A Theory of Multiplexity: Sustaining Cooperation with Multiple Relations

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NBER OE Working Group
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Introduction

People are embedded in *multiple* social relations

- friend, co-author, business, political, borrow-lending, etc.

They are not isolated, the pattern of existing networks likely to affect the formation of another

When and why do different networks overlap?

Despite a few empirical/case studies, theoretical probing is still in the early stages

Introduction

> *It is striking that 20 years after Fischer's (1982) classic study of networks in North California communities, so few large-scale studies investigate the* ***multiple, overlapping networks*** *of different types of relationships that his research so admirably chronicled.*

–Mcpherson et al. "Birds of a feather: Homophily in social networks" (2001)

Another 20 years passed...

> *There has been relatively little work on multilayer and multiplex networks to date,* $\cdots$*, without a method* $\cdots$*, we are unlikely to recover the true effect of each network on the outcome of interest, possibly leading to incorrect conclusions.*

–Jackson et al. "Why understanding multiplex social network structuring processes will help us better understand the evolution of human behavior" (2019)

This Paper

Studies the interaction of different social relations

We treat **formation of multiple** relationships as strategic decisions

Question: Given the current network, when a new relationship arises, will agents link with a friend, or a stranger?

– What **network patterns** will affect this choice?

We call the tendency to link with friends **multiplexity**

Key Findings

Multiplexity enhances cooperation by reinforcing incentives on **every** existing relationships between friends

Friend vs. stranger tradeoff: multiplexity vs. community enforcement

Inefficient network formation: "multiplexity trap" can occur

Strong incentives to multiplex when network features:

1. low degree dispersion
2. positive assortativity

What relationships to multiplex? Less important ones

Empirical evidence supports theoretical predictions

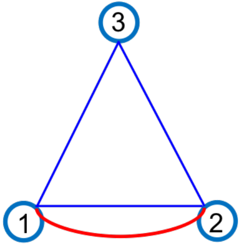
Model: Networks

K relationship networks $G = (G^1, \ldots, G^K)$

- $ij \in G^k$ if agents i and j are linked in relationship k
- undirected: $ij \in G^k$ iff $ji \in G^k$

Example: $G = (G^1, G^2)$

- $G^1 = \{12, 23, 13\}$
- $G^2 = \{12\}$



Relationship - Repeated Prisoner's Dilemma with Variable Stake

Cooperation (stage game) arrives randomly over time on each link/relationship

- Poisson arrival rate λ
- i.i.d. across links/relationships

In each stage game, the pair of agents:

1. simultaneously propose the *stakes* of cooperation (ϕ_i, ϕ_j), $\phi = \min\{\phi_i, \phi_j\}$ is used for the game
2. simultaneously choose *cooperate* or *defect*

	Cooperate	Defect
Cooperate	$\phi,\ \phi$	$-\phi,\ \phi + \phi^2$
Defect	$\phi + \phi^2,\ -\phi$	$0,\ 0$

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Cooperate	$c\phi,\ \phi$	$-\phi,\ \phi+\phi^2$
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Compatibility index c_{ij}^k: vary across pairs / relationships

Equilibrium Conditions

Equilibrium characterized by $\{\phi_{ij}^k\}_{ij \in G^k}$ that satisfy

"No deviation conditions"

$$\phi_{ij}^k + (\phi_{ij}^k)^2 \leq \phi_{ij}^k + \int_0^\infty e^{-rt} \lambda \mathbf{d}t \sum_{j',k'} \phi_{ij'}^{k'} 1_{\{ij' \in G^{k'}\}}$$

- Perfect monitoring
- Deviation punished by "grim trigger" strategy

Maximal Stakes of Cooperation (MSC)

Look for the “maximal equilibrium” among all SPNE

Every link achieves its MSC

- Every agent gets the highest payoff
- Always exists: ϕ's across links are complements

Compare MSC across different network structures

Example 1: Single Link/Relationship

Single relationship, single link

Agents cooperate if future benefit is large enough

$$\cancel{\phi} + \phi^2 \leq \cancel{\phi} + \phi \int_0^\infty e^{-rt} \lambda \mathbf{d}t$$

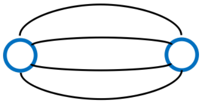
$$\text{i.e. } \phi^2 \leq \phi \frac{\lambda}{r}$$

Therefore, maximal self-enforcing stake of cooperation (**MSC**)

$$\phi^{MSC} = \frac{\lambda}{r}$$

Example 2: Multiplexity

$K = 4$ relationships on one pair



Agents cooperate if

$$\cancel{\phi} + \phi^2 \leq \cancel{\phi} + 4\phi \int_0^\infty e^{-rt} \lambda \, \mathbf{d}t$$

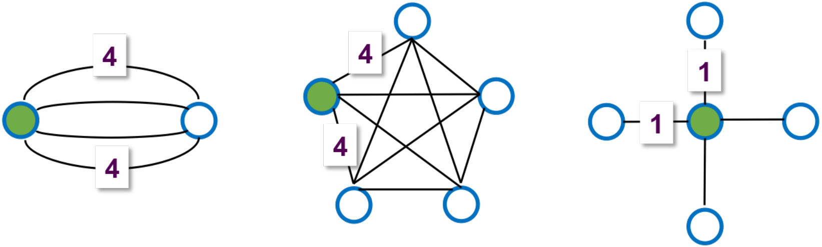
$$\text{i.e. } \phi^2 \leq 4\phi \frac{\lambda}{r}$$

Therefore, maximal self-enforcing stake of cooperation (**MSC**)

$$\phi^{MSC} = 4 \frac{\lambda}{r}$$

Reinforcing Effect: Every relationship benefits from a higher MSC!

Multiplexity vs. Community Enforcement



Multiplexity and community enforcement both enhances cooperation

But the latter relies more on the rest of the network

In the 3rd (star network): $\phi^{MSC} = \mathbf{1}\frac{\lambda}{r}$

Green agent still have 4 links, but her neighbors are not connected, and have less incentives

Endogenous Network Formation

Start with a given network

When there is a new relationship, add it to a friend, or a stranger?

When to multiplex? When not?

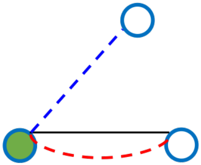
Decision rule

- Add link that maximizes equilibrium payoff
- Myopic regarding further link dynamics

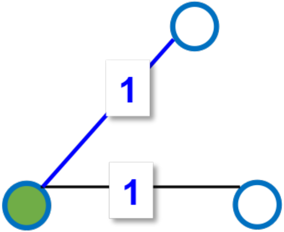
A Simple Example

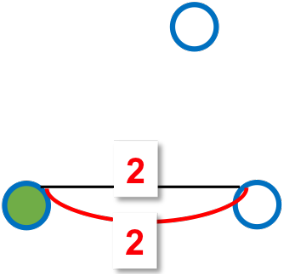
The green node has a new link to add.

Q: Link to a friend, or a stranger?



A Simple Example



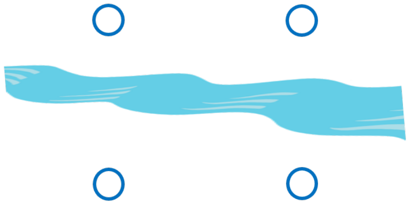


Multiplexity dominates!

Multiplexity can dominate even when it's **more efficient** to link with a stranger

Multiplexity trap

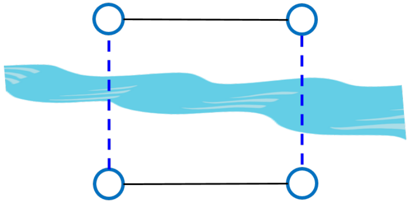
Consider the following example:



Two types of relationships

1. Kid care: easier done within village
2. Trade: more gain across

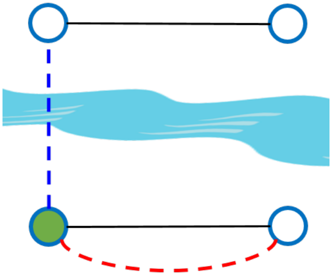
Efficient Network



Efficiency: Kid care (solid) within village, Trade (dashed) across.

Not necessarily the equilibrium network!

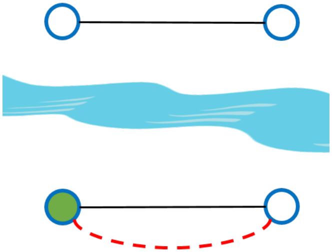
Equilibrium Network



Start with one relationship (e.g. kid care), isolated pairs

Green villager now need to add a Trade link, with whom?

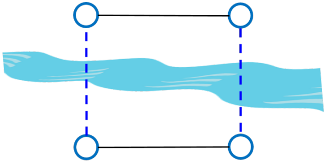
Equilibrium Network



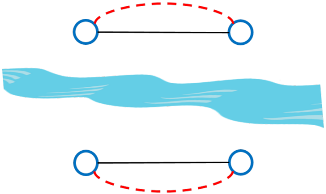
Prefers to multiplex, as long as it's not too efficient to trade across the river!

Efficiency vs. Equilibrium

(Kid care: solid;   Trade: dashed)



(a) **Efficiency**



(b) **Equilibrium**

Agents prefer to multiplex even when it's more efficient to link with a stranger!

Multiplexity Trap: General Statement

Generally, allow links/relationships to vary in compatibilities

- Payoff is $c_{ij}^k \times \phi_{ij}^k$ on relationship (i, j, k)
- When $\max c_{ij}^k / \min c_{ij}^k < \frac{1+\sqrt{5}}{2} \approx 2.618$, multiplexity trap occurs

Proposition (Multiplexity Trap)

Starting from any society G_0 of isolated pairs.

Every agent always strictly prefers to multiplex, i.e., to add new relationships with her neighbor.

So the network will remain as a couple of isolated pairs forever.

"Compatibility misallocation": incentives to multiplex could be so strong that agents fail to link with more compatible partners.

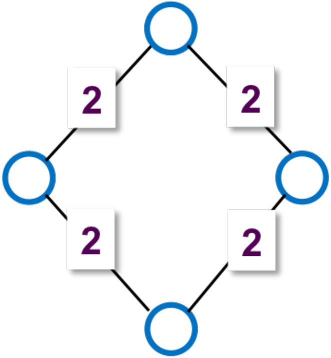
Exploring More Complicated Network Structures

In particular, two global features for any given network

- **Degree dispersion**: Do agents have similar, or very different, degree value?
- **Assortativity**: Do agents link with others who have similar degree value? Or the opposite?

Regular Network (No Degree Dispersion): An Example

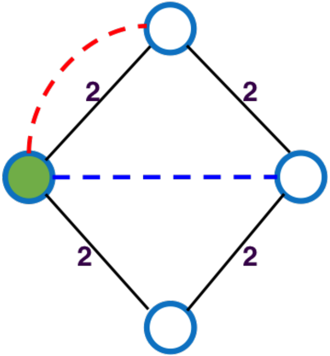
Regular network: everyone has the same degree (2 here)



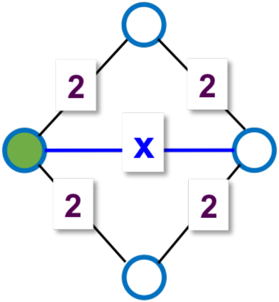
Property: same MSC on every link.

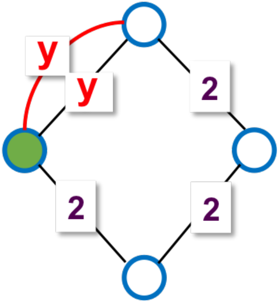
Regular Network (No Degree Dispersion): An Example

Green agent has one link to add. Link with whom?



Regular Network: Multiplexity Dominates





Both x and $y > 2$; but $x < y$

Moreover, 2 links have size y on the right!

The **Multiplex Effect**: more links benefit from the increased MSC

Multiplexity Preferred in Regular Network

Proposition

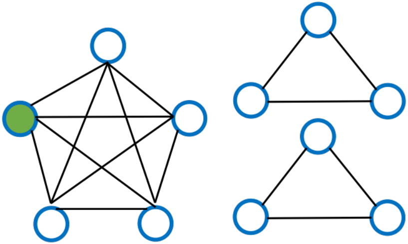
Starting from any regular network G_0. Every agent strictly prefers to multiplex.

Multiplexity dominates in networks with *low degree dispersion*.

- Intuition: agents are "similar" in terms of degrees, hence the tendency to multiplex dominates

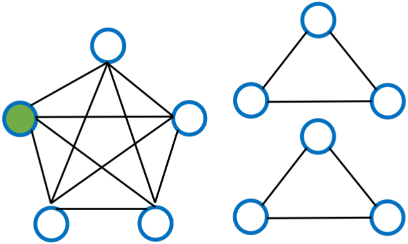
How about networks with large degree dispersion?

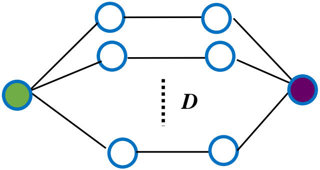
Large Degree Dispersion: Two Cases



- Positive assortativity
- Multiplexity dominates

Large Degree Dispersion: Two Cases





- Positive assortativity
- Multiplexity dominates

- Negative assortativity
- Green prefers to link with Purple ($\forall D > 6$)

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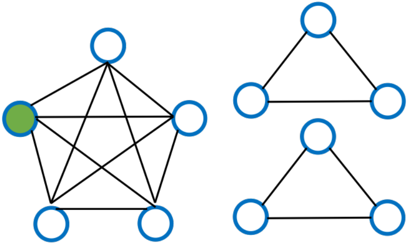
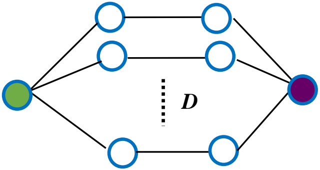




- Positive assortativity
- Multiplexity dominates

- Negative assortativity
- Green prefers to link with Purple ($\forall D > 6$)

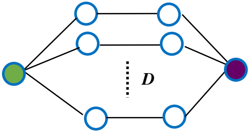
Lesson: To counter the multiplexity tendency, asymmetry in degree between neighbors is key!

Varying Importance of Relationships

Now different relationships could have varying importance

Denoted by c, i.e. payoff from cooperation is $c\phi$

The negative assortative example again



D^*: threshold beyond which linking with stranger is preferred

Proposition

$D^*(c)$ decreases in c. (More willing to link with a stranger with more important relationships)

Lesson: What relationships to multiplex? Less important ones.

Empirical Evidence: Testable Hypothesis from the Model

1. Multiplexity prevails in networks (Hypothesis 1)

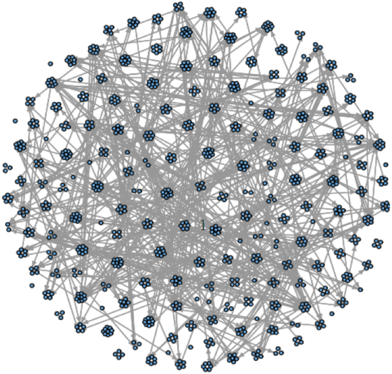
2. Multiplexity more likely to appear in networks that have
   a. low degree dispersion (Hypothesis 2a)
   b. positive assortativity (Hypothesis 2b)

Empirical Evidence: Data

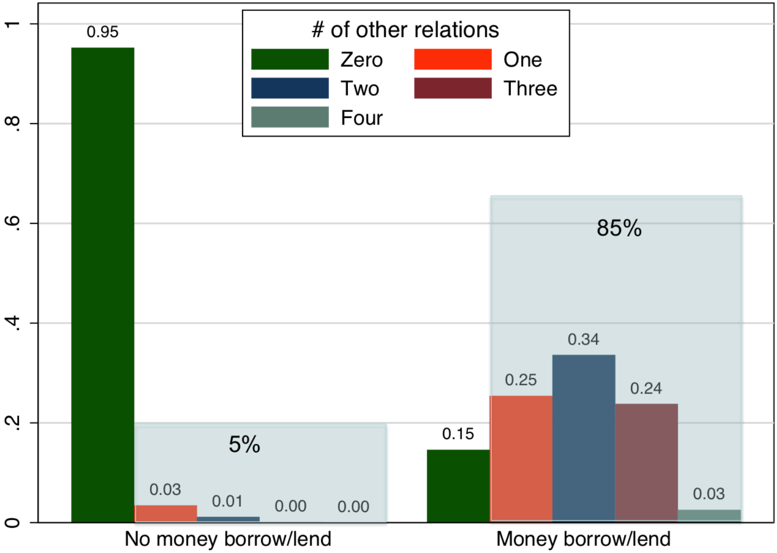
75 rural villages in Karnataka, southern India
Banerjee, Chandrasekhar, Duflo, Jackson (2013)

Several types of relationships:

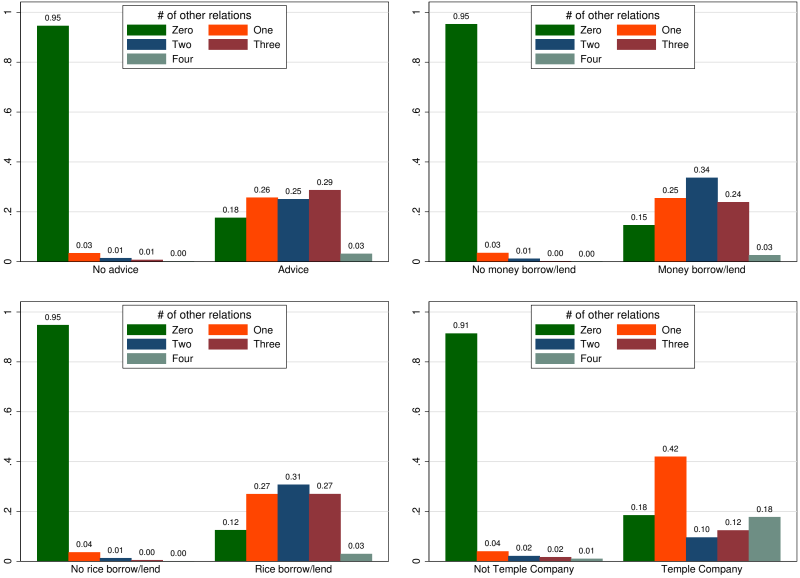
- visit (come and go)
- go to temple together
- seek/provide advice
- borrow/lend money
- borrow/lend kerosene or rice
- ...



Baseline Evidence of Multiplexity: Borrow/Lend Money



Baseline Evidence of Multiplexity: Other Relationships



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1. Multiplexity prevails in networks (Hypothesis 1)

2. Multiplexity is more likely in networks that have
   a. low degree dispersion (Hypothesis 2a)
   b. positive assortativity (Hypothesis 2b)

Empirical Methodology

We view each village as independent observations

Measure "multiplexity" for each village first

Then regress "multiplexity" on network features (e.g. degree dispersion and assortativity)

Empirical Methodology (1): Measuring Multiplexity

For each village v, conduct the following regression

$$Relation_{ij}^{-k,v} = \alpha_0^v + \alpha_1^v Relation_{ij}^{k,v} + \varepsilon_{ij}^v \quad (1)$$

- Regression conducted at household pair ij level.
- Independent variable: whether having relationship k (Yes = 1).
- Dependent variable: whether having any of other relationships (Yes = 1).

Estimate α_1^v for each village v. We use this as a measure for the degree of multiplexity in village v.

Empirical Methodology (2): Main Regression

Then conduct the following regression

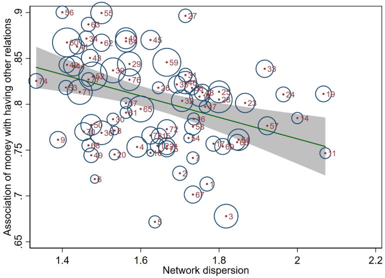
$$Multiplex_v = \beta_0^v + \beta_1 D_v + \varepsilon_v \tag{2}$$

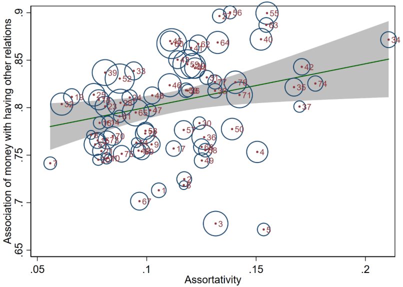
- Dependent variable: multiplexity in village v
  ($\widehat{\alpha}_1^v$ in regression 1)
- Independent variable: *degree dispersion* or *assortativity* in village v

Determinants of Multiplexity

	(1)	(2)	(3)	(4)	(5)	(6)
Relationship k	Money		Rice		Advice	
Panel A: Multiplexity based on all pairs						
Dispersion	**-0.117*** **		-0.0570		**-0.135*** **	
	(0.0353)		(0.0359)		**(0.0435)**	
Assortativity		**0.459** **		**0.383***		0.131
		(0.184)		**(0.206)**		(0.235)
Observations	75	75	75	75	75	75
R-squared	0.142	0.070	0.036	0.052	0.143	0.004

Determinants of Multiplexity





Robustness Check

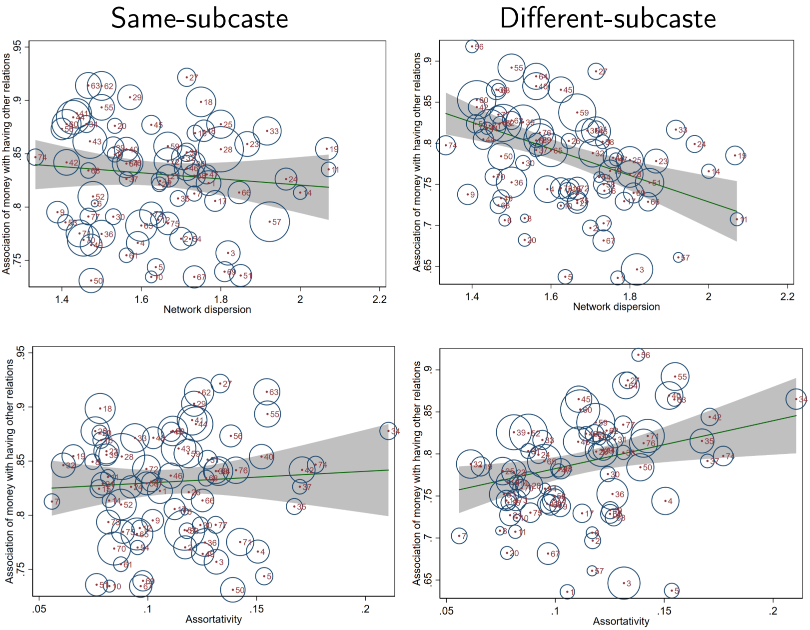
Conduct the regressions for two subsamples: same-subcaste pairs and different-subcaste pairs

- Within subcaste: cooperation mostly driven by unmodelled factors (e.g. religion)
- Across subcastes: incentive issue matters; our theory applies

Prediction: results shall be significant for different-subcaste pairs, but not for same-subcaste pairs.

Robustness Check: same vs. different subcastes

	(1)	(2)	(3)	(4)	(5)	(6)
Relationship k	Money		Rice		Advice	
Panel A: Multiplexity based on different subcaste pairs						
Dispersion	**-0.162*** **		**-0.0628***		**-0.192*** **	
	(0.0420)		**(0.0360)**		**(0.0607)**	
Assortativity		**0.570*** **		**0.445***		0.209
		(0.213)		**(0.231)**		(0.317)
Observations	75	75	75	75	75	75
R-squared	0.191	0.085	0.036	0.065	0.148	0.006
Panel B: Multiplexity based on same subcaste pairs						
Dispersion	-0.0287		-0.0450		-0.0566	
	(0.0371)		(0.0492)		(0.0396)	
Assortativity		0.107		0.264		0.0453
		(0.208)		(0.252)		(0.258)
Observations	75	75	75	75	75	75
R-squared	0.010	0.004	0.017	0.015	0.027	0.000



Same-subcaste
Different-subcaste
Association of money with having other relations
Network dispersion
Association of money with having other relations
Network dispersion
Association of money with having other relations
Assortativity
Association of money with having other relations
Assortativity

Summary

A tractable framework for the formation of multiplex network

Multiplexity vs. community enforcement

Multiplexity trap can occur, leads to (permanently) isolated societies and compatibility-mismatches

Strong incentives to multiplex when network features

1. low degree dispersion
2. positive assortativity

Help us understand why different societies exhibit different patterns of multiplexity

What relationship to multiplex? Less important ones.

Empirical evidence supports theoretical predictions

Interesting Future Work

So far: how multiplexity is affected by the structure of networks.

To be explored: other conditions, such as

- Political/social/economic conditions
- Interaction with formal institution (market, government, etc.)

Understand the great divergence: Why multiplexity is more persistent in China than in, say, the Western world?

Thank you!

Backup Slides

Related Literature

Description of multiplexity/embeddedness in sociology literature

- Fischer 1982, Uzzi 1997, etc.

Multi-market contact in IO literature

- Bernheim, Whinston 1990
- Li, Powell 2018

Correlation among different relations in social networks

- Banerjee, Chandrasekhar, Duflo, Jackson 2018
- Atkisson, Gorski, Jackson, Holyst 2019
- Joshi, Mahmud, Sarangi 2019

Multilayer analysis in complex system literature

- Kivela, Arenas, Barthelemy, Gleeson, Moreno, Porter 2014

Discussion

Multiplexity as increasing the intensity of interaction

Perfect monitoring

Myopic agents

Symmetric vs. asymmetric cooperation

Implications to organization design

With Varying Importances of Relationship