The "New" Economics of Trade Agreements: From Trade Liberalization to Regulatory Convergence?*

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Abstract

We study the incentives governments have to negotiate "new trade agreements," i.e., agreements that constrain not only governments' choices of tariffs, but also their domestic regulatory policies. We focus on horizontal product standards, i.e., those that impose requirements along a horizontal dimension of product differentiation. We introduce differences in ideal products across countries and consider cases in which product choices do not and do confer externalities on other national consumers. In addition to characterizing the features of the optimal new trade agreement in each environment, we ask whether detailed negotiations about regulatory rules are needed for global efficiency or whether an "old trade agreement" augmented by some "policed decentralization" of regulatory procedures can achieve the same outcomes.

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1 Introduction

Trade negotiations at the multilateral, regional and bilateral levels have been remarkably successful at reducing the traditional barriers to trade in the post-war period. The World Bank reports a weighted average applied tariff rate on all products traded in the world of less than 2.6% in 2017. In 1939 average applied tariffs were 23.3% in France, 32.6% in Germany, 29.6% in the United Kingdom and 13.3% in the United States, and even higher in many smaller countries (see Bown and Irwin, 2015). Quota restrictions, which were ubiquitous in earlier periods, have all but disappeared.

With this success, the trade community has shifted its attention to various non-tariff barriers (NTB's) that leave world markets still far from integrated. And among the NTB's that receive the most scrutiny are impediments to trade that arise from differences in domestic regulations or what Sykes (1999a,1999b) has termed 'regulatory heterogeneity.' International disciplines for regulatory procedures lie at the heart of the Technical Barriers to Trade (TBT) Agreement and the Sanitary and Phytosanitary (SPS) Agreement that were concluded as part of the Uruguay Round of trade negotiations, and they have provided the primary impetus for the Transatlantic Trade and Investment Partnership (TTIP) negotiations between the United States and Europe.

National governments regulate commercial behavior for a myriad of reasons. Regulations support cultural and social norms, address environmental, health and safety issues, confront problems arising from asymmetric information between producers and consumers, and protect society from systemic risks in the financial sector, the telecommunications sector, the IT sector, and a host of others. But the trade community has long recognized that governments can use their regulatory authority to pursue mercantilist objectives as well. Regulations can baldly favor domestic firms over foreign firms, or they can be facially neutral but still impose the greatest costs on exporting firms and impede global competition. Moreover, as the economic literature on trade agreements has emphasized, if governments do not cooperate in setting their national policies, and if they neglect the interests of consumers and firms that are not part of their constituencies, then global inefficiencies will emerge even in the absence of any protectionist intent (see, for example, Bagwell and Staiger, 2002, and Grossman, 2017).

Lamy (2015, 2016) highlights a particular form of international externality that arises from regulatory dissonance. Firms that must satisfy different regulations for their various destination markets must produce different versions of their products, often at substantial cost in the form of foregone economies of scale. He argues that as the precautionary motive for trade regulation designed to protect consumers' health, safety and values displaces the protectionist motive that served to insulate producers from competition, the leveling of the trade playing field will become less about eliminating protective barriers and more about reducing differences between policies that have legitimate aims. The new landscape for trade negotiations requires, in his view, harmonization, or at least convergence, in regulatory measures. Yet, as Sykes (1999, 2000) cogently argues, international differences in incomes, cultures, risk preferences and tastes generally justify regulatory heterogeneity, even if we admit the extra cost of satisfying a multitude of different rules. He notes that only very exceptionally will cooperation suggest the desirability of complete harmonization.

The writings of Lamy on the one hand and Sykes on the other raise the immediate question of what is the appropriate trade-off in international trade agreements between heterogeneous tastes across international borders and the extra costs imposed by disparate regulations.

In this paper, we begin the task of answering this question. We consider a trading environment in which individuals residing in different countries hold dissimilar preferences over the characteristics of goods and services that reflect their idiosyncratic histories, cultures, and religions. National governments can impose regulations when economically justified in order to serve the interests of their constituents. Yet, disparate regulations impose costs on firms, and ultimately consumers, the more so the greater are the cross-country differences in product standards. We characterize a "new trade agreement" (NTA) that achieves global efficiency by stipulating not only the cooperative trade taxes that formed the heart of an "old trade agreement" (OTA), but also how governments should optimally set their standards in the light of the international externalities they create. We focus in this paper entirely on what we call "horizontal product standards"; i.e., those that regulate a horizontal dimension of product differentiation. In other words, we consider standards for product attributes that are neither better nor worse, but just different—such as the form of auto safety features, the shape of electric plugs or perhaps the use of genetically modified organisms in the production process—perhaps reflecting differences in local conditions, histories, and values. Thus, our analysis will not have much to say about regulatory differences concerning pollution emissions or violations of labor rights, for which most would agree that less is better but countries differ in their marginal valuations, perhaps due to their different stages of development. Of course, we consider these issues to be important as well, and their treatment in trade agreements will be a topic for our future research.

Our model extends Venables (1987), which is a model of trade in horizontally differentiated products under conditions of monopolistic competition and in the presence of a competitively produced 'outside' good. Whereas Venables and subsequent authors incorporate a single dimension of product differentiation that generates a love of variety, we introduce a second dimension of differentiation along which the residents of different countries have different ideals. An individual pays a utility cost from consuming any good that differs from her ideal along this dimension, where the loss of utility enters as a 'demand shifter' in a familiar CES formulation. We allow firms in the differentiated product sector to tailor their brands to the alternative destination markets, either to cater to consumers tastes and thereby stimulate demand, or to satisfy standards imposed by the local regulatory authority. Although firms can supply different versions of their brands, they bear a fixed cost of design adaptation or from maintaining separate facilities, as suggested in the writings of Lamy.

In our first pass, we assume that an individual's utility depends only on the characteristics of the goods she consumes herself. However, we recognize that the motive for government regulation becomes stronger in environments where the choices of which goods to purchase and consume confer externalities. Moreover, such externalities are natural for the types of horizontally differentiated goods and services that we have in mind. Drivers care not only about the safety features of the cars they drive, but also about the features of other cars on the road. Individuals who care about modes of production for cultural or religious reasons are likely to care about how goods consumed by others around them have been produced. And the functioning of the electric grid, the internet, and the financial sector depend on choices made by all consumers inasmuch as they affect compatibility and network externalities. Accordingly, after characterizing an NTA in a trading environment without consumption externalities, we revisit the issue for settings were such externalities are pervasive.

Our model incorporates shipping costs that generate home-market effects, as in Krugman (1980) and in the original Venables (1987) paper. As a consequence, firms sell relatively more in their local market than in their export market. This affects their optimal design decisions. Profit-maximizing firms cater especially to local tastes given the relatively greater importance of that market to their bottom line. Given the extra fixed costs of designing second products that are very different from the core products sold domestically, firms in our model sell products in their export market that are further from the offshore ideal than the products offered there by local firms. In other words, exporters worldwide have legitimate cost reasons to produce goods that are less appealing to local consumers than those offered by local producers. And while local governments may not care about the profits of foreign producers, they do care about the prices and variety of goods available to their constituents. Accordingly, our model features an economic rationale for regulatory heterogeneity and even for "discriminatory" treatment of goods from different origins; we thus validate Sykes' concerns about the inefficiencies of complete harmonization.

In Section 3, we begin by characterizing an NTA that achieves global efficiency in a setting with international preference heterogeneity but no consumption externalities. We find as usual that net trade taxes should be set to zero in an efficient trade agreement to avoid wedges in the marginal rates of substitution between different goods in different countries. Moreover, consumption subsidies are needed as in other settings with monopolistic competition and an outside good (see, for example, Helpman and Krugman, 1989, pp. 137–145) to compensate for the distortion otherwise caused by markup pricing in one sector and competitive pricing in the other. However, provided that consumption subsidies are subject to national treatment (similar subsidies for local and imported goods), there is no need to stipulate the levels of such subsidies in a trade agreement; the governments subject to national treatment will unilaterally set the subsidy rates needed to offset market power. Finally, the consummate NTA can stipulate the characteristics of goods from all sources in all markets. But the products that firms would design and sell to maximize profits in a world without regulation have exactly the characteristics that are globally efficient when consumption externalities are absent. Therefore, an NTA need not formalize detailed rules in this environment, it is enough that they stipulate that governments refrain from regulation.

Next, we ask whether an NTA is needed to achieve global efficiency or whether certain OTAs that respect governments' sovereignty in setting standards can do the trick, perhaps with what Sykes (1999a) terms "policed decentralization"; i.e., provisions such as national treatment that constrain broad aspects of governments' regulatory choices. First, in Section 3.1, we consider standard setting under a free-trade agreement (FTA) that requires national treatment for consumption subsidies but otherwise leaves governments completely free to choose their domestic policies. We find in this setting a strong incentive for "regulatory protectionism"; in the Nash equilibrium, each government leaves its local firms free from regulation but imposes onerous burdens on import goods in an attempt to effect delocation. In other words, the motive for a tariff agreement that Ossa (2011) identified for the Venables model becomes a motive for rules about regulation once tariffs have been fixed to zero. This confirms Sykes' (1999b) intuition that a need for regulatory cooperation may arise because governments are constrained in the use of their preferred protectionist instruments. We also show in Section 3.2 that an OTA with non-zero net tariffs can improve upon the outcome of an FTA, by introducing a tariff-revenue concern in standard setting. But no OTA that allows governments complete sovereignty in setting standards can achieve the first best.

The delocation motive for onerous standards suggests that discriminatory treatment may be the primary cause of the inefficiencies. So in Section 3.3 we consider an FTA with a national treatment provision that applies not only to consumption subsidies, but also to standards. If each government can set at most a single standard that must apply equally to local goods and imports, the outcome is never first best. This finding is obvious, perhaps, because the first best does not involve similar characteristics for the goods sold in a market from different sources; these characteristics will differ to reflect the different adaptation costs for firms with different home markets. So we allow the governments to set multiple standards, provided that they are equally available to all. Such an OTA also fails to secure the globally-efficient outcome, because the governments have no incentive to offer as an option the standard that is efficient for foreign firms. The resulting Nash equilibrium of an FTA with multiple standards set according to national treatment provides an example of Sykes' (1999b) "facially neutral regulatory protectionism."

An alternative to negotiating rules about regulatory cooperation (and also to the nondiscrimination associated with national treatment, which still leaves open the possibility of regulatory protectionism) is a provision for mutual recognition.[1] Under mutual recognition, which we consider in Section 3.4, each government is left free to set a standard or multiple standards while pledging to accept for import any goods or services that meet the standards of their country of origin.[2] When each government can set a single standard and commits to mutual recognition, the outcome again is not first best. In such circumstances, either firms satisfy the standard of their native country for export sales, in which case all firms produce only one version of their brand, or else firms elect to meet the standard of the destination market, in which case all products sold in the same market bear identical characteristics. In either case, there are only two types of goods supplied to the world market, whereas efficiency mandates that there should be four. However, when governments

[1] Costinot (2008) was the first to formally compare national treatment (NT) and mutual recognition (MR) as alternative instititutions for address incomplete international contracting over standards. He studied an international duopoly with one firm in each country in which governments have a legitimate reason for regulations in the face of consumption externalities but also a profit-shifting motive to favor their local firms. In his setting, neither institution can reproduce the optimal complete contract, but NT tends to perform better for goods characterized by high levels of externalities and MR better for goods characterize by low levels of externalities.

[2] In practice, agreements have placed certain legal limits on when firms can invoke mutual recognition. We discuss these limits and their (in)efficacy in Section 3.4 below.

can designate multiple standards, an OTA that includes a provision for mutual recognition does generate an efficient outcome. In the Nash equilibrium, each government announces (at least) two standards, one that maximizes profits for its firms in their local sales and the other that maximizes profits for its firms in their export sales. When the importing government is bound to accept goods that bear these latter characteristics, the outcome is the same as emerges with no regulation whatsoever, which we have argued is first best in a Venables world without consumption externalities.

Finally, in Section 4, we allow for (negative) consumption externalities. In this setting, the optimal NTA has positive net tariffs, to induce individuals to substitute toward local goods that confer relatively smaller externalities (because they are closer to the local ideal) and away from import goods that confer larger externalities. The requisite consumption subsidy is larger than the one that only offsets the monopoly distortion; in combination with the positive net tariff it generates greater consumption of local goods and less consumption of import goods than when the consumption externality is not present. Finally, the optimal standards—while not fully harmonized across countries and not similar for imports and domestic goods—are no longer the same as what profit-maximizing firms would design on their own. Without regulation, firms in both country have insufficient incentive to differentiate the local and export versions of their brands, because consumer demands are insufficiently sensitive to deviations from the local ideal when individuals ignore the adverse effects of their product choices. The optimal NTA calls for standards that induce all firms to design products closer to the ideal in the destination markets compared to what they would choose if unconstrained to maximize profits. Interestingly, the efficient standards are *more* lenient for imports than for local products, reflecting the differential costs that the different firms face in meeting strict regulations.

In Section 4.2, we revisit the question of whether an OTA with mutual recognition can replicate the efficient outcome of an NTA, but this time in the presence of consumption externalities. We answer this time in the negative; even if consumption externalities are entirely local in geographic scope, an NTA with detailed rules about countries' national regulations is needed to achieve global efficiency.

2 The Model

In this section, we extend the two-country model of Venables (1987) to allow for product standards and the possibility that trade agreements might call for regulatory cooperation. The Venables model features costly trade in horizontally-differentiated products. Trade costs generate home-market effects à la Krugman (1980) that create a "delocation" motive for unilateral policies to increase the presence of local producers. The model has been used previously by Helpman and Krugman (1989) to study trade policy for monopolistically-competitive industries and by Bagwell and Staiger (2015) to examine the incentives that countries have to negotiate reciprocal tariff cuts in such settings.[3]

Our model departs from the earlier literature by introducing international taste differences. We characterize each good with two dimensions of product differentiation. Along one dimension, consumers worldwide display a common Dixit-Stiglitz love of variety. Along the other dimension, the consumers in each country share an ideal characteristic that is different from the characteristic most preferred in the other nation. Not only do consumers bear a utility cost from consuming a version of a product different from their ideal, but they might also care about the type of product consumed by their compatriots. In other words, a firm's choice of characteristic affects consumer welfare directly and might also impinge upon a country's welfare via a consumption externality. We assume that firms can tailor different versions of their brands to suit local tastes and norms, but they face (fixed) costs of product adaptation that increase with the distance in the relevant characteristic space between their offerings to the two markets. Regulation might arise from a government's interest in altering the composition of goods available to local consumer and from its desire to address the consumption externality.

Broadly, we have in mind situations where local conditions in a country dictate the optimal safety features of a product, such as for example with tire chains or automobile headlights; and where regulation of these features may be warranted due to externalities generated by potentially dangerous use of an inappropriate product. As another example, consumers in a country may prefer a certain type of Fair Trade coffee and derive greater utility from drinking coffee that is produced with methods closer to their ideal. If individuals suffer disutility when compatriots buy coffee that is produced in a manner different from what they deem to be ethically appropriate, and if consumers fail to account for the impact of their purchases on fellow citizens' utilities, then regulations that enforce local norms might again be warranted. Similar circumstances might arise from an idiosyncratic national taste for eco-friendly packaging, for organic foods, or for products that do not contain genetically modified organisms.[4]

In principle, the consumption externalities that we have in mind might have global dimensions; consumers in a country might also care about the types of goods that are purchased abroad. However, for analytical clarity, we restrict attention here to non-pecuniary externalities (if any) that are purely local in their geographic scope.

2.1 Demand

Citizens of two countries, Home and Foreign, consume a homogeneous good and a set of horizontally-differentiated products. There are N^J identical consumers in country J. The representative consumer maximizes a quasi-linear utility function,

$$U^J = 1 + C_Y^J + \log\left(C_D^J\right) \ , \quad J \in \{H, F\} \ , \tag{1}$$

[3] See also Ossa (2011), who was the first to study the motivation for trade agreements in a "new" trade model with monopolistic competition.

[4] Podhorsky (2015) studies the global inefficiencies that may arise when countries non-cooperatively administer voluntary certication programs in the presence of imperfect consumer information about product characteristics.

where C_Y^J is per-capita consumption of the homogeneous good Y in country J and C_D^J is a sub-utility index for per-capita consumption of the differentiated products.[5] We designate good Y as numeraire and let P^J denote the appropriate (utility-based) price index for differentiated products in country J in units of the numeraire. Then utility maximization subject to a budget constraint implies

$$C_D^J = \frac{1}{P^J} \ , \quad J \in \{H, F\} . \tag{2}$$

The optimal consumption plan yields indirect utility to the representative consumer of

$$V\left(P^J, I^J\right) = I^J - \log P^J, \quad J \in \{H, F\} , \tag{3}$$

where I^J is per capita disposable income in country J.

The goods that comprise the bundle C_D^J have two distinctive characteristics. One characteristic makes each good unique and renders every pair as CES-substitutes with an elasticity of substitution greater than one, so that consumers covet variety. The other characteristic of a good i, denoted a_i^J, positions the variant sold in country J on a scale $[0, 1]$ along which local consumers have an ideal variety, $\hat{a}^J$. As described above, we may think of this characteristic as indexing different safety features whose efficacy reflects local conditions or as indexing alternative methods of production that are perceived differently on ethical grounds by the two cultures. An individual's utility from consuming the differentiated good i may depend not only on the characteristic a_i^J of the good she consumes herself, but possibly also on the characteristic of the goods consumed by others around her. Letting c_i^J denote the representative individual's consumption of good i in country J and $c_{i\mu}^J$ denote the mean consumption by all "other" consumers in the same country, we assume

$$C_D^J = \left\{ \sum_{i \in \Theta^J} \left[A - \xi \left(a_i^J - \hat{a}^J\right)^2 \right] \left(c_i^J\right)^\beta - (1-\xi)\left(a_i^J - \hat{a}^J\right)^2 \left(c_{i\mu}^J\right)^\beta \right\}^{\frac{1}{\beta}} \ , J \in \{H, F\} , \tag{4}$$

with parameters $A > 1$, $\xi \in [0, 1]$ and $\beta \in (0, 1)$, where Θ^J represents the set of varieties available in country J.[6] Here, ξ measures (inversely) the scope of the consumption externality; when $\xi = 1$, an individual cares only about the characteristic a_i^J of the good i that she consumes herself, whereas when $\xi = 0$, she cares about the types of all goods consumed in her country and only negligibly about the sort that she purchases herself. In any case, the representative consumer benefits less from a quantity of consumption of brand i the further is the characteristic a_i^J from the nation-specific

[5] We use the logarithmic form for sub-utility in order to simplify some of the expressions below. All of our substantive conclusions would apply as well if we were instead to take

$$U^J = C_Y^J + \frac{1}{\theta}\left(C_D^J\right)^\theta, \quad J \in \{H, F\}, \theta \in (0, 1) \ ,$$

which would imply a constant elasticity of demand for the bundle of differentiated products, with elasticity $\varepsilon = 1/1\,(1-\theta) > 1$.

[6] To conserve on notation, we are imposing that only a single version of brand i is available for sale in each country. This follows naturally as an optimal strategy for firms, given that product differentiation is costly and that all individuals in country J share the same taste parameter, $\hat{a}^J$.

ideal characteristic, $\hat{a}^J$. We assume, without further loss of generality, that $\hat{a}^H > \hat{a}^F$.

To solve for the demands of a representative consumer in country J, we proceed in the manner suggested by Dixit and Stiglitz (1977), while further assuming that each consumer takes the average national consumption of each brand as given. Letting p_i^J denote the price paid for variety i with characteristic a_i^J in country J, the representative consumer with income I^J solves the problem

$$\max_{\{c_i^J\}} C_Y^J + \log\left[\sum_{i\in\Theta^J} A_i^J \left(c_i^J\right)^\beta - (1-\xi)\left(a_i^J - \hat{a}^J\right)^2\left(c_{i\mu}^J\right)^\beta\right]^{\frac{1}{\beta}}$$

$$s.t. \qquad \sum_{i\in\Theta^J} p_i^J c_i^J + C_Y^J \leq I^J \ ,$$

taking $c_{i\mu}^J$ as given, where $A_i^J \equiv A - \xi\left(a_i^J - \hat{a}^J\right)^2 > 0$ acts as a demand shifter on own consumption, c_i^J. As we establish in the appendix, manipulating the associated first-order conditions and setting $c_i^J = c_{i\mu}^J$ (as must be true when the representative consumer purchases the average quantity) yields the *per capita* demand for brand i in country J,

$$c_i^J = \left(A_i^J\right)^\sigma \left(p_i^J\right)^{-\sigma}\left(\mathcal{P}^J\right)^{\sigma-1}, \ J \in \{H, F\}, \tag{5}$$

and the *aggregate* demand, $N^J c_{i,}^J$, where

$$\mathcal{P}^J \equiv \left[\sum_{i\in\Theta^J}\left(A_i^J\right)^\sigma\left(p_i^J\right)^{1-\sigma}\right]^{-\frac{1}{\sigma-1}}, \ J \in \{H, F\}, \tag{6}$$

is an aggregator of all differentiated-good prices that shifts the demand for each such product and $\sigma = 1/(1-\beta)$ is the price elasticity of demand for each variety. We will refer to $\mathcal{P}^J$ as the *brand-level price index* inasmuch as it is the index that figures in consumers' choices about their allocation of spending across brands.

Notice that (6) gives the usual formula for the price index of differentiated goods in the Venables model. Also, when $\xi = 1$ and thus there are no consumption externalities, this same price index satisfies $\mathcal{P}^J C_D^J = \sum_{i\in\Theta^J} p_i^J c_i^J$, so $\mathcal{P}^J = P^J$, the *industry-level price index* that enters the indirect utility function in (3) and that determines allocation of spending between differentiated products and the numeraire good in (2).

However, in the presence of consumption externalities (i.e., when $\xi < 1$), the brand-level price index and the industry-level price index are not the same. Rather, the two are related by

$$\mathcal{P}^J = \left[\frac{\sum_{i\in\Theta^J}\frac{\hat{A}_i^J}{A_i^J}\left(A_i^J\right)^\sigma\left(p_i^J\right)^{1-\sigma}}{\sum_{i\in\Theta^J}\left(A_i^J\right)^\sigma\left(p_i^J\right)^{1-\sigma}}\right]^{\frac{\sigma}{\sigma-1}} P^J, \ J \in \{H, F\}. \tag{7}$$

where $\hat{A}_i^J \equiv A - \left(a_i^J - \hat{a}^J\right)^2$. Since $\xi < 1$ implies $\hat{A}_i^J < A_i^J$, we have $\mathcal{P}^J < P^J$; i.e., the industry-level price index that determines utility and the aggregate spending on differentiated products is greater than the brand-level index that guides individual consumption choices at the variety level. As a result, consumers spend less on the bundle of differentiated products than they would with the same prices if the externality were absent. In other words, the negative externality from others' consumption choices diminishes each consumer's enthusiasm for the group of differentiated goods. At the same time, when $\xi < 1$ there is a *relative* distortion of consumption across brands *away from* varieties whose characteristics are closer to the local ideal and *towards* those whose characteristics are relatively far from the ideal. This can be seen from (5), which implies that the ratio $c_i^J/c_{i'}^J$ of consumption of two brands i and i' is $c_i^J/c_{i'}^J = \left[\left(A_i^J\right)^\sigma \left(p_i^J\right)^{-\sigma}\right] / \left[\left(A_{i'}^J\right)^\sigma \left(p_{i'}^J\right)^{-\sigma}\right]$. When the brands bear the same price, as we shall see is true in equilibrium for brands originating from the same location, $c_i^J/c_{i'}^J = \left(A_i^J/A_{i'}^J\right)^\sigma$. Then, if variety i is further from the local ideal than variety i', $c_i^J/c_{i'}^J$ is *decreasing* in ξ. In other words, the relative consumption of i compared to i' will be higher when $\xi = 1$ than when $\xi < 1$ and the ratio will rise as the externality-component in utility grows stronger.

2.2 Supply

The two countries have fixed endowments of a single factor of production that we call "labor." Their labor supplies, L^H and L^F, are sufficiently large to ensure positive output of the numeraire good in each country in all circumstances that we examine. The numeraire good is produced with constant returns to scale and traded in a perfectly-competitive world market. Firms in each country can produce one unit of output with one unit of labor, which fixes the common wage rate at one.

The differentiated products are produced and traded under conditions of monopolistic competition. Firms enter freely in both countries and design a product that is unique along the dimension that generates love of variety. Once the fixed costs have been paid, any firm in any location can produce with constant returns to scale, using λ units of labor per unit of output. Thus, the constant marginal cost of production in each country is λ. The fixed costs depend on their design choices along the second dimension of horizontal differentiation. If the firm selling brand i offers a variant with the characteristic a_i^H in the home market, and a variant with characteristic a_i^F in the foreign market, then it pays a total fixed cost of $K + \kappa \left(a_i^H - a_i^F\right)^2$. In other words, the baseline design cost is K. In addition, if it tailors different versions of its brand to local tastes, the firm bears an extra design cost that is increasing and convex in the distance between them in the relevant characteristic space.

Firms face variable trade costs, including both transport costs and trade taxes (or subsidies). The transport costs take the familiar "iceberg" form; that is, $1 + \phi$ units must be shipped for delivery of one unit. For now, we also allow both governments to impose both tariffs (or import subsidies) and export taxes (or export subsidies). Let τ^J be the *ad valorem* tariff imposed on imports by country J, $J = H, F$, and let e^J denote the *ad valorem* tax imposed on goods that exit its ports. In each case, a negative value of the tax represents a subsidy. We summarize the trade impediments faced by a firm located in country J with the variable ι^J, which is one plus the ad valorem cost of serving the market in $\widetilde{J}$; that is[7]

$$\iota^J = 1 + \phi + e^J + \tau^{\widetilde{J}},\ J = H, F \tag{8}$$

For simplicity, we assume that there are no fixed costs of trade, neither on the importing nor the exporting side.

As is well known (see, for example, Helpman and Krugman, 1989, pp. 137-145), in settings such as this one, the monopoly-pricing distortion in the differentiated-product sector creates an efficiency-enhancing role for consumption subsidies. As we will later confirm, the consumption externality in our model introduces a second possible rationale for such subsidies. For these reasons, we allow for the possibility that governments may tax or subsidize the consumption of differentiated products. If a firm in country J producing brand i sets a factory-gate price of q_i^J, then local consumers pay $p_i^J = (1 - s^J)q_i^J$ per unit for this good, where s^J is the *ad valorem* subsidy (tax if negative) for consumption of differentiated-products offered by the government to local consumers in country J. The same *ad valorem* consumption subsidy applies for purchases of imported brands, once those goods clear customs. Therefore, consumers in J pay $p_{i'}^J = (1 - s^J)\iota^{\widetilde{J}} q_{i'}^{\widetilde{J}}$ for a brand i' imported from country $\widetilde{J}$, where $q_{i'}^{\widetilde{J}}$ is the factory-gate price charged by the offshore producer of that brand.[8]

We turn next to firms' pricing decisions, for the moment taking product characteristics as given. Each firm treats the price indices $\mathcal{P}^H$ and $\mathcal{P}^F$ as fixed when setting its price. As can be confirmed from (5), this means that each firm perceives a constant price elasticity of demand for its brand equal to $-\sigma$ in both markets, regardless of the product characteristics associated with its brand and the policies in place. In this light, it is intuitive and easily established that each firm finds it optimal to set a single factory-gate price for its brand, regardless of the characteristic embodied in a particular version of its product or where it is sold. Specifically, the profit-maximizing factory-gate price for all firms, regardless of location, is

$$q = \frac{\sigma}{\sigma - 1}\lambda\ , \tag{9}$$

which is, as usual, a fixed markup over (the common) marginal cost. Then, the consumer price of a typical local brand in country J is[9]

$$p_J^J = q\left(1 - s^J\right),\ J = H, F, \tag{10}$$

[7] We adopt the notation $\tilde{J}$ to reference the country that is "not J"; for example, if $J = H$, then $\tilde{J} = F$. In writing (8), we implicitly assume that transportation services are freely traded. We could instead assume that export taxes are levied on gross exports including those lost in transport, in which case $\iota^J = (1 + \phi)\left(1 + e^J\right) + \tau^{\tilde{J}}$. This alternative specification would yield similar results.

[8] Our modeling of the consumption subsidy is thus consistent with the GATT/WTO non-discrimination rule of national treatment. And, as we demonstrate below, non-discriminatory consumption subsidies (possibly in combination with regulatory standards) are compatible with worldwide efficiency in our model.

[9] To clarify the notation, $p_{J'}^J$ represents the consumer price in country J of a brand emanating from a factory in country J'.

while the consumer price of an imported brand in country J is

$$p_{\tilde{J}}^{J} = \left(1 - s^{J}\right) q\iota^{\tilde{J}}, \ J = H, F. \tag{11}$$

Consider now a firm's decision about product design for the goods it will sell on its local and export markets. This decision may be constrained by government regulation, but to identify the impetus for regulatory intervention, we begin by supposing that firms have free rein in designing their products. A firm producing brand i in country J earns profits of

$$\pi_{iJ} = (q - \lambda)\left[N^{J} c_{iJ}^{J}\left(a_{iJ}^{J}\right) + (1+\phi) N^{\tilde{J}} c_{iJ}^{\tilde{J}}\left(a_{iJ}^{\tilde{J}}\right)\right] - \left[K + \kappa\left(a_{iJ}^{H} - a_{iJ}^{F}\right)^{2}\right]$$

where $c_{iJ}^{J}(\cdot)$ and $c_{iJ}^{\tilde{J}}(\cdot)$ come from (5) and where we have suppressed for the moment the functional dependence of consumption on the local price index and on the two countries' fiscal policies.[10] The firm maximizes these profits with respect to its choices of a_{iJ}^{H} and a_{iJ}^{F}, while also setting the profit-maximizing price recorded in (9).

The trade-off facing each firm is clear. To maximize sales and thus operating profit, it would design each variant to match local tastes, i.e., $a_{i}^{J} = \hat{a}^{J}$ and $a_{i}^{\tilde{J}} = \hat{a}^{\tilde{J}}$. However, a small change in a_{i}^{J} away from the ideal characteristic for market J costs the firm only a second-order loss in local sales, while generating a first-order savings in design costs.[11] The same is true for a small change in $a_{i}^{\tilde{J}}$ with respect to export sales. Accordingly, the unregulated firm maximizes profits by designing its offerings so that $\hat{a}^{H} > a_{iJ}^{H} > a_{iJ}^{F} > \hat{a}^{F}$. Since all firms in country H make the same design choices as do all firms in country F, we use the notation a_{J}^{J} and $a_{J}^{\tilde{J}}$ to denote the optimal, unregulated product characteristics of a brand that is produced in country J and offered to local and offshore consumers, respectively.

2.3 Equilibrium

To complete the description of equilibrium, we need labor-market clearing conditions for each country and zero-profit conditions for all firms. The former are trivial, because all spending not allocated to differentiated products falls on the homogeneous good and all labor not used to produce differentiated products finds work producing the homogeneous good. We can therefore determine

[10] We insert the subscript J on c_{iJ}^{H}, c_{iJ}^{F}, a_{iJ}^{H}, and a_{iJ}^{F}, to remind the reader that the sales and designs of a firm located in J may differ from those of a firm located in $\tilde{J}$, due to the different trade impediments they face. Since we recognize that all firms in a given location make the same decisions, we will subsequently drop the i subscript and use c_{J}^{H}, c_{J}^{F}, a_{J}^{H}, and a_{J}^{F} to refer to these common choices. That is, for example, c_{J}^{H} is the consumption in country H of a brand emanating from country J.

[11] Using (5), the loss in sales from a small change in a_{i}^{J}, evaluated at $a_{i}^{J} = \hat{a}^{J}$, is

$$\left.\frac{\partial N^{J} c_{iJ}^{J}\left(a_{iJ}^{J}\right)}{\partial a_{iJ}^{J}}\right|_{a_{i}^{J}=\hat{a}^{J}} = -2N^{J}\sigma\xi\left(A_{i}^{J}\right)^{\sigma-1}\left(p_{iJ}^{J}\right)^{-\sigma}\left(\mathcal{P}^{J}\right)^{\sigma-1}\left(a_{i}^{J} - \hat{a}^{J}\right) = 0.$$

Meanwhile, the cost savings from this change is $2\kappa\left(a_{i}^{J} - a_{i}^{\tilde{J}}\right)$, which is non-zero whenever $a_{i}^{J} \neq a_{i}^{\tilde{J}}$. Since $\hat{a}^{H} > \hat{a}^{F}$ by assumption, it is optimal for firms in both countries to narrow the design differences between their two offerings so that the characteristics both fall in the interior of the range, $\left(\hat{a}^{H}, \hat{a}^{F}\right)$.

C_Y^J and output of the homogeneous good residually. We turn now to determining the number of producers in each country, as dictated by free entry.

Notationally, we use a boldface variable to denote the vector containing all values of the variable in the world; for example, $\boldsymbol{p} = \left(p_H^H, p_H^F, p_F^H, p_F^F\right)$ is the vector of prices paid by all consumers for goods from all sources, $\boldsymbol{a} = \left(a_H^H, a_H^F, a_F^H, a_F^F\right)$ is the vector of characteristics of the differentiated goods designed by firms in all countries for all markets, and $\boldsymbol{n} = \left(n^H, n^F\right)$ is the vector of the numbers of differentiated goods produced in all countries, where n^J is the number in country J.

Now we can solve the model as follows. First, $\boldsymbol{p}$ is fully determined by the markup-pricing equation (9) that determines q, the trade-impediment equations (8) that determine $\boldsymbol{\iota}$, and the consumer-price equations (10) and (11) that determine p_H^J and p_F^J as functions of q, $\iota^{\tilde{J}}$, and s^J. Next, we can use the formulas for the brand-level price indices in (6) to solve for $\mathcal{P}^J$ as a function of the prices, the number of brands in each country, and the product characteristics of the goods sold there. Suppressing the dependence on prices (since these can be solved separately), we can write $\mathcal{P}^J = \mathcal{P}^J\left(\boldsymbol{n}, a_H^J, a_F^J\right)$. Then we can use the demand functions (5) to write the zero-profit conditions,

$$N^J c_J^J\left(a_J^J, \mathcal{P}^J\left(\boldsymbol{n}, a_H^J, a_F^J\right)\right) + (1+\phi)\, N^{\tilde{J}} c_J^{\tilde{J}}\left(a_J^{\tilde{J}}, \mathcal{P}^{\tilde{J}}\left(\boldsymbol{n}, a_H^{\tilde{J}}, a_F^{\tilde{J}}\right)\right) = \frac{K + \kappa\left(a_J^H - a_J^F\right)^2}{q-\lambda}, \quad J = H, F, \tag{12}$$

where the left-hand side gives the total output that a representative firm in country J produces to meet demand in its local and export markets while the right-hand side is the total fixed cost paid by such a firm divided by the operating profits it earns per unit.[12] Solving the zero-profit conditions gives the number of brands in each country as a function of the vector of product characteristics and the brand-level price indices. Then, using $\mathcal{P}^J = \mathcal{P}^J\left(\boldsymbol{n}, a_H^J, a_F^J\right)$ to substitute for the price indices, we have

$$n^J = n^J(\boldsymbol{a}), \quad J = H, F, \tag{13}$$

the number of brands in each country expressed simply as a function of the vector of product characteristics.[13]

Finally, the equilibrium in an unregulated world market is found by solving the four first-order conditions for the choices of a_H^H and a_H^F by firms producing in H and the choices of a_F^H and a_F^F by firms producing in F. When computing these first-order conditions, the firms take the number and composition of competitors, $\mathbf{n}$, and the price indices $\mathcal{P}^H$ and $\mathcal{P}^F$ as given.

Before moving on, we offer two observations about the unregulated equilibrium that will prove useful later on. First, suppose that we start at the unregulated equilibrium and then we make a small change in any $a_{J'}^J$, as for example might be induced by a (marginally-binding) product standard in country J. This will not change any prices, but it will change the equilibrium numbers of firms in each country. Recall that we have labeled the countries such that $\hat{a}^H > \hat{a}^F$. Then we

[12] Again, we have suppressed the direct dependence of demands on prices, because we have incorporated this dependence in the definitions of the functions $c_J^J(\cdot)$ and $c_J^{\tilde{J}}(\cdot)$.

[13] Of course, we may have $n^J = 0$ if $\pi_i^J < 0$ when firms in $\tilde{J}$ enter freely and price optimally.

have the following Lemma.[14]

Lemma 1 *Let trade taxes and consumption subsidies take any values such that $\iota^H > 1$ and $\iota^F > 1$ and consider the unregulated equilibrium with the profit-maximizing choices of characteristics, $\boldsymbol{a}$. Beginning at this equilibrium, a small increase in any product characteristic $a^J_{J'}$ induces exit by home firms $\left(dn^H/da^J_{J'} < 0\right)$ and entry by foreign firms $\left(dn^F/da^J_{J'} > 0\right)$ for all $J \in \{H, F\}$ and $J' \in \{H, F\}$.*

To understand the intuition, consider the effects of a small increase in the characteristic of the good produced by home firms for the home market. Since a^H_H maximizes profits for home firms, a marginal change has no effect on home-firm profits for given brand-level price index, $\mathcal{P}^H$. But recall that $a^H_H < \hat{a}^H$, because home firms move the design feature away from the ideal to conserve on fixed costs. Therefore, $\mathcal{P}^H$ falls for given $\boldsymbol{n}$. Given the home bias in consumption induced by the impediments to trade when $\iota^H > 1$, a fall in the home price index has a relatively more powerful (negative) effect on the profits of home firms, which earn a disproportionate share of the profits in the home market, than it does on the profits of foreign firms. So, home firms exit and foreign firms enter. A similar argument applies to a small increase in a^H_F, because $a^H_F < \hat{a}^H$ as well.

Now consider the effects of an increase in the product characteristic of the good produced by foreign firms for the foreign market. Again, this has no direct effect on maximized profits. But $a^F_F > \hat{a}_F$, so a marginal increase in this characteristic moves it further from the foreign ideal, raising the foreign price index $\mathcal{P}^F$ for given $\boldsymbol{n}$. An increase in $\mathcal{P}^F$ raises profits relatively more for foreign firms than for home firms, since foreign firms too earn a disproportionate share of profits in their local market. The change in characteristic induces entry by foreign firms, which in turn generates exit by home firms. A similar argument applies to a small increase in a^F_H, because $a^F_H > \hat{a}^F$ as well.

Second, beginning again at the unregulated equilibrium with profit-maximizing choices of all product characteristics, we note the effects of a small change in some $a^J_{J'}$ on the pair of brand-level price indices, $\mathcal{P}^H$ and $\mathcal{P}^F$. The total effect combines the direct effect, noted above, plus the indirect effects of the induced changes in the numbers of brands, as described in Lemma 1. Considering these effects, we can prove the following.

Lemma 2 *Let trade taxes and consumption subsidies take any values such that $\iota^H > 1$ and $\iota^F > 1$ and consider the unregulated equilibrium with the profit-maximizing choices of characteristics, $\boldsymbol{a}$. Beginning at this equilibrium, a small change in any product characteristic $a^J_{J'}$ has no first-order effect on the home brand-level price index $\left(d\mathcal{P}^H/da^J_{J'} = 0\right)$ or on the foreign brand-level price index $\left(d\mathcal{P}^F/da^J_{J'} = 0\right)$.*

Intuitively, and focusing once again on the impact of a small change in a^H_H starting from the set of profit-maximizing values, $\boldsymbol{a}$, we have observed already that there is no first-order direct effect on any firms' profits. It follows that, if the price indices do not change, all firms will continue to earn zero profits, as is required for a monopolistically-competitive equilibrium. The proof of Lemma 2 in the appendix establishes that this is indeed the case.

[14] See the appendix for the proof of all claims not provided in the text.

2.4 National Welfare Measures

In this section, we develop expressions for national welfare as functions of the governments' policy instruments. Recall from (3) that, for the representative consumer in country J, $V^J = I^J - \log P^J$. Per capita disposable income in country J is the sum of an individual's labor income, L^J/N^J, and her share of rebated tax revenues (or of subsidy financing), since aggregate profits are zero in each country. To develop an expression for tax revenues, we calculate aggregate imports in country J,

$$M^J(\boldsymbol{a},\boldsymbol{p}) = n^{\widetilde{J}} N^J c^J_{\widetilde{J}}\left(a^J_{\widetilde{J}}, \mathcal{P}^J\left(\boldsymbol{n}, a^J_H, a^J_F\right)\right) \tag{14}$$

and exports from country J,

$$E^J(\boldsymbol{a},\boldsymbol{p}) = n^J N^{\widetilde{J}} c^{\widetilde{J}}_J\left(a^{\widetilde{J}}_J, \mathcal{P}^{\widetilde{J}}\left(\boldsymbol{n}, a^{\widetilde{J}}_H, a^{\widetilde{J}}_F\right)\right) \ . \tag{15}$$

Then aggregate tax revenues in country J can be written as

$$R^J = \tau^J q M^J(\boldsymbol{a},\boldsymbol{p}) + e^J q E^J(\boldsymbol{a},\boldsymbol{p}) - s^J q N^J \left\{n^J c^J_J\left(a^J_J, \mathcal{P}^J\left(\boldsymbol{n}, a^J_H, a^J_F\right)\right) + \iota^{\widetilde{J}} n^{\widetilde{J}} c^J_{\widetilde{J}}\left(a^J_{\widetilde{J}}, \mathcal{P}^J\left(\boldsymbol{n}, a^J_H, a^J_F\right)\right)\right\},$$

the difference between trade tax proceeds and consumption subsidy outlays. The representative consumer receives a lump-sum rebate (or pays a lump-sum tax) of R^J/N^J.

Now we define the "world" price, ρ^J, of the exports from country J as the offshore price once export taxes have been collected, but before transport costs, import tariffs and consumption subsidies have been imposed by the importing country. That is,

$$\rho^J = \left(1 + e^J\right) q. \tag{16}$$

Notice that world prices are independent of the characteristics of the differentiated products (just as consumer prices), and we will soon see that they are independent of any product standards. For these reasons, governments cannot use their regulatory policies to manipulate the terms of trade. While this feature of our model is special, it is also convenient, because it allows us to focus on the other motives for standard setting that are novel in this setting.[15] Using the definitions of world prices and a shorthand for consumption of local brands in country J, $c^J_J(\boldsymbol{a},\boldsymbol{p}) \equiv c^J_J\left(a^J_J, \mathcal{P}^J\left(\boldsymbol{n}(\boldsymbol{a},\boldsymbol{p}), a^J_H, a^J_F\right)\right)$, we have[16]

$$R^J = \left(\rho^J - q\right) E^J(\boldsymbol{a},\boldsymbol{p}) + \left(p^J_{\widetilde{J}} - q\phi - \rho^{\widetilde{J}}\right) M^J(\boldsymbol{a},\boldsymbol{p}) - \left(q - p^J_J\right) N^J n^J(\boldsymbol{a},\boldsymbol{p})\, c^J_J(\boldsymbol{a},\boldsymbol{p})$$

or $R^J = R^J(\boldsymbol{a},\boldsymbol{p},\boldsymbol{\rho})$ for short. Since $I^J = L^J/N^J + R^J/N^J$, we have now expressed per capita income in country J as a function of product characteristics and domestic and world prices, or

[15] The efficient treatment of a terms-of-trade motive for domestic policies by international trade agreements has been studied by Bagwell and Staiger (2001).

[16] To derive this expression, we use $e^J q = \rho^J - q$, $\tau^J q = q\left(p^J_{\tilde{J}}/p^J_J - \phi\right) - \rho^{\tilde{J}}$, $s^J q = q - p^J_J$, and $s^J q \iota^{\tilde{J}} = p^J_{\tilde{J}}\left(q/p^J_J - 1\right)$. These pricing relationships all follow from the definitions of the world prices and the relevant price arbitrage conditions.

(recalling the dependence of prices on fiscal policies) as a function of product characteristics and tax policies. Also, the industry-level price index is a function of product characteristics a_H^J and a_F^J, of the numbers of varieties $n^H(\boldsymbol{a}, \boldsymbol{p})$ and $n^F(\boldsymbol{a}, \boldsymbol{p})$, and of the local consumer prices, p_H^J and p_F^J, or $P^J = P^J(\boldsymbol{a}, \boldsymbol{p})$. Thus, aggregate welfare is

$$\Omega^J \equiv N^J V^J = L^J + R^J(\boldsymbol{a}, \boldsymbol{p}, \boldsymbol{\rho}) - N^J \log P^J(\boldsymbol{a}, \boldsymbol{p}) \tag{17}$$

which we can write as $\Omega^J(\boldsymbol{a}, \boldsymbol{p}, \boldsymbol{\rho})$ for short.

Notice that $\partial\Omega^J/\partial\rho^J = E^J(\boldsymbol{a}, \boldsymbol{p})$ and $\partial\Omega^{\tilde{J}}/\partial\rho^J = -M^{\tilde{J}}(\boldsymbol{a}, \boldsymbol{p})$, so that $\partial\Omega^J/\partial\rho^J + \partial\Omega^{\tilde{J}}/\partial\rho^J = 0$. Therefore, as in other contexts, a movement in world prices orchestrates a lump-sum transfer of income between countries via the income effects of the movements in the terms of trade.

2.5 Equilibrium in Regulated Markets

Now, we introduce government regulation. To highlight the efficient outcomes, we allow the two governments to stipulate directly the characteristics of the various goods sold in their markets. Later, we will mention circumstances under which the governments can achieve similar outcomes by announcing ranges of permissible characteristics, rather than precise specifications. Also, we do not insist on similar requirements for local and imported products. Rather, we discuss the desirability of so-called *national treatment* in different contexts. With these features of the regulatory regime in mind, we denote by $\bar{a}_H^J$ and $\bar{a}_F^J$ the characteristics mandated by the government of country J for local sales of products emanating from firms located in Home and Foreign, respectively.

In what follows, we focus on equilibria under which active firms in both locations choose to serve both markets. This is not guaranteed with regulation in place, because the required standard in the export market may be so different from that in the local market that firms cannot earn sufficient profits to cover the fixed cost of providing such disparate products. However, it is intuitive and easy to establish that firms will opt to serve both markets for any pair of feasible standards provided that κ/λ is sufficiently small. To avoid a taxonomy, we take this to be the case.

Notice that regulation of product characteristics has no effect on any firm's pricing behavior, as dictated by (9), or on the relationship between producer and consumer prices, as described by (10) and (11). Therefore, the functional relationships between the brand-level price index $\mathcal{P}^J$ and the number of varieties available from each source plus the characteristics of those varieties remains unchanged; i.e., $\mathcal{P}^J = \mathcal{P}^J\left(\boldsymbol{n}, \bar{a}_H^J, \bar{a}_F^J\right)$, where the function $\mathcal{P}^J(\cdot)$ is the same as for the unregulated market. So too do the forms of the individual demand functions, $c_J^J\left(\bar{a}_J^J, \mathcal{P}^J\left(\boldsymbol{n}, \bar{a}_H^J, \bar{a}_F^J\right)\right)$ and $c_{\tilde{J}}^J\left(\bar{a}_{\tilde{J}}^J, \mathcal{P}^J\left(\boldsymbol{n}, \bar{a}_H^J, ar_F^J\right)\right)$ remain the same. It follows, finally, that the equilibrium number of varieties in each country is given by

$$n^J = n^J(\bar{\boldsymbol{a}}), \;\; J = H, F,$$

where the function $n^J(\cdot)$ is the same as the one in (13). In short, product standards in either country have no effect on the prices of traded or local brands, but they do affect the number of varieties emanating from each country.

We can also express the national welfare of each country in a setting with government stipulation of product characteristics. Note that all prices in the model, be they consumer or producer prices and be they domestic or world prices, are unaffected by the choice of product standards. They continue to bear the same relationships to the production and shipping technologies and the various tax and subsidy instruments as with unregulated markets. Then it is easy to see that aggregate national welfare in country J when global standards are $\bar{\mathbf{a}}$ is given by

$$\Omega^J = \Omega^J(\bar{\boldsymbol{a}}, \boldsymbol{p}, \boldsymbol{\rho}) = L^J + R^J(\bar{\boldsymbol{a}}, \boldsymbol{p}, \boldsymbol{\rho}) - N^J \log P^J(\bar{\boldsymbol{a}}, \boldsymbol{p})$$

where the functions $R^J(\cdot)$, $P^J(\cdot)$, and $\Omega^J(\cdot)$ are the same as in the environment with no product standards.

2.6 Global Welfare

Finally, we develop a measure of world welfare. The measure will be useful for finding the cooperative policies that achieve global efficiency under different types of trade agreements.

We begin by noting from (8) that all four local consumer prices defined in (10) and (11) are pinned down for any s^H and s^F once $\tau^H + e^F$ and $\tau^F + e^H$ are known. But (16) then implies that movements in the world prices can be generated while holding all local prices fixed by adjusting export subsidies and import tariffs together, while holding $\tau^H + e^F$ and $\tau^F + e^H$ constant. An increase in a country's import tariff coupled with an increase in the partner's export subsidy orchestrates a lump-sum transfer from the exporting country's treasury to that of the importing country. Given the quasi-linear form of preferences that we have assumed, the availability of (a perfect substitute for) lump-sum transfers ensures that the efficient policies maximize the sum of home and foreign welfare. Moreover, world prices drop out from the sum of home and foreign revenues, $R^H(\boldsymbol{a}, \boldsymbol{p}, \boldsymbol{\rho}) + R^F(\boldsymbol{a}, \boldsymbol{p}, \boldsymbol{\rho})$, so that we can write

$$\Omega(\boldsymbol{a}, \boldsymbol{p}) \equiv \sum_J L^J + \sum_J q\left(\tau^J + e^{\widetilde{J}}\right) M^J(\boldsymbol{a}, \boldsymbol{p}) - \sum_J N^J q s^J \left[n^J(\boldsymbol{a}, \boldsymbol{p})\, c^J_J(\boldsymbol{a}, \boldsymbol{p}) + n^{\widetilde{J}}(\boldsymbol{a}, \boldsymbol{p})\, \iota^{\widetilde{J}} c^J_{\widetilde{J}}(\boldsymbol{a}, \boldsymbol{p})\right] - \sum_J N^J \log P^J(\boldsymbol{a}, \boldsymbol{p}) \quad (18)$$

Evidently, world welfare depends on regulatory standards and local prices, but not on world prices.

In an "old trade agreement" (that includes a subsidies agreement) the two governments choose the net trade taxes, $z^H \equiv \tau^H + e^F$ and $z^F \equiv \tau^F + e^H$, and the subsidy policies, s^H and s^F, so as to maximize $\Omega(\boldsymbol{a}, \boldsymbol{p})$, while allowing each government to choose its own regulatory standards. The sovereign choices of standards might be totally unconstrained, or they might be subject to institutional rules such as *national treatment* (standards in a country must be the same for locally produced and imported products) or *mutual recognition* (each country must accept any product that meets the standards in the other country). Under a "new trade agreement," the governments negotiate a globally efficient set of product standards, $\bar{\boldsymbol{a}} \equiv \left(\bar{a}_H^H, \bar{a}_F^H, \bar{a}_H^F, \bar{a}_F^F\right)$ along with efficient net trade taxes and consumption subsidies.

In the next section, we assume away any consumption externalities; i.e., we take $\xi = 1$ in the individual sub-utility function, (4). We begin by characterizing the new trade agreement that achieves global efficiency in this environment and then proceed to compare it to various forms of old agreements. We consider how the new agreement must be modified in the presence of consumption externalities in Section 4 below.

3 A New Trade Agreement when $\xi = 1$

When consumption of goods different from the local ideal confers no externalities, the brand-level price index $\mathcal{P}^J$ that enters into individual demands for varieties coincides with the industry-level price index P^J that guides the allocation of spending between differentiated goods and the homogeneous good. Moreover, with $\xi = 1$, we can write the brand-level and industry-level price indices as

$$P^J(\boldsymbol{a},\boldsymbol{p})^{1-\sigma} = \mathcal{P}^J(\boldsymbol{a},\boldsymbol{p})^{1-\sigma} = n^H\left(A_H^J\right)^\sigma\left(p_H^J\right)^{1-\sigma} + n^F\left(A_F^J\right)^\sigma\left(p_F^J\right)^{1-\sigma}$$

and we have by (2), (10), and (11) that

$$\begin{aligned} qs^J\left[n^J(\boldsymbol{a},\boldsymbol{p})\,c_J^J(\boldsymbol{a},\boldsymbol{p}) + n^{\tilde{J}}(\boldsymbol{a},\boldsymbol{p})\,\iota^{\tilde{J}}c_{\tilde{J}}^J(\boldsymbol{a},\boldsymbol{p})\right] &= \frac{s^J}{1-s^J}\left[n^J p_J^J c_J^J(\boldsymbol{a},\boldsymbol{p}) + n^{\tilde{J}} p_{\tilde{J}}^J c_J^J(\boldsymbol{a},\boldsymbol{p})\right] \\ &= \frac{s^J}{1-s^J}\ , \end{aligned}$$

where the second equality reflects the fact that total spending on differentiated goods equals one when $P = \mathcal{P}$. Inserting this expression into (18) gives

$$\Omega(\boldsymbol{a},\boldsymbol{p}) \equiv \sum_J L^J + \sum_J qz^J M^J(\boldsymbol{a},\boldsymbol{p}) - \sum_J N^J \log P^J(\boldsymbol{a},\boldsymbol{p}) - \sum_J N^J \frac{s^J}{1-s^J}. \tag{19}$$

An efficient NTA maximizes $\Omega(\boldsymbol{a},\boldsymbol{p})$ in (19) with respect to $\boldsymbol{z}$, $\boldsymbol{s}$ and $\boldsymbol{a}$.

In the appendix, we show that global efficiency requires $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$, as is also the case in other, simpler models of monopolistic competition that lack a motive for regulatory standards (see Campolmi et al., 2018). The intuition is straightforward. The efficient consumption subsidies offset the monopoly distortion that arises due to markup pricing of differentiated products alongside competitive pricing of the homogeneous good. Without the subsidy, the relative consumer price of differentiated products would exceed the marginal rate of transformation in production and consumers would purchase too little of these goods. Meanwhile, net trade taxes different from zero can only harm world welfare once the optimal consumption subsidies are in place, because they

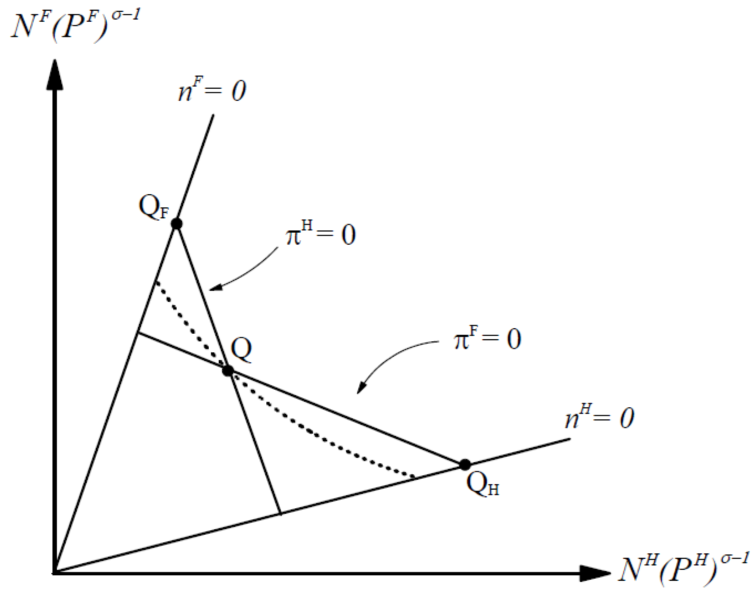

Figure 1: Optimal NTA

distort consumers' allocation of spending between domestic and imported varieties.

To see how the globally-efficient product characteristics are determined, we borrow Figure 1 from Venables (1987). Our Figure 1 is drawn with $N^H\left(P^H\right)^{\sigma-1}$ and $N^F\left(P^F\right)^{\sigma-1}$ on the axes and fixes the product characteristics at the levels that would emerge without government regulation and with $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$. The downward-sloping line labelled $\pi^H = 0$ gives the combinations of $N^H\left(P^H\right)^{\sigma-1}$ and $N^F\left(P^F\right)^{\sigma-1}$ that are consistent with zero profits for home firms, in the light of (12). It has a slope equal to

$$\left.\frac{d\left[N^F\left(P^H\right)^{\sigma-1}\right]}{d\left[N^F\left(P^H\right)^{\sigma-1}\right]}\right|_{\pi^H=0} = -\left(1+\phi\right)^{\sigma-1}\left(\frac{A_H^H}{A_H^F}\right)^{\sigma} < -1\ ,$$

where the inequality follows from the fact that $\phi > 0$ and that $A_H^H > A_H^F$ at the profit-maximizing choices, say $\tilde{a}_H^H$ and $\tilde{a}_H^F$. Similarly, the downward-sloping line labelled $\pi^F = 0$ gives the combinations of $N^H\left(P^H\right)^{\sigma-1}$ and $N^F\left(P^F\right)^{\sigma-1}$ that are consistent with zero profits for foreign firms when their two versions have characteristics $\tilde{a}_F^H$ and $\tilde{a}_F^F$. This line has a slope equal to

$$\left.\frac{d\left[N^F\left(P^H\right)^{\sigma-1}\right]}{d\left[N^F\left(P^H\right)^{\sigma-1}\right]}\right|_{\pi^F=0} = -\frac{1}{\left(1+\phi\right)^{\sigma-1}}\left(\frac{A_F^H}{A_F^F}\right)^{\sigma} > -1.$$

Also depicted in the figure are combinations of $N^H\left(P^H\right)^{\sigma-1}$ and $N^F\left(P^F\right)^{\sigma-1}$ that imply $n^H = 0$ and $n^F = 0$, respectively. These combinations are readily derived from the expressions for P^F and P^H. As shown in the figure, the $n^H = 0$ locus is a ray from the origin with slope $(1+\phi)^{1-\sigma}\left(A^H_F/A^F_F\right)^\sigma\left(N^F/N^H\right)$, while the $n^F=0$ locus is a ray from the origin with slope $(1+\phi)^{\sigma-1}\left(A^H_H/A^F_H\right)^\sigma\left(N^F/N^H\right)$. Price indices that lie inside the cone bounded by these two rays imply $n^H>0$ and $n^F>0$. For illustrative purposes, we have depicted the intersection of the two zero-profit lines as falling inside the cone, hence the equilibrium sans regulation is at Q, with active producers in both countries.

Finally, the figure shows a dotted curve through the point Q. The points on this curve are combinations of $N^H\left(P^H\right)^{\sigma-1}$ and $N^F\left(P^F\right)^{\sigma-1}$ that deliver the same global welfare Ω as at point Q. It is straightforward to show that the slope of the iso-welfare curve at any point is given by $-\left(P^F/P^H\right)^{\sigma-1}$ and that the curve is globally convex, as drawn. Moreover, when Q falls inside the cone defined by $n^H=0$ and $n^F=0$, the slope of the iso-welfare curve through Q lies between the slope of the $\pi^H=0$ line and that of the $\pi^F=0$ line. An infinitesimal change in any product characteristic away from the profit-maximizing levels has no first-order effect on the any firms' profits and therefore no effect on the outcome; in other words, the first-order condition for maximizing Ω is satisfied at Q. Moreover, a small but finite change in some product characteristic would shift the zero-profit line for the affected firms out and to the right; either we would slide up and to the left along the initial $\pi^H=0$ line, or down and to the right along the initial $\pi^F=0$. In either case, world welfare would fall. In other words, the second-order conditions for maximizing Ω are satisfied *locally* at Q.

But consider now a large change in some characteristic, moving for example a^H_F far away from the foreign firms' profit-maximizing choice. The further is a^H_F from $\tilde{a}^H_F$, the greater is the shortfall of foreign firms' profits relative to the maximum achievable level and so the greater is shift in the zero-profit line for these firms. A large shift might take us all the way to point Q_F, where all foreign firms exit the market.[17] If global welfare at Q_F were greater than that at Q, an NTA with onerous regulations for foreign firms that cause all to exit would deliver greater global welfare than one that leaves them free to choose their profit-maximizing characteristics, as underlies the trading equilibrium at Q. Moreover, the trade negotiators can achieve even higher global welfare than at Q_F by reoptimizing the choice of standards that apply to home firms in the light of the absence of foreign competitor. In the appendix, we denote the point of greatest global welfare when $n^F=0$ as Q'_F. Then we prove that the point Q'_F always yields a smaller sum of utilities than does point Q, when the latter point lies the international diversification cone. The trade negotiators cannot change any single product characteristic from its profit-maximizing level and improve thereby on the outcome at Q.

An agreement on product standards might introduce regulations that force both home and foreign firms to design products different from those that maximize profits. Then both curves would shift up and to the right relative to their locations in Figure 1. But the new intersection would necessarily lie outside the dotted line through E. It follows that this too would reduce world welfare. In short, any deviation from the characteristics that home and foreign firms would

[17] Depending on the location of Q, we might not be able to go so far, if we first hit the boundaries of the product-characteristics space.

pick to maximize their profits induces an adjustment in the number and composition of firms in the market that harms global welfare. Evidently, the profit-maximizing product characteristics are globally efficient when coupled with zero net trade taxes and markup-offsetting consumption subsidies.

We summarize our findings in

Proposition 1 *Suppose $\xi = 1$ and let $\widetilde{\boldsymbol{a}}$ be the vector of product characteristics that result from profit-maximizing design choices in an unregulated equilibrium when $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$. Then the maximum world welfare is achieved in a monopolistically-competitive equilibrium when $z^H = z^F = 0$, $s^H = s^F = 1/\sigma$, and $\bar{\boldsymbol{a}} = \widetilde{\boldsymbol{a}}$.*

How could such a globally efficient outcome be achieved with an NTA? First, the agreement would need to stipulate zero net trade taxes on all goods. This is true as well of an OTA in a setting with only one dimension of product differentiation and an international-shared taste for variety; see Campolmi et al. (2018). Without such a provision, the governments would have unilateral incentive to use trade policies to induce delocation, as is well known from the work of Venables (1987) and Ossa (2010). That is, they would try to use trade instruments to increase the share of local firms in the global market, since these firms supply goods at lower delivered prices by avoiding shipping costs and, in our context, also deliver products that are more consonant with local tastes. Second, the agreement could cover product standards; i.e., it could require the home government to set its product standards such that $\left(\bar{a}_H^H, \bar{a}_F^H\right) = \left(\tilde{a}_H^H, \tilde{a}_F^H\right)$ and the foreign government to set its standards such that $\left(\bar{a}_H^F, \bar{a}_F^F\right) = \left(\tilde{a}_H^F, \tilde{a}_F^F\right)$. Finally, the agreement could stipulate that $s^H = s^F = 1/\sigma$. However, this last provision would not be needed, because as we show in the appendix, each government faces a unilateral incentive to set its consumption subsidy at the indicated level when it selfishly maximizes local welfare, given an environment with zero net trade taxes and efficient product standards.

Notice that the trade agreement just described specifies the fine details of each country's product characteristics and that it violates principles of national treatment as regards product standards. Clearly, having identical design requirements for goods produced in different countries is inefficient in our setting, because the home-market effect implies that firms should optimally tailor their locally-sold brand closer to local tastes, and then they face different design costs for serving their export market as compared to firms that are local in that market. However, the coincidence of globally-efficient product standards with the characteristics that firms would anyway choose if given free rein provides flexibility in the design of the efficient NTA, and indeed the agreement can be written in a way that respects national treatment. Suppose, for example, the agreement were to require the home government to permit the range of product characteristics $\left[\tilde{a}_F^H, \tilde{a}_H^H\right]$ and the foreign government to allow the range of characteristics $[\tilde{a}_F^F, \tilde{a}_H^F]$. Such an agreement treats local and offshore firms symmetrically in each market, so it satisfies national treatment. Faced with such (symmetric) freedom of choice, the firms would make their (different) profit-maximizing choices, and global efficiency would be achieved. A different agreement that achieves the same economic effect would have both governments commit not to regulate product characteristics at all.

We summarize our characterization of an efficient NTA in the following corollary to Proposition 1.

Corollary 1 *Suppose $\xi = 1$ and let $\tilde{\boldsymbol{a}}$ be the vector of product characteristics that result from profit-maximizing design choices in an unregulated equilibrium when $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$. Then global efficiency is attained by an international agreement that stipulates trade policies such that all net trade taxes are zero and that regulates product characteristics such that $\bar{\boldsymbol{a}} = \tilde{\boldsymbol{a}}$. Alternatively, global efficiency is attained by an international agreement that requires net trade taxes of zero and requires that all firms be free from regulation in all product markets.*

It might be tempting to conclude that no NTA is needed at all; i.e., that a cooperative trade agreement to maximize joint welfare can be silent on issues of product standards in the absence of consumption externalities. Such a conclusion is not warranted. In the next section, we compare the efficient NTA with an "old" agreement that dictates free trade and markup-offsetting consumption subsidies, but that places no restraint on governments' regulatory choices. We will find that when stripped of their ability to use trade policy to effect delocation, the two governments have strong incentives to use their regulatory practices for such a purpose.

3.1 Benchmark: An Old Free-Trade Agreement without National Treatment

In this section, we study the unilateral incentives that governments have for regulating product characteristics in the context of an OTA that calls for free trade ($\boldsymbol{\tau} = \boldsymbol{e} = 0$) and consumption subsidies. We assume the latter are administered on the basis of national treatment to offset monopoly pricing ($s^H = s^F = \boldsymbol{s} = 1/\sigma$), as they would be in a (new) trade agreement that achieves global efficiency. By examining such an environment, we will begin to understand why governments need to cooperate on standard setting in a NTA.

With $\tau^J = e^J = 0$ and $s^J = 1/\sigma$, the government of each country J seeks to maximize its own domestic welfare with respect to its choice of a_H^J and a_F^J. In this context, domestic welfare is given by

$$\Omega^J(\boldsymbol{a}, \boldsymbol{p}, \boldsymbol{\rho}) = L^J - N^J \log P^J(\boldsymbol{a}, \boldsymbol{p}) - N^J \frac{1}{\sigma - 1} \ ,$$

so the objective of each government is simply to minimize the local price index. We do not impose national treatment on the governments' choices at this point, although we will return to this issue in Section 3.3 below. We aim to characterize the Nash equilibrium that results when the governments choose their regulatory policies freely and noncooperatively.

Let us return to Figure 1, which shows product characteristics at their profit-maximizing levels, and ask whether the home government has an incentive to impose regulations when it is free to do so. Consider first the possibility that it might regulate its local firms; i.e., it might require home products to have characteristics $\bar{a}_H^H$ different from the profit-maximizing choices. Any regulation that requires a different product characteristic than the profit-maximizing choice—be it one that is closer to the home ideal of $\hat{a}^H$ or one that is further away—would reduce profits for the typical home firm. Therefore, the introduction of such a policy would shift the $\pi^H = 0$ line to the right. As is clear from the figure, such regulation would result in a *higher* domestic price index, P^H, after the entry and exit of firms in each country that would be needed to restore zero profits for all firms. Clearly, any such standard would reduce home welfare.

But now consider the possibility that the home government might regulate the characteristics of import products. No matter whether the government sets $\bar{a}_F^H$ a bit closer to $\hat{a}^H$ or a bit further away, a binding regulation reduces profits for foreign firms upon impact (i.e., before any adjustment in the numbers of firms), inasmuch as it forces them to choose characteristics discretely different from those that maximize profits. Thus, the $\pi^F = 0$ curve shifts to the right, resulting in a *lower* domestic price index, P^H, and a higher foreign price index P^F. In this case, the deviation from no regulation is welfare improving for the home country at the expense of the foreign country.

How do we understand the welfare improvement that comes from regulating foreign firms? Consider first a standard $\bar{a}_F^H$ that requires foreign suppliers to produce goods that are a bit closer to the home ideal. Such a regulation would benefit home consumers directly, because it delivers to them products that they find more appealing without changing any prices. At the same time, Lemma 1 tells us that, when the dust settles on the new equilibrium, there will be fewer home firms and more foreign firms than before. In other words, the policy induces what might be termed *anti-delocation*; i.e., the departure of local firms in deference to offshore firms. But the deleterious effects of the anti-delocation do not fully reverse the beneficial effect from having a more suitable imported product, as revealed by the fact that P^H ultimately must fall.[18]

Now consider a home standard $\bar{a}_F^H$ that requires foreign producers to produce goods a bit further from the home ideal. In this case, the direct effect on the welfare of home consumers is negative. But this time delocation would occur; according to Lemma 1, home firms would enter while foreign firms would exit. Evidently, the benefits from delocation outweigh the cost of the diminished appeal of imports to consumers, because—as the figure shows—a small reduction in $\bar{a}_F^H$ from the profit-maximizing level also would cause P^H to fall.

In short, starting from the efficient outcome that could be achieved by an NTA, governments that are free to regulate products differently according to their source will see an incentive to apply pernicious standards to imports products. The incentive for regulation might be either to mandate products that appeal more to local consumers or to induce delocation. In fact, near the efficient characteristics, both incentives for regulation exist at once. Evidently, the globally efficient outcome cannot be achieved with a free-trade agreement that is silent about regulation.

Where does the process of non-cooperative regulation lead us? We note first that, no matter what pair of standards apply to imports in the two countries, it is a best response for each govern-

[18] How could it be that regulation that negatively impacts Foreign firm profits ultimately leads to entry of additional Foreign and exit by Home firms? The answer lies in the asymmetric effects of competition in the Home market. When a_F^H increases closer to the Home ideal, this decreases the Home price index and so increases competition in the Home market. Such enhanced competition is detrimental to profits for all firms, but especially so for Home firms that rely on the Home market for a relatively larger share of their profits. Hence, some Home firms exit. In the adjustment in firm numbers, the Home price index rises above its level after the impact effect alone, due to anti-delocation. But it does not return to its initial, high level.

ment to allow its local firms to choose their profit-maximizing characteristics free from regulation, or else to mandate exactly the profit-maximizing choices. Then, as we show in the appendix, for every pair of standards that applies to local products (or for any pair of profit-maximizing choices, if local products are unregulated), each government has a unilateral incentive to push the standard that applies to its imports to an extreme. Suppose, for example, that the standards for local products are some $\bar{a}_H^H$ and $\bar{a}_F^F$, that the foreign government has some standard $\bar{a}_H^F$ for exports by home firms and that, with these standards, the profit-maximizing choice of foreign firms would be to design a product with characteristic $\left(a_F^H\right)'$ for its sales to the home market. If the home government contemplates a standard $\left(a_F^H\right)''$ for imports, then if $\left(a_F^H\right)'' > \left(a_F^H\right)'$, home welfare would be greater if it were to set instead a standard even larger than $\left(a_F^H\right)''$, whereas if $\left(a_F^H\right)'' < \left(a_F^H\right)'$, home welfare would be greater if it were to set instead a standard even smaller than $\left(a_F^H\right)''$. Each government's incentive for pushing the standards to the extreme persists until either it reaches a boundary of the product space and can go no further, or else one of the governments manages to capture the entire world market for its local firms. We summarize more formally as follows.

Proposition 2 *Suppose $\xi = 1$, $\tau^H = \tau^F == e^H = e^F = 0$ and $s^H = s^F = 1/\sigma$. Suppose governments are free to choose any standards for local products and for imported products, without need for national treatment. Then, in the Nash equilibrium of the standard-setting game, either (i) $n^J = 0$ for some $J \in \{H, F\}$, or (ii) $a_H^F \in \{0,1\}$ and $a_F^H \in \{0,1\}$. The equilibrium level of global welfare is less than that attained under an NTA.*

We offer one further observation about the Nash equilibrium in product standards under an OTA. Recall that the initial motive for regulating imports might be either better suitability (if the regulation moves the characteristic of the imported product closer to the local ideal) or delocation (if the regulation moves the characteristic of the imported product further from the local ideal), in each case when evaluated locally near a policy of no regulation. When we evaluate instead near the Nash equilibrium, the delocation motive always operates on the margin. Take, for example, the home government. If it pushes the standard for imported products down toward or to $\bar{a}_F^H = 0$, it will reduce the number of foreign firms monotonically while tolerating a product less and less suitable for local tastes, so in this case clearly delocation is the only operative motive. On the other hand, if it pushes the standard for imported products up toward or to $\bar{a}_F^H = 1$, the number of foreign firms will respond non-monotonically; at first it will rise, but eventually it will fall. The Nash equilibrium always comes on the falling part of the curve (see Figure **??** in the appendix). Either the standard forces all foreign firms to exit the market (in which case the delocation motive clearly is operating at the margin) or else the home government pushes the standard for import products beyond the home ideal of $\hat{a}^H$ all the way to $\bar{a}_F^H = 1$. Clearly, the marginal incentive for raising the import standard so high cannot be product appeal, because a more moderate standard would be deliver products better suited to local tastes. So, the only reason for pushing the standard to such an extreme would be delocation.

3.2 Benchmark: A Smarter OTA without National Treatment

In Section 3.1, we considered the outcome of regulatory competition under a free-trade agreement (FTA) that stipulates zero tariffs and zero export subsidies while allowing governments free rein in their choices of product standards for local and imported products. We found that the combination of free trade and regulatory autonomy creates strong incentives for the pernicious use of standards on imports. In this section, we show that the countries often can achieve higher joint welfare by using an OTA that departs from free trade. However, a "smarter OTA"—one with offsetting tariffs and export subsidies—can never be designed so as to deliver the first best.

The key to designing a smarter OTA is to use trade taxes and subsidies to dampen the incentives for delocation. In a setting with positive tariffs and export subsidies, a change in regulatory policy that generates entry by local firms and exit by foreign firms imposes a cost in the form of lost revenue for the local tax authority. This adverse revenue effect runs counter to the favorable implications of delocation for the local price index. In some circumstances, a smarter OTA can be designed to deliver less extreme standards for imported products that results under and FTA and thereby achieve a higher level of global welfare.

To illustrate the possibility of a smarter OTA, let us take an initial equilibrium under the FTA with active firms in both countries and with $\bar{a}_F^H = 1$ and $\bar{a}_H^F = 0$. Suppose we were to depict the zero-profit lines for home and foreign firms when all firms are free to choose their profit-maximizing characteristics for sales in their local market but are subject to these extreme regulations in their export markets. In such circumstances, each zero-profit would be downward sloping, just as in Figure 1. Moreover, it will often be the case that the $\pi_H = 0$ line would have a (negative) slope greater than one in absolute value, and the $\pi_F = 0$ line would have a (negative) slope less than one in absolute value, just as in the earlier figure.[19]

Now suppose that we contemplate a trade agreement with zero *net* tariffs, just as with an FTA, but now with $\tau^H = \tau^F = -e^H = -e^F = \tau > 0$. As we know, equilibrium prices and quantities depend only on net trade taxes and so are independent of τ. Home welfare in these circumstances would be given by

$$\Omega^H = L^H + \tau q \left(M^H - E^H\right) - N^H \log\left(P^H\right) - N^H \frac{1}{\sigma - 1} \ ,$$

where aggregate home imports are

$$M^H = N^H n^F \left(A_F^H\right)^\sigma \left(p_F^H\right)^{-\sigma} \left(P^H\right)^{\sigma-1}$$

[19] The slope of the $\pi_H = 0$ line is -$(1+\phi)^{\sigma-1}\left(A_H^H/A_H^F\right)^\sigma$, which often will be greater than one in absolute value, because the extreme standard of $\bar{a}_H^F = 0$ often implies that A_H^F is small. Similarly, the slope of the $\pi_F = 0$ line is -$(1+\phi)^{1-\sigma}\left(A_F^H/A_F^F\right)^\sigma$, which often will be less than one in absolute value, because the extreme standard of $\bar{a}_F^H = 1$ often implies that A_F^H is small. For example, it is possible to show that -$(1+\phi)^{\sigma-1}\left(A_H^H/A_H^F\right)^\sigma < -1$ and -$(1+\phi)^{1-\sigma}\left(A_F^H/A_F^F\right)^\sigma > -1$ when $1 \geq \hat{a}^H \geq 0.5 \geq \hat{a}^F > 0$ and A is sufficiently large.

and aggregate home aggregate exports are

$$E^H = N^F n^H \left(A_H^F\right)^\sigma \left(p_H^F\right)^{-\sigma} \left(P^F\right)^{\sigma-1} \ .$$

Would the home government still wish to apply the extreme standard of $\bar{a}_F^H = 1$ in such circumstances, as it would with free trade? Recall that under the FTA, the delocation motive operates on the margin. Were the home country to slightly ease its regulation of imports to something a bit less than $\bar{a}_F^H = 1$, it would induce entry by foreign firms and exit by home firms; i.e., it would reverse the last bit of delocation. The increase in n^F would contribute to greater imports. Also, since $\bar{a}_F^H$ now is closer to $\hat{a}^H$, import products would be more attractive and the increase in A_F^H also contributes to greater imports. Finally, the shift of $\bar{a}_F^H$ away from the level that minimizes the local price index P^H eases competition in the home market, which further contributes to a rise in imports. Overall, the easing of standards causes imports to rise. Meanwhile, the fall in the number of home firms and the fall in the foreign price index spell a reduction in home exports. The expansion in home imports and the contraction of home exports generate an increase in home tax revenues, as tariff collections rise and export subsidy outlays fall.

The net effect on home welfare combines the adverse effect of the cut in $\bar{a}_F^H$ on the home price index and the favorable effect on total tax revenues. Note, however, that the marginal welfare loss from an increase in P^H is independent of τ, whereas the marginal gain from the increased tax revenues rises linearly with τ. It follows that there must exist a τ large enough that the positive effect dominates.[20] In short, when τ is sufficiently large, the home government's best response to any set of foreign standards will be to choose a standard for imports strictly less than one. By analogous arguments, the foreign government will choose an import standard $\bar{a}_H^F$ that is strictly greater than zero. In other words, the positive tariffs and positive export subsidies induce both governments to moderate their regulation of imports. Finally, if the home and foreign zero profit lines under an FTA are, respectively, steeper and flatter than a line with slope minus one, global welfare will be higher under a smart trade agreement with $\boldsymbol{\tau} > 0$ than under an FTA with $\boldsymbol{\tau} = 0$.

Although countries may be able to design a smarter OTA that improves upon an FTA, there are no values of $\tau^H = -e^F$ and $\tau^F = -e^H$ that would permit an OTA without national treatment to deliver the first-best level of global welfare. To see this, begin at the profit-maximizing standards illustrated in Figure 1. Suppose first that τ^H and e^H are set to be positive and consider the welfare effects of a small increase in $\bar{a}_F^H$. By Lemma 1 foreign firms would enter and home firms would exit. By Lemma 2, there would be no first-order change in either price index. Meanwhile, the increase in $\bar{a}_F^H$ from the level that is profit-maximizing for foreign firms makes the import product more attractive to home consumers. Together, the increases in n^F and A_H^F imply that imports M^H would rise, which would generate a gain in tariff revenues. Meanwhile, the exit by home firms reduces home exports, so home outlays for export subsidies would fall. In combination, the home countries tax revenues grow, with no first-order affect on its price index. This combination represents a gain

[20] Since M^H and E^H depend only on net trade taxes and thus are independent of τ, the gain in tax revenues generated by a reduction in $\bar{a}_F^H$ grows linearly with τ, without bound.

in welfare for the home country and hence we have that no positive τ^H and τ^F exist to discourage deviation from the first-best standards. Suppose instead that the countries set τ^H and τ^F to be negative. In that case, the home government could deviate by reducing its standard $\bar{a}_F^H$ slightly below the efficient level and raise domestic welfare with an increase in trade tax revenues and no first-order effect on the home price index.[21] So, negative tariffs (with positive export taxes) also do not discourage deviations in standard setting. Evidently, a smarter OTA, no matter how smart, cannot deliver the first best.

We summarize the arguments of this section in the following proposition.

Proposition 3 *Suppose $\xi = 1$, $s^H = s^F = 1/\sigma$ and governments are free to choose any standards for local products and for imported products, without need for national treatment. If parameters are such that $n^H > 0$ and $n^F > 0$ and that the $\pi^H = 0$ line has a slope greater than one in absolute value and the $\pi^H = 0$ line has a slope less than one in absolute value in the Nash equilibrium with an FTA, then there exists an OTA with $\tau^H = \tau^F = -e^H = -e^F = \tau > 0$ that yields higher world welfare than the FTA. However, there does not exist any smart OTA with $\tau^H = -e^F$ and $\tau^F = -e^H$ that achieves the first-best level of world welfare.*

3.3 Benchmark: An FTA with National Treatment

Evidently, governments have powerful incentives to use standards as instruments for delocation under an FTA that is silent on regulatory practice. Our findings suggest a potential benefit from provisions for national treatment that would prevent governments from targeting stringent standards solely at import goods. In this section, we examine whether national treatment can be used in our setting to achieve global efficiency without need for more explicit negotiations about product standards. We begin by assuming that each country can impose only a single standard that applies to both domestically-produced goods and imports. We then turn to the possibility that the governments can name two standards, one intended to be attractive to local firms and the other to offshore firms, but with the restriction imposed by national treatment that firms are free choose to meet either standard regardless of their nationality.

As in Section 3.1, we suppose that the countries have agreed to free trade and consumption subsidies that exactly offset the markup pricing, i.e., $\tau^J = e^J = 0$ and $s^J = 1/\sigma$ for $J = H, F$. Under national treatment with a single standard $\bar{a}^J$ in country J, all firms serving that market face the same demand shifter, $A^J = A - \left(\bar{a}^J - \hat{a}^J\right)^2$. This feature simplifies the price indices, which become

$$P^J = \left(A^J\right)^\sigma \left[n^J \left(p_J^J\right)^{1-\sigma} + n^{\tilde{J}} \left(p_{\tilde{J}}^J\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \text{ for } J = H, F.$$

[21] When $\bar{a}_F^H$ is reduced below the profit-maximizing level for foreign firms, n^F falls, A_F^H falls, and n^H rises. So imports fall, exports rise, and the sum of outlays for import subsidies and proceeds from export taxes will rise. Meanwhile, the home price index is unaffected to first order, so the deviation must be beneficial to the home country.

Solving this pair of equations for the number of firms in each location gives

$$n^J = \frac{\left(A^{\tilde{J}}\right)^{\sigma}\left(p^{\tilde{J}}_{\tilde{J}}\right)^{1-\sigma}\left(P^J\right)^{1-\sigma} - \left(A^J\right)^{\sigma}\left(p^J_{\tilde{J}}\right)^{1-\sigma}\left(P^{\tilde{J}}\right)^{1-\sigma}}{\left(A^H\right)^{\sigma}\left(A^F\right)^{\sigma}\left[\left(p^H_H\right)^{1-\sigma}\left(p^F_F\right)^{1-\sigma} - \left(p^H_F\right)^{1-\sigma}\left(p^F_H\right)^{1-\sigma}\right]}.$$

It follows that firms are active in both countries if and only if $\left(A^H/A^F\right)^{\sigma}(1+\phi)^{\sigma-1} > P^H/P^F > \left(A^H/A^F\right)^{\sigma}(1+\phi)^{1-\sigma}$.

Assuming for the moment that firms indeed are active in both countries, we can use the two zero-profit conditions to solve for the equilibrium price indices. We find

$$\left(P^J\right)^{\sigma-1} = \frac{\sigma\left[K+\kappa\left(a^H-a^F\right)\right]}{N^J\left(A^J\right)^{\sigma}}\left[\frac{\left(p^{\tilde{J}}_J\right)^{1-\sigma} - \left(p^{\tilde{J}}_{\tilde{J}}\right)^{1-\sigma}}{\left(p^H_H\right)^{1-\sigma}\left(p^F_F\right)^{1-\sigma} - \left(p^H_F\right)^{1-\sigma}\left(p^F_H\right)^{1-\sigma}}\right], \quad J = H, F. \tag{20}$$

In a Nash equilibrium, each government chooses its standard to minimize its price index, given the standard of the other. The best-response functions that follow from the first-order conditions imply

$$\frac{\kappa\left(\bar{a}^H-\bar{a}^F\right)}{K+\kappa\left(\bar{a}^H-\bar{a}^F\right)^2} = \frac{\sigma\left|\bar{a}^J-\hat{a}^J\right|}{A-\left(\bar{a}^J-\hat{a}^J\right)^2},\ J = H, F. \tag{21}$$

The resulting Nash equilibrium regulations under national treatment, which we denote by $\bar{a}^H_{nt}$ and $\bar{a}^F_{nt}$, have the property that $\left|\bar{a}^J_{nt}-\hat{a}^J\right|$ is common in the two countries, i.e., the equilibrium home standard is the same distance from the home ideal as is the equilibrium foreign standard from the foreign ideal. This in turn implies that $A^H = A^F$; the demand shifters facing firms in the two countries are the same. Accordingly, $n^H > 0$ and $n^F > 0$ under the equilibrium standards if the countries are not too different in size. National treatment does indeed limit the scope for delocation.

It is obvious that an FTA with national treatment and a single standard in each country cannot achieve the globally efficient outcome of an NTA; the latter requires that firms from the two locations serve a given market with *different* products. What is more interesting is the fact that the standards under an FTA with national treatment are independent of sizes of the countries and of the magnitude of shipping costs. This is so, because the price index for country J that is consistent with zero profits for all firms is multiplicatively separable in the size of that country, a term that reflects all consumer prices, and a term that depends on the pair of regulations, $\bar{a}^H_{nt}$ and $\bar{a}^F_{nt}$; see (20). Given this multiplicative separability, the country sizes and shipping costs do not affect the marginal incentives for either government to choose a standard as a best response to the other, even though they do affect the welfare level that each attains in equilibrium. The insensitivity of the equilibrium standards under an FTA with national treatment to N^H, N^F and ϕ contributes to the inefficiency of the equilibrium outcome under such an arrangement inasmuch as globally efficient standards of an NTA certainly do vary with these conditions of the market.

It is tempting to think that the inefficiency of an FTA with national treatment derives only from the fact that we have restricted governments to choose a single standard, whereas the globally efficient outcome requires different standards for goods emanating from different sources. To check this hypothesis, we now allow each government in an FTA to specify a choice of standards, $\bar{a}_1^J$ and $\bar{a}_2^J$, and to allow firms to satisfy either one.[22] If national treatment is sufficient for global efficiency without need for further restrictions on regulation, then the Nash equilibrium of such a standard-setting game ought to achieve the efficient outcome. In fact, it does not.

The problem that arises in such an environment is that each government wants to reduce the profits of foreign firms relative to domestic firms in order to effect delocation. As we have seen, this leads each government to prescribe extreme characteristics for imported products in the absence of national treatment. But, when national treatment applies, the offshore firms can avoid the adverse consequences of extreme standards by choosing to conform to the more moderate standard that local firms obey. The foreign firms cannot be induced to accept a level of profits below what they could achieve under the standard targeted for domestic firms, and so no additional delocation is possible beyond what can be achieved with a single standard. Accordingly, neither government can unilaterally achieve higher domestic welfare by offering a second standard than what it can achieve with only one. Faced with this knowledge, its best response always includes a strategy of announcing $\bar{a}_{nt}^J$ alone, or else it can announce $\bar{a}_{nt}^J$ along with a second standard that is sufficiently extreme as to be ignored by all firms.

We summarize in

Proposition 4 *Suppose $\xi = 1$, $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$. Suppose each government is free to choose any standard or set of standards as long as they are offered to all firms irrespective of origin.. Then, in the Nash equilibrium of the standard-setting game, the outcome is equivalent to one in which each government names a single standard, $\bar{a}_{nt}^H$ and $\bar{a}_{nt}^F$ with the property that $\hat{a}^H - \bar{a}_{nt}^H = \bar{a}_{nt}^F - \hat{a}^F$. The equilibrium standards are independent of N^H, N^F, and ϕ and do not achieve the maximal level of global welfare that is attained by an NTA.*

In short, national treatment alone cannot get the countries out of the prisoner's dilemma that arises from the urge to delocate.

3.4 Benchmark: An FTA with Mutual Recognition

Countries might instead rely on a provision for mutual recognition in an effort to neutralize the delocation motives for standard setting. Under mutual recognition, each government respects the legitimacy of the other country's regulatory aims; therefore, any product that meets standards in an exporting country is considered acceptable for sale in the importing country. Mutual recognition gives exporting firms the choice of whether to meet the standards of the destination market or their local country.[23]

[22] Introducing the possibility of additional allowable products in each country—including that of a continuous range of products—would not alter the conclusions, inasmuch as there will always be one product specification intended for home firms and another (possibly the same) intended for foreign firms.

[23] In practice, the presumption of mutual recognition may be rebutted by a government that can show that its different standards are justified and not introduced as a means to impede or disadvantage non-local firms.

The European Union has explicitly introduced mutual recognition into its customs treaty as an alternative to detailed rules to harmonize standards (see Ortino, 2007, p.310). In its 1985 White Paper on completing the internal market, the European Commission argued that "... the alternative [to mutual recognition] of relying on a strategy based totally on harmonization would be over-regulatory, would take a long time to implement, would be inflexible and could stifle innovation." Mutual recognition in the European context has been interpreted by the European Court of Justice to oblige acceptance of another member's standards whenever a producer is already established in its home country and when it lawfully provides goods or services to the home market that are similar to the ones it intends to supply abroad (Ortino, 2007, p. 312). We will come back to this latter requirement below, after we examine how well mutual recognition can perform in comparison to an agreement that includes more detailed rules on product standards.

We begin again with the case of a single such standard in each country. In this setting, the home and foreign governments announce standards $\bar{a}_H$ and $\bar{a}_F$, respectively. Mutual recognition implies that a firm located in J that wishes to sell in $\widetilde{J}$ has the choice to satisfy either the destination standard $\bar{a}_{\widetilde{J}}$ or to satisfy the standard $\bar{a}_J$ that applies to goods sold in its own market. We ask, What standards will the governments set in a noncooperative equilibrium, if subject to an FTA with zero trade taxes, with consumption subsidies that exactly offset the markup pricing, and with mutual recognition?

Faced with a choice of product characteristics for their export sales, firms compare the extra variable profits they can earn with a variant that meets the standards in the importing country with the savings in design costs that comes from producing a common variant for both markets. A firm located in J will make the former choice if and only if

$$N^{\widetilde{J}}(q-\lambda)(1+\phi)\left(p_J^{\widetilde{J}}\right)^{-\sigma}\left(P^{\widetilde{J}}\right)^{\sigma}\left\{\left[A-\left(\bar{a}_{\widetilde{J}}-\hat{a}^{\widetilde{J}}\right)^2\right]^{\sigma}-\left[A-\left(\bar{a}_J-\hat{a}^{\widetilde{J}}\right)^2\right]^{\sigma}\right\}\geq\kappa\left(\bar{a}_{\widetilde{J}}-\bar{a}_J\right)^2.$$

Clearly, the option to meet the standards of the importing country will be relatively attractive when κ is small and the option to invoke mutual recognition will be attractive when κ is large. We take each case in turn.[24]

If κ is small and the two governments anticipate that all firms will elect to meet standards in their destination markets in order to take advantage of the extra demand that comes with producing a version more suitable for local tastes, then we are back in a world of national treatment. All firms in J produce one version of their brand with characteristic $\bar{a}_J$ for sales in their local market and a second version with characteristic $\bar{a}_{\widetilde{J}}$ for their export sales. The incentives facing the two governments in setting standards are exactly as in Section 3.3, and the outcome is the same. In particular, the Nash equilibrium regulations are the pair of standards $\bar{a}_{nt}^H$ and $\bar{a}_{nt}^F$ that constitute mutual best responses, i.e., that satisfy (21) for $J=H$ and $J=F$.

However, if κ is large, the incentives facing the governments are different than with national

[24] There are also intermediate cases when firms in one country produce two versions of their brand and firms in the other invoke mutual recognition, or when some firms in country make one choice and others do the opposite, and all are indifferent. To conserve on space and the reader's patience, we do not consider these intermediate cases here.

treatment. When each government anticipates that firms exporting to its market will invoke mutual recognition, it realizes that its own standard will only affect design choices by native firms. Accordingly, it selects the standard that maximizes profits for these firms.[25] For a firm in country J, the profit-maximizing characteristic is the one that satisfies

$$N^J \left(A_J^J\right)^{\sigma-1} \left(P^J\right)^{\sigma-1} \left(p_J^J\right)^{-\sigma} \left|\bar{a}_J - \hat{a}^J\right| = N^{\widetilde{J}} (1+\phi) \left(A_J^{\widetilde{J}}\right)^{\sigma-1} \left(P^{\widetilde{J}}\right)^{\sigma-1} \left(p_J^{\widetilde{J}}\right)^{-\sigma} \left|\bar{a}_J - \hat{a}^{\widetilde{J}}\right|$$

or, using the fact that, with free trade, $p_J^{\widetilde{J}} = (1+\phi)\, p_J^J$,

$$N^J \left(A_J^J\right)^{\sigma-1} \left(P^J\right)^{\sigma-1} \left|\bar{a}_J - \hat{a}^J\right| = N^{\widetilde{J}} (1+\phi)^{1-\sigma} \left(A_J^{\widetilde{J}}\right)^{\sigma-1} \left(P^{\widetilde{J}}\right)^{\sigma-1} \left|\bar{a}_J - \hat{a}^J\right|. \tag{22}$$

We can solve for $N^H \left(P^H\right)^{\sigma-1}$ and $N^F \left(P^F\right)^{\sigma-1}$ using the two zero-profit conditions, as we have done before, and then substitute back into (22) to derive the best-response functions for the two governments,

$$\begin{aligned} &\left(A_J^J\right)^{\sigma-1} \left[\left(A_{\widetilde{J}}^{\widetilde{J}}\right)^{\sigma} - (1+\phi)^{1-\sigma} \left(A_J^{\widetilde{J}}\right)^{\sigma}\right] \left|\bar{a}_J - \hat{a}^J\right| = \\ &\qquad (1+\phi)^{1-\sigma} \left(A_J^{\widetilde{J}}\right)^{\sigma-1} \left[\left(A_J^J\right)^{\sigma} - (1+\phi)^{1-\sigma} \left(A_{\widetilde{J}}^J\right)^{\sigma}\right] \left|\bar{a}_J - \hat{a}^{\widetilde{J}}\right| \text{ for } J = H, F. \end{aligned} \tag{23}$$

Evidently, $\hat{a}^H - a_H^{mr} = a_F^{mr} - \hat{a}^F$, where a_J^{mr} is the standard set by country J in a Nash equilibrium with mutual recognition.[26] That is, the two standards are equidistant from the local ideals, just as with national treatment. Also, the standards chosen when mutual recognition is invoked do not depend on the sizes of the two countries, just as with national treatment, although now they do depend on the size of the shipping costs. Of course, mutual recognition with a single standard in each country does not achieve the first best, because global efficiency requires four different types of products (two different types from each of two different countries), whereas mutual recognition with one standard per country gives rise to only two.

So now we allow each government to set two standards, instead of just one. The government of country J announces $\bar{a}_J^1$ or $\bar{a}_J^2$. Firms located in that country must produce a version with one of these characteristics for local sales, but they can choose to meet any of the four legal standards for their sales in country $\widetilde{J}$.

By familiar arguments, each government will choose the product characteristics that maximize

[25] The argument is the same as before. The local price indexes are determined by the intersection of a pair of zero-profit lines, as in Figure 1. The slope of the zero-profit line for home firms in the space of $N^H \left(P^H\right)^{\sigma-1}$ and $N^F \left(P^F\right)^{\sigma-1}$ is $(1+\phi)^{\sigma-1} \left(A_H^H / A_H^F\right)^{\sigma}$, except that now A_H^H and A_H^F are determined by the home standard, $\bar{a}_H$. Similarly, for foreign firms the zero profit line has slope $-(1+\phi)^{1-\sigma} \left(\frac{A_F^H}{A_F^F}\right)^{\sigma}$ that is determined by $\bar{a}_F$. By the same arguments as before, the home government chooses the $\bar{a}_H$ (now a single number) that maximizes home firm profits; any other choice would yield a zero-profit line shifted up and to the right, which would deliver a higher price index, P^H. This would be the same product that home firms would choose themselves, if they were only allowed one type of product. Analogous arguments apply to $\bar{a}_F$, which must be the profit-maximizing choice by a representative foreign firm.

[26] This statement follows from the fact that $\hat{a}_H - \bar{a}_H = \bar{a}_F - \hat{a}_F$ implies $\bar{a}_H - \hat{a}^F = \hat{a}^H - \bar{a}_F$ and thus $A_H^H = A_F^F$ and $A_F^H = A_H^F$.

profits for its representative national firm. But these are just the pair of standards that would emerge under a globally efficient NTA. We conclude that the governments have a viable alternative to negotiating a detailed NTA when $\xi = 1$; instead they can negotiate an FTA and agree to mutual recognition of their partner's standards.

Moreover, the same efficient outcome can be attained if each government designates a range of permissible products, $\left[\bar{a}_J^1, \bar{a}_J^2\right]$, so long as the range in each country includes the products that it would produce under an efficient NTA. Under mutual recognition, firms would choose for local and export sales those characteristics that maximize profits in each market and then invoke mutual recognition for the exports. But, in this case, the product design and all sales and market composition would be the same as under the efficient NTA.

We note one caveat to these arguments. Recall the terms of the European Union treaty, as interpreted by the European Court of Justice. Under that treaty, a firm can invoke mutual recognition in its export market only if it also supplies a similar good to its local market. In our setting, globally efficiency requires firms to supply *different* goods in the two markets. If an OTA includes mutual recognition but also a restriction such as applies in the European Union, then firms would need to sell some minimal amounts of the variants they export to local consumers in order to qualify for legal sales abroad. This too would introduce an inefficiency. The efficient outcome can be achieved in our setting only by an FTA that places no such restrictions on the invocation of mutual recognition.

We state

Proposition 5 *Suppose $\xi = 1$, $\tau^H = \tau^F = e^H = e^F = 0$ and $s^H = s^F = 1/\sigma$. Suppose that each government is free to choose two or more standards for local sales and that firms can invoke mutual recognition for export sales of any product that can legally be sold in its native market. Then, in the Nash equilibrium of the standard-setting game, each government will set two or more standards and the outcome is the same as in the globally efficient NTA.*

4 A New Trade Agreement when $\xi < 1$

We turn now to settings with $\xi < 1$. When consumption of less-than-ideal goods confers a negative externality on other nationals, the brand-level price index $\mathcal{P}^J$ that enters the demand for varieties differs from the industry-level price index P^J that guides the allocation of spending between sectors and that figures in the assessment of indirect utility. With our formulation of the utility function, the parameter ξ does not affect the *total* disutility that the representative consumer bears from local consumption of less-than-ideal varieties, but only the *composition* of that total between own and others' consumption. Accordingly, the size of ξ does not impact the efficient quantities of per-brand consumption, the efficient product characteristics, or the efficient numbers of home and foreign firms; rather, ξ determines the policies needed to achieve these optimal outcomes.

Leveraging this observation, we build on the results from the previous section, where we have characterized the efficient magnitudes and the policies needed to achieve them when $\xi = 1$. Now, we characterize the policies needed to achieve these same outcomes when $\xi < 1$. Once we have identified the efficient policies, we can describe the optimal NTA and ask whether other trade agreements might be able to achieve the same results.

4.1 Efficiency when $\xi < 1$

In order to characterize the policies that are needed to achieve global efficiency when $\xi < 1$, we first introduce notation for the efficient magnitudes. In particular, we apply a superscript E to denote an efficient outcome. For example, the efficient characteristics for a good produced in some country J' and consumed in J is $a_{J'}^{JE}$ and the efficient per capita consumption of such a goods $c_{J'}^{JE}$. Similarly, the efficient numbers of home and foreign firms are n^{HE} and n^{FE}. As before, boldface symbols without country indices denote vectors of all global values; e.g., $\mathbf{n}^E = \left(n^{HE}, n^{FE}\right)$.

An NTA that achieves global efficiency when $\xi < 1$ specifies trade policies and consumption subsidies to implement the efficient numbers of firms, $\mathbf{n}^E$, and the efficient per capita consumption levels of each brand in each country, $\boldsymbol{c}^E$, given the efficient product characteristics, $\boldsymbol{a}^E$.[27] We first characterize the trade policies and consumption subsidies that deliver the efficient per-brand consumption levels and the efficient numbers of home and foreign firms, when product characteristics are set at their efficient levels. Once we have characterized the necessary taxes and subsidies, we will consider whether regulatory standards are in fact required to ensure that firms provide the optimal product designs.

Let $p_J^{JE}(\xi)$ and $p_{\tilde{J}}^{JE}(\xi)$ denote the consumer prices in country J that induce the representative consumer there to purchase the efficient quantities, c_J^{JE} and $c_{\tilde{J}}^{JE}$. Specifically, we need $p_J^{JE}(\xi)$ and $p_{\tilde{J}}^{JE}(\xi)$ such that

$$c_J^{JE} = \left[A - \xi\left(a_J^{JE} - \hat{a}^J\right)^2\right]^\sigma \left(p_J^{JE}(\xi)\right)^{-\sigma} \left(\mathcal{P}^J\left(p_J^{JE}(\xi), p_{\tilde{J}}^{JE}(\xi); \boldsymbol{n}^E, a_J^{JE}, a_J^{JE}\right)\right), \quad J = H, F \tag{24}$$

and

$$c_{\tilde{J}}^{JE} = \left[A - \xi\left(a_{\tilde{J}}^{JE} - \hat{a}^J\right)^2\right]^\sigma \left(p_{\tilde{J}}^{JE}(\xi)\right)^{-\sigma} \left(\mathcal{P}^J\left(p_J^{JE}(\xi), p_{\tilde{J}}^{JE}(\xi); \boldsymbol{n}^E, a_J^{JE}, a_{\tilde{J}}^{JE}\right)\right), \quad J = H, F. \tag{25}$$

Note that the requisite prices—in contrast to the quantities—do depend on the size of the externality. Inserting the efficient consumption quantities into the zero-profit conditions (12) delivers the efficient numbers of home and foreign firms.

Now we can use (24) and (25) to express the efficient consumer prices for an arbitrary ξ in terms of the efficient prices for the case when $\xi = 1$. Letting $p_J^{JE}(1)$ and $p_{\tilde{J}}^{JE}(1)$ denote these latter

[27] The efficient consumption level for the numeraire good must also be achieved, but this is ensured with efficient consumption of differentiated products by satisfaction of the budget constraints.

prices, we have

$$p_J^{JE}(\xi) = p_J^{JE}(1) \left[\left(\frac{A - \xi \left(a_J^{JE} - \hat{a}^J\right)^2}{A - \left(a_J^{JE} - \hat{a}^J\right)^2} \right)^\sigma \left(\frac{\mathcal{P}^J(p_J^{JE}(\xi), p_{\widetilde{J}}^{JE}(\xi); \mathbf{n}^E, a_J^{JE}, a_{\widetilde{J}}^{JE})}{P^{JE}} \right)^{\left(\frac{\sigma-1}{\sigma}\right)} \right]$$

and

$$p_{\widetilde{J}}^{JE}(\xi) = p_{\widetilde{J}}^{JE}(1) \left[\left(\frac{A - \xi \left(a_{\widetilde{J}}^{JE} - \hat{a}^J\right)^2}{A - \left(a_{\widetilde{J}}^{JE} - \hat{a}^J\right)^2} \right)^\sigma \left(\frac{\mathcal{P}^J(p_J^{JE}(\xi), p_{\widetilde{J}}^{JE}(\xi); \mathbf{n}^E, a_J^{JE}, a_{\widetilde{J}}^{JE})}{P^{JE}} \right)^{\left(\frac{\sigma-1}{\sigma}\right)} \right]$$

where P^{JE} is the efficient (brand-level and industry-level) price index in country J when $\xi = 1$.

In the appendix, we establish that for $\xi < 1$, $p_J^J(\xi) < p_J^J(1)$ and $p_{\widetilde{J}}^J(\xi) > p_{\widetilde{J}}^J(1)$; i.e., for efficiency, consumers in each country must face higher prices for import goods and lower prices for domestic goods when consumption externalities are present as compared to when they are not. Intuitively, it is desirable to raise the prices of import goods relative to those of domestic goods, because individuals overconsume imported brands that are far from the local ideal and underconsume local brands that are closer to the ideal, inasmuch as they ignore the externalities they confer on fellow nationals.

Now we can use the relationship between prices and tax policies to compute the net trade taxes and the consumption subsidies that generate the consumer prices needed for efficiency. First, we have[28]

$$\tau^{JE}(\xi) + e^{\widetilde{J}E}(\xi) = (1+\phi) \left[\frac{A_{\widetilde{J}}^{JE}(\xi) / \hat{A}_{\widetilde{J}}^{JE}}{A_J^{JE}(\xi) / \hat{A}_J^{JE}} \right] - 1 > 0,\ J = H, F, \tag{26}$$

where the inequality in (26) follows from the fact that local brands have efficient characteristics closer to the ideal in their country than do imported brands.[29] The efficient consumption subsidies then are given by

$$s^{JE}(\xi) = \frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right) \left[1 - \left(\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}} \right) \left(\frac{\mathcal{P}^J(p_J^{JE}(\xi), p_{\widetilde{J}}^{JE}(\xi); \mathbf{n}^E, a_J^{JE}, a_{\widetilde{J}}^{JE})}{P^{JE}} \right)^{\left(\frac{\sigma-1}{\sigma}\right)} \right],$$

$$J = H, F. \tag{27}$$

[28] Recall the definition of $\hat{A}_{J'}^J = A - \left(a_{J'}^J - \hat{a}^J\right)^2 = A_{J'}^J(1)$.

[29] The fact that $\hat{a}^H > a_H^{HE} > a_F^{HE}$ implies

$$\frac{A - \xi\left(\hat{a}^H - a_F^{HE}\right)^2}{A - \left(\hat{a}^H - a_F^{HE}\right)^2} > \frac{A - \xi\left(\hat{a}^H - a_H^{HE}\right)^2}{A - \left(\hat{a}^H - a_H^{HE}\right)^2}$$

and similarly $\hat{a}^F < a_F^{FE} > a_H^{FE}$ implies

$$\frac{A - \xi\left(a_H^{FE} - \hat{a}^F\right)^2}{A - \left(a_H^{FE} - \hat{a}^F\right)^2} > \frac{A - \xi\left(a_F^{FE} - \hat{a}^F\right)^2}{A - \left(a_F^{FE} - \hat{a}^F\right)^2}.$$

The first term on the right-hand side in (27) is, as before, the subsidy needed to offset the markup pricing of differentiated products. The second term is positive because, as we confirm in the appendix, $A_J^{JE}(\xi) < \hat{A}_J^{JE}$ and $\mathcal{P}^{JE} < P^{JE}$. It may seem surprising that the optimal consumption subsidy is *larger* in the presence of a consumption externality than in its absence. But the larger consumption subsidy generates extra demand for local brands, while the combined consumption subsidy and net trade tax discourage consumption of import brands, as is optimal considering the greater externality that imports cause. In other words, the combination of tax policies delivers $p_J^J(\xi) < p_J^J(1)$ and $p_{\tilde{J}}^J(\xi) > p_{\tilde{J}}^J(1)$, as we have seen is needed for efficiency in the presence of consumption externalities.

Finally, as we confirm in the appendix, the efficient consumption subsidies and net trade taxes in combination with the vector of efficient product characteristics deliver

$$P^J(\mathbf{a}^E, \mathbf{p}^J(\xi)) = P^{JE} \text{ for } J = H, F;$$

i.e., the industry-level price indices are the same as when there are no externalities. We also establish that the additional consumption subsidies and net trade taxes implied by efficient intervention in the presence of consumption externalities are revenue neutral, implying that global welfare under the efficient policies is given by

$$\begin{aligned}\Omega\left(\mathbf{a}^E, \mathbf{p}^E(\xi)\right) &= \sum_J L^J - \sum_J N^J \log P^J\left(\mathbf{a}^E, \mathbf{p}^E(\xi)\right) - \sum_J N^J \frac{1}{\sigma - 1} \\ &= \sum_J L^J - \sum_J N^J \log P^{JE} - \sum_J N^J \frac{1}{\sigma - 1} \ ,\end{aligned}$$

which is independent of ξ. This outcome reflects the fact that the optimal policies induce consumers to internalize the externalities caused by their spending decisions and so protect the world economy from any loss of utility.

We turn now to the efficient product characteristics. Recall that, with $\xi = 1$ an NTA can but need not specify particular standards. Instead, the governments can commit to leave markets unregulated and then firms will choose the efficient characteristics when maximizing profits. We ask now whether the details of product regulation need to be specified in an NTA in the presence of consumption externalities.

To see that product standards indeed are required in an optimal NTA when $\xi < 1$, we evaluate the change in profits for a small change in design around $\mathbf{a}^E$ when the efficient taxes are in place. We know that profits are maximized at $\mathbf{a}^E$ when $\xi = 1$ (no externality), so the first-order changes in profits are zero in such circumstances. When $\xi < 1$, by contrast, $\frac{\partial \pi_{iH}}{\partial a_{iH}^F} > 0 > \frac{\partial \pi_{iH}}{\partial a_{iH}^H}$ and $\frac{\partial \pi_{iF}}{\partial a_{iF}^F} > 0 > \frac{\partial \pi_{iF}}{\partial a_{iF}^H}$ when evaluated at $\mathbf{a}^E$; i.e., firms in both countries will insufficiently differentiate the local and export versions of their brands in the absence of binding regulations, compared to what is globally efficient. Efficient regulation forces firms in each country to tailor their products closer to the ideal in each of their destination markets, relative to what they would choose on their own. This follows from the fact that firms respond to market demands and consumer demands are insufficiently sensitive to deviations from the local ideal when buyers ignore the adverse affects of their decisions on their compatriots' well-being.

We summarize with

Proposition 6 *Suppose $\xi < 1$. Then maximum world welfare requires $z^J > 0$ for $J = H, F$, $s^J > 1/\sigma$ for $J = H, F$, and regulatory standards in each country that induce firms to design products closer to the ideal in their destination markets compared to their profit-maximizing design choices.*

Notice an interesting implication of Proposition 6 for the efficient standards, which follows from the ranking of efficient product characteristics, namely $\hat{a}^H > a_H^{HE} > a_F^{HE}$ and $a_H^{FE} > a_F^{FE} > \hat{a}^F$. We record this observation in

Corollary 2 *When $\xi < 1$, efficient regulatory standards require native producers to supply goods tailored more closely to local tastes than what is required of offshore producers.*

This feature of efficient regulation may seem surprising, but it has a natural interpretation in our context. It simply reflects the more favorable benefit-to-cost ratio that results from moving local brands closer to the local ideal as compared to that for imported brands, in view of the greater market potential that firms enjoy in their local markets in the presence of shipping costs. In other words, the feature of efficient regulation highlighted in Corollary 2 is not about treating locally produced brands differently than imported brands, but rather about easing the regulatory burden imposed on "small firms," i.e., firms with small sales in the market, which in a world with home-market effects applies to firms located abroad. We emphasize, however, that the more lenient treatment of imports with respect to product standards must be coupled with additional taxes (in the form of positive net trade taxes) that shift demand away from these goods inasmuch as they impose the greatest consumption externalities.

4.2 Benchmark: An OTA with Mutual Recognition when $\xi < 1$

In Section 3.4, we demonstrated that in the absence of consumption externalities, global efficiency can be achieved under an OTA without the need for detailed international rules on product standards, provided that each government can set (at least) two standards subject to the principle of mutual recognition. In this section, we revisit the same question, asking whether an OTA with mutual recognition can generate the globally efficient outcome when consumption externalities are present. In keeping with Costinot (2008), we will answer the question in the negative.

Recall that when $\xi = 1$ and an OTA allows each country to announce two standards subject to mutual recognition, each government selects as its two standards one that is profit maximizing for its firms' local sales and the other that is profit maximizing for its firms' export sales. Each country selects these standards, because its own incentives are aligned with those of its firms. If a country chooses the profit-maximizing standards for its own firms, those firms have no reason to select any other option than the one intended for them, even though they have the freedom under mutual recognition to choose any of the four standards available in the world. And by choosing product characteristics for each market to maximize their profits, each country's firms make choices that minimize the country's industry-level price index.

When consumption choices confer externalities, as they do with $\xi < 1$, the firms' profit-maximizing choices of product attributes no longer correspond to the efficient standards, and this changes everything. To see why, suppose we start with the efficient standards, and ask whether any firm or government has an incentive to deviate. There are two problems that now arise. First, since none of these standards are set at profit-maximizing levels, firms may not self select into the standard that would be efficient for them, and will not do so if there is a better option for them among the four efficient standards from which they can choose (a possibility that is more likely when the externality is large and the efficient standards are far from their profit-maximizing levels). Putting this problem to the side, let us suppose that each firm in J prefers to sell in market J' a good with characteristic $a_J^{J'E}$ than one with any of the three other elements of $\boldsymbol{a}^E$ that it might chose for this market. Now consider the incentives facing the home government. Instead of setting the efficient standard a_H^{FE} for its firms' export sales, suppose it were to announce a standard slightly closer to the one that would maximize its firms' profits given the other three standards in place. Such a (small) change in standard presumably would not induce any foreign firm to select a different standard to obey in either market, nor would the home firms elect to sell at home something different from a_H^{HE}. With this deviation, the home country would gain from delocation but would bear none of the cost associated with the externality-generated product inflicted on foreign consumers.

Hence, when $\xi < 1$ and each government is permitted to set two or more standards, an OTA with mutual recognition cannot deliver the efficient outcome. And of course, allowing only one standard to be chosen under mutual recognition cannot possibly achieve efficiency given that efficiency entails four separate standards. We may conclude that the effectiveness of mutual standards in an OTA for achieving efficiency is limited to situations in which their are no important externalities that are motivating the market regulation.

5 Conclusions

Old trade agreements cover traditional protectionist instruments, such as tariffs and quotas. New trade agreements extend international cooperation to a broader set of policy instruments, including domestic regulations and product standards. In this paper, we have studied the need for NTAs in an environment with horizontal product differentiation and cross-countries differences in consumer assessments of the ideal product attributes. We first characterized the optimal NTA in a setting where consumption choices affect only the consumer herself and later introduced the possibility that consumers care about products purchased by their fellow nationals. We also asked whether an OTA with national treatment of product standards or with mutual recognition of product standards could replicate the globally efficient outcome that results from international cooperation on regulation.

When individual consumption choices do not confer any local externalities, the optimal NTA in a familiar setting of monopolistic competition with an outside good dictates zero net tariffs on all differentiated products and standards that deliver the same product characteristics as those that maximize firms' profits for their home and export sales. Alternatively, the optimal NTA can provide for zero net tariffs and an absence of regulation in both countries. Without an international agreement to refrain from regulation, governments have incentives to impose onerous standards on foreign firms in an attempt to induce delocation. An OTA with national treatment cannot achieve the first best, because the governments lack unilateral incentives to offer foreign firms the opportunity to produce the profit-maximizing varieties for their export sales. An OTA with mutual recognition can replicate the optimal NTA, provided that governments can announce multiple standards and that exporting firms can invoke the clause even for variants of their brand that they do not sell at home.

In the presence of consumption externalities—even ones that do not cross international borders—the requirements for cooperation are more severe. In the absence of regulation, consumers over-consume the goods that are far from the national ideal and under-consume brands that are closer to the ideal. In the face of these demands, firms design products that are further from the ideal in destination markets than is socially optimal. The optimal NTA combines positive net tariffs that switch demands from import goods to local goods that are closer to the country's ideal with product standards that force all firms to deviate less from these ideals despite the extra fixed costs of doing so. In this setting, neither national treatment nor mutual recognition suffices for an OTA that leaves governments with sovereignty over local regulations to achieve a globally efficient outcome.

We have identified some examples of goods that might be subject to horizontal product regulation. But vertical regulations also are prevalent: governments have good reason to regulate pollution, product safety, and other aspects of product quality, including (or perhaps especially) in service sectors. We aim to characterize the optimal NTA in settings with vertical product differentiation in a future, companion paper.

References

[1] Bagwell, Kyle and Staiger, Robert W., 2001, "Domestic Policies, National Sovereignty, and International Economic Institutions," *Quarterly Journal of Economics* 116, 519-562.

[2] Bagwell, Kyle and Staiger, Robert W., 2002, *The Economics of the World Trading System*, Cambridge: The MIT Press.

[3] Bagwell, Kyle and Staiger, Robert W., 2015, "Delocation and Trade Agreements in Imperfectly Competitive Markets," *Research in Economics* 69, 132-156.

[4] Bown, Chad P. and Irwin, Douglas A., 2015, "The GATT's Starting Point: Tariff Levels circa 1947," NBER Working Paper 21782, Cambridge MA.

[5] Campolmi, Alessia, Fadinger, Harald, and Forlati, Chiata, 2018, "Trade and Domestic Policies in Models with Monopolistic Competition," http://fadinger.vwl.uni-mannheim.de/Research_files/CFF_2018_1004.pdf.

[6] Costinot, Arnaud, 2008, "A Comparative Institutional Analysis of Agreements on Product Standards," *Journal of International Economics* 75, 197-213.

[7] Dixit, Avinash and Stiglitz, Joseph E., 1977, "Monopolistic Competition and Optimal Product Diversity," *American Economic Review* 67, 297-308.

[8] Grossman, Gene M., 2017, "The Purpose of Trade Agreements," ch. 7 in K. Bagwell and R.W. Staiger, *The Handbook of Commercial Trade Policy*, Amsterdam: North Holland.

[9] Helpman, Elhanan and Krugman, Paul R., 1989, *Trade Policy and Market Structure*, Cambridge: The MIT Press.

[10] Krugman, Paul R., 1980, "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review* 70, 950-959.

[11] Lamy, Pascal, 2015, "The New World of Trade," Jan Tumlir Lecture, https://pascallamy.eu/2015/03/09/colloque-le-bien-commun-futur-paradigme-de-la-gouvernance-de-mers/.

[12] Lamy, Pascal, 2016, "The Changing Landscape of International Trade," The Frank D. Graham Lecture, https://pascallamyeu.files.wordpress.com/2017/02/2016-04-07-lamy-princeton-graham-lecture-final.pdf.

[13] Ortino, Matteo, 2007, "The Role and Functioning of Mutual Recognition in the European Market of Financial Services," *The International and Comparative Law Quarterly* 56, 309-338.

[14] Ossa, Ralph, 2011, "A 'New Trade' Theory of GATT/WTO Negotiations," *Journal of Political Economy* 119, 122-152.

[15] Podhorsky, Andrea, 2013, "Certification Programs and North-South Trade," *Journal of Public Economics* 108, 90-104.

[16] Sykes, Alan O., 1999a, "The (Limited) Role of Regulatory Harmonization in International Goods and Services Markets," *Journal of International Economic Law* 2, 49-70.

[17] Sykes, Alan O., 1999b, "Regulatory Protectionism and the Law of International Trade," *The University of Chicago Law Review* 66, 1-46.

[18] Sykes, Alan O., 2000, "Regulatory Competition or Regulatory Harmonizaton? A Silly Question?," *Journal of International Economic Law* 3, 257-264.

[19] Venables, Anthony, 1987, "Trade and Trade Policy with Differentiated Products: A Chamberlinian-Ricardian Model," *Economic Journal* 97, 700-717.

6 Appendix

In this Appendix we provide proofs of all claims not established in the body of the paper.

6.1 Derivation of Demands

Here we derive an explicit expression for the domestic industry-level price index P that enters (2) and (3), and the utility-maximizing domestic consumption levels c_i^H for each brand i given in (5) The derivation for the analogous foreign magnitudes is similar.

As in the body of the paper, for ease of notation, we define

$$A_i^H \equiv A - \xi \left(a_i^H - \hat{a}^H\right)^2; \quad \hat{A}_i^H \equiv A - \left(a_i^H - \hat{a}^H\right)^2,$$

and hence by (1) and (4) utility is given by

$$U^H = 1 + C_Y^H + \log\left(\left\{\sum_{i\in\Theta^H} A_i^H \left(c_i^H\right)^\beta - (1-\xi)\left(a_i^H - \hat{a}^H\right)^2 \left(c_{i\mu}^H\right)^\beta\right\}^{\frac{1}{\beta}}\right).$$

The first-order conditions for the utility-maximizing choice of c_i^H imply

$$(C_D^H)^{-\beta} A_i^H (c_i^H)^\beta = p_i^H c_i^H.$$

Summing over i yields

$$(C_D^H)^{-\beta} \sum_i A_i^H (c_i^H)^\beta = \sum_i p_i^H c_i^H.$$

We define P so that

$$P^H C_D^H = \sum_i p_i^H c_i^H.$$

Then

$$P^H = (C_D^H)^{-\beta-1} \sum_i A_i^H (c_i^H)^\beta.$$

Also, from the first-order conditions,

$$\begin{aligned}
c_i^H &= (p_i^H)^{\frac{1}{\beta-1}} (A_i^H)^{\frac{-1}{\beta-1}} (C_D^H)^{\frac{\beta}{\beta-1}} \\
(c_i^H)^\beta &= (p_i^H)^{\frac{\beta}{\beta-1}} (A_i^H)^{\frac{-\beta}{\beta-1}} (C_D^H)^{\frac{\beta\beta}{\beta-1}} \\
A_i^H (c_i^H)^\beta &= (p_i^H)^{\frac{\beta}{\beta-1}} (A_i^H)^{\frac{-1}{\beta-1}} (C_D^H)^{\frac{\beta\beta}{\beta-1}}.
\end{aligned}$$

Hence we have

$$P^H = (C_D^H)^{-\beta-1} (p_i^H)^{\frac{\beta}{\beta-1}} (A_i^H)^{\frac{-1}{\beta-1}} (C_D^H)^{\frac{\beta\beta}{\beta-1}} = (C_D^H)^{\frac{1}{\beta-1}} (p_i^H)^{\frac{\beta}{\beta-1}} (A_i^H)^{\frac{-1}{\beta-1}}.$$

Note that with $c_i = c_{i\mu}^H$ we can write

$$C_D^H = \left[\sum_i \hat{A}_i^H \left(c_i^H\right)^\beta\right]^{\frac{1}{\beta}} = (C_D^H)^{\frac{\beta}{\beta-1}} \left[\sum_i \hat{A}_i^H (p_i^H)^{\frac{\beta}{\beta-1}} (A_i^H)^{\frac{-\beta}{\beta-1}}\right]^{\frac{1}{\beta}},$$

and therefore

$$(C_D^H)^{\frac{-1}{\beta-1}} = \left[\sum_i \hat{A}_i^H (p_i^H)^{\frac{\beta}{\beta-1}} (A_i^H)^{\frac{-\beta}{\beta-1}}\right]^{\frac{1}{\beta}},$$

which implies

$$C_D^H = \left[\sum_i \hat{A}_i^H (p_i^H)^{\frac{\beta}{\beta-1}} (A_i^H)^{\frac{-\beta}{\beta-1}}\right]^{\frac{-(\beta-1)}{\beta}}.$$

Substituting yields

$$P^H = \left[\sum_i \hat{A}_i^H (p_i^H)^{\frac{\beta}{\beta-1}} (A_i^H)^{\frac{-\beta}{\beta-1}}\right]^{\frac{-1}{\beta}} (p_i^H)^{\frac{\beta}{\beta-1}} (A_i^H)^{\frac{-1}{\beta-1}}$$

or finally using $\sigma \equiv \frac{1}{1-\beta}$

$$P^H = \frac{\sum_i (p_i^H)^{1-\sigma} (A_i^H)^\sigma}{\left[\sum_i \hat{A}_i^H (p_i^H)^{1-\sigma} (A_i^H)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}}. \tag{A1}$$

Using the relationship between in P^H and $\mathcal{P}^H$ implied by (7), it is then straightforward to derive the expression for $\mathcal{P}^H$ implied by (6)

$$\mathcal{P}^H \equiv \left[\sum_i \left(A_i^H\right)^\sigma \left(p_i^H\right)^{1-\sigma}\right]^{-\frac{1}{\sigma-1}}$$

To derive the accompanying expression for c_i^H, we note that the first-order conditions for two distinct differentiated goods i and i' imply

$$\frac{A_i^H (c_i^H)^{\beta-1}}{A_{i'}^H (c_{i'}^H)^{\beta-1}} = \frac{p_i^H}{p_{i'}^H}$$

and hence

$$p_i^H c_i^H = (A_{i'}^H)^{\frac{1}{\beta-1}} (p_{i'}^H)^{\frac{1}{1-\beta}} c_{i'}^H (A_i^H)^{\frac{1}{1-\beta}} (p_i^H)^{\frac{\beta}{\beta-1}}.$$

Summing over i and using $\sigma \equiv \frac{1}{1-\beta}$ yields

$$\sum_i p_i^H c_i^H = (A_{i'}^H)^{-\sigma} (p_{i'}^H)^\sigma c_{i'}^H \times \left[\sum_i (A_i^H)^\sigma (p_i^H)^{1-\sigma}\right].$$

Using $P^H C_D^H = \sum_i p_i^H c_i^H$ and plugging $C_D^H = (P^H)^{-1}$ into this expression yields

$$c_i^H = (A_i^H)^\sigma (p_i^H)^{-\sigma} \frac{1}{\sum_i (A_i^H)^\sigma (p_i^H)^{1-\sigma}} = (A_i^H)^\sigma (p_i^H)^{-\sigma} (\mathcal{P}^H)^{\sigma-1}.$$

6.2 Proof of Lemma 1

Lemma 1 *Let trade taxes and consumption subsidies take any values such that $\iota^H > 1$ and $\iota^F > 1$ and consider the unregulated equilibrium with the profit-maximizing choices of characteristics, a. Beginning at this equilibrium, a small increase in the product characteristic of any firm for any market induces exit by home firms and entry by foreign firms ($dn^H/da^J_{J'} < 0$ and $dn^F/da^J_{J'} > 0$ for all $J \in \{H, F\}$ and $J' \in \{H, F\}$).*

Proof To prove Lemma 1, we make use of the zero-profit conditions

$$N^J c_J^J(a_J^J, \mathcal{P}^J\left(\boldsymbol{n}, a_H^J, a_F^J\right)) + (1+\phi)\, N^{\tilde{J}} c_J^{\tilde{J}}(a_J^{\tilde{J}}, \mathcal{P}^{\tilde{J}}\left(\boldsymbol{n}, a_H^{\tilde{J}}, a_F^{\tilde{J}}\right)) = \frac{K + \kappa\left(a_J^H - a_J^F\right)^2}{q-\lambda},\ J = H, F.$$

We prove the claims of Lemma 1 for standards in the home country market, with the proof for standards in the foreign country market proceeding in an analogous fashion.

6.2.1 $\frac{dn^H}{da_H^H} < 0$ and $\frac{dn^F}{da_H^H} > 0$

Totally differentiating the zero profit conditions with respect to n^H, n^F and a_H^H yields

$$N^H \left[\frac{\partial c_H^H}{\partial a_H^H} da_H^H + \frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial a_H^H} da_H^H + \frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^H} dn^H + \frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^F} dn^F\right]$$
$$+ (1+\phi) N^F \left[\frac{\partial c_H^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^H} dn^H + \frac{\partial c_H^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^F} dn^F\right] = \left[\frac{2\kappa(a_H^H - a_H^F)}{q-\lambda}\right] da_H^H \quad (28)$$

$$N^F \left[\frac{\partial c_F^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^H} dn^H + \frac{\partial c_F^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^F} dn^F\right]$$
$$+ (1+\phi) N^H \left[\frac{\partial c_F^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial a_H^H} da_H^H + \frac{\partial c_F^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^H} dn^H + \frac{\partial c_F^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^F} dn^F\right] = 0. \quad (29)$$

But the home firm chooses a_H^H to satisfy the first-order condition for profit maximization

$$\frac{\partial \pi_H^H}{\partial a_H^H} = (q-\lambda) N^H \frac{\partial c_H^H}{\partial a_H^H} - 2\kappa(a_H^H - a_H^F) = 0,$$

which we may substitute into (28) to arrive at the home and foreign totally differentiated zero-profit conditions evaluated at the profit-maximizing choices:

$$N^H\left[\frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial a_H^H}da_H^H+\frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^H}dn^H+\frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^F}dn^F\right]+(1+\phi)N^F\left[\frac{\partial c_H^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^H}dn^H+\frac{\partial c_H^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^F}dn^F\right]=0 \tag{30}$$

$$N^F\left[\frac{\partial c_F^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^H}dn^H+\frac{\partial c_F^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^F}dn^F\right]+(1+\phi)N^H\left[\frac{\partial c_F^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial a_H^H}da_H^H+\frac{\partial c_F^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^H}dn^H+\frac{\partial c_F^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^F}dn^F\right]=0. \tag{31}$$

Solving (31) for dn^F, substituting into (30) and simplifying yields

$$\frac{dn^H}{da_H^H}=\left[\frac{-\frac{\partial \mathcal{P}^H}{\partial a_H^H}\frac{\partial \mathcal{P}^F}{\partial n^F}}{\left[\frac{\partial \mathcal{P}^H}{\partial n^H}\frac{\partial \mathcal{P}^F}{\partial n^F}-\frac{\partial \mathcal{P}^H}{\partial n^F}\frac{\partial \mathcal{P}^F}{\partial n^H}\right]}\right]. \tag{32}$$

The denominator of the expression in (32) is strictly positive provided for $\iota^H>1$ and $\iota^F>1$ (a condition stated in the lemma), while the term in the numerator is composed of the product of two negative terms and hence is positive as well. Hence, $\frac{dn^H}{da_H^H}<0$ as claimed in Lemma 1.

To establish that $\frac{dn^F}{da_H^H}>0$, we solve (31) for dn^H and substitute the resulting expression into (30) and simplify to arrive at

$$\frac{dn^F}{da_H^H}=\left[\frac{\frac{\partial \mathcal{P}^H}{\partial a_H^H}\frac{\partial \mathcal{P}^F}{\partial n^H}}{\left[\frac{\partial \mathcal{P}^H}{\partial n^H}\frac{\partial \mathcal{P}^F}{\partial n^F}-\frac{\partial \mathcal{P}^H}{\partial n^F}\frac{\partial \mathcal{P}^F}{\partial n^H}\right]}\right] \tag{33}$$

which is positive.

6.2.2 $\frac{dn^H}{da_F^H}<0$ and $\frac{dn^F}{da_F^H}>0$

Totally differentiating the zero profit conditions with respect to n^H, n^F and a_F^H yields

$$N^H\left[\frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial a_F^H}da_F^H+\frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^H}dn^H+\frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^F}dn^F\right]$$
$$+(1+\phi)N^F\left[\frac{\partial c_H^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^H}dn^H+\frac{\partial c_H^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^F}dn^F\right]=0 \tag{34}$$

$$N^F\left[\frac{\partial c_F^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^H}dn^H+\frac{\partial c_F^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^F}dn^F\right]$$
$$+(1+\phi)N^H\left[\frac{\partial c_F^H}{\partial a_F^H}da_F^H+\frac{\partial c_F^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial a_F^H}da_F^H+\frac{\partial c_F^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^H}dn^H+\frac{\partial c_F^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^F}dn^F\right]=\left[\frac{2\kappa(a_F^H-a_F^F)}{q-\lambda}\right]da_F^H. \tag{35}$$

But the foreign firm chooses a_F^H to satisfy the first-order condition for profit maximization

$$\frac{\partial \pi_F^H}{\partial a_F^H} = (q-\lambda)(1+\phi)N^H \frac{\partial c_F^H}{\partial a_F^H} - 2\kappa(a_F^H - a_F^F) = 0,$$

which we may substitute into (35) to arrive at the home and foreign totally differentiated zero-profit conditions evaluated at the profit-maximizing choices:

$$N^H \left[\frac{\partial c_H^H}{\partial \mathcal{P}^H} \frac{\partial \mathcal{P}^H}{\partial a_F^H} da_F^H + \frac{\partial c_H^H}{\partial \mathcal{P}^H} \frac{\partial \mathcal{P}^H}{\partial n^H} dn^H + \frac{\partial c_H^H}{\partial \mathcal{P}^H} \frac{\partial \mathcal{P}^H}{\partial n^F} dn^F \right] + (1+\phi) N^F \left[\frac{\partial c_H^F}{\partial \mathcal{P}^F} \frac{\partial \mathcal{P}^F}{\partial n^H} dn^H + \frac{\partial c_H^F}{\partial \mathcal{P}^F} \frac{\partial \mathcal{P}^F}{\partial n^F} dn^F \right] = 0 \tag{36}$$

$$N^F \left[\frac{\partial c_F^F}{\partial \mathcal{P}^F} \frac{\partial \mathcal{P}^F}{\partial n^H} dn^H + \frac{\partial c_F^F}{\partial \mathcal{P}^F} \frac{\partial \mathcal{P}^F}{\partial n^F} dn^F \right] + (1+\phi) N^H \left[\frac{\partial c_F^H}{\partial \mathcal{P}^H} \frac{\partial \mathcal{P}^H}{\partial a_F^H} da_F^H + \frac{\partial c_F^H}{\partial \mathcal{P}^H} \frac{\partial \mathcal{P}^H}{\partial n^H} dn^H + \frac{\partial c_F^H}{\partial \mathcal{P}^H} \frac{\partial \mathcal{P}^H}{\partial n^F} dn^F \right] = 0. \tag{37}$$

Solving (36) for dn^F, substituting into (37) and simplifying yields

$$\frac{dn^H}{da_F^H} = \left[\frac{-\frac{\partial \mathcal{P}^H}{\partial a_F^H} \frac{\partial \mathcal{P}^F}{\partial n^F}}{\left[\frac{\partial \mathcal{P}^H}{\partial n^H} \frac{\partial \mathcal{P}^F}{\partial n^F} - \frac{\partial \mathcal{P}^H}{\partial n^F} \frac{\partial \mathcal{P}^F}{\partial n^H} \right]} \right]. \tag{38}$$

As before, the denominator of the expression in (38) is strictly positive provided for $\iota^H > 1$ and $\iota^F > 1$, while the term in the numerator is composed of the product of two negative terms and hence is positive as well. Hence, $\frac{dn^H}{da_F^H} < 0$ as claimed in Lemma 1.

To establish that $\frac{dn^F}{da_F^H} > 0$, we solve (36) for dn^H and substitute the resulting expression into (37) and simplify to arrive at

$$\frac{dn^F}{da_F^H} = \left[\frac{\frac{\partial \mathcal{P}^H}{\partial a_F^H} \frac{\partial \mathcal{P}^F}{\partial n^H}}{\left[\frac{\partial \mathcal{P}^H}{\partial n^H} \frac{\partial \mathcal{P}^F}{\partial n^F} - \frac{\partial \mathcal{P}^H}{\partial n^F} \frac{\partial \mathcal{P}^F}{\partial n^H} \right]} \right] \tag{39}$$

which is positive.
QED

6.3 Proof of Lemma 2

Lemma 2 *Let trade taxes and consumption subsidies take any values such that $\iota^H > 1$ and $\iota^F > 1$ and consider the unregulated equilibrium with the profit-maximizing choices of characteristics, a. Beginning at this equilibrium, a small change in any product characteristic $a_{J'}^J$ has no first-order effect on the home brand-level price index $\left(d\mathcal{P}^H/da_{J'}^J = 0\right)$ or on the foreign brand-level price index $\left(d\mathcal{P}^F/da_{J'}^J = 0\right)$.*

Proof The proof follows from the derivative expressions in the proof of Lemma 1. In general, the eight derivatives boil down to the following two calculations that need to be performed for all

$J \in \{H, F\}$ and $J' \in \{H, F\}$, where D^J is an indicator variable that is equal to 1 for $J = H$ and equal to -1 for $J = F$:

$$\begin{aligned}
\frac{d\mathcal{P}^J}{da^J_{J'}} &= \frac{\partial\mathcal{P}^J}{\partial a^J_{J'}} + \frac{\partial\mathcal{P}^J}{\partial n^H}\frac{dn^H}{da^J_{J'}} + \frac{\partial\mathcal{P}^J}{\partial n^F}\frac{dn^F}{da^J_{J'}} \\
&= \frac{\partial\mathcal{P}^J}{\partial a^J_{J'}} + \frac{\partial\mathcal{P}^J}{\partial n^H}\left(\frac{-D^J \times \frac{\partial\mathcal{P}^J}{\partial a^J_{J'}}\frac{\partial\mathcal{P}^{\tilde{J}}}{\partial n^F}}{\left[\frac{\partial\mathcal{P}^H}{\partial n^H}\frac{\partial\mathcal{P}^F}{\partial n^F} - \frac{\partial\mathcal{P}^H}{\partial n^F}\frac{\partial\mathcal{P}^F}{\partial n^H}\right]}\right) + \frac{\partial\mathcal{P}^J}{\partial n^F}\left(\frac{D^J \times \frac{\partial\mathcal{P}^J}{\partial a^J_{J'}}\frac{\partial\mathcal{P}^{\tilde{J}}}{\partial n^H}}{\left[\frac{\partial\mathcal{P}^H}{\partial n^H}\frac{\partial\mathcal{P}^F}{\partial n^F} - \frac{\partial\mathcal{P}^H}{\partial n^F}\frac{\partial\mathcal{P}^F}{\partial n^H}\right]}\right) \\
&= \frac{\partial\mathcal{P}^J}{\partial a^J_{J'}}\left(1 - \frac{\left[\frac{\partial\mathcal{P}^H}{\partial n^H}\frac{\partial\mathcal{P}^F}{\partial n^F} - \frac{\partial\mathcal{P}^H}{\partial n^F}\frac{\partial\mathcal{P}^F}{\partial n^H}\right]}{\left[\frac{\partial\mathcal{P}^H}{\partial n^H}\frac{\partial\mathcal{P}^F}{\partial n^F} - \frac{\partial\mathcal{P}^H}{\partial n^F}\frac{\partial\mathcal{P}^F}{\partial n^H}\right]}\right) = 0
\end{aligned} \tag{40}$$

$$\begin{aligned}
\frac{d\mathcal{P}^{\tilde{J}}}{da^J_{J'}} &= \frac{\partial\mathcal{P}^{\tilde{J}}}{\partial n^H}\frac{dn^H}{da^J_{J'}} + \frac{\partial\mathcal{P}^{\tilde{J}}}{\partial n^F}\frac{dn^F}{da^J_{J'}} \\
&= \frac{\partial\mathcal{P}^{\tilde{J}}}{\partial n^H}\left(\frac{-D^J \times \frac{\partial\mathcal{P}^J}{\partial a^J_{J'}}\frac{\partial\mathcal{P}^{\tilde{J}}}{\partial n^F}}{\left[\frac{\partial\mathcal{P}^H}{\partial n^H}\frac{\partial\mathcal{P}^F}{\partial n^F} - \frac{\partial\mathcal{P}^H}{\partial n^F}\frac{\partial\mathcal{P}^F}{\partial n^H}\right]}\right) + \frac{\partial\mathcal{P}^{\tilde{J}}}{\partial n^F}\left(\frac{D^J \times \frac{\partial\mathcal{P}^J}{\partial a^J_{J'}}\frac{\partial\mathcal{P}^{\tilde{J}}}{\partial n^H}}{\left[\frac{\partial\mathcal{P}^H}{\partial n^H}\frac{\partial\mathcal{P}^F}{\partial n^F} - \frac{\partial\mathcal{P}^H}{\partial n^F}\frac{\partial\mathcal{P}^F}{\partial n^H}\right]}\right) \\
&= \frac{\partial\mathcal{P}^J}{\partial a^J_{J'}}\left(\frac{\left[\frac{\partial\mathcal{P}^{\tilde{J}}}{\partial n^F}\frac{\partial\mathcal{P}^{\tilde{J}}}{\partial n^H} - \frac{\partial\mathcal{P}^{\tilde{J}}}{\partial n^F}\frac{\partial\mathcal{P}^{\tilde{J}}}{\partial n^H}\right]}{\left[\frac{\partial\mathcal{P}^H}{\partial n^H}\frac{\partial\mathcal{P}^F}{\partial n^F} - \frac{\partial\mathcal{P}^H}{\partial n^F}\frac{\partial\mathcal{P}^F}{\partial n^H}\right]}\right) = 0.
\end{aligned} \tag{41}$$

QED

6.4 Proof of Proposition 1

Proposition 1 *Suppose $\xi = 1$ and let $\widetilde{\boldsymbol{a}}$ be the vector of product characteristics that result from profit-maximizing design choices in an unregulated equilibrium when $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$. Then the maximum world welfare is achieved in a monopolistically-competitive equilibrium when $z^H = z^F = 0$, $s^H = s^F = 1/\sigma$, and $\bar{a} = \widetilde{\boldsymbol{a}}$.*

Proof We begin with the expression for world welfare when $\xi = 1$:

$$\Omega = \sum_J L^J - N^H \log(P^H) - N^F \log(P^F) + qz^H n^F N^H c^H_F + qz^F n^H N^F c^F_H - N^H \frac{s^H}{1 - s^H} - N^F \frac{s^F}{1 - s^F}.$$

We first prove that global efficiency requires $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$. We then turn to the efficiency of $\bar{a} = \widetilde{\boldsymbol{a}}$.

Evaluating the derivatives of Ω with respect to net trade taxes and consumption subsidies at the levels $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$ yields

$$\frac{d\Omega}{dz^H}|_{z^H = z^F = 0,\ s^H = s^F = 1/\sigma} = -\frac{N^H}{P^H}\frac{dP^H}{dz^H} - \frac{N^F}{P^F}\frac{dP^F}{dz^H} + qn^F N^H c^H_F$$

$$\frac{d\Omega}{dz^F}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma} = -\frac{N^H}{P^H}\frac{dP^H}{dz^F} - \frac{N^F}{P^F}\frac{dP^F}{dz^F} + qn^H N^F c_H^F$$
$$\frac{d\Omega}{ds^H}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma} = -\frac{N^H}{P^H}\frac{dP^H}{ds^H} - \frac{N^F}{P^F}\frac{dP^F}{ds^H} - N^H\left(\frac{\sigma}{\sigma-1}\right)^2$$
$$\frac{d\Omega}{ds^F}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma} = -\frac{N^H}{P^H}\frac{dP^H}{ds^F} - \frac{N^F}{P^F}\frac{dP^F}{ds^F} - N^F\left(\frac{\sigma}{\sigma-1}\right)^2.$$

To establish that $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$ are efficient when $\xi = 1$, we show that $\frac{d\Omega}{dz^H}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma} = 0$ and $\frac{d\Omega}{ds^F}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma} = 0$, with $\frac{d\Omega}{dz^F}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma} = 0$ and $\frac{d\Omega}{ds^F}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma} = 0$ then following under analogous arguments.[30]

Efficient net trade taxes $z^H = z^F = 0$ We first show that $\frac{d\Omega}{dz^H}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma} = 0$, noting that p_H^H, p_H^F and p_F^F are independent of z^H with z^H impacting directly only the price of the Foreign brand in the Home market p_F^H. As noted in the text, total per capita spending on differentiated goods equals one when $P = \mathcal{P}$ (as is the case for $\xi = 1$), and so we have

$$n^H p_H^H c_H^H + n^F p_F^H c_F^H = 1;\quad n^H p_H^F c_H^F + n^F p_F^F c_F^F = 1. \tag{42}$$

Using (42) we can then write

$$\begin{aligned} -\frac{N^H}{P^H}\frac{dP^H}{dz^H} &= \left(\frac{1}{\sigma-1}\right)N^H\left[p_H^H c_H^H\frac{dn^H}{dz^H} + p_F^H c_F^H\frac{dn^F}{dz^H}\right] - \left(\frac{\sigma-1}{\sigma}\right)qn^F N^H c_F^H \\ -\frac{N^F}{P^F}\frac{dP^F}{dz^H} &= \left(\frac{1}{\sigma-1}\right)N^F\left[p_H^F c_H^F\frac{dn^H}{dz^H} + p_F^F c_F^F\frac{dn^F}{dz^H}\right], \end{aligned}$$

and therefore

$$\frac{d\Omega}{dz^H}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma} = \left(\frac{1}{\sigma-1}\right)\left[[p_H^H N^H c_H^H + p_H^F N^F c_H^F]\frac{dn^H}{dz^H} + [p_F^F N^F c_F^F + p_F^H N^H c_F^H]\frac{dn^F}{dz^H}\right] + \frac{q}{\sigma}n^F N^H c_F^H.$$

When $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$ we also have

$$\begin{aligned} p_H^H &= \left(\frac{\sigma-1}{\sigma}\right)q = p_F^F \\ p_F^H &= \left(\frac{\sigma-1}{\sigma}\right)q(1+\phi) = p_H^F, \end{aligned} \tag{43}$$

and therefore

$$\frac{d\Omega}{dz^H}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma} = \left(\frac{q}{\sigma}\right)\left[[N^H c_H^H + (1+\phi)N^F c_H^F]\frac{dn^H}{dz^H} + [N^F c_F^F + (1+\phi)N^H c_F^H]\frac{dn^F}{dz^H} + n^F N^H c_F^H\right]. \tag{44}$$

[30] Note to discussants: Recall footnotes 17 and 19; we have not fully nailed down the SOC as yet.

Our goal is to show that the right-hand side of (44) is equal to zero. Evidently, as (44) makes clear, this will be true if, beginning from $z^H = z^F = 0$ and $s^H = s^F = 1/\sigma$, a small increase in the net tariff on Home imports generates additional tariff revenue (in the amount $n^F N^H c_F^H$) that is just offset by the loss in differentiated goods production associated with the induced entry and exit (in the amount $[N^H c_H^H + (1+\phi) N^F c_H^F] \frac{dn^H}{dz^H} + [N^F c_F^F + (1+\phi) N^H c_F^H] \frac{dn^F}{dz^H}$).

To derive expressions for $\frac{dn^H}{dz^H}$ and $\frac{dn^F}{dz^H}$, we use the Home and Foreign zero-profit conditions

$$N^H c_H^H(P^H(z^H, n^H, n^F)) + (1+\phi)\, N^F c_H^F(P^F(n^H, n^F)) = \frac{K + \xi\left(a_H^H - a_H^F\right)^2}{q - \lambda} \tag{45}$$

$$N^F c_F^F(P^F(n^H, n^F)) + (1+\phi)\, N^H c_F^H(P_F^H(z^H), P^H(z^H, n^H, n^F)) = \frac{K + \xi\left(a_F^H - a_F^F\right)^2}{q - \lambda}, \tag{46}$$

where we have used that $P^J = \mathcal{P}^J$ for $\xi = 1$, and where we have suppressed the dependency of consumption and price indices on product characteristics and have made explicit the direct dependency of consumption, prices and price indices on z^H. Totally differentiating (45) and (46) yields

$$\frac{dn^H}{dz^H} = \frac{(1+\phi)\frac{dc_F^H}{dp_F^H}\frac{dp_F^H}{dz^H}\left[\left(\frac{N^H}{N^F}\right)\frac{dc_H^H}{dP^H}\frac{dP^H}{dn^F} + (1+\phi)\frac{dc_H^F}{dP^F}\frac{dP^F}{dn^F}\right] - \frac{\partial P^H}{\partial z^H}\frac{dP^F}{dn^F}\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F} - (1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right]}{\left[\frac{dP^H}{dn^H}\frac{dP^F}{dn^F} - \frac{dP^H}{dn^F}\frac{dP^F}{dn^H}\right]\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F} - (1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right]}, \tag{47}$$

and

$$\frac{dn^F}{dz^H} = \frac{-(1+\phi)\frac{dc_F^H}{dp_F^H}\frac{dp_F^H}{dz^H}\left[\left(\frac{N^H}{N^F}\right)\frac{dc_H^H}{dP^H}\frac{dP^H}{dn^H} + (1+\phi)\frac{dc_H^F}{dP^F}\frac{dP^F}{dn^H}\right] + \frac{\partial P^H}{\partial z^H}\frac{dP^F}{dn^H}\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F} - (1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right]}{\left[\frac{dP^H}{dn^H}\frac{dP^F}{dn^F} - \frac{dP^H}{dn^F}\frac{dP^F}{dn^H}\right]\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F} - (1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right]}. \tag{48}$$

Substituting (47) and (48) back into (44) and rearranging then yields

$$\frac{d\Omega}{dz^H}\Big|_{z^H = z^F = 0,\ s^H = s^F = 1/\sigma} = 0 \qquad \Leftrightarrow$$

$$\begin{aligned}
&[N^H c_H^H + (1+\phi) N^F c_H^F]\Big\{(1+\phi)\frac{dc_F^H}{dp_F^H}\frac{dp_F^H}{dz^H}\left[\left(\frac{N^H}{N^F}\right)\frac{dc_H^H}{dP^H}\frac{dP^H}{dn^F} + (1+\phi)\frac{dc_H^F}{dP^F}\frac{dP^F}{dn^F}\right] \\
&\qquad - \frac{\partial P^H}{\partial z^H}\frac{dP^F}{dn^F}\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F} - (1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right]\Big\} \\
&- [N^F c_F^F + (1+\phi) N^H c_F^H]\Big\{(1+\phi)\frac{dc_F^H}{dp_F^H}\frac{dp_F^H}{dz^H}\left[\left(\frac{N^H}{N^F}\right)\frac{dc_H^H}{dP^H}\frac{dP^H}{dn^H} + (1+\phi)\frac{dc_H^F}{dP^F}\frac{dP^F}{dn^H}\right] \\
&\qquad - \frac{\partial P^H}{\partial z^H}\frac{dP^F}{dn^H}\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F} - (1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right]\Big\} \\
&+ n^F N^H c_F^H\left[\frac{dP^H}{dn^H}\frac{dP^F}{dn^F} - \frac{dP^H}{dn^F}\frac{dP^F}{dn^H}\right]\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F} - (1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right] = 0.
\end{aligned}$$

We now make use of the following:

$$\begin{aligned}\frac{dc_H^H}{dP^H} &= (\sigma-1)\frac{c_H^H}{P^H};\ \frac{dc_H^F}{dP^F}=(\sigma-1)\frac{c_H^F}{P^F};\ \frac{dc_F^F}{dP^F}=(\sigma-1)\frac{c_F^F}{P^F};\\ \frac{dc_F^H}{dP^H} &= (\sigma-1)\frac{c_F^H}{P^H};\ \frac{dc_F^H}{dp_F^H}=-\sigma\frac{c_F^H}{p_F^H},\end{aligned}$$

and also

$$\begin{aligned}\frac{dP^H}{dn^F} &= \frac{1}{1-\sigma}P^H p_F^H c_F^H;\ \frac{dP^H}{dn^H}=\frac{1}{1-\sigma}P^H p_H^H c_H^H;\ \frac{dP^F}{dn^F}=\frac{1}{1-\sigma}P^F p_F^F c_F^F;\ \frac{dP^F}{dn^H}=\frac{1}{1-\sigma}P^F p_H^F c_H^F;\\ \frac{\partial P^H}{\partial z^H} &= P^H n^F c_F^H(\sigma-1)\frac{q}{\sigma};\ \frac{dp_F^H}{dz^H}=(\sigma-1)\frac{q}{\sigma}.\end{aligned}$$

With this the above can be simplified to

$$\frac{d\Omega}{dz^H}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma}=0 \qquad \Leftrightarrow$$

$$c_H^H[n^F q\frac{\sigma-1}{\sigma}c_F^F-1]-(1+\phi)\,c_H^F[n^F q\frac{\sigma-1}{\sigma}\,(1+\phi)\,c_F^H-1]=0.$$

But using (42) and (43) we then have

$$\begin{aligned}& c_H^H[n^F q\frac{\sigma-1}{\sigma}c_F^F-1]-(1+\phi)\,c_H^F[n^F q\frac{\sigma-1}{\sigma}\,(1+\phi)\,c_F^H-1]\\ =\ & -c_H^H n^H q\frac{\sigma-1}{\sigma}\,(1+\phi)\,c_H^F+(1+\phi)\,c_H^F n^H q\frac{\sigma-1}{\sigma}c_H^H\\ =\ & 0.\end{aligned}$$

This establishes that global efficiency requires $z^H=z^F=0$.

Efficient consumption subsidies $s^H=s^F=1/\sigma$ We next show that $\frac{d\Omega}{ds^F}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma}=0$, noting that p_H^F and p_F^F are independent of s^H with s^H impacting directly only the prices of the Home and the Foreign brand in the Home market, p_H^H and p_F^H. Again using (42) we can then write

$$\begin{aligned}-\frac{N^H}{P^H}\frac{dP^H}{ds^H} &= \left(\frac{1}{\sigma-1}\right)N^H\left[p_H^H c_H^H\frac{dn^H}{ds^H}+p_F^H c_F^H\frac{dn^F}{ds^H}\right]+qn^H N^H c_H^H+qn^F(1+\phi)N^H c_F^H\\ -\frac{N^F}{P^F}\frac{dP^F}{ds^H} &= \left(\frac{1}{\sigma-1}\right)N^F\left[p_H^F c_H^F\frac{dn^H}{ds^H}+p_F^F c_F^F\frac{dn^F}{ds^H}\right],\end{aligned}$$

and therefore

$$\begin{aligned}\frac{d\Omega}{ds^H}|_{z^H=0=z^F,\ s^H=\frac{1}{\sigma}=s^F} = \left(\frac{1}{\sigma-1}\right)&\left[[p_H^H N^H c_H^H+p_H^F N^F c_H^F]\frac{dn^H}{ds^H}+[p_F^F N^F c_F^F+p_F^H N^H c_F^H]\frac{dn^F}{ds^H}\right]\\ &+q[n^H N^H c_H^H+n^F(1+\phi)N^H c_F^H]-N^H\left(\frac{\sigma}{\sigma-1}\right)^2.\end{aligned}$$

Using (43) and (42) then delivers

$$\frac{d\Omega}{ds^H}|_{z^H=0=z^F,\ s^H=\frac{1}{\sigma}=s^F} = \left(\frac{q}{\sigma}\right)\left[[N^Hc_H^H+(1+\phi)N^Fc_H^F]\frac{dn^H}{ds^H}+[N^Fc_F^F+(1+\phi)N^Hc_F^H]\frac{dn^F}{ds^H}-\frac{1}{q}N^H\left(\frac{\sigma}{\sigma-1}\right)^2\right]. \tag{49}$$

Our goal is to show that the right-hand side of (49) is equal to zero.

To derive expressions for $\frac{dn^H}{ds^H}$ and $\frac{dn^F}{ds^H}$, we again use the Home and Foreign zero-profit conditions, which we now write as

$$N^Hc_H^H(p_H^H(s^H),P^H(s^H,n^H,n^F))+(1+\phi)\,N^Fc_H^F(P^F(n^H,n^F))=\frac{F+\kappa\left(a_H^H-a_H^F\right)^2}{q-\lambda} \tag{50}$$

$$N^Fc_F^F(P^F(n^H,n^F))+(1+\phi)\,N^Hc_F^H(P_F^H(s^H),P^H(s^H,n^H,n^F))=\frac{F+\kappa\left(a_F^H-a_F^F\right)^2}{q-\lambda}. \tag{51}$$

Totally differentiating (50) and (51) yields

$$\begin{aligned}\frac{dn^H}{ds^H} &= \frac{(1+\phi)\frac{dc_F^H}{dp_F^H}\frac{dp_F^H}{ds^H}\left[\left(\frac{N^H}{N^F}\right)\frac{dc_H^H}{dP^H}\frac{dP^H}{dn^F}+(1+\phi)\frac{dc_H^F}{dP^F}\frac{dP^F}{dn^F}\right]-\frac{\partial P^H}{\partial s^H}\frac{dP^F}{dn^F}\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F}-(1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right]}{\left[\frac{dP^H}{dn^H}\frac{dP^F}{dn^F}-\frac{dP^H}{dn^F}\frac{dP^F}{dn^H}\right]\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F}-(1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right]} \\ &\quad -\frac{\frac{dc_H^H}{dp_H^H}\frac{dp_H^H}{ds^H}\left[\frac{dc_F^F}{dP^F}\frac{dP^F}{dn^F}+(1+\phi)\left(\frac{N^H}{N^F}\right)\frac{dc_F^H}{dP^H}\frac{dP^H}{dn^F}\right]}{\left[\frac{dP^H}{dn^H}\frac{dP^F}{dn^F}-\frac{dP^H}{dn^F}\frac{dP^F}{dn^H}\right]\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F}-(1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right]},\end{aligned} \tag{52}$$

and

$$\begin{aligned}\frac{dn^F}{ds^H} &= \frac{-(1+\phi)\frac{dc_F^H}{dp_F^H}\frac{dp_F^H}{ds^H}\left[\left(\frac{N^H}{N^F}\right)\frac{dc_H^H}{dP^H}\frac{dP^H}{dn^H}+(1+\phi)\frac{dc_H^F}{dP^F}\frac{dP^F}{dn^H}\right]+\frac{\partial P^H}{\partial s^H}\frac{dP^F}{dn^H}\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F}-(1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right]}{\left[\frac{dP^H}{dn^H}\frac{dP^F}{dn^F}-\frac{dP^H}{dn^F}\frac{dP^F}{dn^H}\right]\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F}-(1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right]} \\ &\quad +\frac{\frac{dc_H^H}{dp_H^H}\frac{dp_H^H}{ds^H}\left[\frac{dc_F^F}{dP^F}\frac{dP^F}{dn^H}+(1+\phi)\left(\frac{N^H}{N^F}\right)\frac{dc_F^H}{dP^H}\frac{dP^H}{dn^H}\right]}{\left[\frac{dP^H}{dn^H}\frac{dP^F}{dn^F}-\frac{dP^H}{dn^F}\frac{dP^F}{dn^H}\right]\left[\frac{dc_H^H}{dP^H}\frac{dc_F^F}{dP^F}-(1+\phi)^2\frac{dc_H^F}{dP^F}\frac{dc_F^H}{dP^H}\right]}.\end{aligned} \tag{53}$$

Substituting (52) and (53) back into (49), using the price derivatives recorded above and in addition noting that

$$\frac{\partial P^H}{\partial s^H}=-P^H\frac{\sigma}{\sigma-1};\ \frac{dp_F^H}{ds^H}=-q(1+\phi);\ \frac{dp_H^H}{ds^H}=-q,$$

and using as well the expressions for efficient prices in (43), we then have

$$\frac{d\Omega}{ds^H}|_{z^H=z^F=0,\ s^H=s^F=1/\sigma}=0.$$

This establishes that global efficiency requires $s^H=s^F=1/\sigma$.

Efficient standards $\bar{a} = \widetilde{\boldsymbol{a}}$ We next prove that global efficiency is achieved when we also have $\bar{a} = \widetilde{\boldsymbol{a}}$. With net trade taxes and consumption subsidies set at their efficient levels $z^H = z^F = 0$ and $s^H = s^F = \frac{1}{\sigma}$, the expression for world welfare when $\xi = 1$ becomes

$$\Omega = \sum_J L^J - N^H \log(P^H) - N^F \log(P^F) - \frac{N^H + N^F}{\sigma - 1}. \tag{54}$$

The first-order conditions are

$$\frac{d\Omega}{da^J_{J'}} = -\frac{N^H}{P^H}\frac{dP^H}{da^J_{J'}} - \frac{N^F}{P^F}\frac{dP^F}{da^J_{J'}} = 0 \text{ for all } J \in \{H, F\} \text{ and } J' \in \{H, F\},$$

and by Lemma 2 these conditions are satisfied at the profit-maximizing characteristics choices.

This establishes that the first-order conditions for global efficiency are satisfied at the profit maximizing characteristics choices $\widetilde{\boldsymbol{a}}$.

Second-order conditions We now consider in detail the second order conditions for efficiency, focusing on the planner's choice of standards. To illustrate why this problem raises particular questions about the second-order conditions, we first derive the slope of the world welfare contours in Figure 1. With net tariffs and consumption subsidies fixed at the efficient levels, world welfare when $\xi = 1$ is given by:

$$\Omega = \sum_J L^J - N^H \log(P^H) - N^F \log(P^F) - N^H \frac{1}{\sigma - 1} - N^F \frac{1}{\sigma - 1}.$$

Using

$$\begin{aligned} P^H &\equiv \left(\frac{[N^H (P^H)^{\sigma-1}]}{N^H}\right)^{\frac{1}{\sigma-1}} \\ P^F &\equiv \left(\frac{[N^F (P^F)^{\sigma-1}]}{N^F}\right)^{\frac{1}{\sigma-1}}, \end{aligned}$$

we now transform the expression for world welfare to the equivalent expression

$$\Omega = \sum_J L^J - N^H \log(\left(\frac{[N^H (P^H)^{\sigma-1}]}{N^H}\right)^{\frac{1}{\sigma-1}}) - N^F \log(\left(\frac{[N^F (P^F)^{\sigma-1}]}{N^F}\right)^{\frac{1}{\sigma-1}}) - N^H \frac{1}{\sigma - 1} - N^F \frac{1}{\sigma - 1},$$

or

$$\Omega = \sum_J L^J - \frac{1}{\sigma - 1}\{N^H \log\left([N^H (P^H)^{\sigma-1}]\right) + N^F \log\left([N^F (P^F)^{\sigma-1}]\right) - N^H[\log(N^H) - 1] - N^F[\log(N^F) - 1]\}. \tag{55}$$

Totally differentiating yields

$$\frac{d[N^F(P^F)^{\sigma-1}]}{d[N^H(P^H)^{\sigma-1}]}|_{d\Omega=0} = -\left(\frac{P^H}{P^F}\right)^{1-\sigma}. \tag{56}$$

According to (56), for $\sigma > 1$, the slope is flatter than -1 to the right of the N^F/N^H ray (where $P^H > P^F$) and it is steeper than -1 to the left of the N^F/N^H ray (where $P^H < P^F$). Figure 1 depicts the world welfare indifference curve passing through the point labeled Q, which corresponds to the equilibrium under profit-maximizing choices of product characteristics when net tariffs and consumption subsidies are set at the efficient levels.

This raises the question whether the second-order conditions for the planner's choice of standards are globally met. Specifically, we seek conditions under which the point labeled Q in Figure 1 is preferred to the extremes where either the planner sets product attributes to maximize global welfare when $n^F = 0$ or $n^H = 0$.

To explore this question, we first define the following variables:

$$\begin{aligned}
Y &\equiv [N^F(P^F)^{\sigma-1}]; \quad X \equiv [N^H(P^H)^{\sigma-1}] \\
Z_H &\equiv \frac{K+\kappa(a_H^H - a_H^F)^2}{q-\lambda}; \quad Z_F \equiv \frac{K+\kappa(a_F^H - a_F^F)^2}{q-\lambda} \\
\mu_H &\equiv (1+\phi)^{\sigma-1}\left(\frac{A_H^H}{A_H^F}\right)^{\sigma} > 1; \quad \mu_F \equiv (1+\phi)^{\sigma-1}\left(\frac{A_F^F}{A_F^H}\right)^{\sigma} > 1 \\
B_H &\equiv \frac{Z_H}{\lambda^{-\sigma}(1+\phi)^{1-\sigma}(A_H^F)^{\sigma}}; \quad B_F \equiv \frac{Z_F}{\lambda^{-\sigma}(A_F^F)^{\sigma}}.
\end{aligned}$$

Then we have

$$\begin{aligned}
\pi^H &= 0: \quad Y = B_H - \mu_H X \\
\pi^F &= 0: \quad Y = B_F - \frac{1}{\mu_F}X
\end{aligned}$$

The point Q in Figure 1 is defined by $\pi^H = 0$ and $\pi^F = 0$ yielding

$$X = \frac{B_H - B_F}{\mu_H - \frac{1}{\mu_F}}; \quad Y = \frac{\mu_H B_F - \frac{1}{\mu_F}B_H}{\mu_H - \frac{1}{\mu_F}},$$

where these expressions are evaluated at the profit-maximizing product characteristic choices for both home and foreign firms. Notice that we have $\mu_H > \frac{1}{\mu_F}$, so we must have $B_H > B_F$ for $X > 0$ at the point Q.

Now let $\mu_H^{'}$ be the slope of the Home zero profit line and and $B_H^{'}$ be its intercept when the planner sets the attributes $\bar{a}_H^H$ and $\bar{a}_H^F$ for home produced goods *at the levels that maximize global welfare when* $n^F = 0$. Note that $Y = \mu_H^{'}\left(\frac{N^F}{N^H}\right)X$ is the equation that satisfies $n^F = 0$ in these circumstances. We solve for the corresponding $Q_F^{'} = \left(X^{'}, Y^{'}\right)$, where

$$X^{'} = \frac{B_H^{'}}{\mu_H^{'}\left(1+\frac{N^F}{N^H}\right)}; \qquad Y^{'} = \frac{B_H^{'}}{1+\frac{N^H}{N^F}}$$

Global welfare at this $Q_F^{'}$ is

$$\Omega_{Q_F^{'}} = -\left(N^H + N^F\right)\log B_H^{'} + N^H \log \mu_H^{'} + N^H \log\left(1+\frac{N^F}{N^H}\right) + \log N^F \log\left(1+\frac{N^H}{N^F}\right)$$

Suppose that when the planner sets $z^H = 0$, it is possible for her to find a a_F^F and a_F^H with $a_F^F < a_F^H$, *while leaving the standards for home firms as above,* such that when $n^F > 0$ firms in both countries earn zero profits. Take an arbitrary pair of such standards, $\check{a}_F^F$ and $\check{a}_F^H$ and call the resulting point $\check{Q} = \left(\check{X}, \check{Y}\right)$. Notice, of course, that these standards are not optimal for the planner when firms are active in both countries. At the point of intersection of the zero profit lines,

$$\check{X} = \frac{B_H^{'} - \check{B}_F}{\mu_H^{'} - \frac{1}{\check{\mu}_F}}, \qquad \check{Y} = \frac{\mu_H^{'}\check{B}_F - \frac{1}{\check{\mu}_F}B_H}{\mu_H^{'} - \frac{1}{\check{\mu}_F}}$$

Note that the $B_H^{'}$ and $\mu_H^{'}$ are the same as above (since we haven't changed the standards facing home firms), while we use a check above the B_F and μ_F to remind ourselves that these are associated with the arbitrary standards, $\check{a}_F^F$ and $\check{a}_F^H$. The resulting global welfare is

$$\Omega_{\check{Q}} = -N^H \log\left(B_H^{'} - \check{B}_F\right) - N^F \log\left(\mu_H^{'}\check{B}_F - \frac{1}{\check{\mu}_F}B_H^{'}\right) + \left(N^H + N^F\right)\log\left(\mu_H^{'} - \frac{1}{\check{\mu}_F}\right)$$

The difference is

$$\begin{aligned}\Omega_{\check{Q}} - \Omega_{Q_F^{'}} &= N^H \log\frac{\mu_H^{'}B_H^{'} - B_H^{'}/\check{\mu}_F}{\mu_H^{'}B_H^{'} - \mu_H^{'}\check{B}_F} + N^F \log\frac{\mu_H^{'}B_H^{'} - B_H^{'}/\check{\mu}_F}{\mu_H^{'}\check{B}_F - B_H^{'}/\check{\mu}_F} - N^H\log(1+\frac{N^F}{N^H}) - N^F\log(1+\frac{N^H}{N^F}) \\ &= N^H \log\frac{D_1 + D_2}{D_1} + N^F \log\frac{D_1+D_2}{D_2} - N^H\log(1+\frac{N^F}{N^H}) - N^F\log(1+\frac{N^H}{N^F})\end{aligned}$$

where $D_1 \equiv \mu_H^{'}B_H^{'} - \mu_H^{'}\check{B}_F > 0$ and $D_2 \equiv \mu_H^{'}\check{B}_F - B_H^{'}/\check{\mu}_F > 0$.

To show $\Omega_{\check{Q}} - \Omega_{Q_F^{'}} \geq 0$, requires

$$\left(N^H\right)^{N^H}\left(N^F\right)^{N^F}\left(D_1 + D_2\right)^{N^H + N^F} - \left(N^H + N^F\right)^{N^H+N^F}\left(D_1\right)^{N^H}\left(D_2\right)^{N^F} \geq 0$$

Now normalize so that $N^H + N^F = 2$ and re-arrange to get,

$$\left(N^H\right)^{N^H}\left(2 - N^H\right)^{2-N^H} - 4\left(\frac{D_1}{D_1+D_2}\right)^{N^H}\left(\frac{D_2}{D_1+D_2}\right)^{2-N^H} \geq 0$$

Note that $\left(D_1\right)^{N^H}\left(1-D_1\right)^{2-N^H}$ is maximized at $D_1/\left(1-D_1\right) = N^H/\left(2-N^H\right) \Rightarrow \frac{D_1}{D_1+D_2} =$

$N^H/2$ and $\frac{D_2}{D_1+D_2} = \left(2 - N^H\right)/2$. So the expression above is greater than or equal to

$$\left(N^H\right)^{N^H} \left(2 - N^H\right)^{2-N^H} - 4\left(\frac{N^H}{2}\right)^{N^H} \left(\frac{2-N^H}{2}\right)^{2-N^H} = 0$$

So we have proven that $\Omega_{\check{Q}} - \Omega_{Q'_F} \geq 0$, i.e., the planner prefers $\check{Q}$ to Q'_F for arbitrary $\check{a}^F_F$ and $\check{a}^H_F$ such that $n^F > 0$ and all firms break even. But Q is the social optimum when all firms are active. Clearly $\Omega_Q \geq \Omega_{\check{Q}}$. So

$$\Omega_Q - \Omega_{Q'_F} \geq 0$$

An analogous argument shows that Q also welfare-dominates an extreme where the planner sets attributes to maximize global welfare when $n^H = 0$.

Unilateral incentives to deviate from efficient consumption subsidies Finally, we show that there is no need for an NTA that stipulates zero net trade taxes on all goods and covers product standards to also cover consumption subsidies provided that National Treatment (NT) is imposed, as we observed in the text. To this end, we position the home and foreign consumption subsidies initially at the efficient level $1/\sigma$, and ask whether a country has a unilateral incentive to deviate (with trade taxes and standards all held to efficient levels). A first observation is that the world prices are functions of trade taxes but independent of consumption subsidies (and standards) in this model, so there is no need to negotiate over consumption subsidies for purposes of eliminating terms-of-trade manipulation (also true of standards). Hence we need only consider the incentive to use consumption subsidies for purposes of delocation.

With net trade taxes set to zero, the home country's choice of consumption subsidy s^H will impact p^H_H and p^H_F according to

$$p^H_H = (1 - s^H)q; \quad p^H_F = (1 - s^H)(1 + \phi)q,$$

and similarly the foreign country's choice of consumption subsidy s^F will impact p^F_F and p^F_H according to

$$\begin{aligned} p^F_F &= (1 - s^F)q; \\ p^F_H &= (1 - s^F)(1 + \phi)q. \end{aligned}$$

Focusing on the home country choice of s^H and beginning from the efficient point, in the context of Figure 1 a slight increase in s^H will shift both the home zero profit line and the foreign zero profit in (toward the y-axis). Totally differentiating the home zero profit line with respect to s^H and $(P^H)^{\sigma-1}$ yields

$$\frac{d\left[N^H(P^H)^{\sigma-1}\right]}{ds^H}\Big|_{\pi^H=0} = \frac{-\sigma(P^H)^{\sigma-1}}{(1 - s^H)}.$$

Hence, the home zero profit line shifts in (toward the y-axis in Figure 1) with a small increase in

s^H by the amount $\frac{-\sigma(P^H)^{\sigma-1}}{(1-s^H)}$. But totally differentiating the foreign zero profit line with respect to s^H and $(P^H)^{\sigma-1}$ yields

$$\frac{d\left[N^H(P^H)^{\sigma-1}\right]}{ds^H}|_{\pi^F=0} = \frac{-\sigma(P^H)^{\sigma-1}}{(1-s^H)}.$$

Hence, the foreign zero profit line shifts in with a small increase in s^H by the exact same amount $\frac{-\sigma(P^H)^{\sigma-1}}{(1-s^H)}$. This implies that $(P^F)^{\sigma-1}$ is left unchanged by the increase in s^H, and hence implies that foreign welfare (which is given by $\Omega^F = L^F - N^F \log\left(P^F\right) - N^F \frac{1}{\sigma-1}$) is unaffected by the small increase in s^H. But given that s^H was initially positioned at the efficient level, it is impossible for home welfare to rise if foreign welfare does not fall. We may thus conclude that the home country cannot improve its welfare with a small unilateral deviation from $s^H = \frac{1}{\sigma}$. And with

$$\frac{d\left[N^H(P^H)^{\sigma-1}\right]}{ds^H}|_{\pi^H=0} = \frac{-\sigma(P^H)^{\sigma-1}}{(1-s^H)} = \frac{d\left[N^H(P^H)^{\sigma-1}\right]}{ds^H}|_{\pi^F=0}$$

starting from any level of s^H, it is easy to see that the same argument applies globally for unilateral deviations from $s^H = \frac{1}{\sigma}$ of any size.

Therefore, we may conclude that in the presence of NT, an NTA does not need to cover the consumption subsidies for each country.

QED

6.5 Proof of Proposition 2

Proposition 2 *Suppose $\xi = 1$, $\tau^H = \tau^F = e^H = e^F = 0$ and $s^H = s^F = 1/\sigma$. Suppose governments are free to choose any standards for local products and for imported products, without need for national treatment. Then, in the Nash equilibrium of the standard-setting game, either (i) $n^J = 0$ for some $J \in \{H, F\}$, or (ii) $a_H^F \in \{0,1\}$ and $a_F^H \in \{0,1\}$. The equilibrium level of global welfare is less than that attained under an NTA.*

Proof We look for the Nash equilibrium choices of product standards in an FTA without NT. By an FTA, we mean that the two governments are constrained to set $\tau^J = 0$, $e^J = 0$, and we also have $s^J = 1/\sigma$.[31].

Consider the outcome from free entry when $a_H^F = 0$, $a_F^H = 1$ and a_H^H and a_F^F are at their profit-maximizing levels in response to these extreme standards for imports. There are three possible outcomes: (i) $n^H > 0$ and $n^F > 0$; (ii) $n^H > 0$ and $n^F = 0$; (iii) $n^F > 0$ and $n^H = 0$.

Case (i): If $n^H > 0$ and $n^F > 0$ when $a_H^F = 0$, $a_F^H = 1$ and a_H^H and a_F^F are at their profit-maximizing levels in response to these extreme standards for imports, neither government can induce "complete delocation"; i.e., exit by all firms in the other country. As long as there are active firms in both countries, each government has an incentive to push its standard for import goods

[31] While the NTA could constrain consumption subsidies to their efficient levels $s^J = 1/\sigma$, by the result proved just above there is no need for such a constraint as long as NT is imposed on consumption subsidies.

to the extreme, since doing so (given the other government's policy) always reduces the local price index by the arguments in Figure 1. Given the pair of extreme standards for import goods, the Nash response for each government is to set the standard for local products equal to the profit maximizing level.

Case (ii): Now the home government can induce complete delocation and it has an incentive to do so. It will set its standard for import products high enough to ensure $n^F = 0$. There will be a range of standards that achieve this, including $a_F^H = 1$; all of them are best responses so any can be part of a Nash equilibrium (with the same consequences for other variables). But given that a_F^H is chosen such that $n^F = 0$, the incentives facing the foreign government are different. It does not use a_H^F to induce delocation, since such a strategy is bound to fail. Instead it "accepts" that all differentiated products will be imported and it trades off the desirability of the import products given local tastes and variety. By setting $a_H^F = \hat{a}^F$, it maximizes A_H^F, the local demand shifter in Foreign. By setting a_H^F at the profit maximizing level for home firms, it maximizes variety. It will choose a standard somewhere between these two. Arguing in this way, it is straightforward to establish that the best response for a_H^F is strictly between $\hat{a}^F$ and a_H^H. Similarly, the best response for a_H^H will be strictly between a_H^F and $\hat{a}^H$.

Case (iii) is similar.

Notice that we have structured our arguments above under the implicit assumption that a country can always hurt the firms of its trading partner most by moving its standard all the way in its own direction – and past its local ideal, rather than going all the way in its trading partner's direction – and past its trading partner's local ideal. This feature is not essential for the statement of Proposition 2, but it could be guaranteed under natural parameter restrictions (e.g., that $1 \geq \hat{a}^H \geq 0.5 \geq \hat{a}^F \geq 0$).

On the interplay between better suitability and delocation In the text following the statement of Proposition 2, we also discussed the interplay between the two motives for regulation – better suitability and delocation – featured by our model, and we claimed that when evaluated near the Nash equilibrium the delocation motive always operates on the margin. Here we expand on the interplay between better suitability and delocation in the context of standard setting and establish this claim.

To this end, it is first helpful to express $\frac{dn^H}{da_F^H}$ and $\frac{dn^F}{da_F^H}$ evaluated at an arbitrary a_F^H. Following the same steps as in Appendix section 6.2.2 but not requiring a_F^H to satisfy the first-order condition for profit maximization yields the following expressions for $\frac{dn^H}{da_F^H}$ and $\frac{dn^F}{da_F^H}$ evaluated at an arbitrary a_F^H:

$$\frac{dn^H}{da_F^H} = \left[\frac{\left[\frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^F} + (1+\phi)\frac{\partial c_H^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^F}\right]\left(N^H(1+\phi)\frac{\partial c_F^H}{\partial a_F^H} - \frac{2\kappa(a_F^H - a_F^F)}{q-\lambda}\right)}{\left[\frac{\partial \mathcal{P}^H}{\partial n^H}\frac{\partial \mathcal{P}^F}{\partial n^F} - \frac{\partial \mathcal{P}^H}{\partial n^F}\frac{\partial \mathcal{P}^F}{\partial n^H}\right]\left[\frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial c_F^F}{\partial \mathcal{P}^F} - (1+\phi)^2\frac{\partial c_F^H}{\partial \mathcal{P}^H}\frac{\partial c_H^F}{\partial \mathcal{P}^F}\right]} \right] + \left[\frac{-\frac{\partial \mathcal{P}^H}{\partial a_F^H}\frac{\partial \mathcal{P}^F}{\partial n^F}}{\left[\frac{\partial \mathcal{P}^H}{\partial n^H}\frac{\partial \mathcal{P}^F}{\partial n^F} - \frac{\partial \mathcal{P}^H}{\partial n^F}\frac{\partial \mathcal{P}^F}{\partial n^H}\right]} \right] \tag{57}$$

$$\frac{dn^F}{da_F^H} = \left[\frac{-\left[\frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^F} + (1+\phi)\frac{\partial c_H^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^F}\right]\left(N^H(1+\phi)\frac{\partial c_F^H}{\partial a_F^H} - \frac{2\kappa(a_F^H - a_F^F)}{q-\lambda}\right)}{\left[\frac{\partial \mathcal{P}^H}{\partial n^H}\frac{\partial \mathcal{P}^F}{\partial n^F} - \frac{\partial \mathcal{P}^H}{\partial n^F}\frac{\partial \mathcal{P}^F}{\partial n^H}\right]\left[\frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial c_F^F}{\partial \mathcal{P}^F} - (1+\phi)^2\frac{\partial c_F^H}{\partial \mathcal{P}^H}\frac{\partial c_H^F}{\partial \mathcal{P}^F}\right]} \right] + \left[\frac{\frac{\partial \mathcal{P}^H}{\partial a_F^H}\frac{\partial \mathcal{P}^F}{\partial n^H}}{\left[\frac{\partial \mathcal{P}^H}{\partial n^H}\frac{\partial \mathcal{P}^F}{\partial n^F} - \frac{\partial \mathcal{P}^H}{\partial n^F}\frac{\partial \mathcal{P}^F}{\partial n^H}\right]} \right] \tag{58}$$

It is clear that the term $\left[\frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial \mathcal{P}^H}{\partial n^F} + (1+\phi)\frac{\partial c_H^F}{\partial \mathcal{P}^F}\frac{\partial \mathcal{P}^F}{\partial n^F}\right]$ is negative while the terms $\left[\frac{\partial \mathcal{P}^H}{\partial n^H}\frac{\partial \mathcal{P}^F}{\partial n^F} - \frac{\partial \mathcal{P}^H}{\partial n^F}\frac{\partial \mathcal{P}^F}{\partial n^H}\right]$ and $\left[\frac{\partial c_H^H}{\partial \mathcal{P}^H}\frac{\partial c_F^F}{\partial \mathcal{P}^F} - (1+\phi)^2\frac{\partial c_F^H}{\partial \mathcal{P}^H}\frac{\partial c_H^F}{\partial \mathcal{P}^F}\right]$ are positive, so the sign of the first term in (57) will be opposite the sign of $\left(N^H(1+\phi)\frac{\partial c_F^H}{\partial a_F^H} - \frac{2\kappa(a_F^H - a_F^F)}{q-\lambda}\right)$ while the sign of the first term in (58) will be the same as the sign of $\left(N^H(1+\phi)\frac{\partial c_F^H}{\partial a_F^H} - \frac{2\kappa(a_F^H - a_F^F)}{q-\lambda}\right)$. And as Lemma 1 confirms, the sign of the second term in (57) is negative while the sign of the second term in (58) is positive.

Evaluated at the profit-maximizing choice of a_F^H, the associated first-order condition assures that

$$N^H(1+\phi)\frac{\partial c_F^H}{\partial a_F^H} - \frac{2\kappa(a_F^H - a_F^F)}{q-\lambda} = 0$$

and so the first term in each of the expressions (57) and (58) is zero, and the expressions collapse to those given in (38) and (39) respectively. But when these expressions are evaluated at a level of a_F^H above the profit-maximizing choice, we have $N^H(1+\phi)\frac{\partial c_F^H}{\partial a_F^H} - \frac{2\kappa(a_F^H - a_F^F)}{q-\lambda} < 0$ making the first term in (57) positive and therefore working to overturn the second term in (57), and making the first term in (58) negative and therefore working to overturn the second term in (58). And when these expressions are evaluated at a level of a_F^H below the profit-maximizing choice, we have $N^H(1+\phi)\frac{\partial c_F^H}{\partial a_F^H} - \frac{2\kappa(a_F^H - a_F^F)}{q-\lambda} > 0$ making the first term in (57) negative and therefore working to reinforce the second term in (57), and making the first term in (58) positive and therefore working to reinforce the second term in (58).

Now consider Figure **??**, which depicts n^H and n^F as a function of a_F^H. To draw the n^H and n^F curves, we use expressions (57) and (58). The point in the figure labeled a_F^{H1} is where n^F takes its maximum value, and the point in the figure labeled a_F^{H2} is where n^H takes its minimum value. According to (57) and (58) evaluated at the profit maximizing levels of a_F^F and a_H^F, $a_F^{H1} < a_F^{H2}$ as depicted. Also depicted in the figure is the local ideal $\hat{a}^H$. And finally, as noted in the figure, P^H falls as we move away from the profit-maximizing level a_F^H in either direction.

Several observations follow from Figure **??**. Moving left from the profit maximizing level a_F^H, P^H falls due to the delocation associated with the fall in a_F^H, with n^F falling and n^H rising as foreign firms are delocated to the home-country market. So the incentive for the home country to defect toward the left from the efficient profit maximizing a_F^H is due to delocation. But moving right from the profit maximizing level a_F^H, P^H falls despite the fact that initially n^F is rising and n^H is falling. So the incentive to defect toward the right from the efficient profit maximizing a_F^H is initially – in the interval $((a_F^H, a_F^{H1})$ – *not* due to delocation; it is due instead to the direct impact on P^H of having imports adopt a characteristic that is a little closer to the Home ideal $\hat{a}^H$, and

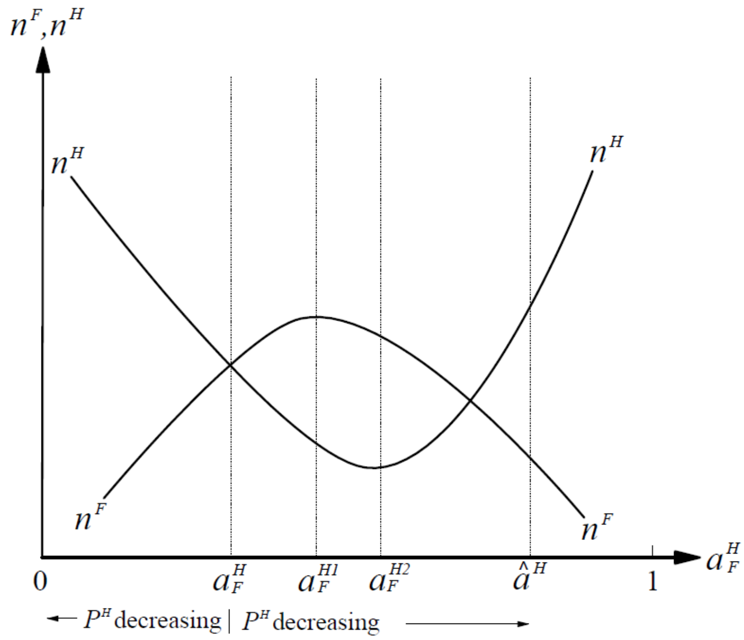

Figure 2: Numbers of Firms as Function of a_F^H

this direct impact dominates the (anti-) delocation effects here. Once we move into the interval (a_F^{H1}, a_F^{H2}), *both* n^H and n^F are falling with further increases in a_F^H, so again the incentive for the home country to keep raising a_F^H in this interval to lower P^H is not due to delocation, but must still be due to the domination of the direct impact on P^H of having imports adopt a characteristic that is a little closer to the Home ideal $\hat{a}^H$. In the interval $(a_F^{H2}, \hat{a}^H)$, we now have delocation *and* the direct impact described above both helping to push P^H lower. But for the interval $(\hat{a}^H, 1)$, the direct effect is now going the wrong way so it is the delocation effect that dominates at this point and keeps P^H falling.

This illustrates why setting tariffs in a way that perfectly offsets the P^H-reducing incentives of the home government with countervailing revenue incentives will not be possible, because the P^H-reducing incentives themselves are not tied monotonically to the trade volume effects – and hence the potential trade tax revenue effects – of standards choices, and only reflect trade volume effects in a consistent way as a_F^H approaches the extremes of 0 or 1. So while the judicious choice of (efficient) trade tax/subsidies can reduce the Nash distortions in standards from their extreme levels, it cannot eliminate these distortions completely, an observation we formalize in Proposition 3.

Finally, notice that Figure **??** shows the number of foreign firms as being still positive at $\hat{a}^H$, which, if a general property, would mean that *only* the delocation motive operates in the neighborhood of the case (ii) Nash equilibrium. On the other hand, if n^F hits zero at a standard smaller than $\hat{a}^H$, then the "last little bit of standard" could provide benefits both via delocation and via product suitability. It can be shown that both possibilities can arise. Hence the product suitability motive may or may not be operative on the margin in the Nash equilibrium, but the delocation motive is *always* operative.

QED

6.6 Proof of Proposition 6

Proposition 6 *When $\xi < 1$ and there is a consumption externality, the efficient net trade taxes and consumption subsidies as defined in (26) and (27) involve positive net trade taxes and consumption subsidies that are higher than in the absence of a consumption externality, and it is efficient to impose regulatory standards on firms that induce firms to select product characteristics closer to each country's ideal relative to the profit-maximizing choices of these product characteristics.*

Proof In the text we derived the following expressions which implicitly define the efficient prices for $\xi \in [0,1]$:

$$p_H^{HE}(\xi) = p_H^{HE}(1) \cdot \left[\left(\frac{A_H^{HE}(\xi)}{\hat{A}_H^{HE}} \right) \left(\frac{\mathcal{P}^H(p_H^{HE}(\xi), p_F^{HE}(\xi); \mathbf{n}^E, a_H^{HE}, a_F^{HE})}{P^{HE}} \right)^{\left(\frac{\sigma-1}{\sigma}\right)} \right]$$

$$p_F^{HE}(\xi) = p_F^{HE}(1) \cdot \left[\left(\frac{A_F^{HE}(\xi)}{\hat{A}_F^{HE}} \right) \left(\frac{\mathcal{P}^H(p_H^{HE}(\xi), p_F^{HE}(\xi); \mathbf{n}^E, a_H^{HE}, a_F^{HE})}{P^{HE}} \right)^{\left(\frac{\sigma-1}{\sigma}\right)} \right]$$

$$p_F^{FE}(\xi) = p_F^{FE}(1) \cdot \left[\left(\frac{A_F^{FE}(\xi)}{\hat{A}_F^{FE}} \right) \left(\frac{\mathcal{P}^F(p_F^{FE}(\xi), p_H^{FE}(\xi); \mathbf{n}^E, a_F^{FE}, a_H^{FE})}{P^{FE}} \right)^{\left(\frac{\sigma-1}{\sigma}\right)} \right]$$

$$p_H^{FE}(\xi) = p_H^{FE}(1) \cdot \left[\left(\frac{A_H^{FE}(\xi)}{\hat{A}_H^{FE}} \right) \left(\frac{\mathcal{P}^F(p_F^{FE}(\xi), p_H^{FE}(\xi); \mathbf{n}^E, a_F^{FE}, a_H^{FE})}{P^{FE}} \right)^{\left(\frac{\sigma-1}{\sigma}\right)} \right],$$

where P^{JE} is the efficient (brand-level and industry-level) price index in country J when $\xi = 1$. We claimed that for $\xi < 1$, $p_H^{HE}(\xi) < p_H^{HE}(1)$, $p_F^{HE}(\xi) > p_F^{HE}(1)$, $p_F^{FE}(\xi) < p_F^{FE}(1)$ and $p_H^{FE}(\xi) > p_H^{FE}(1)$. We also derived expressions for the efficient net trade taxes,

$$\begin{aligned} \tau^{HE}(\xi) + e^{FE}(\xi) &= (1+\phi) \cdot \left[\frac{\left(\frac{A_F^{HE}(\xi)}{\hat{A}_F^{HE}} \right)}{\left(\frac{A_H^{HE}(\xi)}{\hat{A}_H^{HE}} \right)} - 1 \right] \\ e^{HE}(\xi) + \tau^{FE}(\xi) &= (1+\phi) \cdot \left[\frac{\left(\frac{A_H^{FE}(\xi)}{\hat{A}_H^{FE}} \right)}{\left(\frac{A_F^{FE}(\xi)}{\hat{A}_F^{FE}} \right)} - 1 \right], \end{aligned}$$

and the efficient consumption subsidies

$$\begin{aligned} s^{HE}(\xi) &= \frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right)\left[1 - \left(\frac{A_H^{HE}(\xi)}{\hat{A}_H^{HE}}\right)\left(\frac{\mathcal{P}^H(p_H^{HE}(\xi), p_F^{HE}(\xi); \mathbf{n}^E, a_H^{HE}, a_F^{HE})}{P^{HE}}\right)^{\left(\frac{\sigma-1}{\sigma}\right)}\right] \\ s^{FE}(\xi) &= \frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right)\left[1 - \left(\frac{A_F^{FE}(\xi)}{\hat{A}_F^{FE}}\right)\left(\frac{\mathcal{P}^F(p_F^{FE}(\xi), p_H^{FE}(\xi); \mathbf{n}^E, a_F^{FE}, a_H^{FE})}{P^{FE}}\right)^{\left(\frac{\sigma-1}{\sigma}\right)}\right], \end{aligned}$$

and we claimed that for $\xi < 1$, $s^{HE}(\xi) > \frac{1}{\sigma}$ and $s^{FE}(\xi) > \frac{1}{\sigma}$. Each of these claims follow provided that

$$\left[\left(\frac{A_H^{HE}(\xi)}{\hat{A}_H^{HE}}\right)\left(\frac{\mathcal{P}^H(p_H^{HE}(\xi), p_F^{HE}(\xi); \mathbf{n}^E, a_H^{HE}, a_F^{HE})}{P^{HE}}\right)^{\left(\frac{\sigma-1}{\sigma}\right)}\right] < 1 \tag{59}$$

$$\left[\left(\frac{A_F^{HE}(\xi)}{\hat{A}_F^{HE}}\right)\left(\frac{\mathcal{P}^H(p_H^{HE}(\xi), p_F^{HE}(\xi); \mathbf{n}^E, a_H^{HE}, a_F^{HE})}{P^{HE}}\right)^{\left(\frac{\sigma-1}{\sigma}\right)}\right] > 1 \tag{60}$$

$$\left[\left(\frac{A_F^{FE}(\xi)}{\hat{A}_F^{FE}}\right)\left(\frac{\mathcal{P}^F(p_F^{FE}(\xi), p_H^{FE}(\xi); \mathbf{n}^E, a_F^{FE}, a_H^{FE})}{P^{FE}}\right)^{\left(\frac{\sigma-1}{\sigma}\right)}\right] < 1 \tag{61}$$

$$\left[\left(\frac{A_H^{FE}(\xi)}{\hat{A}_H^{FE}}\right)\left(\frac{\mathcal{P}^F(p_F^{FE}(\xi), p_H^{FE}(\xi); \mathbf{n}^E, a_F^{FE}, a_H^{FE})}{P^{FE}}\right)^{\left(\frac{\sigma-1}{\sigma}\right)}\right] > 1, \tag{62}$$

which we now prove.

To prove this, we first prove another claim made in the text, namely, that under the efficient consumption subsidies and net trade taxes and the implied vector of efficient prices (which we denoted by $\mathbf{p}^E(\xi)$), and in combination with the vector of efficient product characteristics (which we denoted by $\mathbf{a}^E$), we have

$$P^H(\mathbf{a}^E, \mathbf{p}^E(\xi)) = P^{HE}; \quad P^F(\mathbf{a}^E, \mathbf{p}^E(\xi)) = P^{FE},$$

where recall that we have defined P^{JE} as the efficient (brand-level and industry-level) price index in country J when $\xi = 1$. To show that $P^H(\mathbf{a}^E, \mathbf{p}^E(\xi)) = P^{HE}$ (the steps to show $P^F(\mathbf{a}^E, \mathbf{p}^E(\xi)) = P^{FE}$ are analogous), we first write P^{HE} as

$$P^{HE} = \left[n^{HE}(\hat{A}_H^{HE})^{\sigma}(p_H^{HE}(1))^{1-\sigma} + n^{FE}(\hat{A}_F^{HE})^{\sigma}(p_F^{HE}(1))^{1-\sigma}\right]^{\frac{-1}{\sigma-1}},$$

where we have used $A_H^{HE}(\xi = 1) = \hat{A}_H^{HE}$ and $A_F^{HE}(\xi = 1) = \hat{A}_F^{HE}$. Then, using the definition of

$\mathcal{P}^H$ and the relationship between $\mathcal{P}^H$ and P^H, we have

$$P^H(\mathbf{a}^E, \mathbf{p}^E(\xi)) = \frac{\left[\mathcal{P}^H(p_H^{HE}(\xi), p_F^{HE}(\xi); \mathbf{n}^E, a_H^{HE}, a_F^{HE})\right]^{-(\sigma-1)}}{\left[n^{HE}\frac{\hat{A}_H^{HE}}{A_H^{HE}(\xi)}(A_H^{HE}(\xi))^\sigma (p_H^{HE}(\xi))^{1-\sigma} + n^{FE}\frac{\hat{A}_F^{HE}}{A_F^{HE}(\xi)}(A_F^{HE}(\xi))^\sigma (p_F^{HE}(\xi))^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}}.$$

Plugging the expressions for $p_H^{HE}(\xi)$ and $p_F^{HE}(\xi)$ into the denominator of the above expression and simplifying then yields

$$\begin{aligned}
& \frac{\left[\mathcal{P}^H(p_H^{HE}(\xi), p_F^{HE}(\xi); \mathbf{n}^E, a_H^{HE}, a_F^{HE})\right]^{-(\sigma-1)}}{\left[n^{HE}\frac{\hat{A}_H^{HE}}{A_H^{HE}(\xi)}(A_H^{HE}(\xi))^\sigma (p_H^{HE}(\xi))^{1-\sigma} + n^{FE}\frac{\hat{A}_F^{HE}}{A_F^{HE}(\xi)}(A_F^{HE}(\xi))^\sigma (p_F^{HE}(\xi))^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}} \\
= & \frac{\left(P^{HE}\right)^{-(\sigma-1)}}{\left[n^{HE}\frac{\hat{A}_H^{HE}}{A_H^{HE}(\xi)}(A_H^{HE}(\xi))^\sigma (p_H^{HE}(1))^{1-\sigma}\left(\frac{A_H^{HE}(\xi)}{\hat{A}_H^{HE}}\right)^{1-\sigma} + n^{FE}\frac{\hat{A}_F^{HE}}{A_F^{HE}(\xi)}(A_F^{HE}(\xi))^\sigma (p_F^{HE}(1))^{1-\sigma}\left(\frac{A_F^{HE}(\xi)}{\hat{A}_F^{HE}}\right)^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}} \\
= & \left[n^{HE}(\hat{A}_H^{HE})^\sigma (p_H^{HE}(1))^{1-\sigma} + n^{FE}(\hat{A}_F^{HE})^\sigma (p_F^{HE}(1))^{1-\sigma}\right]^{\frac{-1}{\sigma-1}} \\
= & P^{HE}.
\end{aligned}$$

With $P^H(\mathbf{a}^E, \mathbf{p}^E(\xi)) = P^{HE}$ established, we now establish the claim in (59), with each of the other three claims in (60)-(62) following under analogous arguments. Using $P^H(\mathbf{a}^E, \mathbf{p}^E(\xi)) = P^{HE}$ and the relationship between $\mathcal{P}^H$ and P^H, we have

$$\begin{aligned}
& \left(\frac{A_H^{HE}(\xi)}{\hat{A}_H^{HE}}\right)\left(\frac{\mathcal{P}^H(p_H^{HE}(\xi), p_F^{HE}(\xi); \mathbf{n}^E, a_H^{HE}, a_F^{HE})}{P^{HE}}\right)^{\left(\frac{\sigma-1}{\sigma}\right)} \\
= & \left(\frac{A_H^{HE}(\xi)}{\hat{A}_H^{HE}}\right)\left(\frac{\mathcal{P}^H(p_H^{HE}(\xi), p_F^{HE}(\xi); \mathbf{n}^E, a_H^{HE}, a_F^{HE})}{P^H(\mathbf{a}^E, \mathbf{p}^E(\xi))}\right)^{\left(\frac{\sigma-1}{\sigma}\right)} \\
= & \left(\frac{A_H^{HE}(\xi)}{\hat{A}_H^{HE}}\right)\left[\frac{n^{HE}\frac{\hat{A}_H^{HE}}{A_H^{HE}(\xi)}(A_H^{HE}(\xi))^\sigma (p_H^{HE}(\xi))^{1-\sigma} + n^{FE}\frac{\hat{A}_F^{HE}}{A_F^{HE}(\xi)}(A_F^{HE}(\xi))^\sigma (p_F^{HE}(\xi))^{1-\sigma}}{n^{HE}(A_H^{HE}(\xi))^\sigma (p_H^{HE}(\xi))^{1-\sigma} + n^{FE}(A_F^{HE}(\xi))^\sigma (p_F^{HE}(\xi))^{1-\sigma}}\right] \\
= & \left[\frac{n^{HE}(A_H^{HE}(\xi))^\sigma (p_H^{HE}(\xi))^{1-\sigma} + n^{FE}\left[\frac{\frac{A_H^{HE}(\xi)}{\hat{A}_H^{HE}}}{\frac{A_F^{HE}(\xi)}{\hat{A}_F^{HE}}}\right](A_F^{HE}(\xi))^\sigma (p_F^{HE}(\xi))^{1-\sigma}}{n^{HE}(A_H^{HE}(\xi))^\sigma (p_H^{HE}(\xi))^{1-\sigma} + n^{FE}(A_F^{HE}(\xi))^\sigma (p_F^{HE}(\xi))^{1-\sigma}}\right] \\
< & 1
\end{aligned}$$

where the inequality follows for $\xi < 1$ from the ranking of efficient product characteristics.

Finally, in the text we also claimed that the additional consumption subsidies and net trade taxes implied by efficient intervention in the presence of the consumption externality are revenue neutral, implying that global welfare under the efficient policies when $\xi < 1$ is given by

$$\begin{aligned}\Omega\left(\mathbf{a}^E, \mathbf{p}^E(\xi)\right) &= \sum_J L^J - \sum_J N^J \log P^J\left(\mathbf{a}^E, \mathbf{p}^E(\xi)\right) - \sum_J N^J \frac{1}{\sigma-1} \\ &= \sum_J L^J - \sum_J N^J \log P^{JE} - \sum_J N^J \frac{1}{\sigma-1} \ ,\end{aligned}$$

the same level of global welfare that is reached under efficient policies when $\xi = 1$.

To confirm that the additional consumption subsidies and net trade taxes implied by efficient intervention in the presence of the consumption externality are revenue neutral, note that the trade tax revenue goes from zero under the efficient policies when $\xi = 1$ to the amount

$$\sum_J N^J q(1+\phi) \cdot \left[\frac{\left(\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}\right)}{\left(\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}\right)} - 1 \right] \times [n^{\tilde{J}E} c_{\tilde{J}}^{JE}] \tag{63}$$

under the efficient policies when $\xi < 1$: the increase in trade tax revenue is therefore given by (63). The increase in consumption subsidy payments is given by

$$\begin{aligned}\sum_J \{ N^J q \left(\frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right) \left[1 - \left(\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}\right) \left(\frac{\mathcal{P}^J(p_J^{JE}(\xi), p_{\tilde{J}}^{JE}(\xi); \mathbf{n}^E, a_J^{JE}, a_{\tilde{J}}^{JE})}{P^{JE}} \right)^{\left(\frac{\sigma-1}{\sigma}\right)} \right] \right) \times \\ \left[n^{JE} c_J^{JE} + n^{\tilde{J}E}(1+\phi+\tau^{JE}(\xi) + e^{\tilde{J}E}(\xi)) c_{\tilde{J}}^{JE} \right] - N^J q \frac{1}{\sigma} \left[n^{JE} c_J^{JE} + n^{\tilde{J}E}(1+\phi) c_{\tilde{J}}^{JE} \right] \}\end{aligned}$$

which can be simplified to

$$\begin{aligned}\sum_J \{ -N^J q \frac{1}{\sigma} n^{\tilde{J}E}(1+\phi) c_{\tilde{J}}^{JE} + N^J q \left(\frac{\sigma-1}{\sigma}\right) n^{JE} c_J^{JE} + N^J q n^{\tilde{J}E}(1+\phi) \frac{\left(\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}\right)}{\left(\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}\right)} c_{\tilde{J}}^{JE} - \\ N^J q \left(\frac{\sigma-1}{\sigma}\right) \left(\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}\right) \left(\frac{\mathcal{P}^J(p_J^{JE}(\xi), p_{\tilde{J}}^{JE}(\xi); \mathbf{n}^E, a_J^{JE}, a_{\tilde{J}}^{JE})}{P^{JE}} \right)^{\left(\frac{\sigma-1}{\sigma}\right)} \times \\ \left[n^{JE} c_J^{JE} + n^{\tilde{J}E}(1+\phi) \frac{\left(\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}\right)}{\left(\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}\right)} c_{\tilde{J}}^{JE} \right] \}.\end{aligned}$$

Hence, in going from $\xi = 1$ to $\xi < 1$ the change in revenue implied by the efficient trade taxes and consumption subsidies is given by

$$\Delta Rev = \sum_J \{ N^J q(1+\phi) \cdot \left[\frac{\left(\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}} \right)}{\left(\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}} \right)} - 1 \right] \times [n^{\tilde{J}E} c_{\tilde{J}}^{JE}] + N^J q \frac{1}{\sigma} n^{\tilde{J}E}(1+\phi) c_{\tilde{J}}^{JE} -$$

$$N^J q \left(\frac{\sigma - 1}{\sigma} \right) n^{JE} c_J^{JE} - N^J q n^{\tilde{J}E}(1+\phi) \frac{\left(\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}} \right)}{\left(\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}} \right)} c_{\tilde{J}}^{JE} +$$

$$N^J q \left(\frac{\sigma - 1}{\sigma} \right) \left(\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}} \right) \left(\frac{\mathcal{P}^J(p_J^{JE}(\xi), p_{\tilde{J}}^{JE}(\xi); \mathbf{n}^E, a_J^{JE}, a_{\tilde{J}}^{JE})}{P^{JE}} \right)^{\left(\frac{\sigma-1}{\sigma}\right)} \times$$

$$\left[n^{JE} c_J^{JE} + n^{\tilde{J}E}(1+\phi) \frac{\left(\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}} \right)}{\left(\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}} \right)} c_{\tilde{J}}^{JE} \right] \},$$

which simplifies to

$$\Delta Rev = q \left(\frac{\sigma - 1}{\sigma} \right) \sum_J N^J \{ n^{\tilde{J}E}(1+\phi) c_{\tilde{J}}^{JE} \cdot \left[\left(\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}} \right) \left(\frac{\mathcal{P}^J(p_J^{JE}(\xi), p_{\tilde{J}}^{JE}(\xi); \mathbf{n}^E, a_J^{JE}, a_{\tilde{J}}^{JE})}{P^{JE}} \right)^{\left(\frac{\sigma-1}{\sigma}\right)} - 1 \right] +$$

$$n^{JE} c_J^{JE} \cdot \left[\left(\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}} \right) \left(\frac{\mathcal{P}^J(p_J^{JE}(\xi), p_{\tilde{J}}^{JE}(\xi); \mathbf{n}^E, a_J^{JE}, a_{\tilde{J}}^{JE})}{P^{JE}} \right)^{\left(\frac{\sigma-1}{\sigma}\right)} - 1 \right] \}.$$

Using $P^H(\mathbf{a}^E, \mathbf{p}^E(\xi)) = P^{HE}$ and the relationship between $\mathcal{P}^H$ and P^H, we then have

$$\Delta Rev = q \left(\frac{\sigma - 1}{\sigma} \right) \sum_J \frac{N^J}{n^{HE}(A_H^{HE}(\xi))^\sigma (p_H^{HE}(\xi))^{1-\sigma} + n^{FE}(A_F^{HE}(\xi))^\sigma (p_F^{HE}(\xi))^{1-\sigma}} \times$$

$$\{ n^{\tilde{J}E}(1+\phi) c_{\tilde{J}}^{JE} \cdot \left[n^{JE} \left[\frac{\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}}{\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}} - 1 \right] (A_J^{JE}(\xi))^\sigma (p_J^{JE}(\xi))^{1-\sigma} \right] +$$

$$n^{JE} c_J^{JE} \cdot \left[n^{\tilde{J}E} \left[\frac{\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}}{\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}} - 1 \right] (A_{\tilde{J}}^{JE}(\xi))^\sigma (p_{\tilde{J}}^{JE}(\xi))^{1-\sigma} \right] \},$$

which can be rewritten as

$$\Delta Rev = q\left(\frac{\sigma-1}{\sigma}\right)\sum_J \frac{N^J}{n^{HE}(A_H^{HE}(\xi))^\sigma(p_H^{HE}(\xi))^{1-\sigma}+n^{FE}(A_F^{HE}(\xi))^\sigma(p_F^{HE}(\xi))^{1-\sigma}}\times$$

$$[n^{\tilde{J}E}n^{JE}]\times[\frac{c_J^{JE}c_{\tilde{J}}^{JE}}{(\mathcal{P}^J(p_J^{JE}(\xi),p_{\tilde{J}}^{JE}(\xi);\mathbf{n}^E,a_J^{JE},a_{\tilde{J}}^{JE}))^{\sigma-1}}]$$

$$\{(1+\phi)p_J^{JE}(\xi)\left[\frac{\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}}{\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}}-1\right]+p_{\tilde{J}}^{JE}(\xi)\left[\frac{\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}}{\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}}-1\right]\},$$

which implies $\Delta Rev = 0$ if an only if

$$(1+\phi)p_J^{JE}(\xi)\left[\frac{\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}}{\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}}-1\right]+p_{\tilde{J}}^{JE}(\xi)\left[\frac{\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}}{\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}}-1\right]=0.$$

But substituting in the expressions for $p_J^{JE}(\xi)$ and $p_{\tilde{J}}^{JE}(\xi)$ yields

$$(1+\phi)p_J^{JE}(\xi)\left[\frac{\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}}{\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}}-1\right]+p_{\tilde{J}}^{JE}(\xi)\left[\frac{\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}}{\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}}-1\right]$$

$$=(1+\phi)\lambda\left(\frac{\mathcal{P}^J(p_J^{JE}(\xi),p_{\tilde{J}}^{JE}(\xi);\mathbf{n}^E,a_J^{JE},a_{\tilde{J}}^{JE})}{P^{JE}}\right)^{\left(\frac{\sigma-1}{\sigma}\right)}\times\{\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}-\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}+\frac{A_J^{JE}(\xi)}{\hat{A}_J^{JE}}-\frac{A_{\tilde{J}}^{JE}(\xi)}{\hat{A}_{\tilde{J}}^{JE}}\}$$

$$=0.$$

QED