Financial Frictions and the Wealth Distribution

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Motivation

Our goal

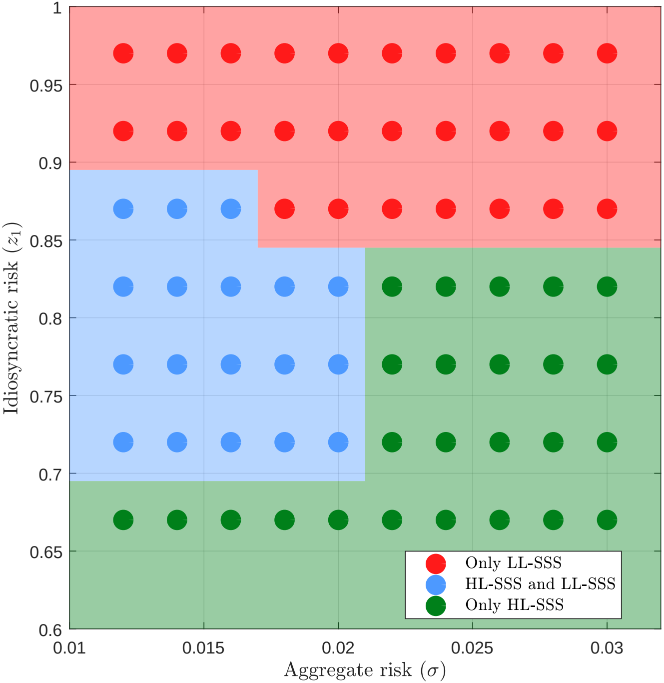
We investigate how, in a HA-model with financial frictions, idiosyncratic individual shocks interact with exogenous aggregate shocks to generate:

1. highly nonlinear behavior,
2. endogenously time-varying volatility and levels of leverage, and
3. endogenous aggregate risk.

- To do so, we postulate, compute, and estimate a continuous-time model *à la* Basak and Cuoco (1998) and Brunnermeier and Sannikov (2014) with a financial expert and a non-trivial distribution of wealth among households.

Four main results

- Multiple stochastic steady states or SSS(s):
  - Depending on the the volatility of the idiosyncratic and aggregate shocks, we can have one high-leverage SSS, one low-leverage SSS, or both.
  - Why? Interaction of precautionary behavior by households with desire to issue debt by the financial expert.
  - Higher micro turbulence leads to higher macro volatility, more inequality, and more leverage.
- Strong state-dependence on the responses of endogenous variables (GIRFs and DIRFs) to aggregate shocks.
- Long spells at different basins of attraction.
  - Multimodal and skewed ergodic distributions of endogenous variables, with endogenous time-varying volatility and aggregate risk.
- Thus, key importance of heterogeneity and breakdown of "quasi-aggregation."


Only LL-SSS
HL-SSS and LL-SSS
Only HL-SSS
Idiosyncratic risk (z_1)
Aggregate risk (σ)
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.01
0.015
0.02
0.025
0.03

Methodological contribution

- New approach to (globally) compute and estimate with the likelihood approach HA models:
  1. Computation: we use tools from machine learning.
  2. Estimation: we use tools from inference with diffusions.
- Strong theoretical foundations and many practical advantages.
  1. Deal with a large class of arbitrary operators efficiently.
  2. Algorithm that is easy to code, stable, and massively parallel.

The firm

- Representative firm with technology:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

- Competitive input markets:

$$w_t = (1-\alpha) K_t^{\alpha} L_t^{-\alpha}$$
$$rc_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}$$

- Aggregate capital evolves:

$$\frac{dK_t}{K_t} = (\iota_t - \delta)\, dt + \sigma dZ_t$$

- Instantaneous return rate on capital dr_t^k:

$$dr_t^k = (rc_t - \delta)\, dt + \sigma dZ_t$$

The expert I

- Representative expert holds capital $\widehat{K}_t$ and issues risk-free debt $\widehat{B}_t$ at rate r_t to households.
- Expert can be interpreted as a financial intermediary.
- Financial friction: expert cannot issue state-contingent claims (i.e., outside equity) and must absorb all risk from capital.
- Expert's net wealth (i.e., inside equity): $\widehat{N}_t = \widehat{K}_t - \widehat{B}_t$.
- Together with market clearing, our assumptions imply that economy has a risky asset in positive net supply and a risk-free asset in zero net supply.

The expert II

- The law of motion for expert's net wealth $\widehat{N}_t$:

$$\begin{aligned} d\widehat{N}_t &= \widehat{K}_t dr_t^k - r_t \widehat{B}_t dt - \widehat{C}_t dt \\ &= \left[\left(r_t + \widehat{\omega}_t \left(rc_t - \delta - r_t \right) \right) \widehat{N}_t - \widehat{C}_t \right] dt + \sigma \widehat{\omega}_t \widehat{N}_t dZ_t \end{aligned}$$

where $\widehat{\omega}_t \equiv \frac{\widehat{K}_t}{\widehat{N}_t}$ is the leverage ratio.

- The law of motion for expert's capital $\widehat{K}_t$:

$$d\widehat{K}_t = d\widehat{N}_t + d\widehat{B}_t$$

- The expert decides her consumption levels and capital holdings to solve:

$$\max_{\left\{\widehat{C}_t, \widehat{\omega}_t\right\}_{t \geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-\widehat{\rho} t} \log(\widehat{C}_t) dt \right]$$

given initial conditions and a NPG condition.

Households I

- Continuum of infinitely-lived households with unit mass.
- Heterogeneous in wealth a_m and labor supply z_m for $m \in [0, 1]$.
- $G_t(a, z)$: distribution of households conditional on realization of aggregate variables.
- Preferences:

$$\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma} - 1}{1-\gamma} dt \right]$$

- We could have more general Duffie and Epstein (1992) recursive preferences.
- $\rho > \widehat{\rho}$. Intuition from Aiyagari (1994) (and different from BGG class of models!).

Households II

- z_t units of labor valued at wage w_t.
- Labor productivity evolves stochastically following a Markov chain:
  1. $z_t \in \{z_1, z_2\}$, with $z_1 < z_2$.
  2. Ergodic mean of z_t is 1.
  3. Jump intensity from state 1 to state 2: λ_1 (reverse intensity is λ_2).
- Households save $a_t \geq 0$ in the riskless debt issued by experts with an interest rate r_t. Thus, their wealth follows:

$$da_t = (w_t z_t + r_t a_t - c_t)\, dt = s\,(a_t, z_t, K_t, G_t)\, dt$$

- Optimal choice: $c_t = c\,(a_t, z_t, K_t, G_t)$.
- Total consumption by households:

$$C_t \equiv \int c\,(a_t, z_t, K_t, G_t)\, dG_t\,(da, dz)$$

Market clearing

1. Total amount of labor rented by the firm is equal to labor supplied:

$$L_t = \int z dG_t = 1$$

   Then, total payments to labor are given by w_t.

2. Total amount of debt of the expert equals the total households' savings:

$$B_t \equiv \int a dG_t (da, dz) = \widehat{B}_t$$

   with law of motion $d\widehat{B}_t = dB_t = (w_t + r_t B_t - C_t) dt$.

3. The total amount of capital in this economy is owned by the expert:

$$K_t = \widehat{K}_t$$

   Thus, $d\widehat{K}_t = dK_t = \left(Y_t - \delta K_t - C_t - \widehat{C}_t\right) dt + \sigma K_t dZ_t$ and $\widehat{\omega}_t = \frac{K_t}{N_t}$, where $\widehat{N}_t = N_t = K_t - B_t$.

4. Also, we get:

$$\iota_t = \frac{Y_t - C_t - \widehat{C}_t}{K_t}$$

Density

- The households distribution $G_t(a, z)$ has density (i.e., the Radon-Nikodym derivative) $g_t(a, z)$.

- The dynamics of this density conditional on the realization of aggregate variables are given by the Kolmogorov forward (KF; aka Fokker–Planck) equation:

$$\frac{\partial g_{it}}{\partial t} = -\frac{\partial}{\partial a}\left(s\left(a_t, z_t, K_t, G_t\right) g_{it}(a)\right) - \lambda_i g_{it}(a) + \lambda_j g_{jt}(a), \;\; i \neq j = 1, 2$$

  where $g_{it}(a) \equiv g_t(a, z_i)$, $i = 1, 2$.

- The density satisfies the normalization:

$$\sum_{i=1}^{2} \int_0^\infty g_{it}(a) da = 1$$

Equilibrium

An equilibrium in this economy is composed by a set of prices $\left\{w_t, rc_t, r_t, r_t^k\right\}_{t\geq 0}$, quantities $\left\{K_t, N_t, B_t, \widehat{C}_t, c_{mt}\right\}_{t\geq 0}$, and a density $\left\{g_t\left(\cdot\right)\right\}_{t\geq 0}$ such that:

1. Given w_t, r_t, and g_t, the solution of the household m's problem is $c_t = c\left(a_t, z_t, K_t, G_t\right)$.
2. Given r_t^k, r_t, and N_t, the solution of the expert's problem is $\widehat{C}_t$, K_t, and B_t.
3. Given $K_t,$ firms maximize their profits and input prices are given by w_t and rc_t.
4. Given w_t, r_t, and c_t, g_t is the solution of the KF equation.
5. Given g_t and $B_t,$ the debt market clears.

Characterizing the equilibrium I

- First, we proceed with the expert's problem. Because of log-utility:

$$\widehat{C}_t = \widehat{\rho} N_t$$

$$\omega_t = \widehat{\omega}_t = \frac{rc_t - \delta - r_t}{\sigma^2}$$

- We can use the equilibrium values of rc_t, L_t, and ω_t to get the wage:

$$w_t = (1-\alpha) K_t^{\alpha}$$

the rental rate of capital:

$$rc_t = \alpha K_t^{\alpha-1}$$

and the risk-free interest rate:

$$r_t = \alpha K_t^{\alpha-1} - \delta - \sigma^2 \frac{K_t}{N_t}$$

Characterizing the equilibrium II

- Expert's net wealth evolves as:

$$dN_t = \underbrace{\left(\alpha K_t^{\alpha-1} - \delta - \widehat{\rho} - \sigma^2 \left(1 - \frac{K_t}{N_t}\right)\frac{K_t}{N_t}\right) N_t dt}_{\mu_t^N(B_t,N_t)} + \underbrace{\sigma K_t}_{\sigma_t^N(B_t,N_t)} dZ_t$$

- And debt as:

$$dB_t = \left((1-\alpha) K_t^{\alpha} + \left(\alpha K_t^{\alpha-1} - \delta - \sigma^2 \frac{K_t}{N_t}\right) B_t - C_t\right) dt$$

- Nonlinear structure of law of motion for dN_t and dB_t.

- We need to find:

$$C_t \equiv \int c\left(a_t, z_t, K_t, G_t\right) g_t\left(a, z\right) dadz$$

$$\frac{\partial g_{it}}{\partial t} = -\frac{\partial}{\partial a}\left(s\left(a_t, z_t, K_t, G_t\right) g_{it}(a)\right) - \lambda_i g_{it}(a) + \lambda_j g_{jt}(a), \; i \neq j = 1, 2$$

The DSS

- No aggregate shocks ($\sigma = 0$), but we still have idiosyncratic household shocks.
- Then:

$$r = r_t^k = rc_t - \delta = \alpha K_t^{\alpha-1} - \delta$$

and

$$\begin{aligned} dN_t &= \left[\left(rc_t - \delta \right) K_t - r_t B_t - \widehat{\rho} N_t \right] dt \\ &= \left(\alpha K_t^{\alpha-1} - \delta - \widehat{\rho} \right) N_t dt \end{aligned}$$

- Since in a steady state the drift of expert's wealth must be zero, we get the steady state capital

$$K = \left(\frac{\widehat{\rho} + \delta}{\alpha} \right)^{\frac{1}{\alpha-1}}$$

and the risk-free rate

$$r = \widehat{\rho} < \rho$$

- The value of N is given by the dispersion of the idiosyncratic shocks (no analytic expression).

How do we find aggregate consumption?

- As in Krusell and Smith (1998), households only track a finite set of n moments of $g_t(a,z)$ to form their expectations.
- No exogenous state variable (shocks to capital encoded in K). Instead, two endogenous states.
- For ease of exposition, we set $n = 1$. The solution can be trivially extended to the case with $n > 1$.
- More concretely, households consider a *perceived law of motion* (PLM) of aggregate debt:

$$dB_t = h(B_t, N_t)\, dt$$

where

$$h(B_t, N_t) = \frac{\mathbb{E}\left[dB_t | B_t, N_t\right]}{dt}$$

A new HJB equation

- Given the PLM, the household's Hamilton-Jacobi-Bellman (HJB) equation becomes:

$$\begin{aligned}\rho V_i(a,B,N) &= \max_c \frac{c^{1-\gamma}-1}{1-\gamma} + s\frac{\partial V_i}{\partial a} + \lambda_i\left[V_j(a,B,N) - V_i(a,B,N)\right] \\ &\quad + h(B,N)\frac{\partial V_i}{\partial B} + \mu^N(B,N)\frac{\partial V_i}{\partial N} + \frac{\left[\sigma^N(B,N)\right]^2}{2}\frac{\partial^2 V_i}{\partial N^2}\end{aligned}$$

$i \neq j = 1,2$, and where

$$s = s(a, z, N+B, G)$$

- We solve the HJB with a first-order, implicit upwind scheme in a finite difference stencil.
- Sparse system. Why?
- Alternatives for solving the HJB? Finite volumes, fem, meshfree methods, ...

An algorithm to find the PLM

1) Start with h_0, an initial guess for h.

2) Using current guess h_n, solve for the household consumption, c_m, in the HJB equation.

3) Construct a time series for B_t by simulating by J periods the cross-sectional distribution of households with a constant time step Δt (starting at DSS and with a burn-in).

4) Given B_t, find N_t, K_t, and:

$$\widehat{\mathbf{h}} = \left\{ \widehat{h}_1, \widehat{h}_2 ..., \widehat{h}_j \equiv \frac{B_{t_j+\Delta t} - B_{t_j}}{\Delta t}, ..., \widehat{h}_J \right\}$$

5) Define $\mathbf{S} = \{\mathbf{s}_1, \mathbf{s}_2, ..., \mathbf{s}_J\}$, where $\mathbf{s}_j = \left\{s_j^1, s_j^2\right\} = \left\{B_{t_j}, N_{t_j}\right\}$.

6) Use $\left(\widehat{\mathbf{h}}, \mathbf{S}\right)$ and a universal nonlinear approximator to obtain h_{n+1}, a new guess for h.

7) Iterate steps 2)-6) until h_{n+1} is sufficiently close to h_n.

A universal nonlinear approximator

- We approximate the PLM with a neural network (NN):

$$h(\mathbf{s};\theta) = \theta_0^1 + \sum_{q=1}^{Q} \theta_q^1 \phi \left(\theta_{0,q}^2 + \sum_{i=1}^{D} \theta_{i,q}^2 s^i \right)$$

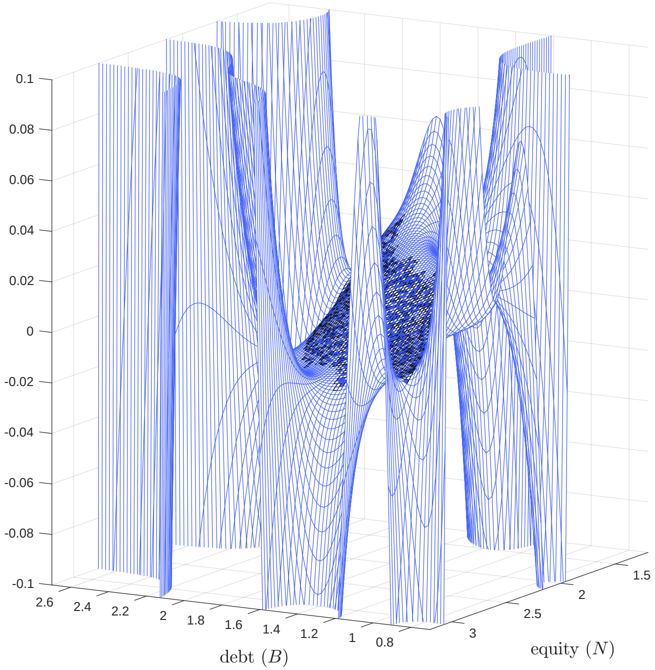
  where $Q = 16$, $D = 2$, and $\phi(x) = \log(1 + e^x)$.

- θ is selected as:

$$\theta^* = \arg\min_{\theta} \frac{1}{2} \sum_{j=1}^{J} \left\| h(\mathbf{s}_j;\theta) - \widehat{h}_j \right\|^2$$

- Easy to code, stable, and good extrapolation properties.

- You can flush the algorithm to a graphics processing unit (GPU) or a tensor processing unit (TPU) instead of a standard CPU.


0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
debt (B)
3
2.5
2
1.5
equity (N)

Two classic (yet remarkable) results

Universal approximation theorem: Hornik, Stinchcombe, and White (1989)

A neural network with at least one hidden layer can approximate any Borel measurable function mapping finite-dimensional spaces to any desired degree of accuracy.

- Assume, as well, that we are dealing with the class of functions for which the Fourier transform of their gradient is integrable.

Breaking the curse of dimensionality: Barron (1993)

A one-layer NN achieves integrated square errors of order $\mathcal{O}(1/Q)$, where Q is the number of nodes. In comparison, for series approximations, the integrated square error is of order $\mathcal{O}(1/(Q^{2/D}))$ where D is the dimensions of the function to be approximated.

- We actually rely on more general theorems by Leshno et al. (1993) and Bach (2017).

Estimation with aggregate variables I

- $D+1$ observations of Y_t at fixed time intervals $[0, \Delta, 2\Delta, .., D\Delta]$:
$$Y_0^D = \{Y_0, Y_\Delta, Y_{2\Delta}, ..., Y_D\}.$$
- More general case: sequential Monte Carlo approximation to the Kushner-Stratonovich equation (Fernández-Villaverde and Rubio Ramírez, 2007).
- We are interested in estimating a vector of structural parameters Ψ.
- Likelihood:
$$\mathcal{L}_D\left(Y_0^D|\Psi\right) = \prod_{d=1}^{D} p_Y\left(Y_{d\Delta}|Y_{(d-1)\Delta};\Psi\right),$$
where
$$p_Y\left(Y_{d\Delta}|Y_{(d-1)\Delta};\Psi\right) = \int f_{d\Delta}(Y_{d\Delta}, B)dB.$$
given a density, $f_{d\Delta}(Y_{d\Delta}, B)$, implied by the solution of the model.

Estimation with aggregate variables II

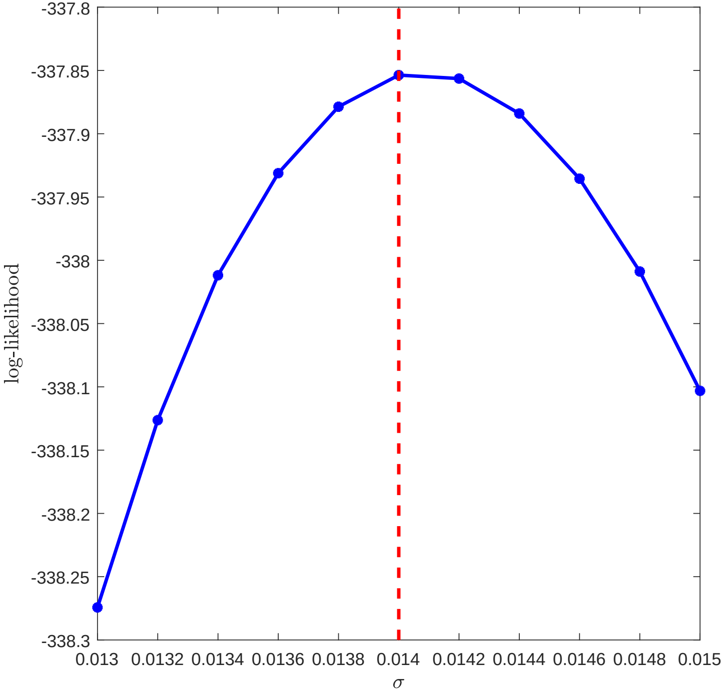
- After finding the diffusion for Y_t, $f_t^d(Y,B)$ follows the Kolmogorov forward (KF) equation in the interval $[(d-1)\Delta, d\Delta]$:

$$\begin{aligned}\frac{\partial f_t}{\partial t} &= -\frac{\partial}{\partial Y}\left[\mu^Y(Y,B)f_t(Y,B)\right]-\frac{\partial}{\partial B}\left[h(B,Y^{\frac{1}{\alpha}}-B)f_t^d(Y,B)\right]\\ &\quad +\frac{1}{2}\frac{\partial^2}{\partial Y^2}\left[\left(\sigma^Y(Y)\right)^2 f_t(Y,B)\right]\end{aligned}$$

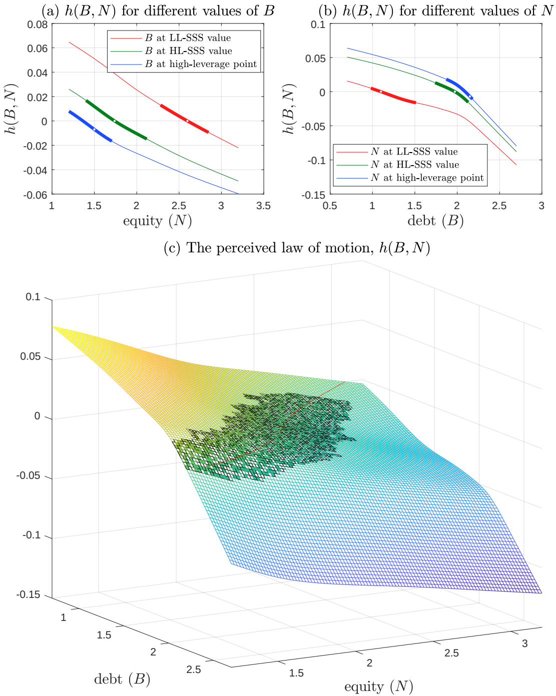
- The operator in the KF equation is the adjoint of the infinitesimal generator generated by the HJB.
- Thus, the solution of the KF equation amounts to transposing and inverting a sparse matrix that has already been computed.
- Our approach provides a highly efficient way of evaluating the likelihood once the model is solved.
- Conveniently, retraining of the neural network is easy for new parameter values.

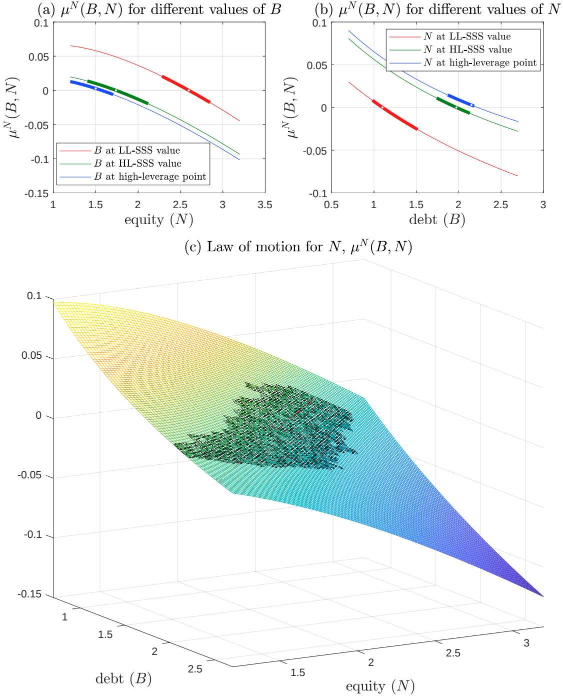
Parametrization

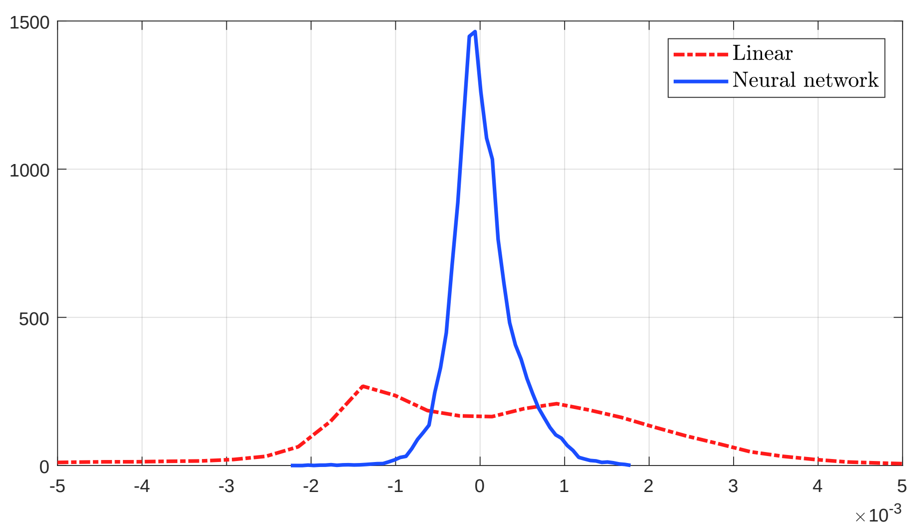
Parameter	Value	Description	Source/Target
α	0.35	capital share	standard
δ	0.1	yearly capital depreciation	standard
γ	2	risk aversion	standard
ρ	0.05	households' discount rate	standard
λ_1	0.986	transition rate u.-to-e.	monthly job finding rate of 0.3
λ_2	0.052	transition rate e.-to-u.	unemployment rate 5 percent
y_1	0.72	income in unemployment state	Hall and Milgrom (2008)
y_2	1.015	income in employment state	$\mathbb{E}(y) = 1$
$\widehat{\rho}$	0.0497	experts' discount rate	$K/N = 2$



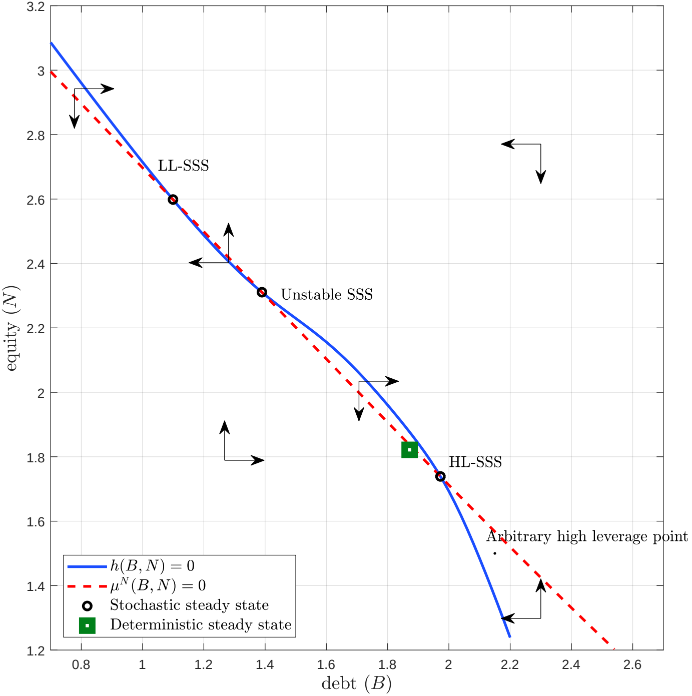
-337.8
-337.85
-337.9
-337.95
-338
-338.05
-338.1
-338.15
-338.2
-338.25
-338.3
log-likelihood
0.013
0.0132
0.0134
0.0136
0.0138
0.014
0.0142
0.0144
0.0146
0.0148
0.015
σ


(a) $h(B,N)$ for different values of B
B at LL-SSS value
B at HL-SSS value
B at high-leverage point
$h(B,N)$
equity (N)
(b) $h(B,N)$ for different values of N
N at LL-SSS value
N at HL-SSS value
N at high-leverage point
$h(B,N)$
debt (B)
(c) The perceived law of motion, $h(B,N)$
debt (B)
equity (N)

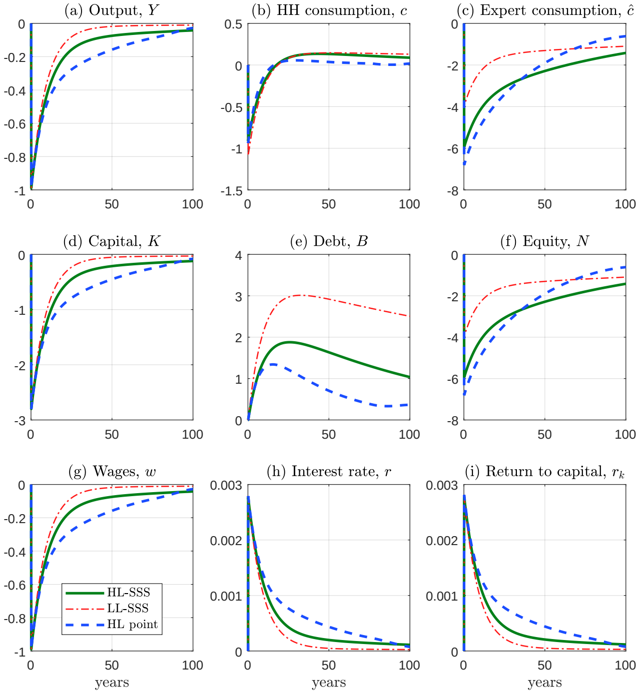

(a) $\mu^N(B,N)$ for different values of B
$\mu^N(B,N)$
equity (N)
B at LL-SSS value
B at HL-SSS value
B at high-leverage point
(b) $\mu^N(B,N)$ for different values of N
$\mu^N(B,N)$
debt (B)
N at LL-SSS value
N at HL-SSS value
N at high-leverage point
(c) Law of motion for N, $\mu^N(B,N)$
debt (B)
equity (N)

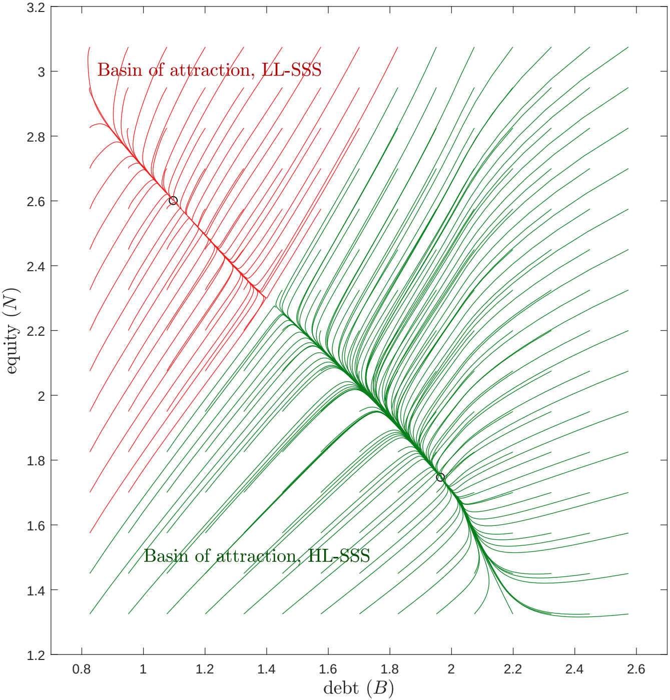


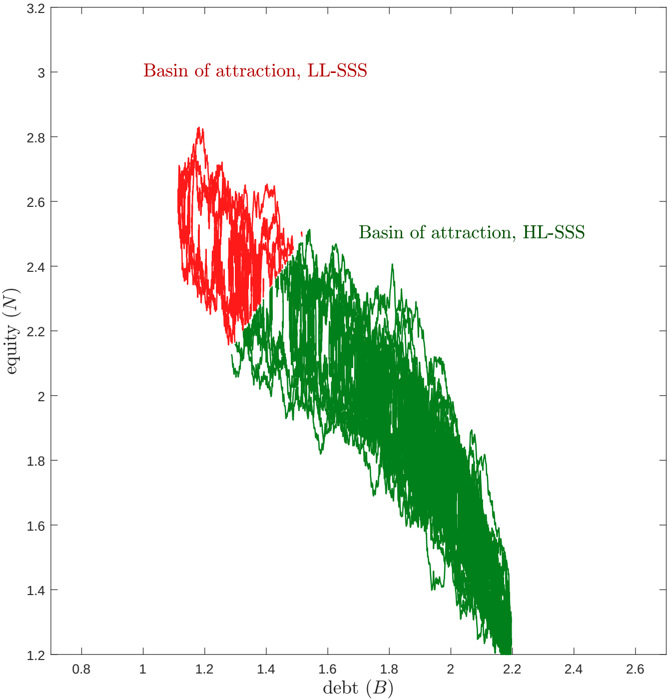
Linear
Neural network
1500
1000
500
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
$\times 10^{-3}$



LL-SSS
Unstable SSS
HL-SSS
Arbitrary high leverage point
$h(B,N)=0$
$\mu^N(B,N)=0$
Stochastic steady state
Deterministic steady state
equity (N)
debt (B)

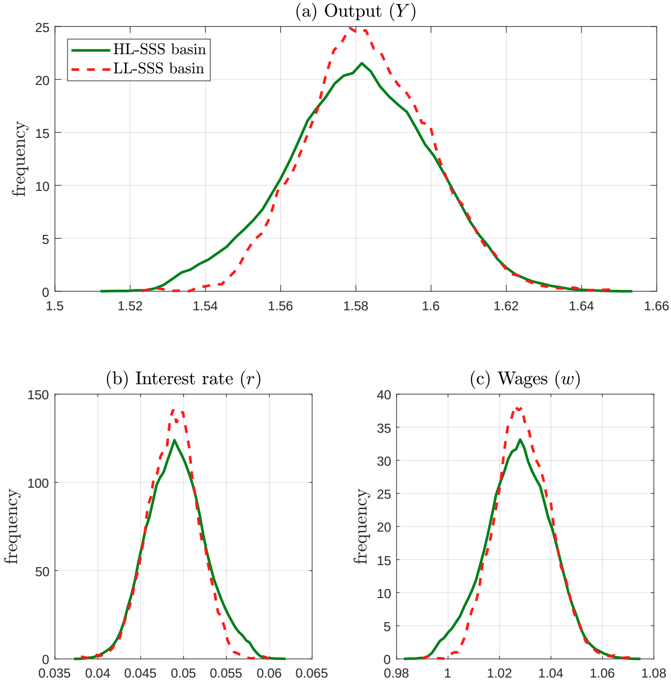

(a) Output, Y
(b) HH consumption, c
(c) Expert consumption, $\hat{c}$
(d) Capital, K
(e) Debt, B
(f) Equity, N
(g) Wages, w
(h) Interest rate, r
(i) Return to capital, r_k
HL-SSS
LL-SSS
HL point
years

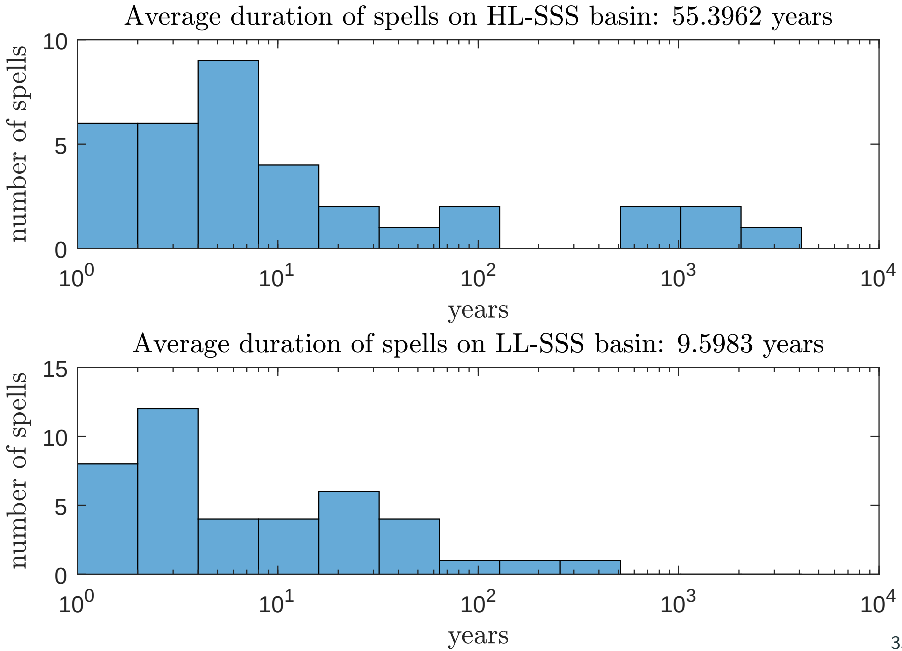

Basin of attraction, LL-SSS
Basin of attraction, HL-SSS
equity (N)
debt (B)
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6

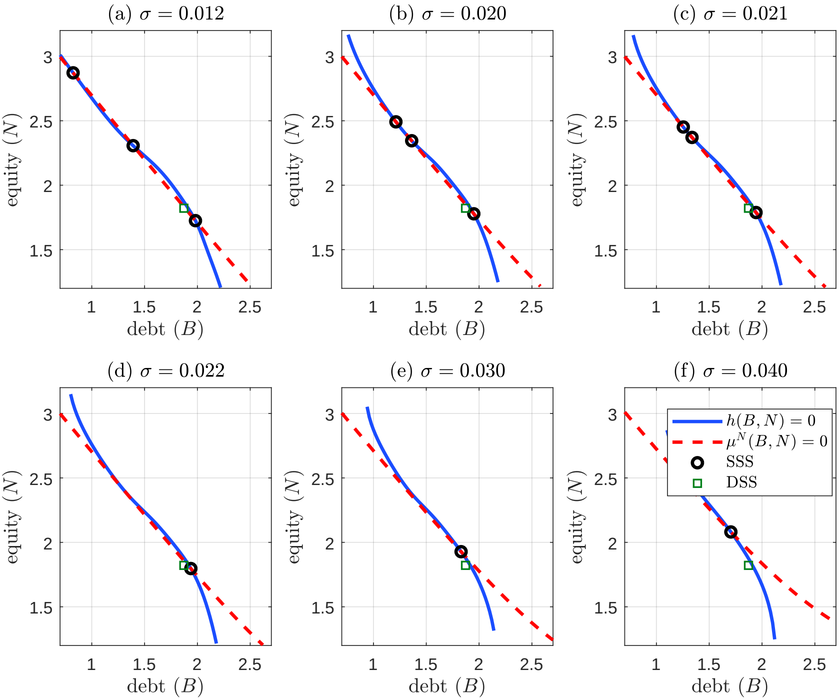

Basin of attraction, LL-SSS
Basin of attraction, HL-SSS
equity (N)
debt (B)
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6

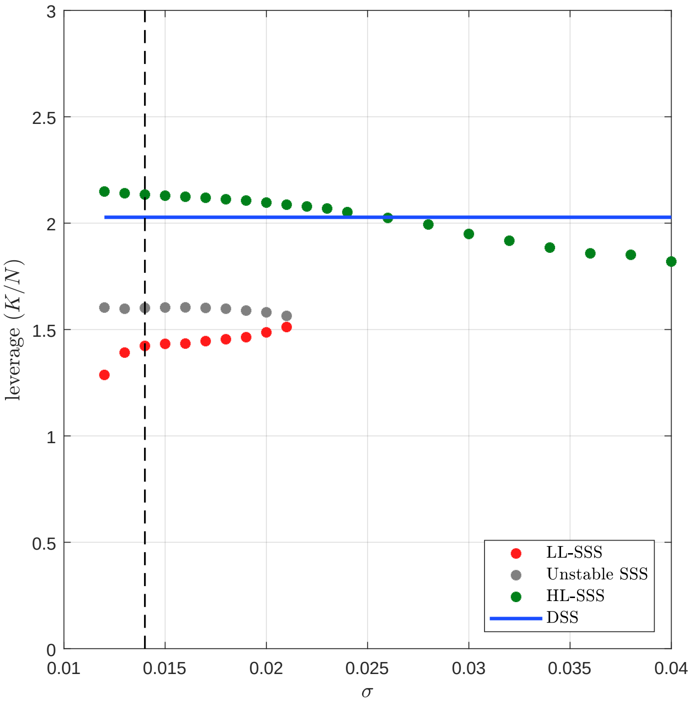
	Mean	Standard deviation	Skewness	Kurtosis
$Y^{\text{basin } HL}$	1.5807	0.0193	-0.0831	2.8750
$Y^{\text{basin } LL}$	1.5835	0.0166	0.16417	3.1228
$r^{\text{basin } HL}$	4.92	0.3360	0.1725	2.8967
$r^{\text{basin } LL}$	4.88	0.2896	-0.0730	3.0905
$w^{\text{basin } HL}$	1.0274	0.0125	-0.0831	2.875
$w^{\text{basin } LL}$	1.0293	0.0108	0.1642	3.1228

Table 1: Moments conditional on basin of attraction.

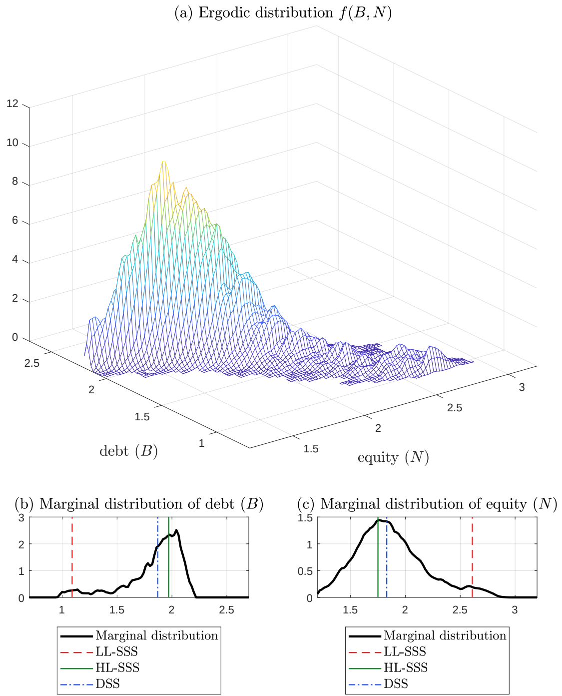

(a) Output (Y)
HL-SSS basin
LL-SSS basin
frequency
25
20
15
10
5
0
1.5
1.52
1.54
1.56
1.58
1.6
1.62
1.64
1.66
(b) Interest rate (r)
frequency
150
100
50
0
0.035
0.04
0.045
0.05
0.055
0.06
0.065
(c) Wages (w)
frequency
40
35
30
25
20
15
10
5
0
0.98
1
1.02
1.04
1.06
1.08

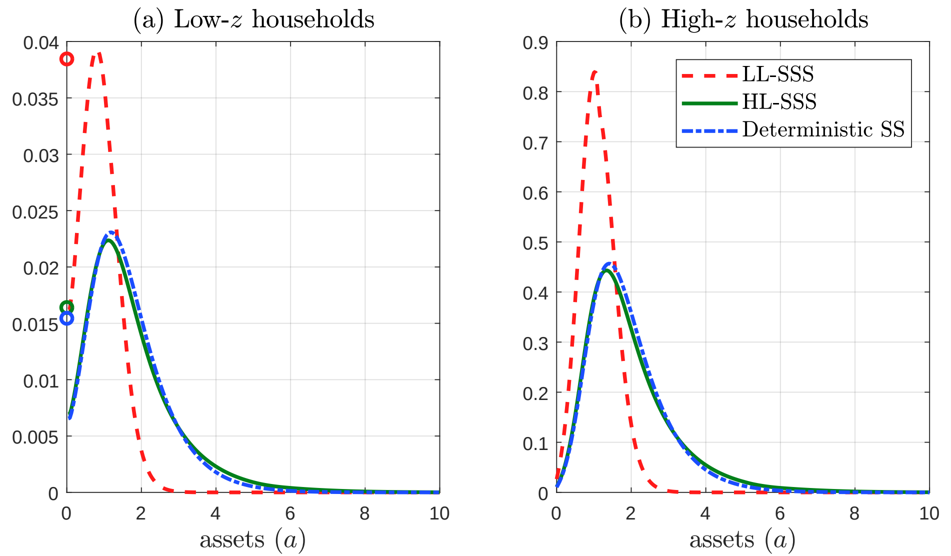

Average duration of spells on HL-SSS basin: 55.3962 years
number of spells
10
5
0
10^0
10^1
10^2
10^3
10^4
years
Average duration of spells on LL-SSS basin: 9.5983 years
number of spells
15
10
5
0
10^0
10^1
10^2
10^3
10^4
years


(a) σ = 0.012
(b) σ = 0.020
(c) σ = 0.021
(d) σ = 0.022
(e) σ = 0.030
(f) σ = 0.040
equity (N)
debt (B)
h(B, N) = 0
μ^N(B, N) = 0
SSS
DSS

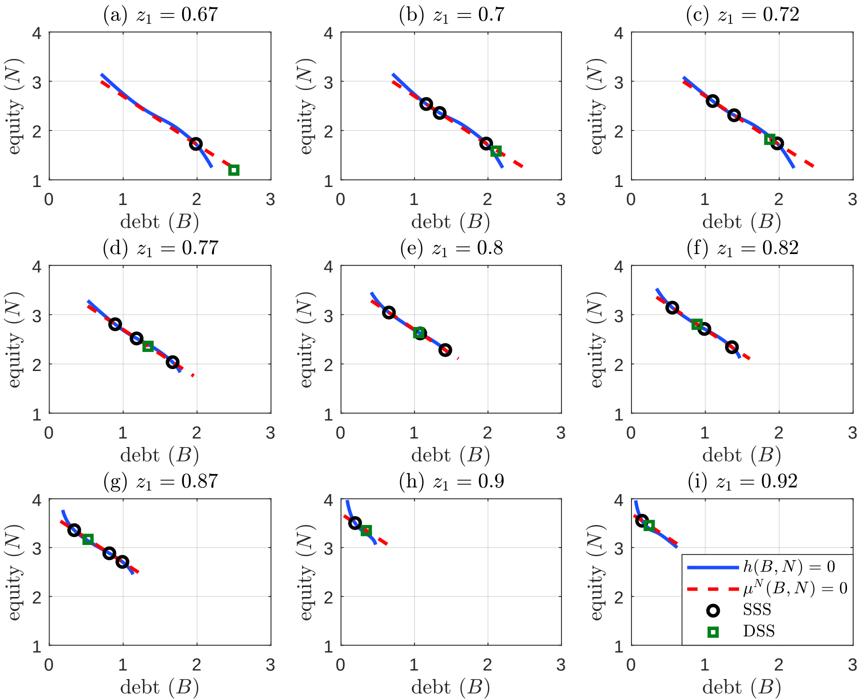


3
2.5
2
1.5
1
0.5
0
0.01
0.015
0.02
0.025
0.03
0.035
0.04
leverage (K/N)
σ
LL-SSS
Unstable SSS
HL-SSS
DSS

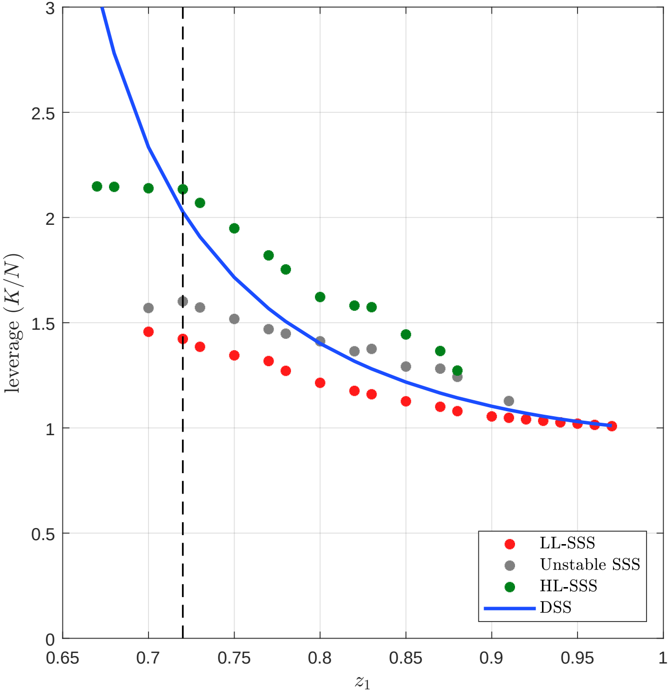

(a) Ergodic distribution $f(B,N)$
12
10
8
6
4
2
0
2.5
2
1.5
1
debt (B)
1.5
2
2.5
3
equity (N)
(b) Marginal distribution of debt (B)
3
2
1
0
1
1.5
2
2.5
Marginal distribution
LL-SSS
HL-SSS
DSS
(c) Marginal distribution of equity (N)
1.5
1
0.5
0
1.5
2
2.5
3
Marginal distribution
LL-SSS
HL-SSS
DSS



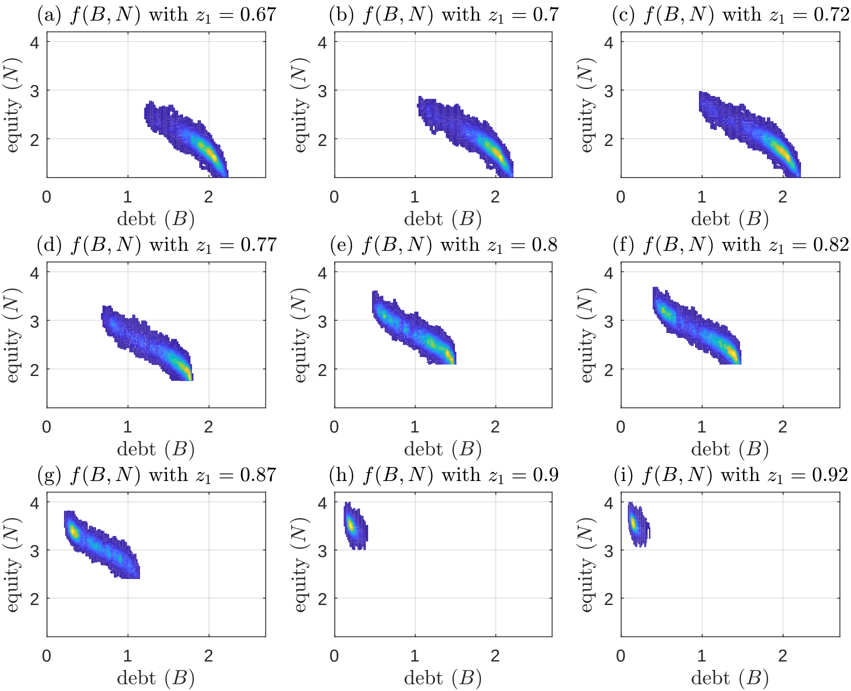
(a) Low-z households
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
2
4
6
8
10
assets (a)
(b) High-z households
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
assets (a)
LL-SSS
HL-SSS
Deterministic SS



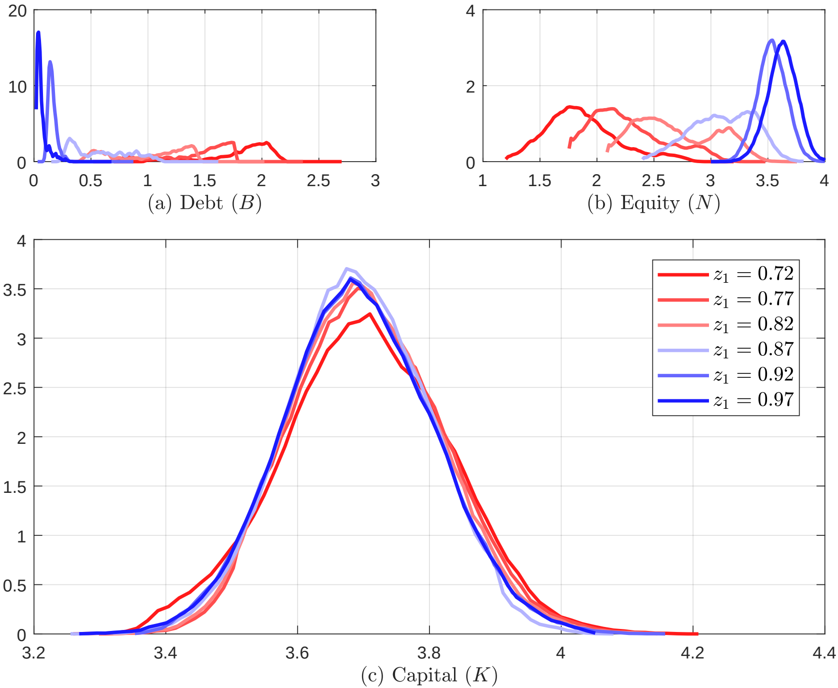
(a) $z_1 = 0.67$
(b) $z_1 = 0.7$
(c) $z_1 = 0.72$
(d) $z_1 = 0.77$
(e) $z_1 = 0.8$
(f) $z_1 = 0.82$
(g) $z_1 = 0.87$
(h) $z_1 = 0.9$
(i) $z_1 = 0.92$
equity (N)
debt (B)
$h(B, N) = 0$
$\mu^N(B, N) = 0$
SSS
DSS



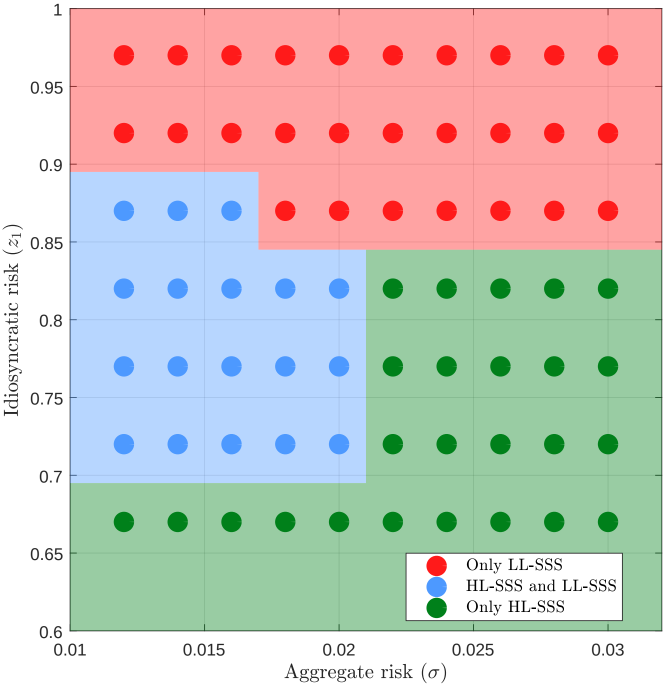
leverage (K/N)
z_1
LL-SSS
Unstable SSS
HL-SSS
DSS



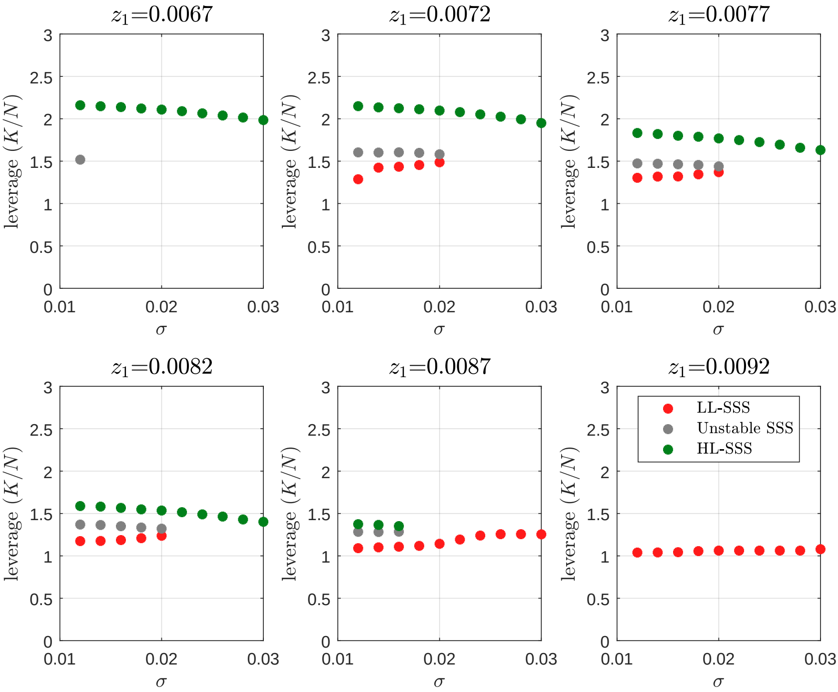
(a) $f(B,N)$ with $z_1 = 0.67$
(b) $f(B,N)$ with $z_1 = 0.7$
(c) $f(B,N)$ with $z_1 = 0.72$
(d) $f(B,N)$ with $z_1 = 0.77$
(e) $f(B,N)$ with $z_1 = 0.8$
(f) $f(B,N)$ with $z_1 = 0.82$
(g) $f(B,N)$ with $z_1 = 0.87$
(h) $f(B,N)$ with $z_1 = 0.9$
(i) $f(B,N)$ with $z_1 = 0.92$
equity (N)
debt (B)



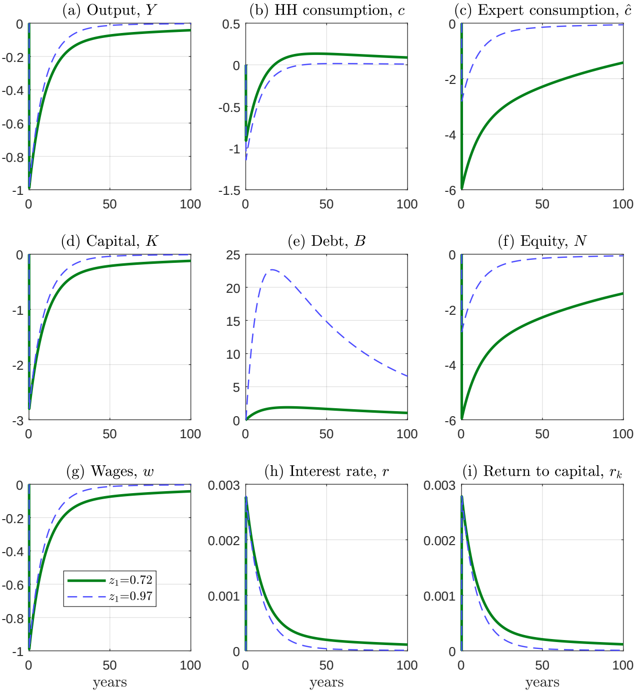
20
10
0
0
0.5
1
1.5
2
2.5
3
(a) Debt (B)
4
2
0
1
1.5
2
2.5
3
3.5
4
(b) Equity (N)
4
3.5
3
2.5
2
1.5
1
0.5
0
3.2
3.4
3.6
3.8
4
4.2
4.4
(c) Capital (K)
$z_1 = 0.72$
$z_1 = 0.77$
$z_1 = 0.82$
$z_1 = 0.87$
$z_1 = 0.92$
$z_1 = 0.97$

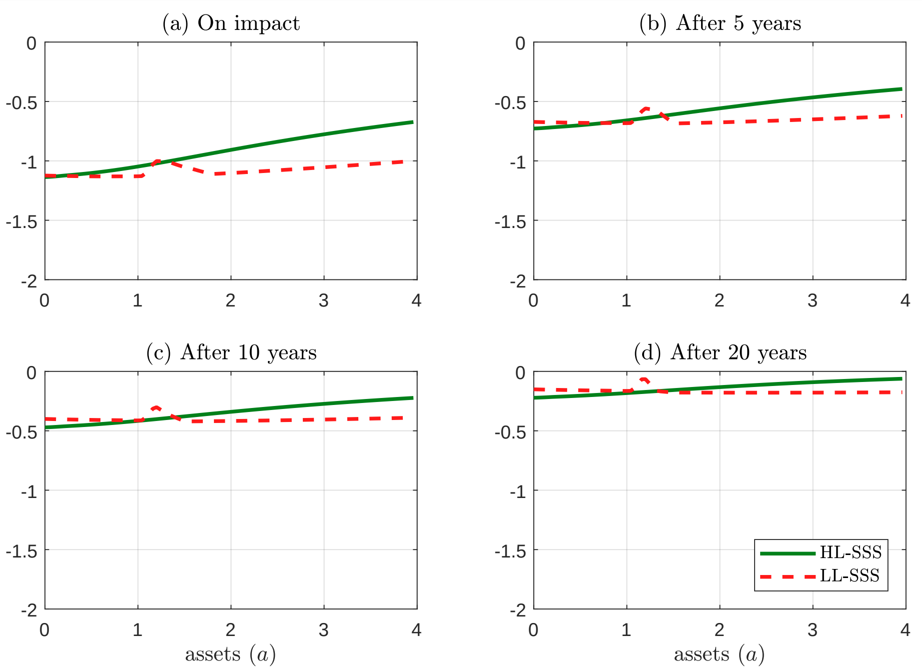


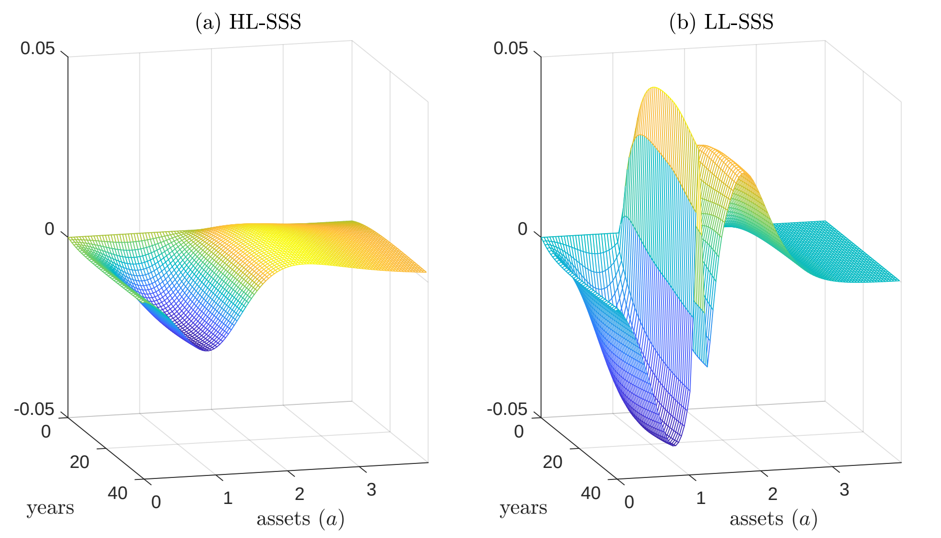
Only LL-SSS
HL-SSS and LL-SSS
Only HL-SSS
Idiosyncratic risk (z_1)
Aggregate risk (σ)
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.01
0.015
0.02
0.025
0.03

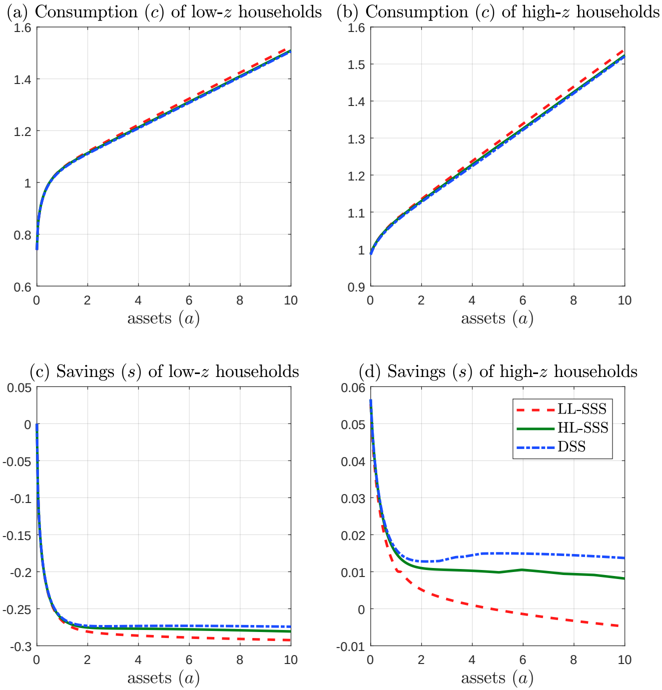


z_1=0.0067
z_1=0.0072
z_1=0.0077
z_1=0.0082
z_1=0.0087
z_1=0.0092
leverage (K/N)
σ
0
0.5
1
1.5
2
2.5
3
0.01
0.02
0.03
LL-SSS
Unstable SSS
HL-SSS


(a) Output, Y
(b) HH consumption, c
(c) Expert consumption, $\hat{c}$
(d) Capital, K
(e) Debt, B
(f) Equity, N
(g) Wages, w
(h) Interest rate, r
(i) Return to capital, r_k
z_1=0.72
z_1=0.97
years
years
years


(a) On impact
(b) After 5 years
(c) After 10 years
(d) After 20 years
0
-0.5
-1
-1.5
-2
0 1 2 3 4
assets (a)
HL-SSS
LL-SSS


(a) HL-SSS
0.05
0
-0.05
0
20
40
years
0
1
2
3
assets (a)
(b) LL-SSS
0.05
0
-0.05
0
20
40
years
0
1
2
3
assets (a)


(a) Consumption (c) of low-z households
(b) Consumption (c) of high-z households
(c) Savings (s) of low-z households
(d) Savings (s) of high-z households
assets (a)
LL-SSS
HL-SSS
DSS

Concluding remarks

- We have shown how a continuous-time model with a non-trivial distribution of wealth among households and financial frictions can be built, computed, and estimated.
- Four important economic lessons:
  1. Multiplicity of SSS(s).
  2. State-dependence of GIRFs and DIRFs.
  3. Long spells at different basins of attraction.
  4. Importance of household heterogeneity.
- Many avenues for extension.