Optimal Monetary Policy in HANK Economies

Sushant Acharya\textsuperscript{a}  Edouard Challe\textsuperscript{b}  Keshav Dogra\textsuperscript{a}

\textsuperscript{a}New York Fed  \textsuperscript{b}CREST & Ecole Polytechnique

The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of New York or the Federal Reserve System
Question and framework

- how does imperfect insurance affect \textbf{optimal} monetary policy?

- \textbf{challenge}: social welfare function aggregates heterogenous (marginal) utilities, each of which is endogenous to policy

- \textbf{solution}: CARA-Normal HANK with closed-form expressions for
  - the aggregate dynamics
  - the (time-varying) distribution of agents
  - the social welfare function
Main results

- optimal policy governed by two forces ($\Rightarrow$ tradeoff)

1. price stability

2. consumption dispersion, as affected by
   - cyclicalilty of income risk (and cumulated effect of)
   - pass-trouch to consumption risk (via time-varying MPC)

- the central bank may tolerate transitory departures from price stability in order to limit the rise in consumption dispersion

- breakdown of “divine coincidence”
Related literature

- quantitative analysis of HANK models
  - **HANK**: Kaplan et al. (2018); Auclert et al. (2018); Debortoli-Gali (2018), Hagedorn et al. (2019)...
  - **HANK & SaM**: Gornemann et al. (2016); Ravn-Sterk (2017, 2018); Challe et al. (2017); Den Haan et al. (2019)...

- optimal monetary policy under perfect insurance
  - **RANK**: Clarida et al. (1999); Woodford (2003); Gali (2008)...
  - **TANK**: Bilbiie (2008)...
  - **RANK & SaM**: Thomas (2008); Faia (2009); Blanchard-Gali (2010); Ravenna-Walsh (2011)

- optimal monetary policy under imperfect insurance
  - **HANK & SaM** with 0-liquidity: Challe (2019)
  - **HANK** with >0 liquidity: Nuño-Thomas (2017); Bhandari et al. (2018)

- CARA-Normal imperfect-insurance models
  - **flex-price**: Calvet (2001); Angeletos and Calvet (2005, 2006)
  - **HANK**: Acharya & Dogra (2018)
Households
Objective and constraint

- objective:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta \theta)^t u(c_t^i, \xi_t^i - \ell_t^i)$$

where

$$u(c_t^i, \xi_t^i - \ell_t^i) = -\frac{1}{\gamma} e^{-\gamma c_t^i} - \rho e^{-\frac{1}{\rho} (\xi_t^i - \ell_t^i)}$$

and

$$\xi_t^i \sim \mathcal{N}(\bar{\xi}, \sigma_t^2) = \text{time endowment}$$

- budget constraint:

$$a_{t+1}^i = \frac{R_{t+1}}{\theta} (a_t^i + w_t \ell_t^i + T_t - c_t^i)$$

- $R_{t+1}/\theta = \text{real riskless return on actuarial bonds}$
Households
Optimality conditions

- assume no aggregate risk ("MIT shock")

- bonds:

\[ e^{-\gamma c_t^i} = \beta R_{t+1} E_t e^{-\gamma c_{t+1}^i} \]

- labor supply:

\[ \ell_t^i = \rho \ln w_t - \gamma \rho c_t^i + \zeta_t^i \]
Households

Policy functions

- conjecture-verify linear consumption function:

\[ c_t(x^i_t) = c_t + \mu_t \times x^i_t \]

\[ \uparrow \quad \uparrow \quad \uparrow \]

agg. cons. MPC state

where

**state** : \[ x^i_t = a^i_t + w_t(\zeta^i_t - \bar{\zeta}) \]

**MPC** : \[ \mu^{-1}_t = 1 + \gamma \rho w_t + \frac{\theta}{R_{t+1}} \mu^{-1}_{t+1} \]

- \( x^i_t \) = quasi cash-on-hand: reflects impact of past asset accumulation \((a^i_t)\) and current labor-endowment shock \((w_t(\zeta^i_t - \bar{\zeta}))\)

- MPC higher when \( \{R_{t+j}\} \) high and/or \( \{w_{t+j}\} \) low
Households
Policy functions

- other policy functions:

  **labor supply**: \( \ell_t(x^i_t, \xi^i_t) = \rho \ln w_t - \gamma \rho c_t(x^i_t) + \xi^i_t \)

  **savings**: \( s_t(x^i_t) = (1 - (1 + \gamma \rho w_t) \mu_t) x^i_t \)

  **wealth**: \( a_{t+1}(x^i_t) = \frac{R_{t+1}}{\theta} (1 - (1 + \gamma \rho w_t) \mu_t) x^i_t \)

- in particular, \( s_t(x^i_t) \) implies that individual state evolves **linearly**:

  \[
  x^i_t = \frac{\mu_{t-1}}{\mu_t} x^i_{t-1} + w_t (\xi^i_t - \bar{\xi})
  \]
Aggregation

- let $f_t(x)$ be the cross-sectional distribution of $x$ (determined later)

- goods:
  \[ \int c_t(x) f_t(x) \, dx = c_t = y_t \]

- labor supply:
  \[ \int l_t(x) f_t(x) \, dx = n_t = \rho \ln w_t - \gamma \rho c_t + \xi \]

- bonds:
  \[ \int a_{t+1}(x) f_t(x) \, dx = 0 \]
Aggregate demand

- individually:

\[ c_t^i = \mathbb{E}_t c_{t+1}^i - \frac{\ln (\beta R_{t+1})}{\gamma} - \frac{\gamma}{2} \mathbb{V}_t c_{t+1}^i \]

- in the aggregate:

\[ c_t = c_{t+1} - \frac{\ln (\beta R_{t+1})}{\gamma} - \frac{\gamma}{2} \left( \mu_{t+1}^2 \sigma_{t+1}^2 \omega_{t+1}^2 \right) \]

\[ \text{intertemp. subst.} \quad \text{income risk} \quad \text{consumption risk} \quad \text{precautionary motive} \]

- \( \mu_t, \sigma_t, \omega_t \) all matter for aggregate demand, and all depend on policy
Aggregate supply

- competitive final-goods firms + monopolistically competitive wholesale firms facing (Rotemberg) pricing frictions

- income/consumption:

\[
c_t = z_t n_t - \frac{\Phi}{2} (\Pi_t - 1)^2
c_t = \frac{z_t n_t}{1 + \frac{\Phi}{2} (\Pi_t - 1)^2}
\]

- NKPC:

\[
(\Pi_t - 1) \Pi_t = \frac{\varepsilon}{\Phi} \left( 1 - \frac{z_t}{w_t} \right) + \frac{1}{R_{t+1}} \left( \frac{c_{t+1} z_t w_{t+1}}{c_t z_{t+1} w_t} \right) (\Pi_{t+1} - 1) \Pi_{t+1}
\]
A Pseudo-RANK

\[
c_t = c_{t+1} - \frac{1}{\gamma} \ln (\beta R_{t+1}) - \frac{\gamma \sigma^2_{t+1} \mu^2_{t+1} \omega^2_{t+1}}{2}
\]

\[
c_t = \frac{z_t n_t}{1 + (\Phi/2) (\Pi_t - 1)^2}
\]

\[
n_t = \rho \ln w_t - \gamma \rho c_t + \zeta
\]

\[
\mu^{-1}_{t} = 1 + \gamma \rho w_t + \left( \frac{\theta}{R_{t+1}} \right) \mu^{-1}_{t+1}
\]

\[
(\Pi_t - 1) \Pi_t = \frac{\varepsilon}{\Phi} \left( 1 - \frac{z_t}{w_t} \right) + \frac{1}{R_{t+1}} \left( \frac{c_{t+1} z_{t+1} \omega_{t+1}}{c_t z_{t+1} \omega_t} \right) (\Pi_{t+1} - 1) \Pi_{t+1}
\]

\[
R_{t+1} = (1 + i_t) / \Pi_{t+1}
\]

\[i_t\] set by central bank; \textbf{how?}
Social welfare function and optimal policy problem

- pseudo-RANK $\Rightarrow$ constraints of the Ramsey planner

- we are missing the Social Welfare Function...

- ...which is endogenous to the time-varying cross-sectional distribution of households across states and ages

- 3 steps

  1. within-cohort cross-sectional distribution at any time $t$
  2. total utility of a particular cohort at any time $t$
  3. aggregation over all currently-alive cohorts at any time $t$
Social welfare function
Step 1: within-cohort distribution

- state of a **newcomer** \( i \) at time \( t \):

\[
x_t^i = w_t (\xi_t^i - \bar{\xi}) \sim \mathcal{N}(0, \sigma_t)
\]

- state of a **survivor** \( i \) at time \( t + 1 \):

\[
x_{t+1}^i = \frac{\mu_t}{\mu_{t+1}} x_t^i + w_{t+1} (\xi_{t+1}^i - \bar{\xi})
\]
\[
= \frac{\mu_t}{\mu_{t+1}} w_t (\xi_t^i - \bar{\xi}) + w_{t+1} (\xi_{t+1}^i - \bar{\xi})
\]
\[
\sim \mathcal{N} \left( 0, \frac{\mu_t^2}{\mu_{t+1}^2} \sigma_t^2 w_t^2 + \sigma_{t+1}^2 w_{t+1}^2 \right)
\]

- **bottom line**: affine savings rule maps normal into normal
Social welfare function
Step 1: within-cohort distribution

- more generally, the state of a HH living from time $t_0$ to $t_1$ is:

$$x_{t_0,t_1}^i \sim \mathcal{N} \left( 0, \mu_{t_1}^{-2} \sum_{j=0}^{t_1-t_0} \mu_{t_0+j}^2 \sigma^2_{t_0+j} + w_{t_0+j}^2 \right)$$

- the corresponding density $f_{t_0,t_1}(x)$ becomes more and more spread out as $t_1$ increases... but that cohort is replaced at rate $1 - \theta$...

- ...and hence $f_t(x)$ (used before) does exist:

$$f_t(x) = \sum_{k=0}^{\infty} (1 - \theta)^k f_{t-k,t}(x) \rightarrow \bar{f}(x)$$
Social welfare function

Step 2: total utility of a cohort

- indirect flow utility of an individual conditional on state:

\[ v_t(x) = u(c_t(x), \zeta - \ell(x, \zeta)) \]

\[ = u(c_t, \zeta - n_t) e^{-\gamma \mu_t} \]

- concavity of \(-e^{-\gamma \mu_t} + \text{dispersion in } x\) ⇒ welfare loss

\[ v(t_0, t_1) = (1 - \theta) \theta^{t_1 - t_0} \int v_{t_1}(x) f_{t_0, t_1}(x) \, dx \]

\[ = (1 - \theta) \theta^{t_1 - t_0} \times u(c_{t_1}, \zeta - n_{t_1}) \times e^{\frac{\gamma^2}{2} \sum_{j=0}^{t_1 - t_0} \mu_{t_0 + j}^2 \sigma_{t_0 + j}^2 + w_{t_0 + j}^2} \]

- mass of cohort
- RANK utility (<0)
- \( \geq 1 \)
Social welfare function

Step 3: Aggregation over cohorts

- aggregate flow utilities over all cohorts alive at time $t$:

\[
U_t = \sum_{k=0}^{\infty} v(t - k, t)
\]

\[
= u(c_t, \bar{\xi} - n_t) \sum_{k=0}^{\infty} (1 - \theta) \theta^k e^{\frac{\gamma^2}{2} \sum_{j=0}^{k} \mu^2_{t-k+j} \sigma^2_{t-k+j} w^2_{t-k+j}}
\]

\[
\begin{align*}
\text{RANK utility (<0)} & \quad \text{consumption dispersion index } \Sigma_t \ (\geq 1)
\end{align*}
\]

- $\Sigma_t$ encodes heterogeneity and evolves recursively:

\[
\Sigma_t = e^{\frac{\gamma^2}{2} \mu^2_{t} \sigma^2_{t} w^2_{t}} (1 - \theta + \theta \Sigma_{t-1})
\]

- RANK: $\Sigma_t = 1 \ \forall t$; HANK: $\Sigma_t$ fluctuates around $\Sigma = \frac{1-\theta}{\beta R - \theta} > 1$
Optimal policy problem

Statement

\[
\max_{\{i_t\}_{t=0}^\infty} \sum_{t=0}^{\infty} \delta^t u (c_t, \bar{\xi} - n_t) \Sigma_t
\]

s.t.

\[
\Sigma_t = e^{\frac{1}{2} \gamma^2 \mu_t^2 \sigma_t^2 w_t^2} (1 - \theta + \theta \Sigma_{t-1})
\]

\[
c_t = \frac{z_t n_t}{1 + (\Phi/2) (\Pi_t - 1)^2}
\]

\[
n_t = \rho \ln w_t - \gamma \rho c_t + \bar{\xi}
\]

\[
c_t = c_{t+1} - \frac{1}{\gamma} \ln \left( \beta \frac{1 + i_t}{\Pi_{t+1}} \right) - \frac{\gamma}{2} \mu_{t+1}^2 \sigma_{t+1}^2 w_{t+1}^2
\]

\[
\mu_t^{-1} = 1 + \gamma \rho w_t + \left( \frac{\theta \Pi_{t+1}}{1 + i_t} \right) \mu_{t+1}^{-1}
\]

\[
(\Pi_t - 1) \Pi_t = \frac{\varepsilon}{\Phi} \left( 1 - \frac{z_t}{w_t} \right) + \left[ \frac{\Pi_{t+1} c_{t+1} Z_t w_{t+1}}{(1 + i_t) Z_{t+1} c_t w_t} \right] (\Pi_{t+1} - 1) \Pi_{t+1}
\]
Optimal policy problem

Solution

- 3 forward-looking constraints $\Rightarrow$ solve sequence problem

- Derive FOCs of the Lagrangian associated with planner’s problem

- Forward/backward-looking system of 13 unknowns (7 endo variables + 6 Lagrange multipliers) for 13 equations (7 FOC + 6 constraints)

- Focus on **timeless** solution – i.e., nothing special about date 0

- Linearise dynamic system around steady state, solve for VARMA representation, parameterise, run IRFs
Parameterisation

- set $z = 1$, target $R = 1.005$, and normalise $n = c = \frac{\xi}{1 + \gamma \rho} = 1$

- **baseline**: HANK with $\Pi = 1$ (i) and counter-cyclical income risk (ii)

- **benchmark**: RANK ($\sigma_t = 0 \ \forall t$) with
  - same $(R, \Pi)$ as baseline (iii)
  - offsetting of income & substitutions effects on labor supply (iv)

- (i)-(iii) require $\delta = 1/R$; (iv) requires $\gamma = 1$

- Frisch (macro) elasticity $\frac{\rho}{n} = \rho = 3 \Rightarrow \bar{\xi} = 1 + \gamma \rho = 4$

- turn-over rate: $1 - \theta = 0.15$ (see Nisticò 2016)

- **NKPC**: $\varepsilon = 6$, $\Phi = 40$ (⇒ slope of NKPC = 0.15)
Parameterisation

- what about the cyclicality of individual risk? (point (ii) above)

- key determinant of aggregate demand under incomplete markets
  (Werning, 2015; McKay et al. 2017; Bilbiie, 2018; Acharya-Dogra 2018)

- depends on $\mu_t \sigma_t w_t$, where $\mu_t$ and $w_t$ are endogenously determined

- assume $\sigma_t = \sigma(y_t)$ and control cyclicality of consumption risk through $\mathcal{E} = \frac{\gamma}{\sigma} \frac{\partial \sigma_t}{\partial y_t}$ (think of HANK & SaM models):

  $$\sigma_t = \sigma(\bar{y}) + \sigma(\bar{y}) \mathcal{E} \hat{y}_t$$

- baseline values: $\sigma(\bar{y}) = 1.5$, $\mathcal{E} = -5 \Rightarrow \sigma(\bar{y}) \mathcal{E} = -7.5$
Optimal response to productivity shock
Optimal response to time-preference shock
Productivity shock: Optimal policy vs. price stability

- **Productivity**
- **Employment**
- **Consumption**
- **Nominal interest rate**
- **Inflation**
- **Wage**
- **MPC**
- **Idiosyncratic risk**
- **Consumption dispersion**
Productivity shock: Alternative income risk cyclicality

- Productivity
- Employment
- Consumption
- Nominal interest rate
- Inflation
- Wage
- MPC
- idiosyncratic risk
- Consumption dispersion
Conclusion

- tractable HANK for optimal monetary policy analysis

- tradeoff between price stability and consumption (in)equality

- 2nd motive implies HANK displays more accommodative response to contractionary productivity shock than RANK

- extensions (in progress):
  - hand-to-mouth households (⇒ MPC heterogeneity)
  - entrepreneurial investment (⇒ other source of idiosyncratic risk)
  - joint optimal fiscal-monetary policy
Productivity shock: Timeless vs. time-0 Ramsey

**Productivity**

- Proportional dev. (%)
  - Quarters

**Employment**

- Proportional dev. (%)
  - Quarters
  - HANK (timeless)
  - HANK (time 0)

**Consumption**

- Proportional dev. (%)
  - Quarters

**Nominal interest rate**

- Level dev. (annual b.p.)
  - Quarters

**Inflation**

- Level dev. (annual p.p.)
  - Quarters

**Wage**

- Proportional dev. (%)
  - Quarters

**MPC**

- Level dev. (p.p.)
  - Quarters

**idiosyncratic risk**

- Level dev. (p.p.)
  - Quarters

**Consumption dispersion**

- Level dev. (p.p.)
  - Quarters
Productivity shock: Timeless vs. time-0 Ramsey