}}\right)}_{\text{balance sheet revaluation}} \\ & + \underbrace{\left(\overline{\frac{dw_0\ell_0^i}{dz_0}} + \frac{1}{1+\bar{r}_1}\overline{\frac{dw_1}{dz_0}}\right)}_{\text{change in non-traded income}} + \underbrace{\bar{a}_0^i \frac{1}{1+\bar{r}_1} \overline{\frac{d(1+r_1)}{dz_0}}}_{\text{income effect}} \\ & + \underbrace{\psi^i c_0^i \frac{1}{1+\bar{r}_1} \overline{\frac{d(1+r_1)}{dz_0}} + \left(\psi^i - 1\right)\bar{w}_0\left(1 - \bar{\tau}^{\ell_0^i}\right)\overline{\frac{d\ell_0^i}{dz_0}}}_{\text{substitution effects}} \Bigg]\end{aligned}$$

given the steady-state labor wedge for household i $\bar{\tau}^{\ell_0^i} \equiv 1 - \frac{-\bar{c}_0^i \Phi^{i'}(\bar{\ell}_0^i)/\Phi^i(\bar{\ell}_0^i)}{(1-\alpha)\bar{z}_0(\bar{\ell}_0)^{-\alpha}k_{-1}^{\alpha}}$.

When households are symmetric in all respects except risk aversion and their portfolio shares, only the balance sheet revaluation channel will redistribute across them. However, a TFP shock can have different effects on prices, profits, and the price of capital than a monetary policy shock. For instance, for χ^W sufficiently large, as is assumed in Proposition 4, we can prove that $\overline{\frac{dP_0}{dz_0}} < 0$, $\overline{\frac{dq_0}{dz_0}} > 0$, and the sign of $\overline{\frac{d\pi_0}{dz_0}}$ depends on parameters. It follows that the effect of a positive TFP shock on the risk premium depends on parameters, because it both redistributes away from levered, high MPR households by raising the real value of their debt burden, redistributes toward these same households by raising the price of capital, and has an ambiguous effect on redistribution through short-run profits.

Corollaries 1-3 generalize as above to the case of TFP shocks, but again the risk premium effects of a TFP shock under the conditions of Proposition 5 are ambiguous. Whatever the sign of this risk premium response, we obtain the following analog of Proposition 6:

Proposition 12. *If monetary policy follows the rule (10) with $\phi = -1$, then*

$$\delta^{k_0}_{z_0\sigma^2} = -\frac{1}{1-\alpha+\chi^x}\zeta_{z_0},$$

given ζ_{z_0} characterized in Proposition 10.

Again, provided the monetary policy rule keeps the real interest rate constant, an increase (decrease) in the risk premium induced by the TFP shock will lower (raise) investment.

C Empirical appendix

In this appendix we provide supplemental material for our empirical analyses provided in section 3.

C.1 The effect of monetary shocks

We first provide supplemental evidence on the empirical effects of a monetary policy shock studied in section 3.1.

C.1.1 Robustness to details of estimation approach

Here we demonstrate that the broad messages of our baseline estimates are robust to the number of lags in the VAR, sample periods used in both stages of the SVAR-IV, variables included in the VAR, and instrument used.

Table 10 summarizes the impact effect of a monetary policy shock on the 1-year Treasury yield (the monetary policy indicator) and the excess return on the S&P 500, as well as the share of the latter driven by news about future excess returns in the Campbell-Shiller

	Current 1-year Treasury yield (*pp*)	Current excess return (*pp*)	Share future excess return news (%)
Baseline	-0.22	2.06	55%
Number of lags in VAR			
4	-0.21	1.95	49%
5	-0.22	1.91	51%
7	-0.23	1.96	59%
8	-0.23	2.03	52%
Sample periods			
VAR: 1/91-6/12, IV: 1/91-6/12	-0.14	1.60	35%
VAR: 7/79-6/12, IV: 1/91-9/01	-0.21	3.21	47%
VAR: 7/79-6/12, IV: 10/01-6/12	-0.17	-2.07	39%
Variable added to VAR			
Excess bond premium	-0.21	2.33	74%
Mortgage spread	-0.24	1.64	50%
3-month commercial paper spread	-0.19	2.31	62%
5-year Treasury rate	-0.17	1.69	75%
10-year Treasury rate	-0.17	1.63	73%
Term spread	-0.21	2.06	62%
Relative bill rate	-0.18	2.68	66%
Change in 3-month Treasury rate	-0.19	2.39	61%
3-month ahead FF as IV	-0.20	2.31	62%

Table 10: robustness of 1 SD monetary shock on current excess returns and components

Notes: series for the Gilchrist and Zakrajsek (2012) excess bond premium, mortgage spread, 3-month commercial paper spread, 5-year Treasury rate, and 10-year Treasury rate are taken from the dataset provided by Gertler and Karadi (2015). The term spread (10-year Treasury rate less 1-month Treasury yield), relative bill rate (difference between the 3-month Treasury rate and its 12-month moving average), and change in the 3-month Treasury rate are constructed using CRSP.

decomposition (24). First, we find that the baseline results using 6 lags in the VAR are little affected if 4-8 lags are used instead. Second, we find that the results are broadly robust to using the same January 1991 - June 2012 period for both the VAR and IV regressions, or limiting the analysis of monetary policy shocks to the first half of the IV sample alone (January 1991 - September 2001). The expansionary monetary policy shock in fact lowers excess returns when using the second half of the IV sample alone (October 2001 - June 2012), but we note that the instrument is weak over this sub-sample (having a first-stage F statistic of 4.67, not shown). Third, we find that news about future excess returns tends to be, if anything, even more important when adding other variables included in the analyses of Bernanke and Kuttner (2005) and Gertler and Karadi (2015) on which we build. Finally,

	1-yr Treasury	CPI	Industrial production	1-mo real rate	1-mo excess return	Dividend/ price
SW [2018] test	0.50	0.62	0.97	0.78	0.68	0.72
Granger causality test	0.07	0.15	0.88	0.12	0.45	0.93

Table 11: tests of invertibility assumed in the VAR

Notes: the first row is the bootstrapped p-value for the null hypothesis that the SVAR-IV and LP-IV impulse responses depicted in Figure 6 are the same 1, 13, 25, and 37 months after shock, using the test statistic provided in Stock and Watson (2018). We construct the variance matrix needed for this statistic using the 10,000 iterations of the wild bootstrap used to construct confidence intervals for our SVAR-IV estimates in the main text. The second row is the p-value for the null hypothesis that the coefficients on 6 lags of the instrument are jointly equal to zero when added to the VAR.

we find that our results are similar when using as the instrument the three-month ahead Fed Funds futures contract instead of the current contract, recalling that this was the other strong instrument indicated by Table 1.

C.1.2 Testing invertibility and comparing SVAR-IV and LP-IV

We now demonstrate that the assumption of invertibility used in the VAR is validated by statistical tests suggested in the literature. Relatedly, we demonstrate that our estimated impulse responses lie within the (quite large) confidence intervals obtained using an alternative local projection instrumental variables approach (LP-IV) at virtually every horizon.

We implement the LP-IV by projecting each outcome variable h months ahead on the 1-year Treasury yield, instrumenting for the latter using the Fed Funds futures surprise also used in our baseline SVAR-IV. Following Stock and Watson (2018), to make this specification comparable with the SVAR-IV and further improve the precision of estimates, we include 6 lags of each of the variables included in the VAR as controls. Moreover, given the serial correlation of the instrument discussed in Ramey (2016) and Stock and Watson (2018), we include a lag of the instrument as an additional control. Figure 6 plots the estimates at each horizon $h \in \{0, \ldots, 47\}$, along with the point-wise 95% heteroskedasticity-robust confidence interval. These estimates are extremely noisy, but their confidence intervals contain our baseline estimates at virtually every horizon for each variable.

Stock and Watson (2018) discuss two tests of the invertibility assumption implicit in the SVAR-IV, the first of which formalizes this comparison of the SVAR-IV and LP-IV estimates. They propose a Hausman-type test statistic of the null hypothesis that invertibility is satisfied by comparing the impulse response at horizon h for a given variable under both approaches. The first row of Table 11 summarizes the p-value for this test in our setting jointly applied at horizons $h \in \{1, 13, 25, 37\}$ for each variable, demonstrating that we cannot reject the

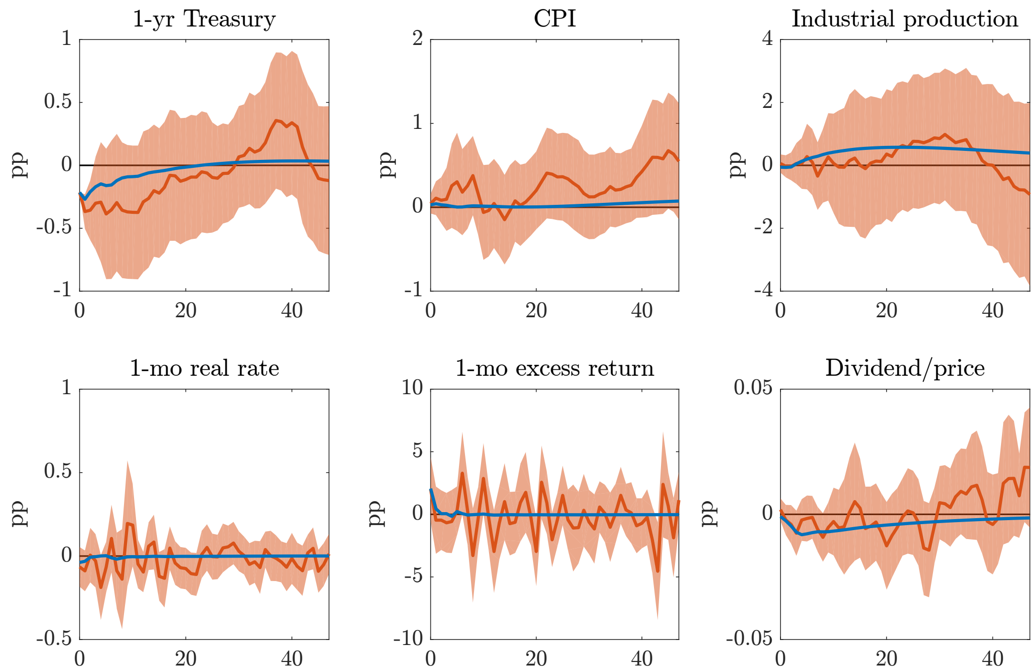

Figure 6: effects of monetary shock using SVAR-IV (blue) versus LP-IV (red)

Notes: heteroskedasticity-robust 95% confidence intervals at each horizon are computed for LP-IV. Since both the SVAR-IV and LP-IV approaches only identify impulse responses up to a normalization on the initial response of the monetary policy indicator (because the structural shock is not observed), for comparability we multiply all LP-IV estimates and confidence intervals by the same constant such that the initial point estimate for the 1-year Treasury is the same as in the SVAR-IV.

null at standard significance levels. They also recommend the use of the complementary Granger causality test discussed in Forni and Gambetti (2014): if invertibility is satisfied, lagged values of the instrument should not have predictive power given the variables included in the VAR. We include 6 lags of our instrument in the VAR and construct an F statistic associated with the null hypothesis that these coefficients are jointly zero for each variable in the VAR. We again cannot reject the null at standard significance levels.

C.2 Micro moments from the SCF

We now provide supplemental details on our measurement of household portfolios using the 2016 SCF described in section 3.3. Table 12 summarizes the assumptions we use on leverage and portfolio shares for each of the components of household net worth reported in the SCF. They are informed by the following analysis.

Moment	Value	Source
Firm net leverage (except private business)	1.5	FA, nonfin corp business
Active managed private business net leverage	1.5	FA, nonfin noncorp business
Non-active managed private business net leverage	4	Axelson, Jenkinson, Stromberg, and Weisbach (2013)
Other mutual fund leverage	1.36	Ang, Gorovyy, and van Inwegen (2011)
Quasi-liquid retirement account equity share	0.57	FA, pension fund holdings
Combination mutual fund equity share	0.67	FA, mutual fund holdings
Other mutual fund equity share	0.67	assumed same as above
Other managed assets equity share	0.67	assumed same as above

Table 12: assumptions used to decompose household net worth in SCF

Notes: references to FA (Financial Accounts of the United States) are for 2016 as reported in Q1 2019 release.

Firm net leverage (except private business). For all household wealth in firm equity except that of private businesses (reflected in stock mutual funds, directly held stocks, and other categories of wealth described below), we use $lev^{firm} = 1.5$. The Q1 2019 Financial Accounts of the United States (FA), nonfinancial corporate business table (S.5.a) reports for 2016 \$41,861.0bn in total assets, \$1,252.5bn in currency and deposit assets, \$199.4 in debt security assets, \$157.9 in loan assets, and \$27,916.7-1,536.7 in equity plus net worth. Hence, we compute net leverage as $\frac{41{,}861.0-1{,}252.5-199.4-157.9}{27{,}916.7-1{,}536.7} = 1.5$.

Active managed private business net leverage. For wealth in actively managed private business, we use $lev^{firm} = 1.5$. The Q1 2019 FA, nonfinancial noncorporate business table (S.4.a) reports for 2016 \$18,688.6bn in total assets, \$1,188.3bn in currency and deposit assets, \$72.8 in debt security assets, \$45.7 in loan assets, \$12.9+11,561.1 in equity plus net worth. Hence, we compute net leverage as $\frac{18{,}688.6-1{,}188.3-72.8-45.7}{12.9+11{,}561.1} = 1.5$.

Non-active managed private business net leverage. For wealth in non-actively managed private business, we use $lev^{firm} = 4$. This is the average leveraged buy-out (LBO) leverage reported by Axelson et al. (2013) in their Table 8. We map non-actively managed private business to LBOs because (i) non-actively managed private business wealth includes private equity (the specific question asked in the SCF is: *Do you (or anyone in your family* *living here) own or share ownership in any other businesses, business investments or other private equity that are not publicly traded and where you do NOT have an active management role?*), and (ii) assets under management in buyout funds comprises more than half of all assets under management in private equity globally (McKinsey (2018)).

Other mutual fund leverage. For wealth in other mutual funds, we use $lev^{inter} = 1.36$. This is the average leverage of long-only hedge funds reported by Ang et al. (2011) in their Table 2B. We map other mutual funds to hedge funds because the specific question asked in the SCF is: *Do you have any other mutual funds, ETFs, hedge funds, or REITs?*

Quasi-liquid retirement account equity share. We assume that 57% of wealth in retirement accounts is held in firm equity (to which we apply $lev^{firm} = 1.5$ characterized above). The Q1 2019 FA, private and public pension funds table (L.117) reports for 2016 \$4,907.9bn in corporate equities, \$3,768.1bn in mutual funds, \$21,197.1bn in total financial assets, and \$,8203.2bn in miscellaneous assets, most of which is unfunded DB entitlements. Hence, we compute the equity share excluding these miscellaneous assets as $\frac{4{,}907.9+3{,}768.1\times 0.67}{21{,}197.1-8{,}203.2} = 0.57$.

Combination mutual fund equity share. We assume that 67% of wealth in combination mutual funds is held in firm equity (to which we apply $lev^{firm} = 1.5$ characterized above). The Q1 2019 FA, mutual fund holdings table (L.122) reports for 2016 \$9,069.9bn in corporate equities and \$13,615.6 in total financial assets. Hence, we compute the equity share as $\frac{9{,}069.9}{13{,}615.6} = 0.67$.

Other mutual fund equity share. We assume that 67% of wealth in other mutual funds is held in firm equity (to which we apply $lev^{inter} = 1.36$ and $lev^{firm} = 1.5$ characterized above). We set 67% to be the same as for combination mutual funds.

Other managed assets equity share. We assume that 67% of wealth in other managed assets is held in firm equity (to which we apply $lev^{firm} = 1.5$ characterized above). We set 67% to be the same as for combination mutual funds.

All other categories of wealth. For all other categories of wealth not mentioned above, we assume they are fully bonds (in zero net supply) or capital (in positive net supply).

Table 13 decomposes the aggregate net worth of U.S. households into claims on capital and bonds using these assumptions.

D Infinite horizon environment and solution

In this appendix we describe the infinite horizon environment studied and our computational algorithm used in section 3.

	$2016bn		
	$\sum_i B^i$	$\sum_i Qk^i$	$\sum_i A^i$
1 Transaction accounts	4,940	0	4,940
2 CDs	620	0	620
3 Stock mutual funds	-3,123	9,062	5,939
4 Tax-free bond mutual funds	1,329	0	1,329
5 Govt bond mutual funds	276	0	276
6 Other bond mutual funds	404	0	404
7 Combination mutual funds	-12	769	757
8 Other mutual funds	-386	1,397	1,011
9 Savings bonds	104	0	104
10 Directly held stocks	-3,019	8,761	5,742
11 Directly held bonds	1,179	0	1,179
12 Cash value life insurance	914	0	914
13 Other managed assets	-53	3,284	3,231
14 Quasi-liquid retirement assets	1,934	13,067	15,001
15 Other misc financial assets	0	659	659
16 Vehicles	0	2,717	2,717
17 Primary residence	0	24,176	24,176
18 Residential RE excl primary residence	0	6,301	6,301
19 Non-residential RE	0	3,694	3,694
20 Actively-managed businesses	-8,538	25,552	17,015
21 Non-active-managed businesses	-6,997	9,329	2,332
22 Other misc non-fin assets	0	559	559
23 Mortgage on primary residence	-8,310	0	-8,310
24 Mortgage excl primary residence	-1,128	0	-1,128
25 Other lines of credit	-127	0	-127
26 Credit card balance	-316	0	-316
27 Installment loans	-1,976	0	-1,976
Vehicle installment	-733	0	-733
28 Other debt	-176	0	-176
29 Total	-22,462	109,327	86,865
30 Total, excl primary residence and vehicles	-13,419	82,434	69,015

Table 13: decomposition of household net worth in SCF

Notes: observations are weighted by SCF sample weights.

D.1 Environment and equilibrium

We first extend the environment described in section 2.1 to the infinite horizon. We closely follow the exposition in that section.

Households The unit measure of households is now organized into a finite set of I groups with measures $\{\lambda^i\}$ such that $\sum_i \lambda^i = 1$, where households are identical within groups. Each household continues to be comprised of a continuum of members $j \in [0, 1]$ supplying a differentiated variety of labor, and there remains full consumption insurance within households. The representative household i has Epstein-Zin preferences (25) with disutility of labor each period (26) following Shimer (2010). Each period, the household faces the resource constraint

$$\begin{aligned} P_t c_t^i + B_t^i + Q_t k_t^i \leq\ & (1-\tau)\int_0^1 W_t(j)\ell_t^i(j)dj - \int_0^1 AC_t^W(j)dj\ + \\ & (1+i_{t-1})B_{t-1}^i + (\Pi_t + (1-\delta)Q_t)k_{t-1}^i \exp(\varphi_t) + T_t^i, \end{aligned} \tag{71}$$

where the Rotemberg (1982) cost of setting the wage for member j is

$$AC_t^W(j) = \frac{\chi^W}{2} W_t \ell_t \left(\frac{W_t(j)}{W_{t-1}\exp(\varphi_t)} - 1\right)^2.$$

Supply-side A union continues to represent each labor variety j across households. Each period, it chooses $W_t(j), \ell_t(j)$ to maximize the social welfare of union members subject to the allocation rule (4) and Pareto weights $\{\mu^i\}$. We now assume for simplicity that the allocation rule and Pareto weights are symmetric: $\ell^i(\ell_t) = \ell_t$ and $\mu^i = 1$. The labor packer combines varieties supplied by the union as in the two-period model, earning profits each period

$$W_t \left[\int_0^1 \ell_t(j)^{(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)} - \int_0^1 W_t(j)\ell_t(j)dj. \tag{72}$$

The representative producer hires ℓ_t units of the labor aggregator in period t and combines it with $k_{t-1}\exp(\varphi_t)$ units of capital rented from households. It further uses $\left(\frac{k_t}{k_{t-1}\exp(\varphi_t)}\right)^{\chi^x} x_t$ units of the consumption good to produce x_t new capital goods, where it again takes k_t as given. Taken together, it earns profits

$$\Pi_t k_{t-1}\exp(\varphi_t) = P_t (z_t\ell_t)^{1-\alpha}(k_{t-1}\exp(\varphi_t))^\alpha - W_t\ell_t + Q_t x_t - P_t\left(\frac{k_t}{k_{t-1}\exp(\varphi_t)}\right)^{\chi^x} x_t. \tag{73}$$

Productivity follows (27).

Policy The government follows a standard Taylor (1993) rule (29) where monetary policy shocks m_t follow (30). The government continues to set $\tau = -\frac{1}{\epsilon - 1}$ and now sets household-specific lump-sum taxes as in (31) given (32). We assume that $\omega_t^a = \omega^a$ and $\omega_t^b = \omega^b$ are constant, and ω_t^c ensures that $\sum_i \lambda^i tr_t^i = 0$. As noted in the main text, we assume that households anticipate this last component of transfers for all households except themselves.

Market clearing Market clearing in goods each period is now

$$\sum_i \lambda^i c_t^i + \left(\frac{k_t}{k_{t-1} \exp(\varphi_t)} \right)^{\chi^x} x_t = (z_t \ell_t)^{1-\alpha} \left(k_{t-1} \exp(\varphi_t) \right)^{\alpha}, \tag{74}$$

in labor is

$$\left[\int_0^1 \ell_t(j)^{(\epsilon-1)/\epsilon} dj \right]^{\epsilon/(\epsilon-1)} = \ell_t, \tag{75}$$

in the capital rental market is

$$\sum_i \lambda^i k_{t-1}^i = k_{t-1}, \tag{76}$$

in the capital claims market is

$$(1-\delta) \sum_i \lambda^i k_{t-1}^i \exp(\varphi_t) + x_t = \sum_i \lambda^i k_t^i, \tag{77}$$

and in bonds is

$$\sum_i \lambda^i B_t^i = 0. \tag{78}$$

Equilibrium Given initial state variables $\{W_{-1}, \{B_{-1}^i, k_{-1}^i\}, i_{-1}, z_0, m_0\}$ and the stochastic processes (27)-(30), the definition of equilibrium naturally generalizes Definition 1:

Definition 3. *An equilibrium is a sequence of prices and policies such that: (i) each household i chooses $\{c_t^i, B_t^i, k_t^i\}$ to maximize (25) subject to (71), (ii) each union j chooses $\{W_t(j), \ell_t(j)\}$ to maximize the utilitarian social welfare of its members subject to the symmetric allocation rule $\{\ell_t^i(j) = \ell_t(j)\}$, (iii) the labor packer chooses $\{\ell_t(j)\}$ to maximize profits (72), (iv) the representative producer chooses $\{\ell_t, x_t\}$ to maximize profits (73), (v) the government sets $\{T_t^i\}$ according to (31)-(32) and i_t according to (29), and (vi) the goods, labor, capital, and bond markets clear according to (74)-(78).*

Since labor varieties and unions j are symmetric, $\ell_t(j) = \ell_t$ and we again drop j going forward.

D.2 First-order conditions

We now outline households' and firms' optimality conditions.

Households Defining the realized real interest rate and real return on capital

$$
\begin{aligned}
1 + r_{t+1} &\equiv (1 + i_t)\frac{P_t}{P_{t+1}}, \\
1 + r^k_{t+1} &=\equiv \frac{(\Pi_{t+1} + (1-\delta)Q_{t+1})\exp(\varphi_{t+1})}{Q_t}\frac{P_t}{P_{t+1}},
\end{aligned}
$$

the representative household i's optimal consumption and savings decisions are characterized by

$$
\begin{aligned}
1 &= \mathbb{E}_t m^i_{t,t+1}(1 + r_{t+1}), \\
1 &= \mathbb{E}_t m^i_{t,t+1}(1 + r^k_{t+1}),
\end{aligned}
$$

given the real stochastic discount factor

$$
m^i_{t,t+1} = \beta \left(ce^i_t\right)^{\gamma^i - 1/\psi} \left(v^i_{t+1}\right)^{1/\psi - \gamma^i} \frac{\left(c^i_{t+1}\right)^{-\frac{1}{\psi}} \Phi(\ell_{t+1})^{1-\frac{1}{\psi}}}{\left(c^i_t\right)^{-\frac{1}{\psi}} \Phi(\ell_t)^{1-\frac{1}{\psi}}}
$$

and certainty equivalent $ce^i_t = \mathbb{E}_t\left[\left(v^i_{t+1}\right)^{1-\gamma^i}\right]^{\frac{1}{1-\gamma^i}}$.

Unions Defining the real wage $w_t \equiv \frac{W_t}{P_t}$, the representative union sets

$$
\begin{aligned}
\sum_i \lambda^i \left(v^i_t\right)^{\frac{1}{\psi}} \left(c^i_t\right)^{-\frac{1}{\psi}} \Bigg[w_t + c^i_t \frac{\Phi'(\ell_t)}{\Phi(\ell_t)} \\
+ w_t \frac{\chi^W}{\epsilon} \Bigg[\frac{w_t}{w_{t-1}\exp(\varphi_t)}\frac{P_t}{P_{t-1}}\left(\frac{w_t}{w_{t-1}}\frac{P_t}{P_{t-1}} - 1\right) \\
- \mathbb{E}_t m^i_{t,t+1}\left(\frac{w_{t+1}}{w_t\exp(\varphi_{t+1})}\right)^2 \frac{P_{t+1}}{P_t}\frac{\ell_{t+1}}{\ell_t}\left(\frac{w_{t+1}}{w_t\exp(\varphi_{t+1})}\frac{P_{t+1}}{P_t} - 1\right)\Bigg]\Bigg] = 0.
\end{aligned}
$$

Producers Defining the real price of capital $q_t \equiv \frac{Q_t}{P_t}$, the representative producer follows

$$
\begin{aligned}
w_t &= (1-\alpha) z_t^{1-\alpha} \ell_t^{-\alpha} \left(k_{t-1}\exp(\varphi_t)\right)^\alpha, \\
q_t &= \left(\frac{k_t}{k_{t-1}\exp(\varphi_t)}\right)^{\chi^x}.
\end{aligned}
$$

D.3 Re-scaled economy

We now characterize the equilibrium conditions of an equivalent, stationary economy obtained by dividing households' resource constraints and market clearing conditions by the price level P_t, and further dividing these conditions as well as the first-order conditions in the prior subsection by z_t. We denote real variables in lower-case (except for the nominal rate i_t) and further defined the re-scaled variables

$$\tilde{c}_t^i \equiv \frac{c_t^i}{z_t}, \;\; \tilde{c}_{t+1}^i \equiv \frac{c_{t+1}^i}{z_{t+1}}, \;\; \tilde{ce}_t^i \equiv \frac{ce_t^i}{z_t}, \;\; \tilde{b}_t^i \equiv \frac{b_t^i}{z_t}, \;\; \tilde{k}_t^i \equiv \frac{k_t^i}{z_t}, \;\; \tilde{k}_t \equiv \frac{k_t}{z_t}, \;\; \tilde{w}_t \equiv \frac{w_t}{z_t}, \tag{79}$$

$$\tilde{m}_{t,t+1}^i \equiv m_{t,t+1}^i \left(\frac{z_{t+1}}{z_t}\right)^{-\gamma}, \tag{80}$$

$$\tilde{k}_{t-1}^i \equiv \frac{k_{t-1}^i}{\exp(\sigma\epsilon_t^z)}, \;\; \tilde{k}_{t-1} \equiv \frac{k_{t-1}}{\exp(\sigma\epsilon_t^z)}, \;\; \tilde{w}_{t-1} \equiv \frac{w_{t-1}}{\exp(\sigma\epsilon_t^z)}. \tag{81}$$

Then the household's optimality conditions and constraints are equivalent to:

$$1 = \mathbb{E}_t \tilde{m}_{t,t+1}^i \exp\left(\gamma^i \left[\sigma\epsilon_{t+1}^z + \varphi_{t+1}\right]\right)(1 + r_{t+1}), \tag{82}$$

$$1 = \mathbb{E}_t \tilde{m}_{t,t+1}^i \exp\left(\gamma^i \left[\sigma\epsilon_{t+1}^z + \varphi_{t+1}\right]\right)(1 + r_{t+1}^k), \tag{83}$$

$$\tilde{m}_{t,t+1}^i = \beta \left(\tilde{ce}_t^i\right)^{\gamma^i - 1/\psi} \left(\tilde{v}_{t+1}^i\right)^{1/\psi - \gamma^i} \frac{\left(\tilde{c}_{t+1}^i\right)^{-\frac{1}{\psi}} \Phi(\ell_{t+1})^{1-\frac{1}{\psi}}}{\left(\tilde{c}_t^i\right)^{-\frac{1}{\psi}} \Phi(\ell_t)^{1-\frac{1}{\psi}}}, \tag{84}$$

$$\tilde{ce}_t^i = \mathbb{E}_t \left[\exp\left(\left(1-\gamma^i\right)\left[\sigma\epsilon_{t+1}^z + \varphi_{t+1}\right]\right)\left(\tilde{v}_{t+1}^i\right)^{1-\gamma^i}\right]^{\frac{1}{1-\gamma^i}}, \tag{85}$$

$$\tilde{c}_t^i + \tilde{b}_t^i + q_t \tilde{k}_t^i = \tilde{w}_t \ell_t^i + \tilde{n}_{t-1}^i, \tag{86}$$

The definition of household wealth inclusive of transfers implies:[42]

$$\tilde{n}_{t-1}^i = \frac{1}{\lambda^i} s_t^i \tilde{k}_{t-1}. \quad . \tag{87}$$

[42]We distinguish wealth inclusive of transfers $n_t \equiv (1 + r_t)b_{t-1}^i + (\pi_t + (1-\delta)q_t)k_{t-1}^i + tr_t^i$ from financial wealth $a_t \equiv (1 + r_t)b_{t-1}^i + (\pi_t + (1-\delta)q_t)k_{t-1}^i$.

The representative union's optimality condition is equivalent to:

$$\sum_i \lambda^i \left(\tilde{v}_t^i\right)^{\frac{1}{\psi}} \left(\tilde{c}_t^i\right)^{-\frac{1}{\psi}} \left[\tilde{w}_t + \tilde{c}_t^i \frac{\Phi'(\ell_t)}{\Phi(\ell_t)} \right.$$
$$+ \tilde{w}_t \frac{\chi^W}{\epsilon} \left[\frac{\tilde{w}_t}{\tilde{w}_{t-1}} \frac{P_t}{P_{t-1}} \left(\frac{\tilde{w}_t}{\tilde{w}_{t-1}} \frac{P_t}{P_{t-1}} - 1 \right) \right. \tag{88}$$
$$\left.\left. - \mathbb{E}_t \tilde{m}_{t,t+1}^i \exp\left(\gamma^i \left[\sigma \epsilon_{t+1}^z + \varphi_{t+1}\right]\right) \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^2 \frac{P_{t+1}}{P_t} \frac{\ell_{t+1}}{\ell_t} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \frac{P_{t+1}}{P_t} - 1 \right) \right]\right] = 0.$$

The representative producer's optimality condition and flow of funds are equivalent to:

$$\tilde{w}_t = (1-\alpha)\ell_t^{-\alpha} \tilde{k}_{t-1}^{\alpha}, \tag{89}$$

$$q_t = \left(\frac{\tilde{k}_t}{\tilde{k}_{t-1}} \right)^{\chi^x}, \tag{90}$$

$$\pi_t \tilde{k}_{t-1} = \alpha \ell_t^{1-\alpha} \tilde{k}_{t-1}^{\alpha}. \tag{91}$$

The specifications of fiscal and monetary policy imply:

$$s_{t+1}^i = (1-\omega_{t+1}^i) \frac{(1+r_{t+1})\tilde{b}_t + (\pi_{t+1} + (1-\delta)q_{t+1})\tilde{k}_t^i \exp(\varphi_{t+1})}{(\pi_{t+1} + (1-\delta)q_{t+1})\tilde{k}_t \exp(\varphi_{t+1})} + \omega_{t+1}^i \lambda^i, \tag{92}$$

$$1 + i_t = (1+\bar{i}) \left(\frac{P_t}{P_{t-1}} \right)^{\phi} m_t. \tag{93}$$

The definitions of real returns remain:

$$1 + r_{t+1} \equiv (1+i_t) \frac{P_t}{P_{t+1}}, \tag{94}$$

$$1 + r_{t+1}^k = \equiv \frac{(\pi_{t+1} + (1-\delta)q_{t+1}) \exp(\varphi_{t+1})}{q_t}. \tag{95}$$

The market clearing conditions are equivalent to:

$$\sum_i \lambda^i \tilde{c}_t^i + \left(\frac{\tilde{k}_t}{\tilde{k}_{t-1}} \right)^{\chi^x} \tilde{x}_t = \ell_t^{1-\alpha} \tilde{k}_{t-1}^{\alpha}, \tag{96}$$

$$\sum_i \lambda^i \tilde{k}_t^i = \tilde{k}_t, \tag{97}$$

$$(1-\delta) \sum_i \lambda^i \tilde{k}_{t-1}^i + \tilde{x}_t = \sum_i \lambda^i \tilde{k}_t^i, \tag{98}$$

$$\sum_i \lambda^i \tilde{b}_t^i = 0. \tag{99}$$

Finally, the evolution of exogenous state variables is:

$$\log m_{t+1} = \rho \log m_t + \varsigma \epsilon_{t+1}^m. \tag{100}$$

After solving this transformed economy, we can simulate prices and quantities in the original economy by reversing the re-scaling in (79)-(81), where z_t follows (27).

D.4 Global solution algorithm

We now outline the computational algorithm used to solve the transformed economy.

Grids The model is solved over a discretized grid of aggregate states S. Each node is defined by the current monetary policy shock $m(S)$, the wealth shares $s^a(S)$ and $s^c(S)$ of groups a and c, the scaled capital chosen in the previous period $\tilde{k}_{-1}(S)$ as chosen in the previous period, as well as the scaled real wage $\tilde{w}_{-1}(S)$ set in the previous period. In the transformed, stationary economy, productivity shocks inclusive of disasters only govern the transition across states. The grid over states is given by a mesh grid over vectors of each state variable. In each dimension we choose a vector length of at least five nodes, where the vector's upper and lower bound are iteratively updated to make sure that the state variables stay well within the chosen limits in ten simulations of 500 years each. We verify that the model solutions are robust to grid boundaries and size for the chosen values.

Expectations and interpolation When forming expectations, we use Gauss-Hermite quadrature for integration. Expectations over future states will typically not lie on the grid, and we use linear interpolation over aggregate states to find variable values for those states. The value functions of the representative household in each group are solved over a vector of individual wealth inclusive of transfers n_{-1}^i, so that households can entertain a range of portfolio and savings choices when optimizing. We use cubic splines to interpolate over the idiosyncratic wealth levels, which also enables us to calculate value function derivatives.

Solution algorithm We look for a stationary solution to the model and use backward iteration until all equilibrium objects converge. We assume that convergence is satisfactory when relative period-to-period changes are smaller than 10^{-6}. For each state S, the solution objects are the price of capital $q(S)$, the nominal rate $i(S)$, the chosen real wage $w(S)$, the inflation rate $\Pi^P(S) = P(S)/P(S_{-1})$, labor supply $\ell(S)$, capital choices of each household group $k^i(S)$, real bond choices of each group $b^i(S)$, and the value functions of each group over a vector of wealth $v^i(n^i_{-1}, S)$.

The solution algorithm starts from an initial guess for $v^i(n^i_{-1}, S)$, $q(S)$, $i(S)$, $w(S)$, $\Pi^P(S)$, and $\ell(S)$ and proceeds as follows.

1. With this guess at hand we can solve each representative household's savings and portfolio choice problem (82)-(86) given its current wealth inclusive of transfers implied by (87), the interest rates implied by (91), (94), (95), and the evolution of state variables implied by (92), (97), and (100).

2. Observing the excess demand for bonds relative to the market clearing condition (99), we adjust $i(S)$ to lower the absolute value of the excess demand, returning to step 1, until excess bond demand relative to the aggregate capital stock is smaller than 10^{-12}.

3. The resulting choice of individual capital holdings, together with the market clearing condition (97), allows us to update the price of capital $q(S)$ according to (90).

4. We use the union's first-order condition (88) to update the wage choice $w(S)$, given labor demand for the representative producer (89).

5. Given the equilibrium nominal rate, we use the Taylor rule (93) to update $\Pi^P(S)$.

6. Finally, we update the value functions $v^i(n^i_{-1}, S)$ by solving the optimization problem (82)-(86) of all representative households for wealth away from the current state.

7. Using the updated equilibrium objects, we define new guesses $v^i(n^i_{-1}, S)$, $q(S)$, $i(S)$, $w(S)$, $\Pi^P(S)$, and $\ell(S)$ and return to step 1. For numerical stability we dampen the updating of most equilibrium objects.

At the conclusion of the algorithm, a policy function for $x(S)$ is implied by the capital accumulation condition (98), and goods market clearing (96) is satisfied by Walras' Law.

The solution code is written in Fortran and parallelized using OpenMP. Convergence can be achieved in less than twenty minutes on a modern computing system with eight cores.

E Additional quantitative results

In this appendix we provide supplementary quantitative results accompanying section 3.

We focus on impulse responses to a productivity shock accompanying the main analyses of monetary shocks in the main text. We again compare in Figure 7 the impulse responses of the model with heterogeneity to a counterfactual economy with a representative agent.

The first row again reports the change in the 1-year nominal bond yield, expected real returns, and expected excess returns. The first panel demonstrates that the central bank following a standard Taylor (1993) rule will cut the nominal interest rate in response to the price deflation induced by this shock. The second and third panels demonstrate that the expected real interest rate and the expected excess returns on capital decline following the shock. The decline in the former is a standard real business cycle response to the shock and also reflects the endogenous monetary easing in this New Keynesian setting. The decline in the latter demonstrates that productivity shocks induce a countercylical risk premium.

The second and third rows demonstrate that redistribution drives the decline in the risk premium following the shock. The first panel of the second row demonstrates that, as in the case of a negative monetary policy shock, realized excess returns on capital are substantially positive on impact and followed by small negative returns in the quarters which follow. Through the lens of Proposition 11 characterized in appendix B, the substantially positive excess returns on impact endogenously redistribute to the high MPR a households who hold levered claims on capital, evidenced in the financial wealth share of a households in the second panel in this row. As described in appendix B, however, the mechanisms are more nuanced than in the case of a monetary shock. On the one hand, unexpected deflation raises the real interest rate, shown in the third panel. On the other hand, the increase in the price of capital raises the return on capital, shown in the first panel of the third row. The effect on profits reflects the competing effects of a higher real wage and higher employment. On balance, the return to capital increases and outweighs the higher realized real interest rate, and wealth redistributes to the high-MPR a households, lowering the risk premium.

The fourth row examines the consequences of this redistribution for the transmission of the shock to the real economy. Comparing the investment response in the model to the representative agent case, we find additional stimulus to investment on impact. The increase in investment is again countervailed by a mitigation of the stimulus to consumption.

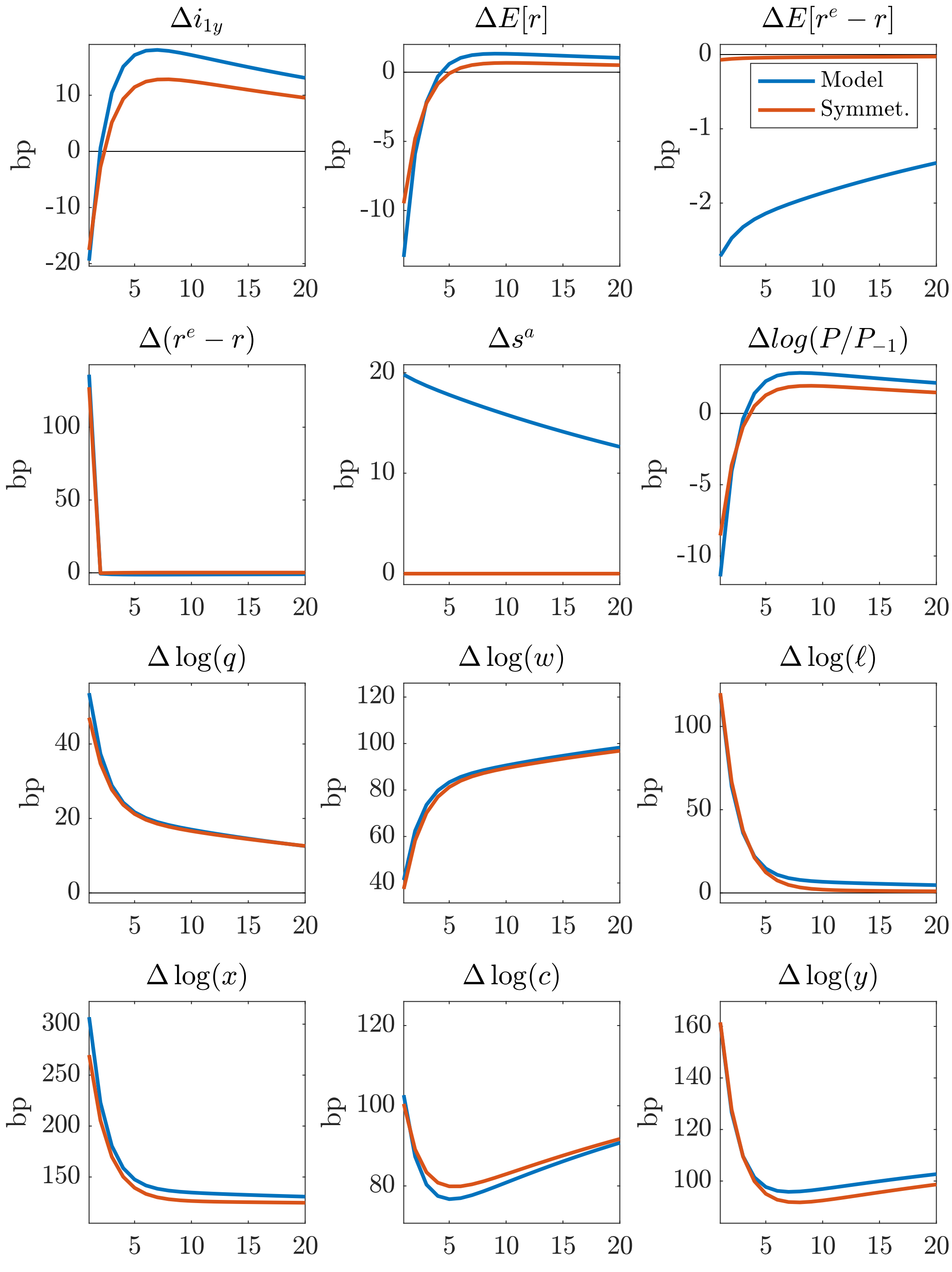

Figure 7: impulse responses to positive productivity shock

Notes: all series are plotted as quarterly (non-annualized) deviations from the stochastic steady-state, except for the 1-year nominal bond yield Δi_{1y}. *bp* denotes basis points (0.01%).