Optimal Monetary Policy in Production Networks*

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Abstract

We study optimal monetary policy in an economy in which firms buy and sell intermediate goods in a multi-sector input-output network. Firms are subject to sectoral productivity shocks and face nominal rigidities. We first show that the flexible-price equilibrium with appropriate sectoral subsidies is first-best efficient. Despite the efficiency of flexible-price allocations, however, generically these allocations cannot be achieved as equilibria under sticky prices. As a result, monetary policy cannot implement the first best in multi-sector input-output network economies. Next, given that monetary policy cannot achieve the first best, we study the second- best optimal monetary policy problem. We find that the optimal monetary policy trades off three types of welfare loss: efficiency loss due to price errors across sectors, efficiency loss due to price dispersion within sectors, and the output gap. Importantly, an output gap arises even though all underlying shocks are efficient.

Keywords: Nominal rigidities, monetary policy, production networks, informational frictions, misallocation.

Preliminary and incomplete

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1 Introduction

In the canonical New Keynesian model, optimal monetary policy is well-known: as long as there are no missing tax instruments, it is optimal to stabilize the price level. Price level stabilization implements flexible price allocations and attains the first best. In the language of the New Keynesian literature, the "divine coincidence" holds: price stability minimizes both inflation and the output gap. The divine coincidence breaks when inefficient, cost-push shocks are introduced and tax instruments to fight these shocks are assumed away. In this case, monetary policy becomes an imperfect instrument to fight inefficient cost-push shocks; as a result the policy maker faces a tradeoff between minimizing the output gap and productive inefficiency due to inflation.

However, even aside from introducing inefficient shocks, the divine coincidence is a special case. This is because in the typical NK model, there is only one sector. That is, even though there is a continuum of firms producing differentiated goods, all firms are technologically symmetric: they are ex-ante identical in terms of how their good contributes to the final good in the economy. This implies that it is optimal to have identical prices across all firms; one can acheive this with price stability and thereby acheive the first best.

If one extends the framework to multiple sectors, even with only efficient shocks, the divine coincidence fails. One cannot replicate flexible-price allocations. This is already well-known and shown to be the case in Aoki (2001); Huang and Liu (2005) and Woodford (2010) in the two sector case, as well as Henderson and Levine when both prices and wages are sticky. However, this literature has focused only on very special cases: two sectors models.

How does the input-output network structure of the economy affect the optimal conduct of monetary policy?

In this paper we instead consider a multi-sector economy in which firms buy and sell intermediate goods. We seek to answer the following question: how should monetary policy respond to sectoral shocks in a multi-sector, input-output network economy?

We consider a multi-sector, general equilibrium model of inter-sectoral trade à la Long and Plosser (1983), and more recently, Acemoglu et al. (2012). Firms operate heterogeneous Cobb-Douglas technologies, employ labor, and demand intermediate goods. Intermediate goods are produced by all sectors; the matrix of cross-sector input requirements defines the input-output production network. A representative household consumes final goods and supplies labor elastically.

We assume that the only payoff-relevant shocks in the economy are shocks to sectoral productivities. We thus abstract from inefficient mark-up, or cost-push, shocks.

In order to accommodate price-setting, we make the typical assumption that each sector consists of a continuum of monopolistically competitive firms, each producing a differentiated good. Goods are aggregated into a sectoral commodity which may then either be used for consumption or as an intermediate good to production. Firms are thus price-setters within a sector, but price-takers across sectors. As in the New-Keynesian tradition, we allow firms to be subject to nominal rigidity. That is, we assume that some firms must make nominal pricing decisions prior to the realization of the aggregate state. This nominal friction opens the door for potential price distortions within the production network.

Finally, we allow for a government with control over two types of policy instruments: a rich set non-state contingent sectoral taxes/subsidies (set by the fiscal authority) and control over state-contingent nominal aggregate demand (set by the monetary authority). Within this framework, we consider optimal monetary policy.

Our methodology for understanding optimal policy builds on the primal approach of the Ramsey literature applied to New Keynesian models, see e.g. Correia et al. (2008); Angeletos and La'O (2019). By the primal approach, we characterize the entire set of allocations that can be implemented as market-based equilibria with the help of the available policy instruments. We then consider choosing the best allocation among that set. We then identify the welfare-maximizing allocation within that set, and back out the prices and policies that implement said allocation.

To begin our analysis we first consider the set of allocations that may be implementable as equilibria in the absence of nominal rigidities. Following the literature, we call this the set of "flexible-price" equilibria. We show that this set is characterized by a set of conditions relating the marginal rates of substitution between goods and their marginal rates of transformation. As typical in these models, we show how fiscal policy and monopolistic markups may drive sectoral wedges within these conditions. Thus, the economy looks similar to that in Bigio and La'O (2018): that is, a production network with sectoral wedges.

The set of flexible-price allocations in our economy has the typical feature that it contains the first best allocation. The first-best allocaiton is the particular flexible-price equilibrium absent any sectoral wedges; it may be acheived as a competitive equilibrium with sector-specific subsidies which eliminate all markups.

We then consider the set of allocations that may be implementable as equilibria in the original economy featuring nominal rigidities. Following the literature, we call this the set of "sticky-price" equilibria. We show how this set, like the set of flexible-price allocations, may be characterized by a set of conditions relating the marginal rates of substitution between goods and their marginal rates of transformation. However, we show how the conduct of monetary policy and the nominal rigidities induce new wedges in these conditions: these wedges are now contingent on sticky-price firms' beliefs and the realized aggregate state.

We first show that while the first best allocation may be implemented under flexible prices, it cannot be acheived when firms face nominal rigidities. The intuition is relatively simple. Consider a two-sector economy with a continuum of firms within each sector and sector-specific productivity shocks. And suppose there are two types of firms within each sector: sticky-price firms that set their prices before knowing nominal demand, and flexible-price firms that set their prices after. The first best planner in this economy would want a uniform price among all firms within each sector, but at the same time would want the relative price *across* the sectors to move in response to changes in productivity.

The question then becomes how to implement the first best allocation. If one wants a uniform price within one sector, the monetary authority can acheive this by targeting a stable price within that sector. Then all firms in that sector would set a uniform price. However, this would imply that there would be some realized error between the sector 1's price and sector 2's price. That is, there will be a realized pricing error between sector 1 and sector 2's relative price. On top of that, there will be price dispersion in sector 2.

This result is in stark contrast to the canonical one-sector New Keynesian model with a continuum of monopolistically competitive price-setting firms. In the typical NK model, the first-best allocation may be implemented under sticky prices with price stability and appropriate tax instruments, in particular subsidy which eliminates the monopolistic mark up. This is because monetary policy may implementsflexible-price equilibria through targeting price stability. In this paper we consider how this simple policy recommendation is affected in a multi-sector economy with an input-output network.

Deviations from optimal policy occur due to missing tax instruments. Generally, one has is a cost-push (markup-shock) and no state-contingent tax to eliminate it. In this case optimal monetary policy is a second-best policy which optimally trades off the output gap with price dispersion.

Our result differs in that there are no missing tax instruments: we allow for the elimination of all mark-ups with sector specific taxes and there are no shocks to mark-ups. Yet, despite this, the first best still cannot be acheived. Instead, an output gap arises naturally due to pricing errors across sectors.

We then consider the second best problem of optimal monetary policy. We express the welfare losses that result from deviations from the first best. We show that welfare losses are the result of three components: (i) volatility of the output gap, i.e. the aggregate labor wedge, (ii) pricing errors across sectors, and (iii) price dispersion within sectors. Consider the first component, the volatility of the output gap. Importantly, an output gap arises even though all shocks are efficient. This is because pricing errors act as distortions. As a result, as these distortions (or errors) aggregate, an aggregate labor wedge arises which translates into an output gap. That is, there is a wedge between the equilibrium real wage and the marginal product of labor.

We characterize the monetary policy that optimally trades off these three components. We find that the optimal policy depends non-trivially on the centrality of sectors, their elasticities of substitution, and their relative price stickiness. We derive a price stability policy with optimal weights on sectors. Our characterization of optimal monetary policy illustrates importance of strategic complementarities.

Related Literature Our paper is part of the growing literature that studies the role of production networks in macroeconomics. Building on the multi-sector model of Long and Plosser (1983), papers such as Horvath (1998), Dupor (1999), Carvalho (2010), and Acemoglu et al. (2012, 2017) investigate whether input-output linkages can transform microeconomic shocks into aggregate fluctuations.[1] Within this literature, our paper builds on the works of Jones (2013), Bigio and La'O (2018), and Baqaee and Farhi (2019), who study misallocation in economies with non-trivial production networks. However, in contrast to these papers, which treat markups and wedges as exogenously-given model primitives, we focus on an economy in which wedges are determined endogenously as a result of firms' optimal price-setting decisions under incomplete information, with our characterization of optimal policy based on the monetary authority's ability to shape the wedges using policy instruments.

[1] See Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) for surveys of the theoretical and empirical literature on production networks.

Even more closely related to our work, in a series of recent papers, Pasten, Schoenle, and Weber (2018a,b) and Ozdagli and Weber (2019) study the production network's role as a possible transmission mechanism of monetary policy shocks. We differ from these papers by providing a closed-form characterization of the optimal monetary policy as a function of the economy's underlying production network and the extent and nature of nominal rigidities.

Our paper also belongs to a small strand of the New Keynesian literature that studies optimal monetary policy in multi-sector economies. In one of the earliest examples of this line of work, Aoki (2001) shows that in a two-sector economy with one sticky and one fully flexible industry (but no input-output linkages), a policy that stabilizes the price of the sticky industry implements the first-best allocation. Mankiw and Reis (2003), Woodford (2003, 2010), and Benigno (2004) generalize this framework by allowing for more general patterns of price stickiness and establish that, while monetary policy cannot in general implement the first best, the (second-best) optimal stabilization policy puts higher weights on stickier industries. Huang and Liu (2005) introduce input-output linkages to a two-sector economy with a final and an intermediate good and show that the second-best policy stabilizes a combination of the price of the two goods. To the best of our knowledge, however, no paper has studied the optimal monetary policy with a general production network.

Finally, our work is related to the literature on policy interventions in social and economic networks. Papers such as Ballester, Calvó-Armengol, and Zenou (2006), Candogan, Bimpikis, and Ozdaglar (2012), and Galeotti, Golub, and Goyal (2019) study how various policies can impact aggregate action or welfare in various strategic games over networks. In a departure from this literature, which models network interactions, payoffs, and policy instruments in a reduced-form manner, we focus on a micro-founded general equilibrium economy and provide a characterization of optimal policy in terms of primitives such as preferences, technologies, and information sets.

Outline The rest of the paper is organized as follows. Section 2 sets up the environment and defines the sticky-price and flexible-price equilibria in our context. Section 3 characterizes these equilibria and establishes our first main result that, in general, monetary policy cannot implement the first-best allocation. In Section 4, we provide a closed-form characterization of optimal policy for a multi-sector economy in terms of the structure of the underlying production network and the extent of price stickiness in the economy. Section 5 contains our quantitative results and Section 6 concludes the paper. All proofs and some additional mathematical details are provided in the Appendix.

2 Framework

Consider a static economy consisting of n industries indexed by $i \in I = \{1, 2, \ldots, n\}$. Each industry consists of two types of firms: (i) a unit mass of monopolistically competitive firms, indexed by $k \in [0, 1]$, producing differentiated goods and (ii) a competitive producer whose sole purpose is to aggregate the industry's differentiated goods to a single sectoral output. The output of each industry can be either consumed by the households or used as an intermediate input for production by firms in other industries. In addition to the firms, the economy consists of a representative household as well as a government with the ability to levy industry-specific taxes and control aggregate nominal demand.

The monopolistically competitive firms within each industry use a common constant returns to scale technology to transform labor and intermediate inputs into their differentiated products. More specifically, the production function of firm $k \in [0,1]$ in industry i is given by

$$y_{ik} = z_i F_i(l_{ik}, x_{i1,k}, \dots, x_{in,k}),$$

where y_{ik} is the firm's output, l_{ik} is its labor input, $x_{ij,k}$ is the amount of sectoral commodity j purchased by the firm, z_i is an industry-specific productivity shock, and F_i is homogenous of degree 1. We assume that labor is an essential input for the production technology of all goods, in the sense that $F_i(0, x_{i1,k}, \dots, x_{in,k}) = 0$ and that $dF_i/dl_{ik} > 0$ whenever all other inputs are used in positive amounts.

The nominal profits of the firm are given by

$$\pi_{ik} = (1 - \tau_i)\, p_{ik} y_{ik} - w l_{ik} - \sum_{j=1}^{n} p_j x_{ij,k}, \tag{1}$$

where p_{ik} is the nominal price charged by the firm, p_j is nominal price of the industry j's sectoral output, w denotes the nominal wage, and τ_i is an industry-specific revenue tax (or subsidy) levied by the government.

The competitive producer in industry i transforms the differentiated products produced by the unit mass of firms in that industry into a uniform sectoral good using a CES production technology

$$y_i = \left(\int_0^1 y_{ik}^{(\theta_i - 1)/\theta_i} dk \right)^{\theta_i/(\theta_i - 1)} \tag{2}$$

with elasticity of substitution $\theta_i > 1$. This producer's profits are thus given by $\pi_i = p_i y_i - \int_0^1 p_{ik} y_{ik} dk$, where p_i is the price of the aggregated good produced by industry i. We include this producer — which has zero value added — only for expositional purposes: it ensures that a homogenous good is produced by each industry, while at the same time allowing for monopolistic competition among firms within the industry.

The preferences of the representative household are given by

$$W(C, L) = U(C) - V(L), \tag{3}$$

where C and L denote the household's final consumption basket total labor supply, respectively. We impose the typical regularity conditions on U and V: they are strictly increasing, twice differentiable, and satisfy $U'' < 0$, $V'' > 0$, and the Inada conditions. The final consumption basket of the household is given by $C = \mathcal{C}(c_1, \dots, c_n)$, where c_i is the household's consumption of the good produced by industry i and $\mathcal{C}$ is homogenous of degree 1. The representative household's budget constraint is thus given by

$$PC = \sum_{j=1}^{n} p_j c_j \leq wL + \sum_{i=1}^{n} \int_0^1 \pi_{ik} dk + T,$$

where $P = \mathcal{P}(p_1, \ldots, p_n)$ is the nominal price of the household's consumption bundle. The left-hand side of the above inequality is the household's nominal expenditure, whereas the right-hand side is equal to the household's total nominal income, consisting of wage income, dividends from owning firms, and lump sum transfers from the government.[2]

In addition to the firms and the representative household, the economy also consists of a government with the ability to set fiscal and monetary policies. The government's fiscal instrument is a collection of industry-specific taxes (or subsidies) on the firms, with the resulting revenue then rebated to the household as a lump sum transfer. Therefore, the government's budget constraint is given by

$$T = \sum_{i=1}^{n} \tau_i \int_0^1 p_{ik} y_{ik} dk,$$

where τ_i is the revenue tax imposed on firms in industry i and T is the net transfer to the representative household. Finally, to model monetary policy, we sidestep the micro-foundations of money and, instead, impose the following cash-in-advance constraint on the household's total expenditure:

$$PC = m, \tag{4}$$

where m — which can be interpreted as either money supply or nominal aggregate demand — is set by the monetary authority.

2.1 Nominal Rigidities and Information Frictions

We model nominal rigidities by assuming that firms do not observe the realized productivity shocks $z = (z_1, \ldots, z_n)$ and, instead, make their nominal pricing decisions under incomplete information. This assumption implies that nominal prices respond to changes in productivities only to the extent that such changes are reflected in the firms' information sets.

Formally, we assume that each firm k in industry i receives a signal $\omega_{ik} \in \Omega_{ik}$ about the economy's aggregate state. The aggregate state includes not only the vector of realized productivity shocks, but also the realization of all signals, that is,

$$s = (z, \omega),$$

where $\omega = (\omega_1, \ldots, \omega_n) \in \Omega$ denotes the realized cross-sectional distribution of signals in the economy and $\omega_i = (\omega_{ik})_{k \in [0,1]}$ denotes the realized cross-sectional distribution of signals in industry i. We use $\mu(\cdot)$ to denote the unconditional distribution of of aggregate state over the set $S = \mathbb{R}^n_+ \times \Omega$.

Since ω_{ik} is the only component of state s that is observable to firm ik, the nominal price set by this firm has to be measurable with respect to ω_{ik}. We capture this measurability constraint by denoting the firm's price by $p_{ik}(\omega_{ik})$. Similarly, we write $p_i(\omega_i)$ and $P(\omega)$ to capture the fact that the nominal prices of sectoral good i and the consumption bundle have to be measurable with respect to the profile of signals in industry i and in the entire economy, respectively.[3]

[2]Since sectoral aggregators are competitive, they make zero profits in equilibrium. Hence, we only need to account for dividends from the monopolistically competitive firms.

[3]More specifically, the sectoral good producer's CES technology implies that $p_i(\omega_i) = (\int_0^1 p_{ik}^{1-\theta_i}(\omega_{ik}) dk)^{1/(1-\theta_i)}$, whereas the consumption good's price index is given by $P(\omega) = \mathcal{P}(p_1(\omega_1), \ldots, p_n(\omega_n))$.

A few remarks are in order. First, note that the above formulation implies that state $s = (z, \omega)$ not only contains all payoff-relevant shocks, but also contains shocks to the aggregate profile of beliefs. Therefore, our framework can accommodate the possibility of higher-order uncertainty, as ω_{ik} may contain information about other firms' (first or higher-order) beliefs. Second, note that we can nest models with "sticky information" (e.g., Mankiw and Reis (2002) and Ball, Mankiw, and Reis (2005)) by assuming that a fraction ρ_i of firms in industry i set their nominal prices under complete information ($\omega_{ik} = z$), whereas the rest of the firms in that industry observe no informative signals ($\omega_{ik} = \varnothing$) and hence set their nominal prices based only on the prior beliefs. Finally, note that while all firms set nominal prices under incomplete information, we do not impose any form of wage rigidities, thus allowing the nominal wage to depend on the entire state s.

In summary, we can represent the economy's price system by the collection of nominal prices and nominal wage at any given state,

$$\varrho = \left\{ \left((p_{ik}(\omega_{ik}))_{k\in[0,1]}, p_i(\omega_i) \right)_{i\in I}, P(\omega), w(s) \right\}_{s\in S}.$$

While nominal prices are set under incomplete information, we assume that firms and the household make their quantity decisions *after* observing the prices and the realization of productivities. As a result, quantities may depend on the entire state s. We thus represent an allocation in this economy by

$$\xi = \left\{ \left((y_{ik}(s), l_{ik}(s), x_{ik}(s))_{k\in[0,1]}, y_i(s), c_i(s) \right)_{i\in I}, C(s), L(s) \right\}_{s\in S},$$

where $l_{ik}(s)$, $x_{ik}(s) = (x_{i1,k}(s), \ldots, x_{in,k}(s))$, and $y_{ik}(s)$ denote, respectively, the labor input, material input, and output of firm k in industry i, $y_i(s)$ is the output of industry i, $c_i(s)$ is the household's consumption of sectoral good i, and $C(s)$ and $L(s)$ are the household's consumption and labor supply, respectively.

We conclude this discussion by specifying how government policy depends on the economy's aggregate state. Recall from equations (1) and (4) that the fiscal and monetary authorities can, respectively, levy taxes and control the money supply. We assume that while the fiscal authority has the ability to levy industry-specific taxes τ_i, these taxes cannot be contingent on the economy's aggregate state. In contrast, the monetary authority can set the money supply as an arbitrary function $m(s)$ of the economy's aggregate state s. This is equivalent to assuming that the monetary authority has the ability to commit, ex ante, to a policy that can, in principle, depend on the realized productivities and the profile of beliefs throughout the economy. Government policy can thus be summarized as

$$\vartheta = \left\{ (\tau_1, \ldots, \tau_n), m(s) \right\}_{s\in S}. \tag{5}$$

Note that our formulation of government's policy instruments in (5) allows for non-contingent taxes to undo distortions due to monopolistic markups, while ruling out state-contingent taxes, as otherwise, the fiscal authority would be able to neutralize the effect of nominal rigidities and implement the first-best allocation without resorting to monetary policy (Correia, Nicolini, and Teles, 2008). As a final remark, we note that it may be far-fetched to assume that the monetary authority can commit to a monetary policy that is contingent not just on the pay-off relevant shocks, z, but also on the entire profile of beliefs, ω. Nonetheless, we make this assumption to show that limits of monetary policy in implementing allocations even under maximum flexibility.

2.2 Equilibrium Definition

We now define our notions of sticky- and flexible-price equilibria. To this end, first note that the market-clearing conditions for labor and commodity markets are given by

$$L(s) = \sum_{i=1}^{n} \int_0^1 l_{ik}(s)dk \tag{6}$$

$$y_i(s) = c_i(s) + \sum_{j=1}^{n} \int_0^1 x_{ji,k}(s)dk = \left(\int_0^1 y_{ik}(s)^{(\theta_i - 1)/\theta_i} dk \right)^{\theta_i/(\theta_i - 1)} \tag{7}$$

for all $i \in I$ and all $s \in S$, whereas the production technology of firm k in industry i requires that

$$y_{ik}(s) = z_i F_i\big(l_{ik}(s), x_{i1,k}(s), \ldots, x_{in,k}(s)\big) \tag{8}$$

for all $s \in S$. Given the above, the definition of a sticky-price equilibrium is straightforward:

Definition 1. A *sticky-price equilibrium* is a triplet $(\xi, \varrho, \vartheta)$ of allocations, prices, and policies such that

(i) firms set nominal prices $p_{ik}(\omega_{ik})$ to maximize expected real value of profits given their information set and optimally choose their inputs to meet;

(ii) the representative household maximizes her utility;

(iii) the government budget constraint is satisfied;

(iv) all markets clear.

We next define our notion of flexible-price equilibria by dropping the measurability constraint on prices imposed on the sticky-price equilibrium in Definition 1. More specifically, we assume that, in contrast to the sticky-price firms, flexible-price firms make their nominal pricing decisions based on complete information of the aggregate state. We can capture this scenario in our framework by simply considering the special case in which all firm-level prices are measurable in the aggregate state, $p_{ik}(s)$. Accordingly, we also adjust our notation for the nominal price of sectoral goods and the consumption bundle by expressing them as $p_i(s)$ and $P(s)$, respectively.

Definition 2. A *flexible-price equilibrium* is a triplet (ξ, δ, ϑ) of allocations, prices, and policies that satisfy the same conditions as those stated in Definition 1, except that all prices are measurable with respect to the aggregate state s.

While not the main focus of our study, the set of flexible-price-implementable allocations serves as a benchmark to which we will contrast equilibria in the presence of nominal rigidities. We conclude with one additional definition, whose meaning is self-evident.

Definition 3. An allocation ξ is *feasible* if it satisfies resource constraints (6), (7), and (8).

3 Sticky- and Flexible-Price Equilibria

In this section, we provide a characterization of the set of all allocations that can be implemented as part of flexible- and sticky-price equilibria. We then use our characterization results to establish that, except for a non-generic set of specifications, the two sets of allocations never intersect, thus implying that, in our multi-sector framework, monetary policy cannot undo the effects of nominal rigidities.

3.1 First-Best Allocation

We start by focusing on the first-best allocation that maximizes household welfare among all feasible allocations. As a first observation, note that, by symmetry, a planner who maximizes social welfare dictates that all firms within an industry choose the same input, labor, and output quantities.[4] The first-best planner's problem thus reduces to maximizing (3) state-by-state subject to resource constraints (6)–(8). The equations characterizing the planner's optimum are straightforward, summarized in the following lemma:

Lemma 1. *The first-best optimal allocation satisfies the following set of equations,*

$$V'(L(s)) = U'(C(s))\frac{d\mathcal{C}}{dc_i}(s)z_i\frac{dF_i}{dl_i}(s) \tag{9}$$

$$\frac{d\mathcal{C}/dc_j}{d\mathcal{C}/dc_i}(s) = z_i\frac{dF_i}{dx_{ij}}(s) \tag{10}$$

for all pairs of industries $i, j \in I$ *and all states* s.

Conditions (9) and (10), along with the resource constraints (6)–(8), hold state-by-state in the first-best allocation. The first condition (9) indicates that for any good, it is optimal to equate the marginal rate of substitution between consumption of that good and labor with the marginal rate of transformation. To see this more clearly, note that left-hand side of (9) is simply the household's marginal disutility of labor. The planner equates this marginal cost with the marginal social benefit. The marginal social benefit consists of three multiplicative components: the marginal product of labor in the production of commodity i, the marginal product of good i in the production of the final good, and the marginal utility of consumption of the final good.

The second condition (10) similarly indicates that the planner finds it optimal to equate the marginal rate of substitution between two goods to their marginal rate of transformation. The marginal rate of substitution on the left-hand side of equation (10) is the ratio of marginal utilities from consumption of either good, whereas the marginal rate of transformation is simply the marginal product of good i in the production of good j, as shown in the right hand side of this condition.

3.2 Flexible-Price Equilibrium

We now turn to the set of allocations that are implementable as flexible price equilibria in this economy. Since the tax instruments $(\tau_1, \ldots, \tau_n)$ are industry-specific and, in a flexible price equilibrium, all firms in the same industry have identical information sets, we can once again drop the firm index k.

[4] Specifically, the planner chooses, $x_{ij} = x_{ij,k}$, $l_i = l_{ik}$, and $y_i = y_{ik}$ for all firms k in industry i.

Proposition 1. *A feasible allocation is part of a flexible-price equilibrium if and only if there exists a set of positive scalars* $(\chi_1^f, \ldots, \chi_n^f)$ *such that*

$$V'(L(s)) = \chi_i^f U'(C(s)) \frac{d\mathcal{C}}{dc_i}(s) z_i \frac{dF_i}{dl_i}(s) \tag{11}$$

$$\frac{d\mathcal{C}}{dc_j}(s) = \chi_i^f \frac{d\mathcal{C}}{dc_i}(s) z_i \frac{dF_i}{dx_{ij}}(s) \tag{12}$$

for all pairs of industries $i, j \in I$ *and all states* s.

The conditions in Proposition 1 are almost identical to those characterizing the first-best allocation in Lemma 1, aside from the set of scalars $(\chi_1, \ldots, \chi_n)$. The first condition (11) indicates that for any good, the marginal rate of substitution between consumption and labor is equal to the marginal rate of transformation, modulo a non-state contingent wedge χ_i^f. Similarly, the second condition equates the marginal rate of substitution between two goods to their marginal rate of transformation, again subject to the wedge χ_i^f. This non-stochastic wedge, which is given by

$$\chi_i^f = (1 - \tau_i) \frac{\theta_i - 1}{\theta_i}, \tag{13}$$

consist of two terms: the tax or subsidy levied by the government and the markup that arises due to monopolistic-competition among firms within each industry. As a result, the scalars $(\chi_1^f, \ldots, \chi_n^f)$ parameterize the power of the fiscal authority. In particular, with sectoral taxes/or subsidies, the fiscal authority can move allocations by inducing wedges in the conditions (11) and (12).

Another immediate consequence of Proposition 1 is that the first-best allocation is implementable as a flexible-price equilibrium. This follows from the observation that equations (11)–(12) reduce to (9)–(10) whenever $\chi_i^f = 1$ for all i. Consequently, the first-best allocation can be implemented as a flexible-price equilibrium with industry-specific subsidies $\tau_i = 1/(1 - \theta_i)$. This, of course, is not surprising: the only distortion in the economy without nominal rigidities arises from monopolistic competition. Therefore, it is optimal for the government to set industry-specific subsidies that are invariant to the economy's aggregate state and undo the monopolistic markups.

3.3 Sticky-Price Equilibrium

With the above preliminary results in hand, we are now ready to characterize the set of equilibrium allocations in the presence of nominal rigidities.

Proposition 2. *A feasible allocation is implementable as a sticky-price equilibrium if and only if there exist positive scalars* $(\chi_1^s, \ldots, \chi_n^s)$*, a policy function* $m(s)$*, and firm-level wedge functions* $\varepsilon_{ik}(s)$ *such that*

(i) the allocation, the scalars $(\chi_1^s, \ldots, \chi_n^s)$*, and the set of wedge functions* $\varepsilon_{ik}(s)$ *jointly satisfy*

$$V'(L(s)) = \chi_i^s \varepsilon_{ik}(s) U'(C(s)) \frac{d\mathcal{C}}{dc_i}(s) z_i \frac{dF_i}{dl_{ik}}(s) \left(\frac{y_{ik}(s)}{y_i(s)}\right)^{-1/\theta_i} \tag{14}$$

$$\frac{d\mathcal{C}}{dc_j}(s) = \chi_i^s \varepsilon_{ik}(s) \frac{d\mathcal{C}}{dc_i}(s) z_i \frac{dF_i}{dx_{ij}}(s) \left(\frac{y_{ik}(s)}{y_i(s)}\right)^{-1/\theta_i} \tag{15}$$

for all firms k*, all pairs of industries* i *and* j*, and all states* s*;*

(ii) the policy function $m(s)$ induces the wedge functions $\varepsilon_{ik}(s)$ given by

$$\varepsilon_{ik}(s) = \frac{\mathrm{mc}_i(s)\mathbb{E}_{ik}[v_{ik}(s)]}{\mathbb{E}_{ik}[\mathrm{mc}_i(s)v_{ik}(s)]} \tag{16}$$

for all firms k, all industries i, and all states s, where

$$\mathrm{mc}_i(s) = m(s)\frac{V'(L(s))}{C((s))U'(C(s))}\left(z_i \frac{dF_i}{dl_i}(s)\right)^{-1} \tag{17}$$

$$v_{ik}(s) = U'(C(s))\frac{d\mathcal{C}}{dc_i}(s)y_i(s)\left(\frac{y_{ik}(s)}{y_i(s)}\right)^{(\theta_i-1)/\theta_i} \tag{18}$$

are, respectively, the firm's nominal marginal cost and stochastic discount factor.

Proposition 2 provides a characterization of the set of sticky-price-implementable allocations in terms of model primitives and the monetary policy instrument $m(s)$. To understand the intuition underlying this result, it is instructive to compare Proposition 2 to its flexible-price counterpart in Proposition 1. First, note that conditions (14) and (15) are identical to (11) and (12), except for a new wedge $\varepsilon_{ik}(s)$. Also, as in Proposition 1, industry-specific wedges $(\chi_1^s, \dots, \chi_n^s)$ are given by (13) and capture the fiscal authority's ability to influence allocations via tax instruments.

The new wedge $\varepsilon_{ik}(s)$ in equations (14) and (15), which is firm-specific and depends on the economy's aggregate state, represents an additional control variable for the government, one that encapsulates the power of monetary policy over real allocations in the presence of nominal rigidities. Similar to the fiscal authority's ability to influence the allocation by setting taxes, the monetary authority can implement different allocations by moving the wedges $\varepsilon_{ik}(s)$ in (14) and (15). This power is non-trivial, but it is also restrained by conditions (16) and (17): unlike the fiscal authority's full control over $(\chi_1^s, \dots, \chi_n^s)$, the monetary authority's choice of the single policy instrument $m(s)$ pins down all wedges $\varepsilon_{ik}(s)$ at the same time.

The constraint on the monetary policy's ability in shaping real allocations can also be seen by the fact that equation (16) implies $\mathbb{E}_{ik}[v_{ik}(s)(\varepsilon_{ik}(s)-1)] = 0$. This means the wedge $\varepsilon_{ik}(s)$ cannot be moved around in an unconstrained manner, as it has to be equal to 1 in expectation irrespective of the policy. This is because these wedges arise only due to "mistakes" by the sticky-price firms in setting their nominal prices. But since firms set their prices optimally given their information sets, they do not make any pricing errors in expectation.

3.4 Power of Monetary Policy

As illustrated in Proposition 2, the monetary authority can use the non-trivial, though restrained, power of monetary policy to implement different real allocations by moving around the wedge $\varepsilon_{ik}(s)$ as a function of the economy's aggregate state. This leads to the natural question of whether monetary policy can fully undo the effect of nominal rigidities. To answer this question, we let $\mathcal{X}^f$ and $\mathcal{X}^s$ denote the set of allocations that are implementable as flexible-price and sticky-price equilibria, respectively. We have the following result:

Theorem 1. *In a multi-sector economy with given preferences and technologies, the sets $\mathcal{X}^f$ and $\mathcal{X}^s$ are disjoint (i.e., $\mathcal{X}^f \cap \mathcal{X}^s = \varnothing$) for a generic set of information structures.*

That is, in general, any allocation implementable as an equilibrium under flexible prices cannot be implemented as an equilibrium under sticky prices. The following result is then immediate.

Corollary 1. *In a multi-sector economy with given preferences and technologies, the first-best allocation is not implementable as a sticky-price equilibrium for a generic set of information structures.*

The intuition for Theorem 1 and Corollary 1 is straightforward. Consider the planner's optimal allocation which can itself be implemented as a flexible-price equilibrium with appropriate industry-level taxes (subsidies). The planner would like relative quantities across industries to move efficiently with productivity shocks, while at the same time ensuring that all firms within each industry produce the same quantity. In order to implement this under flexible prices, relative prices across industries should move with relative productivities, while prices across firms within each industry should be identical. This specific pattern of price movements with productivity shocks is necessary both for flexible-price allocations and in particular for the first-best allocation.

However, inducing this pattern of price movements is in general impossible under sticky prices in a multi-sector economy. In order to ensure that prices are uniform within a particular industry, the monetary authority must target price stability for that industry. This is the typical first-best policy in one-sector New Keynesian models as it implements zero pricing errors within that particular industry. But when there are multiple industries, if monetary policy is used to achieve price stability within one particular industry, it cannot, in general, be used to target price stability in any other industry. That is, monetary policy cannot stabilize prices in all industries at once. And even if it could — for example, because the information structure is such that all firms in any given industry set the same exact price — it is still not sufficient for achieving the first best: by a similar argument, monetary policy cannot in general induce relative prices of any two given industries to move with the corresponding productivity shocks.

While Theorem 1 and Corollary 1 illustrate the limits of monetary policy in a generic multi-sector economy, there are some non-generic, yet important, special cases in which the monetary authority can implement the first-best allocation.

Proposition 3. *If there is a single sticky-price industry i, any flexible-price-implementable allocation can be implemented as a sticky-price equilibrium with a monetary policy that stabilizes the price of i.*

Though focused on a non-generic class of economies, this result nests two important economies as special cases. The first special case is the familiar single-sector New Keynesian model with no markup shocks. As is well-known (and in line with Proposition 3), the first-best allocation can always be implemented by a combination of (i) price stabilization and (ii) an industry-level subsidy that eliminates monopolistic markups, regardless of the nature of information frictions. The second special case is the two-sector model of Aoki (2001), who considers an economy consisting of one flexible industry and one sticky industry subject to Calvo frictions. Aoki shows that stabilizing the price of the sticky industry can implement the first-best allocation. Proposition 3 generalizes this two-sector result to a multi-sector economy with input-output linkages and an arbitrary form of pricing friction.

4 Optimal Monetary Policy

Our results in Theorem 1 and Corollary 1 establish that, in general, monetary policy cannot implement the first-best allocation as a sticky-price equilibrium. In view of these results, we now turn to the study of optimal monetary policy, i.e., the policy that maximizes household welfare over the set of all possible sticky-price-implementable allocations.

In order to obtain closed-form expressions for the optimal policy, we impose a number of functional form assumptions on preferences, technologies, and the nature of price stickiness in the economy. More specifically, we assume that firms in each industry employ Cobb-Douglas technologies to transform labor and intermediate goods into their differentiated products, with the production technology of a firm k in industry i given by

$$y_{ik} = z_i F_i(l_{ik}, x_{i1,k}, \ldots, x_{in,k}) = z_i \zeta_i l_{ik}^{\alpha_i} \prod_{j=1}^{n} x_{ij,k}^{a_{ij}}. \tag{19}$$

In the above expression, l_{ik} denotes the amount of labor hired by the firm, $x_{ij,k}$ is the quantity of good j used as an intermediate input, $\alpha_i \geq 0$ denotes the share of labor in industry i's production technology, z_i is a Hicks-neutral productivity shock that is common to all firms in industry i, and ζ_i is a normalization constant, the value of which only depends on model parameters and is independent of the shocks.[5] The exponent $a_{ij} \geq 0$ in (19) captures the fact that firms in industry i may rely on the goods produced by other industries as intermediate inputs for production. Note that, for all technologies to exhibit constant returns, it must be the case that $\alpha_i + \sum_{j=1}^{n} a_{ij} = 1$.

Input-output linkages in this economy can be summarized by matrix $A = [a_{ij}]$, which, with some abuse of terminology, we refer to as the economy's *input-output matrix*. As is customary in the literature, these input-output linkages can also be represented by a weighted directed graph on n vertices, known as the economy's *production network*. Each vertex in this graph corresponds to an industry, with a directed edge with weight $a_{ij} > 0$ present from vertex j to vertex i if industry j is an input supplier of industry i. We also define the economy's *Leontief inverse* as $L = (I-A)^{-1}$, whose (i, j) element captures the role of industry j as a direct or indirect intermediate input-supplier to industry i.

As in Section 2, we assume a monopolistically competitive market structure within each industry, with the output of firms in industry i aggregated into a sectoral good according to the CES production function (2). We further assume that sector-specific taxes/subsidies in (1) are set to $\tau_i = 1/(1-\theta_i)$ for all i. As discussed in Section 3, this choice undoes the effect of monopolistic markups and guarantees that the flexible-price equilibrium is efficient.

The consumption basket is also a Cobb-Douglas aggregator of the sectoral goods given by

$$\mathcal{C}(c_1, \ldots, c_n) = \prod_{i=1}^{n} (c_i/\beta_i)^{\beta_i},$$

where c_i is the amount of good i consumed and the constants $\beta_i \geq 0$ measure various goods' shares in the household's consumption basket, normalized such that $\sum_{i=1}^{n} \beta_i = 1$. In addition, we assume the

[5] In what follows, we set the value of this constant to $\zeta_i = \alpha_i^{-\alpha_i} \prod_{j=1}^{n} a_{ij}^{-a_{ij}}$. This choice has no bearing on the results, as the sole purpose of this constant is to simplify the analytical expressions.

representative household's preferences (3) are homothetic, with

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad , \quad V(L) = \frac{L^{1+1/\eta}}{1+1/\eta},$$

where recall that C and L denote the household's consumption and labor supply, respectively. Given the static nature of our model, the parameter γ captures both risk aversion and the income elasticity of labor supply, whereas η corresponds to the Frisch elasticity of labor supply.

To specify firms' information structure and the resulting nominal rigidities, we assume all productivity shocks are drawn independently from a log-normal distribution,

$$\log z_i \sim \mathcal{N}(0, \delta^2 \sigma_z^2), \tag{20}$$

with the expected value of all log productivity shocks normalized to zero.[6] Each firm k in industry i then receives a collection of private signals $\omega_{ik} = (\omega_{i1,k}, \dots, \omega_{in,k})$ about the realized productivities given by

$$\omega_{ij,k} = \log z_j + \epsilon_{ij,k} \quad , \quad \epsilon_{ij,k} \sim \mathcal{N}(0, \delta^2 \sigma_i^2), \tag{21}$$

where the noise terms $\epsilon_{ij,k}$ are independent from one another and the productivity shocks.

In this formulation, σ_z^2 measures firms' (common) prior uncertainty about the shocks and σ_i^2 parametrizes the quality of information available to firms in industry i.[7] An increase in σ_i^2 corresponds to an increase in the extent of nominal rigidity of firms in industry i, whereas the extreme case that $\sigma_i^2 = 0$ for all i corresponds to an economy with fully flexible prices. More generally, it is straightforward to verify that

$$\mathbb{E}[\log z_j | \omega_{ik}] = \kappa_i \omega_{ij,k}$$
$$\text{var}[\log z_j | \omega_{ik}] = (1-\kappa_i) \text{var}[\log z_j],$$

where

$$\kappa_i = \sigma_z^2 / (\sigma_z^2 + \sigma_i^2). \tag{22}$$

We thus refer to $\kappa_i \in [0,1]$ as the *degree of price flexibility* of industry i.

The independence assumption imposed on the noise shocks $\epsilon_{ij,k}$ implies that aggregate uncertainty in this economy is solely driven by the productivity shocks $z = (z_1, \dots, z_n)$. As a result, without loss of generality, we can restrict our attention to monetary policies of the form $m(z)$ that only depend on the productivity shocks, as opposed to the entire state of the economy $s = (z, \omega)$.

We conclude by defining the *Domar weight* of industry i as that its equilibrium nominal sales as a fraction of nominal aggregate output, i.e., $\lambda_i = p_i y_i / PC$.

[6] This is merely a normalization, with no bearing on our results.

[7] $\delta > 0$ in equations (20) and (21) is a parameter, which will play a role in the log-linearization of the model in the following subsections.

4.1 Sticky-Price Equilibrium

In this subsection, we use Proposition 2 to characterize the sticky-price equilibrium of the equilibrium described above. We first illustrate that our multi-sector New Keynesian model is isomorphic to a beauty contest game of incomplete information over the production network. We then provide a closed-form representation of equilibrium nominal prices in terms of model primitives and the monetary policy.

To keep the analysis tractable, we work with the log-linearization of the model as $\delta \to 0$, where $\delta > 0$ simultaneously parametrizes the firms' prior uncertainty about (log) productivity shocks (equation (20)) as well as the noise in their private signals (equation (21)). This specific parametrization leads to two desirable features. First, the fact that $\text{var}(\log z_i) = \delta^2 \sigma_z^2$ means that our small-δ approximation is akin to focusing on small departures from the economy's steady-state, as is typical in the New Keynesian literature. Second, scaling $\text{var}(\epsilon_{ij,k})$ with δ^2 ensures that the degree of price flexibility κ_i in equation (22) remains independent of δ.

Proposition 4. *The nominal price set by firm k in industry i is given by*

$$\log p_{ik} = \mathbb{E}_{ik}[\log \text{mc}_i] + o(\delta) \tag{23}$$

$$= \alpha_i \mathbb{E}_{ik}[\log w] - \mathbb{E}_{ik}[\log z_i] + \sum_{j=1}^{n} a_{ij} \mathbb{E}_{ik}[\log p_j] + o(\delta) \tag{24}$$

up to a first-order approximation as $\delta \to 0$.

The above result, which is a consequence of Proposition 2, establishes that, up to a first-order approximation, each firm sets its nominal price equal to its expected marginal cost, given its information set.[8] This is a simple consequence of strategic complementarities between firms within the same industry arising from monopolistic competition.

More importantly, Proposition 4 also illustrates that firms' price-setting behavior can be recast as a "beauty contest" game of incomplete information over the production network (Bergemann, Heumann, and Morris, 2017). More specifically, equation (24) is identical to the first-order conditions of a network game of incomplete information in which firms in industry i choose their log nominal prices to match an industry-specific "fundamental" (given by $\alpha_i \log w - \log z_i$), while simultaneously coordinating with a linear combination of the log prices set by their supplier industries (given by $\sum_{j=1}^{n} a_{ij} \log p_j$). This coordination motive is a direct consequence of strategic complementarities in firms' price-setting behavior in the presence of input-output linkages: all else equal an increase in the price set by firms in an industry increases the incentive of firms in its downstream industries to also raise their prices. As observed by Blanchard (1983) and Basu (1995), such strategic complementarities can amplify the sluggishness of the response of nominal prices to shocks. In our subsequent results, we show that the sluggishness induced by the economy's production network plays a central role in the design of optimal monetary policy.

[8] The relationship between the firm's marginal cost and its nominal price is, in general, more complicated. As we show in the proof of Proposition 2, the nominal price set by the firm is equal to $p_{ik} = \mathbb{E}_{ik}[\text{mc}_i \cdot v_{ik}]/\mathbb{E}_{ik}[v_{ik}]$, where v_{ik} is given by (18). However, in the limit as $\delta \to 0$, this relationship reduces to (23).

We can use the beauty contest representation in Proposition 4 to characterize equilibrium prices. But, note that the right-hand side of (24) is expressed in terms of firms' conditional expectation of nominal wage, which is itself an endogenous variable. That said, our next lemma illustrates that we can simply treat the nominal wage as a policy instrument that is directly controlled by the monetary authority.

Lemma 2. *Suppose $\kappa_i > 0$ for all i. An allocation is implementable by setting the money supply $m(z)$ if and only if it is implementable by setting the nominal wage $w(z)$.*

This lemma therefore establishes a policy isomorphism between setting the money supply and the nominal wage as long as no industry is perfectly sticky. In view of this result, we state all our subsequent results under the assumption that the monetary authority can directly control the nominal wage.

Proposition 5. *The equilibrium vector of log nominal prices is given by*

$$\log p = \big(I - (I - \kappa A)^{-1}(I - \kappa)\big)\,(\mathbf{1}\log w - L\log z) + o(\delta), \tag{25}$$

where $L = (I - A)^{-1}$ is the economy's Leontief inverse and $\kappa = \mathrm{diag}(\kappa_1, \ldots, \kappa_n)$ is a diagonal matrix of price flexibilities defined in (22).

The significance of the above result is threefold. First, it characterizes equilibrium nominal prices as a function of model primitives: the economy's production network, productivity shocks, the extent of nominal rigidities, and the policy set by the monetary authority. This means that equation (25) can be used to investigate the propagation of real and monetary shocks over the production network. Second, Proposition 5 underscores how strategic complementarities arising from the interaction between nominal rigidities and input-output linkages can amplify the sluggishness in price responses to monetary shocks. In particular, equation (25) illustrates that the responsiveness of an industry's nominal price to changes in the monetary instrument (in this case, the nominal wage) is decreasing in not only in the stickiness of that industry, but also that of its direct and indirect upstream suppliers.

4.2 Welfare-Based Policy Objective

As already discussed, we can use the characterization in Proposition 5 to investigate how monetary policy can shape the real allocation in the economy. Determining the optimal policy, however, also requires specifying a policy objective. To this end, we follow Rotemberg and Woodford (1997, 1999) and adopt a welfare-based policy criterion: the monetary authority sets its policy in order to maximize the expected welfare of the representative household in (3).

We express the household's welfare relative to a benchmark with no nominal rigidities, which corresponds to the first-best allocation. More specifically, let W and W^* denote the representative household's welfare in the presence and absence of nominal rigidities, respectively. Similarly, let $\varrho = (p_{ik}, p_i, w)$ and $\varrho^* = (p^*_{ik}, p^*_i, w^*)$ denote the nominal price systems under the two scenarios. Given the indeterminacy of prices in the flexible-price equilibrium, we normalize the nominal wage such that $w^* = w$.[9] We also use $e_{ik} = \log p_{ik} - \log p^*_{ik}$ to denote the "pricing error" of firm k in industry i in the sticky-price economy relative to the benchmark with no nominal rigidities. The cross-sectional average and dispersion of pricing errors within industry i are thus given by

$$\bar{e}_i = \int_0^1 e_{ik} dk \tag{26}$$

$$\vartheta_i = \int_0^1 e_{ik}^2 dk - \left(\int_0^1 e_{ik} dk \right)^2 . \tag{27}$$

We have the following result:

Proposition 6. *The welfare loss due to the presence of nominal rigidities is given by*

$$W - W^* = -\frac{1}{2}\left(\frac{1+1/\eta}{\gamma+1/\eta}\right)\left[\sum_{i=1}^{n} \lambda_i \theta_i \vartheta_i + (\gamma + 1/\eta)\Sigma + \sum_{i=1}^{n} \lambda_i \text{var}_i(\bar{e}_1, \dots, \bar{e}_n) + \text{var}_\beta(\bar{e}_1, \dots, \bar{e}_n)\right] + o(\delta^2), \tag{28}$$

where $\Sigma = (\log C - \log C^*)^2$ *is output gap volatility,* ϑ_i *is price dispersion in industry* i*, and*

$$\begin{aligned} \text{var}_i(\bar{e}_1, \dots, \bar{e}_n) &= \sum_{j=1}^{n} a_{ij} \bar{e}_j^2 - \left(\sum_{j=1}^{n} a_{ij} \bar{e}_j \right)^2 \\ \text{var}_\beta(\bar{e}_1, \dots, \bar{e}_n) &= \sum_{j=1}^{n} \beta_j \bar{e}_j^2 - \left(\sum_{j=1}^{n} \beta_j \bar{e}_j \right)^2 \end{aligned} \tag{29}$$

are the across-industry dispersion of pricing errors from the perspectives of industry i *and the household, respectively.*

Proposition 6 generalizes the well-known expression for welfare loss in single-sector New Keynesian models (e.g., Woodford (2003) and Galí (2008)) to our multi-sector economy with input-output linkages.[10] In particular, equation (28) illustrates that the loss in welfare due to the presence of nominal rigidities manifests itself via four separate terms.

The first term, $\lambda_i \theta_i \vartheta_i$, which is present in the baseline single-sector New Keynesian models, measures welfare losses due to price dispersion *within* each industry i: relative price dispersion in industry i translates into output dispersion and hence misallocation of resources. Note that this term vanishes if all firms in i make their nominal pricing decisions under the same information. Also not surprisingly, the loss due to price dispersion in industry i is weighted by the industry's sales share λ_i in the economy.

The second term on the right-hand side of (28) is also present in the benchmark New Keynesian models and captures loss of welfare due to inefficient supply of labor by the household, i.e., the aggregate labor wedge. This term vanishes as the Frisch elasticity of labor supply $\eta \to 0$.

[9] In view of Lemma 2, this normalization, which is without loss of generality, is equivalent to considering nominal price systems ϱ and ϱ^* in the sticky- and flexible-price equilibria under the same monetary policy. Monetary neutrality in the flexible-price equilibrium guarantees that this choice has no impact on the real allocation.

[10] Euqation (28) also generalizes the corresponding expressions in Woodford (2003, 2010) for a two-sector economy with no input-output linkages. It also reduces to welfare loss in Huang and Liu (2005) when the economy consists of two industries forming a vertical production chain.

In contrast to the first two terms, the third and fourth term on the right-hand side of (28) only appear in multi-sector economies and correspond to welfare losses arising from misallocation of resources *across* industries. To see this, consider the expression $\text{var}_i(\bar{e}_1, \ldots, \bar{e}_n)$ in equation (29). This term measures the dispersion in the average pricing error of i's supplier industries, with higher weights assigned to industries that are more important input-suppliers to i. To be even more specific, suppose i has two suppliers indexed j and r such that $a_{ij} + a_{ir} = 1$. In this case, it is immediate to see that

$$\text{var}_i(\bar{e}_j, \bar{e}_r) = a_{ij}a_{ir}(\bar{e}_i - \bar{e}_j)^2 = a_{ij}a_{ir}\Big(\log(p_j/p_r) - \log(p_j^*/p_r^*)\Big)^2 + o(\delta^2)$$

simply measures the extent to which nominal relative prices of i's inputs diverge from the relative prices that would have prevailed under the flexible-price (and hence efficient) allocation.[11] Finally, note that $\text{var}_i(\bar{e}_1, \ldots, \bar{e}_n) = 0$ whenever industry i has only a single input supplier, as this corresponds to a scenario in which there is no room for misallocation between i's input suppliers.

In summary, Proposition 6 indicates that, in a multi-sector economy, the monetary authority faces an inherent trade-off between minimizing the various losses captured by equation (28). Importantly, as we already establishes in Theorem 1 and Corollary 1, this trade-off can never be circumvented, in the sense that there is no policy that can simultaneously eliminate all forms of misallocation. Also note that, unlike the textbook New Keynesian models, the trade-off faced by the monetary authority arises from the structural properties of the economy's production network as opposed to ad hoc markup shocks.

4.3 Optimal Monetary Policy

We define the *pass-through of monetary policy to industry* i as the sensitivity of i's marginal cost to the nominal wage, i.e.,

$$\rho_i = \frac{d \log \text{mc}_i}{d \log w}. \tag{30}$$

It is easy to verify that that κ_i only depends on the extent of nominal rigidities to industries that are upstream to industry i. In particular, $\rho_i = 1$ if no upstream industry is subject to nominal rigidities, whereas ρ_i becomes smaller the more upstream industries are subject to nominal rigidities. Similarly, we define *monetary non-neutrality of industry* i as

$$\nu_i = 1 - \frac{d \log p_i}{d \log w}. \tag{31}$$

To see why ν_i can be a measure of monetary non-neutrality at industry i, note that $\nu_i = 0$ if industry i's nominal price moves with the nominal wage one-for-one. This would be the case as long as neither i nor any of its upstream industries are subject to any nominal rigidity. In contrast, $\nu_i > 0$, whenever the nominal price of industry i responds to changes in nominal wage sluggishly. Such sluggishness can arise if either i or any of its upstream industries are subject to nominal rigidities.[12]

[11] The interpretation for the term $\text{var}_\beta(\bar{e}_1, \ldots, \bar{e}_n)$ is also identical, with the household replacing industry i as purchaser of various goods.

[12] One can also define a measure of monetary non-neutrality in terms of the monetary policy's effect on real variables (such as industry-level output). While one can transform each representation to another, we find it more convenient to work with the definition in (31).

As already mentioned, both ρ_i and ν_i depend on nominal rigidities in not only industry i but also all its direct and indirect upstream suppliers. This is a consequence of the strategic complementarities in firms' price-setting behavior, highlighted in Proposition 4. In particular, all else equal, more upstream rigidities make a firm's marginal cost less responsive to productivity shocks as well as the monetary policy. Hence, even if the firm can observe the full realization of the shocks, it has less incentives to move its nominal price in accordance with the realized shocks.

We are now ready to present our characterization of optimal monetary policy as a function of the economy's production network and the extent of nominal rigidities throughout the economy.

Recall from our discussion in Subsection 4.2 that the monetary authority faces a trade-off between minimizing the various forms of welfare losses in equation (28). Given the complexity of this expression, and in order to present our results in the most transparent manner, in what follows, we focus on three separate scenarios: in each case the monetary authority is concerned with minimizing the losses in one of the terms on the right-hand side of (28). The optimal policy can then be characterized as a trade-off between the three terms.[13]

Theorem 2. *The monetary authority can stabilize any price index $\sum_{i=1}^n \psi_i \log p_i = 0$, where $\sum_{i=1}^n \psi_i = 1$. Furthermore,*

(a) The policy that minimizes output gap volatility is given by

$$\psi_i^{\text{og}} \propto \lambda_i(1/\kappa_i - 1). \tag{32}$$

(b) The policy that minimizes within-industry misallocation is given by

$$\psi_i^{\text{within}} \propto \lambda_i \theta_i \rho_i (1 - \kappa_i), \tag{33}$$

where ρ_i is the pass-through of monetary policy to industry i.

(c) The policy that minimizes across-industry misallocation is given by

$$\psi_i^{\text{across}} \propto (1/\kappa_i - 1) \sum_{j=0}^n \lambda_j \left[\sum_{r=1}^n a_{jr}\ell_{ri}\nu_r - \left(\sum_{r=1}^n a_{jr}\ell_{ri} \right) \left(\sum_{r=1}^n a_{jr}\nu_r \right) \right], \tag{34}$$

where ν_j is the degree of monetary non-neutrality of j and L is the economy's Leontief inverse.

As a first observation, note that the expressions in (32)–(34) are in general not identical. As a result, in line with Corollary 1, the monetary authority cannot implement the first best by setting all losses equal to zero, and instead, faces a trade-off. Next, note that Theorem 2 illustrates that the optimal policy depends not only on the extent of price rigidities (as captured by $(\kappa_1, \dots, \kappa_n)$) but also on the economy's production network. As a result, it underscores that the nature of optimal policy in our multi-sector framework can be quite distinct from that of baseline single-sector New Keynesian models. To clarify the dependence of optimal policy on model primitives, we next discuss each part of Theorem 2 separately.

[13] See Appendix A for the full characterization of the optimal policy.

The third term on the right-hand side of (35) illustrates that the price stabilization policy of a monetary authority who is purely interested in minimizing output gap volatility targets industries that are (i) larger (higher Domar weight λ_i) and (ii) stickier (higher ρ_i). The first term, on the other hand, provides a characterization of the policy that minimizes price dispersion within each industry. Not surprisingly, it establishes that the monetary authority targets industries that are larger, stickier, and have higher elasticities of substitution, as a higher elasticity translates the same level of price dispersion into larger misallocation. More interestingly, however, it also illustrates that the weight assigned to industry i in the optimal policy is also increasing in the pass-through parameter ρ_s defined in (30). Finally, the middle term in equation (35) characterizes the stabilization policy that minimizes welfare losses arising from price dispersion across various industries. This reduces to the result of Mankiw and Reis (2003) in a horizontal economy.

5 Quantitative Illustration

We use a back of the envelope calculation to derive the optimal price stabilization policy for the U.S. economy.

6 Conclusion

We study optimal monetary policy when firms are arranged in a production network.

A General Form of Optimal Policy

Theorem A.1. *The optimal price stabilization policy* $\sum_{i=1}^{n} \psi_i \log p_i = 0$ *is given by*

$$\psi_s \propto \lambda_s \theta_s \rho_s (1-\kappa_s) + (1/\kappa_s - 1) \sum_{i=0}^{n} \lambda_i \mathrm{cov}_{a_{ij}}(\ell_{js}, \nu_j) + \lambda_s (1/\kappa_s - 1) \left(\frac{\sum_{i=1}^{n} \beta_i \nu_i}{\gamma + 1/\eta} \right), \tag{35}$$

where

$$\mathrm{cov}_{a_{ij}}(\ell_{js}, \nu_j) = \sum_{j=1}^{n} a_{ij} \nu_j \ell_{js} - \left(\sum_{j=1}^{n} a_{ij} \nu_j \right) \left(\sum_{j=1}^{n} a_{ij} \ell_{js} \right),$$

ν_i *is defined in* (31)*, and* ρ_s *is given by* (30).

B Proofs

This appendix contains the proofs and derivations omitted from the main body of the paper. In what follows, we first prove Proposition 2, which we then use to establish Proposition 1. All other results are proved in the same order as they appear in the main text.

Proof of Proposition 2

First, we establish that if an allocation ξ is implementable as a sticky-price equilibrium, then it satisfies equations (14) and (15). To this end, first note that in any sticky-price equilibrium, the first-order conditions corresponding to household's optimization problem are given by

$$V'(L(s)) = \lambda_c(s) w(s) \tag{36}$$

$$U'(C(s)) \frac{d\mathcal{C}}{dc_i}(s) = \lambda_c(s) p_i(\omega_i) \tag{37}$$

for all i, where $\lambda_c(s)$ is the Lagrange multiplier corresponding to the household's budget constraint. From the above it is therefore immediate that

$$V'(L(s)) = \frac{w(s)}{p_i(\omega_i)} U'(C(s)) \frac{d\mathcal{C}}{dc_i}(s)$$

for all i. Furthermore, the fact that $\sum_{i=1}^{n} p_i(\omega_i) c_i(s) = m(s)$ for all $s \in S$ implies that the nominal wage satisfies

$$w(s) = m(s) \frac{V'(L(s))}{C(s) U'(C(s))}, \tag{38}$$

where we are using that the consumption aggregator $\mathcal{C}$ is homogenous of degree 1.

Next, consider the firms' price setting problem. In any sticky price equilibrium, the nominal price set by firm k in industry i is the solution to the optimization problem

$$\max_{p_{ik}} \quad \mathbb{E}_{ik} \left[\mathcal{M}(s) \left((1-\tau_i) p_{ik}(\omega_{ik}) y_{ik}(s) - \mathrm{mc}_i(s) y_{ik}(s) \right) \right] \tag{39}$$

$$\text{s.t.} \quad y_{ik}(s) = \left(p_{ik}(\omega_{ik}) / p_i(\omega_i) \right)^{-\theta_i} y_i(s), \tag{40}$$

where the expectation is taken with respect to the firm's information set and $\mathcal{M}$ denotes the household's nominal stochastic discount factor. Note that we are using the fact that the realized marginal cost of all firms in the same industry are identical, i.e., $\mathrm{mc}_{ik}(s) = \mathrm{mc}_i(s)$ for all $s \in S$ and all firms k in industry i. Plugging in the constraint in to the objective function and taking first-order conditions implies that

$$\mathbb{E}_{ik}\left[\mathcal{M}(s)y_i(s)\left(\frac{p_{ik}(\omega_{ik})}{p_i(\omega_i)}\right)^{-\theta_i}\left((1-\tau_i)\left(\frac{\theta_i-1}{\theta_i}\right)p_{ik}(\omega_{ik}) - \mathrm{mc}_i(s)\right)\right] = 0. \quad (41)$$

On the other hand, note that $U'(C(s))d\mathcal{C}/dc_i(s) - \mathcal{M}(s)p_i(\omega_i) = 0$ for any given industry i, thus implying that the household's nominal stochastic discount factor is given by

$$\mathcal{M}(s) = \frac{1}{p_i(\omega_i)}U'(C(s))\frac{d\mathcal{C}}{dc_i}(s).$$

Plugging the above into equation (41), we obtain

$$\mathbb{E}_{ik}\left[U'(C(s))\frac{d\mathcal{C}}{dc_i}(s)y_i(s)\left(\frac{p_{ik}(\omega_{ik})}{p_i(\omega_i)}\right)^{1-\theta_i}\left((1-\tau_i)\left(\frac{\theta_i-1}{\theta_i}\right) - \frac{\mathrm{mc}_i(s)}{p_{ik}(\omega_{ik})}\right)\right] = 0.$$

Using the demand function $y_{ik}(s) = y_i(s)\left(p_{ik}(\omega_{ik})/p_i(\omega_i)\right)^{-\theta_i}$ one more time, we get

$$\mathbb{E}_{ik}\left[U'(C(s))\frac{d\mathcal{C}}{dc_i}(s)y_i(s)\left(\frac{y_{ik}(s)}{y_i(s)}\right)^{(\theta_i-1)/\theta_i}\left((1-\tau_i)\left(\frac{\theta_i-1}{\theta_i}\right) - \frac{\mathrm{mc}_i(s)}{p_{ik}(\omega_{ik})}\right)\right] = 0.$$

As a result, the nominal price $p_{ik}(\omega_{ik})$ set by firm k in industry i is given by

$$p_{ik}(\omega_{ik}) = \left[(1-\tau_i)\left(\frac{\theta_i-1}{\theta_i}\right)\right]^{-1}\frac{\mathbb{E}_{ik}[v_{ik}(s)\,\mathrm{mc}_i(s)]}{\mathbb{E}_{ik}[v_{ik}(s)]},$$

where $v_{ik}(s)$ is the firms' stochastic discount factor is given by

$$v_{ik}(s) = U'(C(s))\frac{d\mathcal{C}}{dc_i}(s)y_i(s)\left(\frac{y_{ik}(s)}{y_i(s)}\right)^{(\theta_i-1)/\theta_i}.$$

Consequently, the nominal price set by the firm can be written as

$$p_{ik}(\omega_{ik}) = \frac{1}{\chi_i^s\varepsilon_{ik}(s)}\mathrm{mc}_i(s), \quad (42)$$

where $\chi_i^s = (1-\tau_i)(\theta_i-1)/\theta_i$ is a wedge arising due to non-unitary markups and

$$\varepsilon_{ik}(s) = \frac{\mathrm{mc}_i(s)\mathbb{E}_{ik}[v_{ik}(s)]}{\mathbb{E}_{ik}[\mathrm{mc}_i(s)v_{ik}(s)]} \quad (43)$$

is the pricing error due to nominal rigidities. On the other hand, note that the firm's cost minimization implies that

$$\mathrm{mc}_i(s) = w(s)\left(z_i \cdot dF_i/dl_{ik}(s)\right)^{-1}. \quad (44)$$

Therefore, replacing $\mathrm{mc}_i(s)$ in the above equation into (43) and using (38) thus establishes equation (16).

Next, Consequently, we have

$$V'(L(s)) = U'(C(s))\frac{d\mathcal{C}}{dc_i}(s)\frac{w(s)}{p_i(\omega_i)} = U'(C(s))\frac{d\mathcal{C}}{dc_i}(s)\frac{w(s)}{p_{ik}(\omega_{ik})}\left(\frac{y_{ik}(s)}{y_i(s)}\right)^{-1/\theta_i},$$

where the second equality follows from the fact that the demand faced by firm k in industry i satisfies (40). Now replacing for $p_{ik}(\omega_{ik})$ from (42) implies that

$$V'(L(s)) = \chi_i^s \varepsilon_{ik}(s) U'(C(s))\frac{d\mathcal{C}}{dc_i}(s)\frac{w(s)}{\mathrm{mc}_i(s)}\left(\frac{y_{ik}(s)}{y_i(s)}\right)^{-1/\theta_i}.$$

On the other hand, recall that the firm's cost satisfies (44). Therefore, replacing for $\mathrm{mc}_i(s)$ from (44) into the above equation establishes (14).

The proof is therefore complete once we establish (15). Recall that the household's first-order condition requires that (37) is satisfies for all pairs of industries. As a result, for any pairs of industries i and j, we have

$$\frac{d\mathcal{C}}{dc_j}(s) = \frac{p_j(\omega_j)}{p_i(\omega_i)}\frac{d\mathcal{C}}{dc_i}(s) = \chi_i^s \varepsilon_{ik}(s)\frac{d\mathcal{C}}{dc_i}(s)\frac{p_j(\omega_j)}{\mathrm{mc}_i(s)}\left(\frac{y_{ik}(s)}{y_i(s)}\right)^{-1/\theta_i},$$

where once again we are using (40) and (42). On the other hand, whenever industry j is an input-supplier of industry i, the cost minimization of firm ik implies that

$$\mathrm{mc}_i(s) = p_j(\omega_j)\left(z_i \cdot dF_i/dx_{ij,k}(s)\right)^{-1}.$$

The juxtaposition of the last two equations now establishes (15) and completes the proof. □

Proof of Proposition 1

We establish Proposition 1 as a special case of Proposition 2. Recall from Definitions 1 and 2 that a flexible-price equilibrium is sticky-price equilibrium if all productivity shocks are common knowledge. As a result, the right-hand sides of equations (17) and (18) are both measurable with respect to ω_{ik} for all k and all i. Consequently, (16) implies that $\varepsilon_{ik}(s) = 1$ for all $k \in [0,1]$, all $i \in I$, and all $s \in S$. Plugging this into equations (14) and (15) then immediately establishes (11) and (12), thus completing the proof. □

Proof of Theorem 1

Suppose that there exists some feasible allocation that is implementable as an equilibrium under both flexible and sticky prices. By Propositions 1 and 2, this allocation must simultaneously satisfy equations (11)–(12) and (14)–(15). As a result,

$$\chi_i^f = \chi_i^s \varepsilon_{ik}(s)\left(\frac{y_{ik}(s)}{y_i(s)}\right)^{-1/\theta_i}$$

for all firms $(i,k) \in I \times [0,1]$ and all states $s \in S$, where the wedge function $\varepsilon_{ik}(s)$ satisfies equation (16). Since in any flexible price allocation, the output of all firms in the same industry coincide, we have $y_{ik}(s) = y_i(s)$ for all $k \in [0,1]$, and as a result,

$$\chi_i^f = \chi_i^s \varepsilon_{ik}(s)$$

for all $(i,k) \in I \times [0,1]$ and all $s \in S$. Note that, by assumption, the scalars χ_i^f and χ_i^s representing the fiscal policy are assumed to be invariant to the state s and do not depend on the firm index k. Therefore, the above equation implies that $\varepsilon_{ik}(s)$ is also independent of s and k for all i, i.e., $\varepsilon_{ik}(s) = \varepsilon_i$.

On the other hand, note that equation (16) guarantees that $\mathbb{E}_{ik}[v_{ik}(s)(\varepsilon_{ik}(s) - 1)] = 0$, where $v_{ik}(s)$ is given by equation (18). As a result,

$$(\varepsilon_i - 1)\mathbb{E}_{ik}[v_{ik}(s)] = 0$$

for all i. But note that $v_{ik}(s) > 0$ for all $s \in S$ in any feasible allocation, which guarantees that $\varepsilon_i = 1$ for all i. Also, recall from the proof of Proposition 2, that in any sticky price equilibrium, each firm's price and marginal cost satisfy equation (42). Therefore,

$$p_{ik}(\omega_{ik}) = \frac{1}{\chi_i^s}\mathrm{mc}_i(s). \tag{45}$$

The above observation has two implications. First, given that the right-hand side of the above expression is independent of k, it implies that all firms within the same industry set the same nominal price. Thus, we can write $p_{ik}(\omega_{ik}) = p_i(\omega_i)$, with the understanding that $p_i(\omega_i)$ is measurable with respect to the information set of any individual firm k in industry i. Second, equation (45) also implies that the marginal cost of industry i is measurable with respect to the information set of all firms in that industry. Finally, it establishes that, whenever an allocation can be implemented as both a sticky and a flexible price equilibrium, all firms employ constant markups to set their nominal prices. Consequently, we can write i's nominal price as a function of industry i's input prices as

$$p_i(\omega_i) = \frac{1}{\chi_i^s z_i(s)} C_i\big(w(s), p_1(\omega_1), \dots, p_n(\omega_n)\big),$$

where C_i is homogenous of degree 1 and is the convex conjugate and industry i's production function F_i. Dividing both sides of the above equation by the nominal price, we obtain

$$p_i(\omega_i)/w(s) = (\chi_i^s z_i)^{-1} C_i\big(1, p_1(\omega_1)/w(s), \dots, p_n(\omega_n)/w(s)\big). \tag{46}$$

We thus obtain a system of n equations and n relating all industries nominal prices relative to the wage to the productivity shocks and the vector of wedges χ^s. Since we have assumed that labor is an essential input for the production technology of all industries, Theorem 1 of Stiglitz (1970) guarantees that there exists a unique collection of relative prices that solves system of equations (46). Hence, for any given industry i, there exists a function g_i such that the nominal price set by firms in industry i can be written as

$$p_i(\omega_i) = w(s) \cdot g_i(\chi_1^s z_1, \dots, \chi_n^s z_n),$$

where the collection of functions $(g_1, \dots, g_n)$ only depend on the production functions $(F_1, \dots, F_n)$ and are independent of the economy's information structure. But recall that the left-hand side of the above equation is measurable with respect to the information set of all firms in industry i. Therefore, a feasible allocation is implementable as an equilibrium under both flexible and sticky prices only if there exists a function $w(s)$ such that

$$w(s) \cdot g_i(\chi_1^s z_1, \dots, \chi_n^s z_n) \in \sigma(\omega_{ik})$$

simultaneously for all firms k in all industries i, a relationship that can only hold for non-generic information structures. □

Proof of Proposition 3

Suppose all firms in all industries $j \neq i$ set their prices under complete information. This means that the aggregate state s is measurable with respect to ω_{jk} for all $k \in [0,1]$ and all $j \neq i$. Hence, by equation (16), the wedge functions of firms in industry $j \neq i$ are equal to

$$\varepsilon_{jk}(s) = \frac{\text{mc}_j(s)\mathbb{E}_{jk}[v_{jk}(s)]}{\mathbb{E}_{jk}[\text{mc}_j(s)v_{jk}(s)]} = \frac{\text{mc}_j(s)v_{jk}(s)}{\text{mc}_j(s)v_{jk}(s)} = 1 \tag{47}$$

for all s. Let the monetary policy function $m(s)$ be given by

$$m(s) = Mz_i \frac{U'(C(s))C((s))}{V'(L(s))} \frac{dF_i}{dl_i}(s), \tag{48}$$

for some constant M that does not depend on the aggregate state. By equation (17), such a policy induces $\text{mc}_i(s) = M$ for all s. As a result, equation (16) implies that $\varepsilon_{ik}(s) = 1$ for all firms $k \in [0,1]$. This, alongside equation (47), therefore establishes that the above policy can eliminate all wedges arising from nominal rigidities, thus reducing equations (14)–(15) to (11)–(12). In other words, any flexible-price-implementable allocation can be implemented as part of a sticky-price equilibrium.

The proof is therefore complete once we show that the policy in (48) stabilizes the price of industry i. As we already established, such a policy induces $\text{mc}_i(s) = M$ for all s. Thus, by equation (42), $p_{ik}(\omega_{ik}) = M/\chi_i^s$, which means that the nominal price set by the firms in industry i is invariant to the economy's aggregate state. □

Proof of Proposition 4

Recall from equations (42) and (43) in the proof of Proposition 2 that the nominal price set by firm k in industry i is given by

$$p_{ik} = \frac{\mathbb{E}_{ik}[\text{mc}_i \cdot v_{ik}]}{\mathbb{E}_{ik}[v_{ik}]},$$

where v_{ik} is given by equation (18) and we are using the fact that $\chi_i^s = 1$. Consequently,

$$\log p_{ik} - \mathbb{E}_{ik}[\log \text{mc}_i] = \log \mathbb{E}_{ik}\left[e^{\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}] + \log \text{mc}_i - \mathbb{E}_{ik}[\log \text{mc}_i]}\right] - \log \mathbb{E}_{ik}\left[e^{\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}]}\right].$$

Since the standard deviations of log productivity shocks in (20) and noise shocks in (21) scale linearly in δ, it must be the case that $\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}] = o(\delta)$ and $\log \text{mc}_i - \mathbb{E}_{ik}[\log \text{mc}_i] = o(\delta)$ as $\delta \to 0$. As a result,

$$\begin{aligned}\log p_{ik} - \mathbb{E}_{ik}[\log \text{mc}_i] = \log\Big(1 + \mathbb{E}_{ik}\Big[\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}] + \log \text{mc}_i - \mathbb{E}_{ik}[\log \text{mc}_i]\Big] + o(\delta)\Big) \\ - \log\Big(1 + \mathbb{E}_{ik}\Big[\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}]\Big] + o(\delta)\Big),\end{aligned}$$

which in turn implies that $\log p_{ik} - \mathbb{E}_{ik}[\log \text{mc}_i] = \log(1 + o(\delta)) - \log(1 + o(\delta))$. Hence,

$$\log p_{ik} = \mathbb{E}_{ik}[\log \text{mc}_i] + o(\delta)$$

as $\delta \to 0$. Finally, the fact that all firms in industry i have Cobb-Douglas production technologies, as in (19), implies that their marginal cost is given by $\log \mathrm{mc}_i = \alpha_i \log w - \log z_i + \sum_{j=1}^n a_{ij} \log p_j$, thus completing the proof. $\square$

Proof of Proposition 5

As a first observation, we note that since all noise shocks $\epsilon_{ij,k}$ in firms' private signals are idiosyncratic and of order δ, the log-linearization of all industry-level and aggregate variables only depends on the productivity shocks. More specifically, let

$$\log w = \sum_{i=1}^n \phi_i \log z_i + o(\delta) \tag{49}$$

$$\log p_i = \sum_{j=1}^n b_{ij} \log z_j + o(\delta) \tag{50}$$

denote, respectively, the log-linearization of the nominal wage and the nominal price of sectoral good i as $\delta \to 0$. On the other hand, recall from Proposition 4 that the log nominal price set by firm k in industry i is given by (24) up to a first-order approximation as $\delta \to 0$. Therefore,

$$\begin{aligned}\log p_{ik} &= \alpha_i \sum_{j=1}^n \phi_j \mathbb{E}_{ik}[\log z_j] - \mathbb{E}_{ik}[\log z_i] + \sum_{j=1}^n \sum_{r=1}^n a_{ij} b_{jr} \mathbb{E}_{ik}[\log z_r] + o(\delta) \\ &= \alpha_i \sum_{j=1}^n \phi_j \kappa_i \omega_{ij,k} - \kappa_i \omega_{ii,k} + \sum_{j=1}^n \sum_{r=1}^n a_{ij} b_{jr} \kappa_i \omega_{ir,k} + o(\delta),\end{aligned}$$

where κ_i is the degree of price flexibility of firms in industry i in (22). Integrating both sides of the above equation over all firms k in industry i Now, integrating over all k implies that

$$\log p_i = \kappa_i \alpha_i \sum_{j=1}^n \phi_j \log z_j - \kappa_i \log z_i + \kappa_i \sum_{j=1}^n \sum_{r=1}^n a_{ij} b_{jr} \log z_r + o(\delta),$$

where we are using the fact that $\log p_i = \int_0^1 \log p_{ik} dk + o(\delta)$. The juxtaposition of the above equation with equation (50) therefore implies that $b_{ij} = \kappa_i \alpha_i \phi_j - \kappa_i \mathbb{I}_{\{j=i\}} + \kappa_i \sum_{r=1}^n a_{ir} b_{rj}$, which in vector form can be written as

$$B = \mathrm{diag}(\kappa) \alpha \phi' - \mathrm{diag}(\kappa) + \mathrm{diag}(\kappa) AB = \mathrm{diag}(\kappa)(I - A)\mathbf{1}\phi' - \mathrm{diag}(\kappa) + \mathrm{diag}(\kappa) AB,$$

where the second equality is a consequence of the fact that, due to constant returns, $\alpha = (I - A)\mathbf{1}$. Solving for matrix B therefore implies that

$$\begin{aligned}B &= (I - \mathrm{diag}(\kappa)A)^{-1} \mathrm{diag}(\kappa)(I - A) \left(\mathbf{1}\phi' - L\right) \\ &= \left[I - (I - \mathrm{diag}(\kappa)A)^{-1}(I - \mathrm{diag}(\kappa))\right] \left(\mathbf{1}\phi' - L\right).\end{aligned}$$

Multiplying both sides of by $\log z$ and using equations (49) and (50) then establishes (25). $\square$

Proof of Proposition 6

We prove this result in three steps. First, we solve for household welfare in terms of nominal prices and the nominal wage. We then compare the result to welfare under the first-best allocation to obtain an expression for welfare loss, given all nominal prices. Finaally, we provide a quadratic log-approximation to the welfare loss in terms of the distribution of firm-level pricing errors.

Expressing welfare in terms of nominal prices: As our first step, we obtain an expression for welfare as a function of all nominal prices and the wage.

Recall from equation (40) that the output of firm k in industry i is given by $y_{ik} = y_i(p_{ik}/p_i)^{-\theta_i}$, whereas cost minimization implies that the firm's demand for sectoral good j is $x_{ij,k} = a_{ij}y_{ik}\,\mathrm{mc}_i\,/p_j$. Therefore, total demand for sectoral good j by firms in industry i is $\int_0^1 x_{ij,k}dk = a_{ij}\mathrm{mc}_i/p_j \int_0^1 y_{ik}dk = a_{ij}p_iy_i\varepsilon_i/p_j$, where

$$\varepsilon_i = \mathrm{mc}_i p_i^{\theta_i-1}\int_0^1 p_{ik}^{-\theta_i}dk. \tag{51}$$

Hence, market clearing (7) for sectoral good i implies that $p_iy_i = p_ic_i + \sum_{j=1}^n a_{ji}p_jy_j\varepsilon_j$. Dividing both sides by aggregate nominal demand PC and using the fact that $p_ic_i = \beta_iPC$, we obtain

$$\lambda_i = \beta_i + \sum_{j=1}^n a_{ji}\lambda_j\varepsilon_j, \tag{52}$$

where λ_i is the Domar weight of industry i. On the other hand, the household's budget constraint is given by

$$PC = wL + \sum_{i=1}^n \int_0^1 \pi_{ik}dk = wL + \sum_{i=1}^n\left(p_iy_i - \mathrm{mc}_i\int_0^1 y_{ik}dk\right) = wL + \sum_{i=1}^n p_iy_i(1-\varepsilon_i),$$

thus implying that $PC = wL/(1-\sum_{i=1}^n \lambda_i(1-\varepsilon_i))$. Furthermore, note that the household's optimal labor supply requires that $L^{1/\eta} = C^{-\gamma}w/P$. Therefore, solving for household's aggregate consumption and aggregate labor supply from the last two equations, we obtain

$$\begin{aligned} C &= (w/P)^{\frac{1+\eta}{1+\eta\gamma}}\left(1-\sum_{i=1}^n\lambda_i\left(1-\varepsilon_i\right)\right)^{-\frac{1}{1+\eta\gamma}} \\ L &= (w/P)^{\frac{\eta(1-\gamma)}{1+\eta\gamma}}\left(1-\sum_{i=1}^n\lambda_i\left(1-\varepsilon_i\right)\right)^{\frac{\eta\gamma}{1+\eta\gamma}}. \end{aligned} \tag{53}$$

Plugging the above into (3) now characterizes household welfare as a function of all nominal prices and the nominal wage via the expression

$$W = \frac{1}{1-\gamma}(w/P)^{\frac{(1+\eta)(1-\gamma)}{1+\eta\gamma}}\left(1-\sum_{i=1}^n\lambda_i\left(1-\varepsilon_i\right)\right)^{-\frac{1-\gamma}{1+\eta\gamma}}\left(1-\frac{\eta(1-\gamma)}{1+\eta}\left(1-\sum_{i=1}^n\lambda_i\left(1-\varepsilon_i\right)\right)\right), \tag{54}$$

where ε_i is given by (51) and only depends on nominal prices.

Welfare loss: We now use the above expression to determine the first-best welfare W^* under flexible prices, which we then use to calculate the welfare loss arising from nominal rigidities.

Recall that, in the flexible-price equilibrium, all firms in industry i set identical prices and charge no markups, i.e., $\mathrm{mc}_i^* = p_{ik}^* = p_i^*$. Therefore, equation (51) implies that $\varepsilon_i^* = 1$ for all industries i. Plugging this back into (54) leads to $W^* = \frac{1}{1-\gamma}(w/P^*)^{\frac{(1+\eta)(1-\gamma)}{1+\eta\gamma}}$, where recall that, by assumption, $w = w^*$. Hence, we can rewrite (54) as

$$W = W^*(P/P^*)^{\frac{(1+\eta)(\gamma-1)}{1+\eta\gamma}} \left(1 - \sum_{i=1}^{n} \lambda_i \left(1 - \varepsilon_i\right)\right)^{-\frac{1-\gamma}{1+\eta\gamma}} \left(1 + \frac{\eta(1-\gamma)}{1+\eta\gamma} \sum_{i=1}^{n} \lambda_i \left(1 - \varepsilon_i\right)\right). \tag{55}$$

Similarly, we can use (53) to relate aggregate output in the sticky-price equilibrium to that in the flexible-price equilibrium in the form of

$$C = C^*(P/P^*)^{-\frac{1+\eta}{1+\eta\gamma}} \left(1 - \sum_{i=1}^{n} \lambda_i \left(1 - \varepsilon_i\right)\right)^{-\frac{1}{1+\eta\gamma}}. \tag{56}$$

The juxtaposition of the last two equations now implies that welfare in the sticky- and flexible-price equilibria are related to one another via

$$W = W^* \left(C/C^*\right)^{1-\gamma} \left(1 + \frac{\eta(1-\gamma)}{1+\eta\gamma} \left(1 - \left(C/C^*\right)^{-(1+\eta\gamma)} \left(P/P^*\right)^{-(1+\eta)}\right)\right). \tag{57}$$

Second-order approximations: We next derive quadratic log approximations of our main equations, (51), (52), (56), and (57), as $\delta \to 0$.

First consider equation (51). Taking logarithms from both sides and using the fact that $\log \mathrm{mc}_i = \alpha \log w - \log z_i + \sum_{j=1}^{n} a_{ij} \log p_j$ implies that

$$\log \varepsilon_i = \sum_{j=1}^{n} a_{ij}(\log p_j - \log p_j^*) + (\theta_i - 1)(\log p_i - \log p_i^*) + \log \int_0^1 (p_{ik}/p_i^*)^{-\theta_i} dk.$$

Consequently, under a second-order approximation,

$$\log \varepsilon_i = \sum_{j=1}^{n} a_{ij} e_j - e_i + \frac{1}{2} \sum_{j=1}^{n} a_{ij}(1-\theta_j)\vartheta_j + \frac{1}{2}(2\theta_i - 1)\vartheta_i + o(\delta^2), \tag{58}$$

where $\bar{e}_i$ and ϑ_i are the cross-sectional average and dispersion of pricing errors in industry i defined in equations (26) and (27), respectively.

Next, consider equation (52). Recall that $\varepsilon_i^* = 1$ in the flexible-price equilibrium. As a result, Domar weights in the flexible-price equilibrium satisfy $\lambda_i^* = \beta_i + \sum_{j=1}^{n} a_{ji} \lambda_j^*$ and a first-order approximation of Domar weights in the sticky-price equilibrium as $\delta \to 0$ is given by

$$\lambda_i = (1 - \log \varepsilon_i)\lambda_i^* + \sum_{j=1}^{n} \ell_{ji} \lambda_j^* \log \varepsilon_j + o(\delta), \tag{59}$$

where ℓ_{ji} is the (j,i) element of the economy's Leontief inverse $L=(I-A)^{-1}$. As a result,

$$\log\left(1-\sum_{i=1}^{n}\lambda_i(1-\varepsilon_i)\right)=\sum_{i=1}^{n}\lambda_i\log\varepsilon_i+\frac{1}{2}\sum_{i=1}^{n}\lambda_i\log^2\varepsilon_i-\frac{1}{2}\left(\sum_{i=1}^{n}\lambda_i\log\varepsilon_i\right)^2+o(\delta^2)$$
$$=\sum_{i=1}^{n}\lambda_i^*\log\varepsilon_i+\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_j^*\ell_{ji}\log\varepsilon_j\log\varepsilon_i-\frac{1}{2}\sum_{i=1}^{n}\lambda_i^*\log^2\varepsilon_i-\frac{1}{2}\left(\sum_{i=1}^{n}\lambda_i^*\log\varepsilon_i\right)^2+o(\delta^2),$$

where the second equality follows from replacing λ_i by its first-order approximation in (59). By also replacing $\log\varepsilon_i$ by its first-order approximation in (58) and simplifying the result, we obtain the following second-order approximation:

$$\log\left(1-\sum_{i=1}^{n}\lambda_i(1-\varepsilon_i)\right)=-(\log P-\log P^*)+\frac{1}{2}\sum_{i=1}^{n}\lambda_i\theta_i\vartheta_i$$
$$+\frac{1}{2}\sum_{i=1}^{n}\lambda_i\bar{e}_i^2-\frac{1}{2}\sum_{i=1}^{n}\lambda_i\left(\sum_{j=1}^{n}a_{ij}\bar{e}_j\right)^2-\frac{1}{2}\left(\sum_{i=1}^{n}\beta_i\bar{e}_i\right)^2+o(\delta^2). \quad (60)$$

where we are using the fact that $\log P-\log P^*=\sum_{i=1}^{n}\beta_i\bar{e}_i+\frac{1}{2}\sum_{i=1}^{n}\beta_i(1-\theta_i)\vartheta_i+o(\delta^2)$.

Next, consider equation (56). A second-order log-approximation leads to

$$\log C-\log C^*=-\frac{1}{\gamma+1/\eta}(\log P-\log P^*)-\frac{1}{2(1+\eta\gamma)}\left[\sum_{i=1}^{n}\lambda_i\theta_i\vartheta_i\right.$$
$$\left.+\sum_{i=1}^{n}\lambda_i\Big(e_i^2-\Big(\sum_{j=1}^{n}a_{ij}\bar{e}_j\Big)^2\Big)-\left(\sum_{i=1}^{n}\beta_i\bar{e}_i\right)^2\right]+o(\delta^2), \quad (61)$$

where we are using (60) and . Note that the above expression also implies that, up to a first-order approximation,

$$\log C-\log C^*=-\frac{1}{\gamma+1/\eta}(\log P-\log P^*)+o(\delta). \quad (62)$$

Finally, consider the second-order approximation of for welfare loss in (57) as $\delta\to 0$. We have

$$\log(W/W^*)=(1-\gamma)(1+\eta)\left[(\log C-\log C^*)+\frac{1}{\gamma+1/\eta}(\log P-\log P^*)-\frac{1}{2}\eta(\log C-\log C^*)^2\right.$$
$$\left.-\frac{1}{2}\eta\frac{(1+\eta)^2}{(1+\eta\gamma)^2}(\log P-\log P^*)^2-\frac{1+\eta}{\gamma+1/\eta}(\log C-\log C^*)(\log P-\log P^*)\right]+o(\delta^2).$$

To simplify the above, replace for $\log P-\log P^*$ in the last two terms from its first-order approximation in (62) to obtain

$$\log(W/W^*)=(1-\gamma)(1+\eta)\left(\log C-\log C^*+\frac{1}{\gamma+1/\eta}(\log P-\log P^*)-\frac{1}{2\eta}\Sigma\right)+o(\delta^2),$$

where $\Sigma=(\log C-\log C^*)^2$ is the volatility of output gap. Finally, replacing for $\log C-\log C^*$ in terms of its second-order approximation in (61) leads to

$$\log(W/W^*)=-\frac{1}{2}\frac{(1-\gamma)(1+\eta)}{1+\eta\gamma}\left[\sum_{i=1}^{n}\lambda_i\theta_i\vartheta_i+\sum_{j=1}^{n}\lambda_j\bar{e}_j^2-\sum_{i=1}^{n}\lambda_i\Big(\sum_{j=1}^{n}a_{ij}\bar{e}_j\Big)^2-\Big(\sum_{i=1}^{n}\beta_i\bar{e}_i\Big)^2+\frac{1+\eta\gamma}{\eta}\Sigma\right]+o(\delta^2).$$

Further simplification reduces the above to

$$\log(W/W^*) = -\frac{1}{2}\frac{(1-\gamma)(1+1/\eta)}{(\gamma+1/\eta)} \left[\sum_{i=1}^{n} \lambda_i \theta_i \vartheta_i + (\gamma + 1/\eta)\Sigma \right.$$
$$\left. + \sum_{i=1}^{n} \lambda_i \mathrm{var}_i(\bar{e}_1, \ldots, \bar{e}_n) + \mathrm{var}_\beta(\bar{e}_1, \ldots, \bar{e}_n)\right] + o(\delta^2)$$

On the other hand, observe that $W - W^* = W^* \log(W/W^*) + o(\delta^2)$ and $W^* = 1/(1-\gamma) + O(\delta)$. The juxtaposition of these observations with the above equation establishes (28). □

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