Low Interest Rates, Market Power, and Productivity Growth

Ernest Liu, Atif Mian, and Amir Sufi
Introduction

- Secular decline in the long-run real interest rate over past decades
- What is the supply-side response to low interest rates?
  - investment decisions, market concentration, and productivity growth
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r
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Euler equation
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Standard endogenous growth models
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Exogenous growth
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Our model
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Profit share, markup, and concentration
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Business dynamism
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Leader-follower productivity gap
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We find low rates near zero guaranteed to be contractionary

- A model of dynamic competition based on the patent race literature

- Very low interest rate \( r \to 0 \) is guaranteed to be contractionary
- A “fundamental result”: no financial frictions or Keynesian forces

Intuitions: under low \( r \), firms are effectively more “patient”

- For the leader, small prospect of being caught up implies large change in value
- For the follower, low rates motivate investment only if future profits are attainable
- Market leadership becomes endogenously unattainable for the follower
We find low rates near zero guaranteed to be contractionary

- A model of dynamic competition based on the patent race literature

- We find: a reduction in interest rate has an “anti-competitive” effect
  - raises market concentration and profits
  - causes market power to become more persistent
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Model predictions

\[ g(r) \text{ has an inverted-U shape} \]
Model predictions

Other steady-state predictions as $r$ declines:

- Profit share, markups, concentration, leader-follower productivity gap
- Business dynamism, churn, and creative destruction

Short-run predictions:

- Declines in $r$ benefit leaders (relative to followers), especially when initial $r$ is low

$g(r)$ has an inverted-U shape
Model

- Continuous time; a continuum (measure 1) of markets

- Each market has two forward-looking firms competing for profits
  - interest rate $r$: rate at which future payoffs are discounted
    
    $$v(t) = \int_{0}^{\infty} e^{-r\tau} \{\pi(t + \tau) - c(t + \tau)\} d\tau$$

- State variable $s \in \{0, 1, \cdots, \infty\}$: a “ladder” of productivity differences
  - $s = 0$: two firms are said to be “neck-to-neck”
  - $s \neq 0$: one firm is the temporary leader while the other is the follower

- Productivity gap $s$ maps into market structure and flow profits: $\{\pi_s, \pi_{-s}\}_{s=0}^{\infty}$
  - assume $\pi_s$, $-\pi_{-s}$, and $(\pi_s + \pi_{-s})$ are bounded, weakly increasing, and eventually concave
Microfoundation for the static block

- Firm with productivity $z$ has marginal cost of production $\lambda^{-z}$
  - state variable is defined as the (log-)productivity difference $s \equiv |z_1 - z_2|$
- Firms produce imperfect substitutes and face a joint CES demand with unit expenditure:
  $$\max_{q_{i1}, q_{i2}} \left( \frac{q_{i1}^{\sigma-1}}{\sigma} + \frac{q_{i2}^{\sigma-1}}{\sigma} \right)^{\frac{\sigma}{\sigma-1}} \text{ s.t. } p_{i1}q_{i1} + p_{i2}q_{i2} = 1$$
- Bertrand competition $\implies$ flow profits $\pi_s$ are functions of the productivity gap $s$ and not levels
  - homogeneous of degree zero with respect to productivity
- In the limiting case of perfect substitutes ($\sigma = \infty$),
  $$\pi_{-s} = 0, \quad \pi_s = 1 - e^{-s}$$
Microfoundation for the static block

- Firm with productivity $z$ has marginal cost of production $\lambda^{-z}$
  - state variable is defined as the (log-)productivity difference $s \equiv \left|z_1 - z_2\right|$  
- Firms produce imperfect substitutes and face a joint CES demand with unit expenditure:
  \[
  \max_{q_{i1}, q_{i2}} \left( \frac{\sigma^{-1}}{\sigma} q_{i1}^{\sigma^{-1}} + q_{i2}^{\sigma^{-1}} \right)^{\sigma^{-1}} \\
  \text{s.t. } p_{i1} q_{i1} + p_{i2} q_{i2} = 1
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  \[
  \pi_{-s} = 0, \quad \pi_s = 1 - e^{-s}
  \]
- Macro version: within-period consumer utility function $U(t) = \ln Y(t) - L(t)$;
  \[
  \ln Y(t) = \int_0^1 \ln y(t; \nu) d\nu, \quad y(t; \nu) = \left( q_{i1}(t; \nu)^{\frac{\sigma-1}{\sigma}} + q_{i2}(t; \nu)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}; \\
  \text{normalize prices so that the value of total output is one } P(t) Y(t) = 1.
Firms invest in order to enhance market position

- binary decision: incur cost $c$ for Poisson rate $\eta$ to gain productivity

Given investments $\eta_s, \eta_{-s} \in \{0, \eta\}$, the state $s$ evolves to

\[
\begin{cases}
  s + 1 \quad \text{with rate } \eta_s \\
  s - 1 \quad \text{with rate } (\eta_{-s} + \kappa)
\end{cases}
\]

$\kappa < \eta$ is the exogenous rate of catching up

Catch up is gradual: no leapfrogging

Firms are forward-looking and maximize present-discounted-value $v_s$:

\[
rv_s = \pi_s + (\eta_{-s} + \kappa)(v_{s-1} - v_s) + \max \{\eta(v_{s+1} - v_s) - c, 0\}
\]
Symmetric MPE: collection of $\{\eta_s, \nu_s\}_{s=-\infty}^{\infty}$
Symmetric MPE: collection of \( \{\eta_s, \nu_s\}_{s=-\infty}^{\infty} \)
Symmetric MPE: collection of \{\eta_s, \nu_s\}_{s=-\infty}^{\infty}

- Equilibrium induces steady-state distribution \{\mu_s\}_{s=0}^{\infty} of market structure

\[ \eta_s \mu_s = (\eta_{-(s+1)} + \kappa) \mu_{s+1} \]

- Aggregate productivity growth: the average growth rate across market structures

\[ g \equiv \sum_{s=0}^{\infty} \mu_s \mathbb{E}[g_s] \]
Equilibrium structure: leader dominance

**Lemma.** Leader invests (weakly) more than the follower does.
Equilibrium structure: leader dominance

Leader cannot stop investing first—proof by contradiction

- transient monopoly power $\implies$ follower incentive has to be low

```
0 1 2 3 4 State
```
Steady-state, two regions, and growth

- Competitive region: State tends to transition down
- Monopolistic region: State tends to transition up
Steady-state, two regions, and growth

Lemma. In a steady state, productivity growth rate and aggregate investment are increasing in the fraction of markets in the competitive region and decreasing in the fraction of markets in the monopolistic region:

\[
\frac{g}{\ln \lambda} = \left( \sum_{s=1}^{k} \mu_s \right) \times (\eta + \kappa) + \left( \sum_{s=k+1}^{n+1} \mu_s \right) \times \kappa.
\]

- Fraction of markets in the competitive region
- Fraction of markets in the monopolistic region
As $r \to 0$, both regions expand indefinitely

- Traditional expansionary effect: low interest rate raises investments in all states
As \( r \to 0 \), the monopolistic region dominates

**Proposition.** As \( r \to 0 \):

1. The monopolistic region becomes **absorbing**: \( \sum_{s=k+1}^{n+1} \mu_s \to 1 \);
2. Monopoly power becomes **permanently persistent**;
3. Productivity gap between leaders and followers **diverges**: \( \lim_{r \to 0} \sum_{s=0}^{\infty} \mu_s s = \infty \);
4. Aggregate investment drops and productivity growth **slows down**: \( \lim_{r \to 0} g = \kappa \cdot \ln \lambda \).
Value functions and intuition

\[ \lim_{r \to 0} r v_n > 0, \quad \lim_{r \to 0} r v_0 = 0, \]

\[ \lim_{r \to 0} r (v_{k+1} - v_k) > 0. \]
Value functions and intuition

\begin{align*}
l\lim_{r \to 0} rv_n &> 0, \quad l\lim_{r \to 0} rv_0 = 0, \\
l\lim_{r \to 0} r(v_{k+1} - v_k) &> 0.
\end{align*}

- **Leader:**
  - falling to the competitive region is costly
  - keeps investing to ensure such probability is vanishingly small

![Graph showing the transition between competitive and monopolistic regions](graph.png)
Value functions and intuition

\[ \lim_{r \to 0} r v_n > 0, \quad \lim_{r \to 0} r v_0 = 0, \]
\[ \lim_{r \to 0} r (v_{k+1} - v_k) > 0. \]

**Leader:**
- falling to the competitive region is costly
- keeps investing to ensure such probability is vanishingly small

**Follower:**
- leadership is (endogenously) unattainable
- gives up despite being patient
Steady-state implication 1: slowdown in productivity growth

- Secular stagnation literature: level vs growth; demand vs supply;
- Cette, Fernald, Mojon (2015)
- Gutierrez and Philippon (2016, 2017), Lee, Stulz, and Shin (2017): sharp decline of investment relative to operating surplus; investment gap is especially pronounced in concentrated industries
Steady-state implication 2: rise in profits and concentration

Steady-state implication 3: widening productivity gap

Labour productivity: value added per worker (2001-2013)

- productivity gap is widening over time for OECD countries
- slow down in productivity convergence
Steady-state implication 4: decline in business dynamism

Summary: low interest rates are consistent with many stylized facts
Transitional dynamics: growth and markups

**Impulse Response: Productivity Growth**

**Impulse Response: Markups**

![Graph showing impulse response for productivity growth and markups over time.](image-url)
On-impact asymmetric valuation effect

Proportional changes in state-by-state leader value (relative to changes in the average market value of followers)
On-impact asymmetric valuation effect

Relative market value of leaders

starting from low $\gamma$

starting from high $\gamma$

Euler equation

Exogenous growth

Standard endogenous growth models

Aggregation investment

Leader-follower productivity gap

Aggregate investment

productivity growth

$\Delta r$ and $\Delta r$

$\approx 0$

$\Delta r$

low rate

high rate

relative market value

Profit share, markup, and concentration

Business dynamism

Leader-follower productivity gap

productivity growth

interest rate

growth rate

on-impact asymmetric valuation effect
On-impact asymmetric valuation effect

**Proposition.** Consider a decline in the interest rate $-\Delta r$. On impact, as a first-order approximation around $r \approx 0$,

$$- \frac{\Delta V^L}{\Delta r} = \frac{1}{r} \quad \text{and} \quad - \frac{\Delta V^F}{\Delta r} = - \frac{1}{r \ln r}.$$ 

Starting from a low $r$, a further decline in $r$ will

- immediately benefit leaders relative to followers (leaders have longer duration)
- especially when initial $r$ is low (leaders have higher convexity)
Empirical test: long-short portfolio

- Prediction: a decline in interest rate
  - benefits leaders more than followers
  - especially when the level of interest rate is low

Specification:

\[ R_t = \alpha + \beta_0 \cdot i_{t-1} + \beta_1 \cdot \Delta i_t + \beta_2 \cdot \Delta i_t \cdot i_{t-1} + \text{controls} + \epsilon_t \]

- \( R_t \): 90-day return of a value-weighted long-short portfolio
- Leaders defined as top 5% by market cap within Fama-French industries
- Robust to various other specifications: SIC, top 5, EBITDA, sales
Empirical test: long-short portfolio

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  - benefits leaders more than followers
  - especially when the level of interest rate is low

- Data: Compustat, CRSP, 10-year treasury yield, 1980-2017

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### Empirical test: long-short portfolio

<table>
<thead>
<tr>
<th></th>
<th>Portfolio Return</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td>$\Delta i_t$</td>
<td>$-1.150^{***}$</td>
<td>$-3.819^{***}$</td>
<td>$-2.268^{***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.309)</td>
<td>(0.641)</td>
<td>(0.602)</td>
<td></td>
</tr>
<tr>
<td>$\Delta i_t \cdot i_{t-1}$</td>
<td>$0.294^{***}$</td>
<td>$0.117^*$</td>
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<tr>
<td></td>
<td>(0.059)</td>
<td>(0.056)</td>
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<tr>
<td><strong>Controls</strong></td>
<td><strong>N</strong></td>
<td><strong>N</strong></td>
<td><strong>Y</strong></td>
<td></td>
</tr>
<tr>
<td><strong># Obs.</strong></td>
<td>9,016</td>
<td>9,016</td>
<td>9,016</td>
<td></td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.044</td>
<td>0.089</td>
<td>0.228</td>
<td></td>
</tr>
</tbody>
</table>

- Market leaders exhibit relative valuation gains following declines in $r$
  - effect especially strong under low $r$
  - not driven by leverage, HML, cyclicality, P/E ratio

- Return of “leader-portfolio” correlates negatively with “P/E portfolio”
Leaders see higher returns from $-\Delta i$ when $i$ is low
Conclusion

- Low interest rates raise market concentration and reduce creative destruction
  - through strategic and dynamic incentives
  - as $r \to 0$, aggregate investment and growth slows down
    - $g(r)$ has the shape of an inverted-U
  - empirical tests confirm predictions
- A long-run, supply-side perspective of secular stagnation
  - sidestepping short-run, demand-side Keynesian forces
- Developed techniques to analyze asymptotic equilibria of strategic patent races
Appendices

- Distribution of interest rate changes at varying frequencies
- Regression: nonparametric visualization
- Panel regressions
- Portfolio test: full specifications
- Portfolio test: along the yield curve
The panels plot the histograms of interest rate changes in our sample, from daily to annually. The mean and median values for each frequency are as follows:

- **Daily Change in Interest Rate**:
  - Mean: -0.00
  - Median: 0.00

- **Weekly Change in Interest Rate**:
  - Mean: -0.00
  - Median: -0.01

- **Monthly Change in Interest Rate**:
  - Mean: -0.02
  - Median: -0.02

- **Quarterly Change in Interest Rate**:
  - Mean: -0.05
  - Median: -0.03

- **Semi-Annual Change in Interest Rate**:
  - Mean: -0.09
  - Median: -0.13

- **Annual Change in Interest Rate**:
  - Mean: -0.18
  - Median: -0.24
Leaders see higher returns from $\Delta i$ when $i$ is low.
## Testing asymmetric effects: panel specification

<table>
<thead>
<tr>
<th></th>
<th>Stock Return</th>
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<td></td>
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<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Top 5 Percent=$1 \times \Delta i$</td>
<td></td>
<td>-1.187***</td>
<td>-3.881***</td>
<td>-4.415***</td>
<td>-4.182***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.260)</td>
<td>(1.113)</td>
<td>(0.893)</td>
<td>(0.529)</td>
</tr>
<tr>
<td>Top 5 Percent=$1 \times \Delta i \times \text{Lagged } i$</td>
<td></td>
<td>0.293**</td>
<td>0.346***</td>
<td>0.301***</td>
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<tr>
<td></td>
<td></td>
<td>(0.095)</td>
<td>(0.079)</td>
<td>(0.045)</td>
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<tr>
<td>Firm $\beta \times \Delta i$</td>
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<td>14.10***</td>
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<td>(0.795)</td>
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<tr>
<td>R-sq</td>
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<td>0.403</td>
<td>0.403</td>
<td>0.415</td>
<td>0.409</td>
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### Empirical test: long-short portfolio, full specification

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<tr>
<td>( \Delta i_t )</td>
<td>-1.150***</td>
<td>-3.819***</td>
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<td>-3.657***</td>
<td>-3.001***</td>
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<tr>
<td>( i_{t-1} )</td>
<td>0.0842</td>
<td>0.0336</td>
<td>0.160*</td>
<td>0.167*</td>
<td>0.069*</td>
</tr>
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<td>0.096*</td>
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<tr>
<td>Excess Market Return</td>
<td>-0.168***</td>
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<td>High Minus Low</td>
<td>0.0371</td>
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<tr>
<td>( (\Delta i_t &gt; 0) = 1 \times \Delta i_t )</td>
<td>0.341</td>
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<tr>
<td>( (\Delta i_t &gt; 0) = 1 \times \Delta i_t \times i_{t-1} )</td>
<td>-0.102</td>
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<td>PE Portfolio Return</td>
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<tr>
<td>N</td>
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<td>9,016</td>
<td>9,016</td>
<td>9,016</td>
<td>7,402</td>
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<td>0.092</td>
<td>0.196</td>
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Empirical test: long-short portfolio, along the yield curve

<table>
<thead>
<tr>
<th></th>
<th>30-Year</th>
<th>2-Year</th>
<th>10-30 Forward</th>
<th>2-Year &amp; 10-30 Fwd.</th>
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<tbody>
<tr>
<td>( \Delta i_t )</td>
<td>-1.129**</td>
<td>-4.537***</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(0.348)</td>
<td>(0.826)</td>
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<tr>
<td>( \Delta i_t \times i_{t-1} )</td>
<td>0.362***</td>
<td></td>
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<tr>
<td></td>
<td>(0.077)</td>
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<tr>
<td>( \Delta i_{t,0,2} )</td>
<td>-0.584*</td>
<td>-3.535***</td>
<td>-0.126</td>
<td>-2.066*</td>
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<tr>
<td></td>
<td>(0.244)</td>
<td>(0.833)</td>
<td>(0.349)</td>
<td>(0.970)</td>
</tr>
<tr>
<td>( \Delta i_{t,0,2} \times i_{t-1} )</td>
<td>0.280***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.069)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta i_{t,10,30} )</td>
<td>-1.084**</td>
<td>-4.165***</td>
<td>-0.938</td>
<td>-3.138**</td>
</tr>
<tr>
<td></td>
<td>(0.354)</td>
<td>(0.835)</td>
<td>(0.523)</td>
<td>(1.043)</td>
</tr>
<tr>
<td>( \Delta i_{t,10,30} \times i_{t-1} )</td>
<td>0.334***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.080)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>8,006</td>
<td>8,006</td>
<td>8,065</td>
<td>8,065</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.036</td>
<td>0.078</td>
<td>0.021</td>
<td>0.063</td>
</tr>
</tbody>
</table>
Joint profits are increasing in the state:

\[ v_s + v_s > v_{s-1} + v_{(s-1)} \]

\[ \implies v_s - v_{s-1} > v_{(s-1)} - v_s \]

- this implies that \( n \geq k - 1 \)
- \( n \geq k \) follows from the persistence of leadership in state \( k + 1 \)