The long-run effects of monetary policy

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Abstract

A well-worn tenet holds that monetary policy does not affect the long-run productive capacity of the economy. Merging data from two new international historical databases, we find this not to be quite right. Using the trilemma of international finance, we find that exogenous variation in monetary policy affects capital accumulation, and to a lesser extent, total factor productivity, thereby impacting output for a much longer period of time than is customarily assumed. We build a quantitative medium-scale DSGE model with endogenous TFP growth to understand the mechanisms at work. Following a monetary shock, lower output temporarily slows down TFP growth. Internal propagation of the monetary shock extends the slow down in productivity, and eventually lowers trend output. Yet the model replicates conventional textbook results in other dimensions. Monetary policy can have long-run effects.

JEL classification codes: E01, E30, E32, E44, E47, E51, F33, F42, F44

Keywords: monetary policy, interest rates, money neutrality, productivity, hysteresis, trilemma, instrumental variables.

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When coin is in greater plenty, as a greater quantity of it is required to represent the same quantity of goods, it can have no effect either good or bad, taking a nation within itself, any more than it would make an alteration in a merchant’s books; if instead of the Arabian method of notation, which requires few characters, he should make use of the Roman, which requires a great many.

— Of Money in Hume (1758)

1. Introduction

Money has long been believed to be neutral in the long-run, as David Hume noted, and as a voluminous literature on monetary economics has argued more formally with rare exceptions. However, the evidence is less clear cut than the elegant logic of the predominant theories would suggest. Barro (2013) provides evidence that high levels of inflation result in a loss in the rate of economic growth. Work by Caballero, Hoshi, and Kashyap (2008) and Gopinath, Kalemli-Özcan, Karkarabounis, and Villegas-Sánchez (2017) links interest rates to the level of productivity, whereas more recently, Benigno and Fornaro (2018) and Liu, Mian, and Sufi (2018) link low interest rates with the rate of growth of productivity. Our paper follows in this more recent tradition and sets out to investigate the link between monetary policy, the growth rate of productivity, and the growth rate of output in the medium to long term.

In monetary economics, most theories embrace the assumption that an exogenous shock to interest rates has only transitory effects on prices and economic activity (see, e.g., Christiano, Eichenbaum, and Evans, 1999; Ramey, 2016; Coibion, Gorodnichenko, and Ulate, 2017). However, recent research by Jordà, Schularick, and Taylor (2019) suggests that previous measures of monetary shocks might have been endogenous responses to the outlook. They introduce a new instrumental variable based on the trilemma of international finance (see, e.g., Obstfeld, Shambaugh, and Taylor, 2004, 2005; Shambaugh, 2004) to document that monetary shocks have larger and more persistent effects than previously measured, and closer in magnitude to measures obtained with narrative shocks (Romer and Romer, 2004) and market-based, high-frequency identified shocks (Gertler and Karadi, 2015). Moreover, they also document that the effects of monetary policy are state-dependent, extending similar results reported in Tenreyro and Thwaites (2016), Angrist, Jordà, and Kuersteiner (2018), and Barnichon and Matthes (2018).

Using the same instrument as Jordà, Schularick, and Taylor (2019) we show, using historical data since 1870 on 17 advanced economies, that monetary shocks have very long-lasting effects on output and productivity. In particular, we augment the data in Jordà, Schularick, and Taylor (2017) and available at www.macrohistory.net/data, with data on total factor productivity (TFP) and factor

\[\text{See, for example, the discussion of Sidrauski's work by Fischer (1983).}\]
We investigate this proposition from a variety of angles, by correcting for spillover effects, considering different samples, and examining state dependence as a function of the business cycle, the level of inflation, and importantly, credit. We also examine the soundness of our findings to alternative definitions of monetary shocks based on Romer and Romer (2004) for the postwar United States. The main result is robust to all these modifications. We then look under the hood to examine through what channels is monetary policy exacting these changes on output growth. We find that the total hours worked response is relatively stable and small. Thus, the main effects on output growth that we uncover appear to come primarily through the responses of real capital accumulation, and to a lesser extent, of utilization adjusted TFP. These results echo the important early findings by Evans (1992) who cast doubt on the exogeneity of TFP shocks, a key tenet of mainstream DSGE models, finding that money, interest rates, and government spending Granger-caused these impulses.

Motivated by these findings, we build a quantitative medium-scale DSGE model with endogenous TFP growth to understand the mechanisms at work and spell out policy implications. TFP growth deviates from the exogenous trend in response to monetary shocks whenever output is different from potential output. A contractionary monetary policy shock lowers output temporarily producing a slowdown in TFP growth. Under a standard Taylor rule, the temporary slowdown in TFP growth accumulates to yield permanently lower trend levels of output and capital. The model can generate this medium run effect on GDP while replicating the conventional textbook results, namely the utilization rates of labor and capital fall temporarily, and capital to TFP ratio exhibits a hump-shaped response. In our empirical analysis, we corroborate these conventional results (Christiano, Eichenbaum, and Evans, 2005) for the historical sample.

Our paper is related to the seminal work by Cerra and Saxena (2008) who documented that output losses in the aftermath of economic and political crises in low-income and emerging economies are highly persistent. More recently, researchers have also documented similar effects following financial crises for advanced economies (see Martin, Munyan, and Wilson, 2015; Haltmaier, 2013). Fatás and Summers (2018) document strong hysteresis effects of fiscal consolidations for advanced economies following the global financial crisis. We document a causal effect of monetary policy shocks in a historical panel data for advanced economies.

There is mixed evidence of long-run non-neutrality proposition in monetary economics. In the structural VAR literature, Bernanke and Mihov (1998) reject long-run neutrality in their identified responses to monetary policy shocks since the impulse responses are statistically indistinguishable from zero. Mankiw (2001) however interprets the non-recovery of GDP to zero in Bernanke and Mihov (1998)'s estimates as evidence of long-run non-neutrality.

Our paper is also related to the recent literature that emphasizes slow recovery following the Great Recession due to endogenous productivity growth (Anzoategui, Comin, Gertler, and Martinez, 2016) and available at http://www.longtermproductivity.com. We are particularly thankful to Antonin Bergeaud for sharing some of the disaggregated series from their database that we used to construct our own series of adjusted TFP.
2018; Bianchi, Kung, and Morales, 2019), labor force participation (Erceg and Levin, 2014; Galí, 2016), or skill depletion (Kiyotaki and Zhang, 2017). This literature has its origins in the seminal work by Stadler (1990) who developed a business cycle model with endogenous technology and sticky wages to generate a persistent effect of monetary shocks on output. (See also Fatas (2000), and Barlevy (2004).) Of the recent papers, Moran and Queraltó (2018) uses three equation VAR models to emphasize an empirical link between TFP growth and monetary policy shocks. We differ in two important ways. One, we investigate and establish that monetary policy shocks can affect level of GDP, capital stock as well as TFP in a panel of 17 countries spanning more than a hundred years. Two, we use externally identified shocks instead of ordering restrictions to identify monetary policy shocks.

Our analysis further provides answer to questions raised by policymakers including Chair Yellen (2016) recently. Aggregate demand shocks, particularly monetary policy shocks, do indeed have long-lasting effects on the level of output, capital stock and TFP in the economy.

2. DATA AND SERIES CONSTRUCTION

The empirical features motivating our analysis rest on two major international and historical databases. Data on macro aggregates and financial variables, including assumptions on exchange rate regimes and capital controls can be found in www.macrohistory.net/data. This database covers 17 advanced economies reaching back to 1870 at annual frequency. Detailed descriptions of the sources of the variables contained therein, their properties, and other ancillary information are discussed in Jordà, Schularick, and Taylor (2017) and Jordà, Schularick, and Taylor (2019), as well as references therein. Importantly, we will rely on the trilemma instrument discussed in Jordà, Schularick, and Taylor (2016), and more recently, Jordà, Schularick, and Taylor (2019) as the source of exogenous variation in interest rates. We briefly describe the instrument construction below but refer the reader to the last reference for a detailed analysis.

The second important source of data relies on the work by Bergeaud, Cette, and Lecat (2016) and available at http://www.longtermproductivity.com. This historical database adds to our main database observations on capital stock (machines and buildings), hours worked, and number of employees, and the Solow residuals (raw TFP). In addition, we construct time-varying capital and labor utilization corrected series using the procedure discussed in Imbs (1999) with the raw data from Bergeaud, Cette, and Lecat (2016) to construct our own series of utilization-adjusted TFP. We went back to the original sources so as to filter out cyclical variation in input utilization rates in the context of a richer production function that allows for factor hoarding. We explain the details of this correction below.

3“The first question I would like to pose concerns the distinction between aggregate supply and aggregate demand: Are there circumstances in which changes in aggregate demand can have an appreciable, persistent effect on aggregate supply? ... More research is needed, however, to better understand the influence of movements in aggregate demand on aggregate supply.”
2.1. Instrument construction

Jordà, Schularick, and Taylor (2019) developed a quasi-natural experiment utilizing the theory of trilemma in international finance: a country with fixed nominal exchange rates and open capital flows loses monetary independence. Variations in the base country interest rates can be used as an instrumental variable to identify exogenous monetary shocks to an open peg country.

The sample is split into three groups of countries, bases whose currency serve as the anchor for the second group of pegging economies, labeled as the pegs, and floats are the final group of countries that allow their currency to be freely determined in the market. A country $i$ is defined to be in a peg $q_i = 1$ at date $t$ if it maintained a peg at dates $t-1$ and $t$. This conservative definition serves to eliminate opportunistic pegging due to country’s monetary conditions. In order to construct the instrument, changes in short term nominal interest rate in the base country are first sterilized to remove the component that can be explained by economic conditions in that country. The resulting variation is adjusted for capital openness.

More formally: let $\Delta r_i(t)$ denote the changes in short term interest rate in country $i$ at date $t$, $\Delta r_{b(i,t)}$ be the change in short term interest rate in country $i$’s base country $b(i,t)$ at time $t$ and $\hat{\Delta} r_{b(i,t)}$ denote the predictable component of variations in base rate. Recognizing that countries are not perfectly open to capital flows, adjustment for capital mobility is constructed using the continuous version $k_{i,t} \in [0,1]$ of capital openness index of Quinn, Schindler, and Toyoda (2011). The resulting instrument is $z_{i,t} = k_{i,t}(\Delta r_{b(i,t)} - \hat{\Delta} r_{b(i,t)})$.

2.2. Factor utilization correction

The Imbs (1999) correction follows Burnside and Eichenbaum (1996) in endogenizing the capital utilization rate in a partial equilibrium model. We assume perfectly competitive factor markets and an aggregate production function which is constant returns to scale in effective capital and labor:

$$Y_t = A_t (K_t u_t)^{\alpha} (L_t e_t)^{1-\alpha}$$

where $Y_t$ is output, $K_t$ is capital stock, and $L_t$ is total hours worked. $u_t$ and $e_t$ denote the respective factor utilizations. $A_t$ is the utilization adjusted TFP. We assume perfect competition in the input and the output markets. Higher capital utilization increases the depreciation of capital $\delta_t = \delta u_t^\phi$ where $\phi > 1$. As a result, firms choose capital utilization rate optimally. Labor hoarding is calculated assuming instantaneous adjustment of effort $e_t$ against a payment of a higher wage $w(e_t)$, while keeping fixed employment (determined one period in advance). The firm’s optimization problem is given by:

$$\max_{e_t, u_t, K_t} A_t (K_t u_t)^{\alpha} (L_t e_t)^{1-\alpha} - w(e_t) L_t - (r_t + \delta u_t^\phi))K_t$$

See Jordà, Schularick, and Taylor (2019) for implementation details.

We assume that these variables are stationary. In section 4, we construct capital-utilization in a general equilibrium model along with endogenous growth.
Households choose consumption, labor supply and effort to maximize their lifetime utility subject to their budget constraint (with complete asset markets)

\[
\max_{c_t, L_t, e_t} \sum_{t=0}^{\infty} \beta^t \left[ \ln C_t - \chi \left( \frac{(e_t L_t)^{1+\nu}}{1+\nu} \right) \right]
\]

Normalizing the long-run capital-utilization and labor-utilization rates to one, the utilization rates can be derived from following:

\[
u_t = \left( \frac{Y_t}{K_t} \right)^{\frac{\delta}{\alpha}}; \quad e_t = \left( \frac{C_t}{Y_t} \right)^{\frac{1}{1+\nu}} \frac{L_t}{L_t}
\]

where \(Y, C, L\) and \(K\) are the steady-state values of output, consumption, labor and capital.

The Solow residual then can be decomposed into utilization-adjusted TFP and utilization corrections:

\[
TFP_t = \frac{Y_t}{K_t^{1-a}} = A_t \times u_t^{a} e_t^{1-a}
\]

To construct steady state values of \(Y, C, L\) and \(K\), we extract a HP-filter trend from the data series. We will show later that our empirical results are robust to computing moving averages over a 10 year window, using time-varying values of \(a\) constructed from labor-income data, and reasonable parameters of the aggregate capital depreciation rate. Bergeaud, Cette, and Lecat (2016) constructed capital stock for machines and buildings separately using the perpetual inventory method with data on investment in machines and buildings and different depreciation rates. We will show robustness of our results to choosing different depreciation parameters. We wish to emphasize that our estimation of productivity assumes misallocation related-wedges are absent. We have not yet found the data to take into account markups in our productivity estimation. See Basu and Fernald (2002) and Syverson (2011) for extensive discussions on what determines productivity.

3. Monetary shocks have long lasting effects

3.1. Empirical approach

The basic empirical approach relies on local projections (Jordà, 2005) estimated with instrumental variables (LPIV). Several applications of these methods are available in the literature, though a more general discussion of the method can be found in Ramey (2016), Stock and Watson (2018) and Jordà, Schularick, and Taylor (2019). Based on the latter, we estimate in particular:

\[
y_{i,t+h} - y_{i,t-1} = \alpha_{i,h} + \Delta r_{i,t} \beta_h + x_{i,t} \gamma_h + v_{i,t+h}; \quad h = 0, 1, \ldots, H; i = 1, \ldots, N; t = t_0, \ldots, T \tag{1}
\]

where \(y_{i,t+h}\) is the outcome variable for country \(i\) observed \(h\) periods from today; \(\alpha_{i,h}\) are country fixed effects, \(\Delta r_{i,t}\) refers to the instrumented change in the short-term government bond (3-months...
in duration), our stand-in for the policy rate which we instrument with \( z_{i,t} \), the trilemma instrument borrowed from Jordà, Schularick, and Taylor (2019) as discussed earlier; and \( x_{i,t} \) collects all additional controls including lags of the outcome and interest rates, as well as lagged values of other macro aggregates. Moreover, we control for global business cycle effects through a global GDP control variable to parsimoniously soak up time effects. We estimate Equation 1 with instrumental variable methods and report cluster robust standard errors.

Table 1 reports the first-stage regression of the pegging country’s short term interest rate \( \Delta r_{i,t} \) on the instrument \( z_{i,t} \) with controls \( x_{i,t} \), country fixed effects and (robust) clustered standard errors. The t-statistic is well above 3 for full sample and post-WW2 samples illustrating that it is not a weak instrument. We refer the reader to Jordà, Schularick, and Taylor (2019) for detailed discussion on the instrument and proceed henceforth assuming the reader is on board regarding instrument relevance and strength.

<table>
<thead>
<tr>
<th>Pegs (q = 1)</th>
<th>All years</th>
<th>PreWW2</th>
<th>PostWW2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_{i,t} )</td>
<td>0.52***</td>
<td>0.35**</td>
<td>0.56***</td>
</tr>
<tr>
<td>t-statistic</td>
<td>[8.62]</td>
<td>[2.05]</td>
<td>[8.97]</td>
</tr>
<tr>
<td>Obs</td>
<td>672</td>
<td>148</td>
<td>524</td>
</tr>
</tbody>
</table>

Notes: *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \). Full sample: 1890 – 2015 excluding WW1: 1914 – 1919 and WW2: 1939 – 1947. Pre-WW2 sample: 1870-1938 (excluding 1914-1919). Post WW2 sample: 1948 – 2015. These regressions include country fixed effects as well as up to two lags of the first difference in log real GDP, log real consumption, investment to GDP ratio, credit to GDP, short and long-term government rates, log real house prices, log real stock prices, and CPI inflation. In addition we include world GDP growth to capture global cycles. See text.

3.2. Main results

The main story in the paper can be illustrated with the response of real GDP to a shock to domestic interest rates using the trilemma instrument. Before we show the main results, we highlight the value of our instrumental variable by providing a comparison of the response of GDP per capita to a shock in the short-term domestic interest rate calculated using selection-on-observables identification versus identification with the trilemma instrument. This is shown in Table 2. The table reports the coefficient estimates of the impulse response calculated with each identification approach for the full and post-WW2 samples. Note that the label LP-OLS refers to identification via selection, LP-IV to the trilemma instrument identification. The samples are restricted to pegging economies to match the samples in both cases.\(^6\)

Table 2 is organized as follows. We provide the coefficient estimates by row and provide a test of the null hypothesis that LP-OLS and LP-IV estimates are equivalent for the two samples considered:

\(^6\)The plots and inference is robust to using real GDP per capita. See Table A1
Table 2: LP-OLS vs. LP-IV. Attenuation bias of real GDP responses to interest rates.
Trilemma instrument. Matched samples

Responses of real GDP at years 0 to 10 (100 \times \log change from year 0 baseline).

<table>
<thead>
<tr>
<th>Year</th>
<th>(a) Full Sample</th>
<th>OLS-IV</th>
<th>(b) Post-WW2</th>
<th>OLS-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LP-OLS (1)</td>
<td>LP-IV (2)</td>
<td>p-value (3)</td>
<td>LP-OLS (4)</td>
</tr>
<tr>
<td>h = 0</td>
<td>0.08**</td>
<td>-0.04</td>
<td>0.18</td>
<td>0.05**</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.09)</td>
<td></td>
<td>(0.02)</td>
</tr>
<tr>
<td>h = 2</td>
<td>-0.27</td>
<td>-1.63***</td>
<td>0.00</td>
<td>-0.21</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.39)</td>
<td></td>
<td>(0.13)</td>
</tr>
<tr>
<td>h = 4</td>
<td>-0.11</td>
<td>-2.22***</td>
<td>0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(0.56)</td>
<td></td>
<td>(0.21)</td>
</tr>
<tr>
<td>h = 6</td>
<td>-0.01</td>
<td>-2.55***</td>
<td>0.00</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.67)</td>
<td></td>
<td>(0.22)</td>
</tr>
<tr>
<td>h = 8</td>
<td>-0.30</td>
<td>-3.47***</td>
<td>0.00</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.85)</td>
<td></td>
<td>(0.22)</td>
</tr>
<tr>
<td>h = 10</td>
<td>-0.33</td>
<td>-4.20***</td>
<td>0.00</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(0.36)</td>
<td>(1.15)</td>
<td></td>
<td>(0.27)</td>
</tr>
<tr>
<td>h = 12</td>
<td>-0.58</td>
<td>-6.77***</td>
<td>0.00</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>(0.42)</td>
<td>(2.08)</td>
<td></td>
<td>(0.33)</td>
</tr>
</tbody>
</table>

KP weak IV | 68.34 | 69.18 |
H,0: LATE = 0 | 0.00 | 0.00 |
Observations | 607 | 607 |

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. Cluster robust standard errors in parentheses. Full sample: 1890 – 2015 excluding WW1: 1914 – 1919 and WW2: 1939 – 1947. Post WW2 sample: 1948 – 2015. Matched sample indicates LP-OLS sample matches the sample used to obtain LP-IV estimates. KP weak IV refers to the Kleibergen-Paap test for weak instruments. H,0: subATE = 0 refers to the p-value of the test of the null hypothesis that the coefficients for h = 0,...,10 are jointly zero for a given subpopulation. OLS = IV shows the p-value for the Hausmann test of the null that OLS estimates equal IV estimates. See text.
full and post-WW2. The differences between identification schemes could not be starker. Both are economically and statistically, but the LP-IV response is considerably larger at all horizons. We display these results graphically in Figure 1.

Regardless of the sample used, a 1 percentage point increase in domestic short-term interest rates have sizable and long-lasting effects on GDP. In the full sample, GDP declines by over 6 percent relative to period 0 over 12 years. The effect, while still large, is considerably more muted in the post-WW2 sample. The drop 12 years after impact is about half, at 2.7 percent. This is a far cry from traditional notions of long-run neutrality found in the literature.

What is the source of this persistent decline? We decompose the decline in GDP into its components, namely, total hours worked (that is, employees times number of hours per employee); capital stock (the sum of capital measured in machines and buildings); and the Solow residual (using a Cobb-Douglas production function) labeled as the total factor productivity (TFP). Using the Imbs (1999) correction, we shortly decompose this Solow residual into factor-utilization (capital and labor) and the subsequent residual is labeled as the utilization adjusted TFP.

Figure 2 displays the responses of each of these components to the same shock to the domestic short-term interest rate instrumented with the trilemma, both for the full and the post-WW2 samples. The figure displays the responses of total hours worked, capital and raw TFP without error bands to provide a clearer sense of the dynamic paths. A more detailed figure is provided in the appendix A.1, which includes the error bands.

Several features deserve mention. Figure 2 shows that there are similar declines in capital and raw TFP whereas total hours worked exhibits a much flatter pattern. Because capital enters the
**Figure 2:** Baseline response to 100 bps trilemma shock: Real GDP and components

![Figure 2](image)


production function with a smaller weight, it should be clear from the figure that most of the decline in GDP is explained by TFP variable, and then capital, with total hours worked mostly flat. To be sure, the response of hours worked conforms well with the textbook response to a monetary shock. Total hours fall in the short-run, but then recover quickly and remain mostly flat.

Capital accumulation also follows textbook dynamics in the short-run. The response is initially muted but builds up over time. But unlike the textbook model, capital does not appear to recover even 12 years after the shock. Similarly, TFP falls gradually rather than suddenly. Over time, the decline in TFP accelerates, ending at a level $3(1)$ percent lower in full (post WW2) sample by year 12 relative to year 0.

We present the responses of the various components of the Solow residual in Figure 3 for both the full and post-WW2 samples. Utilization rates of labor and capital exhibit cyclical dynamics: falling in the short-run and then recovering back to zero. In a sense, this pattern is mechanical. It reflects the restriction imposed on the calculation that utilization rates must return to zero eventually (Imbs, 1999). Figure 4 presents the evolution of capital to utilization adjusted TFP ratio in response to the trilemma shock. The hump-shaped response replicates the conventional justification for including investment adjustment costs in typical DSGE models (Christiano, Eichenbaum, and Evans, 2005).

One explanation for the long-lasting effects of the monetary shock could be that domestic interest

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7We later show the robustness of using a more commonly used utilization adjusted TFP series constructed by Fernald (2014) for the post-WW2 US, which requires more detailed data than is available for the historical panel of 17 countries. Reassuringly, the results for utilization adjusted TFP using the two methods are qualitatively similar.
Figure 3: Baseline response to 100 bps trilemma shock: TFP and utilization


Figure 4: Baseline response to 100 bps trilemma shock: Capital to utilization adjusted TFP

rates remain elevated for a long period of time as well. In other words, persistence is generated by a delayed response in interest rates. A simple check of this proposition can be done in two steps. Figure 5 shows that the short-term real interest rate does indeed take approximately 8 years to return to zero, while the nominal interest rate returns to zero after 4 years. The response of nominal interest rate is typical of what has been reported often in the literature (see, e.g., Christiano et al., 1999; Ramey, 2016). Secondly, we can calculate the responses of the main variables normalized by the response of interest rates over time to sterilize the dynamics of interest rates themselves. This is no different than calculating a multiplier (see, e.g., Ramey, 2016; Ramey and Zubairy, 2018).

**Figure 5:** Baseline response to 100 bps trilemma shock: Short term real interest rate

![Graph](image)


Thus, in Figure 6 we show the ratio of the cumulative change in GDP to the area under the real interest rate path in Figure 5. In a given period, the difference in the level of real interest rate relative to the counterfactual path measures the tightness of monetary policy. We consider these cumulative gaps as a measure of overall monetary policy tightness (the area under the solid line in Figure 5). By year 12, the multiplier is -1 in the full sample and -0.8 in the post-WW2 sample, virtually identical and sizable. These numbers compare with those obtained in the literature using VAR and Romer and Romer (2004) shocks for the US.\(^8\)

\(^8\)For real GDP per capita, the multiplier is -1.4 in the full sample and -1.3 in the post-WW2 sample by year 12.

\(^9\)In their online appendix (Table A.1), Nakamura and Steinsson (2018) report the multiplier of monthly industrial production to monetary shocks over 36 months to lie between 1 and 2 using the local projections specification for VAR shocks and Romer-Romer shocks.
Figure 6: Cumulative change in real GDP / Cumulative short term expected real interest rate gap


3.3. Is the US different?

The US for most of our historical sample is not a pegging economy (with the exception of the Gold standard years before WW1). It is the quintessential base country for many economies in our sample. For this reason, the trilemma instrument mechanically sets aside any information coming from the US economy during estimation. It is natural to wonder the extent to which US data conforms with the patterns presented so far.

In this section, we examine US data post-WW2. This allows us to incorporate three useful modifications. First, we use higher frequency quarterly data. Second, we use the alternative utilization adjusted series for TFP constructed by Fernald (2014). Third, in order to achieve identification we rely on the instrumental variable constructed by Romer and Romer (2004) based on the Federal Reserve staff’s implied forecast errors for the policy rate, and extended to recent years by Wieland and Yang (2016). It turns out that results based on U.S. data largely confirm the results we reported in the previous section for non-U.S. pegging economies.  

\[ y_{t+h} - y_{t-1} = \alpha_h + \hat{\Delta} r_t \beta_h + x_t \gamma_h + v_{t+h}; \quad h = 0, 1, \ldots, H; i = 1, \ldots, N; t = t_0, \ldots, T \]

where \( y_{t+h} \) is the outcome variable at horizon \( h \), \( \hat{\Delta} r_t \) is the instrumented change in the Federal Funds rate, \( x_t \) is the set of controls that includes contemporary and four lags of log real GDP, log CPI, and changes in federal funds rate. We do not include the contemporary variable when it is same as the dependent variable. We report robust standard errors.
**Figure 7:** Baseline response to 100 bps *Romer and Romer* (2004) shock: US postwar data

![Graphs of Real GDP, (Util. Adj.) TFP index, Hours, bus sector, Capital input](image)


*Figure 7* plots the path of real GDP along with its three components: total hours worked, capital, and utilization adjusted TFP. The responses are qualitatively similar to those in the long-run panel, although the amplitudes are more muted. Quarterly data is naturally noisier than yearly.
**Figure 8:** Baseline response to 100 bps Romer and Romer (2004) shock: US postwar data

![Graph showing baseline response to 100 bps Romer and Romer (2004) shock with US postwar data. The graph includes lines for GDP, Util adj TFP, K, L, and LQ.](image)


The data, but smoothing over a temporary recovery in GDP 4 to 5 years after the shock, real GDP ends nearly one percent lower 8 years after impact. Utilization adjusted TFP and hours exhibit a similar u-shaped pattern, with TFP nearly back to zero by year 8. Strikingly, capital accumulation exhibits a protracted decline over the entire period, ending about 1.25 percent lower after eight years. For comparison, we plot the various components on the same graph in Figure 8.

The data for utilization-adjusted TFP series are based on sectoral data for the US that accounts for heterogeneity across workers and types of capital. Fernald (2014) also notes that there are various other corrections that are not conducted in the quarterly series due to the lack of rich-industry level data. The finding that monetary policy shocks can affect utilization-adjusted TFP echo the Evans (1992)'s critique of using Solow residuals as productivity shocks in RBC models. While the construction of quarterly utilization-adjusted TFP series is detailed and thorough, our analysis suggests caution against using the quarterly-adjusted residuals as “pure” TFP shocks (i.e. perfectly...
orthogonal to demand shocks).\footnote{Ramey (2016) also documents that utilization adjusted TFP series fail Granger causality tests (See Table 11 in her paper). We are grateful to Valerie Ramey for alerting us to that analysis.}

\section{A model of hysteresis}

Impulse responses calculated with standard methods that internally favor reversion to the mean will tend to underestimate the value of the response at longer horizons. By relying on local projections, we allow the data to more directly speak as to its long-run properties. The evidence presented in the previous sections strongly indicate that these long-run effects are important and require further investigation. In order to think through a possible mechanism that explains our empirical findings, we augment a textbook New Keynesian model with endogenous growth in a stylized manner. We present the different components of the model in the next few sections, starting with the production side of the economy.

\subsection{Monopolistically Competitive Producers}

Assume there is a continuum of differentiated intermediated good producers that sell the intermediate good $Y_{kt}$. These goods can be aggregated into a Dixit-Stiglitz final composite good $Y_t$ as follows:

$$Y_t = \left[ \int_0^1 Y_{kt}^{1+\lambda_p} dk \right]^{1+\lambda_p}$$

where $\lambda_p > 0$ is the price markup. The iso-elastic demand for intermediate good $k$ is given by:

$$Y_{kt} = \left( \frac{P_{kt}}{P_t} \right)^{1+\lambda_p} \lambda_p Y_t$$

The zero profit condition for competitive final good producers implies that the aggregate price index is

$$P_t = \left[ \int_0^1 P_{kt}^{1+\lambda_p} dk \right]^{-\lambda_p}$$

Each intermediate good $k$ is produced by a price-setting monopolist using labor $L_{kt}$ and physical capital $K_{kt}$\footnote{In the appendix B, we show that similar results are obtained with samples beginning until at least 1973Q2. However, these persistent effect results are not robust to considering even shorter samples for the US economy. This is consistent with the findings of Coibion, Gorodnichenko, and Ulate (2017).}:

$$Y_{kt} = (Z_t L_{kt})^{1-\alpha} K_{kt}^{\alpha}$$

\footnote{We can append fixed cost in the production function to eliminate steady state profits. This is usually done to justify no entry and exit in the steady state in a DSGE model (Christiano et al. 2005, Justiniano, Primiceri, and Tambalotti 2013, Smets and Wouters 2007). In a micro-founded model of growth, positive rents are needed to incentivize investment in growth (Romer, 1990).}
where $Z_t$ is the aggregate TFP. The variable $Z_t$ denotes a non-stationary TFP series that we describe in the next subsection.\footnote{Relative to the conventional literature, we will allow for an endogenous relationship between TFP growth and output gap.} Firms may not be able to adjust their price in a given period, but they will always choose inputs to minimize total cost each period. The cost minimization yields the input demand functions.

$$W_t = (1 - \alpha) \frac{mc_{kt} F_t^{1-\alpha}}{L_t} \left( \frac{K_t}{L_t} \right)^\alpha$$

$$R^k_t = \alpha \frac{mc_{kt} F_t^{1-\alpha}}{1 - \alpha} \left( \frac{K_t}{L_t} \right)^{\alpha-1}$$

The first order condition implies that the capital labor ratio at the firm level is independent on firm-specific variables:

$$\frac{K_{kt}}{L_{kt}} = \frac{K_t}{L_t} = \frac{\alpha}{1 - \alpha} \frac{W_t}{R^k_t}$$

Thus, (nominal) marginal cost is independent of firm specific variables:

$$P_t mc_{kt} = P_t mc_t = \frac{1}{F_t^{1-\alpha}} \left( \frac{R^k_t}{\alpha} \right)^\alpha \left( \frac{W_t}{1 - \alpha} \right)^{1-\alpha}$$

Each firm $k$ is assumed to set prices on a staggered basis following Calvo (1983). With probability $(1 - \theta_p)$, a firm adjusts its price independent of previous history. A resetting firm chooses $\tilde{P}_{kt}$ to maximize:

$$\mathbb{E}_t \sum_{s=t}^\infty \theta_s^{s-t} Q_{t,s} \left[ \frac{\tilde{P}_{kt}}{P_s} - mc_s \right] Y_s(k)$$

subject to demand for its product

$$Y_{kt} = \left( \frac{P_{kt}}{P_t} \right)^{1+\lambda_p} \alpha \frac{P_t}{\Lambda_t} P_s$$

where the stochastic discount factor in period $s$ relative to period $t$ is given by:

$$Q_{t,s} = \beta^{s-t} \Lambda_s \frac{P_t}{\Lambda_t} P_s$$

and $\Lambda_t$ is the marginal utility of consumption defined later. The first order condition is:

$$\mathbb{E}_t \sum_{s=t}^\infty \theta_p^{s-t} Q_{t,s} \left[ \frac{\tilde{P}_t}{\Pi_{t,s}} - (1 + \lambda_p) mc_s \right] \left( \frac{\tilde{P}_t}{\Pi_{t,s}} \right)^{-\lambda_p} Y_s = 0$$

By the law of large numbers, the law of motion of the aggregate price index $P_t$ is given by:

$$P_t^{\lambda_p} = (1 - \theta_p)(\tilde{P}_t)^{\lambda_p} + \theta_p P_{t-1}^{\lambda_p}$$
4.2. Hysteresis effects

In order to be able to capture the empirical features described in the previous sections, we examine a richer specification of the law of motion for total factor productivity than is conventional. In particular, we assume that the law of motion for $Z_t$ is:

$$\log Z_t = \log Z_{t-1} + \mu_t + \eta \log \left( Y_{t-1} / Y_{t-1}^{f,t-1} \right)$$

where $\mu_t$ is the exogenous component of the TFP growth rate, that may be subject to trend shocks. $Y_{t-1}^{f,t-1}$ is the flexible price level of output in period $t - 1$ conditional on $Z_{t-1}$, and will be referred to as the potential output at time $t - 1$. The second component denotes the endogenous component of TFP growth, where $\eta$ is the elasticity of TFP growth rate with respect to fluctuations in output due to nominal rigidities. We refer to this as the hysteresis elasticity (to be consistent with DeLong and Summers 2012).

The above law of motion allows business cycles to affect TFP growth rate only in the presence of nominal rigidities or inadequate stabilization. For clarity, we employed this reduced form equation. A micro-founded model of innovation and productivity growth that yields this exact representation under monetary policy shocks can be found in the recent literature embedding endogenous growth into DSGE models (Bianchi et al., 2019; Garga and Singh, 2016). The effects of business cycles on TFP growth rate that are unrelated to nominal rigidities can be denoted by time varying values of $\mu_t$, which may depend on other shocks (markup shocks, stationary TFP shocks, discount factor shocks, capital quality shocks etc.). For ease of exposition, we only focus on the hysteresis effects induced by the presence of nominal rigidities and treat $\mu_t$ to be an exogenous series. Movements in this series can be used to induce state-dependent hysteresis effects of monetary policy shocks.

This functional dependence creates a role for hysteresis stabilization by central banks in a reduced form manner (Yellen, 2016). Long-run effects of monetary policy shocks depend on the value of $\eta$. To clarify, in this paper, we are only able to discuss the sign and the magnitude of $\eta$ in response to temporary monetary shocks denoted as $\eta^{mp}$. We assume that $\eta^{mp} > 0$ and the exact quantitative magnitudes can be extracted from the empirical IRFs estimated above.

4.3. Households

Rest of the model components are standard. We briefly summarize them here and leave the detailed derivations to the appendix.

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15 A similar setup was used by Stadler (1990) in his seminal work.
16 Theoretically, there is no apriori reason to expect $\eta$ to be positive. While a ‘cleansing’ effect of recessions may induce counter-cyclicality, recessions may reduce funding access to firms to conduct R&D, skill development, and learning-by-doing. The sign on the cyclicalities of TFP misallocation is also ambiguous and depends on the assumptions in a model.
4.3.1 Households

Each household supplies differentiated labor indexed by \( j \). Household \( j \) chooses consumption \( C_t \), risk-free nominal bonds \( B_t \), investment \( I_t \) and capital utilization \( u_t \) to maximize the utility function, with external habits over consumption:

\[
E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \log(C_{j,s} - hC_{j,s-1}) - \frac{\omega}{1 + v} L_{j,s}^{1+v} \right]
\]

where \( h \) is the degree of habit formation, \( v > 0 \) is the inverse Frisch elasticity of labor supply, \( \omega > 0 \) is a parameter that pins down the steady-state level of hours, and the discount factor \( \beta \) satisfies \( 0 < \beta < 1 \). We assume perfect consumption risk sharing across the households. As a result, household’s budget constraint in period \( t \) is given by

\[
P_t C_t + P_t I_t + B_{t+1} = B_t (1 + i_t) + B_{j,t}^S + (1 + \tau^w) W_t L_{j,t} + \Gamma_t + T_t + R_k^u u_t K_t^u - P_t a(u_t) K_t^u
\]

where \( I_t \) is investment, \( B_{j,t}^S \) is the net cash-flow from household \( j \)'s portfolio of state-contingent securities. Labor income \( W_t L_{j,t} \) is subsidized at a fixed rate \( \tau_w \). Households own an equal share of all firms, and thus receive \( \Gamma_t \) dividends from profits. Finally, each household receives a lump-sum government transfer \( T_t \). Since households own the capital and choose the utilization rate, the amount of effective capital that the households rent to the firms at nominal rate \( R_k^u \) is:

\[
K_t = u_t K_t^u
\]

The (nominal) cost of capital utilization is \( P_t a(u_t) \) per unit of physical capital. As in the literature (Smets and Wouters 2007) we assume \( a(1) = 0 \) in the steady state and \( a'' > 0 \). Following Christiano, Eichenbaum, and Evans (2005), we assume investment adjustment costs in the production of capital. Law of motion for capital is as follows:

\[
K_{t+1}^u = \left[ 1 - S \left( \frac{I_t}{(1 + g_{ss}) I_{t-1}} \right) \right] I_t + (1 - \delta_k) K_t^u
\]

where \( g_{ss} \equiv \bar{q} \) is the steady state growth rate of \( Z_t \). Utility maximization delivers the first order condition linking the inter-temporal consumption smoothing to the marginal utility of holding the risk-free bond

\[
1 = \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} (1 + i_t) \frac{P_t}{P_{t+1}} \right]
\]

The stochastic discount factor in period \( t + 1 \) is given by:

\[
Q_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}}
\]
where $\Lambda_t$ is the marginal utility of consumption given by:

$$
\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \frac{h\beta}{C_{t+1} - hC_t}
$$

The household does not choose hours directly. Rather each type of worker is represented by a wage union who sets wages on a staggered basis. Consequently the household supplies labor at the posted wages as demanded by firms.

We introduce capital accumulation through households. Solving household problem for investment and capital yields the Euler condition for capital:

$$
q_t = \beta \mathbb{E}_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \left( \frac{R^K_{t+1}}{P_{t+1}} u_{t+1} - a(u_{t+1}) + q_{t+1}(1 - \delta_k) \right) \right]
$$

where the (relative) price of installed capital $q_t$ is given by

$$
q_t \left[ 1 - S \left( \frac{I_t}{(1 + g_{ss})I_{t-1}} \right) \right] - S' \left( \frac{I_t}{(1 + g_{ss})I_{t-1}} \right) \frac{I_t}{(1 + g_{ss})I_{t-1}} + \beta \frac{\Lambda_{t+1}}{\Lambda_t} q_{t+1} \frac{1}{(1 + g_{ss})} \left( \frac{I_{t+1}}{I_t} \right)^2 S' \left( \frac{I_{t+1}}{(1 + g_{ss})I_t} \right) = 1
$$

Choice of capital utilization rate yields:

$$
\frac{R^K_t}{P_t} = a'(u_t)
$$

### 4.3.2 Wage Setting

Wage Setting follows Erceg, Henderson, and Levin (2000) and is relatively standard. Perfectly competitive labor agencies combine $j$ type labor services into a homogeneous labor composite $L_t$ according to a Dixit-Stiglitz aggregation:

$$
L_t = \left[ \int_0^1 L_{j,t}^{1+\lambda_w} \, dj \right]^{1+\lambda_w}
$$

where $\lambda_w > 0$ is the nominal wage markup. Labor unions representing workers of type $j$ set wages on a staggered basis following Calvo (1983), taking given the demand for their specific labor input:

$$
L_{j,t} = \left( \frac{W_{j,t}}{W_t} \right)^{-1+\lambda_w} \lambda_w L_t, \quad \text{where} \quad W_t = \left[ \int_0^1 W_{j,t}^{-1} \, dj \right]^{-\lambda_w}
$$

In particular, with probability $1 - \theta$, the type-$j$ union is allowed to re-optimize its wage contract and it chooses $\tilde{W}$ to minimize the disutility of working for laborer of type $j$, taking into account
the probability that it will not get to reset wage in the future. The first order condition for this problem is given by:

\[ E_t \sum_{s=0}^{\infty} (\beta \theta_w)^s \Lambda_{t+s} \left( (1 + \tau_t^W) \bar{W}_t - (1 + \lambda_{t+w}) \omega \frac{L_t^V}{\Lambda_{t+s}} \right) L_{t+1} = 0 \]  

(4)

By the law of large numbers, the probability of changing the wage corresponds to the fraction of types who actually change their wage. Consequently, the nominal wage evolves according to:

\[ W_t^\frac{1}{\bar{w}} = (1 - \theta_w) \bar{W}_t^\frac{1}{\bar{w}} + \theta_w W_{t-1}^\frac{1}{\bar{w}} \]

where the nominal wage inflation and price inflation are related to each other:

\[ \pi_t^w = \frac{W_t}{W_{t-1}} = \frac{w_t}{w_{t-1} \pi_t} \frac{1}{1 + g_t} \]

where \( \pi_t \equiv \frac{p_t}{p_{t-1}} \) is the inflation rate, \( w_t \equiv \frac{W_t}{\pi_{t}Z_t} \) is the productivity adjusted real wage and \( g_t \) is the growth rate of \( Z_t \).

4.4. Government

The central bank follows a Taylor rule in setting the nominal interest rate. It responds to deviations in inflation, output and output growth rate from time-t natural allocations.

\[ \frac{1 + i_t}{1 + i_{ss}} = \left( \frac{1 + i_{t-1}}{1 + i_{ss}} \right)^{\rho_R} \left[ \left( \frac{\pi_t}{\pi_{ss}} \right)^{\phi_\pi} \left( \frac{Y_t}{Y_{t-1}} \right)^{\phi_y} \right]^{1 - \rho_R} \left( \frac{Y_t}{Y_{t-1}} \right)^{\phi_y} e_{t,mp} \]

(5)

where \( i_{ss} \) is the steady state nominal interest rate, \( Y_{t-1}^{\pi,ss} \) is the time-t natural output, \( \rho_R \) determines interest-rate smoothing and \( e_{t,mp} \sim N(0, \sigma_r) \) is the monetary policy shock.

We assume government balances budget every period:

\[ P_i T_i = \tau^p \int_0^1 p_i x_i d_i + \tau^w W_i L_i + P_i G_t \]

where \( G_t \) is the government spending, which is determined exogenously as a fraction of GDP

\[ G_t = \left( 1 - \frac{1}{\lambda^g} \right) Y_i \]

\(^{17}\)We assume imperfect wage indexation in our nominal wage rigidity assumption. We ignore specifying it here for ease of exposition. See appendix for details.
where the government spending shock follows the process:

\[
\log \lambda_s^g_t = (1 - \rho_g) \lambda_s^g + \rho_g \log \lambda_s^g_{t-1} + \epsilon_s^g_t; \quad \epsilon_s^g_t \sim N(0,\sigma_s^g)
\]

4.5. Market Clearing

\[
Y_t = C_t + I_t + a(u_t)K_t + G_t
\]

4.6. Simulations

Since the DSGE model is intentionally standard, we take the parameters estimates from estimates common in the literature Justiniano et al. (2013). We report these in Table 4. The steady state parameters imply (annualized) real interest rate of 2.40%, and an investment-GDP ratio of 17%. For the monetary shock process, we chose the following parameters \(\sigma_r = 0.02\) and \(\rho_R = 0.8\). The short term nominal interest rate increases by about 10 basis points on impact in the three simulations that we report here.

The new parameter in our model, relative to the business cycles literature, is \(\eta\) - the hysteresis elasticity. Table 3 reports the point estimates for \(\eta\) implies by the estimated impulse responses. We take an average of the IRF of raw TFP and the IRF of GDP to the respective monetary policy shock in computing this elasticity. Following the persistent drop in output after the Great Recession in the US, DeLong and Summers (2012) infer that this parameter could be as high as 0.24. In our calibration henceforth, we use the value of 0.18.

**Table 3: Point estimates for hysteresis elasticity \(\eta\)**

<table>
<thead>
<tr>
<th></th>
<th>pegs (trilemma)</th>
<th>US (RR)</th>
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</thead>
<tbody>
<tr>
<td>(\eta)</td>
<td>0.18</td>
<td>0.42</td>
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</table>

Notes: The point estimates are calculated by taking an average of the IRF of raw TFP and the IRF of GDP to the monetary policy shock. Figure 2 and Figure 7 report the IRFs for GDP and TFP.

Figure 9 plots the model-implied impulse responses for output, capital stock, real interest rate and capital to utilization adjusted TFP after a monetary policy shock. Solid line reports the IRFs for endogenous growth model with \(\eta = 0.18\), and dashed line reports IRFs for the comparable exogenous growth benchmark i.e. \(\eta = 0\). The IRFs for output and capital stock are plotted in percent deviations from an exogenous trend. For real interest rate, we plot the actual path of real interest.

\(^{18}\)Note that this exercise is merely illustrative. A detailed investigation is pending until the next iteration of the draft.
rate. Capital to utilization adjusted TFP ratio is in percent deviation from steady state ratio. Time is in quarters.

The model replicates the estimated empirical patterns. There is a persistent decline in capital stock, output and TFP. Furthermore, the endogenous growth model exhibits considerable amplification to the transitory shock because of the large hysteresis elasticity.

We define the accumulated gaps in TFP growth rate as the hysteresis. We next show the path of output, and capital when the central bank sets interest rates following an augmented Taylor rule:

\[
1 + i_t = (1 + i_{t-1})^{\rho_R} \left( \frac{\pi_t}{\pi_{ss}} \right)^{\phi_\pi} \left( \frac{Y_t}{Y_{t-1}^f} \right)^{\phi_y} \left( \frac{H_t}{H_{t-1}^f} \right)^{1-\rho_R} \left( \frac{Y_t/Y_{t-1}}{Y_{t-1}/Y_{t-1-1}} \right) \phi_{dy}^{mp} e_t^{mp} \]

where \(H_t\) is hysteresis and follows the law motion given by:

\[H_t = H_{t-1} + g_t - g_t^f\]

We set \(\phi_H = 0.2\) to plot the graphs. Figure 10 contrasts the path of output and capital stock after a similar 10 bps shock to the federal funds rate. The real interest rate increases by less than in the case of the standard Taylor rule (Equation 5). This is because, under a hysteresis targeting rule, the central bank accommodates above-target inflation at a later time in order to target zero output hysteresis. Expectations of a high inflation rate induce less contraction in the economy as well as a sharp recovery of GDP to the pre-shock trend.

<table>
<thead>
<tr>
<th>Table 4: Parameters</th>
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<tbody>
<tr>
<td><strong>Steady State Parameters</strong></td>
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<td>(\beta)</td>
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<td>Discount factor</td>
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<table>
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<tr>
<th>Parameters Characterizing the Dynamics</th>
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<tr>
<td>(v)</td>
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<tr>
<td>Inverse Frisch elasticity</td>
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<tr>
<td>(\frac{a''(1)}{a'(1)})</td>
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<tr>
<td>Capital utilization cost</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

Notes: The table shows the parameter values of the model for the baseline calibration. See text.
Figure 9: Response of Output, Capital Stock, Real interest rate and Capital to TFP ratio to a 10 bps increase in nominal interest rate

Notes: The figure plots the model-implied IRFs for output, capital stock, real interest rate and capital to utilization adjusted TFP to a transitory shock to the assumed Taylor rule. Solid line reports the IRFs for endogenous growth model with \( \eta = 0.18 \), and dashed line reports IRFs for the comparable exogenous growth benchmark i.e. \( \eta = 0 \). Time is in quarters. IRFs are traced following a one-time exogenous shock in the federal funds rate of about 10 basis points. The IRFs for output and capital stock are plotted in percent deviations from an exogenous trend. For real interest rate, we plot the actual path of real interest rate. Capital to utilization adjusted TFP ratio is in percent deviation from steady state ratio.

5. Robustness and Discussion

In this section, we address some of the concerns relating to our empirical analysis presented above.

5.1. Structural breaks

Fernald (2014) and Gordon (2016) have convincingly argued that there are structural breaks in TFP growth in the US economic trajectory. It is plausible that there are structural breaks in other economies’ TFP growth rates. If these structural breaks implying slowdown in TFP growth occur around the identified monetary shocks, it could bias our results leading us to attribute the persistent effects incorrectly to monetary shocks. To address this concern, we first estimate five structural breaks in TFP growth for each country in our sample using the UD-max statistic of Bai & Perron (1998). We report these estimated structural break dates in the appendix A.4. Then in our baseline specification, we allow output growth to lie in either of the five regimes at horizon zero as well as
Figure 10: Responses of output, capital, real interest rate and inflation rate to a 10 bps increase in nominal interest rate under a Taylor rule and a Hysteresis targeting rule with $\eta = 0.18$

Notes: The figure plots the model-implied IRFs for output, capital, real interest rate and net inflation rate. Time is in quarters. IRFs are traced following a one-time exogenous shock in the federal funds rate of 10 bps (annualized). The IRFs for output and capital stock are plotted in deviations from an exogenous trend. See text for details.

horizon $h$: i.e.

$$y_{i,t+h} = \alpha_{i,h} + \sum_{k=1}^{k} (D_{i,k,t} + D_{i,k,t+h}) + \Delta r_{i,t} \beta_h + x_{i,t} \gamma_h + v_{i,t+h},$$

where $D_{i,k,t+h}$ is country-specific dummy for TFP growth regime $k$ (Bai-Perron) at horizon $h = 0, ..., H - 1$ and $k \in (1, 5)$. Notice that this specification is conservative whereby we allow horizon $h$ regime changes in the estimation along with horizon 0 regimes.

Figure 11 compares the estimated impulse response when including structural breaks in the left panel and the baseline specification of no structural breaks in the right panel. Taking structural breaks in account does not change the result that there are persistent effects on output of monetary shocks.\(^\text{19}\)

\(^{19}\)In the appendix A.2, we report the IRFs when controlling for country specific structural breaks in growth rate of real GDP per capita.
Figure 11: Response to 100 bps trilemma shock with structural breaks: Real GDP

5 breaks in TFP and expected TFP

no breaks in TFP


5.2. Levels/Differences and number of lags

We report the robustness of IRFs estimated in the baseline to adding the control variables $x_{i,t}$ in levels instead of first differences in the left panel of Figure 12, as well as to including up to 5 lags of the control variables in the right panel.
**Figure 12:** Response to 100 bps trilemma shock with controls in levels: Real GDP

![Graph showing response to 100 bps trilemma shock with controls in levels: Real GDP](image)

**Notes:** Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). LP-OLS estimates displayed as a dashed red line, LP-IV estimates displayed as a solid blue line and 1 S.D. and 90% confidence bands. See text and Jordà, Schularick, and Taylor (2019).

### 5.3. Spillover correction for the trilemma instrument

A violation of the exclusion restriction could occur if base rates affect home outcomes through channels other than movements in home rates. Additional influences via such channels are sometimes referred to as spillover effects. These could occur if base rates proxy for factors common to all countries. That said, these factors would have to persist despite having included global GDP to soak up such business cycle variation. We now assess such spillover effects more formally, using two separate approaches: a) a control function approach developed in Jordà, Schularick, and Taylor (2019), and b) controlling for foreign variables at each horizon to orthogonalize any spillover channels from the interest rate channel.

#### 5.3.1 Synthetic control function approach

First test for exclusion restriction violation follows the control function approach developed in Jordà, Schularick, and Taylor (2019). It exploits the presence of subpopulation of floats in our sample which for example contain information about the global factors at the time of the trilemma shock. **Figure 13** shows our spillover-adjusted estimates of response of output to a 100 bps monetary policy shock.
Figure 13: Response to 100 bps trilemma shock with spillover corrections: Real GDP

Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). Post-WW2 sample: 1948–2015. LP-OLS estimates displayed as a dashed red line, LP-IV estimates displayed as a solid blue line and 1 S.D. and 90% confidence bands, LP-IV spillover corrected estimates displayed as a light green shaded area with dashed border, using $\lambda \in [1, 18]$. See text and Jordà, Schularick, and Taylor (2019).

5.3.2 Orthogonalized interest rate channel

A second approach that attempts to provide validity to the exclusion restriction is directly controlling for a primary channel through which the spillover effects may originate. A monetary tightening in the base country may reduce the demand for imports from pegging economy. This reduction in demand my arise from other trading partners when the tightening affects other countries as well. Or the interest rate channel induced contraction in pegging economy by reducing demand for other countries’ output may be subject to spillbacks. If the transmission is primarily driven by increased trade, controlling for global GDP growth can potentially absorb these effects allowing us to orthogonalize domestic demand effects with respect to international spillbacks. We augment the baseline specification as follows:

$$y_{i,t+h} = \alpha_{i,h} + \Delta \hat{r}_{i,t} \beta_{h} + x_{i,t} \gamma_{h} + G_{t+h} \hat{\gamma}_{h} + v_{i,t+h}, \quad \text{for } h = 0, ..., H - 1$$

where $G_{t+h}$ is global GDP growth at time $t+h$. 
Figure 14: Response to 100 bps trilemma shock with foreign variables controlled: Real GDP


Figure 14 produces the results from the estimation with global GDP growth at each horizon. In comparison to the baseline specification, we still find very persistent and significant effects on GDP. In the appendices A.3 and A.4, we show two additional robustness exercises where we control for base country’s business cycle and control for current account dynamics of the pegging country. The results are robust to these additional checks.

5.4. Why might VARs not find persistent effects?

That a misspecified VAR suffers from compounding of model mis-specification errors gives local projections approach a well-known advantage. We provide an example here that suggests why researchers may not be able to estimate such long-run effects that we find when estimating VARs in finite samples and few lags.

Using the estimated IRFs in the baseline model for the full sample over 12 years, we recover the coefficients of an invertible MA(12) process and generate data using this data generating process. Figure 15 compares the IRFs obtained from estimating AR(2), AR(4) and a local projection with two lags. AR(4) process outperforms AR(2) in replicating the true impulse response function coefficients at horizons greater than two. The misspecification errors get compounded at higher horizons under AR(2) than under AR(4), while local projections is not prone to a similar weakness.
Figure 15: Estimated IRFs using LP(p) and AR(p)

Monte Carlo Simulation: AR(2) and AR(4) vs LP (2)
1000 Monte Carlo replications. Sample size: 150 obs

Notes: Solid black line denotes the baseline response of real GDP in pegging economies to a 100 bps shock in domestic interest rate instrumented with the trilemma. Sample: 1890–2015 (World Wars excluded).

6. Conclusion

In this paper, we set out to investigate the widely accepted proposition of long-run money non-neutrality. Using panel-data spanning over a century and trilemma-identified monetary policy shocks, we find that monetary policy shocks indeed have long-run effects on output, capital and TFP. Furthermore, in the quarterly data for the US since 1969, we find that capital stock is permanently lower after a temporary monetary policy shock identified using a separate instrument. At the same time, capital to TFP ratio exhibits a hump-shaped relation as commonly found in the empirical macro-economics literature using vector-autoregression estimation. We reconcile these findings using a simple extension of the Smets and Wouters (2007) medium-scale DSGE model, where TFP growth depends on deviations of output from its flexible-price counterpart. Using the model, we show that a central bank that also targets hysteresis in its policy rule can offset the long-run effects of monetary policy.

References


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A. Appendix: Additional Graphs Related to Trilemma Shock

A.1. Figure 2 IRFs with error bands

Figure A1: Full Sample: Real GDP and components

IRFs to a 100 bps trilemma shocks: 1890-2015

Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). See text.
Figure A2: Post WW2 sample: Real GDP and components

IRFs to a 100 bps trilemma shocks: 1948–2015

Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Post-WW2 sample: 1948–2015. See text.

A.2. IRFs with Structural Breaks in GDP per capita
We show an alternate selection of structural breaks based on real GDP per capita.
**Figure A3:** Response to 100 bps trilemma shock with structural breaks: Real GDP


A.3. IRFs with base country cycle

We test for exclusion restriction by controlling for base country’s gdp growth rate. The specification follows the one described in Section 5.3.2:

\[
y_{i,t+h} = \alpha_{i,h} + \Delta r_{i,t} \beta_h + x_{i,t} \gamma_h + B_{b(i,t),t+h} \hat{\gamma}_h + v_{i,t+h}, \quad \text{for } h = 0, \ldots, H - 1
\]

where \( B_{b(i,t),t+h} \) is gdp growth of base country \( b(i,t) \) at time \( t + h \) where \( i \) is the pegging country.

**Figure A4:** Response to 100 bps trilemma shock with base country’s business cycle controlled: Real GDP

While there is considerable attenuation in the estimated impulse response, the long-run effects are quantitatively and statistically significant.

A.4. IRFs controlling for current account and exchange rate

Another test for exclusion restriction is conducted by controlling for current account of the peg at each horizon. Furthermore, we also control for the exchange rate with respect to the USD. Whenever US is the base country, controlling for the exchange rate with respect to USD is unlikely to affect our results. However, when the base country is DEU or GBR, then this exchange rate can potentially control for spillover effects. The specification is similar to that described in Section 5.3.2:

\[ y_{i,t+h} = \alpha_{i,h} + \Delta r_{i,t} \beta_h + x_{i,t} \gamma_h + (CA_{i,t+h} \times XRUSD_{i,t+h}) \hat{\gamma}_h + \nu_{i,t+h} \quad \text{for } h = 0, ..., H - 1 \]

where \( CA_{i,t+h} \) is the current account and \( XRUSD_{i,t+h} \) is the exchange rate with respect to USD of country \( i \) at time \( t + h \).

**Figure A5:** Response to 100 bps trilemma shock with base country’s business cycle controlled: Real GDP

![Figure A5](image)

**Notes:** Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). Post-WW2 sample: 1948–2015. LP-OLS estimates displayed as a dashed red line, LP-IV estimates displayed as a solid blue line and 1 S.D. and 90% confidence bands. See text.

While there is considerable attenuation in the estimated impulse response, the long-run effects are quantitatively and statistically significant.
Structural Breaks in TFP growth and GDP growth
A.5. First stage t-statistic for each country

Notes: The figures report the first-stage t-statistics for each country during its peg. The change in peg country’s short term interest rate is regressed on the instrument $z$. The instrument is the change in base country’s short term interest which is orthogonal to base country conditions. AVG denotes the t-statistic for the pooled sample.

![Country Specific IRFs over 1890-2015](image_url)
B. **Appendix: Graphs Related to Romer and Romer (2004) Surprises**

B.1. Different sample selection for US

**Figure A6: Response to 100 bps Romer and Romer (2004) shock**

(a) Full sample: 1969Q2: 2008Q3

(b) Sample: 1973Q2: 2008Q3

Figure A7: Response to 100 bps Romer and Romer (2004) shock

(a) Sample: 1979Q3: 2008Q3

(b) Sample: 1984Q1: 2008Q3

Figure A8: Response to 100 bps Romer and Romer (2004) shock

(a) Sample: 1969Q2: 2002Q4

(b) Sample: 1987Q1: 2008Q3

C. Appendix: Related Tables

Table A1: LP-OLS vs. LP-IV. Attenuation bias of real GDP per capita responses to interest rates. 
Trilemma instrument. Matched samples

<table>
<thead>
<tr>
<th>Year</th>
<th>(a) Full Sample</th>
<th>OLS-IV</th>
<th>(b) Post-WW2</th>
<th>OLS-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LP-OLS (1)</td>
<td>LP-IV (2)</td>
<td>p-value (3)</td>
<td>LP-OLS (4)</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td></td>
<td></td>
<td>p-value</td>
</tr>
<tr>
<td>$h=0$</td>
<td>0.07*** (0.03)</td>
<td>-0.07 (0.09)</td>
<td>0.09 (0.09)</td>
<td>0.04** (0.02)</td>
</tr>
<tr>
<td>$h=2$</td>
<td>-0.28* (0.16)</td>
<td>-1.72*** (0.34)</td>
<td>0.00 (0.34)</td>
<td>-0.25* (0.13)</td>
</tr>
<tr>
<td>$h=4$</td>
<td>-0.16 (0.26)</td>
<td>-2.53*** (0.50)</td>
<td>0.00 (0.50)</td>
<td>-0.08 (0.21)</td>
</tr>
<tr>
<td>$h=6$</td>
<td>-0.06 (0.29)</td>
<td>-2.87*** (0.66)</td>
<td>0.00 (0.66)</td>
<td>0.02 (0.23)</td>
</tr>
<tr>
<td>$h=8$</td>
<td>-0.36 (0.29)</td>
<td>-3.55*** (0.84)</td>
<td>0.00 (0.84)</td>
<td>0.10 (0.22)</td>
</tr>
<tr>
<td>$h=10$</td>
<td>-0.40 (0.35)</td>
<td>-4.05*** (1.08)</td>
<td>0.00 (1.08)</td>
<td>0.26 (0.24)</td>
</tr>
</tbody>
</table>

KP weak IV 79.66 84.86

$H_0$: LATE = 0 0.00 0.00 0.00 0.00 0.00 0.00

Observations 607 607 482 482 482 482

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. Cluster robust standard errors in parentheses.
Full sample: 1890 – 2015 excluding WW1: 1914 – 1919 and WW2: 1939 – 1947. Post WW2 sample: 1948 – 2015. Matched sample indicates LP-OLS sample matches the sample used to obtain LP-IV estimates. KP weak IV refers to the Kleibergen-Paap test for weak instruments. $H_0$ : subATE = 0 refers to the p-value of the test of the null hypothesis that the coefficients for $h = 0, \ldots, 10$ are jointly zero for a given subpopulation. OLS = IV shows the p-value for the Hausmann test of the null that OLS estimates equal IV estimates. See text.

D. Appendix: DSGE model

D.1. Stationary Allocation

We normalize the following variables:

\[ y_t = Y_t / Z_t \]
\[ c_t = C_t / Z_t \]
\[ k_t = K_t / Z_t \]
\[ k^u_t = K^u_t / Z_{t-1} \]
\[ I_t = I_t / Z_t \quad \text{capital investment} \]
\[ G_t = G_t / Z_t \quad \text{Govt Spending} \]
\[ w_t = W_t / (Z_t P_t) \]
\[ r^K_t = R^K_t / P_t \]
\[ \lambda_t = \Lambda_t Z_t \]
\[ \hat{f}_t = \Gamma_t / P_t Z_t \]

**Definition 1** (normalized equilibrium). 19 endogenous variables \( \{ \lambda_t, g_{t+1}, i_t, \pi_t, \pi^w_t, X^p_{1t}, X^p_{2t}, X^w_{1t}, X^w_{2t}, c_t, y_t, z_t, k^K_{t+1}, r^K_t, I_t, q_t, u_t, k_t, w_t, L_t \} \), 3 shocks \( \{ \lambda^S_t, \epsilon^{mp}_t, \mu_t \} \) given the natural rate allocation variables \( \{ y^f_t, g^f_{t+1} \} \).

Consumption Euler Equation
\[ 1 = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t (1 + g_{t+1})} \left( 1 + i_t \right) \right] \]
\[ \lambda_t = \frac{1}{c_t - \frac{hc_t}{1+g_t}} - \frac{\theta \beta}{c_{t+1} (1 + g_{t+1}) - hc_t} \]

Price-Setting
\[ \frac{X^p_{1t}}{X^p_{2t}} = \left( \frac{1 - \theta \pi^w_t \pi^m_t}{1 - \theta \pi_t} \right)^{-\lambda_p} \]
\[ X^p_{1t} = \lambda_t y_t mc_t + \theta \pi_t \pi^m_{t+1} X^p_{1t+1} \]
\[ X^p_{2t} = \frac{1}{1 + \lambda_p} \lambda_t y_t + \theta \pi_t \pi^m_{t+1} X^p_{2t+1} \]

Wage-Setting
\[ \frac{X^w_{1t}}{X^w_{2t}} = \left( \frac{1 - \theta \pi^w_t \pi^m_{1w}}{1 - \theta \pi^w_t} \right)^{-\lambda_w + (1 + \lambda_w) \pi^w_{t+1} - \lambda_w \pi^w_{t+1}} \]
\[ X^w_{1t} = \omega L_t^{1+v} + \theta \pi^w_t \left( \frac{\pi^w_{ss}}{\pi^w_t} \right)^{1-\pi^w_t} \pi_{W,t+1}^{\pi^w_{W,t+1}} X^w_{1t+1} \]
\[ X^w_{2t} = \frac{1 + \tau^w}{1 + \lambda^w} \lambda_t w_t L_t + \theta \pi^w_t \left( \frac{\pi^w_{ss}}{\pi^w_t} \right)^{1-\pi^w_t} \pi_{W,t+1}^{\pi^w_{W,t+1}} X^w_{2t+1} \]
\[ \pi_{W,t} = \frac{w_t}{w_{t-1}} \pi_t (1 + g_t) \]

Capital Investment
\[ k^K_{t+1} = \left[ 1 - S \left( \frac{I_t}{I_{t-1} + g_{ss}} \right) \right] I_t + (1 - \delta_k) \frac{k^K_t}{1 + g_t} \]
\[ q_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t (1 + g_{t+1})} \left( r^K_{t+1} u_{t+1} - a(u_{t+1}) + q_{t+1} (1 - \delta_k) \right) \right] \]
\[
q_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \frac{1 + g_t}{1 + g_{ss}} \right) - S' \left( \frac{I_t}{I_{t-1}} \frac{1 + g_t}{1 + g_{ss}} \right) \right] + \beta \lambda_{t+1} \frac{1 + g_{t+1}}{1 + g_{ss}} \left( \frac{I_{t+1}}{I_t} \right)^2 S' \left( \frac{I_{t+1}}{I_t} \frac{1 + g_{t+1}}{1 + g_{ss}} \right) = 1
\]

(17)

Capital utilization rate
\[
k_t = u_t \frac{k_t^l}{1 + g_t}
\]

(18)

Production Technologies
\[
y_t = k_t^\alpha L_t^{1-\alpha}
\]

(20)

Endogenous Growth
\[
g_t = \log Z_t - \log Z_{t-1} = \mu_t + \eta \log(y_{t-1}/y_{t-1}^f)
\]

(23)

Government
\[
\frac{1 + i_t}{1 + i_{ss}} = \left( \frac{1 + i_{t-1}}{1 + i_{ss}} \right)^\rho_R \left[ \left( \frac{\pi_t}{\pi_{ss}} \right)^{\phi_y} \left( \frac{y_t}{y_{t-1}} \right)^{\phi_y} \right]^{1-\rho_R} \left( \frac{y_t/y_{t-1}}{y_{t-1}^f/y_{t-1}^f} \right)^{\phi_{iy}} \epsilon_t^{mp}
\]

(24)

Market Clearing
\[
y_t = c_t + I_t + a(u_t) \frac{k_t^l}{1 + g_t} + \left( 1 - \frac{1}{\lambda_t^g} \right) y_t
\]

(25)

Shocks
\[
\log \lambda_t^g = (1 - \rho_g) \lambda_t^g + \rho_g \log \lambda_{t-1}^g + \epsilon_t^g; \quad \epsilon_t^g \sim N(0, \sigma_g^2)
\]

\[
\epsilon_t^{mp} \sim N(0, \sigma_r)
\]

(26)

(27)

D.2. Steady State

\[
1 = \beta \frac{1}{(1 + g)} \frac{1 + i}{\pi}
\]

\[
\lambda = \frac{(1 + g) - h\beta}{c(1 + g - h)}
\]

\[
mc = \frac{1}{1 + \lambda_p}
\]

\[
\omega L^{1+v} = \frac{1 + \tau^\omega}{1 + \lambda_w} \lambda w L
\]

\[
\pi^\omega = \pi(1 + g)
\]
\[ q = 1 \\
\hat{u} = 1 \\
(1 - \frac{1 - \delta_k}{1 + \delta})k^u = \mathbb{I} \\
1 = \beta \left[ \frac{1}{1 + g} \left( r^K + (1 - \delta_k) \right) \right] \\
k = \frac{k^u}{1 + g} \\
r^K = a'(1) \\
y = k^\alpha L^{1-\alpha} \\
r^k = \frac{a y}{k} \\
w = (1 - \alpha) \frac{y}{L} \\
g = \mu \\
y = c + \mathbb{I} + \left( 1 - \frac{1}{\lambda g} \right) y \]

D.3. Approximate Equilibrium

We log-linearize the variables around the steady state as follows: for any variable \( x \),

\[ \hat{x}_t = \log \left( \frac{x_t}{x} \right) \]

where \( x \) is the steady state, except for the following variables

\[ \hat{g}_{t+1} = \log \left( \frac{1 + g_{t+1}}{1 + g} \right) \\
\hat{i}_t = \log \left( \frac{1 + i_t}{1 + i_{ss}} \right) \]

**Definition 2** (Approximate Equilibrium). An approximate competitive equilibrium in this economy with endogenous growth is defined as a sequence of 16 endogenous variables \( \{ \hat{\lambda}_t, \hat{\lambda}_{t+1}, \hat{i}_t, \hat{p}_{t}, \hat{c}_t, \hat{\pi}_t, \hat{i}_t, \hat{\pi}_w, \hat{m}L_t, \hat{\hat{w}}_t, \hat{\hat{k}}^u_{t+1}, \hat{\hat{i}}_t, \hat{\hat{q}}_t, \hat{\rho}_t, \hat{\hat{k}}_t, \hat{\hat{u}}_t \} \), which satisfy the following equations, for a given sequence of one exogenous shock \( \{ \hat{\epsilon}_{t, mp} \} \) and given sequence of flexible price allocations.

Consumption Euler Equation

\[ (E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - \hat{g}_{t+1}) + \hat{i}_t - E_t \hat{\pi}_{t+1} = 0 \]  \hspace{1cm} (28)
\[
\hat{\lambda}_t = \frac{1+g}{1+g-h\beta} \left[ \hat{g}_t - \left( \frac{1+g}{1+g-h} (\hat{c}_t + \hat{g}_t) - \frac{h}{(1+g)-h} \hat{c}_{t-1} \right) \right] + \mathbb{E}_t \left( \frac{h\beta}{1+g-h\beta} \left[ \left( \frac{1+g}{1+g-h} (\hat{c}_{t+1} + \hat{g}_{t+1}) - \frac{h}{(1+g)-h} \hat{c}_t \right) \right] \right) 
\]

(29)

Price-Setting

\[
\hat{\pi}_t = \frac{\beta}{1+\nu \beta} \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\nu}{1+\nu \beta} \hat{\pi}_{t-1} + \kappa_p \hat{\pi}_t 
\]

(30)

where \( \kappa_p = \frac{(1-\theta_p)(1-\theta_p)}{\theta_p(1+\nu \beta)} \).

Wage-Setting

\[
\hat{\pi}_t^w = \frac{\beta}{1+\nu \beta} \mathbb{E}_t \hat{\pi}_{t+1}^w + \frac{\nu}{1+\nu \beta} \hat{\pi}_{t-1}^w + \kappa_w [ -\hat{\lambda}_t + \nu \hat{L}_t - \hat{\omega}_t ] 
\]

(31)

where \( \kappa_w = \frac{(1-\theta_p)(1-\theta_w)}{\theta_w(1+\nu(1+\nu \beta))} > 0 \)

\[
\hat{\pi}_t^w = \hat{\omega}_t - \hat{\omega}_{t-1} + \hat{\pi}_t + \hat{g}_t 
\]

(32)

Capital Investment

\[
\hat{k}^u_{t+1} = \frac{1}{k^u} \hat{k}_t + \frac{1-\delta_k}{1+g} \left[ \hat{k}^u_t - \hat{g}_{t+1} \right] 
\]

(33)

\[
\hat{q}_t = \left[ \mathbb{E}_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - \hat{g}_{t+1} \right] + \frac{r^K}{r^K + (1-\delta_k)} \hat{r}^K_{t+1} + \frac{1-\delta_k}{r^K + (1-\delta_k)} \mathbb{E}_t \hat{q}_{t+1} 
\]

(34)

\[
\hat{q}_t - S'' (\hat{L}_t - \hat{L}_{t-1} + \hat{g}_t) + \beta S'' (\hat{L}_{t+1} - \hat{L}_t + \hat{g}_{t+1}) = 0 
\]

(35)

parametrize \( S'' \) from Smets and Wouters (2007), or Justiniano et al. (2013) to be between 3 and 5.

Capital utilization rate

\[
\hat{k}_t = \hat{\omega}_t + \hat{k}^u_t - \hat{g}_t 
\]

(36)

\[
\hat{r}^K_{t} = \frac{a''(1)}{a'(1)} \hat{r}_t 
\]

(37)

Production Technologies

\[
\hat{g}_t = a \hat{k}_t + (1-a) \hat{L}_t 
\]

(38)

\[
\hat{r}^K_{t} = \hat{g}_t - \hat{k}_t 
\]

(39)

\[
\hat{\omega}_t = \hat{g}_t - \hat{L}_t 
\]

(40)

Endogenous Growth

\[
\hat{g}_t = \hat{\mu}_t + \eta (\hat{g}_{t-1} - \hat{g}_{t-1}') 
\]

(41)

where \( \eta \) is the hysteresis elasticity.
Government

\[ \hat{I}_t = \rho_R \hat{I}_{t-1} + (1 - \rho_R) \left[ \phi_{\pi} \hat{\pi} + \phi_y (\hat{y}_t - \hat{y}'_t) \right] + \phi_{dy} \left[ (\hat{y}_t - \hat{y}_{t-1}) - (\hat{y}'_t - \hat{y}'_{t-1}) + (\hat{g}_t - \hat{g}'_t) \right] + \varepsilon_{mp}^{mp} \tag{42} \]

Market Clearing

\[ \frac{1}{\lambda^g} \hat{y}_t = \frac{c}{y} \hat{c}_t + \frac{I}{y} \hat{I}_t + a'(1)k \hat{u}_t \tag{43} \]