The Nature of Firm Growth*

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*Any opinions and conclusions expressed herein are those of the author(s) and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed.
Firm heterogeneity and the macroeconomy

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- Could we find more direct evidence?
Can we use administrative panel data now available to better understand “nature” of firm life-cycle dynamics/growth in macro models with firm dynamics ...
Research question

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- **ex-ante heterogeneity**: are firms (“born”) inherently different?

... and their macro implications, i.e., should we care?
What we do

1. Use large administrative panel data to estimate contributions of **ex-post shocks** and **ex-ante heterogeneity** to dispersion in firm size
   - New moments: autocovariance matrix of log employment
   - Statistical approach: estimate an employment process to match moments
   - Structural approach: estimate parameters in a macro firm dynamics model

2. Quantify importance of **ex-ante heterogeneity** for:
   - Firm-level outcomes: entry, survival, up-or-out dynamics
   - Aggregate outcomes: productivity gains from selection; effects of adjustment frictions

3. Changes over time: decline in "dynamism"; disappearance of gazelles
   ⇒ Evidence for shift in distribution of ex-ante profiles
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Estimating the “nature” of firm growth
A statistical model
Estimating a new set of moments: employment autocovariance matrix

- Autocovariance matrix, \([c_{aa'}]\), of firm-level (residual) log employment

\[ c_{aa'} = \text{Cov}\left[ \log \tilde{n}_{ia}, \log \tilde{n}_{ia}' \right] \quad a = 0, 1, \ldots, 19 \]

- We measure this covariance matrix of 210 moments using Census LBD
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- To understand its value, consider two (extreme) thought experiments:

  1. Employment determined only by permanent \textbf{ex-ante heterogeneity}:
    \[ \implies \text{perfect correlation of size over firm’s life-cycle} \]

  2. Employment determined only by iid \textbf{ex-post shocks}:
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Pugsley, Sedláček, Sterk
Nature of Firm Growth
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  2. Employment determined only by iid **ex-post shocks**:  
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- (Borrowing from income dynamics literature) can estimate parameters of employment processes by matching autocovariance patterns
Generalized employment process

\[ \log n_{ia} = \sum_{k=0}^{\alpha} \rho_u^k \theta_i + \rho_u^{a+1} \tilde{u}_i + \rho_v^{a+1} \tilde{v}_i + \sum_{k=0}^{\alpha} \rho_v^k \varepsilon_{ia-k} + z_{ia} \]
Generalized employment process

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\]

Ex-ante profile \((a < 0)\):

\[
\begin{align*}
\theta_i &\sim IID (\mu_\theta, \sigma_\theta) \\
\tilde{u}_i &\sim IID (\mu_{\tilde{u}}, \sigma_{\tilde{u}}) \\
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Ex-post shocks \((a \geq 0)\):

\[ \begin{align*}
\varepsilon_{i,a} &\sim \text{IID} (0, \sigma_\varepsilon) \\
z_{i,a} &\sim \text{IID} (0, \sigma_z)
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- Nests models of Hopenhayn and Rogerson (1993) and Melitz (2003) and allows for richer ex-ante profiles as in e.g. Luttmer (2011)
- Closed-form autocovariance function in model parameters
- Estimate parameters by matching data and process autocovariance (EWMD)
Data and model fit

![Graph showing covariance over firm age]
Data and model fit

- RW
- Ex-ante+RW
- Ex-ante+RW + AR(1)
- Age dep.
- AR(1)
- Panel AR(1)
- Panel AR(2)
- Panel ARMA(1,1)
Data and restricted models

Note: (I) only persistent shocks and initial condition, (II) only cumulated permanent effect and initial condition, (III) baseline with only one initial condition, (IV) baseline without transitory ex-post shocks.
Ex-ante factors and the dispersion of firm sizes by age
Estimating the “nature” of firm growth
A general equilibrium model with firm dynamics
Important advantages of structural model

- accounts for endogenous firm selection
- enables us to address aggregate questions
- can address various frictions and their effect on observed patterns
- show why getting source of heterogeneity right is important
Sketch of model

Environment à la Hopenhayn (1992), Melitz (2003) and Luttmer (2011)
- stationary, no aggregate uncertainty
- general equilibrium closed economy
- endogenous entry and exit
- frictionless factor adjustment
- inelastic labor supply
- demand heterogeneity (Foster et al., 2015; Hottman et al., 2016)
- idiosyncratic shock process as in reduced-form analysis
  ex-ante profiles + ex-post shocks
Estimate by matching autocovariance and life-cycle profiles of exit and size distribution.

Autocovariance matrix, average size and exit rates

Notes: Top panel: Autocovariances of log employment between age $a = h + j$ and age $a = h$ in the data and the model, for a balanced panel of rms surviving up to at least age $a = 19$. Bottom left panel: average employment by age $a$ (unbalanced panel). Bottom right panel: exit rate by age $a$. In the data, about eighteen percent of rms exit between age zero and one. Subsequently, the exit rate gradually declines, stabilizing at older age categories. The model matches this pattern, predicting relatively high exit rates at young ages, but somewhat underestimates the exit rates of young rms. Overall, the model provides a good fit of the three sets of empirical comments, considering that 10 parameters are used to target 249 moments.

Additionally, we consider how the model fits the employment distribution by age and size, which is not directly targeted. Figure 5 shows employment shares of different age/size bins, in the model and in the data. Overall, the model fits this distribution well. The model also provides a similarly good fit of the fractions of rms in each of these bins (not shown).

The associated parameter values are shown in Table 2. The fixed cost is estimated to be 0.54, which is about half the wage of a single employee. The exogenous exit rate is estimated to be about 4.1 percent. Thus, a substantial fraction of rms exits for...
Variance decomposition in estimated model with selection

\[ \text{Var} [\ln n] = \text{Var} [\ln n^{EXA}] + \text{Var} [\ln n^{EXP}] + 2 \text{Cov} [\ln n^{EXA}, \ln n^{EXP}] \]

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Figure 6: Contribution of ex-ante heterogeneity to cross-sectional employment dispersion (rms)

Note: Contributions of ex-ante heterogeneity to the total cross-sectional variance of log employment by age. Reduced-form refers to the estimates from Figure 3, model: covariance decomposition is the decomposition based on the second line in Equation (5). The shaded areas (model: selection band) is constructed based on the first equality in Equation (5) by attributing, in turn, the term \(2 \text{Cov} (\ln \phi^{EXA}, \ln \phi^{EXP})\) fully to the ex-ante component and to the ex-post component.

Figure 6 also plots a selection band based on the first equality in Equation (5). This band is constructed by attributing, in turn, the covariance term either fully to the ex-ante component or fully to the ex-post component. This gives us a sense of how much selection matters in the model. The widening band indicates that selection has an increasingly important impact on the cross-sectional dispersion of rm size as rm age.

Overall, however, the various decompositions re-establish our earlier conclusion that ex-ante heterogeneity is a key source of size dispersion, in particular among younger rms.

4.2 Firm exit

Next, we study the importance of ex-ante heterogeneity for exit, the out part of up-or-out dynamics. One might think that exit is entirely triggered by unexpected ex-post

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Quantifying the importance of ex-ante heterogeneity
How important is ex-ante heterogeneity for firm selection and growth?
Drivers of firm selection

![Graph showing the relationship between age and percent baseline. The graph indicates a decreasing trend in percent baseline as age increases.]
Drivers of firm selection

![Graph showing drivers of firm selection with two lines: one for baseline and another for no ex-post shocks. The x-axis represents age, and the y-axis represents percent. The baseline line starts high and decreases sharply, while the no ex-post shocks line starts lower and decreases gradually.]
Quantify impact of firm selection: firm growth

The graph shows the average employment growth over time, with a clear upward trend indicating a positive impact of firm selection on firm growth.
Quantify impact of firm selection: firm growth

[Graph showing the average employment over age for two scenarios: baseline and no ex-post shocks.]
Quantify impact of firm selection: firm growth

- baseline
- no ex-post shocks
- no selective exit

Age vs. average employment
Quantify impact of firm selection: aggregate output

- Aggregate output can be written

\[ Y \propto \Omega^\eta \tilde{n}^{\frac{\eta}{\eta-1}} \]

- \( \Omega \) is the total mass of firms
- \( \tilde{n} \) is average firm size

Firm selection leads to output gains.

Results show that output would be 38 percent lower without effects of selection altogether.

4 percent lower without effects of selection on ex-post shocks.
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Quantifying the importance of ex-ante heterogeneity
How does source of heterogeneity matter for macro outcomes?
The macro impact of micro-level frictions

Consider two economies:

1. baseline

- restricted version with simple AR(1) process – as in Hopenhayn and Rogerson (1993)
- no (ex-ante) heterogeneity in growth potential
- calibrate both models to the same size distribution

Introduce following adjustment cost into both versions of the model

- Now, must pay cost $\kappa$ to accept next period's demand shock
- “Active” accumulation of demand as in Foster, Haltiwanger and Syverson (2016)
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- Reason: key decisions made by firms depend on firm values, which are forward looking and hence depend on the process
The macro impact of micro-level frictions

Table: Aggregate impact of adjustment costs (percent change)

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- Dispersion of firm values much greater in baseline model
  - fewer “marginal” firms
  - smaller impact of frictions
Quantifying the importance of ex-ante heterogeneity
Apply to estimating changing nature of firm dynamics.
Declines in business dynamism and its macro effects

Recently documented decline in dispersion/skewness of firm growth rates
  – i.e. firms are not growing as fast as they used to
don’t trust me?

Decker et al. (2016) link declining dynamism to
  – slowdown in aggregate employment and productivity growth
Declines in business dynamism and its macro effects

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Decker et al. (2016) link declining dynamism to
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We use our framework to answer whether
- dynamism decline is caused by a change in ex-post shocks
- or ex-ante factors, i.e. are high-growth firms ("gazelles") dying out?
Estimating model on split sample: “gazelles”

- Firm share
- Employment share
- Average size
- Exit rate
Conclusions

- Estimate (and release) the autocovariance structure of firm-level employment over first 20 years of lifecycle for U.S. firms

- Using both statistical and structural approaches, \textit{ex-ante profile heterogeneity} explains \textit{almost half} of within-industry size dispersion, even at long horizons

- Important for firm-level and macro outcomes
  - Selection and age-profile of firm size
  - Aggregate productivity gains from selection
  - Macro effects of micro frictions
  - Decline in dynamism is in part explained by change in ex-ante “gazelles”

- Even more applications in the paper
Thank you!
Two popular processes in firm dynamics literature


\[ \ln n_{i,a} = \mu + \rho \ln n_{i,a-1} + \epsilon_{i,a} \]

\[ \tilde{n}_{i,-1} \sim \text{IID} \left( \mu_{\tilde{v}}, \sigma_{\tilde{v}} \right), \]

\[ \epsilon_{i,a} \sim \text{IID} \left( 0, \sigma_{\epsilon} \right) \]


\[ \ln n_{i,a} = \theta_i \]

\[ \theta_i \sim \text{IID} \left( \mu_{\theta}, \sigma_{\theta} \right) \]
Generalized reduced-form process

\[ \ln n_{i,a} = u_{i,a} + v_{i,a} + w_{i,a} + z_{i,a} \quad a = 0, 1, 2, \ldots \]

Ex-ante components $u_{i,a} - 1$ $v_{i,a}$

Ex-post components $w_{i,a} + z_{i,a}$
Generalized reduced-form process

\[ \ln n_{i,a} = u_{i,a} + v_{i,a} + w_{i,a} + z_{i,a} \quad a = 0, 1, 2, \ldots \]

Ex-ante components:

\[ u_{i,a} = \theta_i + \rho_u u_{i,a-1} \]
\[ v_{i,a} = \rho_v v_{i,a-1} \]

“pre startup” draws \( a < 0 \)

\[ \theta_i \sim IID(\mu_{\theta}, \sigma_{\theta}) \]
\[ u_{i,-1} \sim IID(\mu_{\tilde{u}}, \sigma_{\tilde{u}}) \]
\[ v_{i,-1} \sim IID(\mu_{\tilde{v}}, \sigma_{\tilde{v}}) \]
Generalized reduced-form process

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\ln n_{i,a} = \underbrace{u_{i,a} + v_{i,a}}_{\text{Ex-ante components}} + \underbrace{w_{i,a} + z_{i,a}}_{\text{Ex-post components}} \quad a = 0, 1, 2, \ldots
\]

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\end{align*}
\]

Ex-post components:

\[
\begin{align*}
w_{i,a} &= \rho_w w_{i,a-1} + \varepsilon_{i,a} \\
\end{align*}
\]

with \( w_{i,-1} = 0 \) and for \( a \geq 0 \)

\[
\begin{align*}
\varepsilon_{i,a} &\sim \text{IID} (0, \sigma_{\varepsilon}) \\
z_{i,a} &\sim \text{IID} (0, \sigma_{z})
\end{align*}
\]
Autocovariance function

\[
\text{Cov} \left[ \log n_{ia}, \log n_{a-j} \right] = \left( \sum_{k=0}^{a} \rho_u^k \right) \left( \sum_{k=0}^{a-j} \rho_u^k \right) \sigma_\theta^2 + \rho_u^{2(a+1)-j} \sigma_u^2 + \rho_v^{2(a+1)-j} \sigma_v^2 \\
+ \sigma_\varepsilon^2 \rho_j \sum_{k=0}^{a-j} \rho_w^{2k} + \sigma_z^2 \mathbf{1}_{j=0}.
\]

- autocovariance matrix (over-)identifies persistence and variance parameters
  - \( \rho_u, \rho_v, \rho_w, \sigma_\theta^2, \sigma_u^2, \sigma_v^2, \sigma_\varepsilon^2, \sigma_z^2 \)

- does not identify levels
  - \( \mu_\theta, \mu_\varpi, \mu_\varpi \)
Estimation

- minimum distance estimation using empirical autocovariance
- 210 moments, 8 parameters, identity weighting matrix
- include 4-digit NAICS and cohort fixed effects
- estimate version restricted to AR(1) and other processes
Related literature

Firm dynamics models

Empirical evidence:

Household earnings processes:
- e.g. Lillard and Weiss (1979), MaCurdy (1982), Abowd and Card (1989), Guvenen (2009)
Firm employment data

U.S. Census Bureau Longitudinal Business Database (LBD)

- administrative data
- nearly universal coverage of U.S. employers
- annual data from 1976 until 2012
- observe firms and establishments
- observe employment, age and 4-digit industry

Use only within-industry and within-cohort *residual* employment:

\[ \log n_{i,a,t,j} = \log n_{i,a,t,j} - \mu_j - \lambda_{t-a} \]
ALTERNATIVE STATISTICAL MODELS
### Table: EWMD estimation of alternative firm employment processes

<table>
<thead>
<tr>
<th></th>
<th>(1) Base</th>
<th>(2) RW</th>
<th>(3) RW+Base</th>
<th>(4) Age Dep.</th>
<th>(5) AR(1)</th>
<th>(6) AR(1) + FE</th>
<th>(7) Dynamic Panel Data Models</th>
<th>(8) ARMA(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Panel Data Models</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>ρ_u</td>
<td>0.2184</td>
<td>0.5853</td>
<td>0.2199</td>
<td>0.0002</td>
<td>—</td>
<td>—</td>
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<tr>
<td>ρ_v</td>
<td>0.8323</td>
<td>0.9608</td>
<td>0.8246</td>
<td>0.8123</td>
<td>0.9771</td>
<td>0.9716</td>
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<td>ρ_w/ρ</td>
<td>0.9625</td>
<td>1</td>
<td>0.9492</td>
<td>0.9693</td>
<td>0.9771</td>
<td>0.9716</td>
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<td>ρ_2</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0</td>
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<td>0.0316</td>
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<tr>
<td>σ_θ</td>
<td>0.5545</td>
<td>0.2142</td>
<td>0.5572</td>
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<td>—</td>
<td>3.782</td>
<td>0.8420</td>
<td>1.1019</td>
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<td>σ_u</td>
<td>1.7425</td>
<td>0.7402</td>
<td>1.7305</td>
<td>—</td>
<td>—</td>
<td>0</td>
<td>0.2641</td>
<td>0.3313</td>
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<tr>
<td>σ_v</td>
<td>0.6951</td>
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<td>0.6992</td>
<td>0.7605</td>
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<td>0.7308</td>
<td>0.8420</td>
<td>0.8617</td>
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<td>σ_ε</td>
<td>0.2548</td>
<td>0.2020</td>
<td>0.2408</td>
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<td>σ_x</td>
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<td>—</td>
<td>0.0945</td>
<td>0.276</td>
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<tr>
<td>σ_z</td>
<td>0.2716</td>
<td>0.3313</td>
<td>0.2660</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>γ</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>- 0.4478</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.0120</td>
<td>0.0191</td>
<td>0.0119</td>
<td>0.0083</td>
<td>0.0368</td>
<td>0.0367</td>
<td>0.0367</td>
<td>0.032</td>
</tr>
<tr>
<td># Params</td>
<td>8</td>
<td>7</td>
<td>9</td>
<td>27</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Back
ALTERNATIVES: MODEL FIT
Model fit: Random walk with heterogeneous drift
Model fit: Ex-ante + Ex-post growth rate shocks (random-walk)
Model fit: Ex-ante + Ex-post persistent and permanent shocks
Model fit: Age dependent ex-post shocks
Model fit: AR(1)
Model fit: Panel AR(1) vs. separate AR(1) and FE terms

![Graph showing model fit comparison between data, panel AR(1), and AR(1) + FE terms.]
Model fit: Panel AR(2)
Model fit: Panel ARMA(1,1)
ALTERNATIVES: VARIANCE DECOMPOSITION
“Ex-ante” share of total variance by firm age
EQUILIBRIUM MODEL DETAILS
Idiosyncratic demand shocks

$$\ln n_{i,a} = (u_{i,a} + v_{i,a}) + (w_{i,a} + z_{i,a}) \quad a = 0, 1, 2, \ldots$$

Ex-ante components

Ex-post components

Ex-ante components:

$$u_{i,a} = \theta_i + \rho_u u_{i,a-1}$$
$$v_{i,a} = \rho_w v_{i,a-1}$$

“pre startup” draws $a < 0$

$$\theta_i \sim IID (\mu_\theta, \sigma_\theta)$$
$$u_{i,-1} \sim IID (\mu_{\tilde{u}}, \sigma_{\tilde{u}})$$
$$v_{i,-1} \sim IID (\mu_{\tilde{v}}, \sigma_{\tilde{v}})$$

Ex-post components:

$$w_{i,a} = \rho_w w_{i,a-1} + \varepsilon_{i,a}$$

with $w_{i,-1} = 0$ and for $a \geq 0$

$$\varepsilon_{i,a} \sim IID (0, \sigma_\varepsilon)$$
$$z_{i,a} \sim IID (0, \sigma_z)$$
Parametrization

- demand process as in reduced-form part
- all shocks drawn from normal distribution
- $\beta = 0.96$, $\eta = 6$
- $\frac{f^e}{f} = 0.82$ (Barseghyan and DiCecio, 2011)
- remaining parameters target firm-level values (0-19 years)
  - autocovariance matrix of log employment (persistence and variance)
  - average size and exit rates by age (level $\mu_\theta$ and fixed cost $f$)
  - normalize $\mu_{\tilde{u}} = \mu_{\tilde{v}} = 0$
### Parameters of structural model

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ discount factor</td>
<td>0.96</td>
</tr>
<tr>
<td>$\eta$ elasticity of substitution</td>
<td>6.00</td>
</tr>
<tr>
<td>$f^e$ entry cost</td>
<td>0.44</td>
</tr>
<tr>
<td>$f$ fixed cost of operation</td>
<td>0.539</td>
</tr>
<tr>
<td>$\delta$ exogenous exit rate</td>
<td>0.041</td>
</tr>
<tr>
<td>$\mu_\theta$ permanent component $\theta$, mean</td>
<td>$-1.762$</td>
</tr>
<tr>
<td>$\sigma_\theta$ permanent component $\theta$, st. dev.</td>
<td>1.304</td>
</tr>
<tr>
<td>$\sigma_{\tilde{u}}$ initial condition $u_{t-1}$, st. dev.</td>
<td>1.572</td>
</tr>
<tr>
<td>$\sigma_{\tilde{v}}$ initial condition $v_{t-1}$, st. dev.</td>
<td>1.208</td>
</tr>
<tr>
<td>$\sigma_\epsilon$ transitory shock $\epsilon$, st. dev.</td>
<td>0.307</td>
</tr>
<tr>
<td>$\sigma_z$ noise shock $z$, st. dev.</td>
<td>0.203</td>
</tr>
<tr>
<td>$\rho_u$ permanent component, persistence</td>
<td>0.393</td>
</tr>
<tr>
<td>$\rho_v$ transitory component, persistence</td>
<td>0.988</td>
</tr>
</tbody>
</table>

*set a priori*

*used to target moments*
Model fit: untargeted

Size-age distributions (employment shares)

young (<6)

middle-aged (6 to 10)

older (11 to 19)
Note: Employment shares by firm age and size (employment). Values are expressed as percentages of total employment in firms between 0 to 19 year old firms, both in the data and the model. Data from the Business Dynamics Statistics.
Remaining targets: alternative calibration
Variance decomposition: alternative calibration

![Graph showing variance decomposition with different models: selection effects, covariance decomposition, reduced form, and baseline calibration. The x-axis represents age, and the y-axis represents percent deviation.]
Variance decomposition: model with learning

- Model: Selection effects (learning)
- Model: Covariance decomposition (learning)
- Model: Covariance decomposition (baseline)
- Reduced form
Supportive evidence: firm cohorts

Figure 11: Flattening of the average size profile in the data (rms)

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>6</td>
</tr>
<tr>
<td>1985</td>
<td>8</td>
</tr>
<tr>
<td>1990</td>
<td>10</td>
</tr>
<tr>
<td>1995</td>
<td>12</td>
</tr>
<tr>
<td>2000</td>
<td>14</td>
</tr>
<tr>
<td>2005</td>
<td>16</td>
</tr>
<tr>
<td>2010</td>
<td>18</td>
</tr>
<tr>
<td>2015</td>
<td>20</td>
</tr>
</tbody>
</table>

Note: The left panel plots, by year, average rm employment in different age bins: 0-5 years, 6-10 years, 11-15 years, 16-20 years, and 21-25 years. The right panel plots the same data, but now by overlapping 5-year cohorts, grouped by the birth year of the youngest firm in each cohort. Source: Business Dynamics Statistics.

5.2 Are gazelles dying out?

Our previous analysis suggests that the flattening of the average size profile might be related to ex-ante characteristics of startups. Importantly, our baseline results also suggest that attenuated average size profiles are also associated with lower aggregate output because they indicate a lesser extent of endogenous rm selection. To investigate the underlying changes more directly, we re-estimate the model on the two subsamples. The parameter values and model $t$ are shown in Appendix B.2. The Appendix shows that the model generates a substantial decline in skewness of firm growth rates across the two subsamples.

We compute the fraction of gazelles in the population of firms in both subsamples. This is shown in the left top panel of Figure 12. Among startups, the fraction of gazelles has declined by 17 per cent from a share of 6.4 per cent in the early sample to 5.3 per cent in the late sample. As firms age, the fraction of gazelles increases because gazelles are relatively unlikely to shut down compared to other firms with lower growth potential. This decline is generated entirely endogenously, since we assume symmetric, normally distributed shocks.

Figure 11: Flattening of the average size profile in the data (rms)
How has the “nature” of firm growth changed?

Figure 10 plots the three sets of key moments in the two samples. The top panel shows that the autocovariance function of logged employment (balanced panel) has remained remarkably stable over time. This suggests that the relative importance of ex-ante and ex-post heterogeneity has not changed much. The bottom right panel shows that exit rates have also remained stable, see also Pugsley and "ahin (2016).

What has changed, however, is the profile of average size by age, which is shown in the bottom left panel of Figure 10. Over time, this profile has attenuated. At startup, average size is about 7 employees in both the early and the late sample. However, by...
"Gazelles" and average life-cycle profiles

Figure 13: The impact of disappearing gazelles (rms)

Note: The figure plots the average size profile among all rms in the early sample and the late sample. It also plots a counterfactual average size profile for the early sample, computed by replacing rm share and average size profile of gazelles by their counterparts from the late sample.

We conduct a simple counterfactual exercise. In particular, we note that at any age, the average size among all rms is the mean of the average size of gazelles and non-gazelles, weighted by their respective rm share. We then construct a counterfactual in which we re-compute the average size in the early sample, but with the average size and rm share profiles of the gazelles in the late sample.

The dashed line in Figure 12 plots this counterfactual. It shows that the change in the fraction of gazelles and their average size profile accounts for roughly half of the decline in the average size profile. This is remarkable, given that gazelles account for only about five percent of the startups.

Finally, we evaluate the implications for aggregate output. We find that between the two samples, aggregate output declines by 4.5 percent. Thus, seemingly small changes in the distribution of rms, such as the decline in the (already low) share of high-potential startups, as well as a reduction in their growth potential, emerge as important drivers.
Extensions: learning

Limited information

– what if entrepreneurs do not know ex-ante profiles?

– using estimated parameters and optimal Bayesian learning
  – the majority of information uncovered in first years
  – survival is very informative about large ex-ante heterogeneity!

– extend model to include learning in early years
  – decisions based on (updated) beliefs about demand fundamental
  – main results very similar to baseline model!
Extensions: adjustment costs

**Active accumulation of demand**

- in addition to “passive accumulation of demand” (of $\theta_i$ via $\rho_u$)
- include option to invest into demand accumulation
- as in e.g. Foster, Haltiwanger, Syverson (2016)
- parameter estimates affected:
  - dispersion of $\theta$ lower
- main results very similar to baseline model!