

# A Macro-Finance Model with Sentiment \*

\*\*PRELIMINARY AND INCOMPLETE\*\*

Peter Maxted

Harvard University

October 16, 2019

## Abstract

This paper builds a general equilibrium macroeconomic model that combines diagnostic expectations and financial frictions. Diagnostic expectations are a forward-looking model of extrapolative expectations that overreact to recent news. Frictions in financial intermediation generate nonlinear spikes in risk premia and slumps in investment during periods of financial distress. The calibrated model is solved globally to characterize the full equilibrium dynamics generated by the interaction of sentiment and financial frictions. The model evaluates the causal role of over-optimism in triggering the amplification of financial distress into a full-blown crisis. Boom-bust investment cycles emerge endogenously out of a feedback from sentiment to financial frictions. Under the baseline calibration, the model predicts that financial crises are less likely to occur when expectations are diagnostic than when they are rational.

---

\*I would like to thank Francesca Bastianello, Pedro Bordalo, John Campbell, Daniele d'Arienzo, Emmanuel Farhi, Paul Fontanier, Xavier Gabaix, Nicola Gennaioli, Thomas Graeber, Samuel Hanson, Arvind Krishnamurthy, David Laibson, Matteo Maggiori, Andrei Shleifer, Jeremy Stein, Ludwig Straub, and Stephen Terry for helpful discussion and comments. The computations for this paper were run on the Research Computing Environment supported by the Institute for Quantitative Social Science in the Faculty of Arts and Sciences at Harvard University.

# 1 Introduction

The 2007-2008 Financial Crisis and the Great Recession that followed are a powerful reminder of the adverse effects that financial market turmoil can have on the broader economy. Since the crisis, macro-finance theory has turned to frictions in financial intermediation to account for financial crises and their consequences for the real economy. Financial frictions prevent risks from being shared and funds from flowing to productive investments, especially in times when frictions are acute. Crises are modeled as nonlinearities where occasionally binding constraints on financial intermediaries generate sharp increases in risk premia and contractions in output (e.g., [Brunnermeier and Sannikov \(2014\)](#), [He and Krishnamurthy \(2013\)](#)).

While models of financial frictions can capture spikes in risk premia during financial crises and the transmission of financial sector vulnerabilities into aggregate downturns, the rational expectations version of these models struggles to explain low risk premia in the lead-up to a crisis. The 2007-2008 Financial Crisis is a particularly salient example. By the summer of 2007 it was clear that the housing market was deflating, yet financial market indicators such as credit spreads and the VIX showed little sign of the systemic risks that quickly revealed themselves following the bankruptcy of Lehman in September 2008 ([Gennaioli and Shleifer, 2018](#)).

This evidence is not unique to 2007-2008 episode. Examining an international panel of financial crises, [Krishnamurthy and Muir \(2017\)](#) show that spreads are regularly “too low” before financial crises, and that frothy financial markets predict future crises. [Baron and Xiong \(2017\)](#) find similarly that risk premia are exceedingly low in the lead-up to financial crises due to neglected crash risk. Both papers directly conclude that this pre-crisis evidence is difficult to square with rational models of financial fragility.

Patterns of excessive optimism preceding financial market and macroeconomic downturns appear consistently. [Greenwood and Hanson \(2013\)](#) show that the credit quality of corporate debt issuers deteriorates during credit booms, and that deteriorating issuer quality predicts a tightening of credit markets, a widening of spreads, and low realized returns. Issuer quality is poor following periods of low defaults, consistent with investors extrapolating past default

rates. Taking this result to the macroeconomy, [López-Salido et al. \(2017\)](#) estimate that periods of loose credit and narrow spreads predict a decline in future economic activity once the sentiment-driven financial cycle predictably reverses.

Direct expectations data from professional forecasters supports this evidence of cyclical overreaction to recent economic trends.<sup>1</sup> Using professional forecasts of the Baa-Treasury spread, [Bordalo et al. \(2018a\)](#) find that forecast errors are predictable: when the credit spread is narrow the expected future spread is too narrow, and when the credit spread is wide the expected future spread is too wide. [Bordalo et al. \(2018c\)](#) document a similar pattern of overreaction in professional forecasters' expectations across a variety of macroeconomic outcomes, such as real GDP and consumption growth.

In this paper I propose a macro-finance model consistent with this evidence of non-rational expectations in order to characterize how sentiment interacts with financial frictions to jointly drive financial market and business cycle dynamics. The structure of the macro-finance model is based on [He and Krishnamurthy \(2019\)](#). [He and Krishnamurthy \(2019\)](#) present a continuous-time RBC model augmented with a financial intermediary sector subject to an occasionally binding constraint on its ability to raise equity funding from households. The intermediary sector's funding capacity becomes a key state variable for the economy, capturing the severity of financial frictions. In non-distress periods the model behaves similarly to a frictionless RBC model and is calibrated to represent the U.S. economy. However, a sequence of poor returns leads to financial distress as the intermediary sector moves closer to its funding constraint. In periods of financial distress the nonlinearities arising from financial frictions become quantitatively important, thereby adding a systemic risk element to the model. The model is solved globally in order to fully characterize the nonlinear effect of financial frictions on the economy's dynamics.

I depart from rational expectations by introducing behavioral frictions to this model. This paper develops a method for extending rational models with a continuous-time variant of diagnostic expectations ([Bordalo et al., 2018a](#)). Diagnostic expectations are a forward-

---

<sup>1</sup>Data on the expectations of financial market and macroeconomic professionals is most consistent with this paper's model. As will be detailed later, restrictions on asset-market participation imply that it is sophisticated financial intermediaries who are responsible for pricing risky assets (see equation (22)). Similar extrapolative behavior is exhibited by households, as discussed in [Fuster et al. \(2010\)](#) and [Greenwood and Shleifer \(2014\)](#).

looking model of extrapolative expectations in which agents overweight future states that are representative of recent news (Kahneman and Tversky, 1972). When recent shocks have tended to be positive, sentiment is elevated and agents are over-optimistic about future economic growth. The reverse holds when recent shocks have tended to be negative. Relative to Bordalo et al. (2018a), the innovation of the expectations model in this paper is that it is applicable to endogenous processes. This will imply not only that recent economic performance affects expectations (a standard feature of extrapolative expectations), but also that expectations can then feed back into the dynamical system to alter the future evolution of the processes on which expectations are formed. This methodological advance allows me to derive a number of asset pricing and macroeconomic consequences resulting from the interaction of behavioral and financial frictions.

In financial markets, the model quantifies the causal role of over-optimism in amplifying the vulnerability of financial intermediaries. Elevated sentiment erodes the balance sheets of intermediaries by lowering returns relative to expectations. Because this sentiment-driven build-up of systemic risk is generated by non-rational expectations, it is neglected by agents in the model. With diagnostic expectations – unlike with rational expectations – endogenous market prices will not reflect the true risks building in the financial sector. As a result, the model can generate heightened crash risk in the background of low risk premium environments. Model-based stress tests reveal that it is critical to measure sentiment alongside financial distress in order to quantify systemic risk.

Turning to macroeconomic dynamics, the model demonstrates how diagnostic expectations feed back into financial frictions to generate boom-bust patterns in investment and output growth. A period of positive shocks gets amplified in the short run through a sentiment-driven investment boom. However, this sentiment-driven boom begets its own financial-frictions-driven bust by “firing off” the risk-bearing capacity of the financial sector. Overoptimistic bankers incorrectly price the assets that they hold, causing intermediary balance sheets to deteriorate throughout the boom. Once sentiment subsides, the economy is left with a weakened financial sector that constrains investment. The reverse is true following a period of negative shocks: the sentiment-driven bust begets its own financial-frictions-driven boom. Depressed sentiment causes a short-run drop in investment that coincides with the

financial sector “reloading” its balance sheet through high returns. Once sentiment recovers, the strengthened financial sector will support a long-run investment boom.

Though diagnostic expectations amplify business cycles, they simultaneously stabilize financial cycles. Under the baseline calibration, the model predicts that financial crises are *less likely* to occur when expectations are diagnostic than when they are rational. This is in direct opposition to the typical narrative that extrapolative beliefs create financial instability (e.g., the Financial Instability Hypothesis of [Minsky \(1977\)](#)). Financial-market stability arises in equilibrium because sentiment tracks recent economic shocks, so sentiment is typically depressed as the economy nears a financial crisis. Excessive pessimism amplifies the recession precipitated by a sequence of negative shocks, but also reloads intermediary balance sheets. In short, the way that the economy avoids financial crises is by enduring sentiment-driven recessions.

In addition to developing a model that evaluates the effect of sentiment on financial market and macroeconomic dynamics, this paper makes three methodological contributions to the behavioral macroeconomics and finance literatures. First, the results of this paper highlight the importance of using global solution methods for characterizing the effect of beliefs on economic dynamics. Indeed, the narrative on behavioral macroeconomic dynamics is one of sentiment-driven expansions and slumps (e.g., [Keynes \(1936\)](#)). This is fundamentally an analysis of the cyclical effects of expectations away from the steady state, and global solution methods allow for the complete characterization of the nonlinear dynamical system.

Second, continuous-time methods are particularly applicable for the study of non-rational expectations in dynamic equilibrium models. Over short (instantaneous) horizons, Itô’s lemma provides a straightforward method for calculating forecast errors on endogenous equilibrium objects (e.g., asset returns). At longer horizons, the Kolmogorov forward equation can be used to characterize the distribution over future states that agents perceive, as well as the true equilibrium distribution over future states. These methods will be utilized throughout this paper to elucidate the model’s key equilibrium predictions.

Third, a common downside of introducing behavioral elements into economic models is that the model often needs to be simplified elsewhere in order to maintain tractability. This is not the case here. I provide a microfounded model of non-rational expectations that is used

to generalize a dynamic stochastic general equilibrium macroeconomic model with financial frictions. This allows for the study of non-rational expectations without compromising the equilibrium dynamics on which sentiment can interact.

**Related Literature** The macroeconomic model in this paper follows from a large literature on financial frictions. Seminal work includes [Kiyotaki and Moore \(1997\)](#) and [Bernanke et al. \(1999\)](#). Recent research has tended to use continuous-time methods in order to study global dynamics in models with nonlinearities. Examples include [Adrian and Boyarchenko \(2012\)](#), [Brunnermeier and Sannikov \(2014\)](#), [Di Tella \(2017\)](#), [He and Krishnamurthy \(2013, 2019\)](#), [Maggiore \(2017\)](#), and [Moreira and Savov \(2017\)](#). The contribution of this paper is the introduction of extrapolative expectations.

Diagnostic expectations align this set of models more closely with empirical findings on the behavioral triggers of financial distress. The literature mentioned above provides a partial summary of this evidence. In the cross-section of U.S. banks, [Fahlenbrach et al. \(2017\)](#) show that rapid loan growth predicts equity underperformance and elevated crash risk over the next three years. [Adrian and Brunnermeier \(2016\)](#) find that systemic risk measurements suffer from a “procyclicality pitfall” in which tail risks build up in the background of low-volatility environments. [Mian et al. \(2017\)](#) observe that increases in the household debt-to-GDP ratio predict lower future GDP growth, and also that economic forecasters are over-optimistic at the end of household debt expansions. [Schularick and Taylor \(2012\)](#) conclude that financial crises are often “credit booms gone wrong.”

There is also a growing theoretical literature focusing on behavioral credit cycles and the behavioral triggers of crises. Closest to the current paper, [Bordalo et al. \(2018a\)](#) develop the original model of discrete-time diagnostic expectations and apply these expectations to a simple frictionless macroeconomic model. This work is extended in [Bordalo et al. \(2019b\)](#), who study the business cycle implications of diagnostic expectations in a quantitative heterogeneous firm model.<sup>2</sup> [Gennaioli and Shleifer \(2018\)](#) summarize the research on diagnostic expectations and present a belief-driven narrative of the 2007-2008 Financial Crisis. [Gertler et al. \(2017\)](#) examine how waves of optimism about future capital returns can generate an ex-

---

<sup>2</sup>See [Bordalo et al. \(2019a\)](#) and [Bordalo et al. \(2018b\)](#) for applications of diagnostic expectations to financial markets.

ante buildup of leverage which increases the fragility of the banking sector once expectations disappoint. [Greenwood et al. \(2019\)](#) develop a model of firm financing and default in which extrapolative expectations about past default rates generate boom-bust credit market cycles. [Jin \(2015\)](#) presents a model in which crash risk is extrapolated from recent experiences. [Gennaioli et al. \(2012\)](#) model neglected risk and its effect on financial fragility. [Barberis \(2018\)](#) provides a summary of research on the psychology of extrapolative expectations and their application in macro-finance.

This paper also contributes more broadly to a budding literature on general equilibrium macroeconomic theory augmented with behavioral features. A partial list of recent examples includes [Farhi and Werning \(2017\)](#), [Fuster et al. \(2012\)](#), [Gabaix \(2016a,b\)](#), [García-Schmidt and Woodford \(2019\)](#), [Sims \(2003\)](#), and [Woodford \(2013\)](#). These papers focus on various forms of cognitive limits and typically imply sluggish expectations.<sup>3</sup> Here, the model of diagnostic expectations generates overreaction to recent information.

The organization of this paper is as follows. Section 2 describes the macroeconomic model. In Section 2, only the reduced-form beliefs process is specified. Section 3 outlines the solution strategy and the baseline calibration. The global solution is characterized in Section 4. Section 5 studies the impact of elevated sentiment in generating financial crises and endogenous boom-bust investment dynamics. Section 6 simulates the 2007-2008 Financial Crisis. A microfoundation for diagnostic expectations is provided in Section 7. An extension on bubble-pricking is discussed in Section 8.

## 2 Macro-Finance Model

This paper embeds diagnostic expectations into a macroeconomic model with financial intermediary frictions. The macro-finance model builds on [He and Krishnamurthy \(2019, henceforth HK\)](#). The HK model is one of the first quantitative papers in the continuous-time macro-finance literature, and it successfully replicates the downside macroeconomic risks precipitated by financial instability. However, the authors acknowledge that a failure of their model is its inability to decouple the market price of risk from the probability of a

---

<sup>3</sup>One exception is [Fuster et al. \(2012\)](#), which generates overreaction to recent data.

future financial crisis, as was observed in financial markets preceding the collapse of Lehman.

I extend HK in two ways. First, I generalize the model to allow for diagnostic expectations. Second, I introduce a simple labor income margin in order to improve the model’s quantitative fit. Whenever possible, I adopt the same notation as HK.

## 2.1 Model Setup

Time is continuous with  $t$  denoting the current period. The economy has two sectors: households and financial intermediaries. The economy has two types of capital: productive capital  $K_t$  and housing capital  $H$ .<sup>4</sup> The housing supply is fixed and normalized to  $H \equiv 1$ . The price of a unit of capital is denoted  $q_t$  and the price of housing is denoted  $P_t$ . These prices are endogenous and will be determined in equilibrium.

Only financial intermediaries possess the requisite skills to operate capital, and therefore the intermediary sector directly holds  $K_t$  and  $H$ .<sup>5</sup> Intermediaries fund these purchases by issuing debt and equity to households. The key financial friction in this model is that each intermediary faces an “equity capital constraint” which restricts its ability to raise equity funding. When binding, the intermediary sector must replace its equity funding with additional debt funding.

The economy features an “AK” production technology, with flow output  $Y_t$  given by:

$$Y_t = AK_t. \tag{1}$$

$A$  is a positive constant determining the productivity of capital.  $K_t$  evolves according to:

$$\frac{dK_t}{K_t} = (i_t - \delta)dt + \sigma dZ_t, \tag{2}$$

where  $i_t$  is the endogenous rate of capital installation at time  $t$  and  $\delta$  is the exogenous depreciation rate. The term  $\sigma dZ_t$  is a capital quality shock.<sup>6</sup>  $\{Z_t\}$  is a standard Brownian

---

<sup>4</sup>Section 4.1 provides a discussion of the quantitative benefits of using two types of capital.

<sup>5</sup>In the data, households do own some capital and housing directly. The HK model can be extended to allow households to directly own a share of  $K_t$  and  $H$ , but HK conclude that the quantitative impact is negligible under their preferred calibration. For details, see [He and Krishnamurthy \(2019\)](#).

<sup>6</sup>The RBC literature typically features productivity shocks. Having shocks directly alter capital improves



motion. Capital quality shocks  $\sigma dZ_t$  are the only source of uncertainty in the model.

Investment in capital is subject to quadratic adjustment costs. For a gross capital installation of  $i_t K_t$ , the cost is given by:

$$\Phi(i_t, K_t) = i_t K_t + \frac{\xi}{2} (i_t - \delta)^2 K_t.$$

## 2.2 Diagnostic Expectations

**Overview** I depart from rationality by assuming that all agents have extrapolative expectations about the capital stock. Specifically, this paper extends the model of diagnostic expectations developed in [Bordalo et al. \(2018a, henceforth BGS\)](#). In this section I specify expectations in reduced-form as applied to the macroeconomic model. The microfoundation is provided in [Section 7](#).

Diagnostic expectations are based on Kahneman and Tversky’s representativeness heuristic, defined as follows:

“an attribute is representative of a class if it is very diagnostic; that is, the relative frequency of this attribute is much higher in that class than in the relevant reference class.” ([Tversky and Kahneman, 1983](#), p. 296)

Following BGS, the reference class should reflect the absence of new information. In the context of expectations, this implies that future states become more representative, and are therefore overweighted, when they become more likely to occur in light of incoming data. Put differently, agents overweight future states that are diagnostic of recent news.

This paper’s model of diagnostic expectations features two innovations relative to BGS. First, diagnostic expectations are cast in continuous time. Continuous-time methods provide a valuable toolbox for evaluating the effect of non-rational expectations in dynamic equilibrium models. Additionally, global solutions enable non-rational expectations to be analyzed over the entire state space. Second, the methodology developed here allows for diagnostic expectations to be applied to endogenous Itô processes. This is my main improvement on BGS, who can only apply diagnostic expectations to exogenous AR(N) processes.

---

tractability by reducing the requisite number of state variables. Capital quality shocks are also used by [Gertler and Kiyotaki \(2010\)](#) and [Brunnermeier and Sannikov \(2014\)](#).

A goal for this paper’s formulation of diagnostic expectations is to constitute a tractable and portable methodology that enables rational models to be augmented with diagnostic expectations using a single additional state variable. Importantly, even though I introduce an additional state variable I do not introduce any additional shocks. Capital quality shocks remain the only source of uncertainty in the model.

**Expectations of Capital** All agents have diagnostic expectations about the log capital stock. The log capital stock evolves according to  $dk_t = (i_t - \delta - \frac{\sigma^2}{2})dt + \sigma dZ_t$ . Capital is the fundamental in this economy — capital alone determines output ( $Y_t = AK_t$ ), and capital quality shocks are the sole source of uncertainty. The evolution of capital is endogenous since it depends on the endogenous investment rate  $i_t$  (see equation (2)). This endogeneity introduces expectations-driven macroeconomic dynamics, where non-rational expectations alter investment which in turn affects the growth rate of capital and output. In this way, diagnostic expectations propagate through the equilibrium to affect the endogenous future path of the capital process on which expectations are formed.

The psychology of diagnostic expectations is as follows. The agent has in the back of their mind all necessary information to form correct expectations. However, limited and selective memory means that representative states come to mind more easily. This inflates the perceived likelihood of more representative states, and deflates the perceived likelihood of less representative states. Representative states are those that are diagnostic of incoming data, which in this model corresponds to recent capital quality shocks. This notion is formalized by the following measure of recent information at time  $t$ :

$$\mathcal{I}_t \equiv \int_0^t e^{-\kappa(t-s)} \sigma dZ_s. \quad (3)$$

$\mathcal{I}_t$  is a weighted integral of past shocks to capital, where the weight decays exponentially at rate  $\kappa$  as shocks occur further in the past.<sup>7</sup>  $\mathcal{I}_t > 0$  when recent capital quality shocks have tended to be positive, and  $\mathcal{I}_t < 0$  when recent capital quality shocks have tended to

---

<sup>7</sup>In discrete time, any individual shock can itself represent new information. In continuous time, I integrate over the past sequence of shocks because any individual shock  $\sigma dZ_s$  has only an infinitesimal effect on the capital stock. My specification is similar to the definition of “sentiment” in [Barberis et al. \(2015\)](#).

be negative.  $\mathcal{I}_t$  drifts back to 0 at rate  $\kappa$  in the absence of new shocks. Formally,  $\mathcal{I}_t$  is an Ornstein-Uhlenbeck process. It can be considered the continuous-time analogue of an AR(1) process.

Throughout, I will use hat notation to denote the beliefs of diagnostic agents. The current period is  $t$  and let  $\tau \geq 0$  denote a prediction horizon. Under diagnostic expectations, agents believe that the instantaneous evolution of capital in future period  $t + \tau$  is:

$$\frac{d\widehat{K}_{t+\tau}}{K_{t+\tau}} = (i_t - \delta + \theta \mathcal{I}_t e^{-\kappa\tau})dt + \sigma dZ_t. \quad (4)$$

First consider diagnostic expectations of the instantaneous evolution of capital from period  $t$  to period  $t + dt$  ( $\tau = 0$ ). Parameter  $\theta \geq 0$  governs the extent to which agents “judge by representativeness.”  $\theta = 0$  recovers rationality. When  $\theta > 0$  agents overweight future states that are diagnostic of recent information, resulting in a biased perception of the drift of capital. For this reason, information parameter  $\mathcal{I}_t$  will equivalently be referred to as “sentiment” henceforth.

At more distant prediction horizons, representativeness imparts a diminishing bias on the perceived drift of capital. Equation (4) specifies that the drift bias decays to zero at rate  $\kappa$  with prediction horizon  $\tau$ . This is because information that was representative at time  $t$  slowly dims as the agent simulates the perceived model forward in time.

The psychology of diagnostic expectations suggests that equation (4) should be thought of as an “as if” process. Agents do not consciously calculate the evolution of capital with a biased drift. Instead, agents have the true model in their memory database but selective sampling means that agents are exceedingly drawn to representative states as they form expectations.  $\mathcal{I}_t$  is an unconscious internal parameter that characterizes representativeness, and it is this process of selective recall that results in the biased perception of the capital growth process in equation (4).

One implication of equation (4) is that diagnostic agents have incorrect expectations about their own future expectations. A comparison of equations (3) and (4) shows that diagnostic agents do not perceive that future capital quality shocks will alter the bias of their future expectations. Put differently, the realized future information parameter  $\mathcal{I}_{t+\tau}$  is a

random variable at time  $t$  whereas  $\mathcal{I}_t e^{-\kappa\tau}$  is deterministic at time  $t$ . Therefore  $\mathcal{I}_{t+\tau} \neq \mathcal{I}_t e^{-\kappa\tau}$ , almost surely.

In summary, agents make two mistakes when  $\theta > 0$ . First, they hold incorrect beliefs about the drift of capital. Second, they have incorrect expectations about their own future expectations because they do not understand that they are diagnostic. Figure 1 provides an illustrative example of diagnostic expectations applied to an arithmetic Brownian motion (ABM).<sup>8</sup> The blue line plots the realized sample path of the ABM process. The vertical black line marks time  $t$  when a prediction is made about the future evolution of the ABM. The solid red line plots the diagnostic prediction and the dashed black line plots the rational prediction. Because recent shocks to the ABM have been positive at the time of prediction, diagnostic expectations of the ABM process are biased upward.

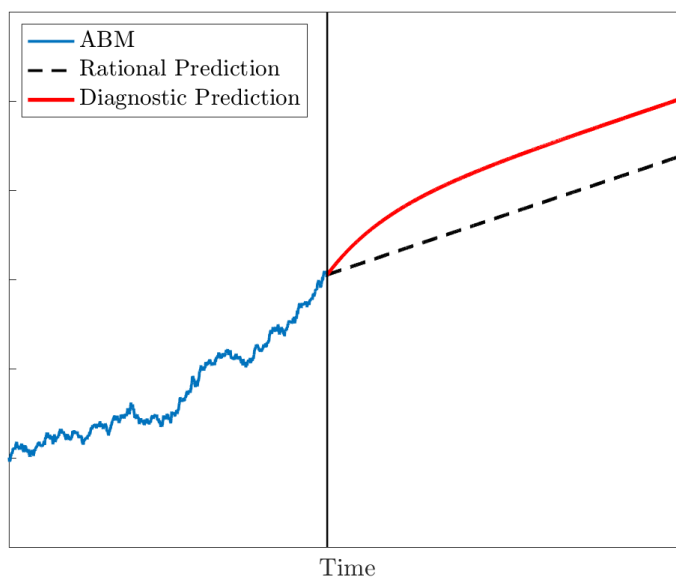


Figure 1: **Diagnostic expectations of arithmetic Brownian motion (ABM)**. The blue line plots the sample path of an ABM from time 0 until time  $t$  when a prediction is made about the future evolution of the ABM. The solid red line plots the diagnostic prediction and the dashed black line plots the rational prediction. In this example, diagnostic expectations are biased upward. This is because recent shocks to the ABM have tended to be positive at the time of prediction. The calibration is illustrative.

<sup>8</sup>Note that log capital  $k_t$  would follow arithmetic Brownian motion if investment rate  $i_t$  were constant.

**Decomposing Diagnostic Expectations** This paper’s formulation of diagnostic expectations reconciles mechanical models of extrapolation with dynamic forward-looking expectations. A decomposition of capital growth process  $\widehat{\frac{dK_t}{K_t}}$  highlights this property:

$$\widehat{\frac{dK_t}{K_t}} = \underbrace{\frac{dK_t}{K_t}}_{\text{Rational}} + \underbrace{\theta \mathcal{I}_t dt}_{\text{Wedge}}. \quad (5)$$

The diagnostic agent’s perception of the capital growth process can be separated into two components: a rational component plus a diagnostic wedge. The rational component is forward looking. The diagnostic wedge is a backward-looking function of past shocks, since  $\mathcal{I}_t \equiv \int_0^t e^{-\kappa(t-s)} \sigma dZ_s$ .

Diagnostic expectations differ from purely mechanical models of extrapolative expectations (e.g., adaptive expectations) because diagnostic expectations depend on the true underlying process. This forward-looking element comes from the rational component of diagnostic expectations. For example, a regime shift such as an unanticipated shock to adjustment cost  $\xi$  would cause a diagnostic agent to update their prediction of  $\widehat{\frac{dK_t}{K_t}}$ . This is because the rational component  $\frac{dK_t}{K_t}$  would change. However, the diagnostic wedge term means that diagnostic expectations are still subject to persistent errors. Though the agent has the true model in the back of their mind (the rational component), future expectations are drawn toward states that are representative in light of recent information (the diagnostic wedge). Thus, diagnostic expectations are characterized by the “kernel of truth” property: diagnostic expectations depend on the true economic process, yet they overreact to recent patterns in the data.

The above discussion specifies the extent to which diagnostic expectations are robust to the Lucas critique. Unlike mechanical models of extrapolation, diagnostic expectations are forward looking and dependent on the underlying economic model. This dependence comes from the rational component of diagnostic expectations. Even with this forward-looking behavior, diagnostic expectations are still subject to persistent extrapolative errors due to the backward-looking diagnostic wedge.

## 2.3 The Financial Intermediary Sector

**Individual Intermediaries** There is a continuum of financial intermediaries. Each intermediary is run by a single banker. In order to purchase capital and housing, intermediaries must raise funds from households. In particular, each intermediary is able to issue risk-free (instantaneous) debt and risky equity to households.<sup>9</sup> Equity issuance is subject to an endogenous equity capital constraint: each intermediary can issue up to  $\epsilon_t$  of equity.

Let  $d\tilde{R}_t$  denote the realized instantaneous return on an intermediary's equity. Equity capital constraint  $\epsilon_t$  evolves according to:

$$\frac{d\epsilon_t}{\epsilon_t} = d\tilde{R}_t. \quad (6)$$

Constraint  $\epsilon_t$  should be thought of as the banker's "reputation." Poor investment returns hurt the banker's reputation and lower the banker's ability to issue equity in the future. The banker does not consume. Instead, the banker has mean-variance preferences over reputation:

$$\hat{\mathbb{E}}_t \left[ \frac{d\epsilon_t}{\epsilon_t} \right] - \frac{\gamma}{2} \widehat{Var}_t \left[ \frac{d\epsilon_t}{\epsilon_t} \right] = \hat{\mathbb{E}}_t \left[ d\tilde{R}_t \right] - \frac{\gamma}{2} \widehat{Var}_t \left[ d\tilde{R}_t \right]. \quad (7)$$

Hat-notation is used to indicate that the banker has diagnostic expectations of asset returns.

This reputation-based constraint behaves similarly to the more standard "net worth" constraint in which the banker's net worth fluctuates as a function of past performance, and the banker's ability to raise future capital depends on net worth. The benefit of the reputation-based constraint is that the banker has no net worth and does not consume, leaving the representative household to consume all of the economy's output. This is the typical assumption in quantitative macroeconomic models without financial frictions, and will allow for a more standard calibration of the model here.<sup>10</sup>

---

<sup>9</sup>Because debt is instantaneous and asset prices are continuous, there will always exist an equity buffer that is large enough to absorb losses and ensure that debt is risk-free.

<sup>10</sup>For further details, see Section 2.2 of [He and Krishnamurthy \(2019\)](#).

**The Aggregate Intermediary Sector** Let  $\mathcal{E}_t$  denote the maximum equity capital that can be raised by the aggregate intermediary sector.  $\mathcal{E}_t$  evolves according to:

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = d\tilde{R}_t - \eta dt + d\psi_t. \quad (8)$$

The first term captures that all bankers behave identically, so aggregate equity capital evolves with the reputation of each individual banker. The second term captures exogenous banker exit, which occurs at rate  $\eta$ . Exit is needed to ensure that bankers don't escape their equity constraint in equilibrium. The final term  $d\psi_t \geq 0$  reflects entry into the intermediary sector. Entry occurs deep in crisis times when reputation is sufficiently low. Bank entry establishes a boundary condition for the model. Details are provided in Section 3.1.

## 2.4 The Household Sector

**Consumption** There is a unit measure of households. Households earn utility over consumption of the output good ( $c_t^y$ ) and housing services ( $c_t^h$ ). The output good is the numeraire. Since households do not hold housing directly, housing services must be rented at endogenous rental rate  $D_t$ .

The household maximizes the value function

$$\widehat{\mathbb{E}} \left[ \int_t^\infty e^{-\rho(s-t)} \frac{C_s^{1-\gamma_h}}{1-\gamma_h} ds \right], \quad (9)$$

where  $C_t$  is a Cobb-Douglas consumption aggregator:

$$C_t = (c_t^y)^{1-\phi} (c_t^h)^\phi. \quad (10)$$

Intratemporal maximization yields the following optimality condition:

$$\frac{c_t^y}{c_t^h} = \frac{1-\phi}{\phi} D_t. \quad (11)$$

**Labor Income** Households supply one unit of labor without disutility. There is an infinite marginal utility cost for providing labor beyond this point. Households receive a wage rate

of  $\mathcal{W}_t$  in exchange for a unit of labor. In equilibrium, households earn a share  $1 - \nu$  of output as labor income:

$$\mathcal{W}_t = (1 - \nu)AK_t. \tag{12}$$

Here I take this reduced-form wage equation as given. A simple microfoundation, based on [Frankel \(1962\)](#), is provided in [Appendix A.1](#). In addition to diagnostic expectations, this labor income margin is where my model differs from HK.

**Capital Goods Production** Investment follows  $q$ -theory. There exists a Capital Goods Producer who is responsible for capital investment. After capital is produced, it is sold directly to the intermediary sector at price  $q_t$ . All profits are passed on to households. The capital goods producer solves:

$$\max_{i_t} q_t i_t K_t - \Phi(i_t, K_t),$$

which results in an equilibrium investment rate of

$$i_t = \delta + \frac{q_t - 1}{\xi}. \tag{13}$$

Equation (13) is important because it highlights the propagation of behavioral and financial frictions to the real economy. The economy's growth rate depends on investment rate  $i_t$ , which in turn depends on capital price  $q_t$ . To the extent that both financial and behavioral frictions affect the capital price  $q_t$ , these frictions will feed back into the endogenous growth rate of output.

## 2.5 Portfolio Choice and Asset Returns

**Household Portfolio Choice** Let  $W_t$  denote aggregate household wealth at time  $t$ . Households can invest in two assets: the debt and equity issued by financial intermediaries. Debt pays a risk-free return of  $r_t$  while equity pays a stochastic return of  $d\tilde{R}_t$ . Reduced-form assumptions will now be made to ensure that intermediaries raise at least  $\lambda W_t$  of debt



funding from households. Households are not the focal-point of this model, and these simplifying assumptions allow the equilibrium leverage of the financial sector to be regulated by exogenous parameter  $\lambda$ .

Each household is comprised of two members, a “debt member” and an “equity member.” At the start of each period the household (i) consumes, and (ii) splits its wealth  $W_t$  between the debt member and the equity member. A share  $\lambda$  of wealth is given to the debt member, who can only invest in the intermediaries’ risk-free debt.<sup>11</sup> Share  $1 - \lambda$  is given to the equity member, who is free to invest in both the debt and the equity of the intermediaries (but is not allowed to make levered investments). Investments pay off at time  $t + dt$ , and returns are pooled before this process is repeated at time  $t + dt$ .

The model will be calibrated such that the equity member always chooses to invest the maximum possible amount in F’s equity in equilibrium.<sup>12</sup> Thus, equity members collectively invest their allocated wealth of  $(1 - \lambda)W_t$  in the intermediaries’ equity, subject to the restriction that they do not purchase more than  $\mathcal{E}_t$ . If the capital constraint binds, equity members place their remaining wealth in bonds. I therefore define

$$E_t \equiv \min\{\mathcal{E}_t, W_t(1 - \lambda)\} \quad (14)$$

as the total equity capital raised by the intermediary sector at time  $t$ .

The risk-free rate  $r_t$  is pinned down by the households’ intertemporal optimization problem. Specifically,

$$r_t = \rho + \zeta \widehat{\mathbb{E}}_t \left[ \frac{dc_t^y}{c_t^y} \right] - \frac{\zeta(\zeta + 1)}{2} \widehat{Var}_t \left[ \frac{dc_t^y}{c_t^y} \right], \quad (15)$$

where  $\zeta = 1 - (1 - \phi)(1 - \gamma_h)$  can be interpreted as the inverse of the EIS.<sup>13</sup> Again, hat notation is used because expectations of the consumption process are diagnostic. Equation

---

<sup>11</sup>One can think of  $\lambda$  as capturing household demand for liquid balances, though this is not formally modeled.

<sup>12</sup>This condition is verified as part of the model solution. For details, see Appendix E.3.

<sup>13</sup>Equating supply and demand in equilibrium,  $c_t^h = 1$  since  $H \equiv 1$ . Setting  $c_t^h = 1$  and plugging equation (10) into (9) gives  $\widehat{\mathbb{E}} \left[ \int_t^\infty e^{-\rho(s-t)} \left( \frac{(c_s^y)^{(1-\phi)(1-\gamma_h)}}{1-\gamma_h} \right) ds \right]$ . Multiplying the value function by positive constant  $\frac{1}{1-\phi}$  shows that we can think of the household as having CRRA preferences over just  $c_t^y$ , with a risk-aversion parameter of  $\zeta = 1 - (1 - \phi)(1 - \gamma_h)$ .

(15) is the standard consumption-based risk-free rate formula in continuous time.<sup>14</sup>

**Intermediary Portfolio Choice** Diagnostic expectations imply that households and intermediaries may not have correct beliefs about equilibrium asset return processes. To start, I postulate that agents expect  $q_t$  and  $P_t$  to evolve according to the Itô processes:

$$\frac{\widehat{dq}_t}{q_t} = \widehat{\mu}_t^q dt + \widehat{\sigma}_t^q dZ_t \quad (16)$$

$$\frac{\widehat{dP}_t}{P_t} = \widehat{\mu}_t^P dt + \widehat{\sigma}_t^P dZ_t. \quad (17)$$

Perceived drift and volatility coefficients are endogenous and will be solved for in equilibrium.

Using price process (16), the return on an investment in productive capital is perceived to be:

$$\widehat{dR}_t^k = \frac{\nu A}{q_t} dt + \frac{\widehat{d}(q_t K_t)}{q_t K_t} - i_t dt = \left( \frac{\nu A}{q_t} + \widehat{\mu}_t^q - \delta + \theta \mathcal{I}_t + \sigma \widehat{\sigma}_t^q \right) dt + \left( \sigma + \widehat{\sigma}_t^q \right) dZ_t. \quad (18)$$

The return on capital is comprised of a dividend component  $\left( \frac{\nu A}{q_t} dt \right)$  and a capital gains component  $\left( \frac{\widehat{d}(q_t K_t)}{q_t K_t} - i_t dt \right)$ . The dividend per unit of capital  $\nu A$  is the output that remains after labor wages are paid. Capital gains are accrued through growth in the price of capital and also through growth in the total of quantity of capital, excluding new investments made by capital goods producers.

Equation (18) illustrates how the perception of capital returns is biased. First, there is the direct effect of diagnostic expectations: capital growth expectations are biased by  $\theta \mathcal{I}_t$ . Second, diagnostic agents have an incorrect understanding of how the economy evolves in equilibrium. This introduces an indirect effect in which diagnostic agents misperceive the endogenous drift and volatility of price process  $dq_t$ .

Proceeding similarly, equation (17) can be used to derive the perceived return on an

---

<sup>14</sup>Because the representative household is split into a debt and equity member, additional assumptions are required in order to generate equation (15). In particular, if the household decides to save an additional dollar then that dollar must be invested in the risk-free bond. The benefit of making these assumptions is that it recovers the standard continuous-time formula for the risk-free rate. See footnote 5 of [He and Krishnamurthy \(2019\)](#) for details.

investment in housing:

$$\widehat{dR}_t^h = \frac{D_t}{P_t} dt + \frac{d\widehat{P}_t}{P_t} = \left( \frac{D_t}{P_t} + \widehat{\mu}_t^P \right) dt + \widehat{\sigma}_t^P dZ_t. \quad (19)$$

The dividend on housing is rental income  $D_t$ . Diagnostic expectations of capital growth indirectly create biased expectations of future rental income growth. Since  $P_t$  is the present discounted value of future dividends, diagnostic expectations generate non-rational perceptions of price process  $dP_t$ .

Introducing some additional notation, let  $\widehat{\pi}_t^k \equiv \left( \frac{\nu A}{qt} + \widehat{\mu}_t^q - \delta + \theta \mathcal{I}_t + \sigma \widehat{\sigma}_t^q \right) - r_t$  denote the perceived risk premium on an investment in productive capital. Similarly, let  $\widehat{\pi}_t^P \equiv \left( \frac{D_t}{P_t} + \widehat{\mu}_t^P \right) - r_t$  denote the perceived risk premium on an investment in housing. Using these definitions, equations (18) and (19) can be rewritten as follows:

$$\begin{aligned} \widehat{dR}_t^k &= \left( \widehat{\pi}_t^k + r_t \right) dt + \widehat{\sigma}_t^k dZ_t \\ \widehat{dR}_t^h &= \left( \widehat{\pi}_t^h + r_t \right) dt + \widehat{\sigma}_t^h dZ_t, \end{aligned}$$

where  $\widehat{\sigma}_t^k \equiv \sigma + \widehat{\sigma}_t^q$  and  $\widehat{\sigma}_t^h \equiv \widehat{\sigma}_t^P$ .

Each banker makes portfolio choices in order to maximize their objective in (7). Let  $\alpha_t^k$  and  $\alpha_t^h$  denote an intermediary's portfolio share of capital and housing, respectively.<sup>15</sup> The intermediary's perceived return on equity is:

$$\widetilde{dR}_t = \alpha_t^k \widehat{dR}_t^k + \alpha_t^h \widehat{dR}_t^h + (1 - \alpha_t^k - \alpha_t^h) r_t dt. \quad (20)$$

From equation (7), the banker solves:

$$\max_{\alpha_t^k, \alpha_t^h} \left[ r_t + \alpha_t^k \widehat{\pi}_t^k + \alpha_t^h \widehat{\pi}_t^h \right] - \frac{\gamma}{2} \left( \alpha_t^k \widehat{\sigma}_t^k + \alpha_t^h \widehat{\sigma}_t^h \right)^2, \quad (21)$$

---

<sup>15</sup>When  $\alpha_t^k + \alpha_t^h > 1$  then the intermediary is taking on leverage. The model will be calibrated such that  $\alpha_t^k + \alpha_t^h > 1$  in equilibrium.

which results in the optimality condition:

$$\frac{\widehat{\pi}_t^k}{\widehat{\sigma}_t^k} = \frac{\widehat{\pi}_t^h}{\widehat{\sigma}_t^h} = \gamma(\alpha_t^k \widehat{\sigma}_t^k + \alpha_t^h \widehat{\sigma}_t^h). \quad (22)$$

Equation (22) states that the intermediary chooses portfolio shares in order to equate the perceived Sharpe ratio on each asset to its risk aversion times its perceived portfolio risk.<sup>16</sup>

As the intermediary sector is required to bear additional risk on its portfolio, it will demand a higher Sharpe ratio as compensation.

**Binding Constraints, Leverage, and Financial Crises** Following HK, financial crises are defined as states in which the equity issuance constraint binds:  $\mathcal{E}_t < W_t(1 - \lambda)$ . A binding constraint generates a sudden and dramatic increase in risk premia, a collapse in asset prices, and impaired economic growth. These crisis nonlinearities arise when the constraint binds for two reasons, as can be seen with equation (22). First, a binding constraint means that intermediaries cannot raise sufficient equity capital and are forced to increase leverage in order to fund asset purchases.<sup>17</sup> This increases  $\alpha_t^k$  and  $\alpha_t^h$ , which causes intermediaries to demand higher perceived Sharpe ratios as compensation for their additional leverage. Second, the constraint generates endogenous amplification of negative capital quality shocks. When the constraint binds, negative shocks will cause the constraint to bind even more tightly, thereby increasing leverage and risk premia even further. This endogenous amplification of shocks in the crisis region increases asset price volatilities  $\widehat{\sigma}_t^k$  and  $\widehat{\sigma}_t^h$ , again increasing the Sharpe ratio demanded by the financial sector.

**Feedback from Behavioral Frictions to Financial Frictions** The intermediary sector's capital capacity  $\mathcal{E}_t$  evolves according to the realized return on equity  $d\widetilde{R}_t$ . However, equation (22) shows that assets are priced according to the perceived return process  $\widehat{d\widetilde{R}_t}$ . To the extent that the perceived return process differs from the realized return process, asset

<sup>16</sup>It is optimal to equate the Sharpe ratio on each asset because returns are perfectly conditionally correlated in this single-shock model.

<sup>17</sup>The model predicts that the market leverage of financial intermediaries is countercyclical. For empirical evidence, see He et al. (2017).

prices will not reflect true fundamentals.<sup>18</sup> When bankers are excessively optimistic, future returns will tend to be lower than expected. As expectations disappoint, these persistent forecast errors will cause the intermediary sector’s capital capacity to deteriorate relative to expectations. Thus, excessive optimism at time  $t$  feeds back to create future equity funding problems. The reverse is true when bankers are excessively pessimistic about future returns. As will be seen from Section 5 onward, this *feedback from behavioral frictions to financial frictions* is the critical interaction underlying many of the model’s key predictions.

## 2.6 Equilibrium

**Definition 1. *Diagnostic Expectations Equilibrium (DEE)*.** A diagnostic expectations equilibrium is a set of prices  $\{q_t, P_t, D_t, r_t, \mathcal{W}_t\}$  and decisions  $\{c_t^y, c_t^h, i_t, \alpha_t^k, \alpha_t^h\}$  such that:

1. Given prices, decisions as specified by (11), (13), (15), and (21) are optimal under diagnostic expectations.
2. The goods market and housing rental market clear:

$$Y_t = AK_t = C_t^y + \Phi(i_t, K_t), \text{ and} \tag{23}$$

$$C_t^h = H \equiv 1.$$

3. The equity issuance constraint is satisfied:

$$E_t = \min\{\mathcal{E}_t, W_t(1 - \lambda)\}.$$

---

<sup>18</sup>The key expectations bias in this economy is the overextrapolation of fundamentals by financial intermediaries. There is a growing empirical literature documenting the extrapolation of fundamentals by financial professionals. A partial list includes [Bordalo et al. \(2018c\)](#), [Bordalo et al. \(2019a\)](#), [Fahlenbrach et al. \(2017\)](#), [Greenwood and Hanson \(2013\)](#), [Gulen et al. \(2019\)](#), and [Pflueger et al. \(2018\)](#).

4. *Asset markets clear with intermediaries holding all capital and housing:*

$$q_t K_t = \alpha_t^k E_t, \text{ and} \quad (24)$$

$$P_t = \alpha_t^h E_t. \quad (25)$$

5. *The total value of assets equals total household wealth:*

$$W_t = q_t K_t + P_t.$$

It should be noted from the model of diagnostic expectations outlined in Section 2.2 that diagnostic expectations generalize rational expectations. Rationality is recovered by setting  $\theta = 0$ . This is formalized in the following definition, which will serve as a benchmark for later comparison.

**Definition 2. *Rational Expectations Equilibrium (REE)*.** *A rational expectations equilibrium is a diagnostic expectations equilibrium in which  $\theta = 0$ .*

## 3 Solution and Calibration

### 3.1 Solution Strategy

In order to solve for an equilibrium I must characterize how any history of shocks  $\{Z_s, s \leq t\}$  maps into equilibrium prices, asset allocations, and beliefs at time  $t$  such that (i) all agents maximize diagnostically expected utility through their consumption and portfolio decisions, and (ii) markets clear. In particular, I consider Markov equilibria with state variables  $K_t$ ,  $\mathcal{E}_t$ , and  $\mathcal{I}_t$ .  $K_t$  captures the overall size of the economy,  $\mathcal{E}_t$  is the financial sector's capital capacity, and  $\mathcal{I}_t$  gives the state of expectations relative to rationality. HK use  $K_t$  and  $\mathcal{E}_t$  as state variables in their rational model. The innovation of this paper is to capture sentiment with state variable  $\mathcal{I}_t$ . When expectations are diagnostic it is not enough to know the current state of the economy ( $K_t$  and  $\mathcal{E}_t$ ); extrapolation means that one must also know the path that the economy took to get to that point ( $\mathcal{I}_t$ ).

The solution can be simplified further by scaling the economy by  $K_t$ .<sup>19</sup> Let

$$e_t \equiv \frac{\mathcal{E}_t}{K_t}.$$

$e_t$  captures the capital capacity of the intermediary sector relative to the size of the overall economy. I will look for price functions of the form  $p_t = \frac{P_t}{K_t} = p(e_t, \mathcal{I}_t)$  and  $q_t = q(e_t, \mathcal{I}_t)$ . The ability to scale the economy by  $K_t$  allows me to numerically solve for an equilibrium as a function of only two state variables:  $e_t$  and  $\mathcal{I}_t$ .  $e_t$  characterizes the severity of financial frictions and  $\mathcal{I}_t$  characterizes the state of beliefs relative to rationality.

**Comparing Diagnostic and Rational Expectations Equilibria** Within the class of Markov equilibria considered here, rational expectations equilibria (REE) can be neatly contextualized within diagnostic expectations equilibria (DEE).<sup>20</sup> This is formalized in the following proposition.

**Proposition 1.** *For any diagnostic expectations equilibrium that is Markov in  $\{K, e, \mathcal{I}\}$ , the price and policy functions for  $\{K, e, \mathcal{I} = 0\}$  compose a rational expectations equilibrium that is Markov in  $\{K, e\}$ .*

*Proof.* Equation (4) specifies that when  $\mathcal{I}_t = 0$ , all agents make decisions that are optimal if  $\mathcal{I} = 0$  in perpetuity. Decisions that are optimal when  $\mathcal{I} = 0$  in perpetuity must also be optimal when  $\theta = 0$  (REE), because in both cases  $\mathcal{I}$  is perceived to have no further effect on the resulting equilibrium.  $\square$

Proposition 1 shows that every diagnostic expectations equilibrium nests its corresponding rational expectations equilibrium. This property makes comparisons between the DEE and the REE simple, as one gets the REE “for free” when solving for the DEE.

One corollary of Proposition 1 is that the REE characterizes the long-run expectations of state variable  $e_t$  held by diagnostic agents.

<sup>19</sup>This is possible because  $P_t$  is linear in  $K_t$ . The price of housing is equal to the discounted value of future housing dividends. In equilibrium, the dividend on housing is given by  $D_t = \frac{\phi}{1-\phi} \left[ A - i_t + \frac{\xi}{2}(i_t - \delta)^2 \right] K_t$ , which is linear in  $K_t$ . Details are provided in Appendix A.3.

<sup>20</sup>While I have not found multiple equilibria in my numerical analysis, no claims of equilibrium uniqueness are made in this paper.

**Corollary 1.** *The ergodic distribution of  $e_t$  that is diagnostically expected by agents in the model is equivalent to the stationary distribution of  $e_t$  that would arise in the corresponding REE.*

*Proof.* Equation (4) specifies that contemporaneous sentiment  $\mathcal{I}_t$  does not alter the diagnostic agent's perception of the evolution of capital as prediction horizon  $\tau \rightarrow \infty$ . In the long run, Proposition 1 therefore obtains.  $\square$

**Boundary Conditions** Boundary conditions are needed to solve for price functions  $q(e_t, \mathcal{I}_t)$  and  $p(e_t, \mathcal{I}_t)$ . As  $e_t \rightarrow \infty$  financial frictions disappear and the equity issuance constraint ceases to affect the equilibrium price and policy functions. The scale invariance properties of the model imply that  $\lim_{e_t \rightarrow \infty} q_e(e_t, \mathcal{I}_t) = \lim_{e_t \rightarrow \infty} p_e(e_t, \mathcal{I}_t) = 0$ . In words, asset prices become insensitive to  $e_t$  as the financial sector moves further from its funding constraint.

A lower boundary is imposed by assuming that bankers enter the intermediary sector deep in crisis times. This is captured by the term  $d\psi_t$  in equation (8). The model assumes that there exists an exogenous minimum reputation level  $\underline{e}$  such that new bankers enter the intermediary sector whenever  $e_t$  reaches  $\underline{e}$ . Thus, state variable  $e_t$  has a reflecting barrier at  $\underline{e}$ . Loosely, this can be thought to capture government intervention deep in crises. It is costly to create new intermediaries because bankers must acquire the skills to operate capital. Specifically, the economy must destroy  $\beta > 0$  units of capital in order for entry to increase aggregate reputation  $\mathcal{E}_t$  by one unit.<sup>21</sup>

The assumption of a reflecting barrier at  $\underline{e}$  pins down boundary conditions for asset price functions  $q(e_t, \mathcal{I}_t)$  and  $p(e_t, \mathcal{I}_t)$ . Prices  $q$  and  $P$  must have a zero derivative with respect to  $e_t$  at  $\underline{e}$ .<sup>22</sup> This implies  $q_e(\underline{e}, \mathcal{I}_t) = 0$  and  $p_e(\underline{e}, \mathcal{I}_t) = \frac{p(\underline{e}, \mathcal{I}_t)\beta}{1+\beta}$ . Details are provided in Appendix A.2.

<sup>21</sup>The capital evolution equations (2) and (4) are altered at  $\underline{e}$  to include this form of capital destruction.

<sup>22</sup>If this were not the case, an arbitrageur could make unbounded profits (almost surely) by betting on a unidirectional change in asset prices at the reflecting barrier. This is because the price shock is of order  $\sqrt{dt}$  while the risk-free rate an arbitrageur borrows at is of order  $dt$ , so the asset price shock dominates borrowing costs.



## 3.2 Calibration

He and Krishnamurthy (2019) build a standard RBC model augmented with a financial intermediary sector. I introduce three new parameters to the HK model. These are the two behavioral parameters  $\theta$  and  $\kappa$ , and the labor income parameter  $\nu$ . The model in this paper nests HK under the calibration  $\theta = 0$  and  $\nu = 1$ .

The macroeconomic model behaves like a standard RBC model when  $e_t$  is far from the constraint, though intermediary frictions become quantitatively important near the crisis region. I follow HK in defining  $e_{distress}$  as the 33<sup>rd</sup> percentile value of  $e_t$  in the model’s stationary distribution.  $e_{distress}$  delineates “normal” periods from “distress” periods. In the calibration to follow,  $e_{distress} \approx 0.65$ . Parameters corresponding to the model’s RBC elements will be calibrated in the part of the state space where  $e_t > e_{distress}$ .

Table 1 presents the baseline calibration, and details are discussed below. When possible, I follow the parameter choices and/or calibration targets of He and Krishnamurthy (2019). In Table 1, parameter values that are marked with an asterisk in the “Choice” column are equivalent to the parameter values of HK. Asterisks in the “Target” column indicate parameters for which the value differs from HK, but the calibration target is the same. The only parameters for which neither the value nor the target aligns with HK are the three new parameters  $\theta$ ,  $\kappa$ , and  $\nu$ .

**RBC Parameters** The household discount rate  $\rho$ , depreciation rate  $\delta$ , and adjustment cost  $\xi$  are relatively standard RBC parameters. My calibration follows HK.

I set  $A = 0.425$  and  $\nu = 0.315$ . Parameters  $A$  and  $\nu$  are calibrated to target the investment-to-capital ratio and the investment-to-output ratio. HK target an investment-to-capital ratio of approximately 9% in the non-distress states. Given the calibration of  $\delta = 0.1$ , equation (13) shows that  $q_t \approx 1$  is necessary to match this calibration target. The dividend on capital is  $\frac{\nu A}{q_t}$ , so  $\nu$  and  $A$  must be jointly calibrated to generate a dividend commensurate with  $q_t \approx 1$ . To separately identify  $A$  and  $\nu$ , I also target an investment-to-output ratio of 20%. Investment-to-output equals  $\frac{i_t K_t}{A K_t}$ , so the calibration that  $i \approx 9\%$  pins down  $A$  accordingly.<sup>23</sup>

---

<sup>23</sup>HK set  $A = 0.133$ . The HK model has no labor income, which can be obtained in my model by

	<b>Parameter</b>	<b>Choice</b>	<b>Target</b>
Panel A: Intermediation Parameters			
$\gamma$	Banker risk aversion	2*	Mean Sharpe ratio
$\lambda$	Debt ratio	0.75*	Intermediary leverage
$\eta$	Banker exit rate	0.143	Prob(crisis)*
$\underline{e}$	Entry barrier	0.08	Max $\mathcal{I} = 0$ Sharpe ratio*
$\beta$	Entry cost	2.8*	Land price volatility
Panel B: Technology Parameters			
$\sigma$	Capital shock	3%*	Consumption volatility
$\delta$	Depreciation rate	10%*	Literature
$\xi$	Adjustment cost	3*	Literature
$A$	Productivity	0.425	Investment-capital ratio*
$\nu$	Capital share	0.315	Investment-output ratio
Panel C: Household Preference Parameters			
$\rho$	Time discount rate	2%*	Literature
$\zeta$	1/EIS	0.725	Interest rate volatility*
$\phi$	Housing share	0.163	Housing-to-wealth ratio*
Panel D: Diagnostic Expectations Parameters			
$\kappa$	Decay of new information	0.139	Slow-moving beliefs
$\theta$	Diagnosticity	0.132	Magnitude of expectations bias
Unconditional Simulated Moments			
	Probability of Crisis		3.18%
	Volatility( $r_t$ )		0.95%
	Mean (Realized Sharpe Ratio)		0.51
	Volatility(Land Price Growth)		11.95%
Non-Distress Simulated Moments			
	Mean $\left(\frac{\text{Investment}}{\text{Capital}}\right)$		9.67%
	Mean $\left(\frac{\text{Investment}}{\text{Output}}\right)$		22.76%
	Mean $\left(\frac{\text{Housing Wealth}}{\text{Total Wealth}}\right)$		45.76%
	Volatility(Consumption Growth)		2.36%
	Volatility(Investment Growth)		5.04%
	Volatility(Output Growth)		2.93%

Table 1: **Baseline calibration.** Model-generated moments are calculated by simulating the model at a quarterly frequency. Simulated growth rates are computed as log changes from quarter  $t - 2$  to quarter  $t + 2$ .

Finally, parameter  $\sigma$  governs the volatility of exogenous capital quality shocks. As in HK, I set  $\sigma = 3\%$ . HK report that from 1975 to 2015 the volatility of investment growth in non-distress periods was 5.79%, and the volatility of consumption growth was 1.24%. In the non-distress region of the model, setting  $\sigma = 3\%$  generates an output growth volatility of 2.93%, an investment growth volatility of 5.04%, and a consumption growth volatility of 2.36%. The model features too much consumption volatility and too little investment volatility, with the choice of  $\sigma = 3\%$  attempting to strike a balance between these two inaccuracies.

**Intermediation Parameters** Parameter  $\gamma$  represents the bankers' risk aversion. I follow HK in setting  $\gamma = 2$ . This implies that bankers demand an average realized Sharpe ratio of 0.51. The closest empirical counterpart is [He et al. \(2017\)](#), who find that the capital ratio of financial intermediaries robustly prices cross-sectional expected returns for a variety of asset classes. They estimate a Sharpe ratio of 0.48 for assets intermediated by the financial sector.

Parameter  $\lambda$  controls the leverage of the financial sector when the capital constraint doesn't bind. Since the financial sector has assets of  $P_t + q_t K_t = W_t$  and equity of  $E_t$ , equation (14) gives a market leverage value of  $\frac{W_t}{E_t} = \frac{1}{1-\lambda}$  in non-crisis states. Again following HK, I set  $\lambda = 0.75$  which generates a leverage ratio of 4 in non-distress states.

**Crisis Parameters** Financial crises are defined as states in which the equity issuance constraint binds. Banker exit rate parameter  $\eta$  is chosen to target a 3% crisis probability. I target the same crisis probability as HK, roughly corresponding to an average of three financial crises every 100 years.

Parameters  $\underline{e}$  and  $\beta$  control the lower boundary condition. In their model without senti-

---

shutting down the labor margin ( $\nu = 1$ ).  $A = 0.133$  implies that HK have an investment-to-output ratio of approximately 67% and a consumption-to-output ratio of less than 30%. An important consequence of these counterfactual ratios is that the HK model struggles to match empirical moments on consumption growth volatility, particularly in periods of financial distress. The interest rate depends on consumption growth (second term in equation (15)), and HK calibrate  $\zeta = 0.13$  (EIS > 7) in order to prevent large changes in the consumption growth rate from generating excessive interest rate volatility. I depart from the HK calibration because, as will be detailed shortly, the EIS becomes particularly important when growth expectations are biased. Though I sacrifice some parsimony of the original HK model by introducing a simple labor income margin, the benefit of doing so is that it allows my model to more accurately match the consumption moments observed empirically.

ment, HK set  $\underline{e}$  such that the Sharpe ratio at  $\underline{e}$  is 6.5.  $\underline{e}$  is set low enough that entry occurs rarely. To align this model's calibration with HK, I set  $\underline{e}$  such that the perceived Sharpe ratio at  $\underline{e}$  and  $\mathcal{I} = 0$  is 6.5.

Parameter  $\beta$  governs the volatility of housing price  $P_t$ . This is because  $\beta$  determines the slope of  $P_t$  at the lower boundary in the crisis state, which in turn affects the slope of  $P_t$  throughout the distress region. HK estimate that the empirical volatility of land price growth has been 11.9% from 1975 to 2015. With  $\beta = 2.8$ , the volatility of land price growth in the model is 11.95%.

**Household Parameters** Parameter  $\phi$  determines the relative value of housing services to the consumption good, which in turn pins down the rental rate  $D_t$  (see equation (11)).  $D_t$  is also the dividend on a housing investment, and  $P_t$  is the discounted value of these future dividend flows. As in HK, I set  $\phi$  to target a non-distress housing-to-wealth ratio of approximately 45%. This requires  $\phi = 0.163$ .<sup>24</sup>

$\zeta$  is the inverse of the EIS, and determines the responsiveness of the interest rate to changes in expected consumption growth and volatility. When expectations are diagnostic, agents misperceive the growth rate of consumption. This implies that  $\zeta$  governs the sensitivity of the interest rate to variation in  $\mathcal{I}_t$ .

The reason that  $\zeta$  plays an important role when expectations are diagnostic is that  $\zeta$  modulates the extent to which sentiment gets incorporated into asset prices. When  $\zeta = 1$ , any bias in consumption growth expectations is passed one-for-one into the risk-free discount rate  $r_t$  (see equation (15)). An important implication is that when  $\zeta = 1$ , asset prices  $q_t$  and  $P_t$  are constant in  $\mathcal{I}_t$ . All bias in cash-flow expectations is exactly offset by the risk-free rate, leaving asset prices completely unresponsive to sentiment. When  $\zeta < 1$ , the interest rate responds less than one-for-one to changes in expected consumption growth. In this case,  $q_t$  and  $P_t$  are increasing in  $\mathcal{I}_t$ .

In the baseline calibration I set  $\zeta = 0.725$ . As in HK, I target a real interest rate volatility of approximately 1%. Appendix Figure 6 plots the ergodic distribution of the risk-free rate in

---

<sup>24</sup>HK set  $\phi = 0.6$ . My calibration has a much higher  $A$  than HK, meaning that my model generates a larger share of output goods relative to housing services. In order to prevent this from driving up the rental rate  $D_t$  paid on housing, my calibration requires a lower value of  $\phi$ .

the baseline calibration. Since  $\zeta < 1$ , asset prices are increasing in  $\mathcal{I}_t$ . This will be important for generating the results in Section 5.2.

**Behavioral Parameters**  $\theta$  governs the extent to which expectations are biased by representativeness and  $\kappa$  governs the decay of  $\mathcal{I}_t$ . These two parameters must be calibrated jointly. The unconditional variance of the diagnostic agent’s bias in output growth expectations is  $\frac{\theta^2 \sigma^2}{2\kappa}$ .<sup>25</sup> Low values of  $\kappa$  must be accompanied by low values of  $\theta$  to prevent growth expectations from being unrealistically biased.

The existing literature offers little guidance for calibrating  $\kappa$ . I set  $\kappa = 0.139$ , equivalent to a half-life of 5 years.<sup>26</sup> This slow-moving sentiment is intended to capture prolonged periods of relatively positive and negative news, such as the Great Moderation, rather than high-frequency volatility.<sup>27</sup> Given  $\kappa$ ,  $\theta = 0.132$  is calibrated such that one standard deviation in  $\mathcal{I}$  corresponds to an output growth bias of 0.75 percentage points. This is consistent with the magnitude of the bias estimated in [Bordalo et al. \(2018c\)](#) (see Appendix F.2 for details).

Appendix B examines the robustness of the results that follow to  $\kappa$  and  $\theta$ . Appendix F.3 outlines relevant properties of Ornstein-Uhlenbeck processes and provides numerical examples under the baseline calibration. Specifically, Appendix F.3 specifies the conditional distribution of  $\mathcal{I}_{t+\tau}$  given  $\mathcal{I}_t$ . It also characterizes the persistence of sentiment by providing a closed-form solution for the distribution of first-hitting times, defined as the first time  $\tau$  at which  $\mathcal{I}_{t+\tau}$  crosses zero.

---

<sup>25</sup>This is a property of Ornstein-Uhlenbeck processes.

<sup>26</sup>Using a different specification than this paper, [Bordalo et al. \(2019a\)](#) estimate that the diagnostic expectations of stock market analysts incorporate the past three years of shocks. Similar findings exist for non-experts. Using survey data from recent homebuyers, [Case et al. \(2012\)](#) argue that slow-moving long-term expectations were the critical driver of the 2000s housing bubble. [Malmendier and Nagel \(2011\)](#) document that the financial market returns experienced over ones lifetime are predictive of future risk taking and beliefs about future returns.

<sup>27</sup>The method can easily be extended such that sentiment contains both a slow-moving component and a high-frequency component. However, this requires an additional state variable. Details are included in Appendix F.5.

# 4 Global Solution: Asset Prices, Policy Functions, and the Stationary Distribution

## 4.1 Prices, Policy Functions, and Forecast Errors

Select prices and policy functions for the DEE under the baseline calibration are shown in Figure 2 (for more, see Appendix Figure 7). The horizontal axis lists (scaled) capital capacity  $e_t = \frac{\mathcal{E}_t}{K_t}$ . All panels plot three curves. The blue curve corresponds to depressed sentiment ( $\mathcal{I}_t = -1.5SD$ ), the red curve corresponds to neutral sentiment ( $\mathcal{I}_t = 0$ ), and the yellow curve corresponds to elevated sentiment ( $\mathcal{I}_t = +1.5SD$ ).

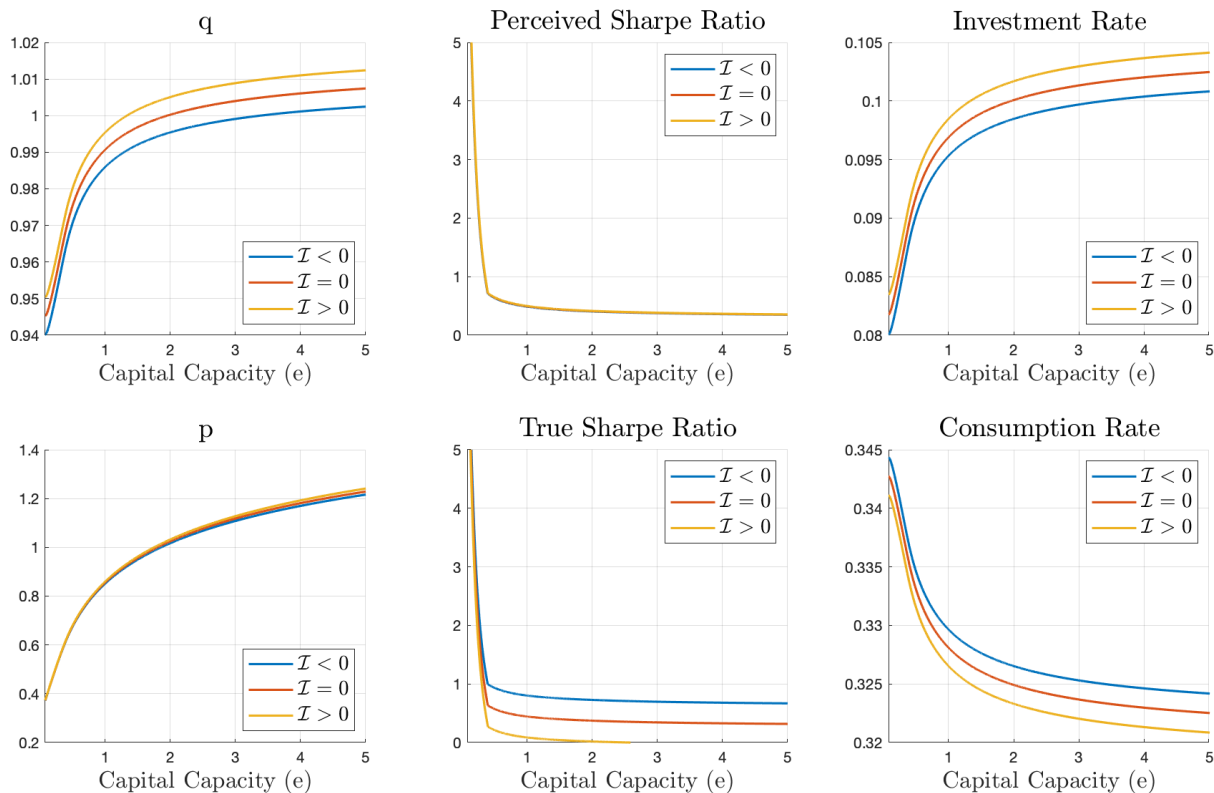


Figure 2: **Selected price and policy functions.** The horizontal axis lists capital capacity  $e = \frac{\mathcal{E}}{K}$ . Each panel plots three curves, corresponding to  $\mathcal{I} = -1.5SD$  (blue),  $\mathcal{I} = 0$  (red), and  $\mathcal{I} = +1.5SD$  (yellow). The model is solved under the baseline calibration in Table 1.

The two leftmost panels of Figure 2 plot asset prices  $q_t$  and  $p_t$ . To understand the effect of financial and behavioral frictions, it is instructive to review two benchmark economies

for comparison: a “no financial frictions” economy (no equity issuance constraint) and a “no behavioral frictions” economy (rational expectations). In the “no financial frictions” benchmark, the scale-invariance properties of this model imply that the price functions are horizontal lines. In the rational expectations benchmark, a direct application of Proposition 1 gives that prices  $q_t$  and  $p_t$  are represented by only the red line ( $\mathcal{I}_t = 0$ ).

Starting with the effect of financial frictions on asset prices, the  $q$  and  $p$  panels show that asset prices are sensitive to the intermediary sector’s funding capacity  $e_t$ . In the crisis region (approximately  $e_t < 0.4$ ), constraints on intermediary risk-bearing capacity cause asset prices to plummet. The Sharpe ratio panels illustrate the nonlinear spike in risk premia that characterizes crisis times. Moving away from the constrained region, asset prices rise as the intermediary sector restores its risk-bearing capacity. As  $e_t$  continues to increase, asset prices eventually asymptote to their value in the “no financial frictions” benchmark.

Importantly, asset prices exhibit what HK refer to as “anticipation effects”: asset prices start to fall well before the equity issuance constraint actually binds. These anticipation effects arise because bankers are unwilling to pay a high price for an asset at time  $t$  if there is a chance that a binding constraint sometime in the future will cause that asset’s price to crash.<sup>28</sup> Anticipation effects mean that financial frictions affect financial market and macroeconomic dynamics well before the constraint actually binds — the anticipation of a binding constraint at some point in the future is enough to drag down current asset prices. It is these anticipation effects that underlie the “financial distress” region of the model, defined as  $e_t$  below its 33<sup>rd</sup> percentile, where constraints don’t necessarily bind but financial friction effects are still present.<sup>29</sup>

The influence of diagnostic expectations on asset prices is illustrated by the separation of the yellow curve and the blue curve from the red curve in the  $q$  and  $p$  panels. Holding  $e_t$

---

<sup>28</sup>In the model, the financial sector’s diagnostic expectations of risk premia will be low when asset prices are high, and vice-versa. This goes against the evidence presented in Greenwood and Shleifer (2014), which shows that investor return expectations are negatively correlated with model-implied returns. However, Greenwood and Shleifer (2014) focus predominantly on the expectations of households. In the model, it is the beliefs of financial intermediaries that are relevant for pricing assets. Adam et al. (2018) provide evidence that professional investors have excess return expectations that covary negatively with the P/D ratio, while individual investors have excess return expectations that covary positively with the P/D ratio.

<sup>29</sup>Consistent with anticipation effects, Baron et al. (2019) find that bank equity declines predict output gaps, even when panics do not materialize.

fixed, asset prices are increasing in  $\mathcal{I}_t$  because bankers are willing to pay more in order to access what they expect to be higher future cash flows. Indeed, asset prices move so that sentiment about future cash flows gets “priced in” in equilibrium. This is illustrated by the Perceived Sharpe Ratio panel, which shows that almost all variation in perceived risk premia is driven by  $e_t$ . The True Sharpe Ratio panel shows the effect of pricing assets based on non-rational expectations of fundamentals. Elevated sentiment lowers subsequent realized returns, and vice-versa.

The Investment Rate and Consumption Rate panels of Figure 2 illustrate how financial and behavioral frictions propagate to the real economy. The growth rate of output is controlled by the endogenous rate of investment, and  $i_t$  is an increasing function of  $q_t$ . The model with neither financial frictions nor behavioral frictions would feature a constant capital price  $q_t$  and therefore a constant rate of investment. Thus, *all variation in investment is due to financial and behavioral frictions*. The investment panel illustrates that investment is low whenever either  $e_t$  is low or  $\mathcal{I}_t$  is low. Correspondingly, the Consumption Rate panel shows that the (scaled) rate of consumption  $\frac{C_t^y}{K_t}$  is high whenever  $e_t$  is low or  $\mathcal{I}_t$  is low. This follows from output market clearing in (23). For a given level of output, if investment is low then consumption must be high in order to clear the output market.

**Aside: Why Two Types of Capital?** At first glance it is puzzling that the model includes two types of capital,  $K_t$  and  $H$ , since these assets are perfectly conditionally correlated. I digress here to outline the quantitative benefits of this setup.

HK attempts to jointly match key macroeconomic and financial-market data. As a macroeconomic model it aims to generate empirically-plausible levels of investment volatility. As a finance model, enough asset price volatility is needed to produce quantitatively significant nonlinearities during periods of financial distress. There is a classic result in production-based asset pricing which says these two goals present a problem. Market values of capital are much more volatile than investment, both across firms and over time. In a standard  $q$ -theory model where investment is closely linked with asset prices, these two facts can only be reconciled with unreasonably high adjustment costs (Campbell, 2017, Ch. 7).

By introducing two types of capital, HK circumvent this issue. The two leftmost panels



of Figure 2 show that  $p_t$  is far more volatile than  $q_t$ . This is because the supply of houses is fixed while  $K_t$  is strongly procyclical.<sup>30</sup> Investment  $i_t$  is a function of  $q_t$ , so the low variance of  $q_t$  allows for the model to match empirical investment volatility with reasonable adjustment costs. Since the intermediary holds both types of capital, the additional volatility provided by  $p_t$  generates an intermediary pricing kernel which is volatile enough to produce significant nonlinearities in financial intermediation.

## 4.2 Ergodic Distribution

Figure 3 plots the economy's stationary distribution over  $e_t$  and  $\mathcal{I}_t$ . The ergodic distribution is solved for numerically using a *Kolmogorov forward equation*.<sup>31</sup> The economy is more likely to be in lighter-colored areas, while dark blue regions are rarely encountered. The dashed gray line marks where the constraint binds. The stochastic steady state occurs at  $e_t = 0.86$  and  $\mathcal{I}_t = 0$ .

The unconditional correlation between  $e_t$  and  $\mathcal{I}_t$  is approximately 0.7. This strong positive correlation arises in equilibrium because  $e_t$  and  $\mathcal{I}_t$  both load positively on the same shocks. Consider a positive capital quality shock. The financial sector uses leverage to take a long position in capital, so the positive shock generates high returns for the financial sector. This increases capital capacity  $e_t$ . The same positive shock also increases sentiment by making future states with high capital more representative.

## 5 Results: Financial Crises and Business Cycles

Now that the solution has been described, I proceed to detail the implications of the interaction that exists between behavioral frictions and financial frictions. To begin, I examine the joint effect of sentiment and financial frictions in generating financial crises.

---

<sup>30</sup>Positive shocks  $\sigma dZ_t$  directly increase  $K_t$ . They also indirectly increase  $K_t$  by increasing investment rate  $i_t$ . The procyclical supply of capital implies that capital supply and demand are positively correlated, insulating asset price  $q_t$  relative to  $p_t$ .

<sup>31</sup>Details are provided in Appendix E.2.

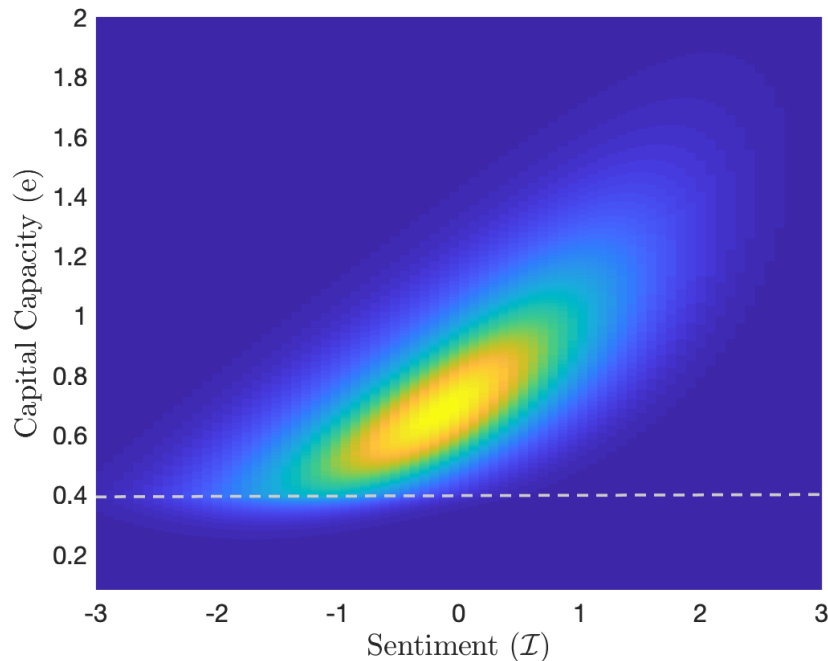


Figure 3: **Ergodic distribution.** Sentiment is reported in standard deviation units. The gray dashed line marks the boundary of the crisis region. The model is solved under the baseline calibration in Table 1.

## 5.1 Sentiment-Driven Financial Crises

The feedback from behavioral frictions to financial frictions means that elevated sentiment can amplify financial fragility in the background of low risk-premium environments. Under diagnostic expectations, measures of financial distress are insufficient for quantifying systemic risk. One must also measure sentiment. This is in contrast to the REE, where state variable  $e_t$  alone is sufficient for characterizing the probability of future financial crises.

To show this result, I conduct the following experiment. The economy is initialized with a capital capacity of  $e_t = e_{distress}$  (the 33<sup>rd</sup> percentile of  $e_t$ ). The risk premia demanded by intermediaries at  $e_{distress}$  will still be moderate. At this level of capital capacity the financial sector is starting to anticipate the possibility of a future crisis, but still has a significant capital buffer remaining before the crisis region is hit.

For each level of sentiment  $\mathcal{I}_t$ , I calculate both the true and the diagnostically expected probability that the economy finds itself in a crisis at some point in the next 1, 2, or 5 years. These “hitting probabilities” are calculated numerically using a *Kolmogorov backward*

equation.<sup>32</sup> Figure 4 graphs the results. The solid lines in Figure 4 plot the true probability of a crisis over different horizons. The dashed lines plot the probability of a crisis that is perceived by agents with diagnostic expectations. Following from Proposition 1, the dots intersecting the dashed line at  $\mathcal{I}_t = 0$  indicate the crisis probabilities in the corresponding REE.

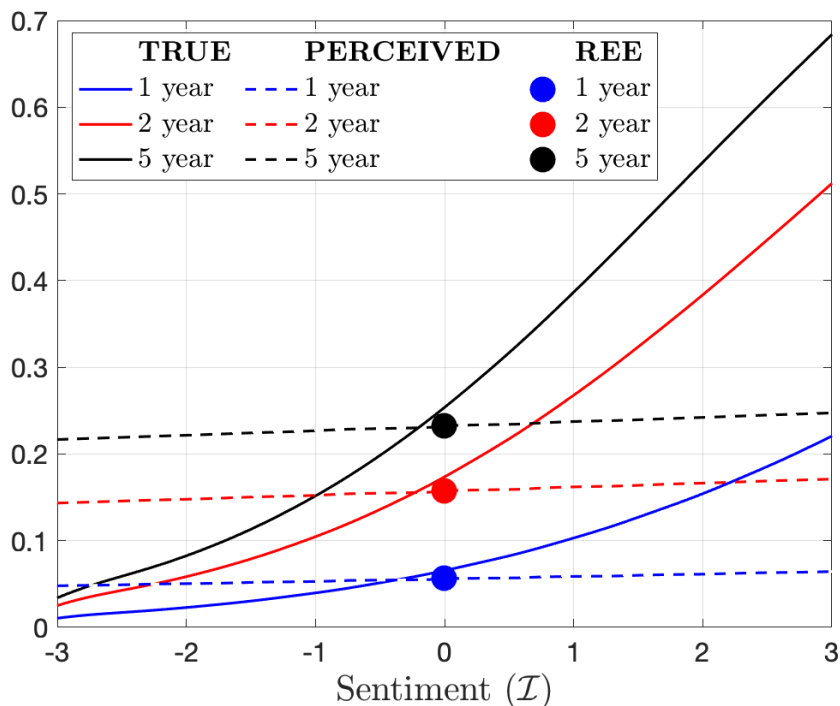


Figure 4: **Crisis hitting probabilities.** Starting from  $e_{distress}$  (33<sup>rd</sup> percentile of  $e_t$ ), solid lines report the true probability of a crisis in the DEE over the next one (blue), two (red), or five (black) years. Dashed lines report perceived crisis probabilities. The dots mark the crisis probabilities in the corresponding REE. Sentiment is reported in standard deviation units.

The REE (dots) features a low probability of initial financial distress devolving into a crisis. In addition to the existing capital buffer, the probability of a crisis is low because intermediaries have a built-in defense mechanism to combat financial crises — a rising risk premium. This is another way of stating the anticipation effects seen in Figure 2. As  $e_t$  moves closer to the crisis region, intermediaries respond by demanding a larger risk premium on capital and housing. This increases intermediary returns, allowing bankers to rebuild their

<sup>32</sup>Details are provided in Appendix E.2.

reputation and move away from the constraint. It therefore takes a rare series of persistently negative shocks to transport the financial sector from the distress region into the crisis region.

This same argument describes why the perceived crisis probabilities (dashed lines) are also moderate. Note first that because diagnostic expectations of fundamentals are priced-in in equilibrium, intermediaries perceive that variation in crash risk is almost entirely driven by  $e_t$ .<sup>33</sup> Since intermediaries are forward looking, they believe that all of the same anticipation effects will operate if  $e_t$  deteriorates further. However, diagnostic agents have incorrect expectations about how the economy evolves in equilibrium. This dislocates the perception of systemic risk from reality.

Elevated sentiment breaks the intermediary sector's crisis defense mechanism and sets the stage for future financial crises. When  $\mathcal{I}_t > 0$  and expectations are excessively optimistic, bankers believe that they are demanding a larger risk premium than they truly are.<sup>34</sup> In particular, when  $\mathcal{I}_t > 0$  then bankers borrow at an elevated interest rate and pay high prices to purchase capital and housing, which they are willing to do because they have excessively-optimistic forecasts of the future cash flows offered by these assets. As expectations disappoint, the intermediary sector does not receive the returns necessary to recover from financial distress. This disappointment of expectations implies that the financial sector remains vulnerable, and a more moderate series of negative shocks is sufficient to move the economy into the crisis region. This explains the neglected crash risk that occurs when  $\mathcal{I}_t > 0$ .

The reverse story explains why perceived crisis probabilities are inflated when  $\mathcal{I}_t < 0$ . In this case, excessive pessimism about future cash flows implies that the intermediary sector borrows at low interest rates and purchases assets cheaply. When cash flows end up being larger than expected, the financial sector makes returns in excess of expectations and quickly rebuilds its reputation.

This discussion highlights why models with rational expectations produce a tight positive correlation between risk premia and crash risk, and therefore struggle to replicate empirical

---

<sup>33</sup>As sentiment increases there is a mild upward slope in the perceived crisis hitting probability. The reason for this is that a crisis occurs when  $e_t < (1 - \lambda)(p_t + q_t)$  (dividing equation (14) by  $K_t$ ). Since  $p_t$  and  $q_t$  are increasing in  $\mathcal{I}_t$ , the RHS of this inequality is increasing in sentiment. This means that a given level of  $e_t$  (here,  $e_t = e_{distress}$ ) is closer to the crisis region when  $\mathcal{I}_t$  is high.

<sup>34</sup>This is not to necessarily say that the risk premium is negative, only that it is not as large as perceived.

patterns of low pre-crisis risk premia. The nonlinearity in financial intermediation generates a dramatic spike in risk premia during crises, but it is precisely this nonlinearity which prevents models with rational expectations from maintaining low risk premia as crisis risk builds. With calibrated levels of risk aversion, the anticipation that asset prices may collapse in a crisis means that intermediaries will demand a larger risk premium ex-ante.

Diagnostic expectations can break the link between risk premia and crash risk. Elevated sentiment increases financial fragility by eroding intermediary balance sheets. However, this fragility is neglected since it emerges from the disappointment of expectations. This allows sentiment-driven financial fragility to build in environments featuring low contemporaneous risk premia.

**Stress Testing** Sentiment-driven amplification of systemic risk is further illustrated through financial intermediary stress tests. In these stress tests I simulate the model at a quarterly frequency and calculate the path of shocks required to generate wide-scale losses in the financial sector under different levels of initial sentiment. The financial sector is started with a capital capacity of  $e_{distress}$  and sentiment  $\mathcal{I}_t \in \{-2SD, -1SD, 0, +1SD, +2SD, REE\}$ . Conditional on each starting location, a constant shock is fed into the system over the following 8 quarters in order to generate the 2-year bank equity loss listed in the lefthand column. Table 2 reports the 8-quarter compounded shock necessary for each level of bank equity losses.

2-yr. Bank Capital	$\mathcal{I}_t = -2SD$	$\mathcal{I}_t = -1SD$	$\mathcal{I}_t = 0$	$\mathcal{I}_t = +1SD$	$\mathcal{I}_t = +2SD$	$REE$
-2 %	-2.1 %	-1.4 %	-0.7 %	0.0 %	0.7 %	-0.8 %
-5 %	-3.4 %	-2.7 %	-2.0 %	-1.3 %	-0.5 %	-2.1 %
-10 %	-5.6 %	-4.8 %	-4.1 %	-3.3 %	-2.5 %	-4.2 %
-15 %	-7.7 %	-6.9 %	-6.1 %	-5.1 %	-3.8 %	-6.2 %
-25 %	-9.8 %	-8.6 %	-7.4 %	-6.2 %	-5.0 %	-7.3 %

Table 2: **Stress testing.** The financial sector is started at  $e_{distress}$  and sentiment  $\mathcal{I}_t \in \{-2SD, -1SD, 0, +1SD, +2SD, REE\}$ . The model is simulated at a quarterly frequency. A constant shock is fed into the system to generate the two-year bank capital losses listed in the leftmost column. The table lists the requisite shock compounded over eight quarters.

As sentiment increases, the disappointment of expectations means that smaller shocks are sufficient to generate a given level of capital losses. This is particularly true for large capital losses. Elevated sentiment generates large, hidden, vulnerabilities in the financial

sector which will not be reflected in the market price of risk. To quantify systemic risk, policymakers need to measure sentiment in addition to financial sector distress.

To contextualize these shock scenarios, note that 8-quarter compounded shocks are distributed approximately  $\text{Lognormal}(0, 2\sigma^2)$ . Consider the 25% equity loss scenario. When  $\mathcal{I}_t = -2SD$  there is a 0.8% percent chance of realizing a series of shocks worse than -9.8%. When  $\mathcal{I}_t = 0$ , there is a 3.5% chance of realizing a series of shocks worse than -7.4%. When  $\mathcal{I}_t = +2SD$ , there is an 11.3% chance of realizing a series of shocks worse than -5.0%. These stress test results show that sentiment plays an essential role in predicting future financial crises given initial distress.

**Behavioral Frictions Before Crises, Financial Frictions in Crises** The empirical literature documents that risk premia are relatively low for prolonged periods as crisis risk builds, and then spike dramatically once a crisis hits (Baron and Xiong, 2017; Brunnermeier and Oehmke, 2013; Krishnamurthy and Muir, 2017; Muir, 2017). A lesson from the model is that both behavioral frictions and financial frictions are required to replicate risk premia around crises. By creating neglected crash risk, diagnostic expectations can produce low pre-crisis risk premia. Nonetheless, slow-moving sentiment alone cannot generate the sudden spike that characterizes crisis times. The model relies on the occasionally binding constraint to generate crisis nonlinearities. Thus, it is the feedback from behavioral frictions to financial frictions that is needed to replicate empirical patterns of risk premia around crises. Ex-ante behavioral frictions set the stage for crisis-time spikes in risk premia driven by financial frictions.

To detail this point, Figure 5 is a contour plot of expected financial intermediary returns ( $\mathbb{E}_t[d\tilde{R}_t]$ ) over the state space.<sup>35</sup> Blue curves represent areas of low returns, and the gradient increases to the red curves which represent high returns. The dashed black line marks  $e_t = e_{distress}$ . Outside of the crisis region, the verticality of the contour lines means that variation in expected returns is driven primarily by sentiment.<sup>36</sup> Crisis nonlinearities are characterized by the contour lines pulling together, as this implies that risk premia are very

---

<sup>35</sup>Note that this figure plots true expected returns, not diagnostic expectations of returns.

<sup>36</sup>This verticality does not exist for diagnostically expected intermediary returns. Diagnostic expectations of intermediary returns are plotted in Appendix Figure 9.

sensitive to small movements in the economy’s state. The critical feature of Figure 5 is that the contour lines feature a sharp right kink as they pull together. This indicates that once the crisis region is hit, variation in risk premia is driven by  $e_t$ .

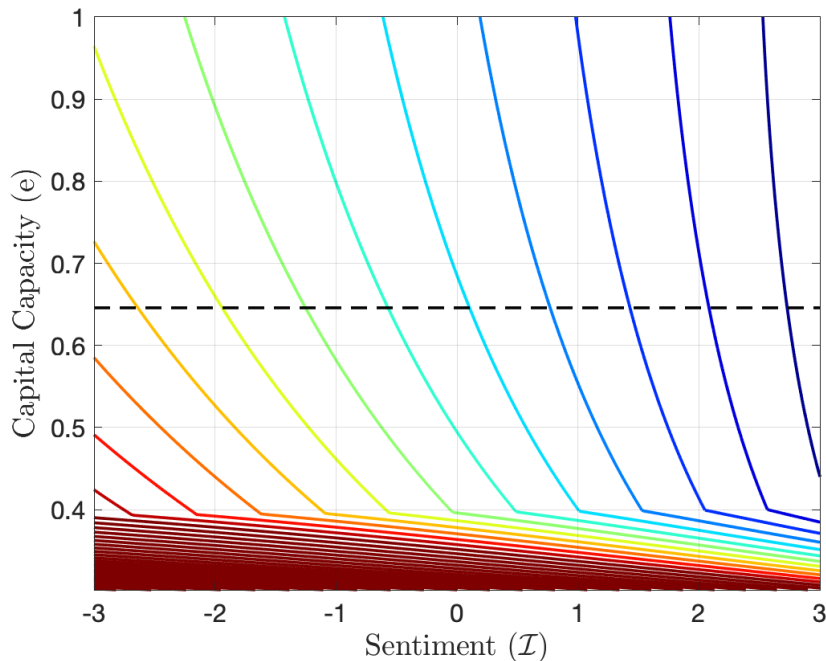


Figure 5: **Expected intermediary returns** ( $\mathbb{E}_t[d\tilde{R}_t]$ ). Each line represents a different level of expected intermediary returns over the state space. Blue curves indicate low returns and red curves indicate high returns. When the intermediary funding constraint does not bind, returns are sensitive to sentiment. Once the funding constraint binds at  $e_t \approx 0.4$ , the contour lines feature a kink. This indicates that in the crisis region, intermediary returns are sensitive to  $e_t$ . Sentiment is reported in standard deviation units.

**Empirical Evidence on Sentiment-Driven Financial Crises** Augmenting HK with diagnostic expectations generates sentiment-driven financial crises. The empirical evidence on financial crisis predictability typically examines the predictive power of growth in credit from the banking sector and/or credit spreads.<sup>37</sup> I let  $e_t$  be the model’s counterpart to credit growth. This follows [Krishnamurthy and Muir \(2017\)](#), who argue that credit growth is an empirical proxy for financial fragility. To map credit spreads into the model, equation (22)

<sup>37</sup>Neither credit growth nor spreads has a direct counterpart in the model because businesses and financial intermediaries are collapsed into a single intermediary sector which holds capital directly. In reality, credit extended by the financial sector becomes liabilities for businesses and households who are the end holders of capital.

establishes an asset pricing equation that will price any asset such that the financial sector has no incentive to take a net position in that asset. Details are included in Appendix D. The reduced-form result is that credit spreads are decreasing in both  $e_t$  and  $\mathcal{I}_t$ . Spreads are decreasing in  $e_t$  due to variation in the risk-bearing capacity of the financial sector. Spreads are decreasing in  $\mathcal{I}_t$  because higher output growth expectations are assumed to lower the diagnostically expected default rate of risky bonds.

The predictions of the DEE align closely with the empirical findings of [Krishnamurthy and Muir \(2017\)](#). [Krishnamurthy and Muir \(2017\)](#) study an international panel of credit spreads and macroeconomic outcomes and find that pre-crisis spreads are “too low” to be explained by rational models. Specifically, after controlling for credit growth they estimate that credit spreads are 23% lower in the five years before a financial crisis. In the model, controlling for credit growth means fixing  $e_t$ . Further conditioning on a future crisis corresponds to conditioning on higher average values of  $\mathcal{I}_t$ . Since spreads are decreasing in  $\mathcal{I}_t$ , the model is able to replicate the finding that spreads are “too low” before financial crises.

Additionally, [Krishnamurthy and Muir \(2017\)](#) find that it is the interaction of low credit spreads and high credit growth that best predicts future financial crises. The model is again consistent with this empirical finding. A low value of  $e_t$  is a necessary condition for a future financial crisis but, as [Figure 4](#) and [Table 2](#) highlight, the predictive power of  $e_t$  is significantly enhanced when combined with high values of sentiment. Further, the model presents a mechanism through which financial market froth predicts future crises — the breakdown of anticipation effects due to asset mispricing.

The model is also consistent with direct evidence of neglected risk exhibited by financial professionals before the 2007-2008 Financial Crisis. [Foote et al. \(2012\)](#) uncover a Lehman Brothers analyst report from 2005 documenting that analysts assigned a 5% probability to the worst-case “Meltdown” scenario of -5% home price growth over the subsequent 3 years, followed by 5% home price growth thereafter. The realized path of home prices was significantly worse. House prices fell a total of 27% from their July 2006 peak to their February 2012 trough (S&P/Case-Shiller National Home Price Index). Surveys of institutional investors by the Investor Behavior Project at Yale University show a neglect of crisis risk up to two months before the collapse of Lehman. An analysis of structured finance



products by [Coval et al. \(2009\)](#) reveals a similar neglect of tail risk by credit rating agencies and investors. The potential for the financial sector to drag down the real economy was also underestimated by professional forecasters. In the summer of 2008 the Fed forecasting staff was asked to prepare a “severe financial stress” forecast. The prediction was a real GDP growth rate of -0.4% for the remainder of 2008, followed by a growth rate of 0.5% in 2009 and 2.6% in 2010 ([Gennaioli and Shleifer, 2018](#)). The true recession was far more severe.

## 5.2 Amplification and the Firing-Reloading of Capital Capacity

I now turn to studying sentiment-driven macroeconomic fluctuations. Expectations influence economic growth through investment, since output is a controlled diffusion with investment rate  $i_t$  alone controlling the growth rate of the economy:  $\frac{dY_t}{Y_t} = (i_t - \delta)dt + \sigma dZ_t$ . The results to follow highlight the benefit of being able to apply diagnostic expectations to endogenous processes. Sentiment propagates through the dynamical system to alter investment rate  $i_t$ , which in turn controls the growth rate of capital. Thus, non-rational expectations about the future level of capital and output will alter the equilibrium path of these endogenous processes.

Readers should note that the calibration of  $\zeta$  is important in this section because  $\zeta$  governs how diagnostic expectations are passed into asset prices versus the risk-free rate. The baseline calibration sets  $\zeta < 1$ , implying that  $q_t$  is increasing in  $\mathcal{I}_t$ . Since investment rate  $i_t$  is a function of  $q_t$ ,  $i_t$  is also increasing in sentiment when  $\zeta < 1$ .

To highlight the impact of sentiment in a conventional way, I use impulse-response functions to study how investment rate  $i_t$  responds to positive and negative economic shocks. I simulate the model at a quarterly frequency and feed in a 20-quarter period of positive capital quality shocks as well as a 20-quarter period of negative capital quality shocks.<sup>38</sup> After this five-year period, shocks are set to zero in perpetuity.

The impulse-response of  $i_t$  is provided in [Figure 6](#). The lefthand panel plots the positive 20-quarter shock, and the righthand panel plots the negative 20-quarter shock. Shocks

---

<sup>38</sup>The magnitude of the shock is set to ensure that the series of quarterly shocks cumulates to a one standard deviation shock over 20 quarters. In particular, I set the quarterly shock to equal  $\pm \frac{\sigma\sqrt{5}}{20}$ . Summing these shocks over 20 quarters gives a 5-year cumulative shock of  $\pm \sigma\sqrt{5}$ . This corresponds to a cumulative one standard deviation shock over 20 quarters, since  $\sigma(Z_{t+5} - Z_t) \sim \mathcal{N}(0, 5\sigma^2)$ .

are fed in from  $t = -5$  to  $t = 0$ . The red curve plots the IRF in the diagnostic expectations equilibrium (DEE). The dashed black curve plots the IRF in the rational expectations equilibrium (REE). Both economies start in their stochastic steady at  $t = -5$ .<sup>39</sup> Appendix Figure 10 plots the corresponding IRFs for log output.

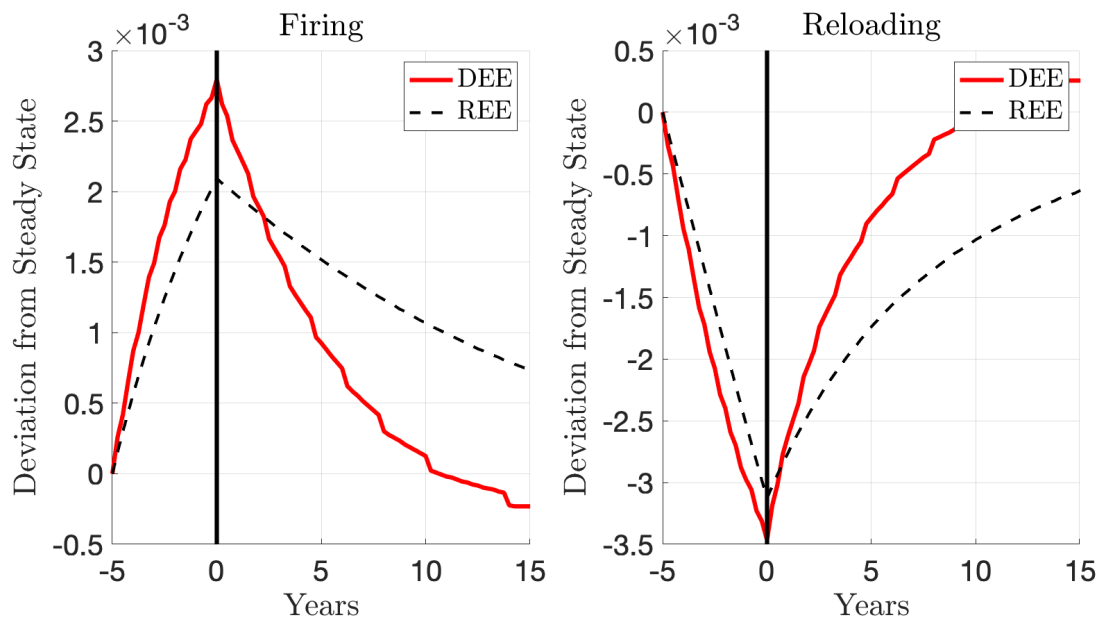


Figure 6: **Investment rate IRFs.** The economy is started at the stochastic steady state at  $t = -5$  before a sequence of shocks is fed into the system. On the lefthand panel there is a five-year sequence of positive shocks, and on the righthand panel there is a five-year sequence of negative shocks. The solid red line plots the response of investment rate  $i_t$  in the DEE. The dashed black line plots the response of  $i_t$  in the REE. The model is simulated at a quarterly frequency.

The striking result from these impulse-response functions is that diagnostic expectations promote boom-bust investment patterns:  $i_t$  exhibits amplified short-run momentum followed by steeper reversals. In the case of positive shocks, this boom-bust pattern arises because the sentiment-driven boom is triggered by the financial sector “firing off” its capital capacity. In short, the boom begets its own bust. In the case of negative shocks, investment features a bust-boom pattern because the sentiment-driven bust allows the financial sector to “reload” its capital capacity. The bust begets its own boom.<sup>40</sup>

<sup>39</sup>The DEE starts at  $e_{-5} = 0.87$  and  $\mathcal{I}_{-5} = 0$ . The REE starts at  $e_{-5} = 0.94$ .

<sup>40</sup>This idea of investment cycles driven by forecast errors goes at least as far back as [Pigou \(1926\)](#). [Beaudry and Portier \(2004\)](#) provide a formalization of this concept.

To detail this result, it is instructive to first review investment dynamics in the REE. When expectations are rational, a series of positive shocks increases  $e_t$ . This increases intermediaries' ability to bear risk, increasing  $q_t$  and therefore increasing  $i_t$ . Once the growth shocks stop at  $t = 0$ ,  $e_t$  is now above its steady state level and slowly recedes to the steady state. As the capital capacity of the financial sector slowly declines, so too does capital price  $q_t$  and investment  $i_t$ . The opposite holds for a series of negative shocks.

Diagnostic expectations add to these dynamics a contemporaneous sentiment effect followed by a feedback from behavioral frictions to financial frictions. Consider the positive shock case in which diagnostic expectations generate a boom-bust investment pattern. Positive shocks from  $t = -5$  to  $t = 0$  elevate sentiment, which further increases capital price  $q_t$ . This causes a sharper investment boom during the period of expansion.

However, this sentiment-driven boom is generated by firing off the financial sector's capital capacity. By increasing asset prices above fundamentals, excessive optimism decreases the financial sector's subsequent returns. This erodes the balance sheets of intermediaries, decreasing asset prices and investment. In this way, the sentiment-driven boom begets its own financial-frictions-driven bust.

The reverse story explains the bust-boom pattern of investment that occurs following a sequence of negative shocks. Negative shocks depress sentiment which drags down investment. However, low interest rates and asset prices mean that the financial sector earns large returns going forward. The sentiment-driven investment bust simultaneously reloads the financial sector's capital capacity. As sentiment recovers the economy is left with a strong financial sector that is able to support high levels of investment. Appendix Figure 11 includes the corresponding IRFs in an economy with diagnostic expectations but no financial frictions. Appendix Figure 11 highlights that it is the interaction of behavioral and financial frictions which generates short-run amplification followed by steep reversals.

**Empirical Evidence on Boom-Bust Investment Dynamics** This pattern of boom-bust investment cycles aligns with recent empirical studies of the credit cycle. Most directly, [Gulen et al. \(2019\)](#) find that elevated credit market sentiment at time  $t$  correlates with a boom in corporate investment over the subsequent year, followed by a long-run contraction in

corporate investment. [López-Salido et al. \(2017\)](#) find that elevated credit-market sentiment at time  $t$  predicts lower GDP growth from year  $t + 2$  to  $t + 3$ . The timing of the interplay between sentiment and financial frictions is consistent with the observation of [Greenwood et al. \(2019\)](#) that the business cycle is disconnected from the credit cycle, with financial fragility arising at the tail-end of economic expansions.

### 5.3 Financial-Market Stability from Beliefs

The analysis in Sections [5.1](#) and [5.2](#) is a conditional analysis. I start at particular points in the  $\{e, \mathcal{I}\}$  state space and trace out the macro-financial consequences that result. I now turn to an unconditional question: does the DEE produce more financial crises than the corresponding REE? Under the baseline calibration, the answer is that the DEE features fewer financial crises than the REE. Diagnostic expectations stabilize financial markets.

Figure [7](#) shows this result visually. The two orange lines (right axis) plot the marginal CDF of state variable  $e_t$  in the DEE and the REE. The blue curve (left axis) plots the CDF of the DEE divided by the CDF of the REE. When the blue curve is less than 1 for any particular value of  $e_t$ , this indicates that the DEE has a lower probability of having capital capacity below that point than the REE. The crisis region is marked by the dashed vertical line at  $e_t \approx 0.4$ . Since the blue curve crosses 1 above the crisis region, this indicates that the DEE produces fewer financial crises than the REE. The blue curve remains above 1 for large values of  $e_t$ , indicating that the DEE also features fewer periods of marked financial sector strength.

One can understand the stabilizing effect of beliefs by superimposing the crisis likelihoods in Figure [4](#) onto the ergodic distribution shown in Figure [3](#). Figure [4](#) illustrates that the part of the state space where the financial sector is distressed and sentiment is elevated is highly predictive of future financial crises. But, the ergodic distribution shows that this part of the state space is rarely encountered. Instead, financial distress almost always coincides with excessive pessimism. Excessive pessimism causes intermediaries to earn a larger risk premium than perceived. This hedges intermediaries against a future financial crisis by reloading the capital capacity of the financial sector.<sup>[41](#)</sup>

---

<sup>41</sup>Though diagnostic expectations prevent financial crises under the baseline calibration, this result can be

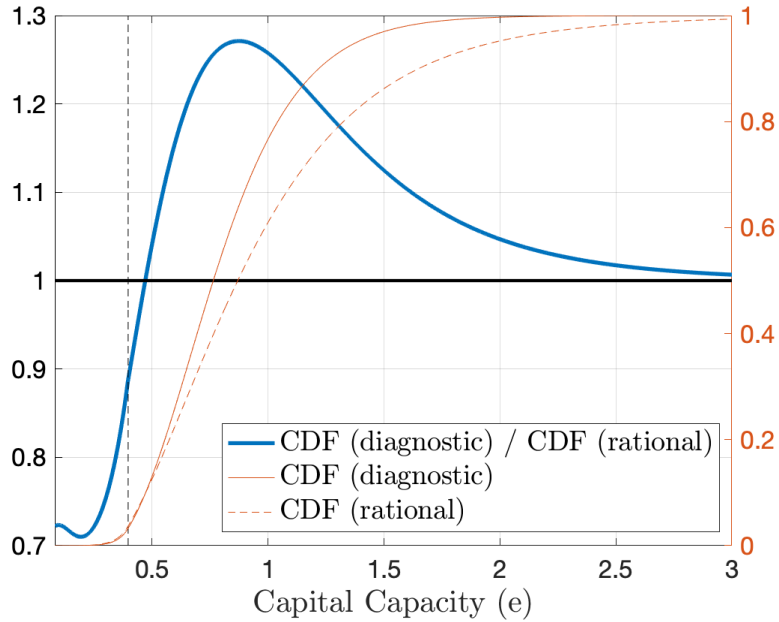


Figure 7: **Financial-market stability from beliefs.** The figure compares the marginal distribution of capital capacity ( $e$ ) for the DEE and the REE. The solid orange curve (right axis) plots the marginal CDF over  $e$  in the DEE. The dashed orange curve plots the marginal CDF over  $e$  in the corresponding REE. The blue curve (left axis) plots the CDF of  $e$  in the DEE divided by the CDF of  $e$  in the REE. The dashed vertical line at  $e \approx 0.4$  marks the boundary of the crisis region.

This conclusion may appear somewhat at odds with the earlier result that elevated sentiment is highly predictive of financial crises. Indeed, much of the empirical literature has found that elevated sentiment is predictive of future financial market downturns and concluded – the model suggests incorrectly – from this finding that extrapolative expectations promote financial instability (Gennaioli and Shleifer, 2018). These two seemingly-contradictory results can be reconciled by recognizing that the former is a conditional prediction while the latter is an unconditional prediction. The model’s conditional prediction is that periods in which the financial sector is vulnerable and sentiment is elevated are highly predictive of financial crises. However, the ergodic distribution illustrates that it is rare for the model to reach these states. The model’s unconditional prediction is that beliefs stabilize the financial sector, because periods of financial distress are highly correlated with depressed sentiment.

---

overturned under alternative calibrations in which the magnitude of perceptual error is increased. Robustness is explored in Appendix B.3.

**Business Cycle Amplification, Financial Cycle Stabilization** Section 5.2 documents that diagnostic expectations amplify business cycles, while the finding here is that diagnostic expectations stabilize financial cycles. Though these conclusions may appear inconsistent, they are intimately linked: the feedback from behavioral frictions to financial frictions amplifies business cycles while simultaneously stabilizing financial cycles.

The intermediary sector has a long position in capital. Balance sheets will typically be strong following a sequence of positive shocks and weak following a sequence of negative shocks. In the case of positive shocks, elevated sentiment amplifies the initial output boom, but does so by “firing off” the capital capacity of intermediaries. In this way, elevated sentiment quickly undoes the balance sheet strength initially generated by the sequence of positive shocks. The reverse is true in the case of negative shocks. Depressed sentiment amplifies the initial investment bust, but this “reloads” capital capacity. Thus, the way that the economy avoids a financial crisis is by going through a sentiment-driven recession.

**Empirical Evidence on Financial-Market Stability from Beliefs** This result highlights a benefit of economic models, especially those with global solutions — the model can be used to compare outcomes from different data generating processes (DEE vs. REE). For the same reason, it is difficult to provide direct empirical evidence on financial-market stability from beliefs. Nonetheless, it is still possible to test the mechanism underlying this result and the empirical evidence in [Bordalo et al. \(2018a\)](#) provides support for this mechanism. BGS analyze professional forecasts of the Baa-Treasury credit spread and conclude that the credit spread mean-reverts quicker than forecasters expect. Forecasters are over-pessimistic about future credit conditions when spreads are wide, and over-optimistic about future credit conditions when spreads are narrow. Since credit spreads are typically wide when the financial sector’s risk-bearing capacity is impaired ([Gilchrist and Zakrajšek, 2012](#)), the BGS spread evidence is consistent with the model’s prediction that recovery from financial distress is typically faster than diagnostically expected. Similarly, [Pflueger et al. \(2018\)](#) document that market risk mean-reverts faster than analyst forecasts, options prices, and loan officers expect.

## 6 Sentiment and the 2007-2008 Financial Crisis

The results in Section 5.1 demonstrate that contemporaneous measures of financial sector distress are insufficient for quantifying systemic risk. The feedback from behavioral frictions to financial frictions means that sentiment plays a critical role in regulating whether or not initial distress devolves into a financial crisis.

**Measuring Sentiment in the Data** A downside of the definition of sentiment parameter  $\mathcal{I}_t \equiv \int_0^t e^{-\kappa(t-s)} \sigma dZ_s$  is that it is based on objective shocks to economic growth, which are difficult to measure directly. While it is true under rational expectations that objective shocks to economic growth equal the realized rate of growth minus the expected rate of growth, this simple property does not hold when expectations are biased.

Proposition 2 below provides a solution:  $\mathcal{I}_t$  can be rewritten in terms of forecast errors.

**Proposition 2.** *Let  $\sigma \widehat{dZ}_t = \frac{dY_t}{Y_t} - \widehat{\mathbb{E}}_t \frac{dY_t}{Y_t} = -\theta \mathcal{I}_t dt + \sigma dZ_t$  denote the economic growth forecast error at time  $t$ . Sentiment  $\mathcal{I}_t$  can be rewritten in terms of forecast errors as follows:*

$$\mathcal{I}_t = \int_0^t e^{(-\kappa+\theta)(t-s)} \sigma \widehat{dZ}_s. \quad (26)$$

*Proof.* See Appendix C. □

Equation (26) is very similar to equation (3), with the difference being that forecast errors are discounted at rate  $\kappa - \theta$  whereas objective shocks in equation (3) are discounted at rate  $\kappa$ .

A feature of equation (26) is that it requires minimal information to calculate. All that is required is the realized rate of economic growth and the forecasted rate of economic growth. One needs no information about the underlying data generating process for the economy, nor the data generating process that agents perceive that the economy follows.

Figure 8 uses the Survey of Professional Forecasters (SPF) to calculate sentiment  $\mathcal{I}_t$  from 1970 through 2018. Because SPF forecasts are collected at a quarterly frequency, Figure 8 is calculated using a discrete-time analogue of equation (26). Additionally, because I do not have an infinite history of past forecast data, I assume that sentiment equals zero in January

1970. Full details are provided in Appendix F.1.

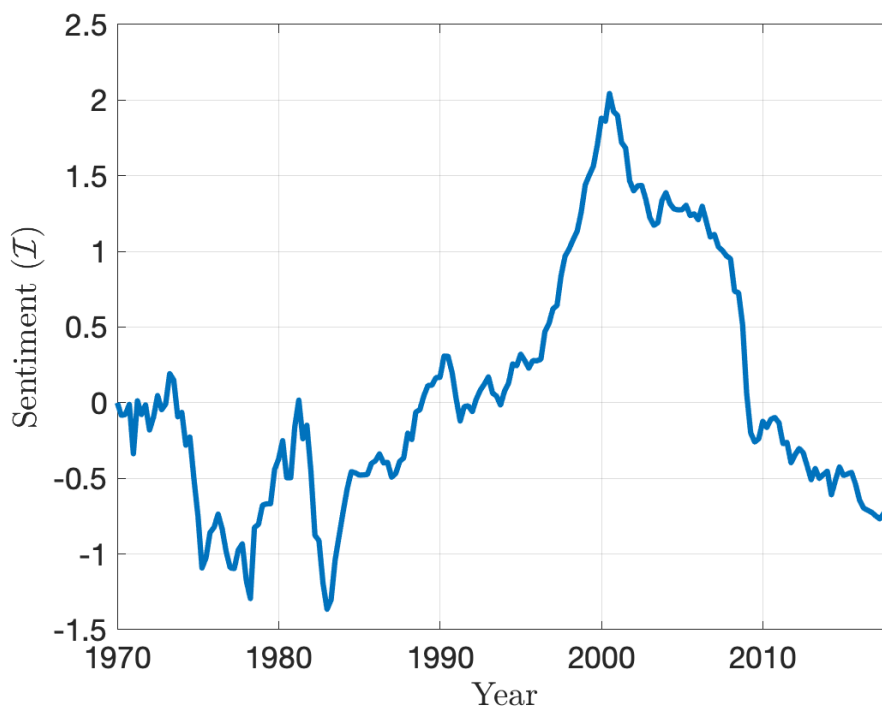


Figure 8: **SPF-measured sentiment.** This figure uses the median SPF forecast error to measure sentiment  $\mathcal{I}_t$  from 1970 through 2018 under the model’s baseline calibration. Sentiment is reported in standard deviation units.

The empirical measure of sentiment in Figure 8 captures what [Kindleberger \(1978\)](#) refers to as “displacement,” namely that financial crises are preceded by large positive shocks to economic fundamentals.<sup>42</sup> Sentiment builds rapidly during the 1990s economic boom, a period characterized by rapid technological innovation and strong economic growth. The bursting of the dot-com bubble and subsequent recession in the early 2000s causes a dip in sentiment, and sentiment continues to unwind slowly throughout the 2000s.<sup>43</sup> Nonetheless, slow-moving sentiment remains elevated until the financial crisis. Linking measured sen-

<sup>42</sup>[Cao and L’Huillier \(2018\)](#) make the observation that the three deepest recessions in developed countries – the Great Recession, the Great Depression, and the Japanese Slump of the 1990s – all occurred approximately 10 years after periods of rapid technological innovation. A similar argument is forwarded by [Gorton and Ordonez \(2019\)](#), who find that credit booms start with a positive shock to productivity, persist for approximately ten years, and end in a bust when followed by a series of negative productivity shocks.

<sup>43</sup>The bursting of the dot-com bubble did not cause a widespread financial crisis even though sentiment was elevated before its burst. This is not inconsistent with the model – especially after such a strong period of economic growth – as elevated sentiment is only predictive of crises when the financial sector is distressed. A similar argument is developed in Section 8.



timent to the crisis hitting probabilities plotted in Figure 4, we see that the 1990s boom fostered the elevated sentiment which then caused a neglect of financial sector vulnerability leading up to the financial crisis.

**Simulating the Financial Crisis** To sharpen this story, the next step is to examine the implications of elevated sentiment within the model. Given a measure of  $\mathcal{I}_t$ , capital quality shocks can be backed-out of forecast errors:  $\sigma dZ_t = \sigma d\widehat{Z}_t + \theta \mathcal{I}_t dt$ . This means that the model can be simulated using the shocks implied by SPF forecast errors.

To conduct this experiment, I use the SPF-implied capital quality shocks to simulate the model at a quarterly frequency from 2003Q1 through 2018Q4. I impose the initial condition that capital capacity  $e_t = e_{distress}$  in 2003Q1, an assumption justified by the elevated credit spreads observed during and after the 2002 stock market downturn. The reader should note that the need for an initial condition indicates a failure of the model. If I start the simulation prior to the mid-90s, the sequence of positive shocks that occurs throughout the remainder of the decade places the financial sector too far above the crisis region. This is a single-shock model, so it is perhaps not surprising that the model cannot fully account for macroeconomic and financial-market trends over the past fifty years.

This paper’s procedure of simulating the model using SPF-implied shocks differs from the standard approach in which the modeler chooses the path of shocks that best aligns their model with the data. Under both rational and diagnostic expectations, forecast errors place strong restrictions on what can be considered a shock. The simulation in this paper takes seriously the restrictions that forecast errors provide.<sup>44</sup>

The lefthand panel of Figure 9 plots the time path of capital capacity  $e_t$  in the DEE and the REE, using the same sequence of SPF-implied shocks. The divergence of the  $e_t$  profiles from 2003 to 2008 shows the impact of elevated sentiment in preventing the financial sector from rebuilding its balance sheet during the mid-2000s. The path of SPF-implied shocks over this period is mildly positive, so capital capacity quickly recovers in the REE. Alternatively, elevated sentiment in the DEE means that balance sheet vulnerability persists throughout the mid-2000s. This leaves intermediaries exposed to the negative shocks that hit during the

---

<sup>44</sup>This approach conforms to an emerging theme in expectations research: expectations data provides an important means for disciplining models (Manski, 2004).

financial crisis.

The righthand panel of Figure 9 illustrates the model’s ability to replicate empirical risk premia. This panel plots the realized risk premium earned by intermediaries in the DEE (red, left axis) and REE (dashed black, left axis), as well as the Baa – 10 Year Treasury spread (blue, right axis). Figure 9 reports the unlevered intermediary risk premium to control for crisis-time variation in leverage. Prior to the crisis, the risk premium in the DEE is excessively low due to elevated sentiment. This is consistent with the narrow credit spreads observed prior to the crisis, which many argue was due to neglected default risk (e.g., Greenwood and Hanson, 2013). Once the crisis hits and intermediaries’ constraint binds, the risk premium in the DEE spikes. Finally, the realized risk premium in the DEE is persistently higher following the crisis, because the crisis de-biases expectations. This pattern also appears in the Baa – 10 Year Treasury spread. Alternatively, the REE exhibits almost no variation in risk premia over the simulated period.<sup>45</sup>

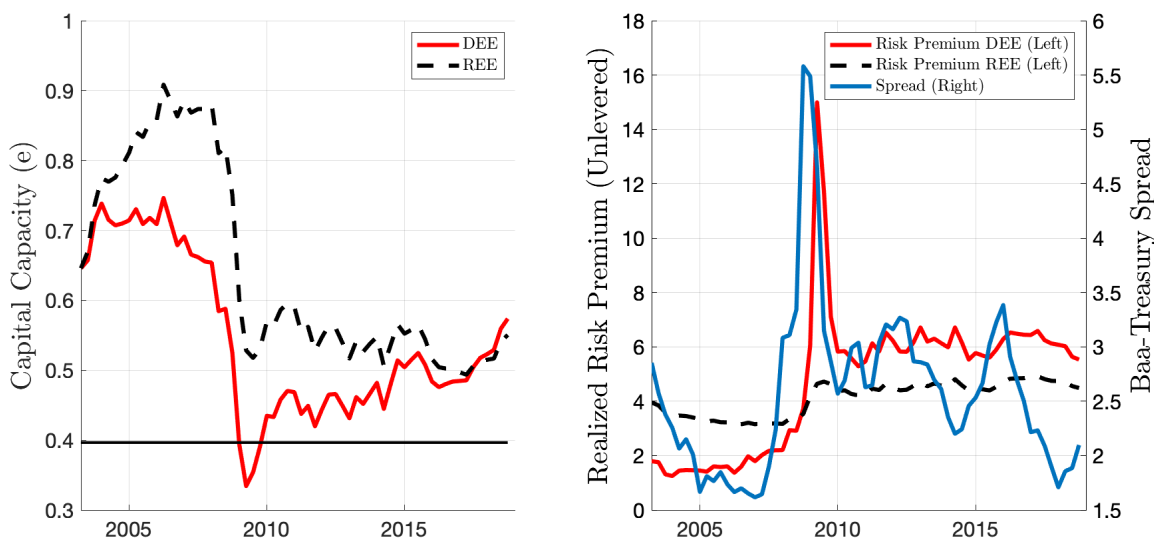


Figure 9: **Simulated Financial Crisis of 2007-2008.** The measure of sentiment calculated in Figure 8 is used to determine the capital quality shocks implied by SPF forecast errors. This sequence of shocks is fed into the model from 2003Q1 through 2018Q4. The lefthand panel plots the path of capital capacity  $e_t$  that results in both the DEE and the REE. The righthand panel plots the model-implied risk premium earned by the financial sector in the DEE and REE (left axis) as well as the Baa – 10 Year Treasury spread (right axis).

<sup>45</sup>Appendix Figure 12 compares the simulated time path of  $e_t$  to the Intermediary Capital Ratio measure constructed in He et al. (2017). Again, the DEE provides a better fit.

## 7 Diagnostic Expectations in Continuous Time

I now provide a microfoundation for the reduced-form expectations process outlined in Section 2.2. The goal for the model of diagnostic expectations specified here is to be a portable extension of existing models [“PEEMish”](Rabin, 2013). Put differently, the expectations model is designed such that rational models can be augmented with diagnostic expectations using a single additional state variable.

**Step 1: Defining the Background Context** Following the terminology of Bordalo et al. (2018a), the first step is to define the “background context” for capital. The background context is the dynamic reference class against which diagnostic expectations are formed. It should be thought of as a counterfactual level of the log capital stock.

The background context reflects the absence of recent information. In equation (3),  $\mathcal{I}_t \equiv \int_0^t e^{-\kappa(t-s)} \sigma dZ_s$  was introduced as an information measure. This leads to the following definition of the background context.

**Definition 3.** Let  $G_t^-$  denote the time-dependent background context of log capital  $k_t$ .  $G_t^-$  is defined as follows:

$$G_t^- = k_t - \mathcal{I}_t.$$

As an Ornstein-Uhlenbeck process,  $\mathcal{I}_t$  has an unconditional mean of zero. Additionally,  $\mathbb{E}_t[\lim_{\tau \rightarrow \infty} (k_{t+\tau} - G_{t+\tau}^-)] = 0$ , meaning that differences between  $k_t$  and  $G_t^-$  are not expected to persist permanently.

Diagnostic expectations are applied to the log of capital for two reasons. Psychologically, it is consistent with Weber’s Law that shocks are perceived as percentage changes rather than level changes. Mathematically, working with log capital ensures that  $\mathcal{I}_t$  is stationary because the diffusion coefficient for log capital is constant.

**Step 2: Modeling Expectations Given the Background Context** Now that I’ve defined the background context  $G_t^-$ , the next step is to specify how agents form expectations. Because time is continuous, I need to specify diagnostic expectations over all future periods

$t + \tau$ , for  $\tau > 0$ . The current period is  $t$ . Let  $h(k_{t+\tau}|k_t, e_t, \mathcal{I}_t)$  denote the true distribution of log capital at time  $t + \tau$  conditional on current state variables. Let  $h(k_{t+\tau}|G_t^-, e_t, \mathcal{I}_t)$  denote the true distribution of log capital at time  $t + \tau$  conditional on current state variables  $e_t$  and  $\mathcal{I}_t$ , but now using counterfactual log capital level  $G_t^-$ .

Let  $k'_{t+\tau}$  denote one possible realization of log capital at time  $t + \tau$ . Following BGS and [Gennaioli and Shleifer \(2010\)](#), the “representativeness” of future state  $k'_{t+\tau}$  is given by the following likelihood ratio:

$$\frac{h(k'_{t+\tau}|k_t, e_t, \mathcal{I}_t)}{h(k'_{t+\tau}|G_t^-, e_t, \mathcal{I}_t)}. \quad (27)$$

The most representative states are the ones exhibiting the largest increase in likelihood based on recent information.

The representativeness heuristic biases expectations because representative states are easier to recall. However, one difficulty with equation (27) is that little is known about distributions  $h(k_{t+\tau}|k_t, e_t, \mathcal{I}_t)$  and  $h(k_{t+\tau}|G_t^-, e_t, \mathcal{I}_t)$  because  $k_t$  is an endogenous process.<sup>46</sup> This difficulty can be overcome by using an instantaneous prediction horizon of  $\tau = dt$ . Specifically, because  $k_t$  is an Itô Process it is instantaneously Gaussian. Taking  $\tau \rightarrow dt$ ,  $h(k'_{t+dt}|k_t, e_t, \mathcal{I}_t) \sim \mathcal{N}\left(k_t + \left[i(e_t, \mathcal{I}_t) - \delta - \frac{\sigma^2}{2}\right] dt, \sigma^2 dt\right)$  and  $h(k'_{t+dt}|G_t^-, e_t, \mathcal{I}_t) \sim \mathcal{N}\left(G_t^- + \left[i(e_t, \mathcal{I}_t) - \delta - \frac{\sigma^2}{2}\right] dt, \sigma^2 dt\right)$ . For this reason I now define diagnostic expectations over prediction horizon  $\tau = dt$ . The prediction horizon will be extended iteratively in Step 3.

Diagnostic expectations overweight states of capital that are representative of recent news. This is formalized by assuming that agents evaluate future levels of log capital according to the distorted density

$$h_t^\theta(k'_{t+dt}|k_t, e_t, \mathcal{I}_t) = h(k'_{t+dt}|k_t, e_t, \mathcal{I}_t) \cdot \left[ \frac{h(k'_{t+dt}|k_t, e_t, \mathcal{I}_t)}{h(k'_{t+dt}|G_t^-, e_t, \mathcal{I}_t)} \right]^{\theta dt} \frac{1}{Z}. \quad (28)$$

Equation (28) modifies a similar formula in BGS, with the key adjustment being that equation (28) defines expectations at  $t + dt$  while the discrete-time formulation of BGS defines expectations at  $t + 1$ . In equation (28), the true conditional probability  $h(k'_{t+dt}|k_t, e_t, \mathcal{I}_t)$  is distorted by the representativeness term in brackets.

---

<sup>46</sup>This is in contrast to BGS, where expectations are only formed over exogenous AR(N) processes.

The extent to which representativeness distorts expectations is governed by parameter  $\theta$ .  $\theta$  is scaled by the prediction horizon  $dt$  because representativeness should impose only an infinitesimal distortion on the perceived distribution of  $k_{t+dt}$  over such a short horizon. The agent knows the current state  $k_t$ , and because  $k_t$  evolves continuously the agent also knows that  $k_{t+dt}$  must be “very close” to  $k_t$ .

Using equation (28), the following proposition characterizes the perceived evolution of capital:

**Proposition 3.** *A diagnostic agent perceives that capital evolves according to*

$$\frac{d\widehat{K}_t}{K_t} = (i(e_t, \mathcal{I}_t) - \delta)dt + \theta \mathcal{I}_t dt + \sigma dZ_t. \quad (29)$$

*Proof.* See Appendix C. □

Proposition 3 shows that at a prediction horizon of  $\tau = dt$ , judging by representativeness biases the perceived drift of  $K_t$ .

### Step 3: The Evolution of Beliefs

In Step 1 I defined the background context  $G_t^-$  and in Step 2 I specified how an agent with diagnostic expectations predicts  $\widehat{dK}_t$ . The final step is to model both the perceived and realized dynamics of expectations over longer horizons. Because capital is endogenous, one only knows the instantaneous distribution of  $k_t$  and therefore future expectations are defined iteratively. In particular, by repeatedly applying the instantaneous Gaussian properties of  $k_t$  I can iteratively define expectations of the economy at  $t + dt$ , then  $t + 2dt$ , then  $t + 3dt$ , etc. This iterative procedure imposes that the law of iterated expectations holds with respect to distorted expectations, consistent with the BGS model.

Diagnostic agents form expectations by drawing from their memory database to simulate the economy forward state-by-state. As the diagnostic agent simulates the economy forward from time  $t$ , the internal representativeness parameter at simulated future time  $t + \tau$  is given

by:

$$\begin{aligned}\mathcal{I}_{t+\tau}^S &\equiv \int_0^t e^{-\kappa(t+\tau-s)} \sigma dZ_s, \text{ or equivalently} \\ &= e^{-\kappa\tau} \mathcal{I}_t.\end{aligned}\tag{30}$$

The superscript  $S$  in equation (30) is used to signify that  $\mathcal{I}_{t+\tau}^S$  is the agent's unconscious internal representativeness state as the agent simulates forward to time  $t + \tau$ . Information that was representative at time  $t$  slowly fades as the perceived model is simulated forward in time. Since no new information is hitting the system as the agent simulates the model forward, no new shocks enter  $\mathcal{I}_{t+\tau}^S$ .

Let  $k'_{t+\tau}, e'_{t+\tau}, \mathcal{I}_{t+\tau}^S$  denote one possible realization of state variables at time  $t + \tau$ . Using equation (30), the simulated background context at  $t + \tau$  can now be defined in an analogous fashion to Definition 3.

**Definition 4.** Let  $k'_{t+\tau}$  denote some simulated level of log capital at future time  $t + \tau$ . The simulated background context at time  $t + \tau$  is defined as follows:

$$G_{t+\tau}^{S,-} = k'_{t+\tau} - \mathcal{I}_{t+\tau}^S.$$

As above, the simulated future background context is defined to reflect the absence of diagnostic information.

Again proceeding in an analogous fashion to Step 2, at time  $t + \tau$  the agent iteratively forms expectations about  $t + \tau + dt$  according to:

$$h_t^\theta(k'_{t+\tau+dt} | k'_{t+\tau}, e'_{t+\tau}, \mathcal{I}_{t+\tau}^S) = h(k'_{t+\tau+dt} | k'_{t+\tau}, e'_{t+\tau}, \mathcal{I}_{t+\tau}^S) \cdot \left[ \frac{h(k'_{t+\tau+dt} | k'_{t+\tau}, e'_{t+\tau}, \mathcal{I}_{t+\tau}^S)}{h(k'_{t+dt} | G_{t+\tau}^{S,-}, e'_{t+\tau}, \mathcal{I}_{t+\tau}^S)} \right]^{\theta dt} \frac{1}{Z}.\tag{31}$$

As with Proposition 3, it follows that the diagnostic agent perceives that capital evolves

according to:

$$\frac{\widehat{dK'_{t+\tau}}}{K'_{t+\tau}} = (i(e'_{t+\tau}, \mathcal{I}_{t+\tau}^S) - \delta)dt + \theta \mathcal{I}_{t+\tau}^S dt + \sigma dZ_{t+\tau}. \quad (32)$$

Future expectations in equation (32) should be contrasted with those of a rational agent who correctly believes that, given  $i_{t+\tau}$ , the drift of  $k'_{t+\tau}$  will be  $i_{t+\tau} - \delta$ . Equation (32) specifies that the effect of diagnostic expectations on the perceived drift of capital at time  $t + \tau$  fades as the agent simulates the evolution of the economy further into the future ( $\tau \rightarrow \infty$ ). Diagnostic expectations capture the overweighting of states that are representative of current economic conditions. As the agent looks further temporally ahead, all states become less representative of economic conditions at time  $t$  so the bias caused by representativeness fades.

It is important to emphasize that equation (32) only stipulates that the diagnostic agent's perception of drift converges to rationality as  $\tau \rightarrow \infty$ . Because the drift has a cumulative effect on the level of  $k_t$ , the diagnostic prediction of the level of capital can diverge increasingly from the rational prediction of  $k_{t+\tau}$  as  $\tau$  increases (e.g., Figure 1).

**Summary** This completes the microfoundation of the reduced-form beliefs process specified in Section 2.2. Extensions are given in Appendix F.5. Appendix F.6 discusses how the discrete-time analogue of this paper's expectations model nests the original BGS model.

To summarize, expectations of the endogenous capital process are formed iteratively in order to make repeated use of the instantaneous Gaussian properties of  $dk_{t+\tau}$ . Step 2 defines how  $\mathcal{I}_t$  affects the expected evolution of the economy from  $t$  to  $t+dt$ . Step 3 then defines how  $\mathcal{I}_t^S$  evolves as expectations are simulated forward. In detail, Step 2 takes  $k_t, e_t, \mathcal{I}_t$  as given and provides a perceived mapping into  $\widehat{k}_{t+dt}$  and  $\widehat{e}_{t+dt}$  given shock  $dZ_t$ . The hat-notation denotes that agents may not properly understand the evolution of these state variables. Step 3 takes  $\mathcal{I}_t$  as given and provides  $\mathcal{I}_{t+dt}^S$ . Then, we can again apply Step 2 (now taking  $\widehat{k}_{t+dt}, \widehat{e}_{t+dt}$ , and  $\mathcal{I}_{t+dt}^S$  as given) to calculate  $\widehat{k}_{t+2dt}$  and  $\widehat{e}_{t+2dt}$  given shocks  $dZ_t$  and  $dZ_{t+dt}$ . Applying Step 3 again gives  $\mathcal{I}_{t+2dt}^S$ . This process is repeated to generate expectations at time  $t + \tau$ , for all  $\tau > 0$ .

I end by discussing why this model of diagnostic expectations can serve as a portable extension of existing rational models. First, equations (29) and (32) illustrate that state variable  $\mathcal{I}_t$  alone is sufficient to characterize the state of expectations relative to rationality. Specifically, for any  $\tau \geq 0$  diagnostic expectations distort the perception of drift by  $\theta \mathcal{I}_{t+\tau}^S$ . Second, the evolution of  $\mathcal{I}_t$  is self-contained.  $\mathcal{I}_t$  can be expressed in differential form as  $d\mathcal{I}_t = -\kappa \mathcal{I}_t dt + \sigma dZ_t$ . Thus, state variable  $\mathcal{I}_t$  plus the shock  $\sigma dZ_t$  are sufficient to calculate  $d\mathcal{I}_t$ . It is these two attributes that make this formulation of diagnostic expectations portable:  $\mathcal{I}_t$  alone characterizes expectations relative to rationality, and  $\mathcal{I}_t$  is sufficient for its own evolution.

## 8 Extension: Sentiment Globally and Bubble-Pricking

Elevated sentiment sets the stage for a future financial crisis while driving a wedge between the true and perceived probability of a crisis occurring. Here I leverage the model's global solution to examine this wedge over the entire state space.

Let  $HP_5(e_t, \mathcal{I}_t)$  denote the true 5-year crisis hitting probability conditional on  $e_t$  and  $\mathcal{I}_t$ . Let  $\widehat{HP}_5(e_t, \mathcal{I}_t)$  denote the perceived crisis hitting probability. Define

$$CEW_5(e_t, \mathcal{I}_t) \equiv HP_5(e_t, \mathcal{I}_t) - \widehat{HP}_5(e_t, \mathcal{I}_t),$$

where CEW stands for "Crisis Expectations Wedge." A positive CEW indicates neglected crash risk, and a negative CEW indicates that crises are overly representative.

Figure 10 plots  $CEW_5(e_t, \mathcal{I}_t)$  over the entire state space of  $e_t$ , conditional on  $\mathcal{I}_t \in \{-3SD, -2SD, \dots, +2SD, +3SD\}$ . The figure shows that CEW depends not only on sentiment, but also on the initial fragility of the financial sector. In particular, the sensitivity of CEW to sentiment is non-monotonic in  $e_t$ .

When  $e_t$  is high and the intermediary sector is well-capitalized,  $CEW_5 \approx 0$  and CEW is insensitive to  $\mathcal{I}_t$ . Though sentiment still affects the future dynamics of  $e_t$ , the intermediary sector is far enough from the constraint that sentiment will unwind before the crisis region is reached. CEW becomes much more sensitive to sentiment as  $e_t$  moves closer to the crisis



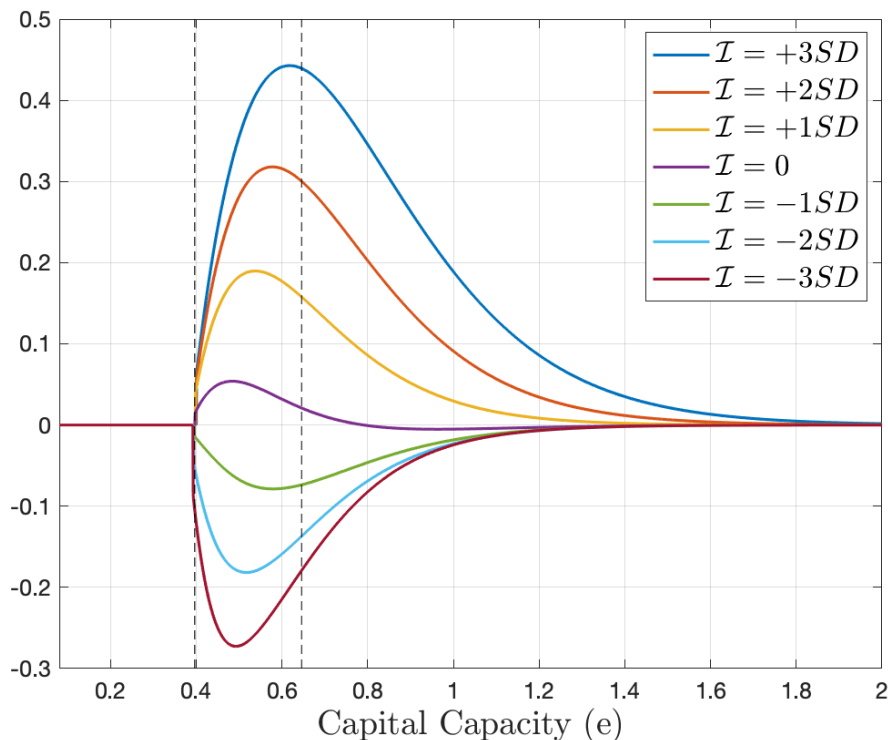


Figure 10: **5-year Crisis Expectations Wedge**. This figure plots the true 5-year crisis hitting probability minus the perceived 5-year crisis hitting probability. Positive values indicate neglected crash risk. The dashed vertical line at  $e \approx 0.4$  marks the boundary of the crisis region, and the dashed vertical line at  $e \approx 0.65$  marks  $e_{distress}$ . The horizontal axis lists initial capital capacity. Different curves correspond to different levels of initial sentiment.

region. Once the intermediary sector starts to show distress, sentiment becomes a critical factor in determining whether or not a financial crisis materializes out of initial vulnerability. However, as  $e_t$  moves even closer to the crisis region, CEW again becomes insensitive to  $\mathcal{I}_t$ . Near the crisis region, the economy is “one shock away” from a financial crisis regardless of the level of sentiment. Finally, when the economy is actually in the crisis region then  $HP_5 = \widehat{HP}_5 = 1$  and  $CEW_5 = 0$ .

**Empirical Evidence on Bursting Bubbles and Systemic Risk** There is an empirical literature characterizing when the bursting of asset price bubbles is and is not predictive of broader systemic risk. For example, the bursting of the dot-com bubble had a relatively muted impact on the real economy, whereas the bursting of the housing bubble resulted in the most severe recession since the Great Depression. [Jordà et al. \(2015\)](#) demonstrate

that the bursting of an asset price bubble is predictive of a deep recession when the bubble coincides with a credit expansion. [Brunnermeier and Schnabel \(2016\)](#) study 400 years of asset price bubbles and bursts, concluding that bursts are more severe when preceded by a lending boom. [Brunnermeier et al. \(2017\)](#) estimate that the contribution of asset bubbles to future systemic risk depends on the vulnerability of the financial sector at the onset of the bubble. This evidence is consistent with the model’s prediction that elevated sentiment is predictive of financial crises only when the intermediary sector is fragile.

**Bubble-Pricking** An important policy debate asks how financial regulators should optimally respond to asset bubbles. My model includes neither monetary policy nor macro-prudential regulation, and therefore one should be cautious in interpreting the following discussion. Nonetheless, [Figure 10](#) contributes to this debate by characterizing where bubble-pricking can be most effective. In particular, the figure illustrates that there are three relevant regions to consider. First, when  $e_t$  is very high and the financial sector is far from its constraint, sentiment-driven asset price booms are not predictive of future systemic risk. In this region, the intermediary sector is well-capitalized and will be able to remain far from its constraint as sentiment unwinds. Second, there is an intermediate region where CEW is very sensitive to sentiment. It is this intermediate region where policymakers should likely focus, as the largest reductions in crisis probabilities can be obtained by unwinding sentiment in this region. Third, for  $e_t$  near the crisis region it may be too late for policymakers to successfully intervene in order to prevent a crisis. When the intermediary sector is “one shock away” from a financial crisis, it may be best for policymakers to avoid being the ones who provide that final shock.

## 9 Conclusion

The 2007-2008 Financial Crisis underscores the importance of studying financial frictions jointly with the non-rational beliefs that can trigger them. This paper develops a general equilibrium macroeconomic model that combines frictions in financial intermediation with diagnostic expectations. This allows the model to examine how the interplay between be-

havioral frictions and financial frictions drives asset pricing and macroeconomic dynamics. When the financial sector is distressed, elevated sentiment amplifies systemic risk and sets the stage for financial crises. Diagnostic expectations generate endogenous boom-bust patterns in investment and output growth by either firing off or reloading the capital capacity of intermediaries. Even with these boom-bust business cycle dynamics, the model also predicts that diagnostic expectations inhibit financial crises. Sentiment is typically depressed when the financial sector is fragile, producing a sentiment-driven recession which allows the intermediary sector to reload its capital capacity before a crisis erupts.

## References

- Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll**, “Income and wealth distribution in macroeconomics: A continuous-time approach,” 2017.
- Adam, Klaus, Dmitry Matveev, and Stefan Nagel**, “Do survey expectations of stock returns reflect risk-adjustments?,” 2018.
- Adrian, Tobias and Markus K Brunnermeier**, “CoVaR,” *American Economic Review*, 2016, *106* (7), 1705–41.
- **and Nina Boyarchenko**, “Intermediary Leverage Cycles and Financial Stability,” *FRB of New York Staff Report*, 2012, (567).
- Aghion, Philippe and Peter W Howitt**, *The economics of growth*, MIT press, 2008.
- Alili, Larbi, Pierre Patie, and Jesper Lund Pedersen**, “Representations of the first hitting time density of an Ornstein-Uhlenbeck process,” *Stochastic Models*, 2005, *21* (4), 967–980.
- Barberis, Nicholas**, “Psychology-Based Models of Asset Prices and Trading Volume,” *Handbook of Behavioral Economics-Foundations and Applications 1*, 2018, p. 79.

- , **Robin Greenwood, Lawrence Jin, and Andrei Shleifer**, “X-CAPM: An extrapolative capital asset pricing model,” *Journal of financial economics*, 2015, 115 (1), 1–24.
- Baron, Matthew and Wei Xiong**, “Credit expansion and neglected crash risk,” *The Quarterly Journal of Economics*, 2017, 132 (2), 713–764.
- , **Emil Verner, and Wei Xiong**, “Salient Crises, Quiet Crises,” *SSRN 3116148*, 2019.
- Beaudry, Paul and Franck Portier**, “An exploration into Pigou’s theory of cycles,” *Journal of monetary Economics*, 2004, 51 (6), 1183–1216.
- Bernanke, Ben S, Mark Gertler, and Simon Gilchrist**, “The financial accelerator in a quantitative business cycle framework,” *Handbook of macroeconomics*, 1999, 1, 1341–1393.
- Bordalo, P, N Gennaioli, R La Porta, and A Shleifer**, “Diagnostic expectations and stock returns,” *Journal of Finance*, 2019.
- Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer**, “Diagnostic expectations and credit cycles,” *The Journal of Finance*, 2018, 73 (1), 199–227.
- , – , – , and **Stephen J. Terry**, “Real Credit Cycles,” *Mimeo*, 2019.
- , – , **Spencer Yongwook Kwon, and Andrei Shleifer**, “Diagnostic bubbles,” 2018.
- , – , **Yueran Ma, and Andrei Shleifer**, “Over-reaction in macroeconomic expectations,” 2018.
- Brunnermeier, Markus and Isabel Schnabel**, “Bubbles and Central Banks: Historical Perspectives,” in “Central Banks at a Crossroads—What Can We Learn from History?,” Cambridge University, 2016, pp. 493–562.
- Brunnermeier, Markus K and Martin Oehmke**, “Bubbles, financial crises, and systemic risk,” in “Handbook of the Economics of Finance,” Vol. 2, Elsevier, 2013, pp. 1221–1288.
- and **Yuliy Sannikov**, “A macroeconomic model with a financial sector,” *American Economic Review*, 2014, 104 (2), 379–421.

– **and** –, “Macro, money, and finance: A continuous-time approach,” in “Handbook of Macroeconomics,” Vol. 2, Elsevier, 2016, pp. 1497–1545.

–, **Simon Rother, and Isabel Schnabel**, “Asset Price Bubbles and Systemic Risk,” 2017.

**Campbell, John Y**, *Financial decisions and markets: a course in asset pricing*, Princeton University Press, 2017.

**Cao, Dan and Jean-Paul L’Huillier**, “Technological revolutions and the three great slumps: A medium-run analysis,” *Journal of Monetary Economics*, 2018, *96*, 93–108.

**Case, Karl E, Robert J Shiller, and Anne Thompson**, “What have they been thinking? Home buyer behavior in hot and cold markets,” 2012.

**Coval, Joshua, Jakub Jurek, and Erik Stafford**, “The economics of structured finance,” *Journal of Economic Perspectives*, 2009, *23* (1), 3–25.

**Duffie, Darrell**, *Dynamic asset pricing theory*, Princeton University Press, 2010.

**Fahlenbrach, Rüdiger, Robert Prilmeier, and René M Stulz**, “Why does fast loan growth predict poor performance for banks?,” *The Review of Financial Studies*, 2017, *31* (3), 1014–1063.

**Farhi, Emmanuel and Iván Werning**, “Monetary policy, bounded rationality, and incomplete markets,” 2017.

**Foote, Christopher L, Kristopher S Gerardi, and Paul S Willen**, “Why did so many people make so many ex post bad decisions? The causes of the foreclosure crisis,” 2012.

**Frankel, Marvin**, “The production function in allocation and growth: a synthesis,” *The American Economic Review*, 1962, *52* (5), 996–1022.

**Fuster, Andreas, Benjamin Hebert, and David Laibson**, “Natural expectations, macroeconomic dynamics, and asset pricing,” *NBER Macroeconomics Annual*, 2012, *26* (1), 1–48.

- , **David Laibson**, and **Brock Mendel**, “Natural expectations and macroeconomic fluctuations,” *Journal of Economic Perspectives*, 2010, *24* (4), 67–84.
- Gabaix, Xavier**, “Behavioral macroeconomics via sparse dynamic programming,” 2016.
- , “A behavioral New Keynesian model,” 2016.
- García-Schmidt, Mariana** and **Michael Woodford**, “Are low interest rates deflationary? A paradox of perfect-foresight analysis,” *American Economic Review*, 2019, *109* (1), 86–120.
- Gennaioli, Nicola** and **Andrei Shleifer**, “What comes to mind,” *The Quarterly journal of economics*, 2010, *125* (4), 1399–1433.
- and – , *A crisis of beliefs: Investor psychology and financial fragility*, Princeton University Press, 2018.
- , – , and **Robert Vishny**, “Neglected risks, financial innovation, and financial fragility,” *Journal of Financial Economics*, 2012, *104* (3), 452–468.
- Gertler, Mark** and **Nobuhiro Kiyotaki**, “Financial intermediation and credit policy in business cycle analysis,” in “Handbook of monetary economics,” Vol. 3, Elsevier, 2010, pp. 547–599.
- , – , and **Andrea Prestipino**, “A macroeconomic model with financial panics,” 2017.
- Gilchrist, Simon** and **Egon Zakrajšek**, “Credit spreads and business cycle fluctuations,” *American Economic Review*, 2012, *102* (4), 1692–1720.
- Gorton, Gary** and **Guillermo Ordonez**, “Good booms, bad booms,” *Journal of the European Economic Association*, 2019.
- Greenwood, Robin** and **Andrei Shleifer**, “Expectations of returns and expected returns,” *The Review of Financial Studies*, 2014, *27* (3), 714–746.
- and **Samuel G Hanson**, “Issuer quality and corporate bond returns,” *The Review of Financial Studies*, 2013, *26* (6), 1483–1525.

- , **Samuel G. Hanson**, and **Lawrence J. Jin**, “Reflexivity in Credit Markets,” *Working Paper*, 2019.
- Gulen, Huseyin**, **Ion Mihai**, and **Stefano Rossi**, “Credit Cycles and Corporate Investment,” *Working Paper*, 2019.
- He, Zhiguo** and **Arvind Krishnamurthy**, “Intermediary asset pricing,” *American Economic Review*, 2013, *103* (2), 732–70.
- and – , “A Macroeconomic Framework for Quantifying Systemic Risk,” *American Economic Journal: Macroeconomics*, 2019.
- , **Bryan Kelly**, and **Asaf Manela**, “Intermediary asset pricing: New evidence from many asset classes,” *Journal of Financial Economics*, 2017, *126* (1), 1–35.
- Jin, Lawrence J**, “A speculative asset pricing model of financial instability,” *Available at SSRN 2524762*, 2015.
- Jordà, Òscar**, **Moritz Schularick**, and **Alan M Taylor**, “Leveraged bubbles,” *Journal of Monetary Economics*, 2015, *76*, S1–S20.
- Kahneman, Daniel** and **Amos Tversky**, “Subjective probability: A judgment of representativeness,” *Cognitive psychology*, 1972, *3* (3), 430–454.
- Karatzas, Ioannis** and **Steven E Shreve**, “Stochastic Differential Equations,” in “Brownian Motion and Stochastic Calculus,” Springer, 1998, pp. 281–398.
- Keynes, JM**, “The General Theory of Employment, Interest and Money,” 1936.
- Kindleberger, Charles P**, *Manias, panics and crashes: a history of financial crises*, Basic Books, 1978.
- Kiyotaki, Nobuhiro** and **John Moore**, “Credit cycles,” *Journal of political economy*, 1997, *105* (2), 211–248.
- Krishnamurthy, Arvind** and **Tyler Muir**, “How Credit Cycles across a Financial Crisis,” *Working Paper*, 2017.

- López-Salido, David, Jeremy C Stein, and Egon Zakrajšek**, “Credit-market sentiment and the business cycle,” *The Quarterly Journal of Economics*, 2017, *132* (3), 1373–1426.
- Maggiore, Matteo**, “Financial intermediation, international risk sharing, and reserve currencies,” *American Economic Review*, 2017, *107* (10), 3038–71.
- Malmendier, Ulrike and Stefan Nagel**, “Depression babies: do macroeconomic experiences affect risk taking?,” *The Quarterly Journal of Economics*, 2011, *126* (1), 373–416.
- Manski, Charles F**, “Measuring expectations,” *Econometrica*, 2004, *72* (5), 1329–1376.
- Mian, Atif, Amir Sufi, and Emil Verner**, “Household debt and business cycles worldwide,” *The Quarterly Journal of Economics*, 2017, *132* (4), 1755–1817.
- Minsky, Hyman P**, “The financial instability hypothesis: An interpretation of Keynes and an alternative to ‘standard’ theory,” *Challenge*, 1977, *20* (1), 20–27.
- Moreira, Alan and Alexi Savov**, “The macroeconomics of shadow banking,” *The Journal of Finance*, 2017, *72* (6), 2381–2432.
- Muir, Tyler**, “Financial crises and risk premia,” *The Quarterly Journal of Economics*, 2017, *132* (2), 765–809.
- Pflueger, Carolin, Emil Siriwardane, and Adi Sunderam**, “A measure of risk appetite for the macroeconomy,” 2018.
- Pigou, Arthur**, “Industrial Fluctuations,” 1926.
- Rabin, Matthew**, “An approach to incorporating psychology into economics,” *American Economic Review*, 2013, *103* (3), 617–22.
- Schularick, Moritz and Alan M Taylor**, “Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008,” *American Economic Review*, 2012, *102* (2), 1029–61.



- Sims, Christopher A**, “Implications of rational inattention,” *Journal of monetary Economics*, 2003, 50 (3), 665–690.
- Tella, Sebastian Di**, “Uncertainty shocks and balance sheet recessions,” *Journal of Political Economy*, 2017, 125 (6), 2038–2081.
- Tversky, Amos and Daniel Kahneman**, “Extensional versus intuitive reasoning: The conjunction fallacy in probability judgment.,” *Psychological review*, 1983, 90 (4), 293.
- Vasicek, Oldrich**, “An equilibrium characterization of the term structure,” *Journal of financial economics*, 1977, 5 (2), 177–188.
- Woodford, Michael**, “Macroeconomic analysis without the rational expectations hypothesis,” *Annu. Rev. Econ.*, 2013, 5 (1), 303–346.

**Appendix** All appendices can be found on the author's webpage:  
<https://scholar.harvard.edu/maxted/publications>