Announcement-Specific Decompositions of Unconventional Monetary Policy Shocks & Their Macroeconomic Effects

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Abstract

I propose to identify announcement-specific decompositions of asset price changes into monetary policy shocks based on intraday time-varying volatility. This approach is the first to accommodate changes in both the nature of shocks and the state of the economy across announcements. I compute daily historical decompositions with respect to three monetary policy shocks for the U.S. from 2007-2018. I derive expressions for the asymptotic variance of such historical decompositions and apply them to assess the statistical significance of notable announcements. Only a handful spark significant shocks, and I discuss the characteristics of those announcements in detail. For many announcements, asset purchase shocks lower corporate borrowing costs, but spreads increase in response to both asset purchases and forward guidance. Turning to the real economy, I find that the asset purchase shock has significant effects on consumer and professional expectations of inflation and GDP growth. I compute dynamic responses of inflation and GDP growth; asset purchases have significant expansionary effects, while Fed Funds shocks and forward guidance do not.

Keywords: high-frequency identification, time-varying volatility, monetary policy shocks, forward guidance, quantitative easing
JEL codes: E44, E52, E58, C32, C58

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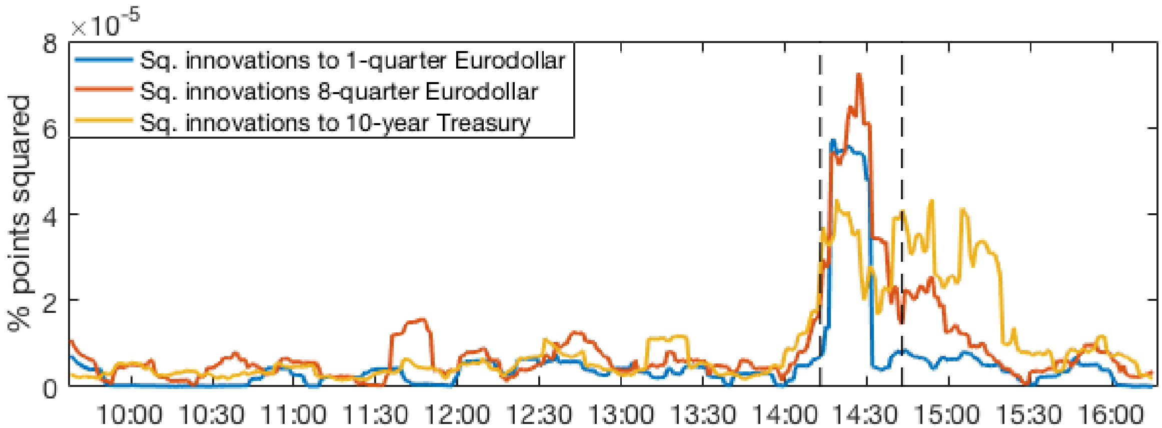
1 Introduction

Since the work of Kuttner (2001), high-frequency movements in asset prices have been used extensively to identify monetary policy shocks. However, with the shift towards more open communication starting in the 1990s across many central banks, the possibility of multiple dimensions of policy has complicated the task of identifying such shocks. Existing approaches either assume that each asset price considered responds only to a single shock over a certain window (e.g., Krishnamurthy & Vissing-Jorgensen (2011), Gertler & Karadi (2015)), or compute decompositions estimated across announcement dates (e.g., Gurkaynak, Sack & Swanson (2005) (hereafter GSS), Swanson (2017), Nakamura & Steinsson (2018), Inoue & Rossi (2018)). Neither approach is suited to the zero-lower-bound (ZLB) period and unconventional monetary policy.

The former approach either assumes the presence of a single shock or imposes exclusion restrictions across assets or factors (e.g., one price responds to target rate shocks, another to news shocks). The latter approach, computing time-invariant decompositions, follows the highly influential work of Nelson & Siegel (1987) and GSS, extracting factors from a set of asset price movements. However, the loadings on the factors are time-invariant, which means that recovered shocks can differ across announcements only in scale, not in their relative impact across asset prices. For example, this means that the asset purchase shock of the announcement of the first round of quantitative easing, QE1, would be restricted to impact a range of prices in exactly the same manner as that of QE2, despite the fact the announced measures targeted different assets and occurred at times when the economy, and thus important elasticities, may have been in very different states. The use of factors presents the further challenge of interpretability; factors are identified only up to orthogonal rotations, meaning no "Fed Funds", "forward guidance", or "asset purchase" shocks can be identified without further structural assumptions. Swanson (2017) makes important progress on this last point, by using judicious exclusion and event constraints.

I propose to identify announcement-specific decompositions of asset price movements to identify monetary policy shocks without assuming time-invariance across announcements. To do so, I treat all asset price movements over the course of an announcement day as responses to a series of news shocks. In the period following a monetary policy announcement, these news shocks can be interpreted as monetary policy shocks. This means that a full day of intraday data can be used to identify an announcement-specific decomposition of asset price movements into news shocks, and thus monetary policy shocks. Figure 1 plots 15-minute moving-averages of squared innovations to three interest rates (based on the model in Section 3) for September 21, 2011, the day of the Federal Open Market Committee (FOMC)

Figure 1: Realized volatility of innovations on September 21, 2011



15-minute moving average of squared innovations from my baseline model for September 21, 2011, a VAR(7) with 1-quarter Eurodollar rates, 8-quarter Eurodollar rates, and 10-year Treasury yields. Reference lines indicate the conventional 30-minute event window.

announcement that launched Operation Twist. There are clearly asset price movements besides the change across the usual 30-minute event window (14:13 to 14:43) which may offer previously unexploited identifying variation. To identify the decomposition, I apply results from Lewis (2019), based on the simple assumption that the shock volatility varies, with some persistence, over the course of the day. The stark intraday volatility patterns evident in Figure 1 motivate this identification approach. The shocks are identified up to labeling, which follows naturally in most cases.

I apply this approach to each scheduled FOMC announcement from 2007-2018. I estimate a triplet of shocks: “Fed Funds”, “forward guidance”, and “asset purchase”. To assess which announcements led to significant monetary policy shocks in each dimension, I compute historical decompositions of asset prices to the end of the day. I derive novel expressions for the asymptotic variance of historical decompositions, and use these to assess the statistical significance of the shocks (via their cumulative impact on relevant interest rates). I find that several shocks that appear significant based on 30-minute windows have no discernible effect by the day’s end, possibly distorting the results of studies that have employed such windows. I instead focus on the end-of-day decompositions. Even among the most notable unconventional policy announcements, few spark significant shocks; those that do are generally the launch of policies or their extension, when markets widely believed them to be coming to an end; subtle revisions appear less important.

For each day, I compute the high-frequency responses of corporate debt and equities to the shocks, finding some evidence that asset purchases in particular bring down corporate borrowing costs; however, these effects rarely persist to the day’s end. I form a time-series of my shock measures, and use these to conduct daily regressions using corporate debt measures, as in Swanson (2017); the findings are qualitatively similar.

While several papers have considered such financial market responses responses, little work has assessed the lower-frequency response of macroeconomic aggregates, the ultimate variables of interest for central banks, to interpretable unconventional policy shocks. I use my shock measures to estimate the response of consumers' and professional forecasters' expectations to unconventional policy at the monthly frequency. I find that asset purchase shocks in particular raise both consumer and professional expectations of inflation and professional expectations of output growth. In general, unconventional policy shocks raise the dispersion of macroeconomic expectations; asset purchases also raise the dispersion of interest rate expectations, while the other shocks have the opposite effect. Most importantly, I use the shocks to compute dynamic responses of both realized inflation and output up to 12 months, and find that asset purchases significantly raises both inflation and output growth.

Many previous studies have assessed the response of financial variables to unconventional U.S. monetary policy shocks. Among them, the only paper, to my knowledge, to separately identify forward guidance and asset purchase shocks is Swanson (2017). His results are broadly comparable with my estimates. For asset purchases, my findings also align with those of Krishnamurthy & Vissing-Jorgensen's event studies (2011, 2013). Other notable studies include Swanson (2011), Campbell, Evans, Fisher, & Justiniano (2012), and Coenen et al (2017).

The impact of non-Fed Funds shocks on professional expectations has been examined by Campbell, Evans, Fisher, & Justiniano (2012), Nakamura & Steinsson (2018), and Del Negro, Giannoni, & Patterson (2015). My results align best with those of the latter paper, which is the only one to attempt to separate forward guidance from asset purchases, albeit using dummies for each occuring. This paper is the first to additionally consider the response of consumer expectations.

While relatively little work has estimated the impact of unconventional policy on macroeconomic variables, Baumeister & Bennati (2013), Gambacorta, Hofmann, & Peersman (2014), Lloyd (2018), and Inoue & Rossi (2018) are exceptions. However, none of these papers has separated and simultaneously identified forward guidance and asset purchase shocks, making the present paper, to my knowledge, the first to offer a comparative analysis of the two. The first three papers identify a range of different shocks ("spread compression"; "balance sheet"; "signaling" and "portfolio balance" respectively) in VARs using sign and exclusion restrictions. Inoue & Rossi (2018) estimate local projections for two policy dimensions corresponding to the slope and curvature factors from a Nelson & Siegel (1987) decomposition; they do not have a way to separately identify forward guidance and asset purchase shocks. Baumeister & Bennati (2013) is the only paper to allow for a time-varying nature of shocks (in a parametric sense), using a time-varying parameters model. Gambacorta, Hofmann, & Peersman's (2014) findings for their balance sheet shock align well with the significant effects I find for my asset purchase shock.

The remainder of the paper is organized as follows. Section 2 discusses the identification problem and previous methodologies in more detail before outlining my approach and data. Section 3 presents the results across announcement days, describes the findings for notable FOMC announcements in detail, and characterizes the properties of the time-series of the implied shocks. Section 4 analyzes the high-frequency and daily responses of financial variables to the shocks. Section 5 computes the response of both expectations and realized values of macroeconomic aggregates to the measures. Section 6 concludes.

2 Intraday identification of monetary policy shocks

In this section, I first motivate the use of announcement-specific decompositions and argue that they can be computed using intraday data. I then discuss how shocks can be identified within a day's data based on time-varying volatility. I outline my approach to labeling the recovered shocks. Finally, I briefly sketch my implementation of the identification scheme.

2.1 The case for intraday identification

High-frequency identification of monetary policy shocks draws on the event-study methodology of empirical finance, as described by Campbell, Lo, and MacKinlay (1997). Those authors write abnormal returns, η_{it} for security i at time t, as

$$\eta_{it} = R_{it} - E\left[R_{it} \mid \mathscr{I}_{t-1}\right], \tag{1}$$

where $\mathscr{I}_{t-1}$ is the information set available at $t-1$ and $E\left[R_{it} \mid \mathscr{I}_{t-1}\right]$ is based on some model. Monetary policy shocks can thus be measured as the unpredictable change in an interest rate future, Treasury yield, or some basket of such asset prices around an announcement. Much recent work computes the price change from 10 minutes prior to an announcement to 20 minutes following; this measure can be either used directly, originating with Kuttner (2001), or as an instrument for some latent monetary policy shock (e.g., Gertler & Karadi (2015)).

A central identification assumption is that η_{it} is exogenous and does not include any endogenous response to additional contemporaneous shocks. For monetary policy, this generally requires that the only shock realized during the 30-minute announcement window is the shock of interest. This only makes sense if there is a single dimension of monetary policy, and thus a single "policy shock". However, substantial evidence suggests that there are in fact multiple dimensions of monetary policy, communicated through interest rate decisions, monetary policy statements, and press conferences, as discussed by GSS, for example. During the Great Recession, the dimensionality of policy became explicit, as the FOMC complemented a ZLB Fed Funds rate with forward guidance about the future Fed Funds rate and large scale asset purchases. Thus, for the recent period, η_{it} cannot be considered to represent a single exogenous policy shock unless one assumes that asset i responds contemporaneously to only a single type of news about monetary policy. Otherwise, η_{it} will contain a mixture of the shocks to different dimensions of monetary policy realized at time t.

The solution to this problem is to explicitly model η_{it} as a combination of different news shocks, ϵ_t, and decompose it accordingly. In particular, stacking n abnormal returns η_{it} into a vector η_t, the problem can be represented as

$$\eta_t = H\epsilon_t, t = 1, \ldots, T, \tag{2}$$

where ϵ_t is an $n \times 1$ vector of orthogonal shocks and H is a constant $n \times n$ invertible matrix. However, since ϵ_t is mean-zero, up to second moments, (2) provides only $\left(n^2 + n\right)/2$ identifying equations in n^2 parameters, so additional identifying information is needed. This is the familiar SVAR identification problem. A popular approach exploits heteroskedasticity, as proposed by Rigobon (2003), and applied to monetary policy and asset prices by Rigobon & Sack (2003) and Rigobon & Sack (2004). Multiple variance regimes generate additional identifying equations. This scheme has been widely extended, for example by Boyarchenko, Haddad, & Plosser (2015) and Gürkaynak, Kısacıkoğlu, & Wright (2018). Other options include imposing short-run exclusion or sign restrictions on parts of H. For all of these approaches, H must be constant across the data.

To estimate H consistently – and thus estimate ϵ_t consistently – requires a sample of many monetary policy announcement days, with changes in asset prices collected for each announcement and pooled to estimate H. Thus, η_t is replaced by a series η_d, with a single observed asset price change for each announcement date d (and similarly for ϵ_d) so

$$\eta_d = H\epsilon_d, d = 1, \ldots, D.$$

However, H must be constant for the entirety of the sample for any existing identification approach to be valid. Considering the Great Recession and the changes it saw in the conduct of monetary policy, as well as sometimes rapid changes in the state of the economy, it seems implausible that during this period the structural relationships of the economy remained constant. Indeed, after the Fed Funds rate hit the zero lower bound (ZLB) in December 2008, the state of the economy prompted the FOMC to introduce two novel dimensions of policy. Moreover, from announcement to announcement, the nature of the shocks received by markets may have changed; for example, there is no reason to believe that the elasticity of asset prices with respect to a horizon-based forward guidance announcement (e.g., the "mid-2015" language of September 2012) was the same as with respect to conditional forward guidance (e.g., the 6.5% unemployment and subdued inflation expectations language of December 2012). In fact, many interesting questions center on the relative efficacy of such subtle changes, which are hard to study with a time-invariant H matrix, allowing only the scale of shocks to change over time. Finally, if one wanted to estimate H across only calendar-based forward guidance announcements, for example, the sample would be far too small to justify any notion of consistency. These problems are compounded by the fact that many existing studies (e.g., Swanson (2017)) use samples dating back to far before the onset of the Recession.

For these reasons, I take a novel approach. Rather than viewing each day's monetary policy shock as being reflected in the change in asset prices over some window, I view intraday asset prices as responses to a continuous stream of news shocks. Monetary policy shocks are a subset of the day's news shocks – those that hit markets as a result of a monetary policy announcement. There are movements in interest rates even when no official announcements are made. Under rational expectations and efficient markets, such asset price movements must represent some form of news as well. On announcement days, these movements likely represent *speculation* about the content of the forthcoming announcement.

This re-framing of the problem as one of identifying high-frequency news shocks, which are present throughout the day, has significant implications. Since a single trading day generally contains many changes in given asset prices, reflecting many news shocks, a day-specific decomposition, H_d, can be identified – using only that day's fluctuations in asset prices. In particular, I model

$$\eta_{d_t} = H_d \epsilon_{d_t}, t = 1, \ldots, T, d = 1, \ldots D, \tag{3}$$

where t indexes time within a given announcement date, d. Thanks to the infill-asymptotic argument (e.g., Cressie (1993)) common in analysis of intraday financial data, identifying moments can be consistently estimated over the fixed time period of a trading day. This means that the set of intraday news shocks, ϵ_{d_t}, and thus the intraday monetary policy shocks, can be consistently estimated without assuming that $H_d \equiv H$ (constant across days) and instead assuming that H_d is constant across news shocks only within day d. In the case of the Great Recession and the constantly-changing nature of unconventional monetary policy, this provides the flexibility needed to characterize the potentially time-varying effects of monetary policy via H_d.

This approach also does not require the researcher to specify a fixed window over which to compute shocks. The appropriate length of such a window has been a topic of much debate. A full path of intraday shocks can be recovered, and then arbitrary subsets of those shocks can be studied, making clear the implications of focusing on a particular window.

On the other hand, this exercise is complicated by the presence of noise and other features of intraday data, which may play less of a role when simple 30-minute windows are used. However, without exploiting intraday time series, it would never be possible to consistently estimate the effects of infrequent events (e.g., the effect of conditional guidance as opposed to calendar-based). Proposition 2 below addresses the role of noise and such concerns motivate a number of robustness checks in my analysis, which I present in Section 3.3.

2.2 Identification via time-varying volatility

Having argued that equation (3) represents the intraday response of asset prices to news shocks, it remains to identify H_d, or, equivalently, the news shocks ϵ_{d_t}. While many identification approaches impose assumptions on H_d (exclusion or sign restrictions), to do so is unappealing in general as H_d is the object of interest and in this case in particular because it is hard to argue that some asset prices systematically respond more slowly to forward guidance or asset purchase shocks, for example. Swanson (2017) proposes a scheme to identify Fed Funds, forward guidance, and asset purchase shocks, but his assumptions are not well-suited to the intraday framework I adopt. While his assumption that front month interest rate futures do not respond to unconventional monetary policy is applicable, his assumption to distinguish forward guidance and asset purchases, that large scale asset purchase shocks played a minimal role prior to January 2009 (around the beginning of QE1), has no analog for intraday identification post-2008.

These factors lead me to consider statistical identification, in particular identification based on time-varying volatility. As previously discussed, identification via heteroskedasticity has proven popular for identifying asset price responses to news and policy shocks. However, traditional identification via heteroskedasticity requires the specification of variance regimes, ideally *ex ante* based on external information. It is not obvious how to define intraday periods of high volatility across announcement dates. In particular, on some days asset prices appear to be very volatile only immediately following monetary policy announcements, while on others, the volatility is elevated at various times throughout the day. While Rigobon (2003) contends that misspecification of regimes does not hinder consistent estimation, Lewis (2018) argues that if such misspecification leads to the structural variances being similar across regimes, a weak identification problem is likely to arise. Multiple dimensions of monetary policy present a prime case for weak identification if shock volatilities move together even if the individual volatility changes are pronounced. Alternatively, variance regimes could be estimated daily, but Lewis (2019) argues that such estimated regimes are endogenous with respect to the structural shocks, which introduces bias into estimates.

I therefore identify (3) based on time-varying volatility (TVV-ID), following the results of Lewis (2019). This strategy identifies the model based on the autocovariance of $\eta_{d_t}\eta_{d_t}'$, which, provided ϵ_{d_t} is a martingale difference sequence, exploits persistence in the time-varying volatility of ϵ_{d_t} for identification. This result generalizes the parametric identification results of Rigobon (2003) and Sentana & Fiorentini (2001) (who argue for identification from a continuous path of reduced-form variances) to a non-parametric identification result based simply on the presence of persistent time-varying volatility in the structural shocks, which may take a near-arbitrary form. The argument makes no requirement that any reduced form covariances of η_{d_t} be supplied as identifying moments, a requirement which drives both the Rigobon and Sentana & Fiorentini approaches to exploit their particular parametric forms (discrete regimes and a deterministically-recoverable GARCH-type process, respectively).

More formally, Assumption 1 lays out assumptions for TVV-ID. I henceforth suppress d subscripts for compactness; all observations and parameters remain date-specific, unless otherwise noted.

Assumption 1. *For every* $t = 1, 2, \ldots, T$,

1. *H is fixed, full-rank, and has a unit diagonal,*
2. *σ_t is an $n \times 1$ stationary stochastic process,*
3. $E(\varepsilon_t \mid \sigma_t, \mathcal{F}_{t-1}) = 0$ and $\mathrm{Var}(\varepsilon_t \mid \sigma_t, \mathcal{F}_{t-1}) = \Sigma_t$,
4. $\Sigma_t = diag\left(\sigma_t^2\right), \sigma_t^2 = \sigma_t \odot \sigma_t$,
5. $\mathrm{Var}\left(\sigma_t^2\right) < \infty$,
6. $\mathrm{Var}\left(\varepsilon_t \varepsilon_t'\right) < \infty$.

I assume stationarity of σ_t for clarity and coherence with empirical practice, unlike in the statements of Lewis (2019). The first assumption is a standard requirement for identification of models of the form (2). The third requires that ϵ_t is a martingale difference sequence with respect to the filtration $\mathcal{F}_{t-1} = \{\varepsilon_1, \ldots \varepsilon_{t-1}, \sigma_1, \ldots \sigma_{t-1}\}$ and σ_t, a form of the standard assumption that ϵ_t are uncorrelated and orthogonal to observables; the conditioning on σ_t

means that the volatility cannot help to predict the sign of shocks. The fourth stipulates orthogonality of shocks, and the final two assumptions are regularity conditions.

Under these assumptions, the autocovariance of squared residuals provides identifying equations. Define L and G to be elimination and selection matrices respectively.[1] Lewis (2019) shows that for $\zeta_t = vech\left(\eta_t \eta_t'\right)$,

Proposition 1. *Under Assumption 1,*

$$Cov\left(\zeta_t, \zeta_{t-p}\right) = L\left(H \otimes H\right) G M_p \left(H \otimes H\right)' L', \; p > 0 \tag{4}$$

where

$$M_p = E\left[\sigma_t^2 vec\left(\varepsilon_{t-p}\varepsilon_{t-p}'\right)\right] - E\left[\sigma_t^2\right] E\left[\sigma_t^2\right]' G'.$$

Define $\tilde{M}_p = \left[\begin{array}{cc} M_p & E\left[\sigma_t^2\right] \end{array}\right]$. Using the decomposition (4) of the autocovariance of ζ_t, the following theorem holds (Theorem 2 of Lewis (2019)) using (4) as identifying equations:

Theorem 1. *Under Assumption 1, equation (4) holds. Then H and $\tilde{M}_p$ are jointly uniquely determined from (4) and $E\left(\zeta_t\right)$ (up to labeling of shocks) provided rank $\left(\tilde{M}_p\right) \geq 2$ and $\tilde{M}_p$ has no proportional rows.*

Briefly, the rank condition will hold provided there is at least one dimension of time-varying volatility in the data.[2] The proportionality condition will hold provided that, for no two dimensions of σ_t^2, say $\sigma_{it}^2, \sigma_{jt}^2$, all respective autocovariances with $vec\left(\varepsilon_{t-p}\varepsilon_{t-p}'\right)$ are related by the ratio $E\left[\sigma_{it}^2\right] / E\left[\sigma_{jt}^2\right]$. Like other identification arguments based on heteroskedasticity, this implies that $n-1$ dimensions of σ_t^2 must vary. H is identified from (4) up to a labeling of the shocks, which I discuss below. Unlike factor models (e.g., GSS, Swanson (2017), Inoue & Rossi (2018)), H represents a truly unique decomposition, and is not only unique up to orthogonal rotations. For additional details and discussion of these results, see Lewis (2019).

In the current context, where all asset price movements are viewed as responses to news shocks, with those during certain parts of the day interpreted as monetary policy shocks, it may be of concern that there is some substantial difference between news shocks that are monetary policy shocks and those that are not. This might imply a different H for those shocks that are not monetary policy shocks. These other news shocks will generally be of lower variance, consistent with a possible "noise" interpretation when no meaningful new information is reaching markets. Proposition 2 demonstrates that, provided these other

[1] This means $vech\left(K\right) = Lvec\left(K\right)$ and $vec\left(KDK'\right) = \left(K \otimes K\right) Gd$ where $d = diag\left(D\right)$.

[2] This is true barring an extremely degenerate case, where all columns of M_p are proportional to$E\left[\sigma_t^2\right]$.

news shocks have relatively lower variance than monetary policy shocks, identification of H is asymptotically unaffected.

Proposition 2. *Suppose that for news shocks during non-announcement periods, $\eta_t = H_N \epsilon_t$, and for monetary policy shocks following monetary policy announcements, $\eta_t = H_{MP}\epsilon_t$, with $H_N \neq H_{MP}$; assume that within each period, the σ_t^2 process is stationary with respective means $\sigma_N^2, \sigma_{MP}^2$. Then if $\frac{\sigma_{i,N}^2}{\min_j \sigma_{j,MP}^2} \to 0$, for all $i = 1, \dots, n$, the H identified by Theorem 1 from full-sample moments is H_{MP}, provided the monetary policy shocks are not measure zero.*

While only an asymptotic result, this proposition suggests the impact on identification of news shocks with distinct properties will not be fatal to the extent that asset price movements during other periods of the day are assumed to be relatively low-variance.

2.3 Labeling of shocks

As with any statistical identification scheme, the obtained intraday shocks, ϵ_t, and thus the columns of H are only identified up to ordering. It remains to label the shocks according to the dimensions of monetary policy. In the present application, as discussed below, I consider a baseline model containing front-quarter Eurodollar (ED) futures, 2-year/8-quarter ED futures, and 10-year Treasury yields, to span short-term and medium term expectations of the Fed Funds rate and a major liquid market impacted by asset purchases. My labeling procedure assumes that the three recovered shocks are the current Fed Funds rate shock, a forward guidance shock, and an asset purchase/QE shock. They could alternatively be viewed, less structurally, as simply shocks at three different points on the yield curve. I select the labeling of the shocks such that the matrix of R^2 values from the regression of each innovation on each shock is as close as possible to the identity (with rows in order of increasing horizon and columns (*Fed Funds*, *Forward guidance*, *Asset purchase*)). This obtains the labeling most compatible with the assumption that the front future is best-explained by the Fed Funds shock (consistent with Swanson's (2017) identification), the 2-year future is best-explained by the forward guidance shock, and the 10-year Treasury is best-explained by the asset purchase shock. The first point is standard; the latter two are compatible with the horizon of forward guidance announcements (generally in the two year range), and the type and maturity of assets included in much of QE.[3] For observations prior to the first suggestions of large scale asset purchases in November 2008, the final shock may

[3]For two announcements, I alter the labeling decision due to the fact that notable asset purchase shocks were not prominently reflected in long-term rates due to announcement-specific factors.

need to be interpreted slightly differently, as a second dimension of news orthogonal to that which drives medium-term expectations.

Note that while news shocks raise concerns about invertibility of VAR residuals (e.g., Sims (2012), Plagborg-Møller (2018)), this is not a problem for the news shocks I describe above. Intuitively, invertibility fails when observable series do not fully capture the state variables of the economy, for example, unobservable news shocks. However, while the shocks described above include news shocks, if markets are efficient, these shocks are contemporaneously reflected in the observed asset price series, so they can be recovered from the reduced-form innovations.

2.4 Implementation

TVV-ID permits a wide range of estimation approaches. Indeed, any estimator that either explicitly (e.g., GMM estimation of (4)) or implicitly (a quasi-maximum likelihood approach) fits an autocovariance of the volatility process to the $\eta_t \eta_t'$ data can be used. Given the minimal nature of Assumption 1, the class of possible likelihoods is very general. Lewis (2019) compares the performance of the entirely non-parametric GMM approach to a quasi-maximum likelihood (QML) approach based on an AR(1) SV model, along with many alternatives in a simulation study, concluding that the AR(1) SV model performs very and is quite robust to misspecification. Given this finding, I estimate H throughout this paper using QML based on the AR(1) SV model. I adopt the EM algorithm developed in Lewis (2019), which extends prior work by Chan & Grant (2016) and Bertsche & Braun (2018) to accommodate correlation across dimensions of the SV process. Moreover, note that even though this implementation fits moments besides the autocovariance and mean of ζ_t, similar logic to Proposition 2 applies to many of the additional full-sample moments implied and estimated by the AR(1) SV model. While a continuous volatility model may seem unnatural for prices that are sometime constant for extended periods, my innovations are VAR residuals, discussed below, which generally vary even when prices themselves do not.

3 The intraday shocks

To obtain a vector η_t of unpredictable asset price innovations, I begin with a vector of observed asset prices,

$$y_t = \begin{pmatrix} ED1_t \\ ED8_t \\ T10_t \end{pmatrix},$$

the front-quarter ED rate, 8-quarter ED rate, and 10-year Treasury yield. I consider each scheduled FOMC announcement date from January 2007 to December 2018. For each day, the sample spans 9:30am to 4:15pm. These three assets are chosen to span important parts of the yield curve; additional justifications for this model (in particular as opposed to a factor model), as well as details of the data more generally, are discussed in Section 1 of the Supplement. I first-difference the data for stationarity due to possible cointegration of rates across the term structure, as noted by Campbell & Shiller (1987). I then estimate a VAR(p) to obtain the unpredictable innovations to each series. p is chosen separately for each day using the Hannan & Quinn (1979) information criterion, which consistently estimates VAR order. The optimal p ranges from 1 to 15 with a median of 2. I thus obtain η_t as

$$\Delta y_t = A_0 + \sum_{l=1}^{p} A_l \Delta y_{t-l} + \eta_t. \tag{5}$$

Using the estimated residuals $\hat{\eta}_t$, I estimate (3), fitting an AR(1) SV process to the variance of $\hat{\epsilon}_t$, via QML.

3.1 Results

I group and present the results in three categories. My central results are based on historical decompositions computed daily. These characterize the cumulative causal effect of each type of shock on each asset price for each day, at all horizons from 10 minutes prior to the FOMC announcement. First, however, as an overview, I document the distributions of the H_d estimates, which represent an instantaneous pass-through to rates. I then present the key historical decompositions. To aid in assessing the statistical significance of these cumulative effects, I derive new expressions for the asymptotic variance of historical decompositions, allowing for frequentist inference that takes into account the fact that the shocks themselves are estimated with uncertainty that is not independent of the estimates of the structural parameters. I plot historical decompositions, with these novel standard errors, for twelve days with notable unconventional monetary policy announcements, along with summary statistics for the 96-day super-sample. Finally, I compute inter-announcement time-series of structural shocks and compare these to a timeline of key historical events.

The distribution of H across announcements

Comparing across announcement days shows substantial heterogeneity in the instantaneous response of asset prices to the three monetary policy shocks. Table 1 reports summary statistics for each free element of H, including the frequency with which one-sided tests reject

Table 1: Summary statistics for $\hat{H}$

	mean	q_{10}	median	q_{90}	positive	positive sig. 5%	negative	negative sig. 5%
$H_{ED8,FF}$	0.58	-0.02	0.31	1.23	78	44	15	2
$H_{T10,FF}$	0.18	-0.06	0.09	0.57	70	33	23	1
$H_{ED1,FG}$	0.00	-0.02	0.00	0.05	65	21	28	9
$H_{T10,FG}$	0.32	-0.00	0.24	0.35	86	82	10	6
$H_{ED1,AP}$	0.01	-0.01	0.00	0.04	58	10	35	4
$H_{ED,AP}$	0.15	-0.37	0.02	0.82	58	20	38	11

Estimates of $\hat{H}$ from AR(1) SV model. Shocks labeled so that the Fed Funds shock best predicts the 1-quarter Eurodollar rate, the forward guidance shock the 8-quarter Eurodollar rate, and the asset purchase shock the 10-year Treasury. H is unit-diagonal normalized based on this labeling. Estimates reflect the percentage point response to a shock that increases the reference rate by 1%. Responses of the 10-year Treasury are scaled to the day's constant-maturity Treasury yield. The right panel tabulates the signs and reports one-sided tests. For three dates, the front Eurodollar and Fed Funds shock are dropped due to zero intraday movement in the front contract.

zero. Figure 6 in the Supplement plots histograms of the estimates. Broadly speaking, the results accord with theory. A positive (contractionary) Fed Funds shock, for the most part, raises expectations of future short rates (8-quarter Eurodollar) and 10-year Treasury yields, often to a statistically significant extent; this implies sensible behaviour for expectations of future short rates and accords with any term structure model for the 10-year Treasury. The forward guidance shock on average has zero impact on front Eurodollar rates and a positive impact on 10-year Treasury yields. The presence of a small positive impact on 1-quarter Eurodollar rates for many days is consistent with the fact that for some announcement days, there is a further scheduled announcement before the contract expires, leaving scope for some forward guidance effect. The asset purchase shock has zero effect on average on 1-quarter Eurodollar rates, as expected. The sign and significance of the impact of the asset purchase shock on the 8-quarter Eurodollar rates is quite variable. This is consistent with the possibility that over the course of the Recession the character of the shocks varied, or that forward guidance and asset purchases may have at times been seen as complements and at others as substitutes, as discussed in more detail under the correlated shocks robustness check. It is important to remember that these elasticities are only contemporaneous at a high frequency, and, while indicative of short-run dynamics, may not give a complete picture of responses at either a 30-minute window, or by the end of the day, for example. I now turn to analyzing those relationships.

Historical decompositions

The object of policy interest is not the instantaneous response of interest rates to a single minute's realization of a monetary policy shock, since that is unlikely to play a role in shifting macroeconomic aggregates. To have any meaningful effect on financial conditions or slower-moving macro variables like inflation or unemployment, a response must be persistent and account for the true size of the surprise registered by markets, a process which may take time. In this intraday setting, this question can only be addressed using historical decompositions. Furthermore, previous work has often used a 30-minute window in the event study framework, but acknowledged that full effects likely take longer. Since I am able to simultaneously estimate ϵ_t throughout the day, an additional benefit of computing historical decompositions is that I can assess the extent to which focusing on a 30-minute window may be misleading, relative to considering end-of-day measures.

Historical decompositions are computed based on impulse response functions. The structural impulse response function at horizon h can be computed as

$$\Phi^h = H, h = 0,$$
$$\Phi^h = \sum_{l=1}^{min(h,p)} A_l \Phi^{h-l}, h = 1, 2, \ldots.$$

Here, since the data are first-differenced prior to estimating the VAR, the object of interest is the cumulative impulse response function (IRF), $\tilde{\Phi}^h$, which is

$$\tilde{\Phi}^h = \sum_{i=0}^{h} \Phi^i.$$

For simplicity, I factor $\Phi^h \equiv B^h H$ and $\tilde{\Phi}^h \equiv \tilde{B}^h H$, the product of the reduced-form IRF or cumulative IRF with the response matrix, H. The responses at time t to a shock realized at $t - s$ can thus be computed as $\tilde{\Phi}^s \epsilon_{t-s}$. The historical decomposition, Ψ_t, takes into account all shocks realized since some start date, τ, so

$$\Psi_t = \sum_{s=0}^{t-\tau} \tilde{B}^s H \epsilon_{t-s}.$$

These objects are simple to compute once the IRF has been obtained. While an exhaustive literature has considered inference on impulse response functions, using both asymptotic results and bootstrap procedures, the asymptotic results have not been extended to frequentist inference on historical decompositions (the problem is straightforward in a Bayesian context). However, in the present exercise, the statistical significance of these cumulative effects of monetary policy shocks determines the significance of the policy measure. Inference cannot rely on IRF results and ignore the estimation error in ϵ_t, since that error will be correlated across t and with estimation error of the IRF. To that end, Theorem 2 presents what I believe to be a novel result, an analytic expression for the asymptotic variance of Ψ_t. Let the vector θ collect the parameters of H, A_0, and $\{A_l\}_{l=1}^{p}$, and denote the decomposition with respect to shock j as Ψ_{jt}. I focus on the case of *cumulative* IRFs (and thus historical decompositions) as my VARs are estimated in first differences, so cumulation of responses is required to obtain results in terms of interest rates.

Theorem 2. *For consistent and asymptotically normal estimators of θ, the asymptotic variance of the corresponding estimator of Ψ_{jt} is equal to*

$$V_{\Psi_t} = \frac{\partial \Psi_{jt}}{\partial \theta'} V_\theta \frac{\partial \Psi'_{jt}}{\partial \theta},$$

where

1. $V_\theta = \begin{bmatrix} V_A & 0 \\ 0 & V_H \end{bmatrix}$, *where V_A is the asymptotic variance of the VAR coefficients and V_H is the asymptotic covariance of the elements of $\hat{H}$ and*

2. $\frac{\partial \Psi_{jt}}{\partial \theta} = \sum_{s=0}^{t-\tau} \frac{\partial \tilde{\Phi}^s \iota_j \epsilon_{t-s}}{\partial \theta'}$, *where*

$$\begin{aligned} \frac{\partial \tilde{\Phi}^s \iota_j \epsilon_{t-s}}{\partial \theta'} &= \left(\epsilon'_{t-s}\iota_j\right) \otimes I_n \begin{bmatrix} 0_{n^2 \times n} & (H' \otimes I_n) F_s & I_n \otimes \tilde{B}^s \end{bmatrix} \\ &\quad + \tilde{\Phi}^s \iota_j \begin{bmatrix} H^{-1} & H^{-1}(X_{t-s} \otimes I_n) & \left(\eta'_{t-s} \otimes I_n\right)(H'^{-1} \otimes H^{-1}) \end{bmatrix}, \end{aligned}$$

with

(a) $F_0 = 0_{n \times n^2 p}$, $F_s = \sum_{v=1}^{s} \sum_{m=0}^{v-1} J\left(\mathbf{A}'\right)^{v-1-m} \otimes B^m \;\forall s > 0$, *where* $J = \left[I_n 0_{n \times n(p-1)}\right]$ *and*

$$\mathbf{A} = \begin{bmatrix} A_1 & A_2 & \dots & A_{p-1} & A_p \\ I_n & 0 & \dots & 0 & 0 \\ 0 & I_n & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I_n & 0 \end{bmatrix},$$

(b) $X_{t-s} = \begin{bmatrix} y'_{t-1} & y'_{t-2} & \dots & y'_{t-p} \end{bmatrix}$.

(c) ι_j *a selection matrix such that* $\iota_j \epsilon_t = \begin{pmatrix} 0_{j-1} & \epsilon_{jt} & 0_{n-j} \end{pmatrix}'$.

The proof is given in the Appendix. Note that owing to the heteroskedasticity in the models considered here, a heteroskedasticity-robust estimator of V_A should be used, which is given by $V_A = E\left[\left(\tilde{X}_t'\tilde{X}_t\right)\otimes I_n\right]^{-1} E\left[\left(\tilde{X}_t'\otimes\eta_t\right)\left(\tilde{X}_t'\otimes\eta_t\right)'\right] E\left[\left(\tilde{X}_t'\tilde{X}_t\right)\otimes I_n\right]^{-1}$, where $\tilde{X}_t = \begin{bmatrix} 1 & X_t \end{bmatrix}$. Armed with this result, I can construct confidence intervals for historical decompositions for each series in y_t with respect to each shock on each announcement day I consider. While Theorem 2 is written for the cumulative IRF (i.e. for level effects on a regressor that enters the VAR in differences), it can be immediately applied to non-cumulative IRFs by redefining $F_s = \sum_{m=0}^{s-1} J\left(\mathbf{A}'\right)^{s-1-m}\otimes B^m \ \forall s>0$.

Table 2 reports summary statistics for the absolute value of historical decompositions at two different horizons. The first panel reports decompositions at 20 minutes following the announcements due to shocks starting from 10 minutes prior to the announcements (the usual 30-minute event-study window). The second panel reports decompositions at 4:15pm due to shocks starting from 10 minutes prior to the announcements. It documents several facts. First, at both horizons, taking simple changes in asset prices, reported in the first two columns, will over-state the size/effect of a particular shock due to the fact that all movements are taken as due to that particular shock of interest, when observed movements are generally due to a combination of realized shocks. Second, the size of the decompositions due to each shock are, on average, generally comparable across the two horizons. However, as the plots for individual dates below make clear, this obscures sometimes substantial differences for a given day. Finally, despite comparable magnitudes, the effects of the shocks, as quantified by the decompositions, are much more likely to be statistically significant at the 30-minute horizon than at the end-of-day horizon, presenting a second dimension on which the window considered can matter.

Table 3 directly reproduces Table 1 of Swanson (2017), adding two additional dates. It reports the selection of highly notable FOMC announcements I address individually, along with key details. Figure 2 plots the historical decompositions for each of these dates for each interest rate with respect to each of the three shocks. Each column plots responses for a given day, with each panel plotting responses of the indicated rate to the three shocks. For comparison, simple changes, as one would calculate in an event study, are plotted for each rate.[4] Four facts are immediate. First, with the obvious exception of December 2008, when the ZLB was reached, there is virtually no effect of shocks to the current policy rate (conventional monetary policy) on these days, consistent with the rate being at the ZLB. Second, simply computing changes in asset prices would frequently be misleading, to an episode-dependent extent, due to the fact that multiple shocks of meaningful size are generally realized. This

[4] Note that the decompositions will not, in general, add up to this path since the regressions are based on first-differenced data.

Table 2: Summary statistics for historical decompositions

	30-minute window						
	mean $\lvert\Delta P_{ref}\rvert$	median $\lvert\Delta P_{ref}\rvert$	mean decomp.	median decomp.	sig. at 10%	sig. at 5%	sig. at 1%
$ED1, FF$			0.02	0.01	57	53	47
$ED1, FG$	0.02	0.01	0.00	0.00	24	14	8
$ED1, AP$			0.00	0.00	12	5	1
$ED8, FF$			0.01	0.00	21	17	11
$ED8, FG$	0.05	0.03	0.03	0.02	77	73	61
$ED8, AP$			0.01	0.00	33	26	14
$T10, FF$			0.00	0.00	18	12	8
$T10, FG$	0.02	0.01	0.01	0.01	66	56	46
$T10, AP$			0.01	0.01	61	49	43
	End-of-day window						
	mean $\lvert\Delta P_{ref}\rvert$	median $\lvert\Delta P_{ref}\rvert$	mean decomp.	median decomp.	sig. at 10%	sig. at 5%	sig. at 1%
$ED1, FF$			0.02	0.01	24	16	7
$ED1, FG$	0.03	0.01	0.00	0.00	6	2	2
$ED1, AP$			0.00	0.00	2	0	0
$ED8, FF$			0.01	0.00	9	5	2
$ED8, FG$	0.06	0.04	0.03	0.02	24	13	5
$ED8, AP$			0.01	0.00	9	5	2
$T10, FF$			0.00	0.00	7	3	2
$T10, FG$	0.03	0.02	0.01	0.00	18	9	3
$T10, AP$			0.01	0.01	24	13	2

Summary statistics for the historical decompositions of each rate with respect to the three shocks; the top panel considers the decomposition based on shocks occurring between 10 minutes prior to the announcement and 20 minutes following, and the bottom considers 10 minutes prior until 4:15pm. The units are percentage points. The first two columns summarize the absolute values of the simple change in the reference rate over the window. The next two columns repeat the exercise for the absolute value of the historical decompositions. The entries for the response of the 10-year Treasury are scaled by the ratio of the end-of-day constant-maturity zero–coupon 10-year Treasury yield to the end-of-day value in the data. Standard errors are computed using Theorem 2.

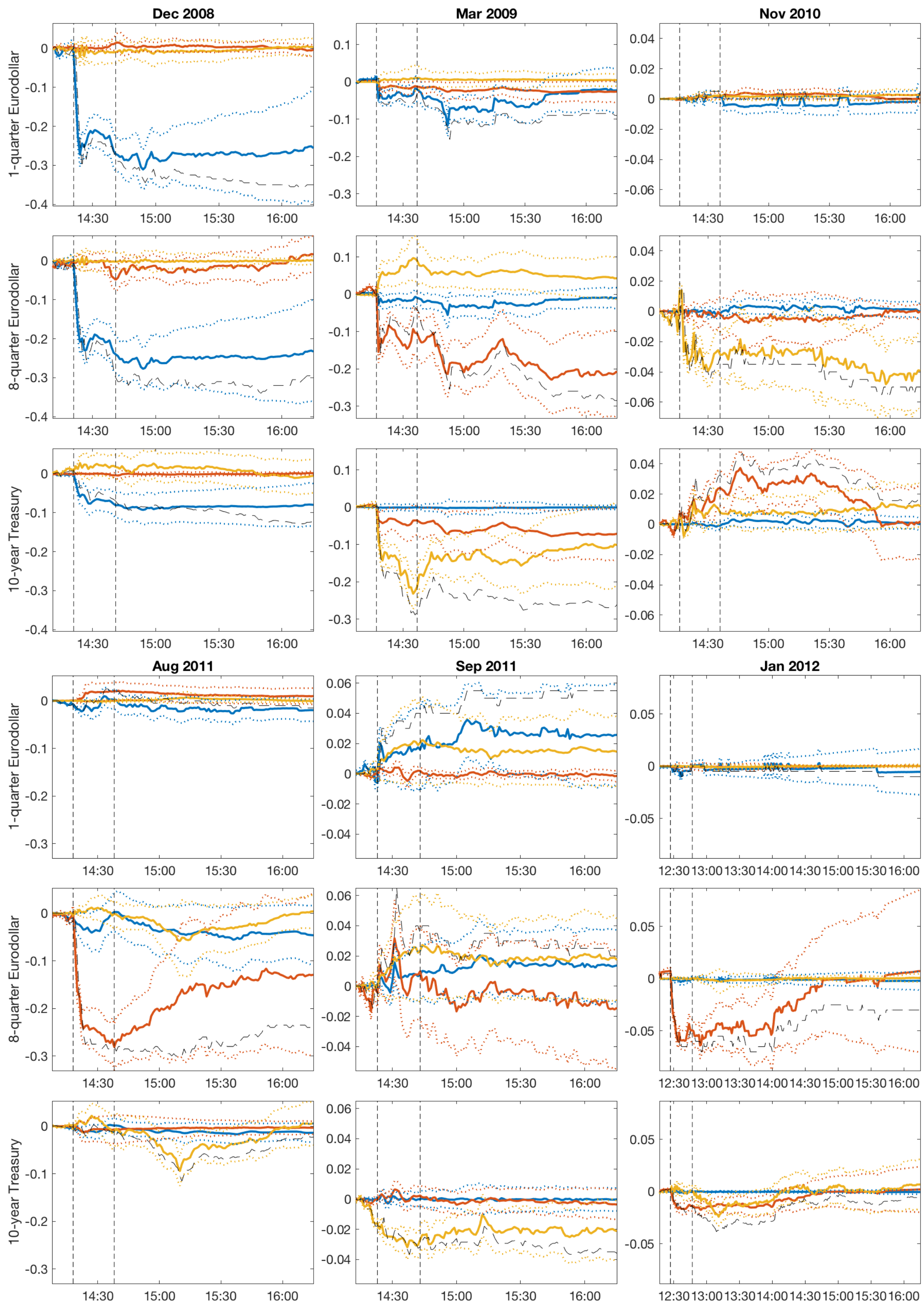
Table 3: Key FOMC announcements 2008-2015

December 2008	FOMC announces that it has cut the FFR to between 0 and 25 basis points (bp), will purchase large quantities of agency debt and will evaluate purchasing long-term Treasuries
March 2009	FOMC announces it expects to keep the federal funds rate between 0 and 25 bp for "an extended period", and that it will purchase \$750B of mortgage-backed securities, \$300B of longer-term Treasuries, and \$100B of agency debt (a.k.a. "QE1")
November 2010	FOMC announces it will purchase an additional \$600B of longer-term Treasuries (a.k.a. "QE2")
August 2011	FOMC announces it expects to keep the federal funds rate between 0 and 25 bp "at least through mid-2013"
September 2011	FOMC announces it will sell \$400B of short-term Treasuries and use the proceeds to buy \$400B of long-term Treasuries (a.k.a. "Operation Twist")
January 2012	FOMC announces it expects to keep the federal funds rate between 0 and 25 bp "at least through late 2014"
September 2012	FOMC announces it expects to keep the federal funds rate between 0 and 25 bp "at least through mid-2015", and that it will purchase \$40B of mortgage-backed securities per month for the indefinite future
December 2012	FOMC announces it will purchase \$45B of longer-term Treasuries per month for the indefinite future, and that it expects to keep the federal funds rate between 0 and 25 bp at least as long as the unemployment remains above 6.5 percent and inflation expectations remain subdued
September 2013	FOMC announces that it will wait to taper asset purchases
December 2013	FOMC announces it will start to taper its purchases of longer-term Treasuries and mortgage-backed securities to paces of \$40B and \$35B per month, respectively
December 2014	FOMC announces that "it can be patient in beginning to normalize the stance of monetary policy"
March 2015	FOMC announces that "an increase in the target range for the federal funds rate remains unlikely at the April FOMC meeting"

Descriptions of announcements are replicated from Swanson (2017), with the addition of December 2008 and September 2013.

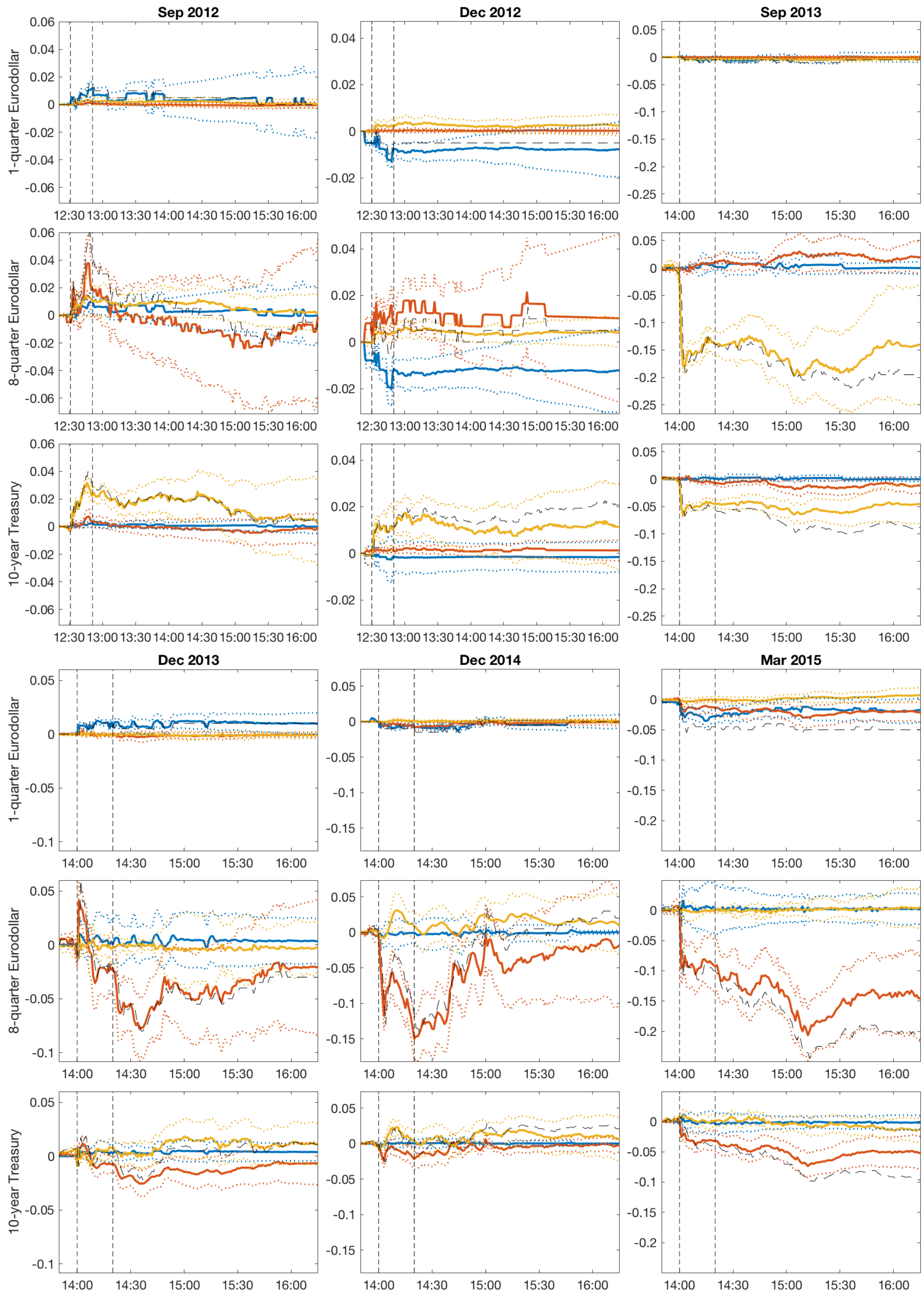
may be the case even on days when explicit statements were only made about one dimension of policy, but market expectations were revised on additional dimensions. Third, focusing on only the 30-minute windows around announcements may be misleading. In some cases, as previous work has speculated, effects do continue to grow before the end of trading, but, more often, effects apparent in the 30-minute window do not persist to the end of the day. From a macroeconomic perspective, the 30-minute window thus may *overstate* the relevant monetary policy shocks. Finally, there are very few announcements and shocks for which there are statistically significant effects by the end of the day. I now briefly interpret the results for each announcement in turn. Magnitudes reported are in percentage points at end-of-day, and significance is discussed at the 5% level.

Figure 2: Historical decompositions of key FOMC announcements



Historical decompositions for the rate series indicated in the left margin with respect to each of the three shocks. 95% confidence intervals are plotted based on the results of Theorem 2. The vertical lines mark the time of the announcement and 20 minutes following the announcement, the end of the conventional analysis window. The black dashed path is the path of the simple change from ten minutes prior to the announcement, the event study estimate. Units are percentage points.

Figure 2b: Historical decompositions of key FOMC announcements (cont'd)



See Figure 2 for notes.

December 2008 The only shock of note is to the Fed Funds rate, which hit the ZLB for the first time. This results in a statistically significant decrease in all three interest rate series. The suggestion that the Fed may purchase Treasuries appears not to surprise markets, having no discernible effect.

March 2009 The first iteration of forward guidance significantly lowers medium-term expectations of short rates (-0.21) and long term rates (-0.07). The announcement of QE1 lowers long-term rates (-0.10), with $p = 0.08$; perhaps puzzlingly, it increases medium-term expectations of short rates. The impacts of these shocks on medium-term expectations of short rates and long-term rates respectively are of comparable magnitude. Note that the former would have been understated (-0.10 instead of -0.21) by using a 30-minute window, while the latter would been have overstated (-0.21 instead of -0.10).

November 2010 The announcement was dominated by the launch of QE2, which significantly lowers medium-term expectations of short rates (-0.04), but does not appear in longer-term rates, which actually rose.[5] Examining contemporary market commentary, it appears that the $600B pledged was towards the upper end of market expectations, but the rate of purchases, $75B per month, was somewhat low relative to expectations; a perceived focus on medium-term securities may also have been disappointing (Anderson & Englander (2010)). Moreover, the apparent non-response of long rates may reflect a trading strategy of "buying the rumour and selling the fact", discussed by commentators prior to the announcement (e.g., Capo McCormick (2010)).

August 2011 The first case of calendar-based guidance ("mid-2013") has a pronounced effect on medium-term expectations of short rates (-0.13), but it is not significant by the end of the day, $p = 0.14$. Using a 30-minute window would substantially inflate this effect (-0.28).

September 2011 The asset purchase shock of "Operation Twist" has a modest but significant downward effect on long-term rates (-0.02). Puzzlingly, it has a near-significant positive effect on medium-term expectations of short rates (0.02). This might reflect a supply response to news of the Fed selling shorter-term Treasury securities, safe assets of comparable maturities to the 8-quarter Eurodollar contract.

[5] Accordingly, this date is one of the two in which the labeling rule appears inaccurate, and the reported results reflect a switch of forward guidance and asset purchase shocks; the mislabeling is a result of the fact that long-term rates (over 7 years) actually rose following the announcement.

January 2012 The extension of guidance to "late-2014" initially causes a dramatic fall in medium-term expectations of short rates, which does not persist to the end of the day (in fact reversing); using a 30-minute window would estimate an effect of -0.05. This is consistent with the fact that many analysts expected language to be extended to some point in 2014 (Blackden (2012), Crutsinger (2012)).

September 2012 Neither the extension of guidance to "mid-2015" nor the announcement of $40B purchases of mortgage-backed securities per month has a significant effect. The extension of guidance was expected, possibly out to *late*-2015 (Kucukreisoglu (2012)). Of course, a purchase of MBS need not lower risk-free rates, so the non-response to the asset purchase shock may be unsurprising. The announcement was also widely-expected, with some sources reporting its magnitude fell short of expectations, while others found it larger than expected, (Klein (2012), Kucukreisoglu (2012), Popper (2012)).

December 2012 Neither the replacement of calendar-based guidance with conditional guidance nor the announcement of $45B in Treasury purchases for the indefinite future has a significant impact on markets, with the latter actually raising long-term rates slightly. While the former was unanticipated, market expectations may have translated the given numbers to the calendar-based horizon already in place (Goldfarb (2012)); the latter was anticipated (Irwin (2012)).

September 2013 The announcement that the Fed would wait to taper asset purchases leads to a significant decrease in long-term rates (-0.05) and a significant decrease in medium-term expectations of short rates (-0.14).[6]

December 2013 The announcement that asset purchases would be tapered has only a minor, non-significant positive effect on long-term rates; a clarification of conditional guidance – that the target rate is unlikely to change until "well past" the time that unemployment falls past 6.5%, leads to a non-significant fall in medium-term expectations of short rates. The former is consistent with the relatively small scale of the tapering ($10B) and the fact that some analysts anticipated the move (Appelbaum (2013)).

December 2014 The announcement that the Fed would be "patient" in normalizing monetary policy ultimately has a minimal impact (-0.02) on medium-term expectations of short rates. Focusing on a 30-minute window would risk overstating the effect (-0.15).

[6]This date is one of the two in which the labeling rule appears inaccurate, and the reported results reflect a switch of forward guidance and asset purchase shocks.

This is consistent with contemporary discourse, with many analysts expecting some revision to the "considerable time" language (Chen & McMahon (2014)).

March 2015 The announcement that rates would stay at the ZLB through at least the April FOMC meeting significantly reduces both medium-term expectations of short rates (-0.15) and long-term rates (-0.05) (and the 1-quarter Eurodollar rate). Using a 30-minute window would substantially underestimate the effect.

While it is unclear whether the correct bar for a meaningful monetary policy shock is necessarily a cumulative response in the respective interest rate that is significant at the 5% level, the subset of episodes that do meet this bar is interesting. In particular, the first forward guidance announcement in March 2009, ("extended period") and the final March 2015 announcement of an additional FOMC cycle at the ZLB pass the bar. On the asset purchase side, Operation Twist (September 2011) led to a significant decrease in long-term rates and the September 2013 decision to delay tapering significantly lowered both long rates and expectations of future short rates, while the November 2010 announcement of QE2 significantly lowered expectations of future short rates. Relaxing the bar for significance to the 10% level admits the QE1 announcement of March 2009.

For forward guidance, this suggests that the calendar-based guidance did not convey significant new information that markets did not already anticipate throughout 2011 and 2012, nor did the switch to conditional guidance change this relationship. Rather, the introduction of forward guidance, an unprecedented move, and its extension beyond the point where markets expected rates to "lift-off" are the two episodes that stand out. The latter accords with the finding of Akkaya, Gürkaynak, Kısacıkoğlu, & Wright (2015) that the potency of forward guidance grows as the distance of the shadow rate from zero shrinks. In a cross-country study, Coenen et al (2017) find that the nature of guidance can imply significantly different effects on bond yields, but their result is not robust to omitting observations confounded by simultaneous asset purchase policies.

For asset purchases, it appears that the effects were not always emphatic at the longer end of the yield curve, at odds with the goal of bringing down longer-term rates. The launch of the policy, as well as its continuation (when markets expected a taper), along with announcements signaling a change in the focus of purchases, are among the most impactful moves by the FOMC.

Bauer, Lakdawala, & Mueller (2019) study the effects of monetary policy uncertainty, and argue that changes in uncertainty around monetary policy shocks can explain why some strongly impact asset prices, while others do not. Lower uncertainty amplifies the effects of shocks. Among the dates in Table 3, the announcements that I find to be associated with significant shocks are precisely those that the authors associate with large falls in monetary policy uncertainty.[7] This suggests that their story of uncertainty explaining which shocks are most impactful is consistent with my results.

Finally, I investigate whether there are any dates not considered "notable" above that sparked shocks significant at the 1% level. For the Fed Funds rate, there are several additional dates, since Table 3 focuses on unconventional policy announcements. They are September 2007 (50 bp cut), March 2008 (75 bp cut, but at least 100 bp expected according to e.g., Goodman & Pan (2008)), April 2008 (25 bp cut), June 2008 (no change in face of rising inflation), January 2016 (no rate hike), and July 2017 (no rate hike); all but March 2008 are expansionary. For forward guidance, June 2007 (inflation described as "predominant policy concern") and March 2014 (6.5% unemployment reached and "considerable time" language dropped) were contractionary, while March 2017 (rate hike, but no revision to further anticipated hikes in medium-term, see e.g., Riccadonna, Shulyatyeva, & Yamarone (2017)) was expansionary. Finally, for asset purchases, additional contractionary shocks occurred in October 2013 (language about tightening financial conditions justifying asset purchases removed) and December 2016 (rate hike, but no change to asset purchases). For the most part, these findings are align with important revisions to the relevant dimensions of FOMC statements.

A new monetary policy shock series

While comparison of the decompositions for these notable announcements presents interesting results in its own right, many questions can only be answered by aggregating these findings into a time-series of inter-announcement shocks to be used in further analysis. To do so requires a stance on first the horizon at which effects will be measured and second the units by which shocks will be scaled. For macroeconomic purposes, I adopt a series defined by the end-of-day horizon, based on the fact that to pass through to the macroeconomy, effects must be at least somewhat persistent; however, I additionally present a series based on 30-minute windows in this section for comparison. I normalize each daily shock by using the historical decomposition of the 1-quarter Eurodollar for ϵ_{FFt}, 8-quarter Eurodollar for ϵ_{FGt}, and the 10-year Treasury for ϵ_{APt}. Together, these values form the time series of 96 announcement dates.

Table 4 reports the correlation of the shocks constructed using these decompositions with simple changes in the relevant asset prices for the 30-minute window and the end-of-

[7]They additionally find the August 2011 forward guidance shock to be large and associated with a large fall in uncertainty; this is consistent with my historical decomposition, but the width of my confidence intervals means the effect is not significant by the end of the day.

Table 4: Correlation of shock measures

	30-min decomp.			End-of-day decomp.		
	ε_{FFt}	ε_{FGt}	ε_{APt}	ε_{FFt}	ε_{FGt}	ε_{APt}
End-of-day decomp.	0.97	0.76	0.75	–	–	–
30-min. change	0.98	0.85	0.87	0.96	0.64	0.71
End-of-day change	0.96	0.72	0.65	0.98	0.86	0.79

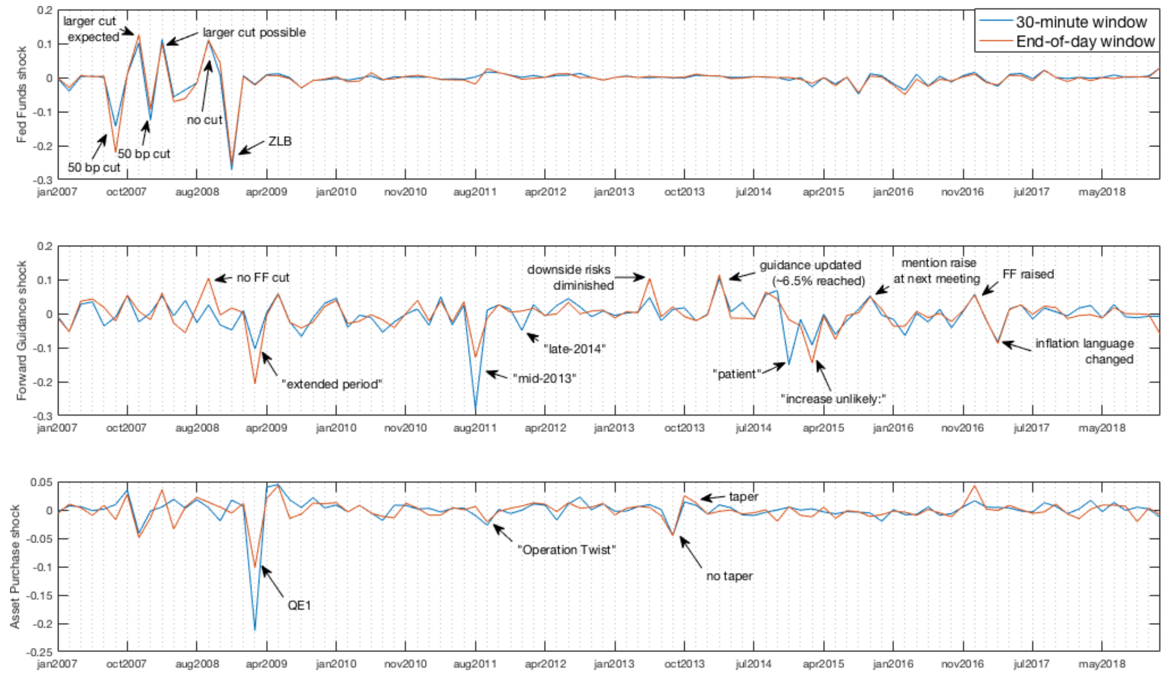
Shock measures are computed by forming time-series of the historical decompositions of the reference rates at either the 30-minute or end-of-day horizon. The 30-minute and End-of-day changes are simply the change in the reference rate over the specified window.

day window. On one hand, the Fed Funds shock series appear to be fairly consistent across all measures, likely due to the fact that most Fed Funds shocks occur prior to the ZLB period, when the other shocks are less active (and event study assumptions are broadly valid). The similarity across horizons also suggests that such shocks are quite persistent. Because the forward guidance and asset purchase shocks are more likely to appear in conjunction, there is more discrepancy between simple changes, ignoring the need to decompose asset price movements in the face of multiple contemporaneous shocks, and decomposition measures. The greater discrepancy across horizons for these shocks also suggests that they are more likely to either wash out over the course of the day or take some time to be fully incorporated by markets. This may be reflective of the fact that these shocks (and the language that triggers them) are of a more complex nature than a comparatively "up or down" change in the Fed Funds rate.

Figure 3 plots the time series for these shocks for the full sample, annotated with important historical events. Broadly speaking, the behaviour of these shocks accords with narrative evidence and expectations. There are large realizations for the Fed Funds shock prior to the ZLB, and then minimal movement until lift-off in December 2015. The largest forward guidance shocks generally correspond with the most notable episodes. The most puzzling feature is some fluctuations in the asset purchase shock prior to the introduction of that measure in the policy discourse in the fall of 2008.

The paths of the forward guidance and asset purchase shocks can be compared to the 2009-2015 paths reported in Figure 1 of Swanson (2017). For forward guidance, the Swanson series notably allocates most of the first announcement, in March 2009, to asset purchases instead. One of his largest forward guidance shocks is associated with the announcement of a 1-quarter extension of QE1 (September 2009); that shock is much more moderate in my series. The results agree on a substantial forward guidance shock with the introduction of calendar guidance (August 2011), but my series do not pick up Swanson's puzzling contractionary shock at the next meeting, which was dominated by Operation Twist. Swanson's

Figure 3: Time-series of shock measures



Shock measures are computed by forming time-series of the historical decompositions of the reference rates at either the 30-minute or end-of-day horizon. Units are percentage points of the reference rates. Large fluctuations that correspond to notable announcements or statement features are labeled.

series picks up a puzzling large contractionary guidance shock following the "taper tantrum" (June 2013). The series agree with a contractionary shock with the updated guidance following unemployment reaching 6.5% in March 2014, with similar shocks at subsequent meetings. Finally, the "patient" and "increase unlikely" shocks at the turn of 2014-2015 appear across series (although the end-of-day "patient" shock is much smaller).

Turning to asset purchase shocks, all series agree that the announcement of QE1 was the most significant episode. Operation Twist is also notable across series. Swanson picks up a large contractionary "taper tantrum" shock in 2013, which is puzzling given that Bernanke's testimony that provoked the tantrum occurred on May 22nd, while the shock registers at the time of the June 19th FOMC announcement. My series have no such shock. Finally, the series agree on an expansionary shock with the announcement that there would be no immediate taper in September 2013, with contractionary shocks through the confirmation of a taper two meetings later.

3.2 Robustness checks

I consider five principal robustness checks to assess the identification of monetary policy shocks from the data. The first allows for the possibility that surprises to multiple dimen-

Table 5: Robustness of shock measures

	30-min decomp.			End-of-day decomp.		
	ε_{FFt}	ε_{FGt}	ε_{APt}	ε_{FFt}	ε_{FGt}	ε_{APt}
Correlated shocks model	0.97	0.83	0.95	0.97	0.85	0.84
Regimes model	0.97	0.88	0.95	0.95	0.88	0.87
Macro shocks model	1.00	0.97	0.98	0.99	0.96	0.89
Post-announcement only	0.99	0.95	0.88	0.99	0.93	0.72
Fed Funds future	0.99	0.98	0.99	1.00	0.99	0.98

Correlation between the baseline shock time series and those of the five robustness checks, for measures based on both the 30-minute and end-of-day horizons. The sample for the Fed Funds future model consists of only the first 21 announcements.

sions of monetary policy are correlated. To do so, I propose a new identification scheme, which is, to my knowledge, the first approach to allow for possible correlation of structural shocks. Section 4 of the Supplement outlines the argument in detail. Since identification is based on lower frequency moments (covariance of asset price innovations across periods of the day), it also provides a foil for the possibility that the baseline results, which rely on minute-by-minute variation, are contaminated by the noise of intraday data. The second approach uses a simple 2-regime version of identification via heteroskedasticity, where the regimes are 9:30am to 10 minutes prior to the announcement and 10 minutes prior to the announcement to 4:15pm. This provides an additional check on the parametric model (with continuous variance process) adopted in the baseline and concerns over high-frequency noise. The third alternative model adds an additional dimension, S&P 500 returns, in an attempt to capture possible macroeconomic news shocks contained in announcements, otherwise known as "Delphic" forward guidance, in the spirit of Matheson & Stavrev (2014). The fourth alternative addresses the concern of non-stationarity between the pre- and post-announcement periods by simply discarding pre-announcement data and estimating the model on data beginning 10 minutes prior to the announcement. The final check assesses whether the use of Eurodollar contracts confounds the analysis by introducing credit risk in the period where the TED spread was both elevated and volatile (my first 21 announcements, January 2007 to August 2009) by replacing the 1-quarter Eurodollar contract with the 2-month Fed Funds future contract. All robustness checks are discussed comprehensively in Section 2 of the Supplement. Table 5 reports the correlation of the baseline shock measures with those from all alternatives at both 30-minute and end-of-day horizons. The Fed Funds shocks identified are incredibly close across approaches. The forward guidance and asset purchase shocks are generally closely related; lower correlations are often due to one or two outlying announcements, as discussed in detail in the Supplement.

4 High-frequency effects on financial markets

Having computed minute-by-minute measures of monetary policy shocks for each announcement and an aggregate time-series for the period 2007-2018, I now assess the effects of policy on a range of variables of economic interest, starting in this section with financial variables, which are generally available at a higher frequency. First, I consider variables available at the same frequency as the identifying data, which allows for announcement-specific estimates; I then turn to daily data.

4.1 Intraday responses

To conduct announcement-specific regressions, the dependent variable must be available at high enough frequency to make daily estimation reliable. This means only financial variables may be assessed, while the intended effects of monetary policy are generally macroeconomic. However, one aim of the large scale asset purchases was to stimulate the economy by lowering corporate borrowing costs. Unfortunately, corporate debt is not liquid enough to conduct high-frequency analysis using specific bonds. A potential proxy is an investment-grade corporate debt ETF, the price of which aims to track the prices of a basket of Aaa corporate bonds. I consider the minute-by-minute returns of the iShares IBoxx $ Invest Grade Corporate Bond ETF (LQD), the most liquid such ETF throughout the period in question; an *increase* in this variable suggests a *fall* in corresponding bond yields. In the Supplement, I also consider the spread of this return over the minute-by-minute return on the 10-year Treasury. The other dependent variable I consider is the S&P 500 return, proxying for equity markets and capturing some measure of market sentiment. Unfortunately, no ETF or similar index related to MBS is suitably liquid during much of the sample.

I conduct simple regressions of the relevant return on the contemporaneous and possibly lagged values of the three identified shocks at the minute frequency, according to

$$r_t = \omega + \gamma_0 \epsilon_t + \sum_{l=1}^{p} \gamma_l \epsilon_{t-l} + u_t. \tag{6}$$

The number of lags, p, is selected day-by-day using the Hannan-Quinn criterion. I compute HAC standard errors using the equal-weighted-periodogram approach with 8 degrees of freedom, following Lazarus, Lewis, & Stock (2019). Table 6 reports summary statistics for the estimated coefficients on contemporaneous *expansionary* shocks. Since the shocks are generated regressors, affected by estimation error, these coefficients are attenuated and estimated effects should be seen as a lower bound. It is clear that, on average, all three

Table 6: Summary statistics: contemporaneous coefficients for for external regressors

	mean	q_{10}	median	q_{90}	positive	positive sig. 5%	negative	negative sig. 5%
				Corporate return proxy				
ϵ_{FF}	1.38	-0.97	0.65	4.57	65	33	28	3
ϵ_{FG}	2.46	-0.04	1.68	2.85	84	66	12	2
ϵ_{AP}	2.09	0.56	1.84	3.79	94	83	2	0
				S&P 500 returns				
ϵ_{FF}	-0.12	-4.39	0.88	4.87	56	17	37	11
ϵ_{FG}	-1.61	-4.95	-0.48	1.75	37	13	59	34
ϵ_{AP}	-2.12	-5.02	-1.59	0.54	16	3	80	50

The corporate debt return proxy is described in the text. Coefficients are estimated by simple regressions of the respective measure on the current and possibly lagged values of the shocks, plus a constant, equation (6). The units are percentage return per expansionary shock (leading to a 1% *fall* in the reference rate). HAC standard errors are computed following Lazarus, Lewis, & Stock (2019).

shocks move corporate returns, and thus yields, in the anticipated direction: the positive coefficients imply that an expansionary shock raises prices, implying lower yields. The magnitude of the effect is comparable for forward guidance and asset purchases and lower for Fed Funds shocks. This makes sense as forward guidance likely concerns a longer portion of the time to maturity, and asset purchases either directly targeted corporate debt (i.e. QE1), or comparable assets of similar maturity. For both forward guidance and asset purchases, the effect of the majority of announcements is both positive and statistically significant. Turning to the S&P 500, the average effect of Fed Funds shocks is more ambiguous, but the majority of estimates are positive (with some significant), indicating expansionary shocks raise returns, as expected. For the unconventional policy shocks, however, the majority of coefficients are negative (with some significant), indicating, surprisingly, that expansionary shocks lower returns.

However, these results indicate only instantaneous elasticities; the end-of-day historical decompositions are more informative of economically meaningfully effects. Table 7 summarizes these decompositions. The end-of-day responses of the corporate return proxy follow the same pattern as the contemporaneous coefficients, ranging from 2 bp on average for Fed Funds shocks to 7 bp for asset purchase shocks. Given the estimation uncertainty from cumulating the estimated shock series, few are significant. Turning to the S&P 500, the effects are comparable across shocks (7-9 bp), with even fewer significant.

Finally, I focus on the key announcement dates in Table 8. Broadly speaking, the results accord with intuition; on the most stimulatory announcement days (as determined in Section

Table 7: Summary statistics for historical decompositions of external regressors

	mean decomp.	median decomp.	sig. at 10%	sig. at 5%	sig. at 1%
		Corporate return proxy			
ϵ_{FF}	0.02	0.01	8	3	0
ϵ_{FG}	0.06	0.03	17	8	2
ϵ_{AP}	0.07	0.04	18	5	1
		S&P 500 returns			
ϵ_{FF}	0.07	0.01	7	5	2
ϵ_{FG}	0.09	0.03	9	7	1
ϵ_{AP}	0.07	0.05	5	1	0

Historical decompositions are computed using the contemporaneous and possibly lag coefficients estimated in equation (6) and the intraday time-series of shocks. Mean and median decompositions are computed based on absolute value. Units are percentage points. Inference combines the covariance matrix of estimates from (6) with intermediate results from Theorem 2.

3), the launch of unconventional policy (March 2009), the taper delay (September 2013), and the final extension of zero-rate guidance (March 2015), there are sizable (and statistically significant) positive effects, up to nearly a full percentage point for March 2009, suggesting a substantial fall in yields. Turning to the S&P 500, while the signs of instantaneous effects, were, on average, surprising across the sample, for the announcements of importance the evidence appears more in-line with intuition. Hitting the ZLB (December 2008), the launch of forward guidance (March 2009), the delay of tapering (September 2013), and the delay of "lift-off" (March 2015) all see sizable positive effects. The only significant effects are due to forward guidance, at its launch, the taper delay, and the lift-off delay, and asset purchases at the taper delay. Overall, there is clearer evidence of unconventional policy having the desired effect on corporate debt markets than a stimulatory effect on equities, which accords with the objectives of the Federal Reserve. My results display considerable heterogeneity in responses across announcements. This type of evidence was previously available only in non-parametric analysis like that of Krishnamurthy & Vissing-Jorgensen (2011, 2013) for asset purchases, whose results also demonstrate this variation. Such results, however, are unable to separate forward guidance effects from contemporaneous asset purchase effects.

4.2 Daily responses of financial variables

Turning to the inter-day time-series of shocks, I now consider the daily impact of the shocks on corporate debt yields and spreads and S&P 500 returns, as in Swanson (2017). The simple regression takes the form

$$\Delta r_d = \nu + \psi \epsilon_d + u_d, \tag{7}$$

Table 8: End-of-day responses of external regressors on key dates

	Dec 2008	Mar 2009	Nov 2010	Aug 2011	Sep 2011	Jan 2012	Sep 2012	Dec 2012	Sep 2013	Dec 2013	Dec 2014	Mar 2015
						Corporate return proxy						
ϵ_{FF}	0.11	0.02	-0.02	0.25*	0.06	0.01	0.00	0.01	0.00	-0.01	0.00	0.04
ϵ_{FG}	-0.04	0.90***	-0.02	-0.05	0.01	-0.01	0.01	-0.01	0.12*	0.08	0.02	0.49***
ϵ_{AP}	0.00	1.13*	−0.22**	-0.07	0.14	-0.06	-0.03	-0.11	0.65*	-0.18	-0.06	0.12
						S&P 500						
ϵ_{FF}	0.40	0.03	0.03	-0.01	-0.04	0.03	0.00	-0.04	0.00	0.09	0.00	0.14
ϵ_{FG}	0.05	0.81**	0.01	-0.06	-0.13	-0.01	0.00	0.00	0.08*	0.03	0.02	0.44***
ϵ_{AP}	-0.09	0.19	-0.09	0.11	-0.25	0.03	0.03	0.10	0.58*	-0.28	0.03	-0.05

For each dependent variable, end-of-day historical decomposition values are reported for the 12 key announcement dates detailed in Table 3. Significance is starred at the 10, 5, and 1% levels.

where d indexes the announcement dates, with HAC standard errors. Table 9 reports the results. Recall that the shock series is aggregated from end-of-day shock measures, which were shown in Section 3.1 to have considerable estimation error; this means that estimated effects are likely attenuated. I find that yields fall significantly in response to both Fed Funds and asset purchase shocks, but less so in response to forward guidance shocks. This partially aligns with Swanson's finding that asset purchases and not forward guidance matter for yields during the ZLB period (he does not report responses to Fed Funds shocks). The asset purchase coefficients (relative to movements in the 10-year Treasury) are larger, at -1.58 and -1.91, compared to his (normalized by the estimated impact on the 10-year Treasury, $4.51/6.49 = 0.69$ and $5.25/6.49 = 0.80$), and larger than those for Fed Funds shocks. The larger asset price coefficients I obtain here may be related to the fact that Swanson considers 30-minute windows, which, as argued above, may lead to larger shocks and thus smaller coefficients. Both forward guidance and asset purchase shocks increase spreads, with asset purchases having larger coefficients. This again aligns with Swanson (2017), as well as Krishnamurthy & Vissing-Jorgensen (2011, 2013) and Swanson (2011). The responses of the S&P 500 are positive and significant for expansionary Fed Funds shocks, broadly in line with Swanson (2017). The effects of forward guidance have the same sign, but I find expansionary asset purchase shocks to lower stock returns; Swanson finds an insignificant positive effect.

Table 9: Corporate debt responses to monetary policy

	Aaa yield	Baa yield	Aaa spread	Baa spread	S&P 500
ϵ_{FF}	-0.28	-0.23	-0.11	-0.06	6.15
ϵ_{FG}	-0.25^{**}	-0.19	0.47^{***}	0.52^{***}	6.73
ϵ_{AP}	-1.44^{***}	-1.75^{***}	1.24^{***}	0.93^{**}	-24.45^{*}

Coefficients are estimated from equation (7). Coefficients can be interpreted as the response in percentage points to an expansionary shock leading to a 1% fall in the reference rate. The sample spans January 2008 through December 2015, to focus on the Great Recession period and be consistent with the expectations regressions in Section 5. HAC standard errors are calculated following Lazarus, Lewis, & Stock (2019). Significant results are starred at the 10%, 5% and, 1% levels.

5 Low-frequency effects on the macroeconomy

While financial series are available at high frequencies, the macroeconomic aggregates of ultimate importance to central bankers are only available at lower frequencies. Much of the focus of unconventional monetary policy was on managing expectations of future policy and conditions, explicitly so in the case of forward guidance. The theoretical basis for this channel has been explored extensively (e.g., Eggertsson & Woodford (2003), Mckay, Nakamura, & Steinsson (2016)). Accordingly, I first consider the effects of the unconventional monetary policy shocks on various measures of expectations, before turning to the dynamic responses of macroeconomic aggregates. Recall that the shock series is aggregated from end-of-day shock measures, which were shown in Section 3.1 to have considerable estimation error, meaning that the reported effects are likely considerably biased towards zero, and constitute lower bounds.

5.1 Effects on expectations

To assess the extent to which shocks impact expectations empirically, I conduct regressions using survey data from the University of Michigan Survey of Consumers and the Blue Chip Economic Indicators survey of professional forecasters. I limit my focus to the 1-year horizon (on which the Michigan survey is based). For the Michigan surveys, my regressor is a two-period change in the respective measure – that of the subsequent month minus that of the prior month. This accounts for the fact that the timing of the announcement within the interview window varies dramatically between months and ensures that all respondents were interviewed either prior to or following the announcement in question. For the Blue Chip survey, the interviews occur over a two-day period early in the month, so I can more easily determine the relative timing of an announcement and the survey dates. As such, I consider a one-period difference between the first survey conducted following the announcement and the prior survey.[8] The regressions take the form

$$\Delta s_d = \alpha + \beta_0 \epsilon_d + \beta_1 \epsilon_{d-} + \delta w_{d-} + \Delta MR_d + \Delta AP_d + u_d, \tag{8}$$

where d indexes announcement dates, ϵ_{d-} is the shock vector in the prior month (zero if no FOMC meeting), and w_{d-} is a vector of controls during the previous month, including inflation, GDP growth, the unemployment rate, and the Fed Funds rate. ΔMR_d records surprises in macroeconomic releases in the period between surveys, following the methodology of Del Negro, Giannoni, & Patterson (2015) and ΔAP_d records changes in asset prices over the period, following the same paper's methodology. These measures control for other news incorporated by participants between surveys. The lagged shock is necessary to account for the fact that, in the case of the Michigan two-period differences, the prior shock may have arrived during the differencing window. β_0 can be interpreted as the change in one year-ahead expectations of measure s_d per 1% monetary policy shock. The sample period is restricted to focus on the crisis period of interest, using data from January 2008 through December 2015 (Fed Funds rate "lift-off"). This reduces the sample size to $T = 64$. Finally, I additionally consider a higher-frequency proxy of expected inflation, the one-day change in the 10-year TIPS spread over the 10-year Treasury using the regression in (7). I compute HAC standard errors.

Table 10 groups the responses to expansionary monetary policy shocks by dependent variable, and I highlight statistically significant findings below. The response of inflation expectations depends on the measure considered. Consumer expectations rise dramatically (5.79%) in response to an asset purchase shock that lowers 10-year Treasury rates by 1%. This exaggerated response appears to be a slope analog to the finding of Coibion, Gorodnichenko, & Weber (2019) that a substantial share of surveyed U.S. consumers overestimate the *level* of inflation. Puzzlingly, there is a significant *negative* response to the Fed Funds shock (-3.39%). This finding is largely driven by December 2008 and is not robust to dropping that observation. Turning to Blue Chip measures, CPI inflation expectations have a significant positive response to asset purchase shocks (0.84%). The comparatively large response of the Michigan measure may be related to well-documented sensationalized reporting of inflationary fears with respect to QE policies (e.g., Resnikoff (2010)); professional forecasters do not exhibit such overreaction. The TIPS inflation measure responds significantly to forward guidance shocks (0.32%). Michigan unemployment expectations fall dramatically in response to expansionary Fed Funds shocks, about (3.28 s.d.) However, the Blue Chip measure puzzlingly rises with expansionary forward guidance shocks (this result holds in

[8] For the handful of announcements that occur *during* the two-day interview period, I assume that most respondents will wait to complete the survey until following the announcement for reputational reasons.

unconditional correlations as well). The only measure of real output, Blue Chip expected real GDP growth, rises by 1.65% for a 1% asset purchase shock, but falls 1.23% for a Fed Funds shock. The latter is puzzling, but appears robust to sample and specification. Finally, there are no significant effects on expectations of interest rates, although all coefficients have the expected sign for the Blue Chip measures. Overall, it appears that only asset purchase shocks have significant effects in the desired direction on survey measures of both key macro variables, inflation and output.

The clearest antecedents to these estimates are found in Campbell, Evans, Fisher, & Justinano (2012), Del Negro, Giannoni, & Patterson (2015), and Nakamura & Steinsson (2018). The first paper regresses Blue Chip unemployment and inflation expectations on the target and path factors from a GSS decomposition. They find no significant effects, with often puzzling signs. However, their sample only runs from 2007 to 2011, and the general imprecision may be further impacted by an identification approach that potentially confounds forward guidance and asset purchase shocks. Del Negro et al (2015) estimate the impact of unconventional policy announcements on changes in Blue Chip Financial Forecasts (a different panel to my Blue Chip Economic Indicators) of CPI inflation, GDP growth, 3-month Treasury rates, and 10-year Treasury rates using dummies for various characteristics of FOMC announcements, including forward guidance, asset purchases, and macro news. They exploit a panel structure with similar controls. Responses at the 1-year horizon are quite small (and generally insignificant), but roughly comparable. I find expansionary effects of asset purchases on both CPI inflation and GDP growth, as do they. While they find expansionary effects of forward guidance on the same variables at short horizons, by four quarters (my horizon) they are insignificant and near-zero, as in Table 10. My coefficients for both Treasury yields accord with their findings for forward guidance, although they do find small insignificant positive coefficients at 4 quarters for asset purchases. Nakamura & Steinsson (2018) also regress changes in Blue Chip expectations on their "news shock" measure, although their regressions run from 1995-2014 and omit most of 2008 and 2009. In this sample, they find expansionary effects on 3-month Treasury rates and on GDP inflation. Interestingly, they find contractionary effects on output growth, which is compatible with their "news shock" containing more information on macroeconomic news than monetary policy news.

I also examine how monetary policy shocks impact the dispersion of expectations. Depending on the survey, I use different measures of dispersion: variance for Michigan inflation, a measure of relative frequency for categorical Michigan variables, and the spread between the top and bottom 10 forecasts for Blue Chip. It is not ex ante clear for most variables whether shocks should raise or lower uncertainty. Given the lack of experience with uncon-

Table 10: Responses of expectations to monetary policy shocks

	Mich. Inf.	BC CPI Inf.	TIPS Inf. 10Y (daily)	Mich. Unem.	BC Unem.	BC RGDP growth	Mich. Int. Rates	BC 3m Treas.	BC 10Y Treas.
ϵ_{FF}	-3.39^{***}	-0.84	0.10	-3.28^{***}	0.35	-1.23^{***}	0.45	-0.19	-0.45
ϵ_{FG}	0.71	0.10	0.32^{***}	0.11	0.59^{*}	-0.00	1.51	-0.04	-0.60
ϵ_{AP}	5.79^{***}	0.84^{*}	-0.15	1.55	-0.15	1.65^{*}	-4.16	-0.96	-0.56

With the exception of the TIPS column, entries are estimates of contemporaneous coefficients β_0 from equation (8) for monthly changes in 1-year ahead survey expectations from the University of Michigan Survey of Consumers and Blue Chip Economic Indicators. The TIPS column considers the 10-year TIPS-Treasury spread according to equation (7). Coefficients can be interpreted as the response to an expansionary shock leading to a 1% fall in the reference rate. Units are percentage points except for the Michigan unemployment and interest rate series, which are a balance measure standardized to have unit variance, so coefficients represent the response in standard deviations. The sample spans September 2008 through December 2015. HAC standard errors are calculated following Lazarus, Lewis, & Stock (2019). Significant results are starred at the 5% and 1% level.

ventional policy, it is possible that expansionary shocks result in increased uncertainty about the future, as respondents have difficulty assessing the macroeconomic effects of novel measures. On the other hand, policies like forward guidance explicitly aim to resolve uncertainty about the future.

Table 11 reports the results. Both Fed Funds and asset purchase shocks significantly lower the variance of Michigan inflation expectations. Professionals, however, react differently; all three shocks increase the dispersion of Blue Chip CPI Inflation forecasts. Both surveys show an increase in the dispersion of unemployment expectations in response to expansionary asset purchase shocks, but the Blue Chip measure's dispersion falls with expansionary Fed Funds shocks. Blue Chip GDP growth dispersion increases in response to forward guidance shocks. Overall, these results together suggest general uncertainty over the real effects of unconventional monetary policies, especially among professional forecasters. Finally, turning to interest rates, Fed Funds shocks and forward guidance cause the Blue Chip 3-month Treasury forecasts to become less varied, but asset purchase shocks have the opposite effect; asset purchases also increase the dispersion of 10-year Treasury forecasts. It appears that while forward guidance conveys a clear message about medium term rates, forecasters were unsure how to interpret the effects of asset purchase policies. To summarize, with the exception of consumer inflation expectations, expansionary shocks seem to correspond with greater uncertainty over macroeconomic outcomes and long-term interest rates. On the other hand, there is more agreement about the path of shorter-term rates following Fed Funds shocks and forward guidance, as expected. This pattern matches the findings of Andrade, Gaballo, Mengus, & Mojon (2018), who study the Survey of Professional Forecast-

Table 11: Responses of dispersion of expectations to monetary policy shocks

	Mich. Inf.	BC CPI Inf.	TIPS Inf. 10Y (daily)	Mich. Unem.	BC Unem.	BC RGDP growth	Mich. Int. Rates	BC 3m Treas.	BC 10Y Treas.
ϵ_{FF}	-31.14^{***}	1.36^{***}	–	0.12	-0.64^{**}	-0.24	-0.11	-1.68^{**}	0.32
ϵ_{FG}	4.75	1.55^{***}	–	0.07	0.27	1.98^{***}	-0.18	-2.31^{***}	0.33
ϵ_{AP}	-69.71^{***}	3.30	–	0.27^{*}	4.06^{***}	1.79	0.41	4.60^{***}	2.33^{**}

With the exception of the TIPS column, entries are estimates of contemporaneous coefficients β_0 from equation (8) for monthly changes in dispersion measures for 1-year ahead survey expectations from the University of Michigan Survey of Consumers and Blue Chip Economic Indicators. Coefficients can be interpreted as the response to an expansionary shock leading to a 1% fall in the reference rate. For the Michigan inflation series, I use the variance reported with the survey; for the remaining (categorical) Michigan series, I use a dispersion measure equal to one minus the square-root of the sum of the squared relative frequencies of the three response categories. For the Blue Chip series, the measure is the spread between the "Top 10" and "Bottom 10" forecasts. The sample spans January 2008 through December 2015. HAC standard errors are calculated following Lazarus, Lewis, & Stock (2019). Significant results are starred at the 5% and 1% level.

ers and document general agreement on interest rates, but the development of two camps of "pessimistic" and "optimistic" forecasters with respect to macroeconomic outcomes during this period. Coenen et al (2017) document a similar interest rate effect, but also a reduction in uncertainty surrounding inflation, although their analysis focuses on the Eurozone and cross-country panel.

5.2 Effects on macroeconomic aggregates

Finally, I turn to the effects of unconventional monetary policy on macroeconomic aggregates, in particular PCE inflation and real GDP growth. To this point, relatively little work has assessed these impacts, with Baumeister & Bennati (2013), Gambacorta, Hofmann, & Peersman (2014), Lloyd (2018), and Inoue & Rossi (2018) being notable exceptions. However, as discussed in the introduction, none of these papers has separated and simultaneously identified interpretable forward guidance and asset purchase shocks, making this analysis the first of its kind.

I merge my shock measures into a monthly time series with PCE inflation, real GDP growth (based on the Macroeconomic Advisers monthly measure), and the Federal Funds target rate. This yields a time-series of 144 observations. I compute impulse response functions using local projections of the form

$$x_{m+h} = \mu^h + \pi_0^h \epsilon_m + \sum_{l=1}^{6} \pi_l^h \epsilon_{m-l} + \sum_{s=1}^{3} \kappa^h X_{m-s} + u_m^h, h = 0, 1, \ldots, 12, \tag{9}$$

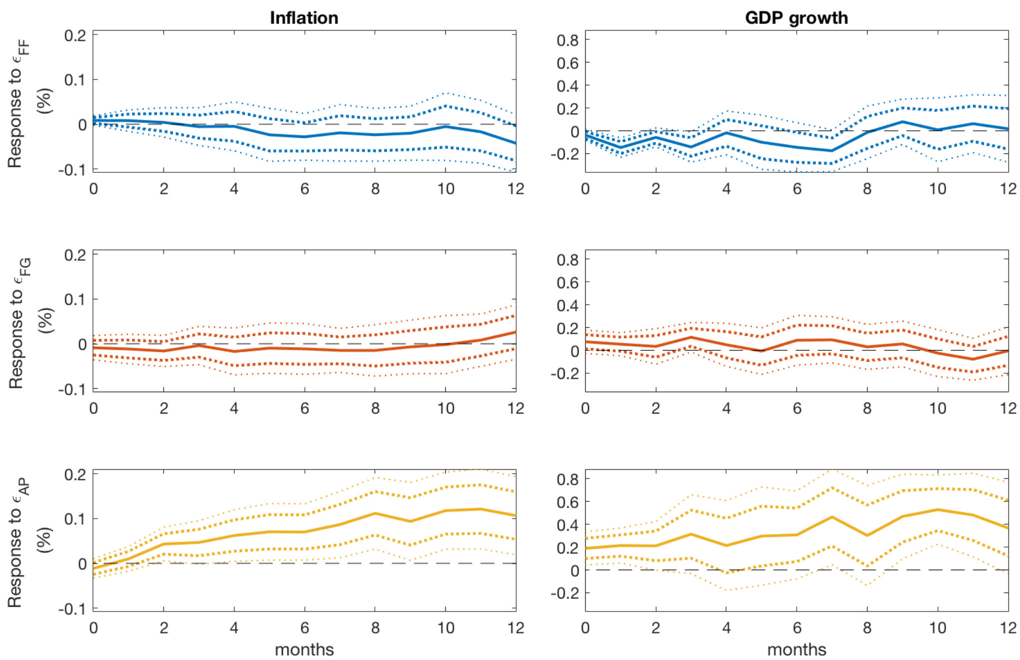
controlling for the previous six months' worth of monetary policy shocks and the prior quarter's macro aggregates (inflation, real GDP growth, and the Fed Funds rate). The coefficient of interest is π_0^h – the effect of a time m shock on x_{m+h}. I consider only horizons up to one year, seeing as, given the limited data, the need for additional leads starts to severely limit the sample, and the imprecision that often affects local projection results is pronounced. I again compute HAC standard errors, and cumulate both responses and standard errors across horizons to obtain cumulative impulse responses.

Figure 4 plots the dynamic response of inflation and real GDP growth to a one standard deviation impulse to each expansionary shock, with 68% and 90% confidence intervals. Inflation displays a small, significant positive response to an expansionary Fed Funds shock on impact, with no significant response thereafter. Forward guidance has no significant impact within the year. On the other hand, a one standard deviation asset purchase shock (one that raises 10-year Treasury yields by 1.8 bp) raises inflation by about 10 bp from 8 months onward, significant after the first month or so. Turning to real GDP growth, it appears at first that an expansionary Fed Funds shock significantly *lowers* growth over the first few months; however, this perplexing result is almost entirely driven by the December 2008 observation, and vanishes if this shock is "zeroed out". Again, forward guidance has no discernible effect, except for a marginal increase on impact. However, the asset purchase shock raises GDP growth by 20 bp on impact and about 50 bp at its peak. The response is significant through 2 months and again further along the horizon, depending on the significance level considered. These results interestingly mimic those obtained for expectations: during the ZLB period, asset purchase shocks appear to have the largest impact on macroeconomic aggregates as well as expectations about them.

In prior work, Inoue and Rossi (2018) do not report mean responses for the "unconventional" period, instead plotting responses for selected announcements. They break down the overall effects of monetary policy as responses to their identified slope and curvature shocks. For both output and inflation, they find that the slope factor drives responses, except in 2012, when the influence of the curvature factor increases. The authors argue that the curvature factor can be seen as a forward guidance shock. These findings do not align with my results, which indicate that, over the same period, a single shock (the asset purchase shock) has pronounced economic effects, while the others do not. It is difficult to compare the results further (for example, to examine the impact of the authors' use of a time-invariant decomposition), since their statistically-identified factors do not have clear economic interpretations along the lines of the three dimensions of monetary policy I consider.

Gambacorta, Hofmann, & Peersman (2014) focus on identifying the effects of balance sheet size shocks in a cross-country panel VAR. Their findings indicate significant stimulatory

Figure 4: Dynamic response of macroeconomic aggregates



Impulse responses are calculated via local projection as in equation (9) using monthly data and the full sample, January 2007 to December 2018. Responses are cumulated to obtain the dynamic responses. Responses are scaled to a one standard deviation expansionary impulse for each shock. 95% HAC confidence intervals are calculated following Lazarus, Lewis, & Stock (2019).

affects for the asset purchase dimension of policy, with effects peaking around six months. The output response is about three times that of inflation, roughly according with my finding of an up to five-times larger response of output.

Finally, Gertler & Karadi (2015) find suggestive evidence that forward guidance serves to amplify shocks to the current policy rate. They do so by comparing responses using the front Fed Funds future as an instrument for the Fed Funds rate to their baseline, which uses three-month ahead futures to instrument for the 1-year Treasury yield. However, their sample runs from 1991-2012, so is dominated by observations outside of the ZLB. Thus, their evidence that forward guidance can offer additional stimulus may be compatible with my finding that it did not have a pronounced impact during the Great Recession. Indeed, since they argue that forward guidance may be effective by *augmenting* policy rate shocks, this distinction accords with the fact that the Fed Funds rate was at the ZLB, so policy rate shocks were not forthcoming.

6 Conclusion

I use intraday data on interest rate movements to recover high frequency time series of monetary policy shocks on announcement days using announcement-specific decompositions. I identify the decompositions based on time-varying volatility. Based on these time-varying decompositions of asset price movements, I find that a small handful of notable FOMC announcements of unconventional measures sparked significant monetary policy shocks. In particular, the leading announcements are the launch of forward guidance (March 2009), the prolonging of forward guidance (March 2015), Operation Twist (September 2011), and the decision to delay tapering (September 2013); the launch of QE1 (March 2009) is significant at a lower level. The fact that these announcements are dominated by the launch or unexpected extension of the policies indicates that the presence of these policy tools, as opposed to subtle refinements of statement language or adjustments of purchases, is what matters to markets.

At high frequency, many announcements, particularly on the asset purchase dimension, raise a proxy for corporate debt returns, and thus lower yields, but the cumulative effects rarely persist by day's end. At the daily frequency, corporate yields also fall significantly with Fed Funds and asset purchase shocks, but spreads rise in response to both forward guidance and asset purchase shocks.

Moreover, I find important macroeconomic effects. Both consumer and professional expectations respond to the shocks at the monthly frequency. Asset purchase shocks are especially impactful, raising expectations of both inflation and real GDP growth; this lends support to the expectations channel in explaining possible effects of unconventional policy shocks. Finally, the dynamic responses of both realized inflation and GDP growth display significant responses to asset purchase shocks, but not to Fed Funds or forward guidance shocks. Taken together, these results offer some of the first evidence on the macroeconomic effects of the Federal Reserve's unconventional monetary policy broken down by policy dimension. They suggest that asset purchase policies in particular were effective with regard to a number of policy objectives.

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A Proofs

Proposition 2. *Suppose that for news shocks during non-announcement periods, $\eta_t = H_N \epsilon_t$, and for monetary policy shocks following monetary policy announcements, $\eta_t = H_{MP}\epsilon_t$, with $H_N \neq H_{MP}$; assume that within each period, the σ_t^2 process is stationary with respective means $\sigma_N^2, \sigma_{MP}^2$. Then if $\frac{\sigma_{i,N}^2}{\min_j \sigma_{j,MP}^2} \to 0$, for all $i = 1, \ldots, n$, the H identified by Theorem 1 from full-sample moments is H_{MP}, provided the monetary policy shocks are not measure zero.*

Proof. Define $W_{MP} \in (0,1]$ as the share of time periods corresponding to monetary policy shocks and $\bar{\sigma}^2 \equiv \min_j \sigma_{j,MP}^2$. Without loss of generality I now work with the re-scaled $\zeta_t/\bar{\sigma}^2$ as the "data". Then

$$E\left[\zeta_t/\bar{\sigma}^2\right] = vech\left(\left(1 - W_{MP}\right) H_N \frac{\Sigma_N}{\bar{\sigma}^2} H_N' + W_{MP} H_{MP} \frac{\Sigma_{MP}}{\bar{\sigma}^2} H_{MP}'\right).$$

Observe that, as $\frac{\sigma_{i,N}^2}{\min_j \sigma_{j,MP}^2} \to 0 \ \forall i = 1, \ldots, n$,

$$E\left[\zeta_t/\bar{\sigma}^2\right] \to \frac{W_{MP}}{\bar{\sigma}^2} H_{MP} \Sigma_{MP} H_{MP}',$$

since $\sigma_N^2/\bar{\sigma}^2 \to 0$.

Turning now to $Cov\left(\zeta_t, \zeta_{t-p}\right)$, for t and/or $t-p$ *not* in the monetary policy shock period,

$$\begin{aligned}
&E\left(\frac{\zeta_t}{\bar{\sigma}^2}\frac{\zeta_{t-p}'}{\bar{\sigma}^2} \mid t \vee t-p \in N\right) \\
&= L\left(H_N \otimes H_N\right) G \frac{E\left[\sigma_t^2 vec\left(\varepsilon_{t-p}\varepsilon_{t-p}'\right) \mid t \vee t-p \in N\right]}{\bar{\sigma}^2 \times \bar{\sigma}^2} \left(H_N \otimes H_N\right)' L' \\
&\to L\left(H_N \otimes H_N\right) G \times 0 \times \left(H_N \otimes H_N\right)' L' = 0,
\end{aligned}$$

since $\sigma_t^2/\bar{\sigma}^2 \to 0$ and/or $\left(\varepsilon_{t-p}\varepsilon_{t-p}'\right) \to 0$, $E\left[\sigma_t^2 vec\left(\varepsilon_{t-p}\varepsilon_{t-p}'\right)\right] \to 0$. On the other hand, for both $t, t-p$ in the monetary policy shock period,

$$\begin{aligned}
&E\left(\frac{\zeta_t}{\bar{\sigma}^2}\frac{\zeta_{t-p}'}{\bar{\sigma}^2} \mid t \wedge t-p \in MP\right) \\
&= L\left(H_{MP} \otimes H_{MP}\right) G \frac{E\left[\sigma_t^2 vec\left(\varepsilon_{t-p}\varepsilon_{t-p}'\right) \mid t \wedge t-p \in MP\right]}{\bar{\sigma}^2 \times \bar{\sigma}^2} \left(H_{MP} \otimes H_{MP}\right)' L'.
\end{aligned}$$

Then since $\sigma^2_{i,MP} \geq \bar{\sigma}^2 \ \forall i$,

$$E\left(\frac{\zeta_t}{\bar{\sigma}^2}\frac{\zeta'_{t-p}}{\bar{\sigma}^2}\right) \to L\left(H_{MP} \otimes H_{MP}\right) G \frac{W_{MP}}{\bar{\sigma}^4} E\left[\sigma^2_t vec\left(\varepsilon_{t-p}\varepsilon'_{t-p}\right) \mid t \wedge t-p \in MP\right] \left(H_{MP} \otimes H_{MP}\right)' L'.$$

Further, since $E\left[\zeta_t/\bar{\sigma}^2\right] \to vech\left(H_{MP}W_{MP}/\bar{\sigma}^2\Sigma_{MP}H_{MP}\right)$,

$$E\left[\frac{\zeta_t}{\bar{\sigma}^2}\right] E\left[\frac{\zeta_t}{\bar{\sigma}^2}\right]' \to L\left(H_{MP} \otimes H_{MP}\right) G \frac{W^2_{MP}}{\bar{\sigma}^4} \sigma^2_{MP}\sigma^{2'}_{MP} G' \left(H_{MP} \otimes H_{MP}\right)' L'.$$

Thus, $Cov\left(\zeta_t, \zeta_{t-p}\right) = L\left(H_{MP} \otimes H_{MP}\right) G M_{p,MP} \left(H_{MP} \otimes H_{MP}\right)' L'$, where

$$M_{p,MP} = \frac{W_{MP}}{\bar{\sigma}^4} E\left[\sigma^2_t vec\left(\varepsilon_{t-p}\varepsilon'_{t-p}\right) \mid t \wedge t-p \in MP\right] - \frac{W^2_{MP}}{\bar{\sigma}^4} \sigma^2_{MP}\sigma^{2'}_{MP} G'.$$

This identifying equation, along with $E\left[\frac{\zeta_t}{\bar{\sigma}^2}\right]$, has the exact same form as in Theorem 1. Thus, the result guaranteeing a unique solution holds, with the conditions stated there for $\tilde{M}_p$ being applied instead to $\tilde{M}_{p,MP} = \left[\begin{array}{cc} M_{p,MP} & \frac{W_{MP}}{\bar{\sigma}^2}\sigma^2_{MP} \end{array}\right]$. □

Theorem 2. *For consistent and asymptotically normal estimators of θ, the asymptotic variance of the corresponding estimator of Ψ_{jt} is equal to*

$$V_{\Psi_t} = \frac{\partial \Psi_{jt}}{\partial \theta'} V_\theta \frac{\partial \Psi'_{jt}}{\partial \theta},$$

where

1. $V_\theta = \left[\begin{array}{cc} V_A & 0 \\ 0 & V_H \end{array}\right]$, *where V_A is the asymptotic variance of the VAR coefficients and V_H is the asymptotic covariance of the elements of $\hat{H}$ and*

2. $\frac{\partial \Psi_{jt}}{\partial \theta} = \sum_{s=0}^{t-\tau} \frac{\partial \tilde{\Phi}^s \iota_j \epsilon_{t-s}}{\partial \theta'}$, *where*

$$\begin{aligned} \frac{\partial \tilde{\Phi}^s \iota_j \epsilon_{t-s}}{\partial \theta'} &= \left(\epsilon'_{t-s}\iota_j\right) \otimes I_n \left[\begin{array}{ccc} 0_{n^2\times n} & \left(H' \otimes I_n\right) F_s & I_n \otimes \tilde{B}^s \end{array}\right] \\ &+ \tilde{\Phi}^s \iota_j \left[\begin{array}{ccc} H^{-1} & H^{-1}\left(X_{t-s} \otimes I_n\right) & \left(\eta'_{t-s} \otimes I_n\right)\left(H'^{-1} \otimes H^{-1}\right) \end{array}\right], \end{aligned}$$

with

(a) $F_0 = 0_{n\times n^2 p}, F_s = \sum_{v=1}^{s}\sum_{m=0}^{v-1} J\left(\mathbf{A}'\right)^{v-1-m} \otimes B^m \ \forall s > 0$, *where* $J = \left[I_n 0_{n\times n(p-1)}\right]$ *and*

$$\mathbf{A} = \begin{bmatrix} A_1 & A_2 & \dots & A_{p-1} & A_p \\ I_n & 0 & \dots & 0 & 0 \\ 0 & I_n & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I_n & 0 \end{bmatrix},$$

(b) $X_{t-s} = \begin{bmatrix} y'_{t-1} & y'_{t-2} & \dots & y'_{t-p} \end{bmatrix}$.

(c) ι_j a selection matrix such that $\iota_j \epsilon_t = \begin{pmatrix} 0_{j-1} & \epsilon_{jt} & 0_{n-j} \end{pmatrix}'$.

Proof. Given the asymptotic variance-covariance matrix of θ, stacking the elements of A_0, A, and H (which is block diagonal, see e.g., Lütkepohl (2005) pg. 110), it follows from the delta method that the variance of Ψ_{jt} can be computed as

$$V_{\Psi_t} = \frac{\partial \Psi_{jt}}{\partial \theta'} V_\theta \frac{\partial \Psi'_{jt}}{\partial \theta}.$$

By the definition of Ψ_t, $\frac{\partial \Psi_{jt}}{\partial \theta'} = \sum_{s=0}^{t-\tau} \frac{\partial \tilde{\Phi}^s \iota_j \epsilon_{t-s}}{\partial \theta'}$, where ι_j is a selection matrix producing a vector of zeros and the j^{th} shock (a matrix of zeros with a 1 in the j, j position). Using the fact that $\frac{\partial Mx}{\partial vec(M)} = x' \otimes I_n$ and the product rule, it follows that $\frac{\partial \tilde{\Phi}^s \iota_j \epsilon_{t-s}}{\partial \theta'} = \left(\epsilon'_{t-s} \iota_j\right) \otimes I_n \frac{\partial \tilde{\Phi}^s}{\partial \theta'} + \tilde{\Phi}^s \iota_j \frac{\partial \epsilon_{t-s}}{\partial \theta'}$. I now evaluate the two partial derivatives.

Begin by noting that, by the product rule and the facts that $\frac{\partial vec(M_1 M_2)}{\partial vec(M_1)} = M'_2 \otimes I_n$ and $\frac{\partial vec(M_1 M_2)}{\partial vec(M_2)} = I_n \otimes M_1$,

$$\begin{aligned} \frac{\partial \tilde{\Phi}^s}{\partial \theta'} = \frac{\partial vec\left(\tilde{B}^s H\right)}{\partial \theta'} &= \begin{bmatrix} 0_{n^2 \times n} & \left(H' \otimes I_n\right) F_s & 0_{n^2 \times n^2} \end{bmatrix} + \begin{bmatrix} 0_{n^2 \times (n^2 p + n)} & I_n \otimes \tilde{B}^s \end{bmatrix} \\ &= \begin{bmatrix} 0_{n^2 \times n} & \left(H' \otimes I_n\right) F_s & I_n \otimes \tilde{B}^s \end{bmatrix}, \end{aligned}$$

where F_s, the derivative of the s-horizon cumulative IRF with respect to the autoregressive coefficients, is given in Lütkepohl (2005) pg. 111 (his F_n) as

$$F_s = \sum_{i=1}^{s} G_i, G_i = \frac{\partial vec\left(B^i\right)}{\partial vec\left(A\right)'} = \sum_{m=0}^{i-1} J \left(\mathbf{A}'\right)^{i-1-m} \otimes B^m,$$

for $s > 0$, with

$$\mathbf{A} = \begin{bmatrix} A_1 & A_2 & \ldots & A_{p-1} & A_p \\ I_n & 0 & \ldots & 0 & 0 \\ 0 & I_n & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & I_n & 0 \end{bmatrix}$$

and

$$J = \left[I_n 0_{n \times n(p-1)} \right].$$

F_0 is trivially equal to $0_{n^2 \times n^2 p}$.

Now turn to the derivative of ϵ_{t-s}. This can be decomposed using the product rule as

$$\frac{\partial \epsilon_{t-s}}{\partial \theta'} = H^{-1} \frac{\partial \eta_{t-s}}{\partial \theta'} + \eta'_{t-s} \otimes I_n \frac{\partial vec\left(H^{-1}\right)}{\partial \theta'},$$

where $\frac{\partial \eta_{t-s}}{\partial \theta'} = \left[\begin{array}{ccc} I_n & X_{t-s} \otimes I_n & 0_{n \times n^2} \end{array} \right]$ and $\frac{\partial vec\left(H^{-1}\right)}{\partial \theta'} = \left[\begin{array}{cc} 0_{n^2 \times (n^2 p + n)} & H'^{-1} \otimes H^{-1} \end{array} \right]$, with $X_{t-s} = \left[\begin{array}{cccc} y'_{t-1} & y'_{t-2} & \ldots & y'_{t-p} \end{array} \right]$, so

$$\begin{aligned} \frac{\partial \epsilon_{t-s}}{\partial \theta'} &= \left[\begin{array}{ccc} H^{-1} & H^{-1}\left(X_{t-s} \otimes I_n\right) & 0_{n \times n^2} \end{array} \right] + \left[\begin{array}{cc} 0_{n \times (n^2 p + n)} & \left(\eta'_{t-s} \otimes I_n\right)\left(H'^{-1} \otimes H^{-1}\right) \end{array} \right] \\ &= \left[\begin{array}{ccc} H^{-1} & H^{-1}\left(X_{t-s} \otimes I_n\right) & \left(\eta'_{t-s} \otimes I_n\right)\left(H'^{-1} \otimes H^{-1}\right) \end{array} \right]. \end{aligned}$$

Finally, combining the partial derivatives with the selection matrix ι_j yields

$$\begin{aligned} \frac{\partial \tilde{\Phi}^s \iota_j \epsilon_{t-s}}{\partial \theta'} = &\left(\epsilon'_{t-s} \iota_j\right) \otimes I_n \left[\begin{array}{ccc} 0_{n^2 \times n} & H' \otimes I_n R_s & I_n \otimes \tilde{B}^s \end{array} \right] \\ &+ \tilde{\Phi}^s \iota_j \left[\begin{array}{ccc} H^{-1} & H^{-1}\left(X_{t-s} \otimes I_n\right) & \left(\eta'_{t-s} \otimes I_n\right)\left(H'^{-1} \otimes H^{-1}\right) \end{array} \right]. \end{aligned}$$

$\square$

Supplemental Materials

Supplemental materials can be found on my personal website, here.