On the Limits of Monetary Policy*

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Abstract

This paper provides theory and evidence that distorted long-term interest-rate expectations represent a fundamental constraint on monetary policy design. Permitting beliefs to depart from those consistent with rational expectations equilibrium breaks the tight link between policy rates and long-term expectations, even when long-term interest rates are determined by the expectations hypothesis of the yield curve. Because bond prices are excessively sensitive to short-term interest rates, the central bank faces an intertemporal trade-off which results in optimal policy being less aggressive relative to rational expectations. More aggressive policy leads to sub-optimal volatility in long-term interest rates and aggregate demand through standard intertemporal substitution effects. These effects are quantitatively important over the Great Inflation and Great Moderation periods of US monetary history.

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1 Introduction

Economists have long debated the limits of monetary policy stabilization. The monetarist controversies of the 1970s [Friedman (1968), Modigliani (1977) and Meltzer (1987)] centered on fundamental questions about a central bank's ability to assess accurately the current state of the economy, to formulate reliable forecasts. With notable exceptions [Orphanides (2003, 2004) and Orphanides and Williams (2012)], these genuine practical concerns received progressively less attention over subsequent decades, perhaps because of the perceived policy success inherent in the Great Moderation, a rising optimism about the efficacy of monetary policy. Amongst policymakers, constrained discretion became a mantra.

Consistent with this, Clarida, Gali, and Gertler (1999) articulate a research program which envisages a highly effective central bank, unhindered by informational problems. The framework variously exemplifies the great power of monetary policy as a stabilization tool. Optimal policy can completely stabilize inflation and the output gap when faced with movements in the natural real rate of interest, so-called Divine coincidence. And should the zero lower bound on nominal interest rates be a relevant constraint on policy actions, commitment to conduct future policy in a certain way can largely mitigate recession [Eggertsson and Woodford (2003)]. These predictions are not only a property of simple models: monetary policy can eliminate almost all inefficient fluctuations in models with multiple nominal and real frictions [Justiniano, Primiceri, and Tambalotti (2013)], financial imperfections [Furlanetto, Gelain, and Sanjani (2017)] and heterogeneous agents [Challe (2017)]; and in such models, commitments about future policy are, if anything, too effective, giving rise to the forward guidance puzzle [Del Negro, Giannoni, and Patterson (2012)].

But the Great Recession raises important questions about this paradigm, and in some quarters, has tempered optimism about the effectiveness of monetary policy. This paper revisits the debate on activist stabilization policy with fresh perspective. We provide theory and evidence that the power of monetary policy rests on the assumption that long-term expectations are anchored. When changes in overnight rates are efficiently transmitted over the term structure of interest rates, central banks have precise control of aggregate demand. Stated differently, when long-run expectations are anchored and insensitive to short-term developments, central banks are free to pursue short-run activist policies. If this pre-condition is not met, there are fundamental limits to what monetary policy can do. Indeed, when expectations are poorly anchored (in a sense to be made precise), we show optimal policy will be less activist, so that interest-rate policy adjustments are considerably smoother, with concomitantly greater variation in real and nominal activity. Acknowledging this has consequences for how we interpret the historical performance of the Federal Reserve.

Figure 1 shows the behavior of long-term forecasts by US professional forecasters. The top panel plots inflation, along with the four-quarter-ahead and five-to-ten year ahead forecasts. The bottom panel shows the same series but for an ex post measure of real interest rates. Long-term inflation expectations exhibit a pronounced downward trend over most of the sample period, with some cyclical variation, while real-rate expectations have no evident trend, though display large cyclical variation, rising in booms, and falling in recessions. Importantly, short- and long-run forecasts co-move with realized variables, consistent with forecasters extrapolating trends from the recent behavior of these series. Also notable, is that by the end of the 1990s long-run expectations become fairly insensitive to short-run developments, suggesting a shift in the expectations formation process. This behavior is difficult to reconcile with standard models used for monetary policy evaluation, which typically assume long-run beliefs about inflation and the real interest rate are constant. It also raises the question: to what extent is monetary policy responsible for these patterns? We address both these issues.

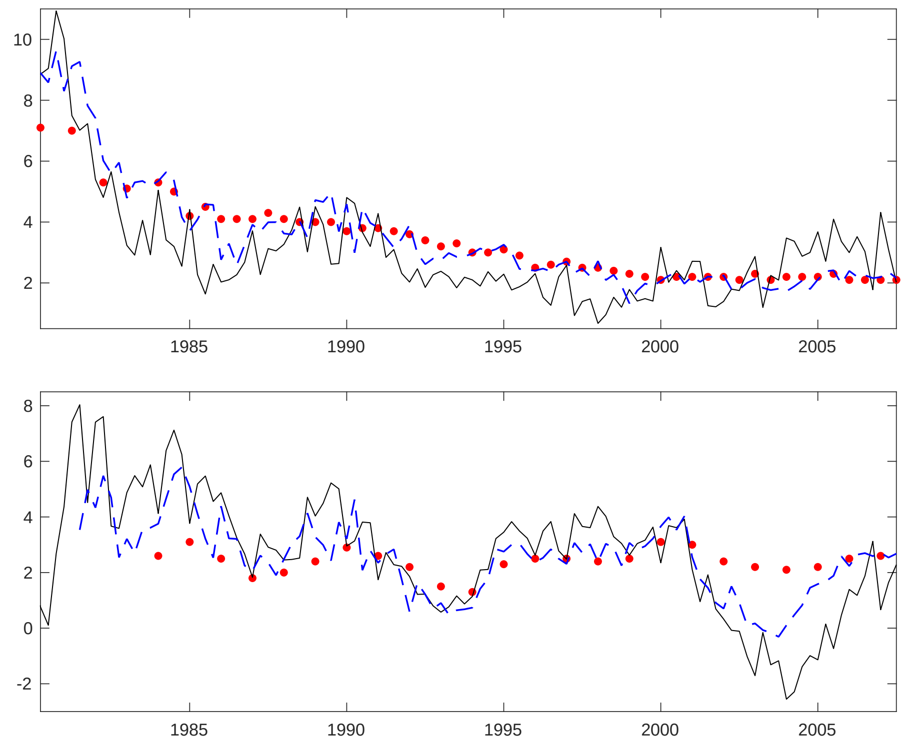

Figure 1: **Long-run expectations**

The how the evolution of survey expectations data for inflation (top panel) and the short-term ex-post real rate of interest (bottom panel). Realized variables are shown in solid black lines, four-quarters ahead forecast in dashed blue lines and five-to-ten years ahead forecasts are in red dots. Source: Blue Chip Economics and Survey of Professional Forecasters.

This paper departs from the standard assumption of full information rational expectations and assumes agents have imperfect knowledge about the long-run, reflecting uncertainty about the mean of inflation and the equilibrium real interest rate. Households and firms seek to detect unobserved shifts in the means of these variables from observed short-run fluctuations. This filtering problem leads to a tight connection between short-term forecast errors and long-run beliefs. The strength of this connection measures the degree of anchoring of expectations.

We embed this expectations formation mechanism into a standard New Keynesian model. Because long-run beliefs matter for consumption, employment and pricing decisions, they become partially self-confirminga property Marcet and Sargent (1989b) call self-referentiality. In equilibrium, subjective beliefs differ from objective beliefs, and exhibit less mean reversion than objective beliefs. Equilibrium beliefs are too responsive to short-term changes relative to what would be optimal given the actual true data-generating process. This informational friction is consistent with extensive evidence that observed measures of expectations exhibit extrapolation bias — the tendency to overweight recent changes when making projections.[1] Our mechanism shares this feature with behavioral theories such as *natural expectations* in Fuster, Laibson, and Mendel (2010) and *diagnostic expectations* in Bordalo, Gennaioli, Ma, and Shleifer (2018). A contribution of this paper is to study extrapolation bias in general equilibrium, where the macroeconomic variables agents are trying to forecast are themselves endogenous and determined in part by aggregate expectations. This wedge between subjective and objective beliefs is therefore endogenous, time-varying and depends on monetary policy and economic disturbances.

An implication of extrapolation bias is the central bank has imprecise control of long-term interest rates and aggregate demand: short-term policy changes reverberate through the whole term structure of interest rates, as agents update their beliefs about the long-run level of nominal rates. Long-term interest rates are excessively sensitive to surprise movements in short-term rates. This property imposes a constraint on stabilization policy.[2] To give focus to the consequences of long-run uncertainty for short-run stabilization policy, we assume the monetary authority is perfectly informed about the state of the economy. As such, the informational friction we consider is fundamentally different to those that pre-occupied the monetarist controversies.

In our framework the limited ability to manage expectations is an inherent feature of the economy. While empirically determined, we take the degree of anchoring as exogenous. Retaining a linear state-space model affords considerable simplification in the theoretical and empirical work, while still affording rich insight. Carvalho, Eusepi, Moench, and Preston (2019) propose a partial equilibrium non-linear model in which the degree of anchoring is endogenous and state dependent. They adduce evidence that long-term expectations were poorly anchored for much of the Great Inflation and Great Moderation periods. Only in the late 1990s do long-term beliefs show declining sensitivity to short-run developments. Our empirical work proxies this changing sensitivity with a structural break in beliefs, leaving an endogenous gain to future research.

In a simple endowment economy we study the implications of distorted beliefs for monetary policy when implemented using simple rules. Extrapolation bias creates a stabilization trade-off. On the one hand, aggressive monetary policy can weaken the the feedback between expectations and outcomes. For example, by lowering aggregate demand it can prevent higher inflation expectations from being incorporated in prices. On the other hand, excess sensitivity of long-term rates to short-term policy changes creates potential instability in aggregate demand. This constraint becomes binding when expectations are poorly anchored. In general, monetary policy is less aggressive relative to what would be prescribed under full information and rational expectations. This basic insight is also a feature of optimal Ram-

[1] Fuster, Laibson, and Mendel (2010), Bordalo, Gennaioli, Ma, and Shleifer (2018) and others, document extensive evidence of extrapolation bias. Decision makers in a range of settings over-weight events considered more representative when making probability assessments.

[2] Gurkaynak, Sack, and Swanson (2005), Crump, Eusepi, and Moench (2015), Campbell, Fisher, Justiniano, and Melosi (2017) and Nakamura and Steinsson (2018) document evidence that monetary disturbances have sizable and significant effects on long-term nominal and real rates of interest.

sey policy in the canonical New Keynesian model.[3] That optimal policy is less aggressive in response to aggregate disturbances under imperfect knowledge relative to rational expectations also contrasts with Orphanides and Williams (2005a), Ferrero (2007) and Molnar and Santoro (2013). The different conclusions arise from assumptions about the transmission mechanism of monetary policy. These earlier papers assume that only current interest rates matter for aggregate demand, rather than the entire path of future expected one-period rates, as in the New Keynesian model.

To establish the quantitative relevance of the information friction, we estimate a medium-scale New Keynesian model on US data over the sample 1964Q1-2007Q3. The sample spans the Great Inflation and the Great Moderation periods, periods during which data exhibit substantial low-frequency movement. In addition to standard macroeconomic variables we use measures of short- and long-term expectations from professional forecasters to discipline beliefs. The proposed expectations formation mechanism captures well the evolution of inflation and interest-rate expectations. The estimation uncovers substantial sensitivity of long-run forecasts to short-term surprises until the late 1990s. Subsequently, expectations appear to have been broadly anchored.

The model identifies the economic sources and relevance of extrapolation bias. Monetary policy surprises drive the wedge between subjective and objective expectations, with the sign and size of the wedge informative about economic developments. Loose monetary policy in the late 1960s and early 1970s led market participants to revise significantly downward their long-term forecast for the real interest rate, generating positive output gaps and, ultimately, the Great Inflation. Monetary policy permitted rising inflation to become entrenched in long-term inflation expectations. Conversely, the Volcker disinflation resulted in a dramatic increase in long-run real-rate expectations with contractionary effects. In both episodes, subjective expectations display weaker mean reversion when compared to the forecast of an outside observer with full knowledge of the true data generating process. As a result, temporary monetary policy shocks had long lasting effects on the economy.

Earlier research by Justiniano, Primiceri, and Tambalotti (2013) and Orphanides and Williams (2012) has similarly emphasized the importance of monetary policy errors in the Great Inflation, concluding optimally chosen policy would have performed substantially better. To speak to this, we conduct counterfactual policy experiments. Suppose that the central bank is faced with: i) the expectations formation process estimated for the first part of the sample; and ii) the estimated shocks, but can now adopt an alternative policy rule chosen to maximize household welfare. Two key results emerge. First, there is evidence of monetary policy mistakes. Optimal policy could have provided a stable nominal anchor (by which we mean, stable long-term inflation expectations), even with poorly anchored expectations. In fact, in this counterfactual, long-term inflation expectations are predicted to remain fairly stable even in the face of sizable markup shocks. Second, the ability of the central bank to conduct short-term stabilization is compromised when compared to optimal policy under rational expectations. While price and wage inflation are less volatile, they remain far from stable, and so too for the output gap. A further counterfactual in which markup shocks are absent, where all business cycle fluctuations are due to movement in the natural rate of

[3] Formally, for beliefs which are sufficiently sensitive to short-run forecast errors, the Lagrange multiplier on the aggregate demand constraint is strictly positive. Under rational expectations, it would be zero.

interest, renders this point stark: optimal policy under learning can only slightly dampen the business cycle. In contrast, and consistent with Justiniano, Primiceri, and Tambalotti (2013), optimal policy under rational expectations can fully stabilize the output gap and inflation. There are limits to monetary policy.

The paper proceeds as follows. Section 2 elucidates the constraints that extrapolation bias impose on policy, in a simple example. Section 3 lays out a medium-scale New Keynesian model with features required for a plausible account of aggregate data. Section 4 develops the theory of beliefs. Section 5 estimates the model and discusses basic properties. Section 6 performs a number of counterfactual experiments to isolate core mechanisms and the central result on optimal policy. Section 7 provides theory of optimal policy in a special case of the model, formalizing insights from the simple example and empirical work. Finally, section 8 concludes.

2 Extrapolation Bias as a Policy Constraint

This section develops a simple example to establish general principles and conclusions. We show: i) extrapolation bias drives a wedge between subjective and objective probability models — dynamics are self-referential; ii) extrapolation bias implies subjective beliefs display less mean reversion than objective beliefs; iii) monetary policy regulates the degree of extrapolation bias; but iv) distorted interest-rate beliefs constrain monetary policy relative to a full information rational expectations analysis. There are limits to what monetary can achieve.

Consider an endowment economy in which a continuum of households $i \in [0,1]$ have log utility. Assets are in zero net supply. Optimal consumption demand is given by

$$c_t(i) = -\beta R_t + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) y_T - \beta \left(\beta R_{T+1} - \pi_{T+1} \right) \right] \tag{1}$$

where $c_t(i)$ is consumption; i_t the nominal interest rate; and π_t the inflation rate. The household's discount factor satisfies $0 < \beta < 1$ and the endowment

$$y_t = \rho_y y_{t-1} + \varepsilon_t$$

with $0 < \rho_y < 1$ and ε_t an i.i.d. mean zero disturbance. The operator $\hat{E}_t^i$ denotes household expectations which we discuss below. All variables are expressed in log-deviation from their non-stochastic steady state.

As in the standard New Keynesian framework, monetary policy influences aggregate demand through intertemporal substitution. Not only does the contemporaneous interest rate matter, i_t, but the entire anticipated future sequence of rates, $E_t^i i_T$. These two objects are connected through the expectations hypothesis of the term structure; assumptions about monetary policy; and assumptions about belief formation. We now explore the consequences of uncertainty about the long-term level of interest rates embodied in the conditional expectations $\hat{E}_t^i i_T$ for short-run aggregate demand management.

Rational Expectations. To frame ideas, suppose monetary policy is given by the instrument rule

$$R_t = \phi \pi_t$$

with $\phi > 1$. Rational expectations equilibrium implies

$$\begin{bmatrix} R_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \Phi_R \\ \Phi_\pi \end{bmatrix} \rho_y y_{t-1} + e_t$$

where Φ_R and Φ_π are composites of model primitives and the vector e_t proportional to ε_t. Long-run beliefs satisfy

$$\begin{aligned} \lim_{T\to\infty} E_t^i \pi_T &= 0 \\ \lim_{T\to\infty} E_t^i r_T &= \lim_{T\to\infty} E_t^i \left(R_T - \pi_{T+1}\right) = 0 \end{aligned}$$

consistent with the long-run inflation target and real rate of interest.

Subjective beliefs. Now suppose households are uncertain about these long-run outcomes because of uncertainty about long-run production possibilities, and imperfect credibility of the central bank's inflation target. Following Kozicki and Tinsley (2001) agents have a 'shifting end-points' forecasting model

$$\begin{aligned} \begin{bmatrix} R_t \\ \pi_t \end{bmatrix} &= \bar{\omega}_t + \begin{bmatrix} \Phi_R \\ \Phi_\pi \end{bmatrix} \rho_y y_{t-1} + e_t \\ \bar{\omega}_t &= \bar{\omega}_{t-1} + u_t \end{aligned}$$

where prior beliefs about transitory shocks and low-frequency developments satisfy $R = E\left[e_t e_t'\right]$, $Q = E\left[u_t u_t'\right]$ and $R = g^2 Q$.

Agents form an estimate ω_t of $\bar{\omega}_t$, using a steady-state Kalman filter. Without loss of generality, assume agents know the monetary policy rule.[4] Expectations are then given by

$$\hat{E}_t R_T = \omega_{t-1}^R + \Phi_R \rho_y^{T-t} y_t = \phi \hat{E}_t \pi_T \tag{2}$$

and beliefs evolve according to

$$\omega_t^R = \omega_{t-1}^R + g\left(R_t - \omega_{t-1}^R - \Phi_R \rho_y y_{t-1}\right).$$

This belief structure and the decision rule (1) represent the optimal Bayesian solution to the intertemporal consumption allocation problem in an edowment economy.

Equilibrium dynamics. Evaluating expectations in the demand, aggregating and imposing goods market clearing, gives the state-space representation of the true data-generating process

$$R_t = -\frac{\beta - \phi^{-1}}{1-\beta} \omega_{t-1}^R + \Phi_R \rho_y y_{t-1} + \varepsilon_t \tag{3}$$

and

$$\omega_t^R = \left(1 - g\frac{1-\phi^{-1}}{1-\beta}\right) \omega_{t-1}^R + g\varepsilon_t$$

Four properties are relevant to later findings. First, because of beliefs, interest-rate dynamics have a time-varying drift. This reflects what Marcet and Sargent (1989a) call self-referential

[4] The appendix provides calculations for the more general case — conclusions are identical.

dynamics. Second, good policy manages the degree of self-referential feedback by influencing the degree to which shifting beliefs affect interest rates. Third, households exhibit 'extrapolation bias' — they fail to account for long-run mean reversion in forming expectations. They believe the low-frequency component of interest rates to have a unit root, while, as discussed further below, in equilibrium the dynamics of beliefs display mean reversion because

$$\left|1 - g\frac{1-\phi^{-1}}{1-\beta}\right| < 1.$$

Extrapolation bias therefore generates a wedge between subjective and objective probability models.[5] The wedge is time-varying and endogenous, depending on monetary policy and shocks, and provides a gauge of the quantitative relevance of the information friction. For example, the wedge in interest-rate expectations is

$$\begin{aligned} \hat{E}_t i_T - E_t i_T &= \left(1 - \frac{\phi^{-1}-\beta}{1-\beta}\right)\omega^i_{t-1} \\ &= \left(1 - \frac{\phi^{-1}-\beta}{1-\beta}\right)\sum_{j=0}^{\infty}\left(1 - g\frac{1-\phi^{-1}}{1-\beta}\right)^j g\varepsilon_{t-1-j} \end{aligned}$$

in any future period $T > t$. The stance of policy matters for both the size of the wedge for given beliefs (the first line), but also the size of the deviation of beliefs from their true long-term mean of zero (the second line). Depending on beliefs and monetary policy, transitory endowment shocks may have long-lived effects. Fourth, when the Kalman gain g approaches zero, beliefs nest rational expectations. That is, rational expectations equilibrium arises when $\omega^i_t = 0$, the wedge vanishes. We now show this constrains policy.

Constraints on Policy. For beliefs and nominal rates to be stationary requires

$$g < \frac{2(1-\beta)}{1-\phi^{-1}}. \tag{4}$$

Hence for a given sensitivity of beliefs to new information, the policy coefficient cannot be too large. And the more sensitive beliefs to surprise movements in interest rates, the tighter the constraint on policy choice. This limits the ability of policy to respond to disturbances. The distortion operating through the term structure of expectations confronts monetary policy with an additional trade-off. The informational friction implies adjustment in short-term rates drive low-frequency developments in long-term interest rates. Aggressive adjustment of short-term rates can generate instability in aggregate demand through excessive volatility in long-term rates. This is a direct consequence of extrapolation bias.

Earlier literature gives a different perspective on policy design. For example, Clarida, Gali, and Gertler (1999) and Woodford (2003) establish rational expectations optimal policy satisfies the Taylor principle: interest-rate policy should be suitably responsive to movements in inflation. Furthermore, Schmitt-Grohe and Uribe (2007) demonstrate that simple interest rates rules which respond aggressively to inflation approximate well optimal policy. The imperfect knowledge and learning literature punctuates these findings: Bomfim, Tetlow,

[5] This is a general property of models of learning dynamics and doesn't depend on the assumption of a perceived unit root, as the empirical model will make clear.

von zur Muehlen, and Williams (1997), Orphanides and Williams (2005b), Ferrero (2007) and Molnar and Santoro (2013) all demonstrate that policies which are more aggressive relative to rational expectations predictions help restrain inflation expectations, and improve short-run stabilization outcomes.

We arrive at the opposing conclusion because of different assumptions on the transmission mechanism of monetary policy. The papers just cited either assume long-term interest rates are known with certainty or completely ignore the transmission mechanism of monetary policy, because the central bank can control aggregate demand directly. In contrast, we use standard New Keynesian microfoundations in which monetary policy operates through the expectations hypothesis of the term structure, but permit distorted beliefs to reflect uncertainty about long-term interest-rate policy.

The assumptions made by these earlier papers amount to analyzing the model

$$\begin{aligned} R_t &= E_t\pi_{t+1} - (1-\rho)\, y_t \\ R_t &= \phi\pi_t \end{aligned}$$

which follows from (1) under rational expectations. The first equation is just the Fisher equation (a direct implication of the aggregate Euler equation in an endowment economy) with a natural real rate determined as

$$r_t = -(1-\rho)\, y_t.$$

The second equation continues to specify monetary policy as a Taylor-type rule. This model predicts long-term interest-rate expectations play no role in determining equilibrium outcomes in the following sense. Researchers using a rational expectations analysis invariably restrict attention to unique bounded solutions which require long-term interest rates to revert to steady state; and those using models of learning dynamics which exhibit extrapolation bias conclude interest-rate policy matters only through its contemporaneous effects. Either there is no uncertainty about long-term interest rates, or there is, but it is irrelevant to equilibrium outcomes.

To be concrete, when inflation and interest-rate expectations satisfy (2), the true data-generating process is given by the system

$$\pi_t = \phi^{-1}\omega_{t-1}^{\pi} + \Phi_\pi y_t$$

and

$$\omega_t^{\pi} = \left(1 + g\phi^{-1} - g\right)\omega_{t-1}^{\pi} + g\Phi_\pi\varepsilon_t.$$

Stability requires satisfaction of the Taylor principle: $\phi > 1$ — beliefs place no further constraints on the choice of policy. This property emerges because dynamics are independent of interest-rate beliefs. Were long-term bonds to be priced, shifting views about their yields are irrelevant to inflation determination. Because of this separation, monetary policy can deliver price stability by choosing an highly aggressive interest-rate rule, as under rational expectations.

While the preceding discussion contemplates policy analysis with simple rules, the sequel demonstrates optimal policy in New Keynesian models is subject to the same constraints. In the policymaker's Ramsey problem, the aggregate demand equation generally has a strictly positive Lagrange multiplier.

3 A Medium-Scale New Keynesian Model

This section states a version of the New Keynesian model widely used for monetary policy analysis. The principle modeling innovation concerns the treatment of expectations formation. This feature, and wanting a tightly-specified empirically plausible model, dictated assumptions on scale. Further details on the microfoundations can be found in Woodford (2003) and Giannoni and Woodford (2004).

Firms. A continuum of monopolistically competitive firms $f \in [0,1]$ each produce differentiated goods, $Y_t(f)$, using the linear production technology in composite labor services, $N(f)$,

$$Y_t(f) = A_t [Z_t N_t(f)]$$

where Z_t, labor-augmenting technological progress, evolves deterministically as $Z_t = \gamma Z_{t-1}$, with $\gamma > 1$, and A_t denotes a stationary technology shock

$$\log A_t = \rho_a \log A_{t-1} + \sigma_a \varepsilon_t^a$$

where ε_t^a is IID $N(0,1)$, $\sigma_a > 0$, and $0 < \rho_a < 1$. Each firm faces a demand curve

$$Y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\theta_{p,t}} Y_t$$

where $\theta_{p,t} > 1$, the elasticity of substitution across differentiated goods, follows an exogenous process

$$\log \left(\frac{\theta_{p,t}}{\theta_p} \right) = \rho_{\theta_p} \log \left(\frac{\theta_{p,t-1}}{\theta_p} \right) + \sigma_{\theta_p} \varepsilon_t^{\theta_p}$$

where $\varepsilon_t^{\theta_p}$ is IID $\mathrm{N}(0,1)$, $\sigma_{\theta_p} > 0$, $0 < \rho_{\theta_p} < 1$ and $E[\theta_{p,t}] = \theta_p$.

Following Calvo (1983) and Yun (1996) a fraction of firms $0 < \xi_p < 1$ cannot optimally choose their price, but reset it according to the indexation rule

$$P_t(f) = P_{t-1}(f) \, \pi_{t-1}^{\iota_p}$$

where $\pi_t = P_t / P_{t-1}$ is the inflation rate, and $0 < \iota_p < 1$. The remaining fraction of firms choose a price $P_t(f)$ to maximize the expected discounted value of profits

$$\hat{E}_t^f \sum_{T=t}^{\infty} \xi_p^{T-t} Q_{t,T} \Gamma_T^f(f)$$

where the stochastic discount factor, $Q_{t,T} = \beta^{T-t} \lambda_T / \lambda_t$, values future profits

$$\Gamma_T^f(f) = Y_T(f) \left((1 - \tau_f) \frac{P_t(f)}{P_T} \left(\frac{P_{T-1}}{P_{t-1}} \right)^{\iota_p} - \frac{W_T}{A_T P_T Z_T} \right)$$

for constant sales revenue tax τ_f, and λ_t the marginal value of household wealth. The conditional expectations of firms, $\hat{E}_t^f$, is discussed below.

Households. A continuum of households $i \in [0,1]$ maximize intertemporal utility

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\frac{C_{H,T}(i)^{1-\sigma}}{1-\sigma} - \varphi_T \int \frac{N_T(i,j)^{1+\phi^{-1}}}{1+\phi^{-1}} dj \right]$$

where

$$C_{H,t}(i) = \frac{C_t(i)}{A_t} - b\frac{C_{t-1}(i)}{A_{t-1}}$$

with $\sigma, \phi > 0$, $0 < b < 1$. Each household comprises a large family, whose members $j \in [0,1]$ supply specialized labor, $N(i,j)$, to the production of each differentiated good j. The large family assumption insures each household member against labor market risk from nominal wage contracting. The dis-utility of labor supply shock is a stationary exogenous process

$$\log\left(\frac{\varphi_t}{\varphi}\right) = \rho_\varphi \log\left(\frac{\varphi_{t-1}}{\varphi}\right) + \sigma_\varphi \varepsilon_t^\varphi$$

ε_t^φ is IID $\mathrm{N}(0,1)$, $\sigma_\varphi > 0$, $0 < \rho_\varphi < 1$ and $E[\varphi_t] = \varphi$. The conditional expectations of households, $\hat{E}_t^i$, is discussed below.

The household's flow budget constraint is

$$C_t(i) + \frac{B_t(i)}{P_t} \leq R_{t-1}\pi_t^{-1}\frac{B_{t-1}(i)}{P_{t-1}} + (1-\tau_w)\int \frac{W_t(j)}{P_t} N_t(i,j)\,dj + \Gamma_t^f - T_t + T_t^w + T_t^f$$

where: R_t is the gross one-period nominal interest rate; $B_t(i)$ holdings of one-period nominal government debt; Γ_t^f dividend payments net of sales taxes; τ_w the labor income tax rate whose proceeds are rebated lump-sum to households as T_t^w; T^f the lump-sum rebate of sales revenue taxes; and T_t lump-sum taxes.[6] Household's optimal consumption and portfolio choice must also satisfy the No-Ponzi condition

$$\lim_{T\to\infty} \hat{E}_t^i \left(\prod_{s=0}^{T-t} R_{t+s}\pi_{t+s}^{-1}\right)^{-1} \frac{B_T(i)}{P_T} \geq 0.$$

Households have market power in the supply of differentiated labor inputs.[7] The demand for labor type j by firm f is

$$N_t(j,f) = \left(\frac{W_t(j)}{Wt}\right)^{-\theta_{w,t}} N_t(f) \tag{5}$$

where

$$N_t(f) = \left[\int_0^1 N_t(j,f)^{\frac{\theta_{w,t}-1}{\theta_{w,t}}}\,dj\right]^{\frac{\theta_{w,t}}{\theta_{w,t}-1}} \text{ and } W_t = \left[\int_0^1 W_t(j)^{1-\theta_{w,t}}\,dj\right]^{\frac{1}{1-\theta_{w,t}}}$$

define the composite labor input used in production and the associated wage rate. The elasticity of demand across differentiated labor inputs satisfies the exogenous process

$$\log\left(\frac{\theta_{w,t}}{\theta_w}\right) = \sigma_{\theta_w}\varepsilon_t^{\theta_w}$$

[6] The assumptions on tax policy ensure an efficient steady state level of output.

[7] The assumption is interpreted as follows. For each type of labor, which is sourced from all households, there is an employment agency that has market power. See Giannoni and Woodford (2004) and Justiniano, Primiceri, and Tambalotti (2013).

and $\theta_{w,t} > 1$ and $\varepsilon_t^{\theta_w}$ is IID N$(0,1)$, $\sigma_{\theta_w} > 0$ and $E[\theta_{w,t}] = \theta_w$. Following Erceg, Henderson, and Levin (2000) a fraction of household members $0 < \xi_w < 1$ cannot optimally reset their wage, but follow the indexation rule

$$W_t(j) = W_{t-1}(j)\,\pi_{t-1}^{\iota_w}\gamma \tag{6}$$

for $0 < \iota_w < 1$. For the remaining fraction, ξ_w, each member j of household i, choose optimally their nominal wage, $W_t(j)$, to maximize

$$\hat{E}_t^i \sum_{T=t}^{\infty} (\xi_w \beta)^{T-t} \left[Q_{t,T}(i) \frac{W_t(j)}{P_T} \left(\frac{P_{T-1}}{P_t - 1} \right)^{\iota_w} \frac{Z_{T-1}}{Z_{t-1}} N_T(i) - \varphi_T \frac{N_T(j)^{1+\phi^{-1}}}{1+\phi^{-1}} \right]$$

subject to (5).[8]

Government Policy. The central bank implements monetary policy using the interest rate rule

$$R_t = (R_{t-1})^{\rho_R} \left[R\,(P_t/P_{t-1})^{1+\phi_\pi} X_t^{\phi_x} \right]^{1-\rho_R} m_t$$

where $\phi_\pi,\ \phi_x \geq 0$, R the steady-state gross interest rate, and $X_t = Y_t/Y_t^n$ denotes the model-theoretic output gap, where Y_t is the level of output and Y_t^n the natural rate of output in a flexible-price equilibrium of the model. Interest-rate policy exhibits inertia and responds to deviations of inflation and output gap from steady-state levels. The steady-state inflation rate is zero; $\log m_t = \sigma_m \varepsilon_t^m$ denotes a mean-zero IID monetary shock.

To give focus to how learning dynamics constrain monetary policy, we assume fiscal policy is Ricardian, and that this is understood by agents. Eusepi and Preston (2018) show that in general learning will imply departures from Ricardian equivalence, with holdings of the public debt perceived as net wealth. The associated wealth effects on aggregate demand can be sizable, which impairs the standard intertemporal substitution channel of monetary policy. We also assume that agents know the tax rules in place, including the rebate of sales and income taxes. Together these assumptions imply agents do not need to forecast various taxes and that debt will not have monetary consequences. This permits focus on how belief distortions affect long-term interest rates and monetary policy design, understanding that imperfect knowledge about fiscal and monetary policy both serve to complicate inflation policy. With this in mind, we consider an economy with zero government debt and balanced budget policy

$$T_t = G_t$$

where exogenous government purchases satisfy

$$\log\left(\frac{G_t}{G}\right) = \rho_G \log\left(\frac{G_{t-1}}{G}\right) + \sigma_G \varepsilon_t^G$$

where ε_t^G is IID N$(0,1)$, $\sigma_G > 0$, $0 < \rho_G < 1$ and $E[G_t] = G$. Motivated by empirical fit we follow Smets and Wouters (2007) and permit correlation between government purchases and technology shocks.

[8] Members supplying labor of type j, being represented by an employment agency, re-optimize at the same time in all households i.

Market Clearing and Equilibrium. We consider a symmetric equilibrium in which all households are identical, even though they do not know this to be true. Given that households have the same initial asset holdings, preferences, and beliefs, and face common constraints, they make identical state-contingent decisions. Similarly, all firms having the opportunity to re-optimize choose an identical re-set price. Equilibrium requires all goods, labor and asset markets to clear providing the restrictions

$$\begin{aligned} \int_0^1 C_t(i)\, di + G_t &= \int_0^1 Y_t(f)\, df \\ \int_0^1 \int_0^1 N_t(i,j)\, di dj &= \int_0^1 N_t(f)\, df \end{aligned}$$

and

$$\int_0^1 B_t(i)\, di = 0$$

with initial condition $B_{-1}(i) = 0$. Given exogenous processes $\{G_t, \theta_{p,t}, \theta_{w,t}, m_t, Z_t, A_t, \varphi_t\}$, equilibrium then is a sequence of prices $\{P_t, W_t, R_t\}$ and allocations $\left\{C_t, N_t, Y_t, T_t, T_t^w, T_t^f, \Gamma_t^f\right\}$ satisfying individual optimality — detailed in the appendix — and market clearing conditions.

4 Beliefs

The appendix shows a first-order approximation to optimal decisions and market clearing conditions give aggregate dynamics

$$A_0 z_t = \sum_{s=1}^{3} A_s \hat{E}_t \sum_{T=t}^{\infty} \lambda_s^{-(T-t)} z_{T+1} + A_4 z_{t-1} + A_5 \varepsilon_t \tag{7}$$

where the vector z_t collects all model variables, the vector ε_t collects exogenous innovations and the matrices A_i, for $i \in 1, ..., 5$, collect relevant model coefficients. This representation holds for arbitrary beliefs, including rational expectations. Dynamics depend on a set of projections into the indefinite future, reflecting the intertemporal decision problems solved by households and firms. The projected variables are those macroeconomic objects taken as given and beyond the control of each decision maker. Firms must forecast real wages and goods price inflation; households must forecast goods price inflation, wage inflation, the real wage, nominal interest rates, and aggregate demand. The discount factors λ_s are the model's unstable eigenvalues, so that the infinite sums encode the usual forward recursion to suppress the effects of explosive roots.

An assumption on belief formation closes the model. We make a number of choices to ensure tractability in estimation and optimal policy exercises. Specifically, we analyze a belief structure that delivers a linear state-space representation of the model so standard likelihood methods can be employed. At the same time, these choices ensure a linear-quadratic optimal policy problem. Importantly, the decision rules under these beliefs represent the optimal Bayesian solution to the microfoundations, an example of internal rationality — see Preston (2005) and Adam and Marcet (2011).

Subjective beliefs. Consistent with the assumption of a symmetric equilibrium, each agent has a common forecasting model

$$z_t = S\bar{\omega}_t + \Phi z_{t-1} + e_t \tag{8}$$
$$\bar{\omega}_t = \rho\bar{\omega}_{t-1} + u_t \tag{9}$$

where Φ is a matrix to be discussed; $0 \leq \rho \leq 1$ a parameter; e_t and u_t IID with $R = E\left[e_t e_t'\right]$ and $Q = E\left[u_t u_t'\right]$. The vector $\bar{\omega}_t$ is an unobserved state, capturing imperfect knowledge about the conditional mean of the process z_t. For example, when forecasting inflation, the unobserved state represents an estimate of the inflation target; when forecasting real variables it reflects fundamental uncertainty about long-term production possibilities. We refer to these terms as low-frequency drift, drift in beliefs, or distorted beliefs. The matrix S is a selection matrix which determines which low-frequency drift is relevant for each macroeconomic variable z_t. The beliefs nest rational expectations as a special case: $\bar{\omega}_t = \bar{\omega}_{t-1} = 0$ when $Q = 0$ — that is the prior belief about the variance-covariance matrix of the drift terms is zero.

The forecasting model implies conditional expectations satisfy

$$E_t z_{t+n} = \Phi^n z_t + \sum_{j=0}^{n} \Phi^j S \rho^{n-j} \bar{\omega}_t.$$

Medium- to long-term forecasts are determined by two components: the first term is the conventional auto-regressive impact of the current state. The second term captures the effects of drifting beliefs on conditional expectations. The empirical work resolves an identification question: which component is more important for projections? For the model to explain the properties of survey data requires either highly persistent exogenous shocks, or highly persistent low-frequency movements in beliefs. We present evidence in support of the latter. In the special case $\rho = 1$ we have an example of a shifting end-point model in the language of Kozicki and Tinsley (2001). Beliefs then satisfy

$$\lim_{n\to\infty} E_t z_{t+n} = (I - \Phi)^{-1} S\bar{\omega}_t. \tag{10}$$

Objective Beliefs. Given an estimate of the unobserved state, ω, we can evaluate expectations required for optimal decisions as

$$E_t \sum_{T=t}^{\infty} \lambda_s^{-(T-t)} z_{T+1} = F_0(\lambda_s) S\omega_t + F_1(\lambda_s) z_t$$

where $F_0(\lambda_s)$ and $F_1(\lambda_s)$ are composites of structural parameters and eigenvalues λ_s. The structural equations (7) then provide

$$\begin{aligned} Z_t &= \left(A_0 - \sum_{j=1}^{3} A_s F_1(\lambda_s)\right)^{-1} \left[\sum_{j=1}^{3} A_s F_0(\lambda_s) S\omega_t + A_4 z_{t-1} + A_5 \varepsilon_t\right] \\ &= T(\Phi^*) S\omega_t + \Phi^* z_{t-1} + \Phi^*_\varepsilon \varepsilon_t \end{aligned}$$

where

$$\begin{aligned}\Phi^* &\equiv \left(A_0 - \sum_{j=1}^{3} A_s F_1(\lambda_s)\right)^{-1} A_4 \\ \Phi^*_\varepsilon &\equiv \left(A_0 - \sum_{j=1}^{3} A_s F_1(\lambda_s)\right)^{-1} A_5\end{aligned}$$

represent a fixed point of the beliefs (8).[9] We therefore assume that agents understand the true dynamics of aggregate variables up to the unobserved mean. This preserves linearity of aggregate belief dynamics and gives focus to the effects of long-run uncertainty for policy design.[10]

Drifts in beliefs are encoded into the intercept of the true data-generating process, and represent the only difference between subjective and objective beliefs in the model. That beliefs affect the true data-generating process, which in turn affects beliefs, is an example of what Marcet and Sargent (1989b) call self-referentiality. When $T(\Phi^*) = I$ beliefs are perfectly validated by the data, generating a self-confirming equilibrium — see Sargent (1999). If $T(\Phi^*) = 0$ we have rational expectations. For intermediate cases, beliefs are partially self-confirming. Such beliefs present a challenge for stabilization policy. Eusepi and Preston (2018b) reviews relevant theory, showing good policy limits self-referential dynamics.

Subjective belief updating. Beliefs are updated using the recursion

$$\begin{aligned}\omega_{t+1} &= \rho\omega_t + \rho P_t (P_t + R)^{-1} S' F_t \\ P_{t+1} &= \rho^2 P_t - \rho^2 P_t (P_t + R)^{-1} P_t + Q\end{aligned}$$

where the matrix P_t is the mean square error associated with the estimate ω_{t+1}. The vector F_t denotes the current prediction error

$$F_t = (z_t - S\omega_{t-1} - \Phi^* z_{t-1}).$$

Following Sargent and Williams (2005), we make the following simplifying assumptions. Rescale the posterior estimate using $P_t = \Xi_t R$ and use the approximation $(I + \Xi_t)^{-1} \simeq I$ for small Ξ_t to give

$$\begin{aligned}\omega_{t+1} &= \rho\omega_t + \rho\Xi_t S' F_t \\ \Xi_{t+1} &= \rho_t^2 \Xi - \rho_t^2 \Xi_t \Xi_t + QR^{-1}\end{aligned}$$

Study the steady state of this filter assuming prior beliefs satisfy the restriction $Q = g^2 R$ for scalar g. Under these assumptions the belief updating equation becomes

$$\omega_{t+1} = \rho\omega_t + \rho\alpha S' F_t$$

[9] Formally an example of the method of undetermined coefficients.

[10] Eusepi and Preston (2011, 2018a, 2018b) adduce theoretical and empirical evidence that together demonstrate learning about intercepts generates empirically relevant variation and creates policy challenges. Learning about the coefficients Φ would make the filtering problem and the state-space representation of the model non-linear.

where $\Xi = \alpha I$ and $0 < \alpha < 1$ is a function of the parameters ρ and g. In the special case $\rho = 1$, $\alpha = g$.

The restriction on prior beliefs about low- and high-frequency components of data is important to policy exercises in the sequel. Because we study counterfactuals in which the central bank implements optimal policy conditional on knowing beliefs, we want beliefs to be endogenous to the policy framework (at least to some extent). As the policy regime changes the transmission of exogenous disturbances and therefore R, scaling the prior variance Q in proportion ensures low-frequency effects of prior beliefs don't change in relative importance. The signal-to-noise ratio is policy invariant.

Evaluating the forecast error implies beliefs are updated as

$$\begin{aligned} \omega_{t+1} &= \rho\omega_t + \rho\alpha S' \left(z_t - S\omega_{t-1} - \Phi^* z_{t-1}\right) \\ &= \left[\rho + \alpha S' \left(T\left(\Phi^*\right) - I\right) S\right] \omega_t + \alpha S' \Phi^*_\varepsilon \varepsilon_t. \end{aligned} \tag{11}$$

As discussed for the simple model, short-term surprises, which are determined by structural shocks, drive long-run beliefs. The model allows us to quantify the sources of low-frequency behavior in macroeconomic variables, including expectations. Subsequent estimation and policy evaluation exercises require beliefs to be stationary. This implies a restriction on the matrix

$$\rho + \alpha S' \left(T\left(\Phi^*\right) - I\right) S$$

whose eigenvalues determine the evolution of the first-order difference equation in beliefs. The degree of extrapolation bias will depend on the size of these eigenvalues relative to ρ.

State-space representation. Finally, combining aggregate dynamics with beliefs, provides the linear state-space representation of the model

$$Z_t = F\left(\Theta\right) Z_{t-1} + Q\left(\Theta\right) \varepsilon_t$$

where Θ defines the set of model parameters with

$$F\left(\Theta\right) = \begin{bmatrix} \Phi^* & T\left(\Phi^*\right) S\rho & T\left(\Phi^*\right) S\alpha \\ 0 & \rho I & \alpha I \\ 0 & S'\left[T\left(\Phi^*\right) - I\right] S\rho & S'\left[T\left(\Phi^*\right) - I\right] S\alpha \end{bmatrix}$$

and

$$Z_t = \begin{bmatrix} z_t \\ \omega_t \\ S' F_t \end{bmatrix} \text{ and } Q\left(\Theta\right) = \begin{bmatrix} \Phi^*_\varepsilon \\ 0 \\ S'\Phi^*_\varepsilon \end{bmatrix}.$$

This permits standard likelihood-based estimation.

5 Estimation and Model Implications

5.1 Estimation

The Data. To estimate model parameters we use thirteen US time series. Five are standard macroeconomic variables: the log-difference of the GDP deflator, the output gap (as measured by the Congressional Budget Office), the three-month Treasury-Bill interest rate, and, following Justiniano, Primiceri, and Tambalotti (2013), two measures of nominal wage growth from NIPA and the BLS Establishment survey.[11] The remaining eight time series are short and long-term professional forecasts of the three-month Treasury-bill rate and inflation. We use these series to discipline beliefs. For each of these two variables, the one-quarter- and four-quarter-ahead forecasts from the Survey of Professional forecasts measure short-term forecasts. The mean one-to-ten-years-ahead and the five-to-ten-years-ahead forecasts from Blue Chip Economics and Financial measure long-term beliefs. Together these short- and long-term data on expectations permit inference on the gain parameter.

The estimation uses quarterly data over the period 1964Q1 to 2007Q3. The end of the sample is chosen to exclude the period when the policy rate is at the zero lower bound on nominal interest rates. Short-term forecasts of inflation are available from 1968Q3; short-term forecasts of nominal interest rates from 1981Q3; long-term forecasts of inflation from 1979Q3 and long-term interest-rate forecasts from 1985Q1.

Two Regimes. The behavior of long-term inflation expectations, and, to a lesser extent, interest-rate expectations, display a significant shift at the end of the 1990s. Long-term expectations become much less sensitive to surprise movements in current inflation and interest rates. Carvalho, Eusepi, Moench, and Preston (2019) account for these patterns as the endogenous outcome of increased central bank credibility which successfully anchors long-term expectations. Shifting credibility reflects a state-dependent Kalman gain, with lower gains associated with higher credibility.

Here we treat this regime change as exogenous. We estimate two Kalman gains: one for the period starting from the beginning of the sample to 1998Q4; and a second for the period from 1999Q1 to the end of the sample. The break-point approximates the point in time when Carvalho, Eusepi, Moench, and Preston (2019) estimate the gain to fall substantially. Exogeneity preserves the linearity of the model. Furthermore, the main goal of this paper is to characterize the constraints imposed on policy by the pre-1999 regime, where expectations are far from their long-run mean, and poorly anchored.

Observation Equation. Conditional on the Kalman gain regime, Section 3 showed the model has a time-invariant state-space representation

$$Z_t = F(\Theta) Z_{t-1} + Q(\Theta) \varepsilon_t \tag{12}$$

where Θ is a vector of parameters and Z the state vector of variables, which include the perceived drifts. The measurement equation

$$Y_t = \mu_t(\Theta) + H_t(\Theta) Z_t + o_t$$

attaches ten measurement errors, o_t, to the eight survey forecasts and the two measures of the nominal wage growth. The vector μ_t contains the long-run mean of the observables. The matrix H_t and μ_t are time varying because of missing observations. We estimate the model using Bayesian inference.[12]

Calibrated Parameters. The quarterly growth rate in technical progress $\gamma = 1.04$ matches the average GDP per-capita growth over the sample. Elasticity of demand across

[11] We use the CBO measure of the output gap to detrend output, not to fit the model-theoretic output gap.

[12] Details are in the appendix.

differentiated goods and labor services, θ_p and θ_w, are both set equal to 5. The parameter ρ which determines the persistence in beliefs is 0.995.[13] And the government spending-to-output ratio is G/Y=0.16.

Prior Distributions. Tables 1, 2 and 3 provide details on the priors. The priors for the exogenous shock processes are the same across variables. The persistence of the autocorrelated processes have a beta distribution with mean 0.5 and standard deviation 0.1; the standard deviations of the innovations and all measurement errors have an inverse-gamma distribution with mean 0.1 and standard deviation of 2. The priors to the parameters of the monetary policy reaction function are based on the Taylor rule — we define the coefficient on inflation as $1 + \phi_\pi$. Given evidence in Hall (1988) and Ravina (2011), the inverse intertemporal elasticity of substitution, σ, has a gamma distribution with mean 1.5 and fairly large standard deviation of 0.6, while the degree of habit persistent has a beta prior with mean 0.35 and standard deviation of 0.1. Turning to price setting, the Calvo adjustment parameters, ξ_p have prior mean which imply contracts have an average duration of two quarters, with a fairly diffuse prior. In contrast the wage rigidity parameter is ξ_w is set to be fairly high (over a year duration) and with a fairly tight prior. The parameters capturing price and wage indexation, ι_p and ι_w, have means 0.5. Following Slobodyan and Wouters (2012), the constant-gain coefficients g and g_{1999} have a gamma distribution with mean 0.035 and standard deviation 0.03.

	Prior			Posterior			
	Dist.	Mean	Std	Mode	Mean	5%	95%
β	Gamma	0.500	0.100	0.209	0.209	0.183	0.236
σ	Gamma	1.50	0.600	8.26	8.41	6.69	10.2
ϕ_n	Gamma	0.500	0.100	0.203	0.212	0.145	0.291
b	Beta	0.350	0.100	0.704	0.695	0.605	0.768
ξ_w	Beta	0.850	0.010	0.885	0.884	0.871	0.897
ι_w	Beta	0.500	0.150	0.598	0.604	0.485	0.717
ξ_p	Beta	0.500	0.100	0.884	0.868	0.781	0.919
ι_p	Beta	0.500	0.150	0.076	0.093	0.035	0.174
ϕ_π	Gamma	0.500	0.250	0.058	0.075	0.027	0.141
ρ_i	Beta	0.500	0.100	0.810	0.811	0.786	0.835
ϕ_x	Normal	0.120	0.050	0.142	0.153	0.114	0.198
$\bar{\pi}$	Gamma	0.500	0.100	0.574	0.589	0.462	0.718
g	Gamma	0.035	0.030	0.073	0.071	0.059	0.084
g_{1999}	Gamma	0.035	0.030	0.007	0.007	0.003	0.013

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table 1: **Prior and Posterior Distribution of Structural Parameters**

Posterior Distributions. Tables 1, 2 and 3 also show the mean, the mode and the 5

[13] The data suggest a unit root would be appropriate. However, to fit the steady-state real rate of interest requires a household discount factor quite close to unity. This makes model dynamics highly sensitive to shifting expectations. A value of ρ slightly below unity effectively discounts expectations, permitting jointly fitting expectations data and the steady-state real rate.

and 95 percentiles of the posterior distribution of parameters. The data are informative. The mean inflation rate is estimated to be about 2.3% per annum. The estimated policy parameters are quite different from prior values. In particular, the inflation response coefficient is only slightly above unity. This possibly reflects our choice to capture dynamics of the Great Inflation and Great Moderation under a single monetary policy regime. The intertemporal elasticity of substitution is remarkably low, within the range 0.1 to 0.15. The Frisch elasticity of labor supply is 0.2, broadly consistent with micro evidence. The price and wage stickiness parameters are equal to 0.9, implying a long duration of price contracts, common to most estimated New Keynesian DSGE models. However, because of real rigidities, the implied slope of the wage Phillips curve is an order of magnitude smaller than the price Phillips curve, with important implications for monetary policy. The learning gain before 1999, g, is estimated to be 0.07 which implies a short-term forecast error of 1 percent leads to a 7 basis point revision in long-term beliefs. Moreover, the gain implies an observation that is five-years old receives a weight of about 15% percent. The post-1999 gain, g_{1999}, is close to zero and implies very little sensitivity to new information (equivalently, a much longer memory of old data).

The shocks have lower persistence than usually found in DSGE models. This reflects the role of learning in soaking up low-frequency variation in data. The small observation errors on survey data indicate the expectation formation mechanism is consistent with observed measures of expectations, with a tight mapping between short-run forecast errors and long-term beliefs. This stands as important validation of the expectations formation mechanism central to our model.

	Prior			Posterior			
	Dist.	Mean	Std	Mode	Mean	5%	95%
ρ_{θ_p}	Beta	0.500	0.100	0.254	0.237	0.155	0.315
ρ_g	Beta	0.500	0.100	0.878	0.873	0.838	0.903
ρ_a	Beta	0.500	0.100	0.946	0.944	0.927	0.958
ρ_φ	Beta	0.500	0.100	0.632	0.617	0.512	0.709
σ_{θ_p}	InvGamma	0.100	2.00	0.216	0.222	0.191	0.258
σ_{θ_w}	InvGamma	0.100	2.00	0.110	0.115	0.091	0.141
σ_g	InvGamma	0.100	2.00	0.834	0.846	0.760	0.941
σ_m	InvGamma	0.100	2.00	0.192	0.194	0.177	0.214
σ_a	InvGamma	0.100	2.00	0.598	0.602	0.346	0.954
σ_φ	InvGamma	0.100	2.00	0.034	0.038	0.021	0.060
$\sigma_{g\gamma}$	Beta	0.500	0.200	0.173	0.233	0.062	0.499

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table 2: **Prior and Posterior Distribution of Shock Processes**

5.2 Model Predictions: Policy Mistakes and the Great Inflation

Figure 2 provides model predictions for inflation, nominal and real interest rates. For each variable, we plot the actual time series, the model's predictions for agent expectations at

	Prior			Posterior			
	Dist.	Mean	Std	Mode	Mean	5%	95%
$\sigma_{o,\pi^{1Q}}$	InvGamma	0.100	2.00	0.103	0.104	0.089	0.120
$\sigma_{o,R^{1Q}}$	InvGamma	0.100	2.00	0.052	0.052	0.045	0.060
$\sigma_{o,\pi^{4Q}}$	InvGamma	0.100	2.00	0.099	0.101	0.088	0.115
$\sigma_{o,R^{4Q}}$	InvGamma	0.100	2.00	0.052	0.054	0.035	0.072
$\sigma_{o,R^{510Y}}$	InvGamma	0.100	2.00	0.061	0.066	0.049	0.086
$\sigma_{o,\pi^{510Y}}$	InvGamma	0.100	2.00	0.027	0.028	0.021	0.036
$\sigma_{o,R^{110Y}}$	InvGamma	0.100	2.00	0.061	0.063	0.052	0.075
$\sigma_{o,\pi^{110Y}}$	InvGamma	0.100	2.00	0.028	0.028	0.022	0.035
σ_{o,w_1}	InvGamma	0.100	2.00	0.535	0.540	0.487	0.596
σ_{o,w_2}	InvGamma	0.100	2.00	0.310	0.316	0.282	0.352
Γ	Normal	1.00	0.500	0.801	0.800	0.772	0.828

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table 3: **Prior and Posterior Distribution of measurement errors**

both the one-to-ten-year and five-to-ten-year horizon, along with the corresponding survey data. For the real rate we use an ex post measure, the difference of the nominal interest rate and inflation, and compute associated real-rate expectations as the difference between the forecasts for nominal interest rates and inflation for each horizon.

The model captures low-frequency developments characteristic of the Great Inflation and Great Moderation periods, as well as the subsequent stabilization of long-term expectations from the late 1990s on-wards. The one-to-ten year (blue) and five-to-ten year (black) model-implied expectations tend to move very closely throughout the sample. Recalling expectations satisfy

$$E_t z_{t+n} = \Phi^n z_t + \sum_{j=0}^{n} \Phi^j S \rho^{n-j} \bar{\omega}_t,$$

this reveals that the evolution of long-term expectations is mainly driven by the second component: drifts, ω_t, affect beliefs at very long-horizons, as opposed to first component, which reflects short-run dynamics. One exception is the downward shift in the one-to-ten year nominal and real rate expectations in the mid-2000, which is not accompanied by a similar drop in the five-to-ten year forecast. This behavior holds true both in the model and in the data and reflects the relative stability of long-run beliefs in the post-1999 period.

The model also accounts for business cycle properties of long-run forecasts, which are clearly correlated with variation in actual inflation and nominal interest rates. Consistent with what Coibion, Gorodnichenko, Kueng, and Silvia (2017) find for different estimates of potential output, long-term real rate expectations display variation correlated with contemporaneous real rates, adducing further evidence in favor of the proposed mechanism of extrapolation bias in belief formation.

Figure 3 offers further insight on the role of beliefs in fitting observed data. The top panel plots short-term nominal interest rates, along with ten-year bond yields, priced using the expectations hypothesis of the term structure under both subjective and objective beliefs — that is, model-consistent expectations. Comparing yields reveals the basic mechanism of the model. Subjective yields display weaker mean reversion than objective yields. An outside observer who knows the true data-generating process would correctly predict long-term yields to fall more quickly from the peak of the Great Inflation and over the subsequent Great Moderation period. Subjective yields adjusted much more slowly, consistent with the professional forecast survey data.

While the discrepancy between subjective and objective yields is often not large, it gives perspective on the economic mechanisms at work in the model, which confirm the insights of the simple endowment economy analysis. The second panel provides a shock decomposition of the wedge between subjective and objective yields. We combine labor preference and technology shocks as 'supply shocks', and price and wage markup shocks as 'markup shocks'. The decomposition reveals that monetary policy shocks drive the discrepancy between subjective and objective yields. Early in the sample, monetary policy is loose relative to the historical policy rule. Because of extrapolation bias, agents project the current low rate to persist further into the future, relative to model-consistent expectations. These policy surprises therefore shift down the whole term structure of interest rates. The decline in long-term yields provides stimulus to the economy. Conversely, from the late 1970s into the 1980s, the Volcker disinflation, policy surprises on the upside, generating a surge in the long-term interest rate. This period observes the largest wedge between subjective and objective expectations, with contractionary consequences for the real economy.

The bottom panel illustrates these effects. The figure plots the output gap, defined as the difference between output and the level of equilibrium output under: flexible prices; no markup shocks; and rational expectations. The black line shows the median prediction for the output gap in the baseline model, while the red line describes a counterfactual model in which agents have rational expectations. Under rational expectations, the wedge between subjective and objective beliefs is zero at all times. The contrasting movements of each measure reveals the 'over-heating' effect of low long-rates during the mid-1960s and 1970s period in the benchmark model. Conversely, while the output gap under rational expectations increases in the early 1980, the rise in long-term rates induces a recession in the baseline model. Manifest is the link between time-varying extrapolation bias and economic fluctuations.[14]

Figure 5 corroborates the role of monetary policy shocks in generating low-frequency movements in the economy. The first two rows plot the variance decomposition of inflation and the short-term interest rate, and, for both these variables, expectations at the one-to-ten-year horizon. The bottom panel reports the variance decomposition of output gap and the real wage. At short-horizons, with the exception of the short-term interest rate, monetary shocks have little impact. Markup shocks account for most of the variance of inflation and real wages, while supply shocks dominate the short-term volatility of long-term expectations and the output gap. The role of monetary policy shocks increases significantly with the forecast horizon for all variables. Looking at two-year horizons and above, monetary shocks explain about 30 percent of real variables such as the output gap and the real wage and up to 60 percent of the variance of long-run expectations of inflation and interest rates.

To understand the economics of these patterns, Figure 6 shows counterfactual simulations

[14] In the counterfactual, the economy is subject to the same shocks and policy as in the baseline case, but expectations are rational.

under different assumptions about shocks and expectations. Each panel shows the five-to-ten year expectations under the benchmark model, the survey data on long-term expectations, along with the counterfactual paths for inflation and long-term inflation expectations. Absent monetary policy shocks, long-term expectations only rise gradually to about 4 percent in 1981 and remain around 3 percent until the early 1990s. Monetary policy shocks are central to generate the Great Inflation. As shown in Figure 3, the low expected path for the short-term interest rate during that period fueled economic activity and inflation, leading to a persistent upward drift in inflation expectations.[15] The middle panel shows markup shocks to also be an important driver of inflation expectations: absent both monetary and markup shocks the path of long-term inflation expectations would have been flat throughout the sample period.

The learning mechanism is crucial to the observed inflation drift. The bottom panel shows the counterfactual evolution of long-term inflation expectations in an economy in which agents have rational expectations. There is no drift in long-term expectations. Monetary and markup shocks are, by themselves, unable to generate any change in long-term beliefs. The experiment reveals the self-referential mechanism that is at the core of the model: the expectations formation mechanism amplifies and propagates disturbances to endogenously deliver the trend. Furthermore, it demonstrates, the monetary policy rule, despite its weaker response to inflation than usually estimated, is able to deliver stability at medium-to-low frequencies when expectations are anchored — that is, when expectations are rational. This property is central to the power of monetary policy in New Keynesian models.[16]

Taken together these findings present a simple narrative: in the late 1960s and early 1970s monetary policy errors led to excessively stimulatory policy which increased demand and inflation. Because the endogenous response of interest rates to variations in inflation was relatively modest, rising inflation became entrenched in inflation expectations. Through self-referentiality, these expectations drove low-frequency developments in inflation. The Volcker disinflation, a necessary corrective, came at considerable output cost, because inflation and interest-rate expectations were unanchored, and stubbornly high. The process of normalizing inflation expectations resolved itself over the Great Moderation period, with beliefs finally being relatively stable from the late 1990s.

Elements of our results appear in earlier literature. Perhaps most closely related is Orphanides and Williams (2006, 2012) where persistent policy mistakes interact with agents's learning, leading to substantial macroeconomic volatility. In a stylized New Keynesian model, policies that are optimal under rational expectations produce substantial economic fluctuations when the central bank mis-measures the output gap and when private agents form expectations according to least-square learning. Policies that are robust under imperfect knowledge display large interest rate inertia and relatively less response to the output gap. Melosi (2017) considers the effects of imperfect information on the part of both the central bank and private agents in a micro-founded New Keynesian model with dispersed information. It explains the increase in inflation and short-term inflation expectations in the early 1970s as a response to central bank loose monetary policy. This policy in turn was triggered by the Fed responding to a large perceived negative demand shock stemming from

[15] the behavior of the real wage over the period mirrors that of the reported output gap.

[16] A similar outcome would obtain if the counterfactual assumed the low Kalman gain that we estimate for the post-1999 period.

overestimating the level of potential output.

5.3 What are these monetary shocks

Given the prominent role of monetary policy shocks in low-frequency movements in inflation, a natural question is how should we interpret these disturbances? While we don't provide foundations, earlier literature provides useful insight on the potential source of monetary policy shocks.

Orphanides (2003, 2004) emphasizes the role of policy errors leading to over-accommodative policy in the 1970s. Policy mistakes reflect imperfect measurement of the current state of the economy and, in particular, measures of the output gap. Sargent (1999) and Primiceri (2006) argue optimal policies designed using the 'wrong model' of the economy generated persistent policy mistakes. Because of incorrect assumptions about the Phillips curve, policymakers in the 1960s thought they could exploit a long-run policy trade off, leading to excessive monetary accommodation and inflation.

Rotemberg (2013) changes tack, formulating a view of the policy mistakes based on the idea of 'penitence'. The Federal Reserve had been highly aggressive toward inflation in the 1950s until the early 1960s and had been: "[...] willing to raise interest rates and bring on recessions to nip even modest inflation rates in the bud. This brought withering criticism for the Federal Reserve on the grounds that the recessions of 1957 and 1960 had been unnecessary."[17] This induced a weaker response to rising inflation in the mid-1960s as Federal Reserve officials were reluctant to create a recession to fight inflation. The view that the expansionary policy was not entirely related to the wrong perception of the state of the economy resonates with Modigliani (1977): "One may usefully recall in this connection the experience of 1965-70 referred to earlier, with the further remark that the existence of excess employment was quite generally recognized at the time, and failure to eliminate it resulted overwhelmingly from political considerations and not from a wrong diagnosis."

Figure 7 shows the evolution of the smoothed estimates of monetary shocks in the model. The black line measures the median prediction, and the grey area includes the 95% coverage interval. The estimated shocks display a mild autocorrelation over the sample (in the range 0.19-0.27) suggesting at time the Federal Reserve surprise market participants for few quarters. The estimated shock sequence, however, does not seem to be at odds with other estimated of monetary surprises, which are shown in the Figure. The red line denotes the extended quarterly measure of the Romer and Romer (2004) shock series updated in Coibion, Gorodnichenko, Kueng, and Silvia (2017), obtained using Greenbook forecasts and other Fed information to eliminate source of endogeneity or anticipatory effects, while the blue line show the measure from Kuttner (2001), based on future contracts. Both measures are positively correlated with our estimated shocks series. Taking into account of estimation uncertainty, the correlation ranges from $0.58 - 0.61$ for the Romer and Romer measure to the interval $0.38 - 0.50$ for the Kuttner measure.

Most importantly, they capture the key historical monetary episodes over the sample. Over the first two decades of the sample, shocks are estimated to be large and volatile. The shock series captures well the negative surprises in the 1970s, the large positive surprises

[17] Rotemberg(2013), p. 65.

in the 1980, during the Volcker disinflation. The shocks also capture the contractionary surprises over the years 1994-1995 and the mid-2000s, and the negative surprises during the 1990 recession and the 2000-2004 period. In other words, the policy surprises driving expected real rate dynamics in the models display a behavior that is comparable to other measures derived with very different methodologies.

6 Optimal Policy Counterfactuals

This section provides a quantitative evaluation of the trade-off confronting policy from belief distortions. Counterfactual analysis shows that optimal policy is unable to jointly stabilize, output, wage and price inflation in contrast to a rational expectations analysis of the model. Importantly, disturbances that result in efficient movements in output are a non-trivial source of variation under optimal policy.

6.1 The Loss Function

Under arbitrary beliefs, the period welfare-theoretic loss is

$$L_t = \lambda_p \left(\pi_t - \iota_p \pi_{t-1}\right)^2 + \lambda_w \left(\pi_t^w - \iota_w \pi_{t-1}\right)^2 + \lambda_x \left(x_t - \bar{b} x_{t-1}\right)^2$$

where the weights

$$\begin{aligned}\lambda_p &= \frac{\theta_p \kappa_p^{-1}}{\theta_p \kappa_p^{-1} + \theta_w \kappa_w^{-1}} \\ \lambda_w &= \frac{\theta_w \kappa_w^{-1}}{\theta_p \kappa_p^{-1} + \theta_w \kappa_w^{-1}}\end{aligned}$$

and

$$\lambda_x = \frac{\phi^{-1} \sigma \left(1 - \beta b\right)^{-1}}{\theta_p \kappa_p^{-1} + \theta_w \kappa_w^{-1}}$$

determine the relative priority given to stabilizing prices, wages and output, and are functions of the slopes of the wage and price Phillips curves, κ_p and κ_w. Finally, the parameter $\bar{b} \leq b$ is a function of structural parameters. Details are found in Giannoni and Woodford (2004).

The derivation of the second-order approximation to household utility is valid under both rational expectations and learning. The architecture of the loss function reflects well-understood sources of inefficiency which arise from monopoly power in goods and labor markets. In our model, equilibrium price and wage markups can vary for two reasons. First, exogenous time variation in the elasticity of demand across differentiated goods and labor services shifts firms' and workers' desired markups. Second, staggered price setting in goods and labor markets means prevailing prices depart from the optimal flexible-price levels, which lead to endogenous variation in markups in response to all aggregate disturbances. Optimal policy mitigates this second source of variation due to nominal rigidities. By stabilizing endogenous variation in markups, policy reduces cross sectional dispersion in price and wage setting, and the associated inefficiencies in supply of goods and labor.

6.2 The Policy Problem

We study two counterfactual optimal policy problems, distinguished by agents' beliefs: they either learn or have rational expectations. In each case the policy maker takes as given the set of equations characterizing private sector behavior and knows the objective probability distribution and so has rational expectations. The policy maker has only short-term interest rates as an instrument of policy.

As both policy problems are linear-quadratic, it is straightforward to solve for the optimal state-contingent path of interest rates that maximizes welfare. However, to assist interpreting the differences in policy across belief structures, we instead look for optimal policy within a class of interest-rate rules. This permits direct comparison of policy rule coefficients, and therefore inference on how drift in long-term interest rates constrain optimal policy. The approach also resolves the question of how to implement optimal policy. It is well known that purely fundamentals-based rules are prone to indeterminacy of rational expectations equilibrium and expectations instability under learning. Furthermore, while the optimal target criterion can in principle be derived under rational expectations, it is rather complicated, involving a large number of leads and lags of various endogenous variables.

Under rational expectations the central bank minimizes the expected discounted loss

$$E_t^{RE} \sum_{T=t}^{\infty} \beta^{T-t} L_T \tag{13}$$

subject to

$$\begin{aligned} z_t &= \Phi^* (\phi) z_{t-1} + \Phi_\varepsilon^* (\phi) \varepsilon_t \\ 0 &= \pi_t - \iota_p \pi_{t-1} + \phi_w (\pi_t^w - \iota_w \pi_{t-1}) + \phi_x (x_t - x_{t-1}) \end{aligned}$$

by choice of policy parameters $\phi = \{\phi_\pi, \phi_x\}$, where the first equation gives the true data-generating process under the special case of rational expectations: $T(\Phi^*) = 0$. With a slight abuse of notation, the first equation implicitly drops the interest rate from the true data-generating process (12). The second equation, interpreted as a target criterion for monetary policy, is chosen because it approximates very well the optimal commitment policy under the timeless perspective.

Under learning the central bank minimizes the discounted loss subject to

$$\begin{aligned} Z_t &= F(\Theta; \phi) Z_{t-1} + Q(\Theta; \phi) \varepsilon_t \\ R_t &= \rho_R R_{t-1} + \phi_\pi (\pi_t - \iota_p \pi_{t-1}) + \phi_w (\pi_t^w - \iota_w \pi_{t-1}) + \phi_x x_t \end{aligned}$$

by choice of policy parameters $\phi = \{\rho_R, \phi_\pi, \phi_w, \phi_x\}$. Recalling

$$Z_t = \begin{bmatrix} z_t \\ \omega_t \\ S' F_t \end{bmatrix}$$

the central bank internalizes the effects of policy on the evolution of beliefs and forecast errors — beliefs are state variables. Ideally the policy rule would nest the rational expectations target criterion (so that in the special case $\phi_\pi \to \infty$ we would have identical policy rules). However, this rule performed poorly. A search of a wide class of rules led to the stated rule.

The counterfactuals make the following assumptions. With the exception of monetary policy shocks, the economy experiences the same sequence of shocks and has the same initial state estimated in the benchmark model. Monetary policy shocks are set equal to zero since purely exogenous variation in interest rates reduces welfare. When computing dynamics under different policies we assume agents know the new transition dynamics associated with the regime. We interpret this thought experiment as one in which agents have inhabited the regime since the distant past. The question of how to design the optimal transition from one regime to another is left for future research. Finally, we assume the gain coefficient is policy invariant. While this means the perceived signal-to-noise ratio is invariant across policy regime (that is, agents perceive the same volatility of long-term drift relative to short-term disturbances), it doesn't imply beliefs are invariant to policy. Because short-run forecast errors are endogenous to policy, long-term expectations will adjust. Policy can't exploit beliefs to deliver any equilibrium of its choosing.

6.3 The Counterfactuals

Figure 8 shows the optimal policy counterfactuals. The successive panels report results for wages, goods prices and the output gap. Consistent with the welfare-theoretic loss function, we report wage inflation net of inflation indexation and goods-price inflation net of indexation. The dashed grey line reports the benchmark economy, while the red and blue lines report dynamics under optimal policy for learning and rational expectations.

Regardless of beliefs, optimal policy provides much greater stabilization of wage and goods-price inflation, particularly over the late 1960s and 1970s. Reflecting greater nominal distortions in labor markets, priority is given to wage stabilization (recall the wage Phillips curve is an order of magnitude smaller than the goods-price Phillips curve). The results provide clear evidence that monetary policy error led to excessive wage and price inflation. Of course greater nominal stabilization comes with a cost. Tighter interest-rate policy leads to declines in real economic activity, early in the sample. However, these policy actions confer later advantage: throughout the Volcker disinflation counterfactual policy delivers higher real activity with lower wage and price inflation — reward for providing a strong nominal anchor, for stabilizing long-term expectations.

Comparing the two belief structures, optimal policy under rational expectations provides greater stabilization of the macroeconomy. For the most part, the rational expectations economy experiences lower wage and price inflation, at reduced output cost. To render the differences more stark, Figure 9 plots the counterfactual outcomes under optimal policy and no markup shocks for the output gap, as well as the model's predictions for the benchmark policy. Because the remaining disturbances all represent efficient movements in the natural rate of output, optimal policy under rational expectations completely stabilizes the output gap.[18] In contrast, the optimal policy under learning is unable to insulate the economy

[18] That output is not fully stabilized early in the samples reflects our assumptions on initial conditions in the counterfactual.

from fluctuations in the natural real rate of interest. While optimal policy clearly moderates fluctuations in real activity — for example, it predicts the Volcker recession to not occur — sizable recessions still result in the mid 1970s and early 2000s.

Beliefs then represent a quantitatively relevant constraint on policy, leading to different conclusions to earlier literature on the monetary history of the United States. For example, Justiniano, Primiceri, and Tambalotti (2013) study historical policy through the lens of a medium-scale model of the kind proposed by Smets and Wouters (2007). They show that once appropriate account is taken of low-frequency movements in hours data, as well as measurement error in wage data, there is little evidence of a fundamental trade-off confronting monetary policy. Exogenous variation in desired markups explains little variation in observed data. A striking implication is that observed fluctuations in the output gap, and events such as the Great Inflation, are the result of policy error.

Our results cast a more positive light on the historical performance of the Federal Reserve. Like the Justiniano, Primiceri, and Tambalotti (2013) analysis, our optimal policy counterfactuals assume the identified monetary policy shocks do not occur. But their absence alone doesn't explain the improvement in stabilization policy. The top panel of Figure 10 plots long-term inflation expectations in three economies: the benchmark model (the blue line); the benchmark model without monetary policy shocks (the black line); and the model under optimal policy (the red line). Also shown are the survey data. Even absent monetary policy shocks, optimal policy improves upon the historical policy rule. While over the early 1970s markup shocks confront optimal policy with unresolveable trade-offs, leading to a rise in long-term inflation expectations, from the mid 1970s optimal policy delivers much greater restraint. Indeed, from this time, long-term expectations almost continually decline, until they stabilize around the late 1990s. The historical rule permits much more elevated, and persistently so, long-term expectations. The Federal Reserve could have provided a much stronger nominal anchor.

The second panel shows the counterfactual yields on a ten-year bond under the rational expectations and learning optimal policies, a striking illustration of the basic insight of the paper: optimal policy under learning is less aggressive than under rational expectations.

7 Intertemporal Trade-offs under Optimal Policy

This section provides some final formal results, to complement the simple example and empirical findings. We do this in a special case of our empirical model, in which there is a frictionless labor market, and purely forward-looking optimal pricing and consumption decisions. The analysis of optimal monetary policy shows that in general the aggregate demand curve is a binding constraint on feasible choices of interest-rate paths, even though this is never true of the equivalent model with rational expectations.

7.1 The Policy Problem

The policymaker minimizes the period loss function

$$L_t = \pi_t^2 + \lambda_x x_t^2$$

where $\lambda_x > 0$ determines the relative weight given to output gap versus inflation stabilization. Feasible sequences of inflation and the output gap must satisfy the aggregate demand and supply equations

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\, x_{T+1} - (R_T - \pi_{T+1} - r_T^n) \right] \tag{14}$$

$$\pi_t = \hat{E}_t \sum_{T=t}^{\infty} (\xi_p \beta)^{T-t} \left[\kappa x_T + (1-\alpha)\, \beta \pi_{T+1} \right] \tag{15}$$

where all variables are interpreted as log-deviations from steady state; x_t is the output gap; r_t^n the natural rate of interest an exogenous process; and $\kappa = (1-\xi_p\beta)(1-\xi_p)/\xi_p$ the slope of the short-run trade-off between inflation and the output gap.[19] Optimal consumption and price-setting requires households and firms to forecast future output, interest rates and inflation. Assume agents have a forecasting model of the form (8) and (9), with

$$z_t = \begin{bmatrix} \pi_t \\ x_t \\ R_t \end{bmatrix} \text{ and } \omega_t = \begin{bmatrix} \omega_t^{\pi} \\ \omega_t^{x} \\ \omega_t^{R} \end{bmatrix}$$

and where $\Phi = 0$ and $\rho = 1$ to give a shifting end-point model.[20]

Subject to aggregate demand and supply, and the evolution of beliefs, the central bank solves the problem

$$\min_{\{x_t, \pi_t, R_t,\ \omega_t\}} E_t^{RE} \sum_{T=t}^{\infty} \beta^{T-t} L_T \tag{16}$$

taking as given initial beliefs, ω_{-1}. Assume that the central bank has rational expectations and has complete information about the true structural relations describing household and firm behavior. The first-order conditions are described in the appendix. Because beliefs are state variables there is no distinction between optimal commitment and discretion. The policy maker can only influence expectations through current and past actions — not through announced commitments to some future course of action.

The first-order conditions constitute a linear rational expectations model.[21] The system can be solved using standard methods. Using results from Giannoni and Woodford (2017), Eusepi, Giannoni, and Preston (2018) we establish conditions on beliefs for a unique bounded rational expectations equilibrium.

Proposition 1. *Let* $\bar{g} = \frac{(1-\alpha\beta)\left(\lambda_x + \kappa^2\right)}{\lambda_x(1-\beta)+\kappa^2}$*. For beliefs* $g \in (0, \bar{g})$ *that satisfy either* $g < 2(1-\beta)$ *or* $g > \beta^{-1} - \beta$ *the optimal policy problem has a unique bounded solution. When* $g < 2(1-\beta)$ *the aggregate demand constraint is not binding, and the associated Lagrange multiplier is*

[19] Derivation of these expressions assume a unity elasticity of intertemporal substitution, and infinite Frisch elasticity of labor supply.

[20] Under rational expectations, because the model is purely forward looking, the minimum state variables solution is a linear function of aggregate disturbances. We therefore assuming a belief structure consistent with this solution.

[21] In an innovative study, Molnar and Santoro (2013) explore optimal policy under learning in a model where only one-period-ahead expectations matter to the pricing decisions of firms. Gaspar, Smets, and Vestin (2006) provide a global solution to the same optimal policy problem but under a more general class of beliefs.

equal to zero. When $g > \beta^{-1} - \beta$ the aggregate demand constraint is binding, and the associated Lagrange multiplier is strictly positive.

Proof. See the appendix. □

This result formalizes the central insight of the paper. When long-term interest rate beliefs are sufficiently sensitive to short-run forecast errors, aggregate demand limits the movements in interest rates. The central bank has imprecise control of long-term interest rates, even though the model satisfies the expectations hypothesis of the term structure. Belief distortions prevent changes in short-term rates being efficiently transmitted to long-term rates relevant for aggregate demand.

A further implication concerns a special case of beliefs. When the gain coefficient converges to zero the optimal policy coincides with optimal discretion under rational expectations. This result is intuitive: for small gains beliefs are almost never revised. Because policy cannot influence beliefs, which is precisely the assumption of optimal discretion, dynamics will correspond to those predicted by optimal discretion. For sufficiently small gains, policy is well approximated by rational expectations equilibrium analysis, and the central bank will have precise control of long-term inflation expectations.

Corollary 1. *In the special case $g \to 0$ optimal policy will give the same dynamic responses to disturbances as optimal discretion under rational expectations.*

This type of result has been discussed by Sargent (1999), Molnar and Santoro (2013) and Eusepi, Giannoni, and Preston (2018). The empirical model reflects this property. After 1998, when agent's beliefs display relatively little sensitivity to forecast errors, monetary policy ensures greater stability of long-term inflation expectations. By providing a nominal anchor, policy permits better stabilization outcomes.

7.2 A Simple Example

To appreciate further aggregate demand as a constraint confronting policy, consider a central bank faced only with i.i.d. shocks to the natural rate r_t^n, and private agent beliefs initially consistent with rational expectations equilibrium so that $\omega_{t-1} = 0$. Because initial forecasts satisfy $E_t z_T = 0$ for all $T > t$, period t equilibrium is determined by the aggregate demand and supply curves (14) and (15) which simplify to

$$\pi_t = \kappa x_t \qquad \text{and} \qquad x_t = -\left(R_t - r_t^n\right).$$

Given a disturbance to the natural rate of interest, complete stabilization is possible in period t. Nominal interest-rate policy must track the natural rate, $R_t = r_t^n$, giving $\pi_t = x_t = 0$. But this implies subsequent movements in long-run interest-rate beliefs according to

$$\omega_t^R = \omega_{t-1}^R + g\left(r_t^n - \omega_{t-1}^R\right).$$

The next-period's stabilization problem — and every subsequent period — is given by the pair of equations

$$\begin{aligned} \pi_{t+1} &= \kappa x_{t+1} \\ x_{t+1} &= -\left(R_{t+1} - r_{t+1}^n\right) - \frac{1}{1-\beta}\beta\omega_t^R \end{aligned}$$

where the final term in the demand equation reflects the restraining effects of long-term interest rates on aggregate demand. Complete stabilization of inflation and the output gap is again possible by having nominal interest rates track long-run expectations and the natural rate of interest.

But is this interplay sustainable? Imposing full stabilization, $x_{t+1} = \pi_{t+1} = 0$, the aggregate demand constraint defines the implicit policy rule

$$R_{t+1} = r^n_{t+1} - \frac{\beta}{1-\beta}\omega^R_t \tag{17}$$

in every period t. Optimal policy not only responds to natural-rate disturbances, but also movements in long-term interest rates, driven by expectations.[22] Substituting into the updating rule for beliefs, ω^R_t, gives

$$\omega^R_{t+1} = \left(1 - \frac{g}{1-\beta}\right)\omega^R_t + g r^n_{t+1}$$

which is a first-order difference equation. Sustainable policy requires the dynamics of beliefs to be stationary. The following restriction must hold

$$g < 2\left(1-\beta\right).$$

For larger gains, stability is not feasible, implying beliefs and interest rates are explosive. This is not a permissible, or at least desirable, feature of optimal policy if only because the zero lower bound on interest rates obviates such solutions.[23]

This restriction is the limit of the condition derived for the simple endowment economy when $\phi \to \infty$, and defines one region of the parameter space for which the optimal Ramsey problem has a unique bounded rational expectations equilibrium in Proposition 1. An important lesson emerges: complete stabilization of inflation is infeasible. A central bank charged with implementing the target criterion $\pi_t = 0$ will fail, because it requires large movements in nominal interest rates. This result holds more generally, placing important bounds to arguments made by Evans and Honkapohja (2006), Woodford (2007) and Preston (2008), that the target criterion approach to implementing policy is robust to alternative assumptions about belief formation.[24] Of course, should the condition on the gain be violated, Proposition 1 shows such beliefs are still consistent with equilibrium, but one in which the central bank optimally accepts some variation in inflation.

Figure 11 provides numerical illustration, plotting the standard deviation of the output gap and interest rate as a function of the constant gain g under optimal policy. Assume the discount factor is $\beta = 0.995$; the frequency of price changes determined by $\xi_p = 0.8$; and the weight on output gap stabilization $\lambda_x = 0.05$. Under these assumptions there is relatively small variation in inflation, so it matters little whether we plot the sum of the output gap

[22] The implied interest rates of a bond of any maturity can be shown to be a function of the long-term interest rate belief. This is an example of the expectations hypothesis of the term structure of interest rates.

[23] While some might not object to nominal explosions, if beliefs about real activity depend on nominal interest-rate forecast errors, there would also be unbounded paths for real variables.

[24] For example, it is equally true when using a Taylor-type rule to implement the target criterion $\pi_t = -\theta^{-1}(x_t - x_{t-1})$ the optimal commitment policy in the canonical New Keynesian model.

and inflation variation or the output gap alone. Only variations in the natural rate, r_t^n, drive economic fluctuations. The figure describes outcomes under the welfare-theoretic loss (16), and under a loss function

$$L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_R R_t^2$$

that also penalizes volatility in the interest rate. Recall optimal discretion corresponds to the case $g = 0$. Under the standard loss function a knife-edge result obtains: for $g < 0.01$ the output gap is fully stabilized even if this induces substantial volatility in the interest rate. For large values of g, the policy maker loses the ability to stabilize the output gap. Feasible policy restricts variation in the policy rate, translating into increasing volatility in the output gap. If the policy maker has some preference for interest-rate stabilization, perhaps reflecting zero-lower bound considerations, then the increase in output volatility occurs continuously with the size of the gain. Even relatively small values of the gain lead to output gap volatility.

Proposition 2. *In the model given by (14) and (15), Divine Coincidence will in general not hold even in a model with only disturbances to the natural real rate of interest.*

The inability of the central bank to stabilize both output gap and inflation in the face of aggregate demand shocks stems from agents' expectations about the policy rate. For example, suppose as in Molnar and Santoro (2013) the policymaker can directly control the output gap as the instrument of policy, and solves the problem

$$\min_{\{x_t, \pi_t, \omega_t^x, \omega_t^\pi\}} E_t^{RE} \sum_{T=t}^{\infty} \beta^{T-t} L_T$$

subject only to the Phillips curve (15), taking as given initial beliefs ω_{-1}^x and ω_{-1}^π. Equivalently, suppose interest-rate beliefs are anchored so that $\omega_t^i = 0$ for all t, giving households rational expectation forecasts of interest rates. Then the Divine Coincidence holds, despite long-term drift in expectations about inflation and real activity.

Corollary 2. *Absent low-frequency drift in interest-rate beliefs, the central bank can directly control aggregate demand, and the Divine Coincidence holds.*

Proof. See the appendix. □

The result underscores the importance of jointly modeling monetary policy, long-term expectations and policy credibility. Central banks which provide a credible nominal anchor — that is, they stabilize long-run inflation expectations — will have tighter control of the macroeconomy. If policy lacks credibility, so that long-term expectations display high sensitivity to short-run surprises, stabilization policy becomes more difficult, as the term structure of interest rates constrains policy actions.

8 Conclusions

[TO BE ADDED]

A Appendix

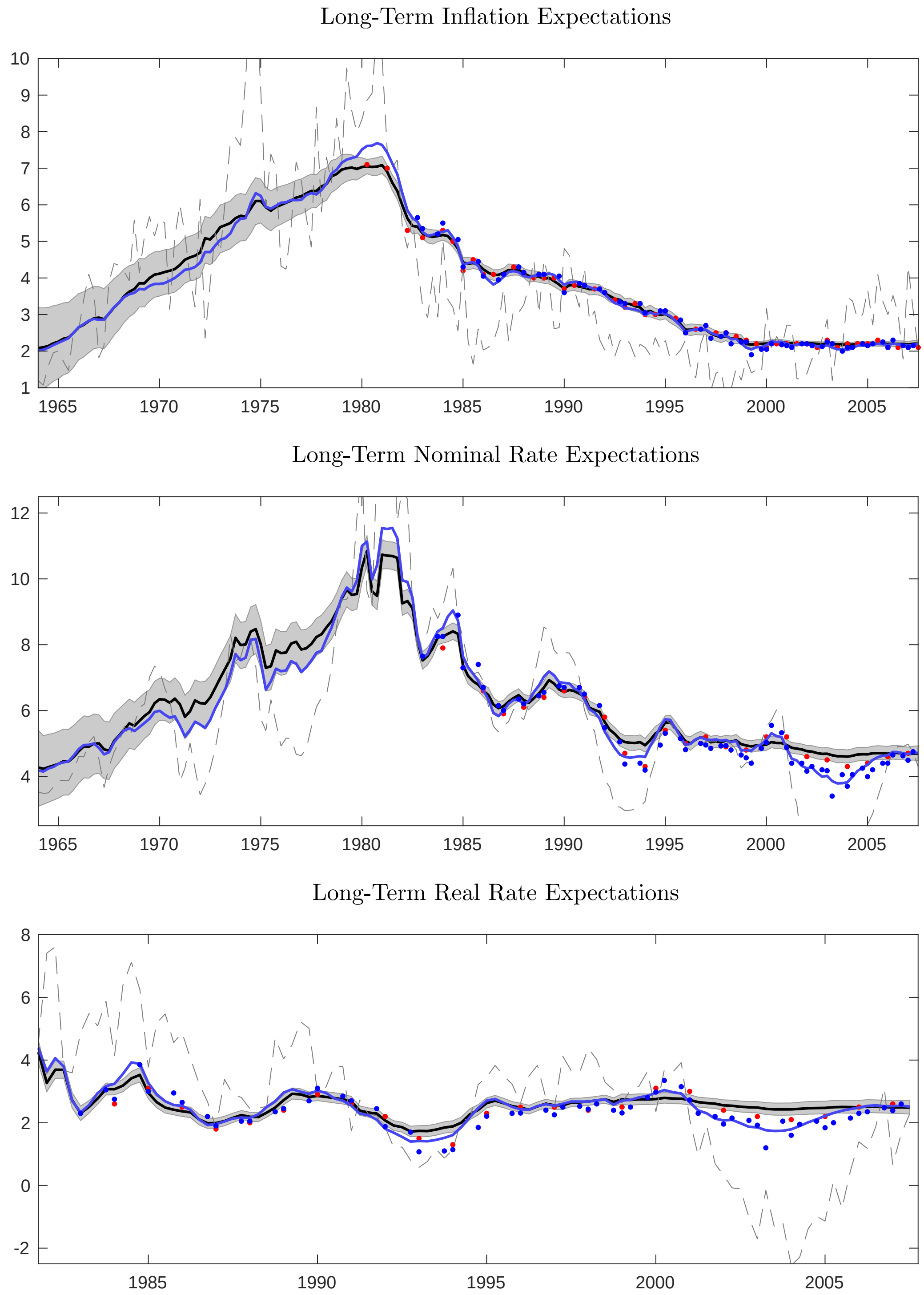

Figure 2: **Baseline Predictions.**

The top and middle panels show the evolution of long-term survey expectations data for inflation and the short-term nominal rate of interest. Actual variable (dashed black), the two survey expectations measures (red and blue dots), the model implied 1-10 year average expectations (the blue line); and the model implied 5-10 year average expectations with 95% posterior probability band (black line). The bottom panel shows long-term expectations for the short-term real rate as implied by both data and model.

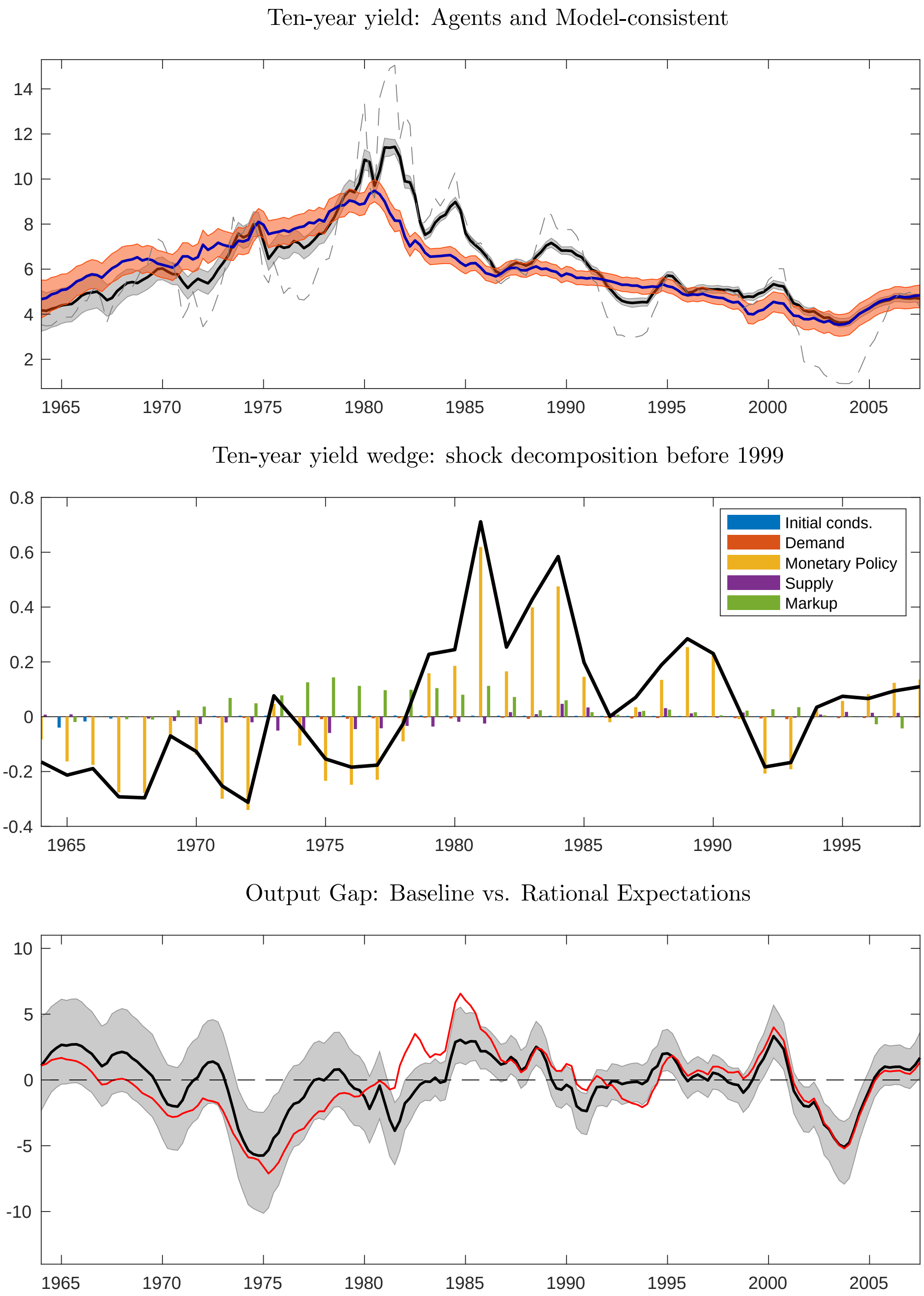

Figure 3: **Over-extrapolation and monetary policy.**

Top panel: the black line defines the 10-year interest rate in the model, while the blue line is the rate that would prevail if the bond was prices under model-consistent expectations. The dashed grey line denotes the short-term interest rate. *Middle panel*: shows the shock decomposition of the wedge between the long-term interest rates showed in panel 1. *Bottom panel*: shows the evolution of the output gap in the baseline model (black line, median) and the model under rational expectations (red line, median).

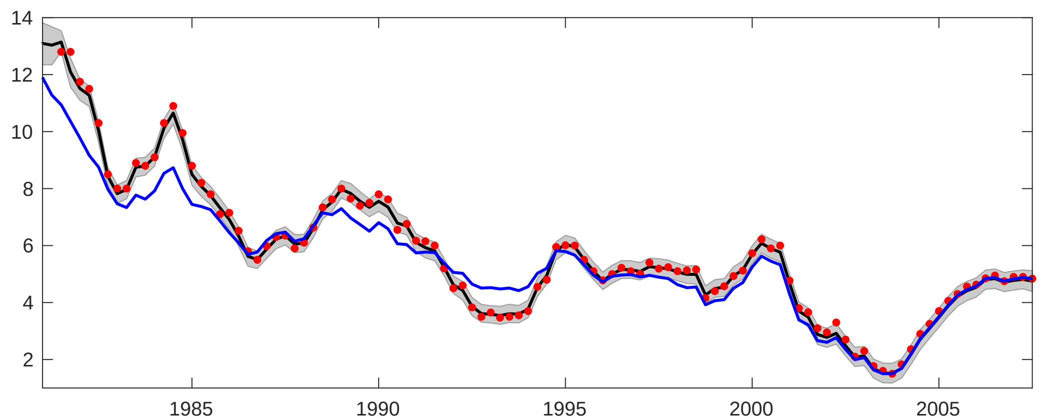

Figure 4: **Short-Term interest rate expectations**

The figure shows the evolution of the four-quarters ahead interest rate expectations. The solid line measures agents' expectations, while the blue line shows the model-consistent expectations. The red dots are survey data.

Figure 5: **Variance Decomposition**

The panels show the variance decomposition of selected variables calculated at the posterior mode.

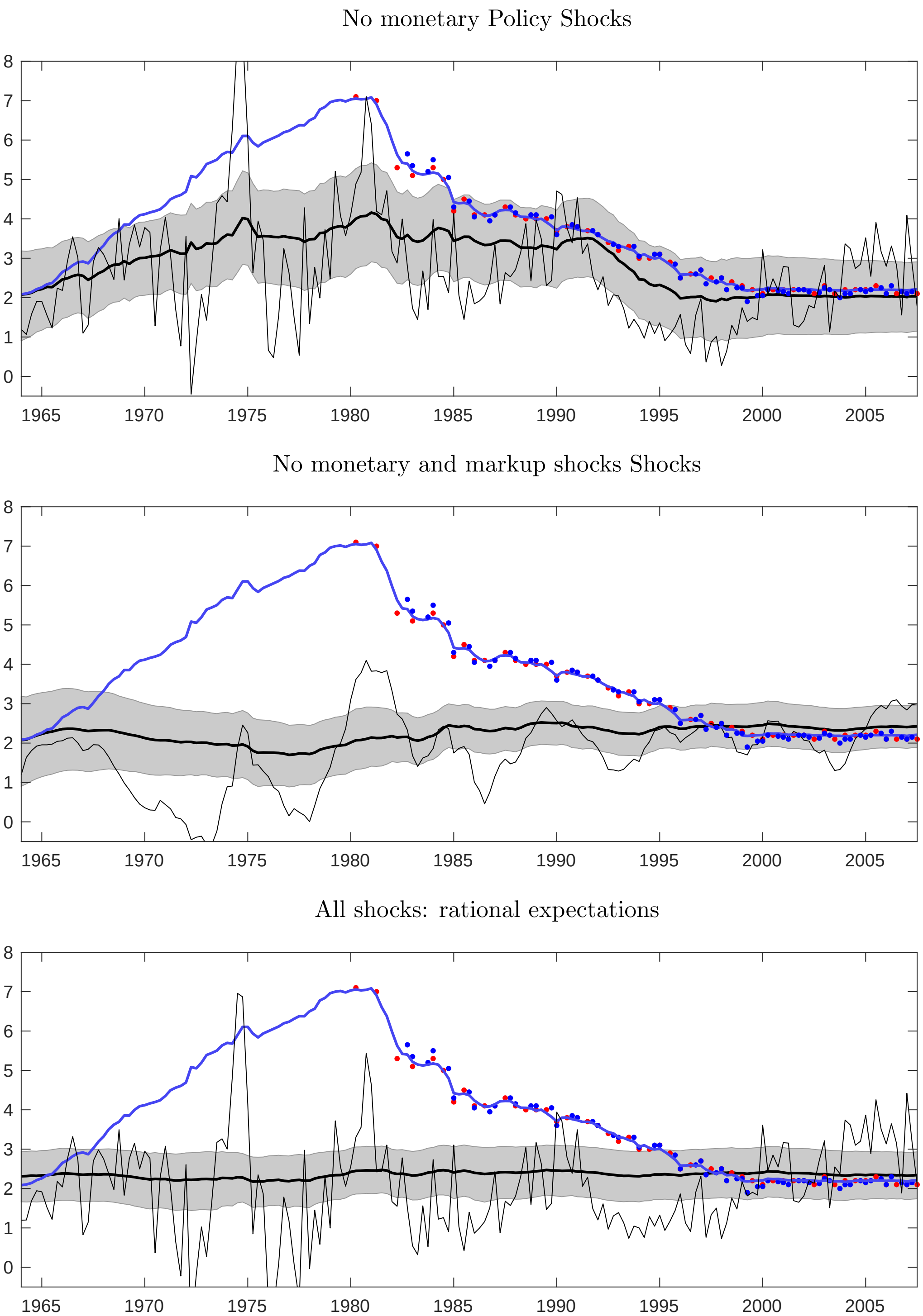

Figure 6: **Long-run Inflation expectations.**

The panels show counterfactual simulations for inflation (grey line) and five-to-ten inflation expectations (black line). The blue line and the blue and red dots describe the evolution of the expectations in the data and in the baseline model. The top panel describe the counterfactual without monetary policy shocks. The middle panel shows path for inflation and inflation expectations without monetary and markup shocks. Finally, the bottom panel describes the counterfactual simulation with rational expectations.

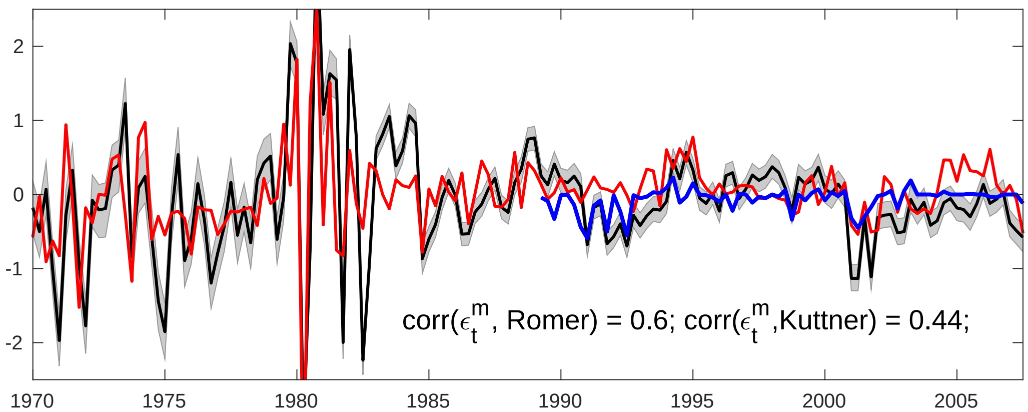

Figure 7: **Monetary policy shocks**

The black line (and shaded area) shows the evolution monetary shocks as implied by the model (smoothed estimates, black line); the red line measures the quarterly measure of the Romer and Romer (2004); the blue displays (quarterly) monetary shocks from Kuttner (2002).

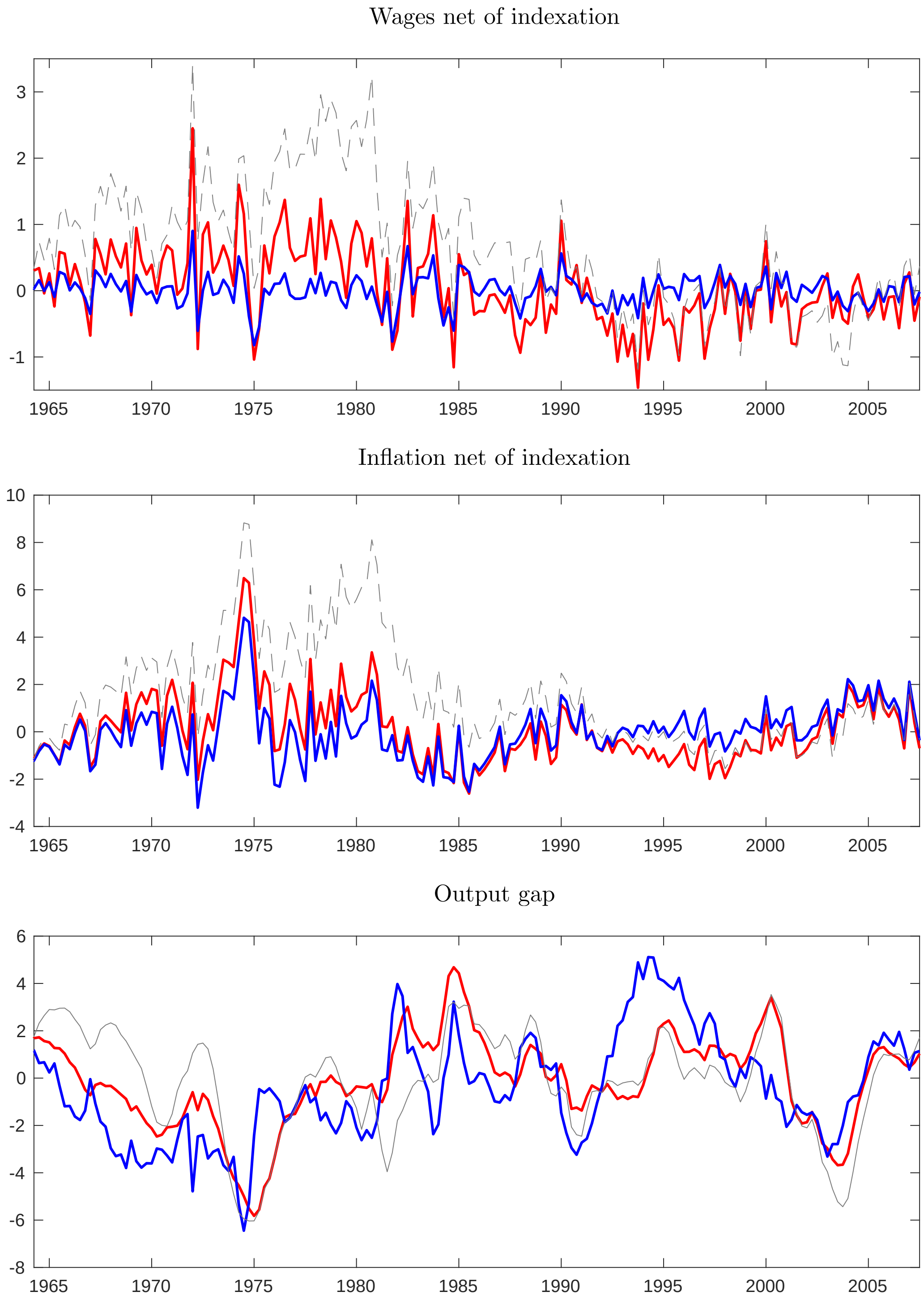

Figure 8: **Optimal Policy.**

The panels describe the evolution of wages net of indexation (top), inflation net of indexation (middle) and output gap (bottom). The grey line describe the baseline model with the historical policy rule; the red line describes optimal policy under learning; the blue line corresponds to optimal policy under rational expectations

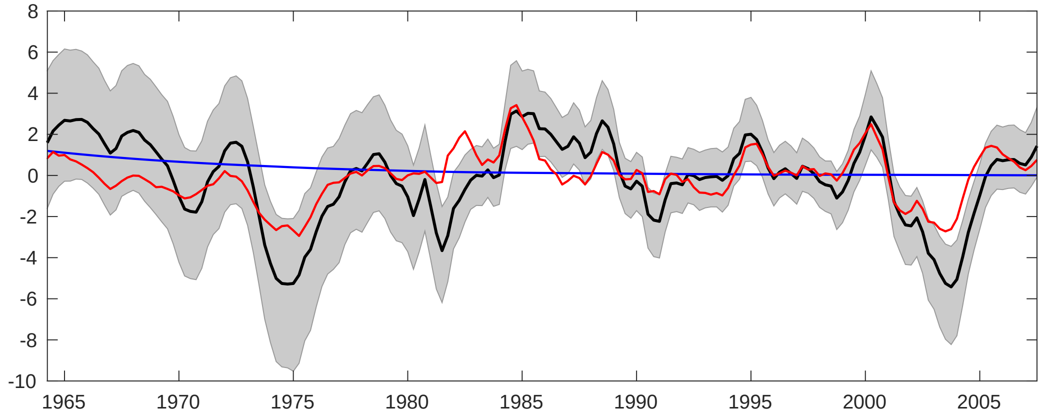

Figure 9: **Optimal Policy: output gap without markup shocks**

The figure shows the evolution of the output gap under optimal policy in absence of markup shocks. The red line show optimal policy under learning; the blue line describes optimal policy under rational expectations; the black line shows the output gap under the baseline model with the historical policy rule.

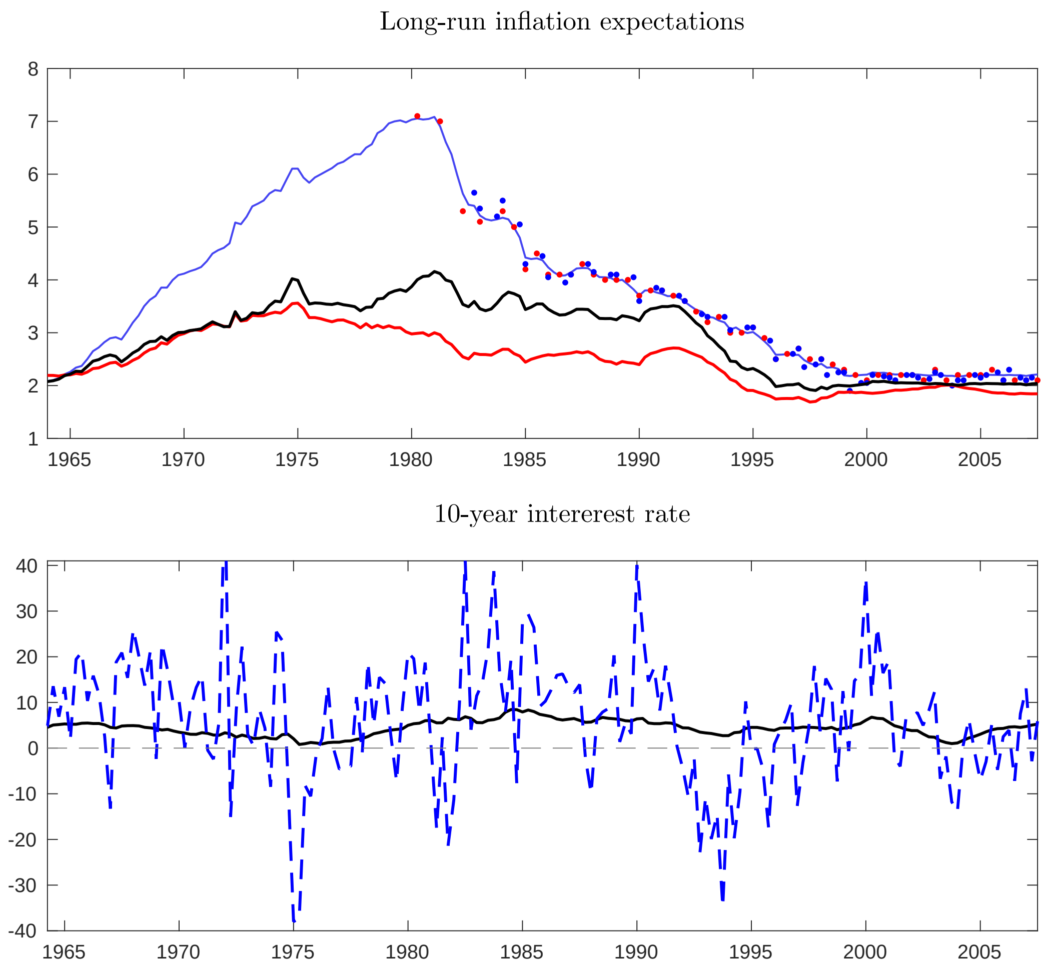

Figure 10: **Long-run trends under optimal policy.**

The top panel describes the evolution of five-to-ten years ahead inflation expectations under optimal policy (red) line, compared with the baseline model (blue line), data (blue and red dots) and the counterfactual simulation without monetary policy shocks. The bottom panel shows the evolution of the 10-year interest rate under optimal policy with learning (black line) and under rational expectations (dashed blue line).

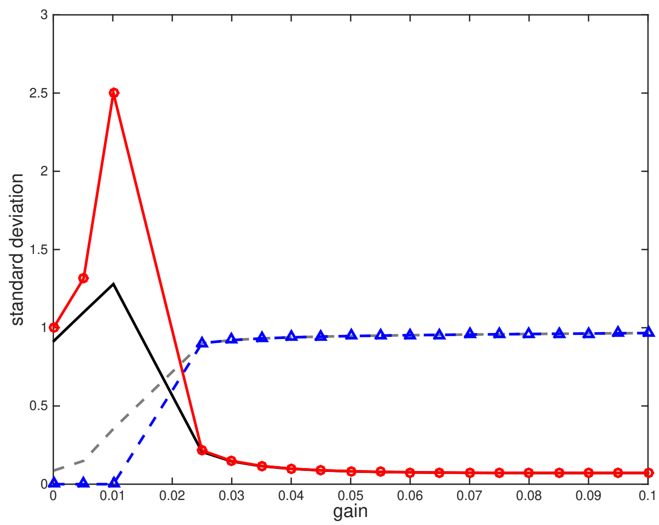

Figure 11: **Volatility as a function of the constant gain.**

This figure show the volatility of output and interest rates as a function of the constant gain $\bar{g}$. The welfare theoretic loss gives the volatility of the interest rate (red circles) and the output gap (blue triangles); while a policy maker with a concern of interest rate volatility delivers the interest rate shown by the black line, and the output gap given by the grey dashed line.

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