THE NATURE OF FIRM GROWTH*

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Abstract

About half of all startups fail within five years, and those that survive grow at vastly different speeds. Using Census microdata, we estimate that most of these differences are determined by ex-ante heterogeneity rather than persistent ex-post shocks. Embedding such heterogeneity in a firm dynamics model shows that the presence of ex-ante heterogeneity (i) is a key determinant of the firm size distribution and firm dynamics, (ii) can strongly affect the macroeconomic effects of firm-level frictions, and (iii) plays an important role in the recently documented decline in business dynamism and the apparent disappearance of high-growth startups ("gazelles").

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1 Introduction

There are enormous differences across firms. On the one hand, many startups fail within the first year and most of those that survive do not grow. On the other hand, a small fraction of high-growth startups, so called "gazelles", makes lasting contributions to aggregate job creation and productivity growth (see e.g. Haltiwanger, Jarmin, Kulick, and Miranda, 2016). While firm dynamics have long been recognized as a key determinant of macroeconomic outcomes, little is known about why firm performance is so different or whether this nature of firm growth affects macroeconomic outcomes.

One view in the literature is that, following entry, firms are hit by ex-post shocks to productivity or demand: some startups are lucky and grow into large firms. An alternative view is that there are ex-ante differences across firms: some types of startups are poised for growth, for example due to a highly scalable technology or business idea.[1]

In this paper, we provide empirical evidence on the relative importance of ex-ante and ex-post heterogeneity in shaping firms' growth paths. We then bring this evidence to a structural firm dynamics model and show that the precise nature of firm growth has strong implications for the macroeconomy and the way in which it is affected by firm-level frictions. Since Hopenhayn and Rogerson (1993), a growing literature uses quantitative heterogeneous-firms models to evaluate the micro- and macro-economic effects of policies and/or frictions. Our results demonstrate the importance of accounting carefully not only for the amount of heterogeneity across firms, but also for its transience and for the moment of its inception, i.e. before or after startup.

To establish these results, we make use of the Longitudinal Business Database (LBD), an administrative panel covering nearly all private employers in the United States from 1976 to 2012. Our central piece of empirical evidence is the cross-sectional autocovariance function of business-level employment by age. We thereby take inspiration from the earnings dynamics literature, which has long recognized that autocovariances help to distinguish shocks from deterministic profiles (see e.g. MaCurdy, 1982; Abowd and Card, 1989; Guvenen, 2009; Guvenen and Smith, 2014). To the best of our knowledge, even the basic autocovariance structure of employment by age has not been systematically documented in the firm dynamics literature which, instead,

[1] Another important dimension of heterogeneity, on which we do not focus in this paper, relates to the role of supply versus demand factors. For evidence on this, see e.g. Hottman, Redding, and Weinstein (2016) and Foster, Haltiwanger, and Syverson (2016).

has focused on the age profiles of average size and exit.[2]

We begin the analysis using a reduced-form statistical model of business-level employment, which allows for the possibility that differences across businesses are a result of both ex-ante heterogeneous growth profiles and ex-post shocks. A major benefit of the statistical model is its simplicity, yielding analytical formulas which help to understand the identification of the key parameters. In particular, it makes clear that crucial information about the extent of ex-ante heterogeneity across firms is contained in the long-horizon autocovariances of business-level employment.

Estimation of the statistical model on the autocovariance matrix reveals a key finding of our study: ex-ante heterogeneity accounts for a large share of the cross-sectional dispersion in employment. In the first year after entry, ex-ante heterogeneity accounts for more than ninety percent of the cross-sectional dispersion in employment. More importantly, even after *twenty* years, ex-ante factors still explain about forty percent of the cohort's employment dispersion.

Next, we take the data to a full-blown structural macroeconomic model with firm dynamics in order to answer other important questions which the statistical model cannot address. The structural model follows the tradition of Hopenhayn (1992), Melitz (2003), and Luttmer (2007), and features endogenous entry, exit and general equilibrium forces. Following the statistical model, we introduce a multi-dimensional idiosyncratic process into this framework, which allows not only for persistent and transitory ex-post shocks, but also for heterogeneity in ex-ante growth and survival profiles. We demonstrate that a combination of ex-ante heterogeneity and ex-post shocks is in fact necessary to obtain a good fit with the empirical autocovariance structure.

While our baseline model contains no explicit frictions, we also consider a version with imperfect information, in the spirit of Jovanovic (1982), in which ex-ante heterogeneity is disentangled from ex-post shocks only gradually. In addition, we consider a version in which firms endogenously invest into demand accumulation subject to adjustment costs. Although these extensions could in principle offer a different perspective on the empirical patterns, this turns out not to be the case. In particular, ex-ante differences still emerge as the key source of heterogeneity.

[2]See e.g. Haltiwanger, Jarmin, and Miranda (2013), Hsieh and Klenow (2014) and Akcigit, Alp, and Peters (2017). Cabral and Mata (2003) also document the evolution of the skewness of the size distribution with age.

We estimate the model by matching not only the autocovariance function of employment at the firm-level, but also the average size and exit profiles, conditional on age. We then use this framework for three purposes: (i) to analyze the extent to which the strong "up-or-out" dynamics in the U.S. economy are driven by a purging of low-growth potential firms rather than a reflection of idiosyncratic business risk, (ii) to understand how the presence of ex-ante heterogeneity can change the macroeconomic effects of micro-level frictions and (iii) to shed more light on the timing and sources of the decline in business dynamism observed over the past decades in the U.S. economy.

First, the model suggests that ex-ante heterogeneity is not only an important determinant of size dispersion, but also of the well-documented "up-or-out" dynamics. That is, the fact that many young firms shut down while surviving businesses grow quickly is in large part driven by ex-ante heterogeneity. The impact of this materializes via selection on ex-ante growth profiles: firms with little growth potential exit, allowing firms with high potential to blossom. Associated with these selection effects on ex-ante factors is a large gain in aggregate output. By contrast, ex-post shocks alone create only small selection effects and in turn only minor aggregate gains.

Second, we use the model to highlight that the presence of ex-ante heterogeneity, which is necessary to match the observed autocovariance structure, has stark implications for the behavior of the aggregate economy. Towards this end, we introduce firm-level adjustment frictions into the baseline model. Interestingly, the baseline is not affected much by these micro frictions. This finding sharply contrasts the result we obtain by introducing the same adjustment friction into the same model but with a more standard shock process which does not allow for rich heterogeneity in ex-ante profiles (and which, therefore, fails to match the autocovariance structure of employment), see e.g. Hopenhayn and Rogerson (1993). In that case, the adjustment friction leads to substantial aggregate losses.

The main reason for the stark differences across the two economies lies in the dispersion of firm values. While the two economies have essentially an identical firm size distribution, the baseline economy has a wider dispersion of firm values owing to the presence of differences in long-run steady state sizes across firms. In contrast, firms in the restricted version of the model essentially grow towards the same long-run steady state resulting in a more compressed distribution of firm values. This, in turn, means that the restricted version of the model is characterized by a larger share of "marginal" firms, i.e. those that are indifferent between adjusting or not (or remaining or exiting the economy). An introduction of micro-level frictions then affects a greater mass of firms and results in considerably stronger aggregate effects. Therefore, our results suggest that the presence of ex-ante heterogeneity in growth profiles, and the associated firm selection process, may dwarf the consequences of micro-level distortions like adjustment costs.

Finally, we use the model to understand how the nature of firm growth has changed over time and whether any such changes can shed light on the observed decline in business dynamism in the U.S. economy. Specifically, we re-estimate the model on two sub-samples, splitting our data in half. The results suggest that the prevalence of ex-ante high-growth firms, gazelles, has substantially declined among the population of startups in the late sample compared to earlier years. In addition, we find that gazelles that do start up in the late sample do not grow as rapidly as their counterparts in the early sample. These changes together account for about half of the observed decline in business dynamism, despite the fact that gazelles account for only about 5 percent of all startups.

Supporting evidence for the conclusion that changes in ex-ante factors are a key driver of the observed decline in business dynamism is contained in the cohort structure of the firm size distribution. In particular, we document that the size profile of *cohorts* of startups born before the mid 1980s is considerably steeper compared to startups born thereafter. This stark difference across cohorts of firms is inconsistent with a potential change in ex-post shocks which would, rather, affect all incumbent businesses at the same time. This observation relates to Sedláček and Sterk (2017), who document strong cohort effects in firm-level employment. They however focus on cyclical variations in entry conditions, whereas the change considered here appears permanent.

A major advantage of the Census data used in this paper is that it spans the population of employers and therefore speaks simultaneously to the micro- and the macro-level. An important next step is to investigate empirically what determines the ex-ante and ex-post differences documented in this paper and use this information to further endogenize firm dynamics in structural models. This, however, will require very different data sources with richer micro information relating to e.g. entrepreneurial skills, business plans, financial characteristics or the organizational structures of firms. Existing studies along these lines include Abbring and Campbell (2005) who study bars in Texas, and Campbell and De Nardi (2009) and Hurst and Pugsley (2011) who present survey evidence that many nascent entrepreneurs do not expect their business to grow large.[3] Our results show that the heterogeneity documented in these studies has important implications at the macro level.

The remainder of this paper is organized as follows. Section 2 presents the data, the reduced-form statistical model, and initial estimates of the importance of ex-ante heterogeneity for size dispersion. Section 3 describes the structural firm dynamics model and the parametrization procedure. Sections 4 to 6 then present the main results. Finally, Section 7 concludes.

2 Evidence from a statistical model

This section takes the first step in analyzing the importance of ex-ante heterogeneity in driving observed differences in employment across firms.[4] Using a statistical model, we estimate the extent to which cross-sectional variation in employment is driven by ex-ante heterogeneity and to what extent it results from ex-post shocks. We begin by describing our data set and the central piece of empirical evidence used in the estimation: the autocovariance function of logged employment, at both the establishment- and firm-level. The simplicity of the statistical model allows us to show analytically how all the relevant model parameters map into the autocovariance function, shedding light on the identification of ex-ante versus ex-post heterogeneity.

2.1 Data

The analysis is based on administrative micro data on employment in the United States, taken from the Census Longitudinal Business Database (LBD). The data cover almost the entire population of employers over the period between 1979 and 2012. We construct a panel of log employment at the establishment- and firm-level

[3]Schoar (2010) makes a distinction between "subsistence" and "transformational" entrepreneurship in this regard. Guzman and Stern (2015) and Belenzon, Chatterji, and Daley (2017) also show that firm growth is partly predictable based on observable characteristics at the time of startup.

[4]See DeBacker, Panousi, and Ramnath (2018) for an analysis of household income risk from owning non-corporate private businesses.

in the year of startup (age zero) up to age nineteen.[5,6] Prior to the analysis, we take out a fixed effect for the birth year of the establishment (or firm) and for its industry classification at the 6-digit level. In order to streamline the discussion, we will use the term "business" whenever we refer to both establishments and firms.

2.2 The autocovariance structure of employment

Figure 1 presents our main piece of empirical evidence: the cross-sectional autocovariance structure of log employment, conditional on age (a). In order to understand this structure more easily, we present the autocovariances in terms of standard deviations (left panels) and autocorrelations (right panels). The figure presents this information for both establishments (top panels), and for firms (bottom panels). Finally, since businesses may exit at any age, we display patterns for a balanced panel (solid line) that includes only businesses that survive for at least 20 years and for an unbalanced panel (dashed line) that includes all businesses in our data set.[7] Interestingly, business exit affects essentially only the cross-sectional employment dispersion by age; the autocorrelations are remarkably similar across the balanced and unbalanced panels.

Let us first focus on the cross-sectional standard deviations by age, shown in the left panels. Standard deviations are between 1 and 1.4 log points for both establishments and firms, indicating large size differences even at young ages. Also, the cross-sectional dispersion generally increases with age and this is true for both the balanced and unbalanced panels. The latter indicates that the observed fanning out of the size distribution with age is not purely driven by selective exit of certain businesses.[8]

The right panels of Figure 1 depict the cross-sectional correlations of logged employment between age a and an earlier age $h < a$. Keeping h fixed, the autocorre-

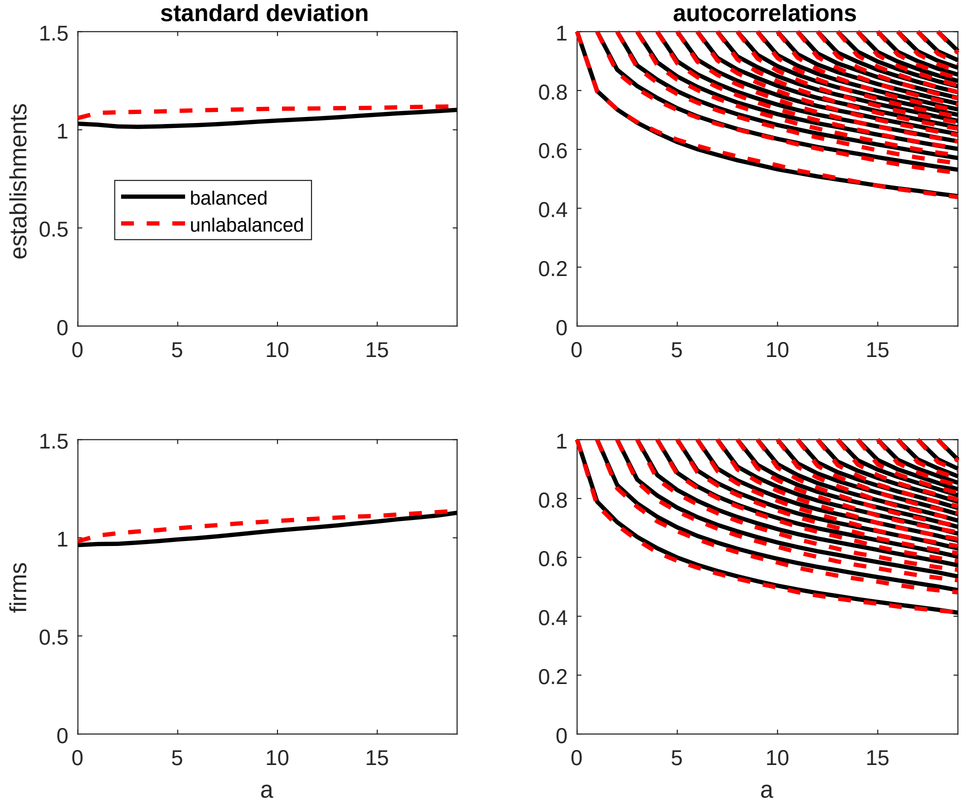
[5]Establishments are the physical units of a firm, located at a specific addresses. A firm can consist of one or multiple establishments. The data are a snapshot taken in the month of March of each year. The age of an establishment is computed as the current year, minus the first year an establishment came into existence. The age of a firm is computed as the age of its oldest establishment.

[6]For brevity, we omit a full description of the panel construction from the LBD microdata from the main text. Please refer to Appendix A for a detailed description of the LBD, the establishment- and firm-level longitudinal links and the construction of the autocovariance matrix.

[7]We focus primarily on the balanced panel of firms for our main results, although we present both here. In Appendix A.2 we describe in more detail the differences in their measurement as well as present the full autocovariance matrices for both panels.

[8]The exception to this pattern is the flat age profile of cross-sectional dispersion for establishments below age five in the balanced panel.

Figure 1: Standard deviations and autocorrelations of log employment by age



Note: The left panels show cross-sectional standard deviations of log employment by age (a) for establishments (top left panel) and firms (bottom left panel). The right panels show cross-sectional correlations of log employment between ages a and age $h \leq a$ for establishments (top right panel) and firms (bottom right panel). "Balanced" refers to a panel of establishments (firms) which survived at least up to age 19, while "unbalanced" refers to a panel of all establishments (firms).

lations decline with age a. For instance, while the autocorrelation between logged employment at ages zero and ten is 0.55, the autocorrelation between ages zero and nineteen is 0.44. Importantly, the long-horizon autocorrelations appear to stabilize at relatively high levels.

On the other hand, for a fixed lag length $a-h$, the autocorrelations are increasing in age. For instance, the correlation of log employment between age zero and age nine is 0.56, whereas the corresponding correlation between age ten and nineteen is 0.73. These empirical patterns contain key information on the relative importance of ex-ante heterogeneity and ex-post shocks, as we will discuss below in detail.

2.3 Employment process

To understand what we can learn from the autocovariances about the importance of ex-ante versus ex-post heterogeneity, we now consider a statistical model of employment which allows for both sources of heterogeneity. The model nests as special cases reduced-form representations of several prominent structural firm dynamics models, such as those of Hopenhayn and Rogerson (1993) and Melitz (2003), while at the same time being flexible enough to fit the observed autocovariance structure well. Appendix B.3 estimates numerous alternative model specifications (including conventional panel data models) showing that our specification, which is grounded in existing firm dynamics models, strikes a balance between model fit and parsimony.

Our baseline employment process features deterministic "ex-ante" profile heterogeneity and "ex-post" shocks. Let $n_{i,a}$ be the employment level of an individual business i at age a and consider the following process for this variable:

$$\ln n_{i,a} = \underbrace{u_{i,a} + v_{i,a}}_{\text{ex-ante component}} + \underbrace{w_{i,a} + z_{i,a}}_{\text{ex-post component}} \tag{1}$$

where

$$\begin{aligned}
u_{i,a} &= \rho_u u_{i,a-1} + \theta_i, && u_{i,-1} \sim iid(\mu_{\tilde{u}}, \sigma^2_{\tilde{u}}), && \theta_i \sim iid(\mu_\theta, \sigma^2_\theta), && |\rho_u| \leq 1,\\
v_{i,a} &= \rho_v v_{i,a-1}, && v_{i,-1} \sim iid(\mu_{\tilde{v}}, \sigma^2_{\tilde{v}}), && && |\rho_v| \leq 1,\\
w_{i,a} &= \rho_w w_{i,a-1} + \varepsilon_{i,a}, && w_{i,-1} = 0, && \varepsilon_{i,a} \sim iid(0, \sigma^2_\varepsilon), && |\rho_w| \leq 1,\\
z_{i,a} &\sim iid(0, \sigma^2_z).
\end{aligned}$$

Here, all shocks are drawn from distributions which are i.i.d. across time and across businesses, and we let μ denote a mean and σ^2 a variance.

In the above process, $\ln n^{EXA}_{i,a} = u_{i,a} + v_{i,a}$ captures the *ex-ante* profile, which is governed by three business-specific parameters that are drawn independently just prior to startup, i.e. at age $a = -1$. The parameter θ_i is a permanent component which is allowed to accumulate gradually with age at a speed governed by ρ_u. The second and third parameter, $u_{i,-1}$ and $v_{i,-1}$, represent two initial conditions. The former allows for the possibility that the path of the ex-ante component starts away from zero. The latter, which is allowed to die out at its own speed ρ_v, enables the curvature of the ex-ante profile to vary over the lifecycle.

Note that this relatively parsimonious specification of ex-ante growth profiles allows for rich heterogeneity. In particular, if $\rho_u < 1$ then the ex-ante component converges to a long-run "steady state" level of $\ln n_{i,\infty}^{EXA} = \theta_i/(1-\rho_u)$. Since this level differs across businesses, the process admits heterogeneity in long-run steady states. Moreover, since initial conditions differ across businesses, we allow for heterogeneity in the paths from initial employment towards the steady states. Finally, since the process includes two separate initial conditions, each with their own persistence parameter, it allows businesses to gravitate towards their steady-state levels at different speeds. The implied ex-ante growth profiles therefore allow for rich heterogeneity.[9]

The *ex-post* shocks enter the model via a second component, $\ln n_{i,a}^{EXP} = w_{i,a} + z_{i,a}$. The process for the ex-post component is constructed such that its expected profile is flat and zero so that it does not capture any of the heterogeneity in ex-ante profiles. Specifically, $w_{i,a}$ captures persistent ex-post shocks, and is modelled as an autoregressive process of order one, with i.i.d. innovations given by $\varepsilon_{i,a}$ and a persistence parameter denoted by $|\rho_w| \leq 1$. Notice that this formulation allows $w_{i,a}$ to follow a random walk, in which case each $\varepsilon_{i,a}$ may be interpreted as a growth rate shock. Because the u and v terms are meant to capture the entire ex-ante profile, we normalize the initial condition of the persistent ex-post shocks to $w_{i,-1} = 0$.

As described earlier, the process above nests various specifications commonly used in the firm dynamics literature to model firm-level shocks. For example, Hopenhayn and Rogerson (1993) assume an AR(1) for firm-level productivity, with a common constant across firms and heterogeneous initial draws. In their baseline model without distortions, the firm-level shocks map one-for-one into employment. We obtain their specification by setting $\rho_u = \sigma_u = \sigma_\theta = \sigma_z = 0$ and $\rho_v = \rho_w$. By contrast, Melitz (2003) allows, like us, for heterogeneity in steady-state levels, but abstracts from ex-post shocks and assumes that steady states are immediately reached. We obtain his process by setting $\sigma_u = \sigma_v = \sigma_z = 0$ and $\rho_u = 0$, which implies that $\ln n_{i,a} = \theta_i$ at any age. Similarly, we obtain the dynamics in Bartelsman, Haltiwanger, and Scarpetta (2013) under the same restrictions, but allowing for $\sigma_z > 0$.[10] Our baseline

[9] By not restricting ρ_u and ρ_v to lay strictly inside the unit circle, we allow in principle for unit roots in the u and v terms. In this case, rather than an ex-ante profile towards some expected long-run size, the ex-ante terms would instead characterize heterogeneous growth rates from some initial size.

[10] Our process also nests specifications commonly assumed in the econometrics literature on dynamic panel data models, see for example Arellano and Bond (1991). This literature typically assumes an autoregressive process, like Hopenhayn and Rogerson (1993), but allows for heterogeneity in the constant θ_i and thus in steady-state levels. Commonly, however, θ_i is differenced out and hence no estimate is provided for σ_θ, a key parameter in our analysis. Moreover, the panel data econometrics literature commonly assumes that $\rho_u = \rho_v = \rho_w$. In our application, it turns out that

process also aligns with models with richer heterogeneity in ex-ante profiles and/or ex-post shocks, as proposed by for example Luttmer (2011) and Arkolakis (2016) and Arkolakis, Papageorgiou, and Timoshenko (2018).[11]

2.4 Estimation strategy and results

In what follows we first discuss several key properties of the model-implied autocovariance function. Next, we present the estimation results and show how our baseline model fits the data. Finally, we provide intuition about the identification of the model parameters and how each of the model components maps into the empirical patterns.

Properties of the autocovariance function. To explain our empirical strategy, we first demonstrate the usefulness of the autocovariance matrix in quantifying the role of ex-ante versus ex-post heterogeneity. All key parameters of the statistical model can be identified from the autocovariance matrix. For any pair of ages, the model-implied cross-sectional covariance of employment can be written as closed-form expression of the model parameters. The covariance of employment of a business at age a and at age $h = a - j$, where $0 \leq j \leq a$ is the lag length, can be expressed as:

$$\mathrm{Cov}\left[\log n_{i,a}, \log n_{i,a-j}\right] = \underbrace{\left(\sum_{k=0}^{a} \rho_u^k\right)\left(\sum_{k=0}^{a-j} \rho_u^k\right)\sigma_\theta^2 + \rho_u^{2(a+1)-j}\sigma_{\tilde{u}}^2 + \rho_v^{2(a+1)-j}\sigma_{\tilde{v}}^2}_{\text{ex-ante components}} \tag{2}$$

$$+ \underbrace{\sigma_\varepsilon^2 \rho^j \sum_{k=0}^{a-j} \rho_w^{2k} + \sigma_z^2 \mathbf{1}_{j=0}}_{\text{ex-post components}}.$$

This result is derived in Appendix B.1. The autocovariance function is a non-linear function of the persistence and variance parameters of the components of the underlying process.[12] We can estimate the parameters of this process by matching

this assumption is too restrictive to provide a good fit of the observed autocovariance matrix (see appendix B.3). Our results thus caution against the use of standard panel data estimators when applied to employment dynamics of young businesses.

[11]For further discussion, please refer to Appendix B.3 where we consider a number of alternative statistical models both as special cases and further generalizations of equation (2).

[12]Note that the mean parameters μ_θ, $\mu_{\tilde{u}}$ and $\mu_{\tilde{v}}$ are not identified by the autocovariance function. These parameters, however, are also not needed to quantify the relative importance of ex-ante versus

the model's autocovariance structure to its empirical counterpart.

To understand the identification, it is useful to consider the case where $|\rho_u|, |\rho_v|, |\rho_w| < 1$ so that the process is covariance stationary in the long run. Then, at an infinite lag length, i.e. letting the age a approach infinity keeping the initial age $h = a - j$ fixed, the autocovariance is:

$$\lim_{a\to\infty} \text{Cov}\left[\ln n_{i,a}, \ln n_{i,h}\right] = \frac{1-\rho_u^{h+1}}{\left(1-\rho_u\right)^2}\sigma_\theta^2.$$

When σ_θ equals zero, i.e. when there is no heterogeneity in steady-state levels, the autocovariance is zero. Thus, long-horizon autocovariances contain valuable information on the presence of ex-ante heterogeneity in steady-state levels. In Figure 1, autocorrelations appear to stabilize at long lag lengths, i.e. at high levels of a given $h = a - j$, suggesting that such heterogeneity is indeed a feature of the data. More intuition on the identification of model parameters is presented below.

Parameter estimates and model fit. We formally estimate the parameters of the process using a minimum distance procedure, as proposed by Chamberlain (1984). Specifically, we minimize the sum of squared deviations of the upper triangular parts of the autocovariance matrix implied by the process, from its counterpart in the data.[13] Because the size of the LBD ensures that each element of the empirical autocovariance matrix is precisely estimated, we assign equal weights to all elements in the estimation procedure. Throughout, our results apply to the balanced panel data set, although they are similar using the unbalanced panel.[14]

Figure 2 shows that the model fit is very good for both establishments and firms, correctly capturing the convexly declining pattern of the autocovariances in the lag length, given the initial age h, and the concavely increasing pattern in age given the lag length $j > 0$. Finally, the model fits the non-monotonic pattern in cross-sectional dispersion by age.

The corresponding parameter estimates are shown in Table 1. A key feature of our baseline process is the presence of dispersion in long-run steady states, governed by σ_θ and ρ_u. The point estimates imply a standard deviation of long-run steady-

ex-post heterogeneity.

[13]For brevity, we defer a detailed discussion of the estimation procedure to Appendix B.2.

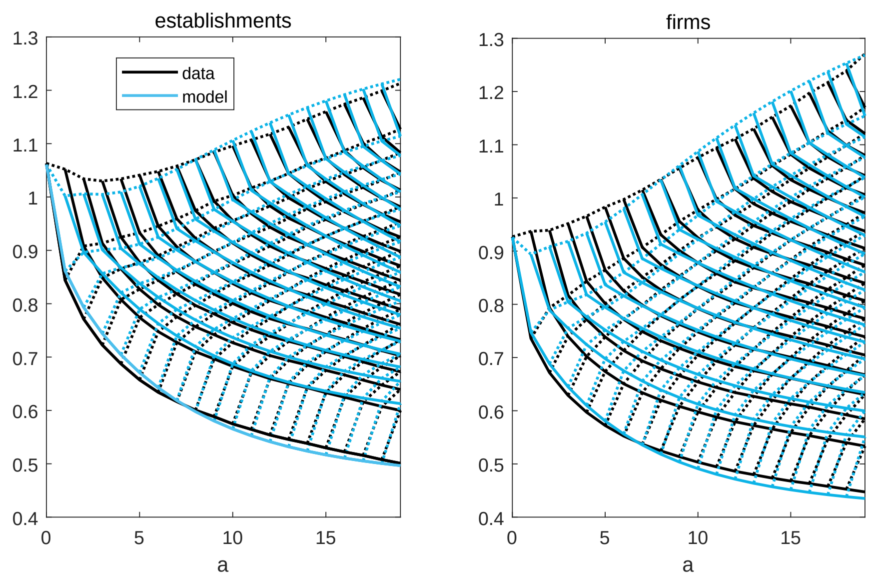
[14]For reference, we include the estimated parameters using the unbalanced panel in Appendix Table B.1 panel A.

Table 1: Parameter estimates from reduced-form model

	ρ_u	ρ_v	ρ_w	σ_θ	$\sigma_{\widetilde{u}}$	$\sigma_{\widetilde{v}}$	σ_ϵ	σ_z
Estabs	0.206	0.842	0.949	0.603	2.046	0.738	0.255	0.262
	(0.002)	(0.001)	(0.001)	(0.001)	(0.017)	(0.002)	(0.001)	(0.001)
Firms	0.218	0.832	0.963	0.555	1.743	0.695	0.255	0.272
	(0.002)	(0.001)	(0.001)	(0.002)	(0.015)	(0.002)	(0.001)	(0.001)

Note: Equally-weighted minimum distance estimates of Equation (2) for both establishments and firms using the balanced panel. See Appendix Table B.1 panel A for estimates using unbalanced panel.

Figure 2: Autocovariance matrices: statistical models versus data



Note: Autocovariance of log employment between age $a = h + j$ and age $h \leq a$ in the data, and in the baseline model. Results are shown for firms and establishments using the balanced panel.

state employment levels of 0.76 for establishments and 0.71 for firms. These values are substantial when considering that the overall cross-sectional dispersion of twenty year old businesses is about 1.3 (see Figure 1). Note also that the data reject the presence of a unit root process, in our sample. Such violations of Gibrat's law have been documented in the literature, in particular among younger firms, see e.g. Haltiwanger, Jarmin, and Miranda (2013).

Mapping model components to the data. We now discuss in more detail the role of each of the model's components in generating the shape of the autocovariance function necessary to match the data. This will also provide further intuition about how the model parameters are identified by the information contained in the autocovariance matrix. We do so by estimating four restricted versions of our baseline model and considering their empirical fit, depicted in Figure 3.

Restricted models I and II (top row) illustrate, respectively, why a combination of permanent ex-ante heterogeneity and ex-post shocks is needed to match the data. In restricted model I, we show a popular specification in the literature on firm dynamics models, which essentially amounts to an AR(1) process with hetreogeneous initial draws but without heterogeneity in long-run steady states.[15] We achieve this by imposing $\rho_v = \rho_w$ and $\rho_u = \sigma_\theta = \sigma_{\widetilde{u}} = \sigma_z = 0$, and re-estimating ρ_w, σ_w and $\sigma_{\widetilde{v}}$. Under this specification, the high long-run autocovariances demand very persistent ex-post shocks. This results in the model-implied autocorrelations being almost linear in age, which conflicts with the non-linear patterns in the data. The presence of ex-ante heterogeneity thus relaxes the need for.[16]

In restricted model II, we shut down all ex-post shocks, allowing only for heterogeneous ex-ante profiles (with only one initial condition). We do so by imposing $\rho_w = \rho_v = \sigma_\epsilon = \sigma_{\widetilde{v}} = \sigma_z = 0$, and re-estimating ρ_w, σ_w and $\sigma_{\widetilde{v}}$. This version fails to match the pattern of increasing employment dispersion with firm age, as *observed* in the data.

Restricted model III illustrates why *both* transitory ex-ante components, u and v, are required to match the data. This version is the same as our baseline except that we set $\rho_v = \sigma_{\widetilde{v}} = 0$, and we re-estimate the remaining parameters. The presence of v enables the model to match the curvature of the autocovariance function, as it allows for different speeds of convergence to the long-run steady state employment levels.

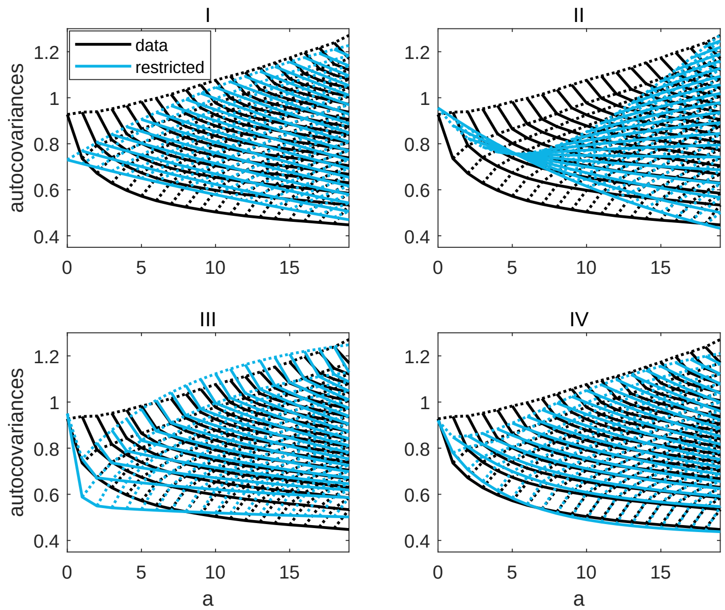
Finally, restricted model IV explores the role of the iid ex-post shock z. In this version, we re-estimate the model imposing $\sigma_z = 0$. The presence of z somewhat improves the fit of the model, by giving an extra kick to the dispersion of employment across firms, in line with the data, but without distorting the higher-order autocovariances.

While our baseline model provides a very good fit to the data, we estimate several

[15]For example, Hopenhayn and Rogerson (1993) consider this process for productivity.

[16]Recent work by Gabaix (2009) and Luttmer (2011) suggests that in order to generate a power-law ergodic firm size distribution that is close to the data, a combination of permanent and persistent shocks may be necessary. This points to a potential trade-off between matching early life-cycle dynamics, as summarized by our autocovariance function, and long-run patterns such as the ergodic firm-size distribution. See Appendix B.3 for estimation results from a variant of our model with a unit root.

Figure 3: Autocovariance matrices: restricted models



Note: Autocovariance of log employment between age $a = h + j$ and age $h \leq a$ in the baseline (for the balanced firm panel estimates), and in the four restricted models. In Model I ρ_w, σ_w and $\sigma_{\widetilde{v}}$ are estimated, while imposing $\rho_v = \rho_w$ and $\rho_u = \sigma_\theta = \sigma_{\widetilde{u}} = \sigma_z = 0$. In Model II ρ_u, σ_θ and $\sigma_{\widetilde{u}}$ are estimated, while imposing $\rho_w = \rho_v = \sigma_\epsilon = \sigma_{\widetilde{v}} = \sigma_z = 0$. Model III is the baseline with the restriction that $\rho_v = \sigma_{\widetilde{v}} = 0$. Model IV is the baseline with the restriction that $\sigma_z = 0$.

extensions and alternatives in Appendix B.3. These include e.g. a generalized AR(1) process with a unit root similar to specifications in Gabaix (2009) or Luttmer (2011), an AR process with age-dependent dispersion of ex-post shocks, and several dynamic panel data models akin to models in Arellano and Bond (1991), including a panel AR(2) model similar to the specification in Lee and Mukoyama (2015). Importantly, none of the alternatives improve on model fit without introducing more parameters, and our conclusions about the importance of ex-ante heterogeneity remain unchanged across specifications.

2.5 The importance of ex-ante and ex-post heterogeneity

With the estimated model in hand, we can quantify the relative importance of ex-ante profiles and ex-post shocks for the cross-section dispersion in employment. This is done based on Equation (2). With the lag length j set to zero, this equation provides a decomposition of the variance of size (log employment), at any given age a, into the contributions of the ex-ante and ex-post components. Figure 4 plots the fraction of the total variance that is accounted for by the ex-ante component. Thick lines denote the age groups used in the estimation, i.e. age zero to nineteen, whereas thin lines represent an extrapolation for businesses at age 20 or above using the point estimates.[17]

Figure 4 shows that for businesses in the year of startup (age zero) the ex-ante component accounts for about 85 percent of the cross-sectional variance in size. The remainder is due to ex-post shocks that materialized in the first year. Considering older age groups, the contribution of ex-ante heterogeneity declines, but remains high. At age twenty, ex-ante factors account for 47 percent of the size variance among establishments, and around 40 among firms. In the data, more than seventy percent of the businesses are twenty years old or younger. Our results show that, among these businesses, ex-ante factors are a key determinant of size. Increasing age towards infinity, the contribution of ex-ante heterogeneity stabilizes at around 45 percent for establishments and 35 percent for firms. Therefore, even among very old businesses ex-ante factors contribute to a large chunk of the dispersion in size.[18]

3 Structural model

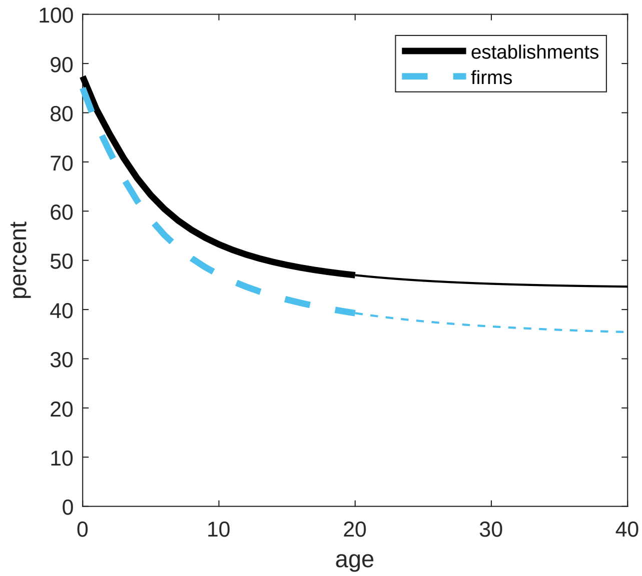
To learn about the implications of our findings for the aggregate economy, in this section we estimate a structural macroeconomic model with firm dynamics. This framework has several advantages relative to the statistical model in Section 2. First, the structural model accounts for selective entry and exit. Second, the structural model allows us to compute aggregates. Third, micro-founding firm decisions allows us to analyze how various frictions (e.g. imperfect information or adjustment costs) affect the observed patterns in the data.

We use the estimated structural model for three distinct purposes. First, we analyze to what extent the strong "up-or-out" dynamics in the U.S. economy are driven by the purging of low-growth-potential businesses or to what extent they reflect id-

[17] We have also computed confidence bands for this decomposition, but these are extremely narrow due to the very large number of data points used in the estimation and the resulting high precision of our point estimates.

[18] Appendix B.3 provides results for a wide range of alternative models showing the robustness of our results. Importantly, regardless of the specified process, ex-ante characteristics explain a significant fraction of early life-cycle employment dispersion. In addition, for the processes that also match the autocovariance structure well, they attribute nearly identical shares of long-run variance to ex-ante characteristics as our baseline model.

Figure 4: Contribution of ex-ante heterogeneity to cross-sectional employment dispersion



Note: Contribution of the ex-ante component, $\ln n_{i,a}^{EXA}$, to the cross-sectional variance of log employment, by age. Thin lines denote age groups not directly used in the estimation. The decomposition is based on Equation (2) with $j = 0$.

iosyncratic business risk. Second, we show that the presence of ex-ante heterogeneity in growth profiles can dramatically change the impact of distortions at the firm level on the macro economy. Finally, we use our framework to provide new insights on the timing and sources of the decline in business dynamism observed over the past decades.[19]

3.1 The model

We consider a closed general equilibrium economy with heterogeneous firms and endogenous entry and exit, as in Hopenhayn and Rogerson (1993). Following Melitz (2003) and others, each firm is monopolistically competitive and faces a demand schedule which is downward-sloping in its price. To model heterogeneity across firms, we embed an idiosyncratic process with the same structure as in Section 2, thereby allowing for differences in both ex-ante profiles and ex-post shocks.

[19]Throughout the analysis we report results for firms. Estimates for establishments are shown in Appendix C.8.

Households. The economy is populated by an infinitely-lived representative household who owns the firms and supplies a fixed amount of labor in each period, denoted by $\overline{N}$. Household preferences are given by $\sum_{t=0}^{\infty} \beta^t C_t$, where $\beta \in (0,1)$ is the discount factor. C_t is a Dixit-Stiglitz basket of differentiated goods given by:

$$C_t = \left(\int_{i \in \Omega_t} \varphi_{i,t}^{\frac{1}{\eta}} c_{i,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}},$$

where Ω_t is the measure of goods available in period t, $c_{i,t}$ denotes consumption of good i, η is the elasticity of substitution between goods, and $\varphi_{i,t} \in [0,\infty)$ is a stochastic and time-varying demand fundamental specific to good i. We consider a stationary economy from now on and simplify notation by dropping time subscripts.

The household's budget constraint is given by $\int_{i \in \Omega} p_i c_i = W\overline{N} + \Pi$, where p_i denotes the price of good i, W denotes the nominal wage and Π denotes firm profits. Utility maximization implies a demand schedule given by $c_i = \varphi_i \left(p_i/P\right)^{-\eta} C$, where P is a price index given $P \equiv \left(\int_{i \in \Omega} \varphi_i p_i^{1-\eta}\right)^{\frac{1}{1-\eta}}$, so that total expenditure satisfies $PC = \int_{i \in \Omega} p_i c_i$.

Incumbent firms. There is an endogenous measure, Ω, of incumbent firms, each of which produces a unique good. Firms are labeled by the goods they produce $i \in \Omega$. The production technology of firm i is given by $y_i + f = n_i$, where y_i is the output of the firm, n_i is the amount of labor input (employment) and f is a fixed cost of operation common to all firms, denominated in units of labor. It follows that firms face the following profit function:

$$\pi_i = p_i y_i - W n_i.$$

Additionally, given the market structure, each firm faces a demand constraint given by

$$y_i = \varphi_i \left(p_i/P\right)^{-\eta} C, \tag{3}$$

which is the demand schedule of the household combined with anticipated clearing of goods markets, which implies $c_i = y_i$.

At the beginning of each period, a firm may be forced to exit exogenously with probability $\delta \in (0,1)$. If this does not occur, the firm has the opportunity to exit endogenously and avoid paying the fixed cost. If the firm chooses to remain in operation, it must pay the fixed cost and in turn it learns its demand fundamental φ_i. Given its production technology and demand function, the firm sets its price p_i (and implicitly y_i, n_i and π_i) to maximize the net present value of profits. The price-setting problem is static and the firm sets prices as a constant markup over marginal costs W:

$$p_i = \frac{\eta}{\eta - 1} W.$$

We let labor be the numeraire so that $W = 1$, and define the real wage $w \equiv W/P$ as the price of labor in terms of the Dixit-Stiglitz consumption basket C. Using this result, we can express profits as $\pi_i = \varphi_i w^{-\eta} C \chi - f$, where $\chi \equiv \frac{(\eta-1)^{\eta-1}}{\eta^\eta}$, and labor demand as $n_i = \varphi_i \left(\frac{\eta}{\eta-1}\right)^{-\eta} w^{-\eta} C + f$. Note that fluctuations in the demand fundamental directly map into the firms' employment levels.

The demand fundamental φ_i is a function of an underlying exogenous Markov state vector, denoted $\mathbf{s}_i$. The value of a firm at the moment the exit decision is taken, denoted V, can now be expressed as:

$$V(\mathbf{s}_i) = \max \left\{ \mathbb{E}\left[\pi(\mathbf{s}_i') + \beta (1 - \delta) V(\mathbf{s}_i') \middle| \mathbf{s}_i \right], 0 \right\}.$$

In the above equation $\mathbf{s}_i'$ denotes the value of the state realized after the continuation decision. Accordingly, we can express the profit, output, employment and exit policies as $\pi_i = \pi(\mathbf{s}_i')$, $y_i = y(\mathbf{s}_i')$, $n_i = n(\mathbf{s}_i')$, and $x_i = x(\mathbf{s}_i)$, respectively.

Firm entry. Firm entry is endogenous and requires paying an entry cost f^e, denominated in units of labor. After paying the entry cost at the beginning of a period, the firm observes its initial level of $\mathbf{s}_i$, at which point it becomes an incumbent. Free entry implies the following condition:

$$wPf^e = \int V(\mathbf{s}) G(d\mathbf{s}),$$

where G is the distribution from which the initial levels of $\mathbf{s}_i$ are drawn.

Aggregation and market clearing. Let $\mu(\mathbf{S})$ be the measure of firms in $\mathbf{S} \in \mathcal{S}$, where is $\mathcal{S}$ is the Borel $\sigma-$algebra generated by $\mathbf{s}$. Given the exit policy, $\mu(\mathbf{S})$ satisfies:

$$\mu(\mathbf{S}') = \int [1 - x(\mathbf{s})] F(\mathbf{S}'|\mathbf{s}) [\mu(d\mathbf{s}) + M^e G(d\mathbf{s})],$$

where M^e denotes the measure of entrants and $F(\mathbf{S}'|\, s)$ is consistent with the transition law for $\mathbf{s}_i$. The total measure of active firms is given by:

$$\Omega = \int \mu(d\mathbf{s}).$$

Labor market clearing implies that total labor supply equals total labor used for production, for the fixed cost, and for the entry cost:

$$\bar{N} = \int y(\mathbf{s}') \mu(d\mathbf{s}') + \int f [1 - x(\mathbf{s})] [\mu(d\mathbf{s}) + M^e G(d\mathbf{s})] + M^e f^e.$$

Stochastic driving process. In line with the reduced-form analysis we allow for the following exogenous idiosyncratic process for the demand fundamental $\varphi_{i,t}$:

$$\begin{array}{lclllll}
\ln \varphi_{i,t} & = & u_{i,t} + v_{i,t} + w_{i,t} + z_{i,t} & & & \\
u_{i,t} & = & \rho_u u_{i,t-1} + \theta_i, & u_{i,-1} \sim iid(\mu_{\tilde{u}}, \sigma^2_{\tilde{u}}) & \theta_i \sim iid(\mu_\theta, \sigma^2_\theta) & |\rho_u| < 1 \\
v_{i,t} & = & \rho_v v_{i,t-1}, & v_{i,-1} \sim iid(\mu_{\tilde{v}}, \sigma^2_{\tilde{v}}) & & |\rho_v| < 1 \\
w_{i,t} & = & \rho_w w_{i,t} + \varepsilon_{i,t}, & w_{i,-1} = 0 & \varepsilon_{i,a} \sim iid(0, \sigma^2_\varepsilon) & |\rho_w| < 1 \\
z_{i,t} & \sim & iid(0, \sigma^2_z), & & &
\end{array}$$

where we re-introduced time indices for clarity. Note that the firm-level state is given by $\mathbf{s}_{i,t} = [u_{i,t}, v_{i,t}, w_{i,t}, z_{i,t}]$. The above process implies that the level of demand faced by a firm is determined by both a idiosyncratic ex-ante profile, captured by $u_{i,t}$ and $v_{i,t}$, as well as ex-post shocks, which enter via $w_{i,t}$ and $z_{i,t}$.

In the model, the ex-ante component reflects the profile for product demand expected immediately after entry, but prior to observing any ex-post shocks. In the baseline specification, we assume that the ex-ante components are observable immediately after paying the entry cost, f_e. By contrast, each period's ex-post demand shocks are observable only after paying the operational cost, f, in that period. Therefore, in this frictionless model employment is based on the current level of demand, while the decision to exit takes into account the entire future demand path, which depends on both ex-ante and ex-post factors. In what follows, we consider extensions to the model that relax the assumptions of perfect information about ex-ante components as well as those of frictionless adjustment.

3.2 Discussion and extensions

An imperfect-information perspective. The baseline model assumes full information, in the sense that firms immediately observe stochastic innovations to the components of the shock process. One may wonder to what extent economic agents, including the firm owners, actually have this much information, and to what extent this affects firms' decisions and the interpretation of the empirical patterns that we have documented.

To investigate these issues, we conduct two exercises, shown in Appendix C.3. First, we document that under the baseline estimates, optimal Bayesian updating enables one to learn about the ex-ante profiles extremely quickly. In fact, most of the uncertainty about ex-ante profiles is resolved in the first year upon entry. Nevertheless, this may still distort firms' decisions. Therefore, as a second exercise we consider a version of the model where firms have imperfect information about their ex-ante profiles, similar to e.g. Jovanovic (1982). While in this version, ex-post shocks are attributed a somewhat larger role, our main conclusions remain unchanged.[20]

Adjustment costs. Ex-post demand shocks are a standard feature of firm dynamics models, since demand conditions may change for various reasons that are beyond the control of the firm. Considering ex-ante heterogeneity across firms also has a strong tradition in the literature. While in certain models ex-ante heterogeneity across firms materialize immediately (see e.g. Melitz, 2003), other studies consider a gradual accumulation of difference, for instance through customer base accumulation (see e.g. Arkolakis, 2016; Luttmer, 2011; Drozd and Nosal, 2012; Gourio and Rudanko, 2014; Perla, 2015).

Our baseline model allows for so-called "passive" accumulation of ex-ante differences through the accumulation of the fixed effect θ_i. Importantly, the parameters

[20] Another interesting possibility is that agents might receive advance information on ex-post shocks, as in the literature on news shocks in macroeconomics, see Beaudry and Portier (2004). If some of the information is already known upon entry, the importance of ex-ante heterogeneity would be even larger than we estimate.

governing this process will be estimated. Nevertheless, in Appendix C.4 we consider endogenous adjustment costs as an alternative. Specifically, in addition to passive demand accumulation, firms can choose to invest into "active" demand accumulation as in e.g. Foster, Haltiwanger, and Syverson (2016). [21] Importantly, while incorporating endogenous adjustment costs affects the parameter estimates, our main results remain essentially unchanged.

The process of firm selection. As in any firm dynamics model with endogenous entry and/or exit, a key channel via which heterogeneity may impact on aggregate outcomes is the process of firm selection. Since we integrate a multi-dimensional idiosyncratic process into the model, selection occurs along several different competing margins. Importantly, there is no one-to-one mapping between a particular value of demand and the survival probability of the respective firm. For example, a currently small and unprofitable startup may survive with high probability if it has sufficiently promising long-run growth potential and only faces poor initial conditions or ex-post shocks. A large part of our analysis is devoted to thoroughly understanding the sources, and (aggregate) consequences, of the process of firm selection.

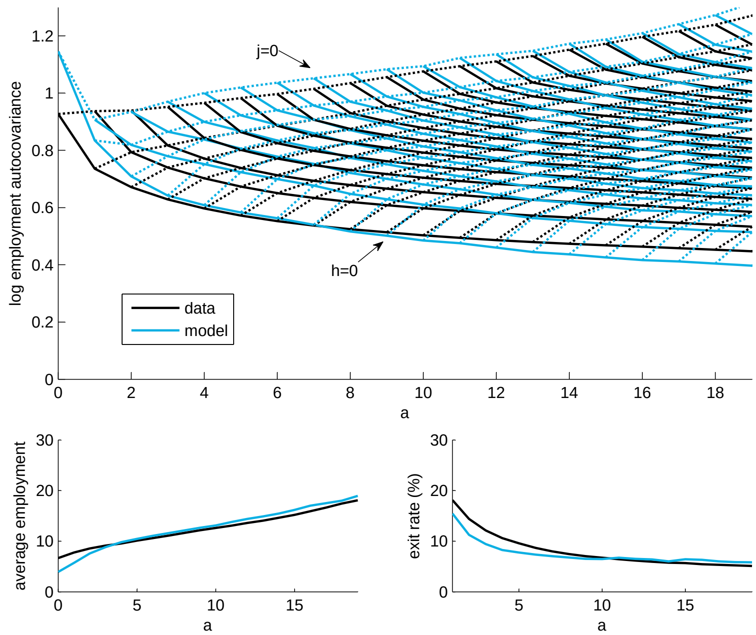
3.3 Parametrization and model fit

We now match the model to our data for firms. Before doing so, we set three parameters a priori, assuming a model period of one year, which corresponds to the frequency of our data. First, the discount factor is set to $\beta = 0.96$, which implies an annual real interest rate of about four percent. Second, we set the elasticity of substitution between goods to $\eta = 6$, which is in the range of values common in the literature. Third, we set the entry cost f_e such that the ratio of the entry cost to the operational fixed cost is $f_e/f = 0.82$, following estimates of Barseghyan and Dicecio (2011).

The remaining parameters are set by matching moments in the data. Details of the numerical solution and simulation procedure are provided in Appendix C.1. Again,

[21] Our baseline model also abstracts from differences in technologies, another form of heterogeneity often considered in the firm dynamics literature. However, given that we match our model to employment data, our model is observationally equivalent to one with heterogeneity in TFP. Moreover, Hottman, Redding, and Weinstein (2016) and Foster, Haltiwanger, and Syverson (2016) have recently investigated the relative importance heterogeneity in demand versus technology. They conclude that demand factors are a major driver of heterogeneity in the data.

Figure 5: Targeted moments: data and structural model (firms)



Notes: Top panel: Autocovariances of log employment between age $a = h + j$ and age $h \leq a$ in the data and the model, for a balanced panel of firms surviving up to at least age $a = 19$. Bottom left panel: Average employment by age a (unbalanced panel). Bottom right panel: exit rate by age a.

we target the upper triangle of the autocovariance matrix of logged employment, by age, for a balanced panel of firms surviving up to at least age nineteen. Now, however, we also target the the age profiles of the exit rate and average size (in an unbalanced panel). In doing so, we assume that all shock innovations are drawn from normal distributions and we normalize the level parameters μ_u and μ_v to zero. In contrast to the reduced-form setup, we further assume that $\rho_v = \rho_w$, which eases the computational burden substantially.[22]

Figure 5 illustrates how the model fits the data. The upper panel shows the autocovariance matrix, while the lower left and right panels show the size and exit profiles by age, respectively. Overall, the model provides a good fit of the three sets of empirical moments (249 altogether), considering that the model consists of only 10 parameters.

Additionally, we consider how the model fits the employment distribution by age

[22]This restriction reduces the number of state variables as firms no longer need to keep track of $w_{i,t}$ and $v_{i,t}$ separately. Table 1 shows that the reduced-form estimates of these persistence parameters are close to each other. Imposing this restriction has only a small cost in model fit, increasing the RMSE from 0.0120 to 0.0171.

Table 2: Parameter values (firms)

parameter		value
		set a priori
β	discount factor	0.96
η	elasticity of substitution	6.00
f^e	entry cost	0.44
		used to target moments
f	fixed cost of operation	0.539
δ	exogenous exit rate	0.041
μ_θ	permanent component θ, mean	-1.762
σ_θ	permanent component θ, st. dev.	1.304
$\sigma_{\widetilde{u}}$	initial condition u_{-1}, st. dev.	1.572
$\sigma_{\widetilde{v}}$	initial condition v_{-1}, st. dev.	1.208
σ_ϵ	transitory shock ϵ, st. dev.	0.307
σ_z	noise shock z, st. dev.	0.203
ρ_u	permanent component, persistence	0.393
ρ_v	transitory component, persistence	0.988

Note: Top three parameters are calibrated as discussed in the main text. The remaining parameters are set such that the model matches the empirical autocovariance of employment and the age profiles of average size and exit rates from age 0 to 19.

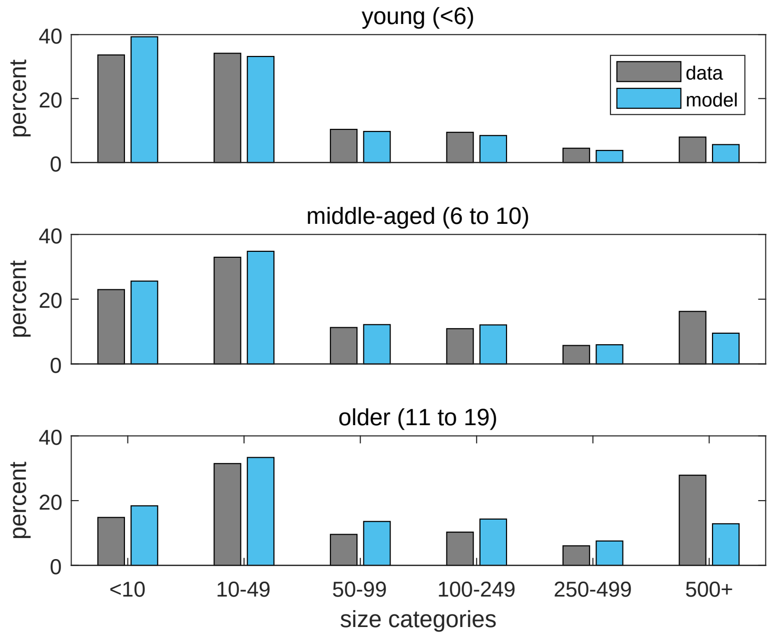
and size, which is not directly targeted. Figure 6 shows employment shares of different age/size bins, in the model and in the data. Overall, the model fits this distribution well.[23]

The associated parameter values for our benchmark model are shown in Table 2. The fixed cost is estimated to be 0.54, which is about half the wage of a single employee. The exogenous exit rate is estimated to be about 4.1 percent. Thus, a substantial fraction of firms exits for reasons unrelated to their fundamentals. However, Figure 5 makes clear that there is also a substantial amount of endogenous exit, as the overall exit rate in the model varies between 15.5 percent at age zero to 5.8 percent at age nineteen.

The remaining parameters are somewhat difficult to interpret individually, especially since the parameter values are for the unconditional distributions, whereas the equilibrium distributions are truncated by selection. However, Appendix C.6 pro-

[23]The only exception is the employment share of very large old firms which is somewhat understated in the model compared to the data. However, Appendix C.5 shows that re-calibrating the model and explicitly targeting the firm size distribution does not change our results.

Figure 6: Employment shares of different age/size bins: model versus data (firms)



Notes: Employment shares by firm age and size (employment). Values are expressed as percentages of total employment in firms between 0 to 19 year old firms, both in the data and the model. Data are obtained from the Business Dynamics Statistics, an aggregated and publicly available version of the LBD over the corresponding time period.

vides an analysis of the sources of identification of the parameters of the process. Importantly, similar to the results in the statistical model, also in the more complex structural model important identifying information about the dispersion of ex-ante differences across firms is obtained from the long-horizon autocovariances.

4 Up-or-out dynamics

We begin our analysis of the structural model by focusing on the well-documented "up-or-out" dynamics of U.S. firms, whereby high rates of firm exit among young businesses are accompanied by strong growth rates of surviving young firms (see e.g. Haltiwanger, Jarmin, and Miranda, 2013). Specifically, we focus on understanding whether firm exit should be thought of as a process that selects firms with high ex-ante potential or rather as a process that reflects idiosyncratic business risk. The advantage of our structural model is that we will also be able to quantify the aggregate gains from such an up-or-out process.

4.1 Selection and the importance of ex-ante heterogeneity

We start by revisiting the importance of ex-ante heterogeneity for the cross-sectional dispersion in employment, but this time we do so within our structural model. The advantage is that we can explicitly account for the role of firm selection which is by construction absent in the statistical model. Defining $\chi \equiv ((\eta-1)/\eta)^{\eta} w^{-\eta} Y$, the employment level of firm i can be expressed as:

$$n_i = \chi \varphi_i^{EXA} \varphi_i^{EXP}, \tag{4}$$

where $\varphi_i^{EXA} = e^{u_i+v_i}$ is the ex-ante component of demand and $\varphi_i^{EXP} = e^{w_i+z_i}$ is the ex-post component. In contrast to the statistical model, however, the ex-ante and ex-post component are no longer orthogonal to each other, due to a correlation induced by endogenous firm selection. This occurs because firms with relatively poor ex-ante conditions can survive only if they were exposed to favorable ex-post shocks and vice versa. Accounting for this correlation, we decompose the variance of logged employment as:

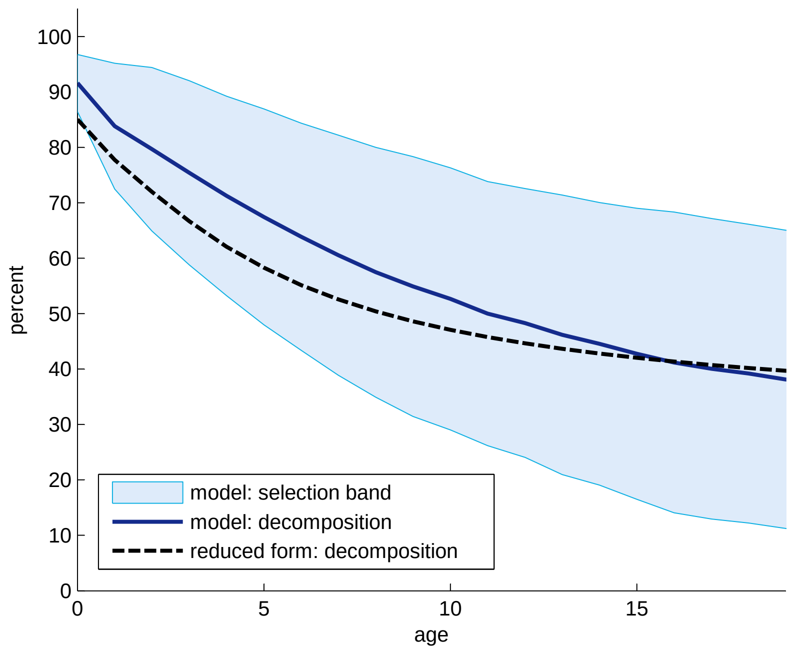
$$\begin{aligned} \text{Var}[\ln n_i] &= \text{Var}\left[\ln \varphi_i^{EXA}\right] + \text{Var}\left[\ln \varphi_i^{EXP}\right] + 2\text{Cov}\left[\ln \varphi_i^{EXA}, \ln \varphi_i^{EXP}\right], \\ &= \text{Cov}\left[\ln \varphi_i^{EXA}, \ln n_i\right] + \text{Cov}\left[\ln \varphi_i^{EXP}, \ln n_i\right]. \end{aligned} \tag{5}$$

In the statistical model, the covariance term $\text{Cov}\left[\ln \varphi_i^{EXA}, \ln \varphi_i^{EXP}\right]$ in the first equality is zero, due to the assumption of independently distributed shocks. However, in the structural model this covariance term tends to be negative. We therefore decompose the variance according to the second equality in Equation (5).[24] Figure 7 depicts the contribution of ex-ante heterogeneity in the structural model (solid line), i.e. $\text{Cov}\left[\ln \varphi_i^{EXA}, \ln n_i\right]/\text{Var}[\ln n_i]$, together with the reduced-form decomposition (dashed line). Both decompositions attribute a similarly large fraction of size dispersion to ex-ante heterogeneity, at any age. The slight differences between the structural and statistical model reflect the fact that the structural model matches more moments and therefore has a somewhat different fit of the autocovariance matrix.

Figure 7 also plots a "selection band" based on the first equality in Equation (5). This band is constructed by attributing, in turn, the covariance term either fully to

[24] Note that when $\text{Cov}\left[\ln \varphi_i^{EXA}, \ln \varphi_i^{EXP}\right] = 0$, it holds that $\text{Var}\left[\ln \varphi_i^{EXA}\right] = \text{Cov}\left[\ln \varphi_i^{EXA}, \ln n_i\right]$ and $\text{Var}\left[\ln \varphi_i^{EXP}\right] = \text{Cov}\left[\ln \varphi_i^{EXP}, \ln n_i\right]$. The decomposition then exactly coincides with the one we used in the reduced-form analysis.

Figure 7: Contribution of ex-ante heterogeneity to cross-sectional employment dispersion (firms)



Note: Contributions of ex-ante heterogeneity to the total cross-sectional variance of log employment by age. "Reduced-form" refers to the estimates from Figure 4, "model: covariance decomposition" is the decomposition based on the second line in Equation (5). The shaded areas ("model: selection band") is constructed based on the first equality in Equation (5) by attributing, in turn, the term $2Cov(\ln \varphi_i^{EXA}, \ln \varphi_i^{EXP})$ fully to the ex-ante component and to the ex-post component.

the ex-ante component or fully to the ex-post component. This gives us a sense of how much selection matters in the model. While the structural model re-establishes our earlier conclusion that ex-ante heterogeneity is a key source of size dispersion, it also highlights the importance of firm selection. The widening selection band indicates that selection has an increasingly important impact on the cross-sectional dispersion of firm size as firms age.

4.2 Firm selection and growth

Having established that firm selection is important for understanding the importance of ex-ante and ex-post shocks, let us now focus on the sources of firm selection. To what extent is firm selection the purging of businesses with low ex-ante growth potential, and to what extent is it a product of idiosyncratic business risk? And what does firm selection along these dimensions imply for firm growth of survivors?

Firm selection. In our structural model, firm selection is a multifaceted process that is affected both by a firm's current fundamentals and by its expectations about how these fundamentals will evolve in the future. This evolution is in turn driven by the relative importance of the ex-ante profile and ex-post shocks.

Therefore, to quantify the importance of ex-ante heterogeneity for exit decisions, we run a counterfactual simulation in which we use the firms' baseline decision rules but we completely shut down ex-post shocks to demand $\epsilon_{i,a} = z_{i,a} = 0$ for all i and a.[25] We do, however, preserve exogenous exit shocks. The resulting average exit rate profile is therefore informative about the extent to which firms shut down because of idiosyncratic risk and to what extent firm exit is driven by ex-ante characteristics. For example, firms may have declining ex-ante demand profiles because of favorable initial condition coupled with a poor long-run growth potential. Such firms will find it economically viable to operate in the initial years, but not later on.

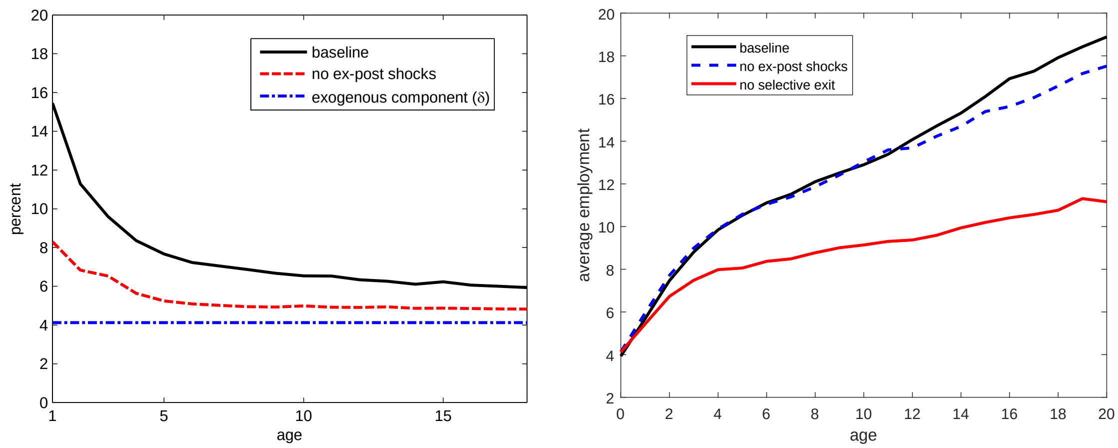
The left panel of Figure 8 shows the age profile of the exit rate in this counterfactual with only ex-ante heterogeneity, together with the exit profile in the baseline model, and the exogenous component of the exit rate, δ. The difference between the baseline exit profile and the constant exogenous exit rate is the endogenous component of the exit rate, i.e. the part that is driven by selection on both ex-ante and ex-post heterogeneity.

As expected, the exit rate is lower without ex-post demand shocks. However, endogenous firm selection remains and the exit rate in the counterfactual declines with age, as in the baseline. Interpreting the difference between the exit rate without ex-post shocks and the exogenous exit rate as the amount of endogenous exit that is driven by selection on ex-ante profiles, the figure shows that selection on ex-ante profiles is quantitatively important. Between 30 and 45 percent of overall endogenous exit is driven by selection on ex-ante profiles.

Firm growth. To examine the effect firm selection has on firm growth, we make use of the same type of counterfactual simulations. First, we consider a counterfactual in which firms are not allowed to shut down endogenously at all, i.e., they exit only at constant exogenous rate δ. The average growth profile of this cohort of startups is depicted in the right panel of Figure 8 as "no selective exit". In contrast to this

[25] This is equivalent to allowing exit to depend only on the ex-ante profile, rather than the ex-ante profile and the moving average of ex-post shocks. These counterfactuals are also partial equilibrium simulations in the sense that we keep aggregate demand fixed.

Figure 8: Exit rates and firm sizes by age



Note: exit rates by age (left panel) in the baseline model, in the counterfactual economy with selection only on ex-ante profile, and in the counterfacual economy with only exogenous exit, i.e. exogenous rate δ. Average size by age (right panel) in the baseline model, in the counterfactual economy without selection and without selection on ex-post shocks. See the main text for a description of these counterfactuals.

counterfactual, the "baseline" employment profile is much steeper. In the absence of firm selection, the average size of twenty year old firms is 50 percent lower compared to the baseline economy. This is exactly because unprofitable (small) firms are allowed to shut down in the baseline economy.

Next, we consider a counterfactual in which we retain the baseline employment policy rules, but we use the exit policy determined only by ex-ante profiles. The resulting average employment profile, depicted as "no ex-post shocks", shows that selection on ex-post shocks accounts for only a very small share of the overall gains from selection.

We thus find that ex-ante heterogeneity is not only an important driver of dispersion in size, but also of the age profiles of exit and average size, especially among younger firms. Therefore, up-or-out dynamics largely reflect the separation of firms with high and low long-run growth potential. An important driver of differences in up-or-out dynamics across countries or different time periods within a country might therefore reflect differences in the types of startups that enter the economy. We will return to this issue in the last section.

4.3 High-growth firms and aggregate gains from selection

Our estimates show a large amount of heterogeneity in ex-ante profiles: some high-potential startups are on steep ex-ante age profiles of demand growth, whereas others are on flat or even downward-sloping age profiles.

High-growth firms. Since at least Birch and Medoff (1994), the literature has emphasized the importance of high-growth firms, labelled as "gazelles", for aggregate job creation and productivity growth. Our structural model allows us to analyze such firms and their contribution to the aggregate economy.

We define gazelles as those startups with an ex-ante projected growth rate of at least 20 percent annually, over the first five years, and an expected employment level of at least 10 workers at some point during their lifetimes.[26]

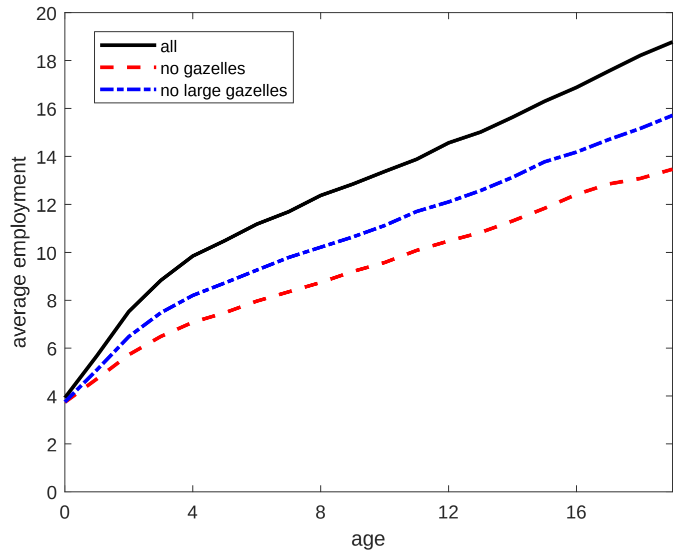
While our definition of gazelles is in line with the literature, we differ from existing studies in an important way: we classify firms according to their *ex-ante* profiles at startup, i.e. those that firms would follow in the absence of ex-post shocks. By contrast, the existing literature has classified firms based on ex-post realizations, since ex-ante profiles are not directly observable. Using ex-post realizations, however, it then follows almost by construction that gazelles contribute disproportionately to aggregate job creation because they are the firms that grew a lot. By contrast, our definition is based on ex-ante profiles and it is therefore less clear whether gazelles end up, ex-post, contributing disproportionately to job creation or not.

The results of our classification show that gazelles account for only 5.4 percent of all startups. However, their impact on the average size profile, and in turn on aggregate outcomes, is much larger. To understand this, we re-compute the average size profile leaving out the gazelles, see Figure 9. Without gazelles, average size is considerably lower and the difference remains large up to at least age twenty. At that age, average size is more than 25 percent lower than in the baseline. In a second counterfactual we leave out only "large gazelles", which are defined as gazelles with a *startup* size of at least 10 workers. In this counterfactual, average size is about 15 percent lower at age twenty compared ot the baseline, despite large gazelles accounting for only about 1 percent of all startups.

These counterfactuals make clear that high-potential startups are indeed impor-

[26]Defining gazelles using not only growth rates but also size excludes firms which grow quickly in percentage terms but nevertheless always stay small in terms of employed workers.

Figure 9: The importance of high-potential startups (firms)



Note: Average size, unbalanced panel and by age, in the baseline in two counterfactuals. See the main text for a description of these counterfactuals.

tant contributors to aggregate output and employment. Moreover, it follows that seemingly small shifts in the distribution of ex-ante profiles of startups may have large consequences, as suggested also by Sedláček and Sterk (2017). Our results further provide a perspective on the findings of Hsieh and Klenow (2014), who report that average size profiles are much flatter in India and Mexico than in the United States. A flat profile can indicate that there are few startups that operate a high-potential business model, or that high-potential startups have relatively low chances of survival.

Aggregate gains from selection. We now use the strucutral model to quantify the aggregate gains from selection. For this purpose we use the same counterfactuals as before, in combination with the following expression for aggregate output:

$$Y = \Omega^{\frac{\eta}{\eta-1}} \chi^{\frac{1}{1-\eta}} \overline{n}^{\frac{\eta}{\eta-1}},$$

where $\overline{n}$ is the average size across all firms; see Appendix C.1 for a derivation. We re-compute average firm size in both of the counterfactuals described in the previous paragraphs (i.e. when there is no selective exit and when ex-post demand shocks are "switched off"). Using these values, we then compute aggregate output based on the above equation.[27]

The productivity gains from selection are large. With only exogenous exit, output is 38 percent lower than the baseline. However, shutting down exit on ex-post shocks, i.e., where selection depends on *only* the ex-ante profiles, reduces output by just 4 percent.

These results imply that up-or-out dynamics are indeed an important contributor to aggregate output. Moreover, a key factor driving these dynamics is selection based on firms' ex-ante growth profiles. By contrast, ex-post shocks alone matter relatively little, especially at younger ages. Note further that our counterfactual exercises are based on distributions conditional on firm entry, i.e. based on demand fundamentals of firms which have already *decided* to begin operating. The impact of ex-ante heterogeneity would likely be even larger if selection before entry were to be included in the counterfactuals.

5 The macroeconomic impact of micro-level frictions

In the previous section, we established that heterogeneity in ex-ante profiles is crucial for understanding the strong up-or-out dynamics in the U.S. economy and for the associated aggregate gains from selection. In this section, we study whether the presence of ex-ante heterogeneity across firms also matters for our understanding of the macroeconomic impact of micro-level frictions.

In the literature, firm dynamics models are often used as laboratories to study quantitatively the impact of firm-level frictions on the macro economy. For example, Hopenhayn and Rogerson (1993) use a firm dynamics model to study the macroeconomic ramifications of firm-level adjustment frictions. We argue that the outcome of such exercises crucially depends on the particular nature of the sources of firm growth.

To make this point, we introduce adjustment costs into our baseline economy and compare it with a restricted version of our model, commonly used in the literature, which does not allow for rich sources of ex-ante heterogeneity. We find that while in the restricted version of the model introducing adjustment costs comes at substantial aggregate losses, our baseline economy is much less sensitive to such micro-level frictions.

[27] Note again that these are partial-equilibrium counterfactuals since we do not recompute χ or Ω.

5.1 Aggregate impact of adjustment costs

In what follows, we first detail the considered adjustment costs and the restricted version of our baseline model. Next, we compare the quantitative impact of introducing the same amount of adjustment costs into the two economies.

Adjustment costs. We introduce a fixed adjustment cost to demand growth, related to e.g. Gourio and Rudanko (2014) or Foster, Haltiwanger, and Syverson (2016). Specifically, we assume that whenever $\varphi' > \varphi$ the incumbent firm has two options: retaining its current level of demand, φ, or paying a cost κ and obtaining the new higher level of demand φ'. The adjustment cost may thus prevent firms from growing their demand and reaching their full potential. One can think this as a cost a firm needs to pay in order to seize a demand growth opportunity, related to for example marketing costs or organizational restructuring. Our formulation has the practical advantage that it does not introduce any additional state variables to the model.

A restricted version of our model. As mentioned above, we introduce this adjustment cost not only in our baseline economy, but also in a "restricted" version of the model without rich ex-ante heterogeneity in growth profiles. The latter model is obtained by setting $\rho_u = \sigma_\theta = \sigma_{\widetilde{u}} = \sigma_z = 0$ and $\rho_v = \rho_w$. These restrictions imply that the underlying process for firm-level demand evolves according to a simple AR(1) process:

$$\ln \varphi_{i,a} = \mu_\theta + \rho_w \ln \varphi_{i,a-1} + \epsilon_{i,a},$$

where $\epsilon_{i,a} \sim N(0, \sigma_\epsilon^2)$, $\varphi_{i,-1} \sim N(0, \sigma_{\widetilde{v}}^2)$. Note that these restrictions are the same as those of model I in Section 2.4. Given these restrictions, it is necessary to reparametrize the model. We do so by matching the same targets as in the baseline with the exception of the autocovariance matrix. Instead, we we follow the literature (see e.g. Hopenhayn and Rogerson, 1993) and target ρ_n and σ_n from the following regression

$$\ln n_{i,a} = \overline{n} + \rho_n \ln n_{i,a-1} + \eta_{i,a}$$

where $\eta_{i,a} \sim N(0, \sigma_n^2)$. The Appendix shows that the model fit, including the implied (untargeted) autocovariance matrix, turns out to be very similar to that of the statistical model in 2.4. Moreover, it also documents that the two economies have essentially an identical size distribution.

Table 3: Aggregate impact of adjustment costs (percent change)

	output	wage	size	exit	firms
restricted model	-3.2	-0.6	-26.2	$+7.2$	$+31.9$
baseline model	-0.1	-0.1	$+2.5$	-0.2	-2.4

Notes: Long-run impact of introducing adjustment costs in the baseline economy and in the restricted version where $\rho_u = \sigma_\theta = \sigma_{\widetilde{u}} = \sigma_z = 0$ and $\rho_v = \rho_w$. In both economies costs amount to 1 percent of output among adjusting firms. Reported values are relative to the respective values when $\kappa = 0$. Output refers to aggregate production, wage is the real wage rate, size is average firm size, exit is the average exit rate and firms refers to the number of incumbent firms.

We now introduce adjustment costs by setting $\kappa > 0$ in both the baseline model and the restricted version such that the average cost paid by adjusting firms is one percent of their output. Table 3 shows the long-run impact of such an introduction, relative to the case when adjustment costs are absent, $\kappa = 0$.

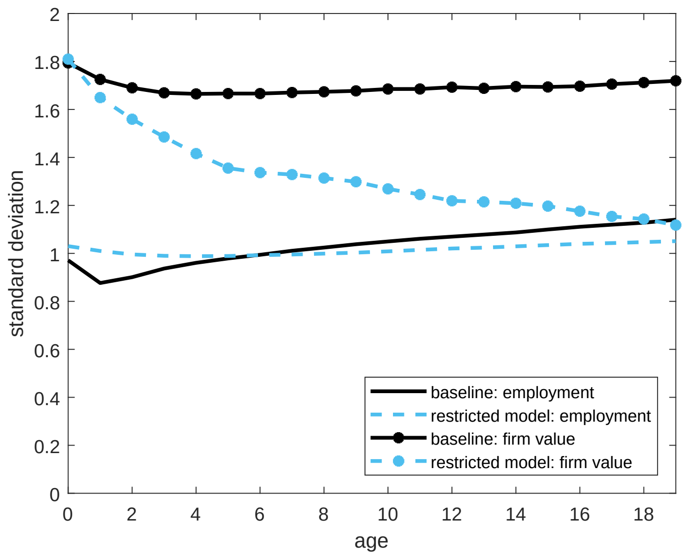
Comparison of the two economies. Let us begin with the restricted version of the model, which predicts substantial aggregate losses induced by the adjustment costs. Firms considerably decrease their demand accumulation which results in a strong decline in average firm size and an increase in firm exit. Given the assumption of a fixed labor supply, this leads to an increase in the number of firms. All these effects result in a decline in the wage and a drop in aggregate output of more than 3 percent. Similar findings have been found in previous studies, see e.g. Hopenhayn and Rogerson (1993).

By contrast, in the baseline model the macroeconomy is largely insensitive to the introduction of adjustment costs. There is a slight reduction in firm values, which puts downward pressure on the real wage. In equilibrium, because of fixed labor supply, firms are larger but fewer. The compositional shift towards larger businesses that are less likely to exit reduces the average exit rate, even as endogenous exit *conditional* on type is virtually unchanged.[28] However, with very little change in the distribution of firm values from the increase in adjustment costs, quantitatively the overall effects are small.

The main reason for the difference between the economies lies in the dispersion of firm values. While the two economies have a similar firm size distribution, the baseline economy is characterized by a highly dispersed distribution of firm values,

[28] See also Karahan, Pugsley, and Şahin (2019) who show the importance of compositional change for the aggregate exit rate.

Figure 10: Dispersion of log employment and firm values



Note: standard deviation of log employment and firm value of all businesses in the baseline and restricted version of our model.

driven by ex-heterogeneity in growth profiles. This is not true for the restricted version of the model which features a larger mass of marginal firms. To illustrate this point, Figure 10 shows the standard deviations of log employment and firm value across all businesses in the baseline and the restricted version of our model. While the dispersion of firm sizes is similar, that of firm values grows apart as incumbent firms age. Indeed, the standard deviation of log employment of all businesses in the baseline economy is 4 percent lower than that in the restricted version of the model. However, the standard deviation of log firm values is 23 percent higher.

Intuitively, firms in the restricted model are moving to the same long-run size and thus their firm values, the net present values of future profits, are much more similar to each other. Firm values, however, are critical to firms' decisions regarding entry, growth and exit. A wider dispersion in firm values generally implies that there are fewer firms indifferent between adjusting or not (or between exiting and continuing). Therefore, our baseline economy, with a wider dispersion in firm values, tends to be less sensitive to a change in adjustment costs.

Our results therefore show that the presence of ex-ante heterogeneity in growth profiles, and the associated firm selection process, may dwarf the consequences of micro-level distortions like adjustment costs. The Appendix considers two other micro-level frictions, changes in entry costs and in the fixed cost of operation. The message arising from these exercises is the similar: the nature of the underlying shock process is crucial for the aggregate implications of micro-level frictions.

6 Changes in the nature of firm growth

Finally, we use our framework to analyze a highly debated secular trend in the U.S. over the few decades, the so-called decline in business dynamism, which has also been associated with the disappointing evolution of aggregate employment and productivity growth, see e.g. Decker, Haltiwanger, Jarmin, and Miranda (2016).

Our framework enables us to identify the *timing* and *sources* of the observed changes in firm dynamics. In particular, we are able to assess whether firm dynamics have changed as a result of changes in ex-post shocks affecting all firms, or whether the observed secular trends are the result of shifts in ex-ante heterogeneity determining the *type* of startups operating in the economy.

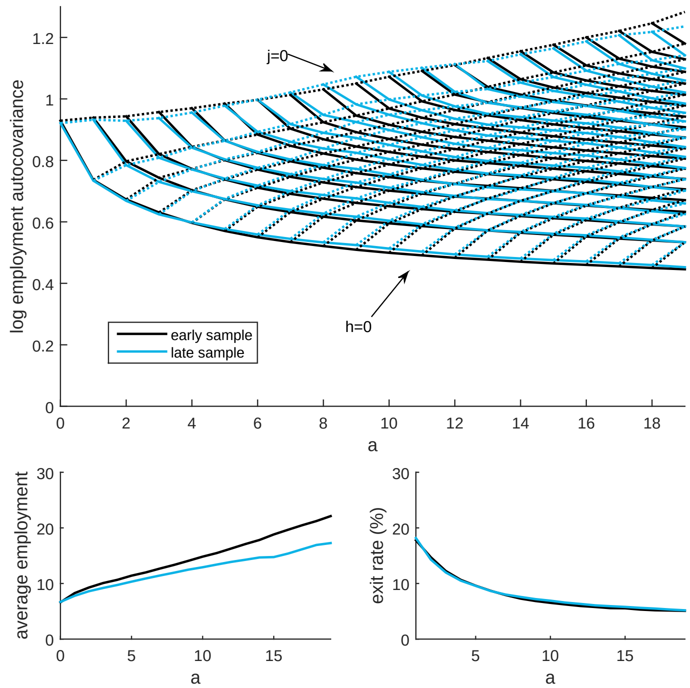
We analyze the changes in firm dynamics by splitting our data into an early sample, including firms born between 1979 and 1985, and a late sample with firms born between 1986 and 1993. Again, we follow all firms up to age 19. We first document changes in the three sets of key moments, the autocovariance function, the average size profile, and the exit profile. Next, we re-estimate our model on the two subsamples and interpret the changes in the data through the lens of our model with a particular focus on gazelles.

6.1 Changes in the data

Figure 11 plots the three sets of key moments in the two samples. The top panel shows that the autocovariance function of logged employment of firms (balanced panel) has remained remarkably stable over time. This suggests that the relative importance of ex-ante and ex-post heterogeneity has not changed much. The bottom right panel shows that exit rates have also remained stable, see also Pugsley and Şahin (2018).

What has changed, however, is the profile of average size by age, which is shown in the bottom left panel of Figure 11. Over time, this profile has flattened. At startup, average size is about 7 employees in both the early and the late sample. However, by age nineteen, average employment has declined by almost 25 percent from an average

Figure 11: Split-sample data moments (firms)



Notes: Top panel: Autocovariances of log employment between age $a = h + j$ and age $h \leq a$ in the early and the late sample, for a balanced panel of firms surviving up to at least age $a = 19$. Bottom left panel: Average employment by age a (unbalanced panel). Bottom right panel: exit rate by age a.

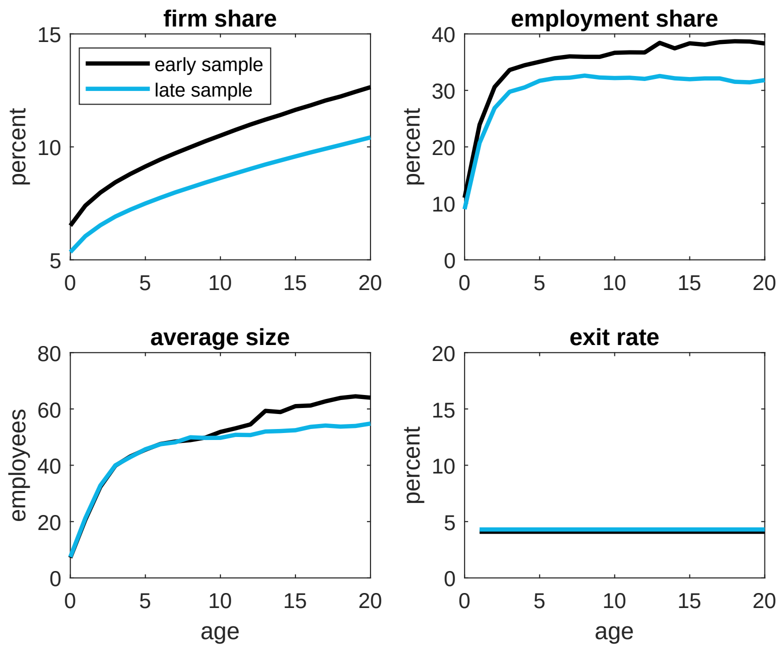
22 workers in the early sample to 17 employees in the late sample. In addition, this divergence in size profiles sets in gradually with age.

6.2 Estimating changes in firm dynamics

To investigate the observed changes in firm dynamics and their aggregate consequences, we first re-estimate the model on the two subsamples.[29] At the end of this section, we also provide some empirical evidence for our findings. The estimated parameter values and model fit are shown in Appendix C.2.

[29] We re-estimate only the parameters of the shock process and the fixed cost parameter. Notably, we keep the elasticity of substitution across goods varieties fixed. Given the evidence in e.g. de Loecker, Eeckhout, and Unger (2017) an interesting extension would be an attempt at simultaneously estimating also a change in the elasticity of substitution. This would, however, likely require additional data sources for measuring market power and we therefore leave it for future research.

Figure 12: Characteristics of gazelles in the early and late sample (firms)



Note: Top panels: share of gazelles in the total number of firms and in total employment. Bottom panels: average size and exit rate profile of gazelles. Gazelles are classified on an ex-ante basis, as those startups with an ex-ante growth rate of at least 20 percent annually, over the first five years, and an associated employment level that exceeds 10 at some point during this period.

Are gazelles dying out and at what cost? In what follows, we focus on (ex-ante identified) gazelles and ask to what extent changes in their characteristics alone can explain the observed decline in business dynamism. This is justified also by the stability of the exit profile across the two subsamples which points to changes in the top of the firm distribution, rather than the bottom where firms exit relatively more frequently.

Towards this end, we compute the fraction of gazelles in the population of firms in both subsamples. This is shown in the left top panel of Figure 12. Among startups, the fraction of gazelles has declined from 6.4 percent in the early sample to 5.3 percent in the late sample. As firms age, the fraction of gazelles increases because gazelles are relatively unlikely to shut down compared to other firms with lower growth potential. Therefore, the gap in the share of gazelles widens with age between the two samples. A similar picture is painted by the top right panel which shows the employment shares, by age. Among startups at age zero, gazelles account for around 9 percent of employment in both the early and the late samples. However, a gap emerges between the two samples as firms age and start fulfilling their ex-ante growth potential.

The bottom left panel shows the average size profile of gazelles. In both sub-samples, gazelles start with around 7 employees, but grow quickly to reach on average about 46 employees by age five. Around age 10, however, the two sub-samples diverge, and a reduction in the average size between the two sub-samples becomes apparent. Thus, in the late sample gazelles on average do not grow as large as in the early sample. Finally, the exit profile, plotted in the bottom right panel, is essentially the same in both samples, as gazelles exit practically only for exogenous reasons.[30]

Our findings thus confirm the concerns that high-growth firms are becoming increasingly rare.[31] While Decker, Haltiwanger, Jarmin, and Miranda (2016) document that the decline in the skewness of firm growth rates occurred around 2000 and primarily in the services, information and high-tech sectors, the sources of these secular changes remain to be identified. While our framework does not provide a definitive answer to this question, it does offer additional new insights. First, we document that the disappearance of gazelles is related to ex-ante factors, suggesting that high-growth firms are in fact dying out. Second, not only are there fewer gazelles, but those that nevertheless start up tend to expand less than high-growth firms of the past.

Note that our results do not exclude the possibility that a change in the responsiveness of firms to (ex-post) shocks is a feature of the observed decline in dynamism, as proposed by Decker, Haltiwanger, Jarmin, and Miranda (2018). In fact, our analysis in Section 5 suggests that changes in the form of ex-ante heterogeneity across firms, as estimated in this Section, has implications for the sensitivity of the macroeconomy to (possibly unchanged) disturbances and frictions.

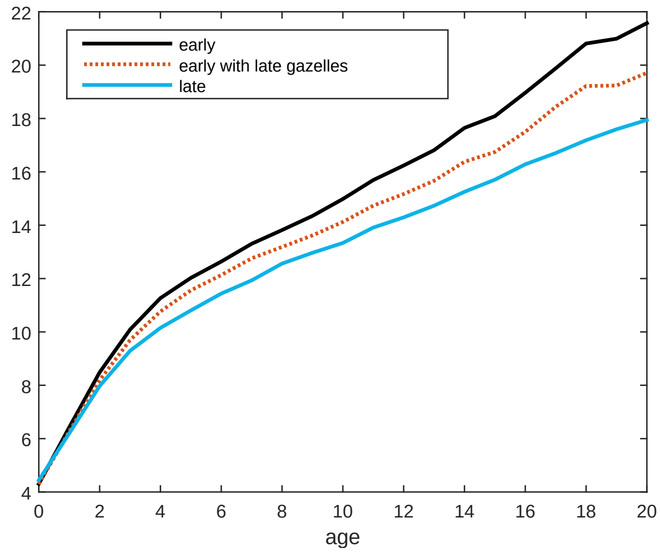
Aggregate implications. While the results clearly suggest a change in the characteristics of high-growth firms, it is unclear to what extent these changes alone can help explain the observed decline in dynamism at the aggregate level. After all, gazelles account for only a small share of all businesses.

To investigate this question, Figure 13 plots the average size profile, in the estimated model over the two sub-samples. As noted before, this profile has flattened.

[30] Consistent with this finding, the Appendix also shows that the model generates a substantial decline in skewness of firm growth rates across the two subsamples. This decline is generated entirely endogenously, since we assume symmetric, normally distributed shocks.

[31] Note that our results pertain firm-level employment. An interesting extension, requiring different data sources, would be to establish similar results for firm-level productivity.

Figure 13: The impact of disappearing gazelles (firms)



Note: The figure plots the average size profile among all firms in the early sample and the late sample. It also plots a counterfactual average size profile for the early sample, computed by replacing firm share and average size profile of gazelles by their counterparts from the late sample.

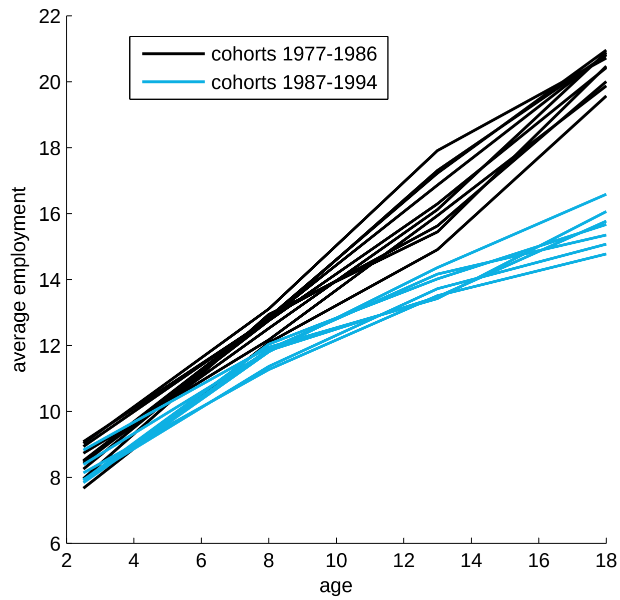
To assess the contribution of disappearing gazelles to this shift, we use the fact that at any age the average size among all firms is the sum of the average size of gazelles and non-gazelles, weighted by their respective firm share. We then construct a counterfactual in which we re-compute the average size in the early sample, but with the average size and firm share profiles of the gazelles in the late sample. The dashed line shows this counterfactual and suggests that changes associated with gazelles alone can account for roughly half of the decline in the average size profile. This is remarkable, given that gazelles account for only about five percent of the startups.

Finally, we evaluate the implications for aggregate output. We find that between the two samples, aggregate output declines by 4.5 percent. Thus, seemingly small changes in the distribution of firms, such as the decline in the (already low) share of high-potential startups, as well as a reduction in their growth potential, emerge as important drivers of *aggregate* changes.[32,33]

[32]Within the model, this decline is entirely driven by a change in output per worker, i.e. labor productivity, since we keep labor supply fixed. In a model version with endogenous labor supply, there could be an associated decline in aggregate employment as well.

[33]Shifts in the number of startups may also have important macroeconomic consequences, see Sedláček (2015).

Figure 14: Flattening of the average size profile in the data (firms)



Note: average firm employment by age according to cohorts, grouped by the birth year of the youngest firm in each cohort. Source: Business Dynamics Statistics.

6.3 Supporting empirical evidence

The above results suggest that changes in ex-ante factors are the key driver of the observed decline in business dynamism. Here we provide *model-free* evidence in support of this conclusion. In particular, if indeed the nature of firm growth has changed over time because of ex-ante factors, we should observe different patterns across different *cohorts* of startups rather changing patterns among all firms across time.

Figure 14 plots average firm size by age for each of the cohorts in our sample, defined by the birth year of the youngest firms in each age category. The figure shows very clearly that the flattening occurred by cohort. In addition, this change was not gradual, but it happened rather abruptly in the late 1980s. These patterns support our results that the flattening of the average size profile was an ex-ante phenomenon, rather than the result of changes in the character of ex-post shocks, which would likely affect *all* firms simultaneously.

The above insights point to potential future avenues of research attempting to identify the reasons behind the disappearance of gazelles. In particular, our results suggest that the disappearance of gazelles was set in motion already in the late 1980's, as opposed to the early 2000's when the change in skewness became most apparent.

An intriguing connection may be made between the demise of gazelle startups and the decline in the aggregate labor share of income, which also started in the late 1980's. For example, Autor, Dorn, Katz, Patterson, and Reenen (2017) suggest that the decline in the labor share was due to an increase in product market concentration, giving rise to "superstar firms". Increased domination of incumbent superstar firms might have made it more difficult for high-potential startups to enter the economy. Or vice versa, a lack of competitive pressure from gazelle startups might have contributed to the increase in market concentration. Alternatively, the late 1980's were also times of large fiscal reforms which may have affected firm dynamics, see e.g. Sedláček and Sterk (forthcoming).

7 Conclusions

We have used data on the population of U.S. firms over several decades to better understand why some startups grow rapidly whereas others remain stagnant or exit quickly. To this end, we documented the autocovariance structure of employment and exploited this structure to estimate firm dynamics models, which allowed us to disentangle heterogeneous ex-ante profiles from ex-post shocks.

We found a dominant role for heterogeneous ex-ante profiles, which capture future potential present at the moment of startup. Much of the firm size distribution, "up-or-out" dynamics and the prevalence of high-growth startups, "gazelles", is determined by ex-ante heterogeneity in growth profiles. Moreover, the presence of such heterogeneity shapes the behavior of the macroeconomy. Indeed, not accounting for the precise nature of firm growth has the potential to dramatically change the macroeconomic predictions of firm dynamics models. Finally, having in mind recent concerns about the disappearance of gazelles, we have investigated potential changes in the nature of firm growth over time. Re-estimating the model using this information, we found a decline in the presence and growth potential of "gazelles" in the population of startups, with important repercussions for aggregate output.

Our results highlight the need for future research on which individuals become entrepreneurs and what decisions such aspiring entrepreneurs make *before or at* startup, as opposed to their behavior *after* the firm has become operational. While the macroeconomic implications of the latter have been studied extensively in the literature, much less is known about how institutional conditions change who becomes an entrepreneur and what types of firms are being created. Our results show that such changes can be of first-order importance for macroeconomic outcomes.

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