Optimal Monetary Policy in HANK Economies

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Question and framework

- how does imperfect insurance affect **optimal** monetary policy?
- **challenge**: social welfare function aggregates heterogenous (marginal) utilities, each of which is endogenous to policy
- **solution**: CARA-Normal HANK with closed-form expressions for
  - the aggregate dynamics
  - the (time-varying) distribution of agents
  - the social welfare function

Main results

- optimal policy governed by two forces ($\Rightarrow$ **tradeoff**)

1. **price stability**
2. **consumption dispersion**, as affected by
   - cyclicalilty of **income** risk (and cumulated effect of)
   - pass-trough to **consumption** risk (via time-varying MPC)

- the central bank may tolerate transitory departures from price stability in order to limit the rise in consumption dispersion

- breakdown of “divine coincidence”

Related literature

- quantitative analysis of HANK models
  - **HANK**: Kaplan et al. (2018); Auclert et al. (2018); Debortoli-Gali (2018), Hagedorn et al. (2019)...
  - **HANK & SaM**: Gornemann et al. (2016); Ravn-Sterk (2017, 2018); Challe et al. (2017); Den Haan et al. (2019)...
- optimal monetary policy under perfect insurance
  - **RANK**: Clarida et al. (1999); Woodford (2003); Gali (2008)...
  - **TANK**: Bilbiie (2008)...
  - **RANK & SaM**: Thomas (2008); Faia (2009); Blanchard-Gali (2010); Ravenna-Walsh (2011)
- optimal monetary policy under imperfect insurance
  - **HANK & SaM** with 0-liquidity: Challe (2019)
  - **HANK** with >0 liquidity: Nuño-Thomas (2017); Bhandari et al. (2018)
- CARA-Normal imperfect-insurance models
  - **flex-price**: Calvet (2001); Angeletos and Calvet (2005, 2006)
  - **HANK**: Acharya & Dogra (2018)

Households

Objective and constraint

- objective:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\theta)^t u(c_t^i, \xi_t^i - \ell_t^i)$$

where

$$u(c_t^i, \xi_t^i - \ell_t^i) = -\frac{1}{\gamma} e^{-\gamma c_t^i} - \rho e^{-\frac{1}{\rho}(\xi_t^i - \ell_t^i)}$$

and

$$\xi_t^i \sim \mathcal{N}(\bar{\xi}, \sigma_t^2) = \text{time endowment}$$

- budget constraint:

$$a_{t+1}^i = \frac{R_{t+1}}{\theta}(a_t^i + w_t \ell_t^i + T_t - c_t^i)$$

- R_{t+1}/θ = real riskless return on actuarial bonds

Households

Optimality conditions

- assume no aggregate risk ("MIT shock")
- bonds:

$$e^{-\gamma c_t^i} \quad = \quad \beta R_{t+1} \mathbb{E}_t e^{-\gamma c_{t+1}^i}$$

- labor supply:

$$\ell_t^i = \rho \ln w_t - \gamma \rho c_t^i + \xi_t^i$$

Households

Policy functions

- conjecture-verify **linear** consumption function:

$$c_t(x_t^i) = \underset{\substack{\uparrow \\ \text{agg. cons.}}}{c_t} + \underset{\substack{\uparrow \\ \text{MPC}}}{\mu_t} \times \underset{\substack{\uparrow \\ \text{state}}}{x_t^i}$$

where

$$\textbf{state} \quad : \quad x_t^i = a_t^i + w_t(\xi_t^i - \bar{\xi})$$

$$\textbf{MPC} \quad : \quad \mu_t^{-1} = 1 + \gamma\rho w_t + \frac{\theta}{R_{t+1}}\mu_{t+1}^{-1}$$

- x_t^i = quasi cash-on-hand: reflects impact of past asset accumulation (a_t^i) and current labor-endowment shock ($w_t(\xi_t^i - \bar{\xi})$)
- MPC higher when $\{R_{t+j}\}$ high and/or $\{w_{t+j}\}$ low

Households

Policy functions

- other policy funtions:

$$\textbf{labor supply} \quad : \quad \ell_t(x_t^i, \xi_t^i) = \rho \ln w_t - \gamma \rho c_t(x_t^i) + \xi_t^i$$

$$\textbf{savings} \quad : \quad s_t(x_t^i) = (1 - (1 + \gamma \rho w_t) \mu_t) x_t^i$$

$$\textbf{wealth} \quad : \quad a_{t+1}(x_t^i) = \frac{R_{t+1}}{\theta} (1 - (1 + \gamma \rho w_t) \mu_t) x_t^i$$

- in particular, $s_t(x_t^i)$ implies that individual state evolves **linearly**:

$$x_t^i = \frac{\mu_{t-1}}{\mu_t} x_{t-1}^i + w_t(\xi_t^i - \bar{\xi})$$

Aggregation

- let $f_t(x)$ be the cross-sectional distribution of x (determined later)
- goods:
$$\int c_t(x)\, f_t(x)\, \mathrm{d}x = c_t = y_t$$
- labor supply:
$$\int l_t(x)\, f_t(x)\, \mathrm{d}x = n_t = \rho \ln w_t - \gamma \rho c_t + \bar{\xi}$$
- bonds:
$$\int a_{t+1}(x)\, f_t(x)\, \mathrm{d}x = 0$$

Aggregate demand

- individually:

$$c_t^i = \mathbb{E}_t c_{t+1}^i - \frac{\ln\left(\beta R_{t+1}\right)}{\gamma} - \frac{\gamma}{2}\mathbb{V}_t c_{t+1}^i$$

- in the aggregate:

$$c_t = c_{t+1} \underbrace{-\frac{\ln(\beta R_{t+1})}{\gamma}}_{\text{intertemp. subst.}} \underbrace{-\frac{\gamma}{2}\underbrace{(\mu_{t+1}^2 \underbrace{\sigma_{t+1}^2 w_{t+1}^2}_{\text{income risk}})}_{\text{consumption risk}}}_{\text{precautionary motive}}$$

- μ_t, σ_t, w_t all matter for aggregate demand, and all depend on policy

Aggregate supply

- competitive final-goods firms + monopolistically competitive wholesale firms facing (Rotemberg) pricing frictions

- income/consumption:

$$c_t = z_t n_t - \frac{\Phi}{2} \left(\Pi_t - 1\right)^2 c_t = \frac{z_t n_t}{1 + \frac{\Phi}{2} \left(\Pi_t - 1\right)^2}$$

- NKPC:

$$\left(\Pi_t - 1\right) \Pi_t = \frac{\varepsilon}{\Phi} \left(1 - \frac{z_t}{w_t}\right) + \frac{1}{R_{t+1}} \left(\frac{c_{t+1} z_t w_{t+1}}{c_t z_{t+1} w_t}\right) \left(\Pi_{t+1} - 1\right) \Pi_{t+1}$$

A Pseudo-RANK

$$
\begin{aligned}
c_t &= c_{t+1} - \frac{1}{\gamma}\ln\left(\beta R_{t+1}\right) - \frac{\gamma\sigma_{t+1}^2 \mu_{t+1}^2 w_{t+1}^2}{2} \\
c_t &= \frac{z_t n_t}{1 + (\Phi/2)\left(\Pi_t - 1\right)^2} \\
n_t &= \rho \ln w_t - \gamma\rho c_t + \bar{\xi} \\
\mu_t^{-1} &= 1 + \gamma\rho w_t + \left(\frac{\theta}{R_{t+1}}\right)\mu_{t+1}^{-1} \\
\left(\Pi_t - 1\right)\Pi_t &= \frac{\varepsilon}{\Phi}\left(1 - \frac{z_t}{w_t}\right) + \frac{1}{R_{t+1}}\left(\frac{c_{t+1} z_t w_{t+1}}{c_t z_{t+1} w_t}\right)\left(\Pi_{t+1} - 1\right)\Pi_{t+1} \\
R_{t+1} &= \left(1 + i_t\right)/\Pi_{t+1}
\end{aligned}
$$

i_t set by central bank; **how?**

Social welfare function and optimal policy problem

- pseudo-RANK $\Rightarrow$ **constraints** of the Ramsey planner
- we are missing the **Social Welfare Function**...
- ...which is **endogenous to the time-varying cross-sectional distribution of households across states and ages**
- **3 steps**
  1. within-cohort cross-sectional distribution at any time t
  2. total utility of a particular cohort at any time t
  3. aggregation over all currently-alive cohorts at any time t

Social welfare function

Step 1: within-cohort distribution

- state of a **newcomer** i at time t:

$$x_t^i = w_t(\xi_t^i - \bar{\xi}) \sim \mathcal{N}(0, \sigma_t)$$

- state of a **survivor** i at time $t+1$:

$$\begin{aligned} x_{t+1}^i &= \frac{\mu_t}{\mu_{t+1}} x_t^i + w_{t+1}(\xi_{t+1}^i - \bar{\xi}) \\ &= \frac{\mu_t}{\mu_{t+1}} w_t(\xi_t^i - \bar{\xi}) + w_{t+1}(\xi_{t+1}^i - \bar{\xi}) \\ &\sim \mathcal{N}\left(0, \frac{\mu_t^2}{\mu_{t+1}^2} \sigma_t^2 w_t^2 + \sigma_{t+1}^2 w_{t+1}^2\right) \end{aligned}$$

- **bottom line**: affine savings rule maps normal into normal

Social welfare function

Step 1: within-cohort distribution

- more generally, the state of a HH living from time t_0 to t_1 is:

$$x^i_{t_0,t_1} \sim \mathcal{N}\left(0, \mu_{t_1}^{-2} \sum_{j=0}^{t_1-t_0} \mu^2_{t_0+j} \sigma^2_{t_0+j} w^2_{t_0+j}\right)$$

- the corresponding density $f_{t_0,t_1}(x)$ becomes more and more spread out as t_1 increases... but that cohort is replaced at rate $1-\theta$...

- ...and hence $f_t(x)$ (used before) does exist:

$$f_t(x) = \sum_{k=0}^{\infty} (1-\theta)\, \theta^k f_{t-k,t}(x) \longrightarrow \bar{f}(x)$$

Social welfare function

Step 2: total utility of a cohort

- indirect flow utility of an individual conditional on state:

$$\begin{aligned} v_t(x) &= u(c_t(x), \xi - \ell(x, \xi)) \\ &= u\left(c_t, \bar{\xi} - n_t\right) \mathrm{e}^{-\gamma \mu_t x} \end{aligned}$$

- concavity of $-\mathrm{e}^{-\gamma \mu_t x}$ + dispersion in $x \Rightarrow$ **welfare loss**

$$\begin{aligned} v(t_0, t_1) &= (1-\theta)\,\theta^{t_1 - t_0} \int v_{t_1}(x)\, f_{t_0, t_1}(x)\, \mathrm{d}x \\ &= \underbrace{(1-\theta)\,\theta^{t_1 - t_0}}_{\text{mass of cohort}} \times \underbrace{u\left(c_{t_1}, \bar{\xi} - n_{t_1}\right)}_{\text{RANK utility } (<0)} \times \underbrace{\mathrm{e}^{\frac{\gamma^2}{2} \sum_{j=0}^{t_1 - t_0} \mu_{t_0+j}^2 \sigma_{t_0+j}^2 w_{t_0+j}^2}}_{\geq 1} \end{aligned}$$

Social welfare function

Step 3: Aggregation over cohors

- aggregate flow utilities over all cohorts alive at time t:

$$
\begin{aligned}
\mathbb{U}_t &= \sum_{k=0}^{\infty} v\left(t-k,t\right) \\
&= \underbrace{u\left(c_t,\bar{\xi}-n_t\right)}_{\text{RANK utility } (<0)} \underbrace{\sum_{k=0}^{\infty}\left(1-\theta\right)\theta^k e^{\frac{\gamma^2}{2}\sum_{j=0}^{k}\mu_{t-k+j}^2\sigma_{t-k+j}^2 w_{t-k+j}^2}}_{\textbf{consumption dispersion index } \Sigma_t \ (\geq 1)}
\end{aligned}
$$

- Σ_t encodes heterogeneity and **evolves recursively**:

$$
\Sigma_t = e^{\frac{\gamma^2}{2}\mu_t^2\sigma_t^2 w_t^2}\left(1-\theta+\theta\Sigma_{t-1}\right)
$$

- **RANK**: $\Sigma_t = 1\ \forall t$; **HANK**: Σ_t fluctuates around $\Sigma = \frac{1-\theta}{\beta R-\theta} > 1$

Optimal policy problem

Statement

$$\max_{\{i_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t u\left(c_t, \bar{\bar{\xi}} - n_t\right) \Sigma_t$$

s.t.

$$\begin{aligned}
\Sigma_t &= e^{\frac{1}{2}\gamma^2 \mu_t^2 \sigma_t^2 w_t^2} \left(1 - \theta + \theta \Sigma_{t-1}\right) \\
c_t &= \frac{z_t n_t}{1 + (\Phi/2)\left(\Pi_t - 1\right)^2} \\
n_t &= \rho \ln w_t - \gamma \rho c_t + \bar{\bar{\xi}} \\
c_t &= c_{t+1} - \frac{1}{\gamma} \ln\left(\beta \frac{1 + i_t}{\Pi_{t+1}}\right) - \frac{\gamma}{2} \mu_{t+1}^2 \sigma_{t+1}^2 w_{t+1}^2 \\
\mu_t^{-1} &= 1 + \gamma \rho w_t + \left(\frac{\theta \Pi_{t+1}}{1 + i_t}\right) \mu_{t+1}^{-1} \\
\left(\Pi_t - 1\right) \Pi_t &= \frac{\varepsilon}{\Phi}\left(1 - \frac{z_t}{w_t}\right) + \left[\frac{\Pi_{t+1} c_{t+1} z_t w_{t+1}}{(1 + i_t) z_{t+1} c_t w_t}\right] \left(\Pi_{t+1} - 1\right) \Pi_{t+1}
\end{aligned}$$

Optimal policy problem

Solution

- 3 forward-looking constraints $\Rightarrow$ solve sequence problem
- derive FOCs of the Lagrangian associated with planner's problem
- forward/backward-looking system of 13 unknowns (7 endo variables + 6 Lagrange multipliers) for 13 equations (7 FOC + 6 constraints)
- focus on **timeless** solution –i.e., nothing special about date 0
- linearise dynamic system around steady state, solve for VARMA representation, parameterise, run IRFs

Parameterisation

- set $z = 1$, target $R = 1.005$, and normalise $n = c = \frac{\bar{\xi}}{1+\gamma\rho} = 1$
- **baseline**: HANK with $\Pi = 1$ (i) and coutercylical income risk (ii)
- **benchmark**: RANK ($\sigma_t = 0\ \forall t$) with
  - same (R, Π) as baseline (iii)
  - offsetting of income & substitutions effects on labor supply (iv)
- (i)-(iii) require $\delta = 1/R$; (iv) requires $\gamma = 1$
- Frisch (macro) elasticity $\frac{\rho}{n} = \rho = 3 \Rightarrow \bar{\xi} = 1 + \gamma\rho = 4$
- turn-over rate: $1 - \theta = 0.15$ (see Nisticò 2016)
- NKPC: $\varepsilon = 6$, $\Phi = 40$ ($\Rightarrow$ slope of NKPC $= 0.15$)

Parameterisation

- what about the cyclicality of individual risk? (point (ii) above)

- key determinant of aggregate demand under incomple markets
  (Werning, 2015; McKay et al. 2017; Bilbiie, 2018; Acharya-Dogra 2018)

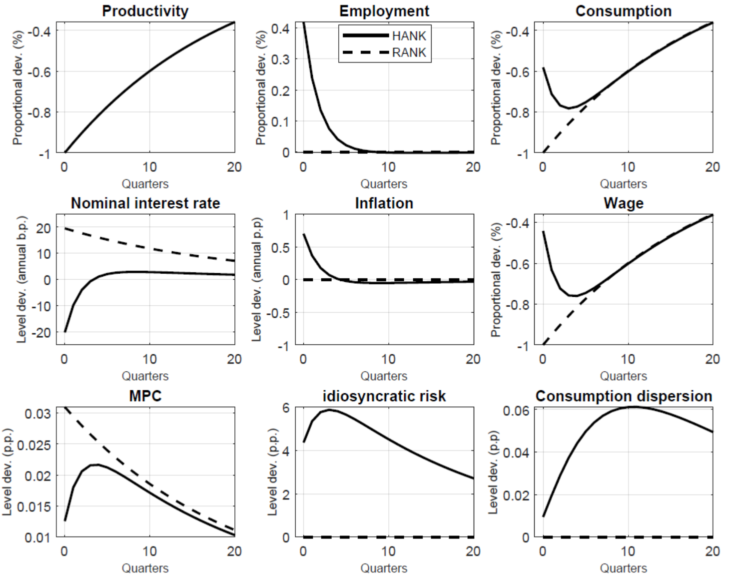
- depends on $\mu_t \sigma_t w_t$, where μ_t and w_t are endogenously determined

- assume $\sigma_t = \sigma(y_t)$ and control cyclicality of consumption risk through $\mathcal{E} = \frac{y}{\sigma}\frac{\partial \sigma_t}{\partial y_t}$ (think of HANK & SaM models):

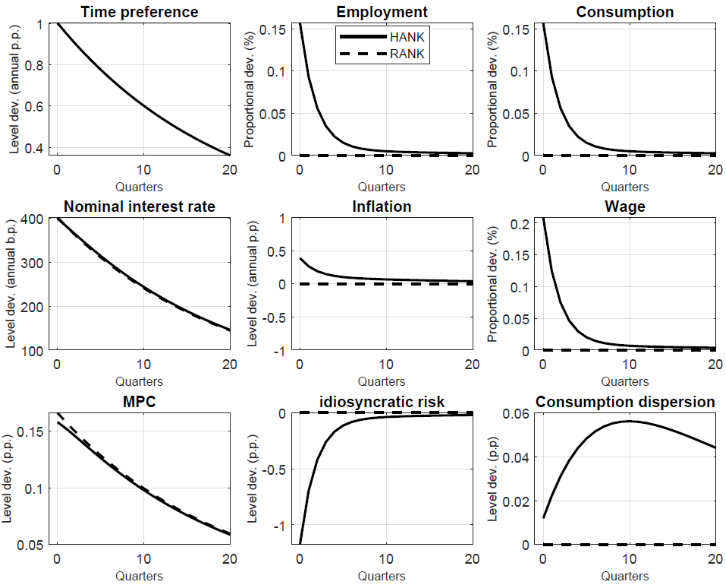
$$\sigma_t \simeq \sigma(\bar{y}) + \sigma(\bar{y})\,\mathcal{E}\hat{y}_t$$

- baseline values: $\sigma(\bar{y}) = 1.5$, $\mathcal{E} = -5 \Rightarrow \sigma(\bar{y})\,\mathcal{E} = -7.5$

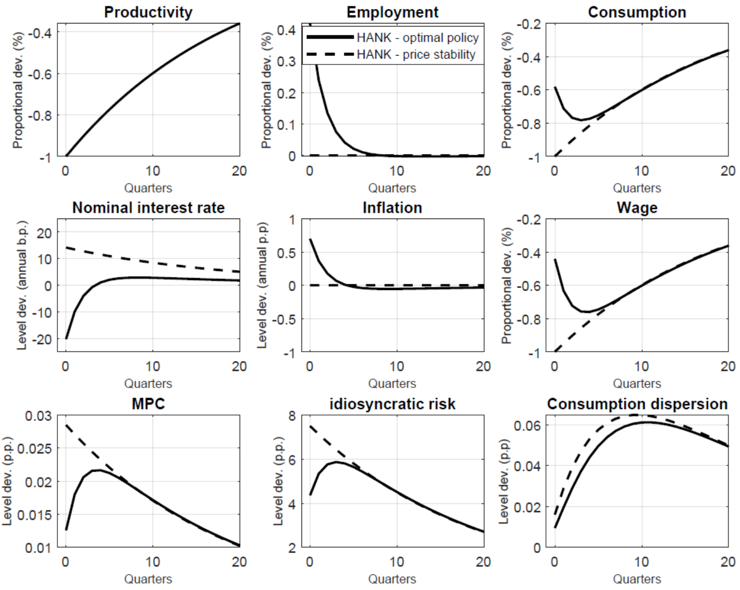
Optimal response to productivity shock



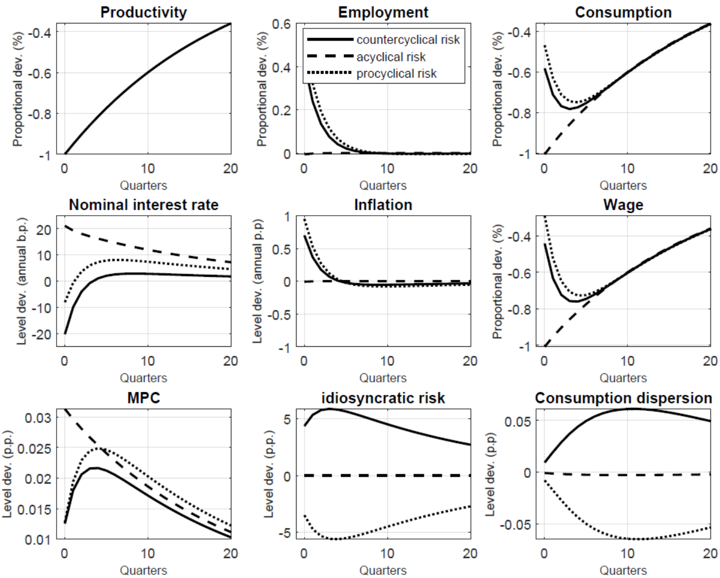
Optimal response to time-preference shock



Productivity shock: Optimal policy vs. price stability



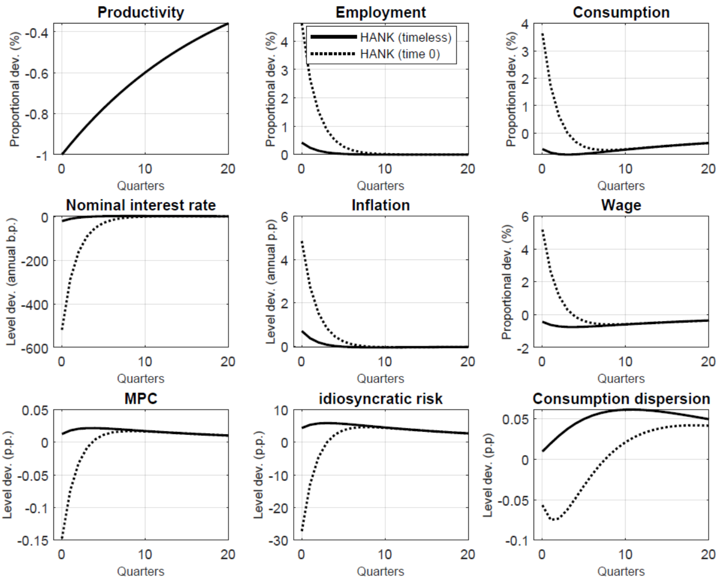
Productivity shock: Alternative income risk cyclicality



Conclusion

- tractable HANK for optimal monetary policy analysis
- tradeoff between price stability and consumption (in)equality
- 2nd motive implies HANK displays more accommodative response to contractionary productivity shock than RANK
- extensions (in progress):
  - hand-to-mouth households ($\Rightarrow$ MPC heterogeneity)
  - entrepreneurial investment ($\Rightarrow$ other source of idiosyncratic risk)
  - joint optimal fiscal-monetary policy

Productivity shock: Timeless vs. time-0 Ramsey



Productivity shock: Timeless vs. time-0 Ramsey