A Sufficient Statistics Approach for Aggregating Firm-Level Experiments *

David Sraer
UC Berkeley, NBER & CEPR

David Thesmar
MIT-Sloan & CEPR

April 22, 2019

Abstract

We consider a dynamic economy populated by heterogeneous firms subject to generic capital frictions: adjustment costs, taxes, and financing constraints. A random subset of firms in this economy receives an empirical "treatment", which modifies the parameters governing these frictions. An econometrician observes the firm-level response to this treatment and wishes to calculate how long-run macroeconomic outcomes would change if *all* firms in the economy were treated. Our paper proposes a simple methodology to estimate this aggregate counterfactual using firm-level evidence only. Our approach takes general equilibrium effects into account, requires *neither* a structural estimation *nor* a precise knowledge of the exact nature of the experiment and can be implemented using simple moments of the distribution of output-to-capital ratios. We provide a set of sufficient conditions under which these formulas are valid and investigate the robustness of our approach to multiple variations in the aggregation framework.

*Sraer: sraer@berkeley.edu; Thesmar: thesmar@mit.edu. We gratefully acknowledge comments and suggestions from Laurent Frésard, Ben Hébert (discussant), Valentin Haddad, Erik Loualliche, Martin Lettau, John Nash as well as seminar participants at UT Austin, UC Berkeley, Columbia, Cornell, HKUST, Maryland, NYU, University of Minnesota, UCLA, Michigan, MIT, Stanford, Yale, Washington University in Saint Louis and at the Econometric Society meetings in Philadelphia. All remaining errors are our own.

1 Introduction

Governments around the world implement a wide range of policies to facilitate business investment and growth. A burgeoning empirical literature seeks to evaluate the effectiveness of these policies using firm-level data and well-identified empirical settings. Some papers look at financial reforms (see for instance Aghion et al. (2007), Bertrand et al. (2007), Ponticelli and Alencar (2016), Larrain and Stumpner (2017)). Others analyze firm response to the availability of subsidized credit (e.g. Lelarge et al. (2010), Banerjee and Duflo (2014), Brown and Earle (2017)), or changes in bank lending behavior (Fraisse et al. (2017), Blattner et al. (2017)). Another set of papers study the effect of capital taxes or subsidies on firm investment and hiring (Yagan (2015), Zwick and Mahon (2015), Giroud and Rauh (2016), Rotemberg (2017)). These papers seek to provide clean empirical estimates of the relative effect these policy interventions have on the *subset* of firms they affect. However, they mostly remain silent on how these firm-level effects would aggregate, were the intervention generalized to a broader set of firms in the economy. In this paper, we develop a simple methodology to estimate such an aggregate counterfactual using firm-level reduced-form evidence. This approach does not require the estimation of a structural model of firm behavior. This methodology can be implemented even when the empiricist does not precisely know how the intervention affects firm-level distortions.

The aggregation exercise we consider is not trivial. First, standard equilibrium effects will typically dampen firm-level responses: for instance, if a policy alleviates credit constraints, its extension to a larger set of firms will increase aggregate labor demand, which in turn will raise the equilibrium wage and mitigate the initial direct effect. Second, the scaling-up of a policy intervention may lead to a reallocation of inputs across heterogeneous firms: as distortions are reduced, capital and labor flow from firms with a low marginal product to firms with a high marginal product, increasing aggregate productivity. Third, the policy treatment effects estimated in a "local" experiment may not carry over in an environment where all firms receive the treatment. In other words, these estimated treatment effects may not be externally valid, so that they may not be used to compute the aggregate counterfactual described above. This paper derives simple sufficient statistics formulas that take these effects into account. It also provides sufficient conditions under which the methodology is valid: importantly, these conditions are weaker than the conditions used in the vast majority of structural papers in the macro-finance literature.

To derive these formulas, we proceed in three steps. First, we set up a steady-state

general equilibrium model with heterogeneous firms that face stochastic productivity shocks and are subject to several forms of distortions: adjustment costs, taxes, and financing frictions. We do not solve the model explicitly, but instead express aggregate output and total factor productivity (TFP) as a function of the joint distribution of output to capital ratios[1] and productivity. In a second step, we consider a small-scale policy intervention that targets a random subset of firms in this economy.[2] This policy treatment affects parameters governing firm-level frictions, although we do not need to specify which ones exactly. Given the exogeneity of this treatment, an econometrician can use standard datasets to recover the treatment effect of this policy on the joint distribution of output to capital ratios and productivity. In a final step, we would simply apply these estimated treatment effects on *all the firms* in the economy and obtain counterfactual changes in output and aggregate TFP through the formulas derived in the first step. However, this final step may not be valid: the treatment effects recovered in the second step are estimated in an economy where a negligible fraction of firms are treated and they may be different in an economy where all firms are treated. For instance, the generalization of a policy may increase the equilibrium wage on the labor market, and firms may respond differently to the policy treatment when facing a higher wage.

A key contribution of our paper is to provide conditions under which these estimated treatment effects on the joint ergodic distribution of output to capital ratios and productivity are *independent* of general equilibrium conditions, and therefore externally valid. This property allows us to safely use, in the aggregation formula, the treatment effects estimated in partial equilibrium. This "scale-invariance" property relies crucially on two key assumptions about technology and frictions. First, the sources of distortions (financing frictions and constraints, tax schedules, adjustment costs) are assumed to be homogeneous of degree one. Intuitively, homogeneity guarantees that frictions remain on average constant on a size-adjusted basis. Hence, a change in general equilibrium, which affects firm size, will not affect distortions. Second, firm-level production is Cobb-Douglas, with either constant or decreasing returns to scale. While these assumptions may appear restrictive, they are almost always satisfied in the structural macro-finance literature (see our extensive review of the literature in Table 1). Additionally, the scale-

[1]The output to capital ratio is commonly called "marginal revenue product of capital" (MRPK) or equivalently "capital wedge" in the misallocation literature (Restuccia and Rogerson (2008), Hsieh and Klenow (2009)).

[2]The focus on small experiments with full randomization is for expositional purposes only. We show in Section 5.2 how our methodology can be applied to cases where the policy experiment is large or affects a sub-group of firms in the population. As long as the treatment is randomized within the sub-group, our methodology can be adapted to recover a "partial" aggregate counterfactual where the policy is extended only to this sub-group of firms.

invariance property only holds at the steady-state of the economy, a notable limitation of our analysis.

The formulas we obtain for changes in steady-state aggregate output and TFP combine parameters of the model (labor share, the elasticity of substitution of goods within an industry, labor supply elasticity) and three sufficient statistics that characterize the joint distribution of productivity and output to capital ratios. The first statistic is the effect of the policy on the average log output to capital ratio. It characterizes the extent to which the treatment affects the aggregate amount of savings available to firms. The second statistic is the treatment effect on the variance of the log output to capital ratio. It measures how the treatment distorts the allocation of capital across firms. The final statistic is the effect of the policy treatment on the covariance of the log output to capital ratio and log productivity. Intuitively, if the treatment reduces this covariance, it makes productive firms relatively less distorted, which boosts aggregate output.

We first develop this result in a model with a simple market structure and a small-scale randomized experiment. We then consider several extensions to show how to adapt our aggregation formulas to more realistic models. We show that the model's key property – the scale invariance of the distribution of log output to capital ratio – remains valid (1) in the presence of persistent difference in productivity across firms (2) in the presence of heterogeneous treatment effects (e.g., across industries) and (3) under alternative industry structure (e.g., heterogeneous industries, roundabout production structure with an input-output matrix, a nested CES aggregator with heterogeneous markups across industries). Each of these extensions yields tractable aggregation formulas. We show how to adapt our framework when the policy experiment is randomized across a non-representative sample of firms in the economy. Finally, we investigate in the paper's Appendix how to account for labor distortions,[3] technological decreasing returns to scale or experiments that affect firm-level productivity.[4]

In our final extension, we go beyond the Cobb-Douglas case and allow for CES production functions. In this case, the distribution of output to capital ratio is no longer independent of general equilibrium conditions. As a result, the aggregation cannot be based only on sufficient statistics recovered in the experiment, since these statistics are no longer externally valid. We can still, however, aggregate the effect of the policy by combining the estimated statistics with two additional sufficient statistics. These statis-

[3]We consider labor distortions generated by exogenous taxes (e.g. payroll taxes) or variations in minimum wage. In these cases, the augmented aggregation formulas include additional sufficient statistics to account for the interaction between capital and labor distortions.

[4]When the experiment can affect firm-level productivity, our aggregation formula require to estimate how firm-level productivity changes interact with and affect distortions.

tics capture how the effect of the policy on the average capital wedge vary with the equilibrium wage on the labor market. A limitation of our approach, in this case, is that the estimation of these additional sufficient statistics require more data than the experimental outcomes, and may thus prove hard to estimate empirically.

While most of our theoretical results are cast in the context of small, randomized experiments, the methodology can easily be extended to large-scale natural experiments, provided that unbiased treatment effects of the policy can be recovered in the data. We show how to adapt our methodology in this context by discussing two particular examples. Ponticelli and Alencar (2016) study the effect of a bankruptcy reform in Brazil on firm-level investment and employment. Identification is obtained by exploiting a measure of city-level bankruptcy court congestion, which is assumed to be exogenous to the unobserved heterogeneity in investment and employment. With this identifying assumption, our methodology can be directly applied to recover the contribution of this reform to changes in aggregate TFP and output in Brazil. Giroud and Rauh (2016) estimate the effect of the corporate income tax on firm-level investment and employment. Identification is obtained by comparing the response to state-level changes in corporate income tax rates for S-corporations and C-corporations. The identifying assumption is that how investment opportunities respond to local changes in taxes is similar for S and C-corps. With this identifying assumption, we show in Section 6how the reduced-form evidence in Giroud and Rauh (2016) can be combined with our methodology to recover, for example, the effect on aggregate TFP and output of a decrease in the corporate income tax rate of 10%.

Our paper first contributes to the growing literature that empirically analyzes firm-level distortions using well-identified settings. Many of the papers cited above estimate the *firm-level* effect of policies promoting business investment but do not speak to how these policies would affect macroeconomic outcomes, were they to be extended to all firms in the economy. Our paper provides a simple framework to answer this question using similar identification strategies but focusing on a set of sufficient statistics typically not estimated in these studies (mean log output to capital ratio, its variance, and its covariance with log productivity). Recent exceptions are Blattner et al. (2017), Rotemberg (2017) and Larrain and Stumpner (2017), who consider an aggregation framework somewhat similar to ours, but in which the distribution of output to capital ratios are assumed to be exogenous, and in particular independent of aggregate conditions. One of our contributions is to provide sufficient conditions under which endogenous capital wedges are in fact independent of the market equilibrium in the steady-state.

Our paper is also broadly related to the literature on misallocation. In a seminal paper building on Restuccia and Rogerson (2008), Hsieh and Klenow (2009) show how to compute TFP losses due to misallocation of inputs using a simple sufficient statistics approach. Hsieh and Klenow (2009), however, abstracts from the origin of distortions and treat distortions (wedges) as primitives in the model. Asker et al. (2014) argue that physical adjustment costs account for an important part of the estimated misallocation in the Hsieh and Klenow (2009) framework. Buera et al. (2011) use a calibrated two sector model with occupation choice and borrowing frictions to show that financial development is responsible for large variations in aggregate TFP across countries, especially in the manufacturing sector. Midrigan and Xu (2014) calibrates a general equilibrium model of firm dynamics with collateral constraints using Korean data and show instead that financial frictions imply fairly small losses from misallocation in their sample.[5] More recently, Edmond et al. (2018) study the welfare costs of markups in a dynamic model with heterogeneous firms and endogenously variable markups and find that one third of the welfare losses due to markups can be attributed to input misallocation. This literature, however, is mostly silent on how policies can improve (or are responsible for) misallocation: it usually simply quantifies the loss in aggregate TFP in the actual economy relative to a frictionless benchmark, which may not be policy relevant.[6] In contrast, our paper offers a methodology to quantify how much improvement in allocative efficiency and production can be obtained through *actual* policies. In that sense, it provides a natural bridge from the applied micro-economics literature, which studies the effect of policies on firm-level outcomes, to this misallocation literature.[7]

Finally, our paper is related to the literature in macroeconomics trying to characterize macroeconomic outcomes using microeconomic data. Hulten (1978) shows that in efficient economies, to a first-order, variations in aggregate TFP are simply equal to the sales-weighted average of firm-level TFP shocks. Baqaee and Farhi (2017a) extend Hulten's theorem to the second-order. More closely related is recent work by Baqaee and Farhi (2017b) who derive general non-parametric formula for aggregating microeconomic shocks in general equilibrium economies with distortions. Their approach diverges from ours in that they use a very general aggregation framework, but, in the

[5]See also Catherine et al. (2018) who uses variations in collateral values generated by shocks to real estate prices to identify the effect of collateral constraints on aggregate TFP and output in the US.

[6]The sufficient statistics approach of Hsieh and Klenow (2009) allows for a comparison of aggregate TFP in the Indian or Chinese economy relative to the US. But there again, it is mostly silent on what policies would be required to allow the Indian or Chinese economy to reach the US counterfactual.

[7]In this dimension, the paper is also related to the large literature on sufficient statistics in economics. A recent contribution looking in a corporate finance context is Davila (2016) who derives the optimal bankruptcy exemption as a function of measurable sufficient statistics.

spirit of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) assume distortions can be represented by exogenous wedges. In contrast, our approach consider *endogenous* wedges and show that, in a large class of models used in firm dynamics and macro-finance, these wedges are invariant to general equilibrium. We think this scale-invariance of the wedge distribution is an important consideration for this literature on aggregation. Finally, our paper is also closely related to Buera et al. (2012), who also use recent empirical evaluations of small scale micro-finance programs to discipline a structural model of entrepreneurship and financial frictions. Like us, Buera et al. (2012) are interested in the scaling up of firm-level interventions and their general equilibrium effects, but they rely on a structural calibrated model, while we propose a sufficient statistics approach to the aggregation exercise.[8]

The paper's structure is as follows. Section 2 lays out the economic model. Section 3 develops our methodology. Section 4 shows that the assumptions of Section 2, which are necessary to this result, are consistent with most of the literature on firm dynamics. Section 5 investigates the robustness of our formulas to various extensions to the basic set-up. The last Section concludes.

2 The Economic Model

2.1 Set-up

The economy is dynamic ($t = 0, 1, \ldots, \infty$). There is no aggregate uncertainty. The economy is at the steady state. We first consider a simple market structure with no industries. We extend the analysis to include heterogeneous industries in Section 5. At each date t, a continuum of monopolists, indexed by $i \in [0;1]$, produce imperfectly substitutable intermediate goods in quantity y_{it} at a price p_{it} (Dixit and Stiglitz (1977)). There is a perfectly competitive final good market, which aggregates intermediate inputs according to a CES technology:

$$Y = \left(\int_i y_{it}^{\theta} di \right)^{\frac{1}{\theta}}, \tag{1}$$

where we omit the t subscript for aggregate output Y as the economy is at the steady state. We use the final good as the numeraire. Profit maximization in the final good

[8] Buera et al. (2011) include an occupational choice problem, so that the size of the productive sector is endogenous. In our baseline model, in contrast, there is no entry decision. We consider in the Appendix a simple example with entry.

market implies that the demand for product i is given by: $p_{it} = \left(\frac{Y}{y_{it}}\right)^{1-\theta}$ and $-\frac{1}{1-\theta}$ is the price elasticity of demand.

To produce, firms combine labor and capital according to a Cobb-Douglas production function: $y_{it} = e^{z_{it}} k_{it}^{\alpha} l_{it}^{1-\alpha}$, where k_{it} is firm i's capital stock in period t, l_{it} labor input, α is the capital share. Log-productivity $z_{it} \in Z$ follows a Markovian process and is i.i.d across firms. In Section 5.1, we extend the model and allow log-productivity to have to have a firm-specific fixed component.

With monopolistic competition and the demand system in Equation (1), firm i revenue in period t is $p_{it} y_{it} = Y^{1-\theta} y_{it}^{\theta}$. We assume that there is no adjustment cost to labor. Thus, labor is a static input. Let w be the steady state wage. Static labor optimization implies that firm i's profit becomes:

$$\pi(z_{it}, k_{it}; w, Y) = \max_{l} \left(Y^{1-\theta} e^{z_{it}} k_{it}^{\theta\alpha} l^{\theta(1-\alpha)} - wl \right) = \Omega \left(\frac{Y^{1-\phi}}{w^{\phi \frac{1-\alpha}{\alpha}}} \right) e^{\frac{\phi}{\alpha} z_{it}} k_{it}^{\phi},$$

where $\phi = \frac{\alpha\theta}{1-(1-\alpha)\theta} < 1$, and Ω is a constant.

We call $\boldsymbol{\Theta}$ the vector of deep structural parameters governing the (real and financial) frictions on capital. For the sake of clarity, we assume in this section that all firms in the economy share the same parameters Θ. We relax this assumption in Section 5 to explore how firm heterogeneity affects our aggregation formulas.

The capital good is the final good, so that its price is also 1. Using the capital good in production leads to physical depreciation at a rate δ. Capital investment in period t is subject to a one period time-to-build. Firms can finance investment using the profits they realize from operations or through external financing. The first source of outside financing is one-period debt. b_{it+1} is the total real payment due to creditors in period $t+1$. To simplify notations, we define $\boldsymbol{x}_{it} = (k_{it}, k_{it+1}, b_{it}, b_{it+1})$. We note r_{it} the interest rate charged by lenders, so that $\frac{b_{it+1}}{1+r_{it}}$ is the proceed from debt financing received in period t. We allow the firm's investment and debt financing at date t to be subject to adjustment costs $\Gamma(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta}, w, Y)$. We also assume that firms pay taxes and receive subsidies: $\mathcal{T}(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta}, w, Y)$ corresponds to the net tax paid by the firm.

Finally, we allow for generic forms of financing frictions. First, equity issuance may be costly, and we note such costs $\mathcal{C}(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta}, w, Y)$. These costs are obviously zero when the firm does not issue equity– i.e. when cash-flows are positive. Second, the amount of outside financing may be constrained, which we capture through a vector of constraint: $\boldsymbol{M}(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta}, w, Y) \leq 0$. Third, the interest rate on debt is described by a function $r()$ such that r_{it}=$r(z_{it}, \boldsymbol{x_{it}}; \boldsymbol{\Theta}, w, Y)$. This function allows for risky debt and may embed costs

of financial distress, such as liquidation costs.

We note e_{it} the cash-flows to equity holders, net of equity issuance costs:

$$\begin{aligned} e_{it} &= \pi(z_{it}, k_{it}; w, Y) - (k_{it+1} - (1-\delta)k_{it}) - \Gamma\left(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta}, w, Y\right) \\ &+ \left(\frac{b_{it+1}}{1 + r(z_{it}, \boldsymbol{x_{it}}; \boldsymbol{\Theta}, w, Y)} - b_{it}\right) - \mathcal{T}(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta}, w, Y) \\ &- \mathcal{C}(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta}, w, Y) \\ &= e(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta}, w, Y) \end{aligned}$$

The timing is standard in models of firm dynamics. At the beginning of period t, productivity z_{it} is realized. The firm then combines capital in place k_{it} with labor l_{it} to produce and receive the corresponding profits. It then selects the next period stock of capital k_{it+1}, pays the corresponding adjustment costs, reimburses its existing debt b_{it} and receives the proceeds from debt issuance $\frac{b_{it+1}}{1+r_{it}}$. Then, the period ends. We allow for non-strategic default: the firm operates in period $t+1$ if and only if its productivity belongs to a "survival set": $z_{it+1} \in \mathcal{Z}(k_{it+1}, b_{it+1}; \boldsymbol{\Theta}, w, Y)$. When $z_{it+1} \notin \mathcal{Z}(k_{it+1}, b_{it+1}; \boldsymbol{\Theta}, w, Y)$, default occurs and the continuation value to the firm's owner is assumed to be zero.[9]

To save on notations, let us temporarily omit the it index. We denote with prime next-period variables. The firm dynamic optimization problem has one exogenous state variable z and two endogenous state variables, k and b. To ensure that policy functions are defined on a compact set, we make the following assumptions:

Assumption 1 (Technical assumptions)**.** *Let Y be the aggregate demand shifter and w the wage. Then, state variables (z, k, b) satisfy the following assumptions:*

1. *The cash-flow function $e(.; \boldsymbol{\Theta}, w, Y)$ is a piecewise continuous function of (z, k, k', b, b')*
2. *Log-productivity $z \in Z$, where Z is compact and convex. The Markovian transition function underlying z is strictly positive, has no atom and satisfies the Feller property.*
3. *Capital takes values in the set K which is assumed to be convex and compact.*
4. *Debt values b are restricted to a compact and convex set B.*

[9] This stark assumption is for the sake of exposition. Our results would carry through for a larger class of default continuation values, as long as they satisfy an homogeneity property similar to the assumptions of proposition 2.

5. *Conditionally on past* (k, b) *and current capital choice* k'*, debt values* b' *are restricted to a correspondence set* $\mathcal{B}$*:*

$$\mathcal{B}(z, k, k', b; \boldsymbol{\Theta}, w, Y) = B \cap \{b' | \boldsymbol{M}(z, k, k', b, b'; \boldsymbol{\Theta}, w, Y) \leq 0\}$$

We assume that the financial constraint $\boldsymbol{M}$ *is such that* $\mathcal{B}$ *is compact, convex and non-empty.*

To ease notations, we write $\mathcal{B}(z, k, k', b; \Theta, w, Y) \equiv \mathcal{B}(z, k, k', b)$*.*

The first item only assumes piecewise continuity because models may include (1) fixed costs of equity issuance (e.g. Hennessy and Whited (2007)) or (2) fixed capital adjustment costs (e.g. Caballero and Leahy (1996)). Most papers typically prove the third item of the above assumption by picking a maximum capital stock that the firm will never find optimal to choose. Decreasing returns to scale ensure that all levels of capital above this maximum cannot be optimal either (e.g. Hennessy and Whited (2007)). The fourth item is typically obtained in corporate finance models through assuming that the return on cash is low compared to the shareholder's discount rate, which bounds debt b from below. b is bounded above through financing frictions (cost of financial distress, collateral or cash-flow constraints).

Define $\mathcal{F}(\Theta, w, Y)$ the set of continuous and bounded functions $f(z, k, b; \Theta, w, Y) : Z \times K \times B \to \mathbb{R}$. Following the rest of the literature, we define the Bellman operator T on the functional space $\mathcal{F}(\Theta, w, Y)$:

$$Tf(z, k, b; \boldsymbol{\Theta}, w, Y) = \left| \begin{array}{l} \max\limits_{(k', b') \in K \times B} e(z, k, k', b, b'; \boldsymbol{\Theta}, w, Y) + \beta \mathbb{E}_{z' \in \mathcal{Z}(k', b'; \boldsymbol{\Theta}, w, Y)}[f(z', k', b'; \boldsymbol{\Theta}, w, Y) | z], \\ \boldsymbol{M}(z, k, k', b, b'; \boldsymbol{\Theta}, w, Y) \leq 0 \end{array} \right. \tag{2}$$

where $\beta < 1$ is the firm's discount rate.

The firm is assumed to maximize the expected present value of equity cash-flows, so that the equity value function is the fixed point of the Bellman operator T. The maximands are the optimal debt and capital policy functions. Under assumptions 1, the contraction mapping theorem holds and the operator T has a unique fixed point in the space of continuous and bounded functions $\mathcal{F}(\Theta, w, Y)$, which is the market value of equity. This unique solution defines unique policy functions so that the problem is well-defined. [10]

[10] These results are broadly used in the literature. They arise from the extension of the results in Stokey

The household side of the economy is stripped down to its essentials. A representative household has GHH preferences (Greenwood et al. (1988)) over consumption and leisure: $u(c_t, l_t) = \frac{1}{1-\gamma}\left(c_t - \frac{l_t^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}}\frac{\bar{w}}{\bar{L}^{\frac{1}{\epsilon}}}\right)^{1-\gamma}$, where c_t is period t consumption, l_t is period t labor supply, ϵ is the Frisch elasticity and $(\bar{w}, \bar{L})$ are normalizing constants. The representative household owns all the firms in the economy, as well as a safe asset that offers real return r. β^H is the representative household's discount rate. In the absence of aggregate uncertainty, at the steady-state, optimal consumption and labor supply decisions imply that $L_t^s = \bar{L}\left(\frac{w}{\bar{w}}\right)^{\epsilon}$ and $\beta^H = \frac{1}{1+r}$, where r is the risk-free rate. Note that since households portfolios are well diversified across firms, we also have $\beta = \frac{1}{1+r}$ even though the model potentially allows for firms' default.

2.2 Introducing Capital Wedges

Instead of solving the model explicitly, we characterize its equilibrium as a function of the distribution of arbitrary objects: capital wedges τ_{it}, which vary over time and across firms. These wedges are defined as the ratio of a firm's marginal revenue product of capital to the frictionless user cost of capital R for firm i in period t. R is fixed throughout the analysis.

Definition 1 (Definition of capital wedges)**.** *For each firm-year observation, we define the capital wedge τ_{it} as:*

$$1 + \tau_{it} = \frac{\alpha\theta}{R}\frac{p_{it}y_{it}}{k_{it}}$$

Capital wedges are *not* welfare-based notion but pure empirical constructs. The capital wedge captures how much the capital stock of a firm deviates from static, frictionless optimization. In our model, firms deviate from the frictionless, static optimum for three reasons: financing frictions, taxes and adjustment costs. Thus, wedges do not generically have a direct welfare interpretation. For instance, adjustment costs or time-to-build in capital can drive a wedge between marginal productivity and cost of capital, even in an efficient economy. This is the case when adjustment costs are simple technological constraints on production. Similarly, financing frictions do not generically have a

and Lucas (1989) from strictly continuous to piecewise continuous cash-flow functions. See Caballero and Leahy (1996) for a version of proof with fixed adjustment costs and Hennessy and Whited (2007) for a version of the proof with fixed equity issuance costs. In both cases, the logic is the same and applies whenever the cash-flow is piecewise continuous.

welfare interpretation. Financing constraints may arise in order to overcome information asymmetries in financing. As a result, capital wedges due to financing constraints may still be consistent with constrained efficiency. A similar argument hold for taxes: non-distortionary taxation may fail to exist, e.g. because of information asymmetries.

Even though they are not welfare-related, wedges are easy to measure: they are proportional to the ratio of output to capital. Both output and capital can be measured in standard firm-level datasets using financial statements. Of course, wedges are *not* exogenous. They are a function of the firm's state variables ($z_,$, k and b), the aggregate state of the economy (summarized by w and Y) and the vector of parameters Θ. In the next section, we show how to map the economy's aggregate output and TFP into the joint distribution of capital wedges and productivity.

2.3 Equilibrium

We focus on two aggregate measures relevant to the literature: total output Y, and aggregate Total Factor Productivity (TFP) defined as $\frac{Y}{K^\alpha L^{1-\alpha}}$, where K and L are the aggregate capital and labor stocks. This TFP definition follows the recent misallocation literature (e.g. Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Moll (2014)) and has an intuitive interpretation (Oberfield (2013)):[11] The maximum output produced in the frictionless economy with aggregate stock of capital K and labor L is: $Y^*(K,L) = \left(\int_i e^{\frac{\theta}{1-\theta}z_i}di\right)^{\frac{1-\theta}{\theta}} K^\alpha L^{1-\alpha}$. As a result, this definition of aggregate TFP leads to a simple decomposition into technology and misallocation:

$$\log(TFP) = \log(Y) - \alpha\log(K) - (1-\alpha)\log(L) = \underbrace{\left(\int_i e^{\frac{\theta}{1-\theta}z_i}di\right)^{\frac{1-\theta}{\theta}}}_{\text{technology}} + \underbrace{\log(Y) - \log(Y^*(K,L))}_{\text{misallocation}}$$

While the measures of productive efficiency we focus on are standard, an important drawback is that they are quite different from welfare. Welfare in our model is given by the utility of the representative agent and is proportional to $\left(C - \frac{L^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}}\frac{\bar{w}}{L^{\frac{1}{\epsilon}}}\right)^{1-\gamma}$ where

[11] This TFP measure is identical to the one defined in Baqaee and Farhi (2017b), whereby factor weights are given by the distortion-weighted cost shares. The aggregate distortion-weighted capital cost share is given by:

$$\widetilde{RK} = \int R(1+\tau_i)k_i di = \int \theta\alpha p_i y_i di = \theta\alpha Y$$

Labor faces no distortion in this baseline model, so that aggregate labor costs are given by: $wL = \int wl_i di = \theta(1-\alpha)Y$. Hence, the distortion-weighted capital share is indeed α, while the distortion-weighted labor share is just $1-\alpha$.

C is steady state consumption and L is aggregate employment. Welfare thus differs from output along several dimensions: it does not account for dis-utility of labor, for investment or for friction-related costs.[12]

We show in Appendix A.1 that equilibrium on the labor and product market of this economy implies the following expression for aggregate output and TFP (e.g. Hsieh and Klenow (2009), Midrigan and Xu (2014)):

$$Y \propto \left(\int_i \left(\frac{e^{z_{it}}}{(1+\tau_{it})^{\alpha}} \right)^{\frac{\theta}{1-\theta}} di \right)^{(1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta}} \tag{3}$$

$$TFP = \frac{Y}{K^{\alpha} L^{1-\alpha}} \propto \frac{\left(\int_i \left(\frac{e^{z_{it}}}{(1+\tau_{it})^{\alpha}} \right)^{\frac{\theta}{1-\theta}} di \right)^{\frac{1-(1-\alpha)\theta}{\theta}}}{\left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1+\tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} di \right)^{\alpha}} \tag{4}$$

where we omit subscripts t for aggregate variables since the economy is at the steady-state.[13] Importantly, note that equations 3 and 4 do *not* solve the model. They simply relate the joint distribution of $(z, \log(1+\tau))$ in an economy with Cobb-Douglas production and CES aggregation through product and labor market equilibrium. In particular, these equations do not tell us anything about the ergodic distribution of τ, which depends on the numerous frictions facing firms in the firm dynamics model. Solving for the general equilibrium of this economy is in general not feasible analytically and requires numerical simulations. However, our methodology does not require solving the model. Instead, we rely on the fact that the distribution of capital wedges is *observed* in the data. As a result, under assumptions that we detail below, an empiricist can use changes in this distribution of capital wedges to infer changes in the macroeconomic outcomes.

2.4 Small Perturbation Approximation

In what follows, we focus on the following case:

Assumption 2. *We assume that* $z_{it} \ll 1$ *and* $\log(1+\tau_{it}) \ll 1$. *All analytical results in the paper rely on second-order Taylor expansions of* z *and* $\log(1+\tau)$ *around their (small) mean.*

[12] If we are willing to assume that physical adjustment costs and other frictional costs can be rebated lump sum to the representative household, we can derive formulas for welfare. These formulas mimic our TFP and output formulas and are based on the similar sufficient statistics. We did not include these formulas for the sake of brevity, but they are available from the authors upon request.

[13] Similarly, the ergodic joint distribution of z and τ does not depend on t,

In this multiplicative set-up, Assumption 2 is equivalent to assuming that $\log(1+\tau_{it})$ and z_{it} are jointly normally distributed, which is the assumption made for instance in Hsieh and Klenow (2009). Since $\log\left(\frac{p_{it}y_{it}}{k_{it}}\right) = \log(1+\tau_{it}) + \text{cst}$, Assumption 2 implies that the log output to capital ratio is also normally distributed. We test the relevance of this assumption using data from BvD AMADEUS Financials for the year 2014. As in Gopinath et al. (2015), we measure $p_i y_{it}$ as the value added of the firm, i.e. the difference between gross output (operating revenue) and materials. We measure the capital stock, k_{it}, with the book value of fixed tangible and intangible. For 6 countries in our sample (France, Spain, Italy, Portugal, Romania and Sweden), we report in Figure 1 normal probability plots, i.e. plots of the empirical c.d.f. of the standardized log output to capital ratios against the c.d.f. of a normal distribution. Figure 1 shows that the log-normality assumption is reasonable.

Note that neither of these assumptions (small deviations or log normality) is necessary. We provide in Appendix B.3 formulas that do not require any of these assumptions. However, assumption 2 proves useful to clarify the logic of our approach, as it summarizes the distribution of wedges in a handful of moments.

The second-order Taylor expansion of equations (3-4) in $\log(1+\tau)_{it}$ and z_{it} around their means leads to simple formulas:

Proposition 1. *Under Assumption 2, at equilibrium, aggregate output and TFP can be written as simple functions of three moments of the joint distribution of log wedges* $\log(1+\tau_{it})$ *and log productivity* z_{it}*:*

$$\log Y = \frac{\alpha(1+\epsilon)}{1-\alpha}\left(-\mu_\tau(\Theta,w,Y) + \frac{1}{2}\frac{\theta}{1-\theta}\left(\alpha\sigma_\tau^2(\Theta,w,Y) - 2\sigma_{\tau z}(\Theta,w,Y)\right)\right) + cst \quad (5)$$

$$\log(TFP) = -\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\sigma_\tau^2(\Theta,w,Y) \quad (6)$$

where $\mu_\tau(\Theta,w,Y)$ *and* $\sigma_\tau^2(\Theta,w,Y)$ *are the mean and variance of the steady-state distribution of log capital wedges.* $\sigma_{\tau z}(\Theta,w,Y)$ *is the covariance between log productivity and log capital wedges at the steady-state. All three moments are generically functions of deep structural parameters* Θ*, aggregate output* Y *and the market clearing wage* w*.*

Proof. See Appendix A.2. □

These formulas illustrate forces already discussed in the literature. Dispersion of wedges impairs aggregate efficiency because it creates capital misallocation (Hsieh and Klenow (2009)). A positive correlation between productivity and wedges also hurts aggregate production: output is lower when the most productive firms experience the

largest distortions (Hopenhayn (2014)). However, in our setting, such a correlation does not affect aggregate TFP. This result emanates from the small deviation assumption (or alternatively, from log normality). For instance, it does not hold in Restuccia and Rogerson (2008), who use a binary distribution for the distribution of distortions.

The above formulas suggest a simple methodology to aggregate firm-level evidence:

1. Measure the treatment effect of a policy experiment on the three moments introduced in Proposition 1 (mean and variance of log wedges, and covariance of log wedges with log productivity). These three moments are easy to compute using firm-level data since log wedges are equal to log revenue-to-capital ratios up to a constant.

2. Plug these treatment effects into formulas (5-6). This would lead to the aggregate effect (in terms of log-changes in aggregate output and TFP) of generalizing the experiment to all firms in the economy. This approach is originally the one of Hsieh and Klenow (2009) who use the variance of log revenue to capital ratios of the US to investigate TFP losses among Indian firms due to misallocation. It has since been taken on in a few recent papers based on quasi-experimental frameworks (Blattner et al. (2017), Larrain and Stumpner (2017), Rotemberg (2017)).

However, this methodology is not necessarily valid. The effect of the policy on the joint distribution of $(z_{it}, \log(1+\tau_{it}))$ in step 1 is estimated in an economy with equilibrium (w, Y). As the policy is extended to all firms in the economy, the economy converge toward a new steady-state (w',Y'). At this new steady-state, the equilibrium joint distribution of $(z_{it}, \log(1+\tau_{it}))$ may differ from the one estimated in the experimental data in the (w,Y) equilibrium. The next section explains this issue in detail and provides a set of sufficient conditions under which the above methodology is valid.

3 Inference and Aggregation of Policy Experiments

In this section, we explain when it is valid to use experimental estimates to compute our aggregate counterfactual. To simplify the exposition, we consider the case of a binary randomized treatment. Our approach can be generalized to continuous treatments and heterogeneous treatment effects (see Section 5).

3.1 The Policy Experiment

An econometrician observes data on an infinite number of firms, of which a random subset is subject to a policy treatment. The treatment is binary: firm i is either treated ($T_i = 1$) or untreated ($T_i = 0$). This treatment is a policy that affects the parameters Θ governing financing constraints, adjustment costs or taxes. Θ_0 (resp. Θ_1) correspond to the parameters of non-treated (resp. treated) firms. The econometrician does not necessarily know how the treatment affects these parameters. However, we assume that she knows that the treatment leaves the following three parameters unchanged: the capital share in production α, the price elasticity of demand θ, and the labor supply elasticity ϵ.

We assume that the econometrician observes the ergodic distribution of firm-level outcomes for both treated and control firms. In the context of our simple model, this implies that the treatment is necessary exogenous: since heterogeneity in our model arises solely through idiosyncratic productivity shocks, the ergodic distribution of outcomes for treated and control firms is independent of initial conditions. Our results extend directly in the presence of persistent productivity shocks (Section 5.1).

3.2 Aggregating the Policy

In this section, we show how to use the experimental data to compute changes in aggregate output and TFP that would result from extending the policy to *all* firms in the economy. An alternative version of this aggregation exercise consists of extending the policy to a larger fraction of firms in the economy. We focus on full aggregation here, and explore partial aggregation in Section 5. We summarize the total aggregation exercise in the paragraph below.

Objective 1 (Aggregation of the Policy)**.** *The aggregation of the treatment consists of computing the log-difference in output and TFP between an economy where* no firms *are treated ($\Theta = \Theta_0$ for all firms) to an economy where* all firms *are treated ($\Theta = \Theta_1$).*

Note (w_0, Y_0, TFP_0) (resp. (w_1, Y_1, TFP_1)) the equilibrium wage, output and TFP in the economy where no firms (resp. all firms) are treated. Then:

$$\Delta \log Y = \log(Y_1) - \log(Y_0) = \frac{\alpha(1+\epsilon)}{1-\alpha}\left(-\Delta\mu_\tau + \frac{1}{2}\frac{\theta}{1-\theta}\left(\alpha\Delta\sigma_\tau^2 - 2\Delta\sigma_{\tau z}\right)\right)$$

$$\Delta \log(TFP) = \log(TFP_1) - \log(TFP_0) = -\frac{\alpha}{2}\left(1 + \frac{\alpha\theta}{1-\theta}\right)\Delta\sigma_\tau^2$$

where:

$$\begin{aligned}\Delta\mu_\tau &= \mu_\tau(\Theta_1, w_1, Y_1) - \mu_\tau(\Theta_0, w_0, Y_0)\\ \Delta\sigma_\tau^2 &= \sigma_\tau^2(\Theta_1, w_1, Y_1) - \sigma_\tau^2(\Theta_0, w_0, Y_0)\\ \Delta\sigma_{z\tau} &= \sigma_{z\tau}(\Theta_1, w_1, Y_1) - \sigma_{z\tau}(\Theta_0, w_0, Y_0)\end{aligned}$$

The equations above make clear the challenge faced by the econometrician. The econometrician does not directly observe the distribution of wedges and productivities *conditional* on the new equilibrium (w_1, Y_1): by definition, this new equilibrium is not observed and is the counterfactual equilibrium we are trying to calculate. Therefore, empirically, the econometrician cannot directly estimate $\Delta\mu_\tau$ using the available evidence, but instead can only estimate the following statistic $\widehat{\Delta\mu_\tau}$, which is a priori not equal to $\Delta\mu_\tau$:

$$\begin{aligned}\widehat{\Delta\mu_\tau} &= \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 1\right) - \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 0\right)\\ &= \mu_\tau(\Theta_1, w_0, Y_0) - \mu_\tau(\Theta_0, w_0, Y_0)\\ &\neq \Delta\mu_\tau = \mu_\tau(\Theta_1, \boldsymbol{w_1}, \boldsymbol{Y_1}) - \mu_\tau(\Theta_0, w_0, Y_0) \quad \text{a priori}\end{aligned}$$

Obviously, a similar issue arises for the other two moments of the joint distribution of $(z, \log(1+\tau))$, $\Delta\sigma_\tau^2$ and $\Delta\sigma_{z\tau}$. The next section presents sufficient conditions under which these statistics are in fact similar.

3.3 Scale Invariance of the Wedge Distribution

This section presents one of our main results: we provide sufficient conditions under which the joint distribution of log-capital wedges and log-productivity is independent of w and Y. As we detail below, these conditions are verified in a large class of models of firm dynamics, commonly used in macro-finance.

Proposition 2 (Distribution of wedges)**.**
Let $S = \frac{Y}{w^{\frac{(1-\alpha)\phi}{(1-\phi)\alpha}}}$ *be the steady state "scale" of the economy. Assume that:*

1. *adjustment costs* $\Gamma()$*, taxes* $\mathcal{T}()$*, funding constraints* $\boldsymbol{M}()$ *and the equity issuance cost function* $\mathcal{C}()$ all *satisfy the following property:*

$$\forall\ (z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y), \quad Q(z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y) = S \times Q\left(z, \frac{\boldsymbol{x}}{S}; \boldsymbol{\Theta}, 1, 1\right) \tag{7}$$

2. *the interest rate $r()$ satisfies the following property:*

$$\forall \ (z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y), \quad r(z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y) = r\left(z, \frac{\boldsymbol{x}}{S}; \boldsymbol{\Theta}, 1, 1\right)$$

3. *The survival set $Z()$ does not depend on aggregate conditions:*

$$Z(k', b'; \boldsymbol{\Theta}, w, Y) = Z(k', b'; \boldsymbol{\Theta}, 1, 1)$$

Then, the joint-distribution of z and $\log(1+\tau)$, which we note $F(z, \tau; \boldsymbol{\Theta}, w, Y)$, does not depend on (w, Y):

$$F(z, \tau; \boldsymbol{\Theta}, w, Y) \equiv F(z, \tau; \boldsymbol{\Theta})$$

Proof. See Appendix A.3 □

This proposition shows that, given parameters Θ, the ergodic distribution of log-capital wedges does not depend on the scale of the economy. This is the key result of the paper. It implies that under the assumptions highlighted in Proposition 2, the wedge distribution observed for a subset of treated firms operating under parameters Θ_1 *does not depend* on the number of treated firms in the economy. In other words, the estimated treatment effect of the policy on the joint distribution of log-capital wedges and log productivity is externally valid: it does not depend on the economy in which it is estimated. This result allows us to estimate the aggregate counterfactual presented in Objective 1 using the experimental data.

This result rests on two key assumptions. First, firm-level production follows a Cobb-Douglas technology. The multiplicative property of Cobb-Douglas technology ensures that firm-level operating profits scale proportionally to $S = \frac{Y}{w^{\frac{(1-\alpha)\phi}{(1-\phi)\alpha}}}$:

$$\pi(z, k; w, Y) = \max_l \{Y^{1-\theta} k^{\theta\alpha} l^{\theta(1-\alpha)} - wl\} = S\pi\left(z, \frac{k}{S}; 1, 1\right).$$

This property is not satisfied, for instance, when production follows a CES technology (Section 5.6) The second key set of assumptions is the homogeneity of the frictions in equation 7. Taken together, these two assumptions ensure, through the contraction mapping theorem, that a firm's optimal policy function and value in an economy (w, Y) scales up with S. Capital wedges, which are simply ratios of sales to capital, are thus invariant to S since both numerator and denominator scale up to S at the optimum policies. Note that an important assumption for Proposition 2 to hold is that the economy is

at the steady-state.[14]

While the assumptions in Proposition 2 may seem restrictive, they cover a large class of models in the literature on corporate investment and in macro-finance. In Section 4, we map a number of standard models in the literature to the assumptions in Proposition 2. We also perform a systematic literature review and find that the overwhelming majority of papers satisfy these assumptions.

3.4 Taking Stock: Aggregation Formulas

In this section, we summarize our methodology for aggregation. If the assumptions in Proposition 2 are satisfied, the methodology proceeds in two steps. First, we estimate the effect of the policy on the three moments of the joint distribution of log capital wedges and log productivity introduced in Proposition 1. For instance, focusing on the average log wedge, one can estimate:

$$\begin{aligned}
\widehat{\Delta\mu_\tau} &= \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=1\right) - \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=0\right) \\
&= \mathbb{E}\left(\log\left(1+\tau\right)|T_i=1\right) - \mathbb{E}\left(\log\left(1+\tau\right)|T_i=0\right) \\
&= \mu_\tau(\Theta_1, w_0, Y_0) - \mu_\tau(\Theta_0, w_0, Y_0) \\
&= \mu_\tau(\Theta_1, w_1, Y_1) - \mu_\tau(\Theta_0, w_0, Y_0) \\
&= \Delta\mu_\tau
\end{aligned}$$

When the assumptions of Proposition 2 are verified, the distribution of τ is independent of w and Y. This implies that $\mu_\tau(\Theta_1, w_1, Y_1) = \mu_\tau(\Theta_0, w_0, Y_1)$. This, in turn, implies that the treatment effect $\widehat{\Delta\mu_\tau}$ estimated in the experiment does provide us with a relevant statistics that can be used in the aggregation exercise. Similarly, $\Delta\sigma^2_\tau$ and $\Delta\sigma_{\tau z}$ can be estimated through:

$$\begin{aligned}
\widehat{\Delta\sigma^2_\tau} &= \text{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=1\right) - \text{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=0\right) = \Delta\sigma^2_\tau \\
\widehat{\Delta\sigma_{z\tau}} &= \text{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i=1\right) - \text{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i=0\right) = \Delta\sigma_{z\tau}
\end{aligned}$$

[14] We can construct a simple example of a model where, *along the transition path*, the distribution of MRPKs is not the same in an economy where a small number of firms are treated and in an economy where all firms are treated. This model is available from the authors upon request.

In a second step, we simply plug these three estimated statistics ($\widehat{\Delta\mu_\tau}$, $\widehat{\Delta\sigma_\tau^2}$ and $\widehat{\Delta\sigma_{z\tau}}$) into the aggregation formulas (8-9). This requires only a calibration of three parameters: the capital share α, the extent of competition θ and the labor supply elasticity ϵ. By combining these calibrated parameters and the three sufficient statistics, we obtain an estimate of the aggregate counterfactual, which does not require a full structural estimation of the firm-level problem 2, nor a precise mapping from the policy to the structural parameters Θ:

$$\Delta \log Y = \log(Y_1) - \log(Y_0) = \frac{\alpha(1+\epsilon)}{1-\alpha}\left(-\widehat{\Delta\mu_\tau} + \frac{1}{2}\frac{\theta}{1-\theta}\left(\alpha\widehat{\Delta\sigma_\tau^2} - 2\widehat{\Delta\sigma_{\tau z}}\right)\right) \tag{8}$$

$$\Delta \log(TFP) = \log(TFP_1) - \log(TFP_0) = -\frac{\alpha}{2}\left(1 + \frac{\alpha\theta}{1-\theta}\right)\widehat{\Delta\sigma_\tau^2} \tag{9}$$

A standard calibration of these parameters in the aggregation formula (8-9) is $\alpha = .33$ (Bartelsman et al. (2013)), $\theta = .8$ (Broda and Weinstein (2006)) and $\epsilon = .5$ (Chetty (2012)). With this calibration, the aggregation formula imply that a policy that increases investment by about 2% at the firm-level and leaves the variance of wedges as well as the correlation of wedges with productivity unchanged would increase output by .63%.[15] A policy that would leave the average wedge and the correlation of wedge and TFP unchanged, but would reduce the dispersion of wedges by 1.29 percentage points would similarly lead to an increase in aggregate output of .63%.

4 Relation with Standard Models of Firm Dynamics

Proposition 2 builds on a number of assumptions about the firm production function (Cobb-Douglas) and the frictions, real and financial, faced by firms (homogeneity condition). In this section, we first show how standard models of firm dynamics map into the assumptions of Proposition 2. We then conduct an extensive review of the recent literature on firms dynamics. Among the 44 papers we discuss, an overwhelming majority satisfy this set of assumptions.

[15] The output to capital ratio is given by: $\Delta \log \frac{py}{k} = -\frac{1-\theta}{1-\theta(1-\alpha)}\Delta \log k$, so that a 2% increase in investment corresponds to a .85% decline in the sales-to-capital ratio ($\Delta\mu_\tau = -.85\%$), which in turn leads to an increase in aggregate output of $\frac{\alpha(1+\epsilon)}{1-\alpha} \times .85\% = .63\%$.

4.1 Adjustment Costs

Consider first the case of adjustment costs. Quadratic adjustment costs to capital, linear adjustment costs, fixed costs that scale either with production, output and capital or discount for capital resale all satisfy the assumptions in Proposition 2. For instance, if $\Gamma()$ is given by:

$$\Gamma(z, \boldsymbol{x}; \Theta, w, Y) = \gamma_1 \frac{(k' - (1-\delta)k)^2}{k} + \gamma_2 k + \mathbb{1}_{\{k'-(1-\delta)k \neq 0\}} (\gamma_3 y + \gamma_4 py + \gamma_5 k) + \gamma_6 k \mathbb{1}_{\{k'-(1-\delta)k<0\}},$$

then, since $y(z, k; \Theta, w, Y) = S \times y(z, \frac{k}{S}; \Theta, 1, 1)$ and $py(z, k; \Theta, w, Y) = S \times py(z, \frac{k}{S}; \Theta, 1, 1)$, it is trivial to show that $\Gamma(z, \boldsymbol{x}; \Theta, w, Y) = S \times \Gamma(z, \frac{\boldsymbol{x}}{S}; \Theta, 1, 1)$

4.2 Financing Frictions

Second, consider the financing side of the model. Our formulation encompass standard models of financing constraints and investment.

Let us start with the interest rate function. For instance, in Michaels et al. (2016) or Gilchrist et al. (2014), debt is risky and in the event that the firm is unable to repay, the lender can seize a fraction $1 - \zeta$ of the firm's fixed assets k. The firm's future market value is not collateralizable, so that a firm's access to credit is mediated by a net worth covenant, which restrains the firm's ability to sell new debt based on its current physical assets and liabilities. Concretely, default is triggered when net worth reaches 0, which defines a threshold value for productivity $\hat{z}$ such that:

$$0 = \kappa_0 S^{1-\phi} e^{\frac{\phi}{\alpha}\hat{z}} k^{\phi} - b + c^k (1-\delta)k, \tag{10}$$

where c^k is the second-hand price of capital, which we treat as a technological parameter. As in Michaels et al. (2016), the right side of the previous equation represents the resources that the firm could raise in order to repay its debt just prior to bankruptcy, which is why its capital is valued at the second-hand price c^k. The wage bill is absent from the previous equation because labor is paid in full, even if the firm subsequently defaults. Finally, the face value of debt discounted at the interest rate $r(z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y)$ must equal the debt holder's expected payoff discounted at the risk-free rate:

$$\frac{1}{1+r}\left[\int_0^{\hat{z}}\left(\kappa_0 S^{1-\phi}e^{\frac{\phi}{\alpha}z'}k'^{\phi}+(1-\zeta)(1-\delta)k'\right)dH(z'|z)+(1-H(\hat{z}|z))b'\right]=\frac{b'}{1+r(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y)} \tag{11}$$

Equations (10) and (11) provides the joint definition for the interest rate function $r(z,\boldsymbol{x};\Theta,w,Y)$, which satisfies the assumption in Proposition 2. Note first that equation (10) can be rewritten as: $0=\kappa_0 e^{\frac{\phi}{\alpha}\hat{z}}\frac{k}{S}^{\phi}-\frac{b}{S}+c^k(1-\delta)\frac{k}{S}$. As a result, it is clear that $\hat{z}(k,b;\Theta,w,Y)=\hat{z}(\frac{k}{S},\frac{b}{S};\Theta,1,1)$. Also, we can rewrite Equation (11) as:

$$\frac{1}{1+r}\left[\int_0^{z(\frac{k'}{S},\widehat{\frac{b'}{S}};\Theta,1,1)}\left(\kappa_0 e^{\frac{\phi}{\alpha}z'}\frac{k'^{\phi}}{S}+(1-\zeta)(1-\delta)\frac{k'}{S}\right)dH(z'|z)+(1-H(\hat{z}|z))\frac{b'}{S}\right]=\frac{\frac{b'}{S}}{1+r(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y)},$$

so that $r(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y)=r(z,\frac{\boldsymbol{x}}{S};\boldsymbol{\Theta},1,1)$.

Similarly, the specification of debt renegotiation in Hennessy and Whited (2007) would also satisfy these assumptions. More generally, these models make the probability of default independent of the scale of the economy S, and the loss given default proportional to S. These properties ensure our assumption about $r()$ in Proposition 2 is satisfied. Obviously, models of risk-free debt, such as Midrigan and Xu (2014), also satisfy the assumption about $r()$.

Our assumption on the cost of equity is also verified in Michaels et al. (2016) and Gilchrist et al. (2014), who posit that equity issuances are subject to an underwriting fees such that there is a positive marginal cost to issue equity:

$$\mathcal{C}(z,\boldsymbol{x};\Theta,w,Y)=\lambda|e(z,\boldsymbol{x};\Theta,w,Y)|\mathbb{1}_{\{e(z,\boldsymbol{x};\Theta,w,Y)<0\}}$$

Given that $e(z,\boldsymbol{x};\Theta,w,Y)=Se(z,\frac{\boldsymbol{x}}{S};\Theta,1,1)$, it is obvious that $\mathcal{C}(z,\boldsymbol{x};\Theta,w,Y)=S\times\mathcal{C}(z,\frac{\boldsymbol{x}}{S};\Theta,1,1)$. Thus, the financing frictions specified in Gilchrist et al. (2014) and Michaels et al. (2016) satisfy the assumptions of Proposition 2. Additionally, it is obvious to see that fixed or quadratic issuance costs would satisfy our assumptions as long as they are appropriately scaled with the size of the firm. For instance, $\psi\frac{e_{it}^2}{k_{it}}\mathbb{1}_{e_{it}<0}$, or $\iota k_{it}\mathbb{1}_{e_{it}<0}$ would fall in this category.

Finally, our formulation of financing frictions also encompasses debt constraints as for instance in Midrigan and Xu (2014) or Catherine et al. (2018). In Midrigan and Xu (2014), debt is assumed to be risk-free through full collateralization: $b'\leq\xi k'$ so that $r(z,\boldsymbol{x};\Theta,w,Y)=r_f$ and producers can only issue claims to a fraction χ of their future profits: $e(z,\boldsymbol{x};\Theta,w,Y)\geq-\chi V(z,\boldsymbol{x};\Theta,w,Y)$. In this case, the vector $\boldsymbol{M}(z,\boldsymbol{x};\Theta,w,Y)$ con-

sists of the last two inequalities, and it is direct to see that both $\boldsymbol{M}$ and $r()$ satisfy the assumptions of Proposition 2. Of course, any combination of the constraints in Midrigan and Xu (2014) and Hennessy and Whited (2007) would also satisfy these assumptions. Note that our model also encompasses debt constraints where debt financing is limited by existing or future cash flows ($b \leq \iota e(z, \boldsymbol{x}; \Theta, w, Y)$).

4.3 Taxes

Standard specifications for the corporate income tax satisfy the assumption of Proposition 2: $\mathcal{T}(z, \boldsymbol{x}; \Theta, w, Y) = \tau \max(0, \pi(z, \boldsymbol{x}; \Theta, w, Y) - \delta k - b)$. However, a progressive tax system would violate our assumptions.

4.4 Recent literature review

In this section, we conduct a systematic literature review to assess if the typical papers on firm dynamics satisfy the conditions in Proposition 2. Given the large number of papers in this literature, we cannot be exhaustive. We limit ourselves to recent and cited papers in this literature: all papers citing Hennessy and Whited (2007), Midrigan and Xu (2014) and Moll (2014), published within a list of twelve journals[16] and with at least 50 Google scholar citations.

We end up with 44 papers. We group the set of assumptions in five categories: production function, adjustment costs, borrowing constraint, equity issuance costs and taxes and report. For each of the 44 papers and each of these 5 categories, we check whether the paper's assumptions satisfy the conditions of Proposition 2 for this category. Table 1 show that modeling choices made in these papers are almost always consistent with our assumptions in Proposition 2. Column 1 verifies if the production function is Cobb-Douglas or not. In *all* papers but one, the Cobb-Douglas assumption is valid. In 5 papers, the authors however added to Cobb-Douglas a non-scalable fixed cost to model operating leverage. That particular dimension does not fit our assumptions and would prevent the results from proposition 2 from holding. In column 2, we look at adjustment costs. In almost all papers, physical adjustment costs (real frictions) satisfy our assumptions. Even when there are fixed costs of investment, the costs are still typically scaled by total sales, which satisfies our assumptions as described in Section 4.1. Columns 3 and 4 focus on financing frictions. The borrowing constraint is almost

[16] American economic review, Econometrica, Journal of Political Economy, Review of Economic Studies, Quarterly Journal of Economics, Journal of Finance, Journal of Financial Economics, Review of Financial Studies, one of the three American Economic Journal and Journal of Monetary Economics.

scale free in all but 3 papers. Equity issuance costs constitute the most frequent deviation from the assumptions of Proposition 2: 9 papers introduce fixed equity issuance cost that does not scale with the size of the firm. Finally, all but 2 papers introduce a standard corporate tax rates with constant and positive rates for firms making positive profits and no tax for firms making negative profits. Overall, most existing models in the recent literature do satisfy the assumptions of Proposition 2.

5 Robustness and Extensions

This section proposes extensions to the methodology developed in Section 3 and the model presented in Section 2. The first four subsections investigate the effect of including additional heterogeneity in frictions, treatment, production and competition. The last extension analyzes the effect of deviating from Cobb-Douglas production. Additional extensions are described in Appendix B for the sake of brevity.

5.1 Productivity Heterogeneity

In this section, we explore the effect of unobserved heterogeneity on aggregation. To do so, we augment the baseline model with to include a firm-specific fixed component in log-productivity. Note first that, in the baseline model, the experiment is always well-identified: the only source of heterogeneity across firms are idiosyncratic productivity shocks; irrespective of their initial productivity levels, a group of firms that receive the treatment at an initial will converge, at the steady-state, to the ergodic distribution for firms with the treatment. In reality, however, there may be sources of persistent heterogeneity across firms. If the experiment targets a particular group of firms that differs in a persistent way from the rest of the economy, then aggregating the policy experiment to the entire economy may be problematic. In this section, we explore what happens when firms differ through (potentially unobservable) persistent productivity levels (e.g., as in Midrigan and Xu (2014)).

We start from our baseline model in Section 2, and now assume that total log-productivity tfp_{it} is now: $\text{tfp}_{it} = z_{it} + z_i^*$, where z_{it} is the same Markovian process as in the baseline model and z_i^* is fixed in time and firm-specific. With this assumption, it is straightforward to show that, even if the treatment is applied to a non-representative sample of firms in the economy, our baseline formulas continue to apply. This result is summarized in the following proposition:

Proposition 3. *Assume that firm log-productivity, tfp_{it}, is given by: $tfp_{it} = z_{it} + z_i^*$, where z_{it} is the Markovian process described in Section 2 and z_i^* is a firm-specific fixed component. Then, even if when the treatment T_i is correlated with z_i^* – and the experimental data is therefore not representative of the entire economy – the sufficient statistics and aggregation formulas of Section 3 remain valid.*

Proof. See Appendix A.4. □

This result is an implication of the Cobb-Douglas technology and homogeneity condition detailed in Proposition 2. Intuitively, in our setting, at the firm optimum, sales, employment and capital stock are all scaled by long-term productivity $e^{z_i^*}$. Hence, the distribution of capital wedges is independent of the distribution of long-term productivities. As a result, our aggregation formulas apply. In particular, even the policy treatment is applied to a subset of firm with a non-representative distribution of z_i^*, the estimated treatment effect on the joint distribution of log-productivity and log-capital wedges will be unbiased estimates of the treatment effect for all firms in the economy. We can therefore use these estimated treatment effects for aggregation.

5.2 Heterogeneous Treatment Effect & Representativeness

In this section, we allow structural parameters Θ, as well as the treatment itself to differ across firms. This extension allows us to investigate two important cases. First, policy treatment may have different intensity across firms in the economy: for instance, in Ponticelli and Alencar (2016), the intensity of the federal bankruptcy reform studied in the paper depends on bankruptcy courts backlog. Second, and related to the previous section, firms receiving a particular policy treatment may systematically differ from the rest of the economy (here in terms of their deep structural parameters Θ).

To focus on the effect of heterogeneous treatment, we keep in this extension the same simple market structure as in the baseline model of Section 2. The only difference with our baseline model is that firms in the economy are now partitioned in Σ groups: in each group $s \in [1, \Sigma]$, the parameters governing frictions, Θ_s, is constant and equal to $\Theta_{s,0}$, but for a small, random subset of firms for which it is $\Theta_{s,1}$. For instance, these groups can be industries or geographic areas.

We note Σ_T the subset of groups for which we observe a treatment effect, i.e. a random subset of firms in this group receive the policy treatment. In such a setting, it is naturally impossible to generalize the experiment to groups $s' \notin \Sigma_T$, since treatment

effects cannot be estimated for these groups.[17] With our sufficient statistics approach, the only meaningful aggregation exercise in this context consists of extending the policy treatment to all of the firms in some of the groups s that belong to Σ_T. The following proposition show hows to estimate this counterfactual:

Proposition 4. *Let Σ_T be the set of groups where a policy experiment is observed.*

1. *For each $s \in \Sigma_T$, we can estimate the following group-specific statistics:*

$$\widehat{\Delta\mu_\tau}(s) = \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 1, i \in s\right) - \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 0, i \in s\right)$$
$$\widehat{\Delta\sigma^2_\tau}(s) = \text{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 1, i \in s\right) - \text{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 0, i \in s\right)$$
$$\widehat{\Delta\sigma_{z\tau}}(s) = \text{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i = 1, i \in s\right) - \text{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i = 0, i \in s\right)$$

2. *Define Σ_A the "aggregation subset": in our aggregate counterfactual, the policy is extended to all firms in each group $s \in \Sigma_A \subseteq \Sigma_T$.*

3. *The effect on total output of extending the policy treatment to all firms in the set of group $s \in \Sigma_A$ is given by:*

$$\Delta \log(Y) = \frac{(1+\epsilon)\alpha}{1-\alpha}\sum_s \gamma_s\left(-\widehat{\Delta\mu_\tau(s)} + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)\left(-2\widehat{\Delta\sigma_{z\tau}(s)} + \alpha\widehat{\Delta\sigma^2_\tau(s)}\right)\right)$$
$$+ \frac{1}{2}\frac{\alpha}{1-\alpha}\left(\frac{\alpha\theta}{1-\theta}\right)(1+\epsilon)var_{\gamma_s}\left(\widehat{\Delta\mu_\tau(s)}\right)$$

where $\gamma_s = \frac{\int_{i\in s} p_i^0 y_i^0 di}{Y^0}$ is the sales share of group s in the absence of treatment if $s \in S_A$ (and is otherwise, for $s \notin S_A$, $\gamma_s = 0$); and $var_{\gamma_s}\left(\widehat{\Delta\mu_\tau(s)}\right) = \sum_{s\in S_A}\gamma_s\left(\widehat{\Delta\mu_\tau(s)}\right)^2 - \left(\sum_{s\in S_A}\gamma_s\widehat{\Delta\mu_\tau(s)}\right)^2$

[17] In this case, a fully fledged structural estimation would be required to estimate the aggregate counterfactuals where all firms – even the ones in the untreated group – would receive the policy treatment.

The effect on aggregate TFP is given by:

$$\begin{aligned}\Delta \log(TFP) = &-\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\sum_s \kappa_s \widehat{\Delta\sigma_\tau^2(s)} \\ &+\alpha\left(1+\frac{\alpha\theta}{1-\theta}\right)\sum_s(\gamma_s-\kappa_s)\left[-\widehat{\Delta\mu_\tau(s)}+\frac{1}{2}\left(\frac{\theta}{1-\theta}\right)\left(\alpha\widehat{\Delta\sigma_\tau^2(s)}-2\widehat{\Delta\sigma_{z\tau}(s)}\right)\right] \\ &+\frac{\alpha}{2}\left(1+\frac{\theta\alpha}{1-\theta}\right)\left(\frac{\alpha\theta}{1-\theta}var_{\gamma_s}\left(\widehat{\Delta\mu_\tau(s)}\right)-\left(1+\frac{\alpha\theta}{1-\theta}\right)var_{\kappa_s}\left(\widehat{\Delta\mu_\tau(s)}\right)\right)\end{aligned}$$

where $\kappa_s = \frac{\int_{i\in s}k_i^0 di}{K^0}$ *is the pre-treatment capital share of group* s *if* $s \in S_A$ *(otherwise, for* $s \notin S_A$, $\kappa_s = 0$*); and* $var_{\kappa_s}\left(\widehat{\Delta\mu_\tau(s)}\right) = \sum_{s\in S_A}\kappa_s\left(\widehat{\Delta\mu_\tau(s)}\right)^2 - \left(\sum_{s\in S_A}\kappa_s\widehat{\Delta\mu_\tau(s)}\right)^2$

Proof. See Appendix A.5. □

These two formulas have a straightforward interpretation. The output formula contains two terms. The first one is a weighted average of within-group output effect. Weights are given by the group's pre-treatment sales share. Within each group, the effect of extending the treatment to all firms in the group on group-level output resembles the effect on total output in our baseline model where there is just one group of firms. The second term accounts for the fact that the extension of the treatment to a certain number of groups of firms in the economy may reallocate output across groups. This will increase the variance of output across groups, which increases total output in our multiplicative setting.

The TFP formula contains three terms. The first one is a weighted average within-group TFP effect. Weights are given by the group's pre-treatment capital shares. The second and third terms arise from the reallocation of output across groups. When the treatment leads to an increase in output for a group whose capital share is smaller than its output share (i.e. a group with too little capital), allocative efficiency increases: this is the second term in the TFP expression. The fact that the treatment is heterogeneous creates additional misallocation across groups: this is the third term in the TFP expression.

Proposition 4 is useful in situations where a policy experiment is implemented on non-representative sample of firms in the economy. In this case, the only aggregate counterfactual that can be estimated is a generalization of the policy to the same set of firms that are in the experiment. For instance, Zia (2008) identifies the effect of credit subsidy by exploiting variations in eligibility conditions to a Pakistani loan subsidy program made to exporting firms. However, a number of industries are never eligible in the sample. In this context, one cannot use the reduced-form evidence to assess the aggregate effect of extending these subsidies to all firms in the economy. Yet, the formulas in

Proposition 4 can be combined with the reduced-form estimates to compute the aggregate effect of extending loan subsidies to all firms within the set of eligible industries (including non-exporting firms, to the extent that exporting status is solely based on productivity).

5.3 Natural Experiments with Heterogeneous Treatment Intensity

A common empirical setting used in the literature on policy evaluation combines an aggregate policy change with heterogeneity in the exposure of firms to this aggregate change. For instance, Ponticelli and Alencar (2016) exploits the heterogeneity in the congestion of civil courts across Brazilian municipalities to evaluate the effect of a bankruptcy reform. Another example is Vig (2013), who evaluates a securitization reform in India that strengthens creditor right by using heterogeneity in the fraction of tangible assets across firms. In a different setting, Larrain and Stumpner (2017) study the effect of capital account liberalization by comparing its effects across industries with different dependence on external finance (in the spirit of Rajan and Zingales (1998)).

This setting differs from our baseline setting for two reasons: (1) the policy reform is already at scale instead of affecting a 0-measure set of firms (2) identification is obtained by comparing groups of firms with heterogeneous exposure to the policy reform instead of comparing treated and control firms. In this setting, the relevant aggregation exercise computes the contribution of the policy change to changes in aggregate output and TFP.

To illustrate how this type of natural experiments fit into our framework, we consider a simple adaptation of the baseline model. The market structure of the economy corresponds to the simple market structure of the baseline model in Section 2. We consider two steady states of the economy. (w_0, Y_0) corresponds to the pre-reform steady state. (w_1, Y_1) corresponds to the post-reform steady state. As in Section 4, there are J group of firms, which are characterized by their exposure to the reform, v_j, for $j \in [1, J]$. We assume that group 1 has no exposure to the reform ($v_1 = 0$) and that groups are ranked in ascending order of treatment intensity.[18] In the initial steady-state (w_0, Y_0), the structural parameters governing frictions are homogenous and equal to Θ_0. In the post-reform steady state, they are given by $\Theta_1(j) = \Theta_0(j) + v_j.d\Theta^{\text{reform}}$. To make the aggregation exercise non-trivial, we assume that the reform may take place at the same time as an

[18]The assumption that group 1 has no exposure to the reform is important, as it provides us with an actual control group. When this assumption is not verified and all groups have exposure to the aggregate reform, then the average effect of the reform across all groups is not identified in the data, and hence, our aggregation exercise can only be done up to a constant corresponding to the effect of the reform on this least affected group.

unexpected shock to firms' productivity: in the pre-reform period, production is given by $y_{it} = e^{z_{it}} k_{it}^{\alpha} l_{it}^{1-\alpha}$ for firm i in period t; in the post reform period, production is given by $y_{it} = Ze^{z_{it}} k_{it}^{\alpha} l_{it}^{1-\alpha}$. Because of this potential unexpected aggregate shock, the aggregate effect of the reform cannot be evaluated by simply comparing changes in aggregate output or TFP between the two steady states, $\log(Y_1) - \log(Y_0)$ and $\log(TFP_1) - \log(TFP_0)$. Instead, we can adjust our baseline methodology to compute the contribution of the reform to the observed changes in aggregate outcomes:

Proposition 5. *1. For each $j \in [1, J]$, we estimate the following group-specific treatment effects by projecting changes in average outcomes of group j, from the pre-reform steady to the post-reform steady-state, on treatment exposure v_j:*

$$\mathbb{E}\left(\log\left(\frac{p_{ij,1}y_{ij,1}}{k_{ij,1}}\right)|i \in j\right) - \mathbb{E}\left(\log\left(\frac{p_{ij,0}y_{ij,0}}{k_{ij,0}}\right)|i \in j\right) = A_1 v_j + A_2 (v_j)^2 + \epsilon_{ij}^1$$

$$\text{Var}\left(\log\left(\frac{p_{ij,1}y_{ij,1}}{k_{ij,1}}\right)|i \in j\right) - \text{Var}\left(\log\left(\frac{p_{ij,0}y_{ij,0}}{k_{ij,0}}\right)|i \in j\right) = A_3 v_j + A_4 (v_j)^2 + \epsilon_{ij}^2$$

$$\text{Cov}\left(\log\left(\frac{p_{ij,1}y_{ij,1}}{k_{ij,1}}, z_{ij,1}\right)|i \in j\right) - \text{Cov}\left(\log\left(\frac{p_{ij,0}y_{ij,0}}{k_{ij,0}}, z_{ij,0}\right)|i \in j\right) = A_5 v_j + A_6 (v_j)^2 + \epsilon_{ij}^3,$$

where 1 corresponds to the steady-state of the post-reform period and 0 the steady-state of the pre-reform period.

2. This defines the following sufficient statistics:

$$\widehat{\Delta\mu_\tau}(j) = \widehat{A_1} v_j + \widehat{A_2} (v_j)^2$$
$$\widehat{\Delta\sigma_\tau^2}(j) = \widehat{A_3} v_j + \widehat{A_4} (v_j)^2$$
$$\widehat{\Delta\sigma_{z\tau}}(j) = \widehat{A_5} v_j + \widehat{A_6} (v_j)^2$$

3. The effect of the reform on aggregate output is then given by:

$$\Delta \log(Y) = \frac{(1+\epsilon)\alpha}{1-\alpha} \sum_j \gamma_j \left(-\widehat{\Delta\mu_\tau(j)} + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)\left(-2\widehat{\Delta\sigma_{z\tau}(j)} + \alpha\widehat{\Delta\sigma_\tau^2(j)}\right)\right)$$
$$+ \frac{1}{2}\frac{\alpha}{1-\alpha}\left(\frac{\alpha\theta}{1-\theta}\right)(1+\epsilon) var_{\gamma_j}\left(\widehat{\Delta\mu_\tau(j)}\right)$$

where $\gamma_j = \frac{\int_{i\in j} p_{ij,0}y_{ij,0}di}{Y_0}$ is the sales share of group j in the pre-reform steady-state and $var_{\gamma_j}\left(\widehat{\Delta\mu_\tau(j)}\right) = \sum_{j\in[1,J]} \gamma_j \left(\widehat{\Delta\mu_\tau(j)}\right)^2 - \left(\sum_{j\in[1,J]} \gamma_j \widehat{\Delta\mu_\tau(j)}\right)^2$

The effect on aggregate TFP is given by:

$$\begin{aligned}\Delta \log(TFP) = &-\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\sum_j \kappa_j \widehat{\Delta\sigma_\tau^2(j)} \\ &+\alpha\left(1+\frac{\alpha\theta}{1-\theta}\right)\sum_j(\gamma_j-\kappa_j)\left[-\widehat{\Delta\mu_\tau(j)}+\frac{1}{2}\left(\frac{\theta}{1-\theta}\right)\left(\alpha\widehat{\Delta\sigma_\tau^2(j)}-2\widehat{\Delta\sigma_{z\tau}(j)}\right)\right] \\ &+\frac{\alpha}{2}\left(1+\frac{\theta\alpha}{1-\theta}\right)\left(\frac{\alpha\theta}{1-\theta}var_{\gamma_j}\left(\widehat{\Delta\mu_\tau(j)}\right)-\left(1+\frac{\alpha\theta}{1-\theta}\right)var_{\kappa_j}\left(\widehat{\Delta\mu_\tau(j)}\right)\right)\end{aligned}$$

where $\kappa_j = \frac{\int_{i\in j} k_{ij,0}di}{K_0}$ *is capital share of group* j *in the pre-reform steady-state and* $var_{\kappa_j}\left(\widehat{\Delta\mu_\tau(j)}\right) = \sum_{j\in[1,J]}\kappa_j\left(\widehat{\Delta\mu_\tau(j)}\right)^2 - \left(\sum_{j\in[1,J]}\kappa_j\widehat{\Delta\mu_\tau(j)}\right)^2$

Proof. See Appendix A.6. □

5.4 Input-Output Linkages

In this section, we allow for heterogeneous industries and input-output linkages. Industries are heterogeneous in terms of price elasticity and treatment effects, and firms in each industry consume inputs produced by all other (potentially treated) sectors. We first describe how to modify the baseline framework. We then show the new aggregation formulas, which end to be quite similar, in spirit, to the baseline formulas.

There are S industries indexed by s. The final good is produced by combining intermediate goods:

$$Y = \Pi_{s=1}^{S} Y_s^{\phi_s} \text{ and } \sum_{s=1}^{S}\phi_s = 1$$

where ϕ_s is the share of each intermediate *goods* in production, and Y_s is the quantity used for final good production. We choose a Cobb-Douglas aggregator here to avoid the accumulation of terms and maintain focus on the effect of the Input-output structure on our aggregation formulas. We explore a more general nested CES case (without input-output structure) in our next robustness check.

There is now a second layer of intermediate production in the economy. Intermediate *good* (s) is produced by combining the intermediate *inputs* produced by firms in industry s, $q_{i,s}$, using a CES technology:

$$Q_s = \left(\int (q_{is})^{\theta_s} di \right)^{1/\theta_s}$$

where Q_s is the quantity of intermediate *good* s produced. However, only a fraction of this production of intermediate good s is used for producing final good: Y_s. The rest, $M_s = Q_s - Ys$, is used in the production process of intermediate input producers in all sectors $q_{i',s'}$, as described below.

Input-output linkages are modeled the following way. Production of *input* (i, s) combines capital, labor and all intermediate *goods* $s \in [1, S]$:

$$q_{i,s} = e^{z_i} k_i^{\alpha_s} l_i^{\beta_s} \Pi_{u=1}^{S} (m_{i,s,u})^{\gamma_{su}}$$

where we assume for convenience constant returns to scale, i.e. $\alpha_s + \beta_s + \sum_{u=1}^{S} \gamma_{su} = 1$. $m_{i,s,u}$ corresponds to the quantity of intermediate *good* produced by sector u that is used for production by firm i in sector s. $\Gamma = (\gamma_{su})_{(s,u)\in[1,S]^2}$ corresponds to the input-output matrix.

Let $M_u = \sum_{s=1}^{S} \int m_{i,s,u} di$ be the total consumption of intermediate good u by all firms in all industries. Then, market clearing condition on good s writes: $Q_s = Y_s + M_s$.

We now introduce a policy experiment, that allows us to measure the effect of the policy treatment in each industry s by comparing treated and untreated firms within each industry. The following proposition shows how to extend the methodology in our baseline case to this more realistic market structure.

Proposition 6 (Input-output Linkages)**.** *Denote ϕ_s^* the linkage-adjusted industry share, the s^{th} element of the vector defined by $(I - \Gamma)^{-1} \boldsymbol{\phi}$, with $\boldsymbol{\phi}$ being the vector of input shares in final production, and Γ the input-output matrix (so $(I - \Gamma)^{-1}$ is the Leontieff inverse). $\alpha^* = \sum_s \alpha_s \phi_s^*$ is the linkage-adjusted capital share. Define aggregate TFP in this economy as $TFP = \frac{Y}{K^{\alpha^*} L^{1-\alpha^*}}$.*

1. *For each industry s, we can estimate the following industry-specific statistics:*

$$\widehat{\Delta\mu_\tau}(s) = \mathbb{E}\left(\log\left(\frac{p_{it} y_{it}}{k_{it}} \right) | T_i = 1, i \in s \right) - \mathbb{E}\left(\log\left(\frac{p_{it} y_{it}}{k_{it}} \right) | T_i = 0, i \in s \right)$$

$$\widehat{\Delta\sigma_\tau^2}(s) = \text{Var}\left(\log\left(\frac{p_{it} y_{it}}{k_{it}} \right) | T_i = 1, i \in s \right) - \text{Var}\left(\log\left(\frac{p_{it} y_{it}}{k_{it}} \right) | T_i = 0, i \in s \right)$$

$$\widehat{\Delta\sigma_{z\tau}}(s) = \text{Cov}\left(\log\left(\frac{p_{it} y_{it}}{k_{it}} \right), z_{it} | T_i = 1, i \in s \right) - \text{Cov}\left(\log\left(\frac{p_{it} y_{it}}{k_{it}} \right), z_{it} | T_i = 0, i \in s \right)$$

2. *The aggregate effect of extending the treatment to all firms in the economy is:*

$$\Delta \log Y = \frac{\alpha^*(1+\epsilon)}{1-\alpha^*} \sum_s \frac{\alpha_s \phi_s^*}{\alpha^*} \left(-\widehat{\Delta \mu_\tau}(s) + \frac{1}{2} \frac{\theta_s}{1-\theta_s} \left(\alpha_s \widehat{\Delta \sigma_\tau^2}(s) - 2\widehat{\Delta \sigma_{z\tau}}(s) \right) \right)$$

$$\Delta \log TFP = -\frac{\alpha^*}{2} \sum_s \kappa_s \left(1 + \frac{\alpha_s \theta_s}{1-\theta_s} \right) \widehat{\Delta \sigma_\tau^2(s)} - \frac{\alpha^*}{2} var_{\kappa_s} \left(\widehat{\Delta \mu_\tau(s)} \right)$$
$$+ \alpha^* \sum_s \left(\frac{\alpha_s \phi_s^*}{\alpha^*} - \kappa_s \right) \left(-\widehat{\Delta \mu(s)} + \frac{1}{2} \frac{\theta_s}{1-\theta_s} \left(\alpha_s \widehat{\Delta \sigma_\tau^2(s)} - 2\widehat{\Delta \sigma_{z\tau}(s)} \right) \right)$$

where $\kappa_s = \frac{\int_{i \in s} k_i^0 di}{K^0}$ *is the capital share of sector* s *in the initial economy and* $var_{\kappa_s} \left(\widehat{\Delta \mu(s)} \right) = \sum_s \kappa_s (\widehat{\Delta \mu_\tau(s)})^2 - \left(\sum_s \kappa_s \widehat{\Delta \mu_\tau(s)} \right)^2$.

Proof. See appendix A.7. □

The output formula is similar to the formula obtained in our baseline model in Section 2. Extending the treatment to all firms in the economy leads to a change in aggregate output that corresponds to a simple weighted average of industry-level changes in output inferred from the experiment. This is because the Cobb-Douglas aggregation framework shuts down reallocation across industries. Industry weights in the output aggregation formula are given by $\frac{\alpha_s \phi_s^*}{\alpha^*}$, so that the effect of the policy on large and capital intensive industries (for which the distortionary effect is larger) accounts for a greater share of variations in total output. The TFP formula has three terms. The first term is close to a weighted average of the baseline TFP formulas for each industry. Industry weights are given by the capital share of each sector. The second term is the effect of capital reallocation across industries: an increase in cross-industry distortions reduces aggregate productivity. When the treatment is homogeneous across industries, this term is 0. Finally, the last term measures the TFP gains that arise when the policy treatment leads to an increase in output in industries that had "too little capital" before the treatment (i.e. industries for which $\alpha_s \phi_s^* / \alpha^* > \kappa_s$.

Input-output linkages appear in this formula in the modified capital share $\alpha^* = \sum_s \alpha_s \phi_s^*$. To develop intuitions on input-output linkages, consider a simplified version of the model where $\alpha_s = \alpha$, $\beta_s = \alpha$, $\theta_s = \theta$ and $\Theta_s = \Theta$, so that the treatment effects are the same across industries. The aggregation formulas simplify to:

$$\begin{cases} \Delta \log Y = \frac{\alpha^*(1+\epsilon)}{1-\alpha^*} \left(-\widehat{\Delta \mu_\tau} + \frac{1}{2} \frac{\theta}{1-\theta} \left(\alpha \widehat{\Delta \sigma_\tau^2} - 2\widehat{\Delta \sigma_{z\tau}} \right) \right) \\ \Delta \log TFP = -\frac{\alpha^*}{2} \left(1 + \frac{\alpha\theta}{1-\theta} \right) \widehat{\Delta \sigma_\tau^2} \end{cases}$$

The only difference with our baseline formula in Section 3 is the use of $\alpha^* = \alpha \times 1' (I - \Gamma)^{-1} \phi$, which is the capital share adjusted for input-output effects. α^* typically depends on the shape of the IO matrix. Intuitively, α^* is larger when large ϕ_s sectors are also sectors intensively used in intermediate production, which results in a larger aggregate effect in our counterfactual.[19].

5.5 Heterogeneous Mark-ups

In this section, we explore the role played by heterogeneous markups in our aggregate counterfactual. To that end, we consider a simple nested CES framework. There are S industries. Each industry s may operate under parameters Θ_s. Industry output Y_s is produced by combining intermediate inputs y_{is} with a CES technology:

$$Y_s = \left(\int_{i \in s} y_{is}^{\theta_s} di \right)^{\frac{1}{\theta_s}}$$

The elasticity of substitution θ_s is industry-specific and implies industry-specific markups. To simplify the exposition, we assume a constant capital share α across industries:

$$y_{is} = e^{z_{is}} k_{is}^{\alpha} l_{is}^{1-\alpha}$$

The final good Y is produced by combining industry output Y_s using a CES technology:

$$Y = \left(\sum_{k=1}^{S} \chi_s Y_s^{\psi} \right)^{\frac{1}{\psi}}, \text{ with } \sum_{s=1}^{S} \chi_s = 1$$

When $\psi = 0$, the intermediate input shares are fixed. As a result, heterogeneous markups across industries are not distortive for industry input allocation (this is the setting of Section 6). When $\psi \neq 0$, however, intermediate input shares are no longer constant and heterogeneous markups across industries create additional misallocation.

In this setting, we consider again the case of industry-specific experiments, where a random subset of firms in each industry receives an industry-specific treatment. The next proposition shows how to aggregate the results from such an experimental setting:

Proposition 7. *In this nested CES framework, the assumptions in Proposition 2 are satisfied. As a result, the joint distribution of z and $log(1+\tau)$ in industry s does not depend on w nor on*

[19] A simple way to view this is to further assume that $\Gamma = \frac{\gamma}{S} J$. In this case, $\alpha^* = \frac{\alpha}{1-\gamma}$. As the share of intermediate goods increase, the apparent capital share blows up because of network effects. For instance, assuming $\gamma = .5$ and a full IO matrix, the IO-adjusted TFP effect would be multiplied by 2 compared to the baseline.

$(Y_{s'})_{s' \in [1,S]}$. *The aggregation exercise can be estimated through the following steps:*

1. *Use the experimental data to estimate industry-level treatment effects:*

$$\widehat{\Delta\mu_\tau}(s) = \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 1, s_i = s\right) - \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 0, s_i = s\right)$$
$$\widehat{\Delta\sigma^2_\tau}(s) = \text{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 1, s_i = s\right) - \text{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 0, s_i = s\right)$$
$$\widehat{\Delta\sigma_{z\tau}}(s) = \text{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i = 1, s_i = s\right) - \text{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i = 0, s_i = s\right)$$

2. *Extending industry-level treatments to all firms in the industry lead to a change in aggregate output:*

$$\begin{aligned}\Delta\log(Y) = &\frac{(1+\epsilon)\alpha}{1-\alpha}\sum_{s=1}^{S}\gamma_s\left[-\widehat{\Delta\mu_\tau(s)} + \frac{1}{2}\left(\frac{\theta_s}{1-\theta_s}\right)\left(-2\widehat{\Delta\sigma_{z\tau}(s)} + \alpha\widehat{\Delta\sigma^2_\tau(s)}\right)\right]\\ &+\frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\alpha\psi}{1-\psi}\right)(1+\epsilon)var_{\gamma_s}\left(\widehat{\Delta\mu_\tau(s)}\right)\\ &+\frac{\alpha\psi}{1-\psi}\sum_{s=1}^{S}(\gamma_s - \iota_s)\left[-\widehat{\Delta\mu_\tau(s)} + \frac{1}{2}\left(\frac{\theta_s}{1-\theta_s}\right)\left(-2\widehat{\Delta\sigma_{z\tau}(s)} + \alpha\widehat{\Delta\sigma^2_\tau(s)}\right)\right]\\ &+\frac{1}{2}\left(\frac{\psi\alpha}{1-\psi}\right)^2\left[var_{\gamma_s}\left(\widehat{\Delta\mu_\tau(s)}\right) - var_{\iota_s}\left(\widehat{\Delta\mu_\tau(s)}\right)\right]\end{aligned}$$

where $\gamma_s = \frac{P^0_s Y^0_s}{P^0 Y^0}$ *is the industry share of revenue.* $\iota_s = \frac{\theta_s\gamma_s}{\sum_{s'}\theta_{s'}\gamma_{s'}}$ *is the mark-up weighted industry share.* $var_{\omega_s}\left(\widehat{\Delta\mu_\tau(s)}\right) = \sum_{s=1}^{S}\omega_s\left(\widehat{\Delta\mu_\tau(s)}\right)^2 - \left(\sum_{s=1}^{S}\omega_s\widehat{\Delta\mu_\tau(s)}\right)^2$ *is the variance of industry treatment effects, weighted by industry weights* $\omega_s \in \{\gamma_s, \iota_s\}$.

Proof. See Appendix A.8. □

To build intuition, consider first the case where markups are constant across industries: $\theta_s = \theta$. This implies: $\gamma_s = \iota_s$. The aggregation formula for output simplifies to:

$$\begin{aligned}\Delta\log(Y) \;=\; &\frac{(1+\epsilon)\alpha}{1-\alpha}\sum_{s=1}^{S}\gamma_s\left[-\widehat{\Delta\mu_\tau(s)} + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)\left(-2\widehat{\Delta\sigma_{z\tau}(s)} + \alpha\widehat{\Delta\sigma^2_\tau(s)}\right)\right]\\ &+\frac{1}{2}\frac{\alpha\psi}{1-\psi}\frac{\alpha}{(1-\alpha)}(1+\epsilon)\left[\text{var}_{\gamma_s}\left(\widehat{\Delta\mu_\tau(s)}\right)\right]\end{aligned}$$

The first line in the previous equation mimics the formula in our baseline model (8), except that the sufficient statistics are replaced with their sales-weighted averages. The second line corresponds to a reallocation effect. It is 0 when the final good aggregator is Cobb-Douglas ($\psi = 0$). When the elasticity of substitution across industries is less than one $\psi < 0$, industry goods are complements in the production of the final good and a policy treatment that increases the variance of wedges across industries hurts aggregate output.

The formula in Proposition 7 contains two additional reallocation terms that capture the effect of heterogeneous markups across industries. First, the treatment effect of industries where $\theta_s < \sum_{s'} \theta_{s'}\gamma_{s'}$ – i.e. industries whose mark-ups are larger than average – gets more weight on the final output effect (third term in output aggregation formula in Proposition 7). The final term in the formula in Proposition 7 is another cross-industry reallocation effect that arises because of heterogeneous markups. If the treatment effect is heterogeneous across industries, it creates misallocation (the $\text{var}_{\gamma_s}\left(\widehat{\Delta\mu_\tau(s)}\right)$ term). However, this effect is mitigated if this heterogeneity is concentrated in high mark-up (low θ_s) sectors. This is because these sectors' output respond less strongly to the treatment.

5.6 CES Production Function

We show in this section how to extend our methodology beyond the case of Cobb-Douglas production function. We keep the same market structure as in Section 2 (no industry, CES aggregation across intermediary input producers). However, we now consider a CES production function:

$$y_{it} = e^{z_{it}} \left(\alpha k_{it}^{\rho} + (1-\alpha) k_{it}^{\rho}\right)^{\frac{1}{\rho}} \tag{12}$$

$\rho = 0$ corresponds to the Cobb-Douglas case of Section 2. Recent finding in the literature suggests, however, that the firm-level elasticity of substitution between labor and capital is less than 1 (e.g., Oberfield and Raval (2014) report a plant-level elasticity of about .5, or $\rho \approx -1$).

The following proposition summarizes the methodology for computing aggregate counterfactuals in this case. Appendix A.9 provides the detailed proofs.

Proposition 8 (CES Production Function)**.** *When production is CES, the aggregation exercise proceeds in four steps:*

1. *Estimate the following three sufficient statistics from the experimental data:*

$$\widehat{\Delta\mu} = E\left(\log\left(\frac{\theta - s_i^L}{s_i^K}\right)|T_i = 1\right) - E\left(\log\left(\frac{\theta - s_i^L}{s_i^K}\right)|T_i = 0\right)$$
$$\widehat{\Delta\sigma_\tau^2} = \text{Var}\left(\log\left(\frac{\theta - s_i^L}{s_i^K}\right)|T_i = 1\right) - E\left(\log\left(\frac{\theta - s_i^L}{s_i^K}\right)|T_i = 0\right)$$
$$\widehat{\Delta\sigma_{\tau z}} = \text{Cov}\left(\log\left(\frac{\theta - s_i^L}{s_i^K}\right)|T_i = 1\right) - E\left(\log\left(\frac{\theta - s_i^L}{s_i^K}\right)|T_i = 0\right)$$

   where s_i^L is the firm-level labor share and s_i^K the firm-level capital share.

2. *Assume that the policy experiments is repeated in a cross-section of economy c – for instance across local labor markets in a country. Then, estimate two additional sufficient statistics, β and γ, that characterize how* treated firms' *log capital wedges respond to variations in wages across these economies. For firm i, in economy c, β and γ are defined in the following way:*

$$\log\left(\frac{\theta - s_{ic}^L}{s_{ic}^K}\right) = cst + \beta \log w_c + \gamma(\log w_c)^2 + u_{ic}$$

   Estimating β and γ requires exogenous variations in wage w across economies.

3. *Compute the additional parameter $a = \frac{s^L}{\theta}$ where s^L is the average labor share in the cross-section. and note $b = 1 - a$.*

4. *Generalizing the policy treatment to all firms in the economy then leads to a change in aggregate output of:*

$$\Delta \log Y = A\widehat{\Delta\mu} + B\widehat{\Delta\sigma_\tau^2} + C\widehat{\Delta\sigma_{\tau z}} + D(\widehat{\Delta\mu})^2$$

   where A, B, C and D are known functions of a,b, β, γ, as well as θ, ϵ and ρ. These functions are reported in Appendix A.9

The first order version of the above formula is simpler and allows to build intuition:

$$\Delta \log Y = -\frac{1 - \frac{s^L}{\theta}}{\frac{s^L}{\theta} + \beta\left(1 - \frac{s^L}{\theta}\right)}\left(\epsilon + \frac{1}{1-\rho}\right)\Delta E \widehat{\log\left(\frac{\theta - s_i^L}{s_i^K}\right)}$$

The equivalent first-order formula in the Cobb-Douglas framework is (Equation 8):

$$-\frac{1-\frac{s^L}{\theta}}{\frac{s^L}{\theta}}(1+\epsilon)\Delta E \log\left(\frac{py_i}{k_i}\right)$$

This CES production case requires three adjustments to the Cobb-Douglas production case. First, and most importantly, distortions are now affected by the general equilibrium of the economy, which is captured at the first-order through the sufficient statistics β. β measures the extent to which the average capital wedge is affected by the equilibrium wage. Proposition 2 has established that with Cobb-Douglas $\beta = 0$, but this does not have to be with CES production. We show in Appendix A.9 that wedges do not depend Y, even with CES production, so that at the first-order, β is the only additional sufficient statistics to estimate relative to the baseline case. Of course, estimating β (and γ, the corresponding second-order term) is an empirical challenge. However, in a simple simulation exercise presented in Appendix 1, the precision loss that results from omitting β and γ in the aggregation formula appear limited (Figure A.1 and A.2).

Second, with CES production and a less-than-one elasticity of substitution between labor and capital ($\rho < 0$), general equilibrium effects leads to a greater dampening of the effects observed in the experimental data. Intuitively, labor cannot be substituted with capital as easily, so that a relaxation of capital frictions has a lower effect in general equilibrium for $\rho < 0$ than $\rho = 0$. This effect corresponds to $\epsilon + \frac{1}{1-\rho} < 1 + \epsilon$ in the above formula. Third, wedges cannot be computed using the sales to capital ratio as in the Cobb-Douglas case: instead, we can uses input shares to measure distortions.

5.7 Additional Results

In Appendix B, we explore additional extensions to our baseline framework. We do not report them in the main text for the sake of brevity. The first extension (Appendix B.1) shows formulas for aggregating treatments designed to increase firm-level productivity (as in, e.g., Bloom et al. (2013a),Bloom et al. (2013b)). Aggregating these experimental evidence is not trivial if firms face capital frictions, as these frictions may interact with the shift in the productivity distribution triggered by the generalization of the experiment. Our framework allows to take these effects into account in a simple way, for the set of frictions described in section 2. This extension highlights in particular that statistics on productivity are not the only ones that empirical researchers should collect, even in an environment where the only exogenous change is on productivity. Instead, sufficient statistics on capital wedges, and their correlation with changes in productivity, need to

be estimated.

A second extension (Appendix B.2) simply explores the effect of decreasing returns to scale and allows for potentially perfect competition. A third extension (Appendix B.3) proposes formulas that do not rest on the fact that wedges are small. These formulas invoke different sufficient statistics, that are easy to compute, but less robust to heteroskedasticity in productivity shocks, which we have not modeled in our framework.

The last two extensions (Appendix B.4 and B.5) introduce labor distortions in our baseline framework. In Appendix B.4, we consider labor distortions represented by exogenous wedges on wage (e.g, because of firm-specific payroll taxes). In this case, the aggregation requires to estimate more than the usual statistics on capital wedges: the covariance between labor and capital wedges becomes an important statistic to inform the aggregate counterfactual. Appendix B.5 provides a micro-foundation for labor wedges through binding firm-level minimum wages on unskilled labor. In this case, the aggregation requires to estimate how capital wedges interact with firm-level minimum wages.

6 Illustrations

In this section, we explain in more detail how our methodology can be applied in practice to two recent papers that investigate the causal effect of distortions on firm behavior using plausibly exogenous variations in policy.

6.1 Reforming Debt Enforcement

Our first illustration is Ponticelli and Alencar (2016). They study the effect of court enforcement on the availability of credit to firm-level investment, employment and sales growth. In a nutshell, the paper runs the following type of regression, where i is an index for firms and j for a municipality:

$$\Delta y_{ij} = \beta T_j + \epsilon_j \tag{13}$$

where $T_j = \log\left(\frac{\text{backlog}_j}{\text{judge}_j}\right)$ is a measure of city-level bankruptcy court congestion. Δy_{ij} is the change of firm-level activity (log employment for instance) around a national bankruptcy reform that Brazil adopted in 2005. The identification strategy rests on the idea that the reform should have a smaller effect on firms located in cities where courts are congested. We would therefore expect β to be negative when y_{ij} measures corporate investment, as firms benefiting from better debt enforcement benefit from easier access

to credit. This is indeed what the paper finds in its Table III.

Our methodology allows us to estimate the aggregate effect of the reform on TFP and output in all of Brazil. Doing this directly using aggregate data would be impossible, as an aggregate shock may have hit Brazil in 2005, confounding the effect of the reform. The cross-sectional approach in equation (13) helps with this as it rests on the comparison between cities and therefore filters out aggregate shocks. Note that, under the paper's identifying assumption, the treatment exposure v_j is *uncorrelated* with city-level exposure to the potential aggregate productivity shock that may have hit Brazil at the same time.

This setting constitutes a direct application of Section 5.3. In a first step, the paper's firm-level data (a large and representative survey of privately-held Brazilian firms) can be used to calculate the change the wedge distribution around the reform. For each firm i at date t, the log capital wedge is computed as the log ratio of sales to physical assets (net book value of Property, Plants and Equipment), or $\log(1+\tau_{it}) = \log \frac{\text{sales}_{it}}{\text{PPE}_{it}}$. With calibrated capital shares α, log TFP can be computed as $z_{it} = \frac{1}{\theta}\log \text{sales}_{it} - \alpha \log \text{PPE}_{it} - (1-\alpha)\log \text{Emp}_{it}$. Within each municipality-year, we then compute (1) the empirical mean of $\log(1+\tau_{it})$, (2) the empirical variance of $\log(1+\tau_{it})$ and (3) the empirical covariance between $\log(1+\tau_{it})$ and z_{it}. We then compute the average of these three statistics within city and across years, separately before and after the treatment, and differentiate, following the exact same procedure as Ponticelli and Alencar (2016) in order to have just one observation per city. We note these three statistics $\Delta\mu_{\tau,j}$, $\Delta\sigma^2_{\tau,j}$ and $\Delta\sigma_{z\tau,j}$.

We can then redefine $v_j = \max_j\{T_j\} - T_j$. v_j is an increasing measure of exposure to the reform and for the least exposed group, $v_j = 0$, as in Section 5.3. To get to the effect of the reform on aggregate TFP and output, we are therefore assuming that the reform has no effect on the city with the highest level of backlog. Equipped with measure of exposure, we can directly apply the methodology of Section 5.3 and estimate the following regressions:

$$\begin{aligned}
\Delta\mu_{\tau,j} &= A_1 v_j + A_2 (v_j)^2 + \epsilon^1_j \\
\Delta\sigma^2_{\tau,j} &= A_3 v_j + A_4 (v_j)^2 + \epsilon^2_j \\
\Delta\sigma_{z\tau,j} &= A_5 v_j + A_6 (v_j)^2 + \epsilon^3_j
\end{aligned}$$

which lead us to the three sufficient statistics needed in Proposition 5:

$$\widehat{\Delta\mu_\tau}(j) = \widehat{A_1 v_j} + \widehat{A_2}(v_j)^2$$
$$\widehat{\Delta\sigma^2_\tau}(j) = \widehat{A_3 v_j} + \widehat{A_4}(v_j)^2$$
$$\widehat{\Delta\sigma_{z\tau}}(j) = \widehat{A_5 v_j} + \widehat{A_6}(v_j)^2$$

Combined with these estimates, the formulas in Proposition 5 would provide us with estimates of the effect of the reform on aggregate TFP and output. In particular, we can quantify these aggregate counterfactuals without specifying precisely how better debt enforcement affects the financing constraint $M()$, the interest rate function $r()$, or the survival set $Z()$. All that we require is that these functions satisfy the assumptions of Proposition 2. Another practical advantage of this method is that we can easily obtain confidence bounds on the aggregate impact, through bootstrapping on the firm-level data set. Relative to a fully structural approach, however, the drawback of our methodology is that we require that the change in firm-level outcomes can be observed over a sufficiently long sample so that the estimated $\Delta\mu_{\tau,j}$, $\Delta\sigma^2_{\tau,j}$ and $\Delta\sigma_{z\tau,j}$ corresponds to the steady-state changes in these outcomes.

6.2 Corporate taxes and economic activity

Our methodology can be also be applied directly to the empirical study of Giroud and Rauh (2016). In this paper, the authors estimate the effect of corporate income tax on corporate outcomes using plant-level U.S. census data and plausibly exogenous variations of plant-level state income tax rates. These variations come mostly from from variation in state-level corporate taxes interacted with the legal form of the firms to which the establishment belongs (S vs. C-corp). [20] This setting allows them to control for a host of fixed effects (firm-level, state-level, etc.).

Their main regression specification is in spirit similar to, for plant p, belonging to firms i at date t:

$$y_{ipt} = \beta t_{it} + X_{ipt} + \epsilon_{ipt}$$

where t_{it} is the firm-level corporate income tax rate, X_{ipt} are the controls including fixed effects and ϵ_{ipt} is the residual, which, conditional on controls, is assumed to be uncorrelated with the tax rate (the identifying assumption).

This empirical setting can be combined with our baseline formulas from Proposition

[20] The paper also uses variations arising through apportionment rules that compute an effective tax rate using sales and employment weights across states.

1 to compute the aggregate effect on output and TFP of increasing the corporate income tax rate from 0% to, say, 20%. Since this is a homogeneous treatment, the baseline formulas of proposition 1 can be used in a two-step approach. First, the sufficient statistics of proposition 1 can be estimated using the Annual Survey of Manufacture (used in the paper). For each plant p, in firm i at date t, the log capital wedge is computed as the log ratio of shipments to capital stock, or $\log(1+\tau_{ipt}) = \log \frac{\text{sales}_{ipt}}{\text{Capital}_{ipt}}$. Log TFP can be computed through $z_{ipt} = \log \text{Shipment}_{ipt} - \alpha \log(\text{Capital}_{ipt}) - (1-\alpha)\log(\#\text{ Employees}_{ipt})$. Second, three plant-level regressions need to be estimated:

$$\begin{aligned}
\log(1+\tau_{ipt}) &= \beta_\tau t_{it} + \gamma' X_{ipt} + \epsilon_{1,ipt} \\
\left(\log(1+\tau_{ipt}) - \log\widehat{(1+\tau_{ipt})}\right)^2 &= \beta_{\tau^2} t_{it} + \gamma' X_{ipt} + \epsilon_{2,ipt} \\
z_{ipt}\left(\log(1+\tau_{ipt}) - \log\widehat{(1+\tau_{ipt})}\right) &= \beta_{z\tau} t_{it} + \gamma' X_{ipt} + \epsilon_{3,ipt}
\end{aligned}$$

where $\log\widehat{(1+\tau_{ipt})} = \widehat{\beta_\tau} t_{it} + \gamma' X_{ipt}$ is the predicted value from the first regression.

These regression coefficients provide us with the three key statistics needed to implement the baseline formulas:

$$\begin{aligned}
\widehat{\Delta\mu_\tau} &= .2 \times \widehat{\beta_\tau} \\
\widehat{\Delta\sigma^2_\tau} &= .2 \times \widehat{\beta_{\tau^2}} \\
\widehat{\Delta\sigma_{z\tau}} &= .2 \times \widehat{\beta_{z\tau}}
\end{aligned}$$

which measure the effect on the distribution of wedges of raising the tax rate from 0 to 20%. Of course, this implementation assumes the effect on first and second moments are linear in t. If the researcher does not believe in such linearity, she can easily implement a less parametric approach (through running non-parametric regressions instead of the linear ones above).

These three sufficient statistics ($\widehat{\Delta\mu_\tau}$, $\widehat{\Delta\sigma^2_\tau}$ and $\widehat{\Delta\sigma_{z\tau}}$) can then be plugged into the baseline aggregation formulas of Proposition 1 to estimate the aggregate TFP and output effect of increasing corporate income tax rates from 0% to 20%. This methodology accounts for general equilibrium effects, but also allows taxes to interact with other frictions such as adjustment costs or financing costs, in a potentially flexible way, without explicitly modeling and estimating these frictions. For instance, it may be argued that taxes, because their impair the ability of firms to hoard equity, indirectly tighten financing constraints (Davila and Hébert, 2018). This effect is accounted for provided

financing constraints satisfy the homogeneity assumptions in Proposition 2. Note also that it is easy to estimate confidence intervals for these macroeconomic variables through bootstrapping on the base sample – i.e. before running the three regressions on wedges.

7 Conclusion

This paper develops a simple sufficient statistics framework to aggregate well-identified firm-level evidence of policy experiments aiming to reduce frictions faced by firms. The methodology proceeds in two steps: (1) using firm-level data, the econometrician estimates the treatment effect of the policy on moments of the joint distribution of productivity, and capital wedges (2) these treatment effects are applied to *all* firms in a general equilibrium model of firm dynamics with real frictions, financial frictions and taxes. Our approach yields simple aggregation formula, that can easily be estimated in quasi-)experimental settings. These formula can easily be extended to more complex economies (e.g, allowing decreasing returns to scale or heterogeneous industries) or partial aggregation exercises where all firms do not receive the treatment.

While variants of this methodology have been used in recent applied work, our paper explicits a set of conditions under which such an approach is valid: (1) intermediate inputs are combined with (nests of) CES aggregators (2) production takes place according to a Cobb-Douglas technology combining labor and capital (3) capital adjustment costs, financing frictions and taxes satisfy an homogeneity condition. While these assumptions may appear restrictive, they are satisfied by a large class of models commonly used in the macro-finance literature.

References

Aghion, P., T. Fally, and S. Scarpetta (2007): "Credit constraints as a barrier to the entry and post-entry growth of firms," *Economic Policy*, 22.

Asker, J., A. Collard-Wexler, and J. D. Loecker (2014): "Dynamic Inputs and Resource (Mis)Allocation," *Journal of Political Economy*.

Banerjee, A. V. and E. Duflo (2014): "Do Firms Want to Borrow More? Testing Credit Constraints Using a Directed Lending Program," *The Review of Economic Studies*, 81, 572–607.

Baqaee, D. and E. Farhi (2017a): "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem," Tech. rep.

——— (2017b): "Productivity and Misallocation in General Equilibrium," Tech. rep.

Bartelsman, E., J. Haltiwanger, and S. Scarpetta (2013): "Cross-Country Differences in Productivity: The Role of Allocation and Selection," *American Economic Review*, 103, 305–334.

Bertrand, M., A. Schoar, and D. Thesmar (2007): "Banking Deregulation and Market Structure: Evidence from France," *Journal of Finance*.

Blattner, L., L. Farinha, and F. Rebelo (2017): "When Losses Turn into Loans: The Cost of Undercapitalized Banks," Tech. rep.

Bloom, N., B. Eifert, A. Mahajan, D. McKenzie, and J. Roberts (2013a): "Does Management Matter? Evidence From India," *Quarterly Journal of Economics*.

Bloom, N., M. Shankerman, and J. V. Reenen (2013b): "Technology Spillovers and Product Market rivalry," *Econometrica*.

Broda, C. and D. Weinstein (2006): "Globalization and the Gains from Variety," *Quarterly Journal of Economics*, 121, 541–585.

Brown, J. D. and J. Earle (2017): "Finance and growth at the Firm Level: Evidence from SBA loans," *The Journal of Finance*, 72.

Buera, F. J., J. P. Kaboski, and Y. Shin (2011): "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 101, 1964–2002.

——— (2012): "The Macroeconomics of Microfinance," Working Paper 17905, National Bureau of Economic Research.

Caballero, R. and J. Leahy (1996): "Fixed Costs: The Demise of Marginal Q," Tech. rep., NBER.

Catherine, S., T. Chaney, Z. Huang, D. Sraer, and D. Thesmar (2018): "Quantifying Reduced-Form Evidence on Collateral Constraints," Tech. rep.

Chetty, R. (2012): "Bounds on Elasticities with Optimization Frictions: A Synthesis of Micro and Macro Evidence on Labor Supply," *Econometrica*, 80, 969–1018.

Davila, E. (2016): "Using Elasticities to Derive Optimal Bankruptcy Exemptions," Tech. rep.

Davila, E. and B. Hébert (2018): "Optimal Corporate Taxation Under Financial Frictions," Tech. rep.

Dixit, A. K. and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67, 297–308.

Edmond, C., V. Midrigan, and D. Xu (2018): "How Costly are Markups?" Tech. Rep. 24800, National Bureau of Economic Research.

Fraisse, H., M. Lê, and D. Thesmar (2017): "The Real Effects of Capital Requirements," Tech. rep.

Gilchrist, S., J. W. Sim, and E. Zakrajšek (2014): "Uncertainty, Financial Frictions, and Investment Dynamics," NBER Working Papers 20038, National Bureau of Economic Research, Inc.

Giroud, X. and J. Rauh (2016): "State Taxation and the Reallocation of Business Activity: Evidence from Establishment-Level Data," Tech. rep.

Gopinath, G., S. Kalemli-Ozcan, L. Karabarbounis, and C. Villegas-Sanchez (2015): "Capital Allocation and Productivity in South Europe," Tech. rep.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988): "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, 78, 402–417.

Hennessy, C. A. and T. M. Whited (2007): "How Costly Is External Financing? Evidence from a Structural Estimation," *The Journal of Finance*, 62, 1705–1745.

Hopenhayn, H. (2014): "On the Measure of Distortions," Tech. rep.

Hsieh, C.-T. and P. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*.

Hulten, C. R. (1978): "Growth accounting with intermediate inputs," *The Review of Economic Studies*, 511–518.

Larrain, M. and S. Stumpner (2017): "Capital Account Liberalization and Aggregate Productivity: The Role of Firm Capital Allocation," *The Journal of Finance*.

Lelarge, C., D. Sraer, and D. Thesmar (2010): "Entrepreneurship and Credit Constraints: Evidence from a French Loan Guarantee Program," *Lerner and Schoar Eds: International Differences in Entrepreneurship*.

Michaels, R., T. B. Page, and T. M. Whited (2016): "Labor and Capital Dynamics under Financing Frictions," .

Midrigan, V. and D. Xu (2014): "Finance and Misallocation: Evidence from Plant-Level Data," *American Economic Review*.

Moll, B. (2014): "Productivity Losses from Financial Frictions: Can Self-financing Undo Capital Misallocation?" *American Economic review*.

Oberfield, E. (2013): "Productivity and misallocation during a crisis: Evidence from the Chilean crisis of 1982," *Review of Economic Dynamics*, 16, 100 – 119, special issue: Misallocation and Productivity.

Oberfield, E. and D. Raval (2014): "Micro Data and Macro Technology," NBER Working Papers 20452, National Bureau of Economic Research, Inc.

Ponticelli, J. and L. S. Alencar (2016): "Court Enforcement, Bank Loans, and Firm Investment: Evidence from a Bankruptcy Reform in Brazil *," *The Quarterly Journal of Economics*, 131, 1365–1413.

Rajan, R. G. and L. Zingales (1998): "Financial Dependence and Growth," *American Economic Review*, 88, 559–86.

Restuccia, D. and R. Rogerson (2008): "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments," *Review of Economic Dynamics*.

Rotemberg, M. (2017): "Equilibrium Effects of Firm Subsidies," Tech. rep.

Stokey, N. and R. Lucas (1989): *Recursive methods in economic dynamics*, Harvard University Press.

Vig, V. (2013): "Access to Collateral and Corporate Debt Structure: Evidence from a Natural Experiment," *The Journal of Finance*, 68, 881–928.

Yagan, D. (2015): "Capital Tax Reform and the Real Economy: The Effects of the 2003 Dividend Tax Cut," *American Economic Review*.

Zia, B. (2008): "Export incentives, financial constraints, and the (mis)allocation of credit: Micro-level evidence from subsidized export loans," *Journal of Financial Economics*, 87, 498–527.

Zwick, E. and J. Mahon (2015): "Tax Policy and Heterogeneous Investment Behavior," Tech. rep.

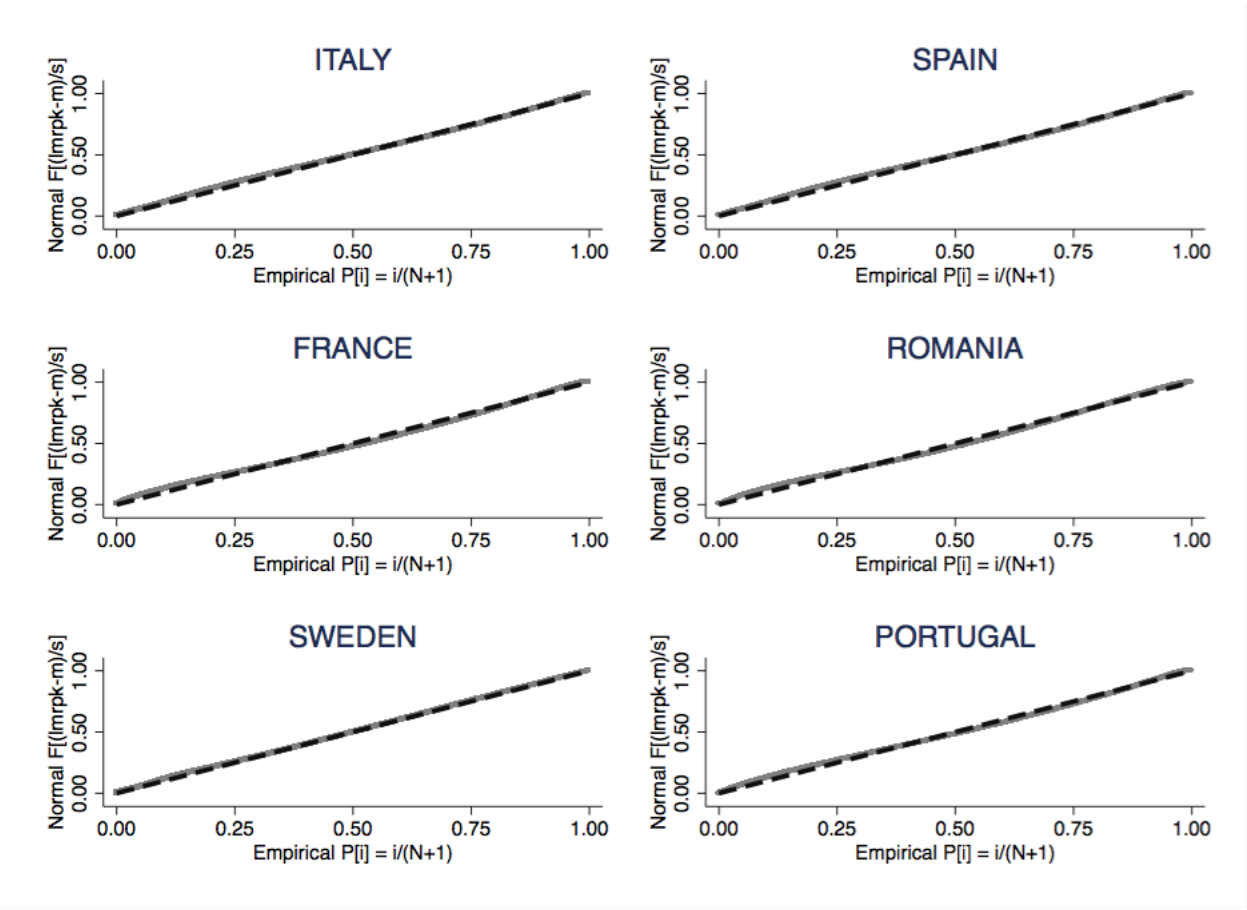
Table 1: Select Literature Review

Authors	Title	Date	(1) Production function function: Cobb-Douglas	(2) Adjustment costs Homogeneous	(3) Borrowing constraint: Homogeneous	(4) Equity issuance: Homogeneous	(5) Taxes schedule: Homogeneous
Panel A: Adjustment Cost papers							
Asker, Collar-Wexler, De Loecker	Dynamic inputs and resource (mis) allocation	2014	Y	Y	-	-	-
Bartelsman, Haltiwanger, Scarpetta	Cross-country differences in productivity: The role of allocation [...]	2013	N	Y	-	-	Y
Basu, Bundick	Uncertainty shocks in a model of effective demand	2017	Y*	Y	-	-	-
Bloom	The impact of uncertainty shocks	2009	Y	Y	-	-	-
Bloom, Bond, Van Reenen	Uncertainty and investment dynamics	2007	Y	Y	-	-	-
Bloom, Floetotto, Jaimovich, Saporta-Eksen, Terry	Really uncertain business cycles	2018	Y	Y	-	-	-
Cooper, Haltiwanger	On the nature of capital adjustment costs	2006	Y	Y	-	-	-
Gourio, Rudanko	Customer capital	2014	Y	Y	-	-	-
Hall	Measuring factor adjustment costs	2004	Y	Y	-	-	-
Peters, Taylor	Intangible capital and the investment-q relation	2017	Y	Y	-	-	-
Panel B: Structural & Dynamic Corporate Finance							
Bolton, Chen, Wang	A Unified Theory of Tobin's q, Corporate Investment Financing [...]	2011	Y	Y	-	Y	-
Bolton, Chen, Wang	Market timing, investment, and risk management	2013	Y	Y	-	Y	-
Cummins, Hassett, Oliner	Investment behavior, observable expectations, and internal funds	2006	Y	Y	-	-	-
DeAngelo, DeAngelo, Whited	Capital structure dynamics and transitory debt	2011	Y	Y	N	N	Y
Gamba, Triantis	The value of financial flexibility	2008	Y*	-	Y	Y	N
Gomes, Schmid	Levered returns	2010	Y*	N	Y	N	Y
Hackbarth, Mauer	Optimal priority structure, capital structure, and investment	2011	-	-	Y	N	Y
Hennessy, Whited	Debt dynamics	2005	Y	Y	Y	Y	N
Hennessy, Whited	How costly is external financing? [...]	2007	Y	-	Y	N	Y
Hennessy, Levy, Whited	Testing Q theory with financing frictions	2007	Y	Y	Y	Y	-
Li, Livdan, Zhang	Anomalies	2009	Y*	Y	-	N	-
Liu, Whited, Zhang	Investment-based expected stock returns	2009	Y	Y	-	-	-
Livdan, Sapriza, Zhang	Financially constrained stock returns	2009	Y*	Y	Y	N	-
Miao	Optimal capital structure and industry dynamics	2005	Y	-	Y	-	Y
Michaels, Page, Whited	Labor and capital dynamics under financing frictions	2018	Y	Y	Y	Y	-
Nikolov, Whited	Agency conflicts and cash: Estimates from a dynamic model	2014	Y	Y	Y	Y	Y
Riddick, Whited	The corporate propensity to save	2009	Y	Y	-	N	Y
Whited, Wu	Financial constraints risk	2006	Y	Y	N	N	-
Panel C: Macro-finance with heterogeneous firms							
Arellano, Bai, Zhang	Firm dynamics and financial development	2012	Y	Y	Y	Y	-
Buera, Shin	Financial frictions and the persistence of history [...]	2013	Y	-	Y	-	Y
Buera, Moll	Aggregate implications of a credit crunch [...]	2015	Y	-	Y	-	-
Cooley, Quadrini	Financial markets and firm dynamics	2001	Y	-	Y	Y	-
Fernandez-Villaverde, Guerron-Quintana, Rubio-ramirez, Uribe	Risk matters: The real effects of volatility shocks	2011	Y	Y	-	-	-
Gilchrist, Sim, Zakrajsek	Uncertainty, financial frictions, and investment dynamics	2014	Y	Y	Y	Y	-
Gomes	Financing investment	2001	Y	Y	Y	N	-
Gopinath, Kalemli-Ozcan, Karabarbounis, Villegas-Sanchez	Capital allocation and productivity in South Europe	2017	Y	Y	N	-	-
Gourio	Disaster risk and business cycles	2012	Y	Y	-	-	-
Gourio, Miao	Firm heterogeneity and the long-run effects of dividend tax reform	2010	Y	Y	-	Y	Y
Itskhoki, Moll	Optimal development policies with financial frictions	2018	Y	-	Y	-	Y
Khan, Thomas	Idiosyncratic shocks and the role of nonconvexities in [...] investment	2008	Y	Y	-	-	-
Khan, Thomas	Credit shocks and aggregate fluctuations[...]	2013	Y	Y	Y	Y	-
Liu, Wang, Zha	Land-price dynamics and macroeconomic fluctuations	2013	Y	Y	Y	-	-
Midrigan, Xu	Finance and misallocation: Evidence from plant-level data	2014	Y	Y	Y	Y	-
Moll	Productivity losses from financial frictions[...]	2014	Y	-	Y	-	-
Percent in line with our assumptions in all 44 papers			98%	98%	93%	79%	95%
Percent in line with our assumptions in papers that have the economic force			82%	97%	88%	59%	85%

Table 1: Literature Review (Continued)

Note: This table checks the validity of our assumption in a select review of 44 recent papers from the literature on firm dynamics. We restrict ourselves to all papers citing Hennessy and Whited (2007), Midrigan and Xu (2014) and Moll (2014), published either in the American economic review, Econometrica, Journal of Political Economy, Review of Economic Studies, Quarterly Journal of Economics, Journal of Finance, Journal of Financial Economics, Review of Financial Studies, one of the three American Economic Journal or Journal of Monetary Economics. To further restrict the scope of the review, we asked that the papers have at least 50 Google scholar citations. We end up with 44 papers, which we classify in 3 broad strands of literature: "adjustment cost papers", in which adjustment costs are the only friction, papers using dynamic corporate finance models (some of them corresponding to structural estimate, some of them being pure theoretical contributions) and macro-finance paper with financing frictions as well as competitive equilibrium modeling. For each of these papers, we then report if our core assumptions are satisfied: Cobb-Douglas production, and homogeneity of taxes, financing and real frictions. We report the results in columns (1)-(5). Y means that the assumption is satisfied, N that it is not. $-$ means that there is no such force in the model (so that our assumption is by default satisfied). Y^* means that the production function is indeed Cobb-Douglas, but the technology also includes *non-scalable fixed costs* of production. In the bottom two lines of the Table, we report the % of papers for which the assumption is satisfied. In the penultimate line, the % is computed among all papers. In the last line, it is computed only *among papers that have the force being in the model*. Hence, in column 3, 88% of the papers that have borrowing constraints satisfy the assumptions of Proposition 2, but this corresponds to 93% of the papers.

Figure 1: Normal probability plot of log-MRPK for firms in Amadeus



Source: BvD AMADEUS Financials, 2014. Note: This figure shows normal probability plots for 6 OECD countries (France, Spain, Italy, Portugal, Romania and Sweden) for the distribution of log-MRPK. Log-MRPK is computed as the ratio of value added (operating revenue minus materials) and total fixed assets.

APPENDIX (FOR ONLINE PUBLICATION)

A Proofs

A.1 Derivation of output and TFP

Let us begin with the output formula. We start from the three equations:

$$\begin{aligned}\theta(1-\alpha)\frac{p_{it}y_{it}}{l_{it}} &= w \\ \theta\alpha\frac{p_{it}y_{it}}{l_{it}} &= R(1+\tau_{it}) \\ p_{it}y_{it} &= Y^{1-\theta}e^{\theta z_{it}}k_{it}^{\theta\alpha}l_{it}^{\theta(1-\alpha)}\end{aligned}$$

Among these equations, the first one is the FOC in labor (assumed frictionless). The second and third ones are *definitions*. The second equation is the definition of the capital wedge τ_{it}. The third equation is the definition of output (monopolistic competition with CES aggregation and Cobb Douglas production). Injecting the top two equations into the last one, we obtain:

$$p_{it}y_{it} = Ye^{\frac{\theta z_{it}}{1-\theta}}\left(\frac{\alpha\theta}{R(1+\tau_{it})}\right)^{\frac{\alpha\theta}{1-\theta}}\left(\frac{(1-\alpha)\theta}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}}$$

Exploiting the fact that $Y = \int p_{it}y_{it}di$, this leads to:

$$1 = \left(\frac{(1-\alpha)\theta}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}}\int e^{\frac{\theta z_{it}}{1-\theta}}\left(\frac{\alpha\theta}{R(1+\tau_{it})}\right)^{\frac{\alpha\theta}{1-\theta}}di$$

hence the market clearing wage is given by:

$$w \propto \left(\int \frac{e^{\frac{\theta z_{it}}{1-\theta}}}{(1+\tau_{it})^{\frac{\alpha\theta}{1-\theta}}}di\right)^{\frac{1-\theta}{(1-\alpha)\theta}}$$

The labor market equilibrium writes:

$$L = \int l_{it}di \propto \int \frac{p_{it}y_{it}}{w}di \propto \frac{Y}{w}$$

Combined with the aggregate labor supply curve $L \propto w^{\epsilon}$, the labor market clearing condition writes:

$$Y \propto w^{1+\epsilon}$$

which, combined with the expression for the market clearing wage gives the formula for output.

Let us now move to the TFP formula. First, note that:

$$TFP = \left(\frac{Y}{L}\right)^{1-\alpha}\left(\frac{Y}{K}\right)^{\alpha}$$

Given the labor market equilibrium written above, $\frac{Y}{L} \propto w$. So we need to compute $\frac{Y}{K}$.

The aggregate capital stock is given by:

$$K = \int k_{it} di \propto \int \frac{p_{it} y_{it}}{1+\tau_{it}} di \propto Y \left(\frac{1}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}} \int \frac{e^{\frac{\theta z_{it}}{1-\theta}}}{(1+\tau_{it})^{1+\frac{\theta\alpha}{1-\theta}}} di$$

We then inject expression for $\frac{Y}{L}$ and $\frac{Y}{K}$ into the TFP formula and obtain:

$$TFP \propto w^{(1-\alpha)\left(1+\frac{\alpha\theta}{1-\theta}\right)} \left(\int \frac{e^{\frac{\theta z_{it}}{1-\theta}}}{(1+\tau_{it})^{1+\frac{\alpha\theta}{1-\theta}}} di \right)^{-\alpha}$$

which after injecting the expression for w, leads to the TFP formula.

A.2 Proof of Proposition 1

First, note $\mu_\tau = \mathbb{E}[\tau]$, $\sigma_\tau^2 = \text{Var}[\tau]$ and $\sigma_{z\tau} = \text{Cov}(z, \tau)$. Since the distribution of wedges is a function of Θ and the aggregate equilibrium (w, Y), so are these moments. However, to save on notations, we omit for now this dependence.

We start with aggregate production (3). $F(z, \tau; \Theta, w, Y)$ is the ergodic distribution of (z, τ) for a firm with structural parameters Θ and facing a wage w on the labor market and total output Y:

$$\log Y = (1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta} \log \left(\int_{z,\tau} \left(\frac{e^z}{(1+\tau)^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z, \tau; \Theta, w, Y) \right)$$

Note $\delta_\tau = \log(1+\tau) - \mu_\tau$, and $u = \frac{\theta}{1-\theta}(z - \alpha\delta_\tau)$. Then:

$$\begin{aligned}
\int_{z,\tau} \left(\frac{e^z}{(1+\tau)^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z, \tau; \Theta, w, Y) &= \mathbb{E}\left(e^{-\frac{\alpha\theta}{1-\theta}\mu_\tau + u} \right) \\
&= e^{-\frac{\alpha\theta}{1-\theta}\mu_\tau} \mathbb{E}\left[e^u \right] \\
&\approx e^{-\frac{\alpha\theta}{1-\theta}\mu_\tau} \mathbb{E}\left[1 + u + \frac{u^2}{2} \right] \\
&\approx e^{-\frac{\alpha\theta}{1-\theta}\mu_\tau} \left(1 + \frac{\text{Var}\left[u\right]}{2} \right) \\
&\approx e^{-\frac{\alpha\theta}{1-\theta}\mu_\tau} \left(1 + \frac{1}{2} \left(\frac{\theta}{1-\theta} \right)^2 \left(\sigma_z^2 - 2\alpha\sigma_{z\tau} + \alpha^2\sigma_\tau^2 \right) \right)
\end{aligned}$$

As a result, at the second-order:

$$\begin{aligned}
\log \left(\int_{z,\tau} \left(\frac{e^z}{(1+\tau)^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z, \tau; \Theta, w, Y) \right) &\approx -\frac{\alpha\theta}{1-\theta}\mu_\tau + \log \left(1 + \frac{1}{2} \left(\frac{\theta}{1-\theta} \right)^2 \left(\sigma_z^2 - 2\alpha\sigma_{z\tau} + \alpha^2\sigma_\tau^2 \right) \right) \\
&\approx -\frac{\alpha\theta}{1-\theta}\mu_\tau + \frac{1}{2} \left(\frac{\theta}{1-\theta} \right)^2 \left(\sigma_z^2 - 2\alpha\sigma_{z\tau} + \alpha^2\sigma_\tau^2 \right)
\end{aligned}$$

which leads to the result. Computation of the TFP formula follows the same logic.

A.3 Proof of Proposition 2

Remember that cash-flows to equity (net of equity issuance costs) are given by the following formula:

$$\begin{aligned} e_{it} &= \frac{\alpha}{\alpha+(1-\alpha)\phi}\left(\frac{(1-\alpha)\phi}{\alpha+(1-\alpha)\phi}\right)^{\frac{1-\alpha}{\alpha}\phi} S_t^{1-\phi} e^{\frac{\phi}{\alpha}z_{it}} k_{it}^{\phi} - (k_{it+1}-(1-\delta)k_{it}) - \Gamma\left(z_{it},\boldsymbol{x}_{it};\boldsymbol{\Theta},w_t,Y_t\right) \\ &\quad + \left(\frac{b_{it+1}}{1+r(z_{it},\boldsymbol{x}_{it};\boldsymbol{\Theta},w_t,Y_t)} - b_{it}\right) - \mathcal{T}(z_{it},\boldsymbol{x}_{it};\boldsymbol{\Theta},w_t,Y_t) - \mathcal{C}(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y), \end{aligned}$$

where $S_t = \frac{Y_t}{w_t^{\frac{(1-\alpha)\phi}{\alpha(1-\phi)}}}$. By combining the different assumptions in Proposition 2, we get that:

$$\begin{aligned} e_{it} &= S_t\left(\frac{\alpha}{\alpha+(1-\alpha)\phi}\left(\frac{(1-\alpha)\phi}{\alpha+(1-\alpha)\phi}\right)^{\frac{1-\alpha}{\alpha}\phi} e^{\frac{\phi}{\alpha}z_{it}}\left(\frac{k_{it}}{S_t}\right)^{\phi} - \left(\frac{k_{it+1}}{S_t}-(1-\delta)\frac{k_{it}}{S_t}\right)\right. \\ &\quad \left. - \Gamma\left(z_{it},\frac{\boldsymbol{x}_{it}}{S_t};\boldsymbol{\Theta},1,1\right) + \left(\frac{\frac{b_{it+1}}{S_t}}{1+r(z_{it},\frac{\boldsymbol{x}_{it}}{S_t};\Theta,1,1)} - \frac{b_{it}}{S_t}\right) - \mathcal{T}(z_{it},\frac{\boldsymbol{x}_{it}}{S_t};\boldsymbol{\Theta},1,1) - \mathcal{C}(z_{it},\frac{\boldsymbol{x}_{it}}{S_t};\boldsymbol{\Theta},1,1)\right) \end{aligned}$$

Therefore, $e(z_{it},\boldsymbol{x}_{it};\boldsymbol{\Theta},w_t,Y_t) = S_t e(z_{it},\frac{\boldsymbol{x}_{it}}{S_t};\boldsymbol{\Theta},1,1)$. We now consider the steady-state of this economy: $w_t = w_{t+1} = w$ and $Y_t = Y_{t+1} = Y$. The Bellman equation 2 becomes:

$$V(z,k,b;\boldsymbol{\Theta},w,Y) = \left|\begin{array}{l} \max\limits_{k',b'} e(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y) + \dfrac{\mathbb{E}_{z'}[V(z',k',b';\boldsymbol{\Theta},w,Y)|z]}{1+r_f} \\ \boldsymbol{M}(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y) \leq 0 \end{array}\right.$$

Let T be the Bellman operator defined in the paper in equation (2). Consider the set of functions $\mathcal{F}$ such that for all $(z,k,b;\boldsymbol{\Theta},w,Y)$, $f(z,k,b;\boldsymbol{\Theta},w,Y) = S\times f(z,\frac{k}{S},\frac{b}{S};\boldsymbol{\Theta},1,1)$. If $f\in\mathcal{F}$, then $\mathcal{B}f\in\mathcal{F}$:

$$\begin{aligned} Tf(z,k,b;\boldsymbol{\Theta},w,Y) &= \left|\begin{array}{l} \max\limits_{k',b'} S\times\left(e(z,\frac{\boldsymbol{x}}{S};\boldsymbol{\Theta},1,1) + \dfrac{\mathbb{E}_{z'}[f(z',\frac{k'}{S},\frac{b'}{S};\boldsymbol{\Theta},1,1)|z]}{1+r_f}\right) \\ S\times\boldsymbol{M}(z,\frac{\boldsymbol{x}}{S};\boldsymbol{\Theta},1,1)\leq 0 \end{array}\right. \\ &= S\times\left|\begin{array}{l} \max\limits_{k',b'}\left\{e(z,\frac{k}{S},\frac{k'}{S},\frac{b}{S},\frac{b'}{S};\boldsymbol{\Theta},1,1) + \dfrac{\mathbb{E}_{z'}[f(z',\frac{k'}{S},\frac{b'}{S};\boldsymbol{\Theta},1,1)|z]}{1+r_f}\right\} \\ \boldsymbol{M}(z,\frac{k}{S},\frac{k'}{S},\frac{b}{S},\frac{b'}{S};\boldsymbol{\Theta},1,1)\leq 0 \end{array}\right. \\ &= S\times\left|\begin{array}{l} \max\limits_{k',b'}\left\{e(z,\frac{k}{S},k',\frac{b}{S},b';\boldsymbol{\Theta},1,1) + \dfrac{\mathbb{E}_{z'}[f(z',k',b';\boldsymbol{\Theta},1,1)|z]}{1+r_f}\right\} \\ \boldsymbol{M}(z,\frac{k}{S},k',\frac{b}{S},b';\boldsymbol{\Theta},1,1)\leq 0 \end{array}\right. \\ &= S\times Tf(z,\frac{k}{S},\frac{b}{S};\boldsymbol{\Theta},1,1) \end{aligned}$$

Since the contraction mapping theorem applies and $\mathcal{F}$ is a compact space, this implies that the value

function V also belongs to $\mathcal{F}$:

$$V(z,k,b;\mathbf{\Theta},w,Y) = S \times V(z,\frac{k}{S},\frac{b}{S};\mathbf{\Theta},1,1).$$

The previous equations also show that, in an economy with scale (w,Y), if (k',b') are the optimal policies for a firm with state variable (z,k,b), then $(\frac{k'}{S},\frac{b'}{S})$ are the optimal policies for a firm with state variables $(z,\frac{k}{S},\frac{b}{S})$ and in the economy with scale $(w=1,\ Y=1)$. As a result, the ergodic distribution of $\frac{k}{S}$ in the economy (w,Y) is equal to the ergodic distribution of k in the economy $(1,1)$.

Remember that, by definition in the steady-state, capital wedges are equal to:

$$(1+\tau_{it}) = \frac{\alpha\theta}{r_f+\delta}\frac{p_{it}y_{it}}{k_{it}} = \frac{\alpha\phi}{(\alpha+(1-\alpha)\phi)(r_f+\delta)}e^{\frac{\phi}{\alpha}z_{it}}\left(\frac{k_{it}}{S}\right)^{\phi}$$

Since the ergodic distribution of $(\frac{k}{S})$ in the economy (w,Y) is the same as the ergodic distribution of k in the economy $(1,1)$ and since the distribution of z is independent of (w,Y), this implies that, in the steady state, the distribution of wedges τ_{it} does not depend on (w,Y) and can be written $G(\tau_i;\Theta)$.

A.4 Proof of Proposition 3

Let the productivity of each be the sum of two terms:

$$z_{it} + z_i^*$$

where z_{it} is the same Markovian process as in the baseline model, while z_i^* is the fixed component, which differs across firms.

In this case, cash-flows to equity holders are:

$$e_{it} = e\left(z_{it}+z_i^*,\boldsymbol{x}_{it};\mathbf{\Theta},w,Y\right) = e^{\frac{\theta}{1-\theta}z_i^*}e\left(z_{it},\boldsymbol{\tilde{x}_{it}};\mathbf{\Theta},w,Y\right) \tag{14}$$

where $\boldsymbol{\tilde{x}_{it}} = \left(\frac{k_{it}}{e^{\frac{\theta}{1-\theta}z_i^*}},\frac{k_{it+1}}{e^{\frac{\theta}{1-\theta}z_i^*}},\frac{b_{it}}{e^{\frac{\theta}{1-\theta}z_i^*}},\frac{b_{it+1}}{e^{\frac{\theta}{1-\theta}z_i^*}}\right)$. This property rests on the homogeneity assumptions 7, as well as the multiplicativity of the Cobb-Douglas production function.

A consequence of this property is that the entire dynamic firm problem can be scaled by $e^{\frac{\theta}{1-\theta}z_i^*}$. Therefore, employment, capital and sales are all proportional to the same optimal policies of a firms with $z_i^*=0$, scaled by $e^{\frac{\theta}{1-\theta}z_i^*}$. This is summarized in the following Lemma:

Lemma 1. *Optimal policies are given by:*

$$\begin{aligned} k_{it} &= e^{\frac{\theta}{1-\theta}z_i^*}\tilde{k}_{it} \\ l_{it} &= e^{\frac{\theta}{1-\theta}z_i^*}\tilde{l}_{it} \\ py_{it} &= e^{\frac{\theta}{1-\theta}z_i^*}\tilde{py}_{it} \end{aligned}$$

where the terms with a tilde correpond to the optimal policies of firms with $z_i^*=0$ – the case investigated in the baseline model.

Proof. The proof is similar to the proof of proposition 2 discussed in Appendix A.3. We first show that the scaling property of $e()$ in Equation 14 carries over to the value function, so that for any state variables $(z^* + z, k, b)$:

$$V(z^* + z, k, b; \mathbf{\Theta}, w, Y) = e^{\frac{\theta}{1-\theta} z^*} V(z, \tilde{k}, \tilde{b}; \mathbf{\Theta}, w, Y)$$

and $(\tilde{k}, \tilde{b})$ correspond to the optimal policies of the dynamic program in the baseline model (Program 2, without long-term heterogeneity). □

Lemma 1 directly implies that wedges $(\frac{py_{it}}{k_{it}})$ are independent of productivity level z_i^*. Hence, Proposition 2 holds even in the presence of a fixed, firm-specific, component of firm productivity and our sufficient statistics on the moments of the distribution of empirical wedges are independent of the distribution of z_i^*'s.

Aggregation formulas and sufficient statistics are unchanged. We only do the proof for output here. Proof for TFP follows the same logic. Start from firm-level sales (omiting time subscript):

$$p_i y_i \propto \frac{Y}{w^{\frac{\theta(1-\alpha)}{1-\theta}}} e^{\underbrace{\frac{\theta(z_i^* + z_i)}{1-\theta} - \frac{\theta\alpha}{1-\theta}\log(1+\tau_i)}_{=v_i^* + v_i}}$$

where $v_i^* = \frac{\theta z_i^*}{1-\theta}$ and $v_i = \frac{\theta}{1-\theta}(z_i - \alpha \log(1+\tau_i))$.

Aggregating the above expression, after injection of the labor market clearing condition $Y \propto w^{1+\epsilon}$, we obtain:

$$\log Y = \frac{(1+\epsilon)(1-\theta)}{\theta(1-\alpha)} \log \int e^{v_i^* + v_i} di + \text{cst}$$

Since v_i^* and v_i are independent, this is equivalent to:

$$\log Y = \frac{(1+\epsilon)(1-\theta)}{\theta(1-\alpha)} \log \int e^{v_i^*} di + \underbrace{\frac{(1+\epsilon)(1-\theta)}{\theta(1-\alpha)} \log \int e^{v_i} di + \text{cst}}_{\text{log output without heterogeneity}}$$

Since the treatment does not affect the distribution of v_i^*, we can differentiate the last expression between the economy with no treatment and the first term drops. The effect of generalizing the treatment is therefore the same as in the absence of constant heterogeneity of productivity across firms and our baseline formula applies.

A.5 Proof of Proposition 4

Output Formula Sales are given by:

$$p_i y_i \propto \frac{Y}{w^{\frac{\theta(1-\alpha)}{1-\theta}}} e^{\underbrace{\frac{\theta z_i}{1-\theta} - \frac{\theta\alpha}{1-\theta}\log(1+\tau_i)}_{=v_i}}$$

and group-level sales:

$$\int_{i\in s} p_i y_i di \propto \frac{Y}{w^{\frac{\theta(1-\alpha)}{1-\theta}}} \underbrace{\int_{i\in s} e^{v_i} di}_{=e^{m(s)}}$$

Aggregating these group-level sales imply that: $w \propto \left(\sum_s e^{m(s)}\right]\Big)^{\frac{1-\theta}{\theta(1-\alpha)}}$. Combined with aggregate labor supply $Y \propto w^{1+\epsilon}$, this implies:

$$\log Y = \frac{(1+\epsilon)(1-\theta)}{\theta(1-\alpha)} \log\left(\sum_s e^{m(s)}\right) + \text{cst}$$

Define γ_s as the sales share of group s in the actual economy:

$$\gamma_s = \frac{\int_{i\in s} p_i^0 y_i^0 di}{Y^0} = \frac{e^{m_0(s)}}{\sum_{s'} e^{m_0(s')}}$$

The difference in log output between an economy where all firms receive the treatment and the initial economy where no firms receive the treatment is given by:

$$\Delta \log Y = \frac{(1+\epsilon)(1-\theta)}{\theta(1-\alpha)} \left[\log\left(\sum_s e^{m_0(s)+\Delta m(s)}\right) - \log\left(\sum_s e^{m_0(s)}\right)\right],$$

where $\Delta m(s) = m(s) - m_0(s)$. We now assume that $\Delta m(s) \ll 1$ so that, at the second-order:

$$\begin{aligned}\Delta \log Y &\approx \frac{(1+\epsilon)(1-\theta)}{\theta(1-\alpha)} \log\left(1 + \sum_s \gamma_s \Delta m(s) + \frac{1}{2}\sum_s \gamma_s (\Delta m(s))^2\right) \\ &\approx \frac{(1+\epsilon)(1-\theta)}{\theta(1-\alpha)} \left(\sum_s \gamma_s \Delta m(s) + \frac{1}{2}\text{var}_{\gamma_s}(\Delta m(s))\right),\end{aligned}$$

where $\text{var}_{\gamma_s}(\Delta m(s)) = \sum_s \gamma_s (\Delta m(s))^2 - (\sum_s \gamma_s \Delta m(s))^2$.

We now need to compute $\Delta m(s)$. We assume small deviation of $v_i = \frac{\theta z_i}{1-\theta} - \frac{\theta\alpha}{1-\theta}\log(1+\tau_i)$ around its mean. Assuming the treatment does not affect the distribution of log productivities, we obtain:

$$\begin{aligned}\Delta m(s) &= \Delta \log \int_{i\in s} e^{v_i} di \\ &\approx \Delta \mathbb{E}[v(s)] + \frac{1}{2}\Delta \text{var}\left(v(s)\right) \\ &= \frac{\theta\alpha}{(1-\theta)}\left(-\Delta\mu_\tau(s) + \frac{1}{2}\frac{\theta}{1-\theta}\left(\alpha\Delta\sigma_\tau^2(s) - 2\Delta\sigma_{z\tau}^2(s)\right)\right)\end{aligned}$$

which we plug back into the output formula and obtain:

$$\Delta \log(Y) = \frac{(1+\epsilon)\alpha}{1-\alpha} \sum_s \gamma_s \left(-\Delta\mu_\tau(s) + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)\left(-2\Delta\sigma_{z\tau}(s) + \alpha\Delta\sigma_\tau^2(s)\right)\right) + \frac{1}{2}\frac{(1+\epsilon)\alpha}{1-\alpha}\left(\frac{\theta\alpha}{1-\theta}\right)\text{var}_{\gamma_s}\left(\Delta\mu_\tau(s)\right)$$

TFP Formula

Start from:

$$\begin{aligned}\log TFP &= -\alpha \log \frac{K}{Y} + (1-\alpha)\log\frac{Y}{L} \\ &= -\alpha \log \frac{K}{Y} + (1-\alpha)\log w + \text{cst}\end{aligned}$$

We know that the labor supply curve is such that: $L \propto w^\epsilon$. Equilibrium in the labor market implies: $Y \propto wL$. Therefore, $w \propto Y^{\frac{1}{1+\epsilon}}$. To get the first term, we start from capital demand:

$$k_i = \alpha\theta\frac{p_i y_i}{R(1+\tau_i)} \propto \frac{Y}{w^{\frac{\theta(1-\alpha)}{1-\theta}}} e^{\underbrace{\frac{\theta z_i}{1-\theta} - \left(1+\frac{\theta(1-\alpha)}{1-\theta}\right)\log(1+\tau_i)}_{=h_i}}$$

Aggregating over the entire economy, we obtain:

$$\frac{K}{Y} \propto \frac{1}{w^{\frac{\theta(1-\alpha)}{1-\theta}}} \sum_s e^{n(s)}$$

where $n(s) = \log \int_{i\in s} e^{h_i} di$.

We can therefore write TFP as:

$$\begin{aligned}\log TFP &= -\alpha \log\left(\sum_s e^{n(s)}\right) + \left(\frac{\theta\alpha(1-\alpha)}{1-\theta} + 1 - \alpha\right)\log w \\ &= -\alpha \log \sum_s e^{n(s)} + (1-\alpha)\left(\frac{\theta\alpha}{1-\theta}+1\right)\frac{1}{1+\epsilon}\log Y\end{aligned}$$

Define κ_s as the capital share of group s in the original economy: $\kappa_s = \frac{\int_{i\in s} k_{is}^0 di}{K^0} = \frac{e^{n(s)}}{\sum_{s'} e^{n(s')}}$

We exploit the facts that:

$$\Delta \log \sum_s e^{n(s)} \approx \sum_s \kappa_s \Delta n(s) + \frac{1}{2}\text{var}_{\kappa_s}\Delta n(s),$$

where $\text{var}_{\kappa_s}(\Delta n(s)) = \sum_s \kappa_s (\Delta n(s))^2 - \left(\sum_s \kappa_s \Delta n(s)\right)^2$. Additionally:

$$\Delta n(s) \approx \left(1+\frac{\theta\alpha}{1-\theta}\right)\left(-\Delta\mu_\tau(s) + \frac{1}{2}\left(1+\frac{\theta\alpha}{1-\theta}\right)\Delta\sigma_\tau^2(s) - \frac{\theta}{1-\theta}\Delta\sigma_{z\tau}(s)\right)$$

which leads to the formula for TFP:

$$\begin{aligned}\Delta \log(TFP) = &-\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\sum_s \kappa_s \Delta\sigma_\tau^2(s) \\ &+\alpha\left(1+\frac{\alpha\theta}{1-\theta}\right)\sum_s(\gamma_s-\kappa_s)\left[-\Delta\mu_\tau(s)+\frac{1}{2}\left(\frac{\theta}{1-\theta}\right)\left(\alpha\Delta\sigma_\tau^2(s)-2\Delta\sigma_{z\tau}(s)\right)\right] \\ &+\frac{\alpha}{2}\left(1+\frac{\theta\alpha}{1-\theta}\right)\left(\frac{\alpha\theta}{1-\theta}\text{var}_{\gamma_s}\Delta\mu_\tau(s)-\left(1+\frac{\alpha\theta}{1-\theta}\right)\text{var}_{\kappa_s}\Delta\mu_\tau(s)\right)\end{aligned}$$

A.6 Proof of Proposition 5

Output Formula

The beginning of the proof is similar to the proof in Appendix A.5. In the initial steady-state, sales are given by:

$$p_i y_i \propto \frac{Y_0}{w_0^{\frac{\theta(1-\alpha)}{1-\theta}}} e^{\underbrace{\frac{\theta z_i}{1-\theta}-\frac{\theta\alpha}{1-\theta}\log(1+\tau_i)}_{=v_i}}$$

and group-level sales:

$$\int_{i\in s} p_i y_i di \propto \frac{Y_0}{w_0^{\frac{\theta(1-\alpha)}{1-\theta}}} \underbrace{\int_{i\in s} e^{v_i} di}_{=e^{m_0(s)}}$$

Aggregating these group-level sales imply that: $w_0 \propto \left(\sum_s e^{m_0(s)}]\right)^{\frac{1-\theta}{\theta(1-\alpha)}}$. Combined with aggregate labor supply $Y_0 \propto w_0^{1+\epsilon}$, this implies:

$$\log Y_0 = \frac{(1+\epsilon)(1-\theta)}{\theta(1-\alpha)}\log\left(\sum_s e^{m_0(s)}\right) + \text{cst}$$

Define γ_s as the sales share of group s in the initial steady-state:

$$\gamma_s = \frac{\int_{i\in s} p_i y_i di}{Y_0} = \frac{e^{m_0(s)}}{\sum_{s'} e^{m_0(s')}}$$

Similarly, in the post-reform steady-state, after input optimization, sales are given by:

$$p_i y_i \propto \frac{Y_1}{w_1^{\frac{\theta(1-\alpha)}{1-\theta}}} Z^{\frac{\theta}{1-\theta}} e^{\underbrace{\frac{\theta z_i}{1-\theta}-\frac{\theta\alpha}{1-\theta}\log(1+\tau_i)}_{=v_i}},$$

so that group-level sales become:

$$\int_{i\in s} p_i y_i di \propto \frac{Y_1}{w_1^{\frac{\theta(1-\alpha)}{1-\theta}}} Z^{\frac{\theta}{1-\theta}} \underbrace{\int_{i\in s} e^{v_i} di}_{=e^{m_1(s)}}$$

which implies the following expression for aggregate output Y_1 :

$$\log Y_1 = \frac{(1+\epsilon)(1-\theta)}{\theta(1-\alpha)} \left(\frac{\theta}{1-\theta} \log(Z) + \log \left(\sum_s e^{m_1(s)} \right) \right) + \text{cst},$$

where $m_1(s) = \int_{i\in s} e^{v_i} di$ in the post-reform steady-state and Z is the confounding, unexpected, aggregate productivity shifter that occurs around the reform. The difference in log output between steady-state 0 and steady-state 1 *that can be attributed to the reform* corresponds to:

$$\Delta \log Y = \frac{(1+\epsilon)(1-\theta)}{\theta(1-\alpha)} \left[\log \left(\sum_s e^{m_0(s)+\Delta m(s)} \right) - \log \left(\sum_s e^{m_0(s)} \right) \right],$$

where $\Delta m(s) = m_1(s) - m_0(s)$ and the unexpected aggregate productivity shock Z has been removed. We now assume that $\Delta m(s) \ll 1$ so that, at the second-order:

$$\begin{aligned} \Delta \log Y &\approx \frac{(1+\epsilon)(1-\theta)}{\theta(1-\alpha)} \log \left(1 + \sum_s \gamma_s \Delta m(s) + \frac{1}{2} \sum_s \gamma_s (\Delta m(s))^2 \right) \\ &\approx \frac{(1+\epsilon)(1-\theta)}{\theta(1-\alpha)} \left(\sum_s \gamma_s \Delta m(s) + \frac{1}{2} \text{var}_{\gamma_s}(\Delta m(s)) \right), \end{aligned}$$

where $\text{var}_{\gamma_s}(\Delta m(s)) = \sum_s \gamma_s (\Delta m(s))^2 - \left(\sum_s \gamma_s \Delta m(s)\right)^2$.

We now estimate $\Delta m(s)$ in the natural experiment:

$$\begin{aligned} \Delta m(s) &= \log \left(\int_{i\in s,1} e^{v_i} di \right) - \log \left(\int_{i\in s,0} e^{v_i} di \right) \\ &\approx \mathbb{E}[v(s)|1] - \mathbb{E}[v(s)|0] + \frac{1}{2} \left(\text{var}\left(v(s)|1\right) - \text{var}\left(v(s)|0\right) \right) \\ &= \frac{\theta\alpha}{(1-\theta)} \left(-\Delta\mu_\tau(s) + \frac{1}{2} \frac{\theta}{1-\theta} \left(\alpha \Delta\sigma_\tau^2(s) - 2\Delta\sigma_{z\tau}(s) \right) \right), \end{aligned}$$

where $\Delta\mu_\tau(s) = \mu_\tau(\Theta_1(s)) - \mu_\tau(\Theta_0)$, $\Delta\sigma_\tau^2(s) = \sigma_\tau^2(\Theta_1(s)) - \sigma_\tau^2(\Theta_0)$ and $\Delta\sigma_{z\tau}(s) = \sigma_{z\tau}(\Theta_1(s)) - \sigma_{z\tau}(\Theta_0)$.

Remember that for group s, the change in structural parameters is proportional to the aggregate change in structural parameters from the reform and the coefficient of proportionality is v_j:

$$\Theta_1(j) = \Theta_0 + v_j . d\Theta^{\text{reform}}$$

As a result, since $d\Theta^{\text{reform}} \ll 1$, we can write:

$$\Delta\mu_\tau(s) \approx v_s \left(\nabla\mu_\tau(0) \cdot d\Theta^{\text{reform}} \right) + \frac{1}{2}(v_s)^2 \left((d\Theta^{\text{reform}})^T \mathbb{H}\mu_\tau(\Theta(0)) d\Theta^{\text{reform}} \right)$$

Therefore, consider the following quadratic regression of changes in log-sales to capital ratio on v_j, the exposure of group j to the aggregate reform (i is a firm identifier, s the group it belongs to, 1 correspond to the post-reform period and 0 to the pre-reform period):

$$\log(\frac{p_{is,1}y_{is,1}}{k_{is,1}}) - \log(\frac{p_{is,0}y_{is,0}}{k_{is,0}}) = Av_s + B(v_s)^2$$

Then we naturally obtain that: $\widehat{\Delta\mu_\tau(s)} = \widehat{A}v_s + \widehat{B}v_s^2$. We can similarly estimate $\widehat{\Delta\sigma_\tau^2(s)}$ by computing the variance of the log-sales to capital ratio for firms in group s after and before the reform, and projecting the change in this variance on v_s and v_s^2. The same logic applies for the covariance of log-capital wedges and productivity, $\widehat{\Delta\sigma_{z\tau}(s)}$.

Formula for TFP

The proof for the TFP formula combines exactly the logic of proof for the output formula, with the TFP formula derived in Appendix A.5 and the estimation procedure of $\widehat{\Delta\mu_\tau(s)}$, $\widehat{\Delta\sigma_\tau^2(s)}$ and $\widehat{\Delta\sigma_{z\tau}(s)}$ described above.

A.7 Proof of Proposition 6

Formula for Output

Because there is perfect competition in the final good market, the demand for industry s bundle coming from the final good market is given by:

$$\phi_s PY = P_s Y_s \Rightarrow Y_s \propto \frac{Y}{P_s},$$

where we have normalized the price of the final good market to 1 ($P = 1$).

Perfect competition in the production of industry bundles leads to the following demand curve for product i in industry s:

$$P_s \left(\frac{q_{is}}{Q_s}\right)^{\theta_s - 1} = p_{is}$$

The first-order condition in the profit of firm i in industry s w.r.t. bundles from industry $j \in [1, S]$ implies that:

$$P_s Q_s^{1-\theta_s} \theta_s \gamma_{sj} \left(q_{is}\right)^{\theta_s} = P_j m_{isj}$$

As a result, the total demand for bundle j from firms in industry s simply comes from aggregating the previous equation across all firms i in industry s:

$$\theta_s \gamma_{sj} P_s Q_s = P_j \underbrace{\int_i m_{isj} di}_{=M_{sj}},$$

where M_{sj} corresponds to the demand for industry j's bundles coming from industry s.

As a result, the total demand for industry j bundles coming from intermediary inputs, $M_j = \sum_{s=1}^{S} M_{sj}$ is simply:

$$P_j M_j = \sum_{s=1}^{S} \theta_s \gamma_{sj} P_s Q_s$$

Remember that the demand for industry j bundles coming from the final good market is Y_j which satisfies $\phi_j Y = P_j Y_j$.

As a result, the total demand for industry j bundle is simply given by:

$$Q_j = M_j + Y_j = \frac{\sum_{s=1}^{S} \theta_s \gamma_{sj} P_s Q_s + \phi_j Y}{P_j} \Rightarrow P_j Q_j = \sum_{s=1}^{S} \theta_s \gamma_{sj} P_s Q_s + \phi_j Y$$

Note $\aleph = (\theta_s \gamma_{sj})_{(j,s)\in[1,S]^2}$, $\boldsymbol{P} = (P_s)_{s\in[1,S]}$, $\boldsymbol{Q} = (Q_s)_{s\in[1,S]}$ and $\boldsymbol{\phi} = (\phi_s)_{s\in[1,S]}$. $\circ$ denotes the Hadamard product of two matrixes and $\oslash$ the Hadamard division. The previous equation can be rewritten as:

$$(I - \aleph)\boldsymbol{P} \circ \boldsymbol{Q} = \boldsymbol{\phi} Y \Rightarrow \boldsymbol{P} \circ \boldsymbol{Q} = \left((I - \aleph)^{-1} \boldsymbol{\phi}\right) Y$$

Therefore, aggregate sales $P_s Q_s$ in each industry s are proportional to Y, although the coefficient is industry-specific.

Turning back to the optimisation problem, the labor first order condition for each firm leads to:

$$P_s Q_s^{1-\theta_s} \theta_s \beta_s \left(y_{is}\right)^{\theta_s} = w l_{is}$$

Aggregating across firm i in industry s, then across industries leads to:

$$wL = \sum_{s=1}^{S} \theta_s \beta_s P_s Q_s$$

Note $\boldsymbol{\theta} = (\theta_s)_{s\in[1,S]}$ and $\boldsymbol{\beta} = (\beta_s)_{s\in[1,S]}$, we have:

$$wL = (\boldsymbol{\theta} \circ \boldsymbol{\beta})' (\boldsymbol{P} \circ \boldsymbol{Q}) = (\boldsymbol{\theta} \circ \boldsymbol{\beta})' \left((I - \aleph)^{-1} \boldsymbol{\phi}\right) Y$$

Given that labor supply is given by $L^s = \bar{L} \left(\frac{w}{\bar{w}}\right)^{\epsilon}$, we see directly that: $Y \propto w^{1+\epsilon}$, which is the first part of the equilibrium.

We now need to compute the equilibrium wage. First order conditions in labor, capital, and inputs are given by:

$$\begin{cases} k_{is} = \alpha_s \theta_s \dfrac{p_{is} q_{is}}{R(1+\tau_i)} \\ l_{is} = \beta_s \theta_s \dfrac{p_{is} q_{is}}{w} \\ m_{isj} = \gamma_{sj} \theta_s \dfrac{p_{is} q_{is}}{P_j} \end{cases}$$

We can use the last three equations to compute firm i's output:

$$p_{is} q_{is} = e^{\theta_s z_i} P_s Q_s^{1-\theta_s} \left(p_{is} q_{is}\right)^{\theta_s} \left(\frac{\alpha_s \theta_s}{R(1+\tau_i)}\right)^{\alpha_s \theta_s} \left(\frac{\beta_s \theta_s}{w}\right)^{\beta_s \theta_s} \left[\prod_{j=1}^{S} \left(\frac{\gamma_{sj} \theta_s}{P_j}\right)^{\gamma_{sj} \theta_s}\right]$$

As a result:

$$p_{is}q_{is} = e^{\frac{\theta_s}{1-\theta_s}z_i} P_s^{\frac{1}{1-\theta_s}} Q_s \left(\frac{\alpha_s\theta_s}{R(1+\tau_i)}\right)^{\alpha_s\frac{\theta_s}{1-\theta_s}} \left(\frac{\beta_s\theta_s}{w}\right)^{\beta_s\frac{\theta_s}{1-\theta_s}} \left[\prod_{j=1}^{S}\left(\frac{\gamma_{sj}\theta_s}{P_j}\right)^{\gamma_{sj}\frac{\theta_s}{1-\theta_s}}\right]$$

We can aggregate the previous equation across all firms i industry s:

$$P_sQ_s = P_s^{\frac{1}{1-\theta_s}} Q_s \left(\frac{\alpha_s\theta_s}{R}\right)^{\alpha_s\frac{\theta_s}{1-\theta_s}} \left(\frac{\beta_s\theta_s}{w}\right)^{\beta_s\frac{\theta_s}{1-\theta_s}} \left[\prod_{j=1}^{S}\left(\frac{\gamma_{sj}\theta_s}{P_j}\right)^{\frac{\gamma_{sj}\theta_s}{1-\theta_s}}\right] \underbrace{\int \frac{e^{\frac{\theta_s}{1-\theta_s}z_i}}{(1+\tau_i)^{\alpha_s\frac{\theta_s}{1-\theta_s}}} di}_{=J_s}$$

The previous equation implies that the price of industry s bundles is proportional to:

$$P_s \propto w^{\beta_s}\left[\prod_{j=1}^{S}\left(P_j\right)^{\gamma_{sj}}\right] J_s^{-\frac{1-\theta_s}{\theta_s}}$$

Taking the logarithm of the previous equation, we get that:

$$\log(P_s) = \beta_s\log(w) - \frac{1-\theta_s}{\theta_s}\log(J_s) + \sum_{j=1}^{S}\gamma_{sj}\log(P_j) + cst,$$

To the second-order, we know that:

$$\frac{1-\theta_s}{\theta_s}\log(J_s) \approx \text{cst} + \alpha_s\left(-\mu_\tau + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha_s\sigma_\tau^2 - 2\sigma_{z\tau}\right)\right)$$

Define: $\log(\mathbf{A}) = \left(\mathbf{ff_s}\left(-\bar{\mathbf{\text{œ}}} + \frac{\mathbf{1}}{\mathbf{2}}\frac{\grave{\mathbf{s}}}{\mathbf{1}-\grave{\mathbf{s}}}\left(\mathbf{ff_s\text{œ}_\text{ø}^2} - \mathbf{2\text{œ}_{z\text{ø}}}\right)\right)\right)_{\mathbf{s}\in[\mathbf{1},\mathbf{S}]}$.
Then, the previous expression lead to the following matrix representation (at the second-order):

$$\log(\mathbf{P}) \approx (I-\Gamma)^{-1}\left(\boldsymbol{\beta}\log(w) - \log(\mathbf{A})\right) + cst$$

Now, remember that because of Cobb-Douglas aggregation in the final good market, $\Pi_{s=1}^{S}P_s^{\phi_s} = \Pi_{s=1}^{S}\phi_s^{\phi_s} = \text{cst}$. Hence, in log vector terms, we have that: $\boldsymbol{\phi}'\log P = cst$. Combining this with the above equation:

$$\log w \approx \frac{\boldsymbol{\phi}'(I-\Gamma)^{-1}\log(\mathbf{A})}{\boldsymbol{\phi}'(I-\Gamma)^{-1}\boldsymbol{\beta}} + cst,$$

which since $Y \propto w^{1+\epsilon}$ leads to the second-order approximation for output:

$$\log Y = (1+\epsilon)\frac{\boldsymbol{\phi}'(I-\Gamma)^{-1}\log(\mathbf{A})}{\boldsymbol{\phi}'(I-\Gamma)^{-1}\boldsymbol{\beta}} + cst,$$

Finally, note that $\alpha_s + \beta_s = 1 - \sum_u \gamma_{su}$, so that $\boldsymbol{\alpha} + \boldsymbol{\beta} = (I-\Gamma)E$, with $E = (1,1,\ldots,1)' \in [1,S]$. This implies:

$$\boldsymbol{\phi}'(I-\Gamma)^{-1}(\boldsymbol{\alpha}+\boldsymbol{\beta}) = \boldsymbol{\phi}'(I-\Gamma)^{-1}(I-\Gamma)E = \boldsymbol{\phi}'E = \sum_{s=1}^{S}\phi_s = 1$$

As a result:

$$\boldsymbol{\phi}'(I-\Gamma)^{-1}\boldsymbol{\beta} = 1 - \boldsymbol{\phi}'(I-\Gamma)^{-1}\boldsymbol{\alpha} = 1-\alpha^*,$$

where α^* is defined in Proposition 6. The rest of the output formula follows directly.

Formula for TFP

First, we show that in an efficient economy with input/output linkages and capital stock K, labor L, total output is such that: $Y^* \propto K^{\alpha^*} L^{1-\alpha^*}$.

An efficiently allocated economy with capital stock K and labor L is defined by:

$$Y^*(K,L) = \begin{vmatrix} \max\limits_{(k_{is}),(l_{is}),(m_{ius})} Y = \prod\limits_{s=1}^{S} Y_s^{\phi_s} \\ Y_s = \underbrace{\left(\int_i q_{is}^{\theta_s}\right)^{\frac{1}{\theta_s}}}_{=Q_s} - \sum\limits_{u=1}^{S} \underbrace{\left(\int_i m_{ius}\right)}_{=M_{us}}, \quad q_{is} = e^{z_{is}} k_{is}^{\alpha_s} l_{is}^{1-\alpha_s} \prod\limits_{u=1}^{S} m_{isu}^{\gamma_{su}} \\ \int_i k_{is} \leq K \quad (\lambda), \qquad \int_i l_{is} \leq L \quad (\mu) \end{vmatrix}$$

The first-order conditions w.r.t. k_{is},l_{is} and m_{isu} are:

$$\begin{cases} \frac{\phi_s}{Y_s} \alpha_s \frac{q_{is}^{\theta_s}}{k_{is}} Q_s^{1-\theta_s} = \lambda \\ \frac{\phi_s}{Y_s} (1-\alpha_s) \frac{q_{is}^{\theta_s}}{l_{is}} Q_s^{1-\theta_s} = \mu \\ \frac{\phi_s}{Y_s} \gamma_{su} \frac{q_{is}^{\theta_s}}{m_{isu}} Q_s^{1-\theta_s} = \frac{\phi_u}{Y_u} \end{cases}$$

The first-order condition for m_{ius} can be written as:

$$\frac{\phi_u}{Y_u} \gamma_{us} q_{iu}^{\theta_u} Q_u^{1-\theta_u} = \frac{\phi_s}{Y_s} m_{ius}$$

Aggregate across all firms in industry u:

$$\frac{\phi_u}{Y_u} \gamma_{us} Q_u = \frac{\phi_s}{Y_s} M_{us}$$

Sum across all industries u:

$$\sum_{u=1}^{S} \frac{\phi_u}{Y_u} \gamma_{us} Q_u = \frac{\phi_s}{Y_s} (Q_s - Y_s)$$

Let $\boldsymbol{Y} = (Y_s)_{s\in[1,S]}$. The previous equation across all industries s implies the following matrix equation:

$$(I - \Gamma')(\boldsymbol{\phi} \circ \boldsymbol{Q} \oslash \boldsymbol{Y}) = \boldsymbol{\phi} \Rightarrow \boldsymbol{\phi} \circ \boldsymbol{Q} \oslash \boldsymbol{Y} = (I - \Gamma')^{-1} \boldsymbol{\phi}$$

Aggregate the first-order condition w.r.t. k_{is} and l_{is} across all firms in industry s:

$$\alpha_s \frac{\phi_s Q_s}{Y_s} = \lambda K_s \quad \text{and} \quad (1-\alpha_s) \frac{\phi_s Q_s}{Y_s} = \mu L_s$$

Sum the previous equations across all industries s, in matrix form:

$$\lambda K = \boldsymbol{\alpha}' (I - \Gamma')^{-1} \boldsymbol{\phi} = \boldsymbol{\phi}' (I - \Gamma)^{-1} \boldsymbol{\alpha} = \alpha^* \quad \text{and} \quad \mu L = (1 - \alpha^*)$$

Now, we derive q_{is} by combining all the first-order conditions:

$$q_{is} = e^{\frac{z_{is}}{1-\theta_s}} \left(\frac{\phi_s}{Y_s}\right)^{\frac{1}{1-\theta_s}} Q_s \left(\frac{\alpha_s}{\lambda}\right)^{\frac{\alpha_s}{1-\theta_s}} \left(\frac{\beta_s}{\mu}\right)^{\frac{\beta_s}{1-\theta_s}} \prod_{u=1}^{S} \left(\frac{\gamma_{su} Y_u}{\phi_u}\right)^{\frac{\gamma_{su}}{1-\theta_s}}$$

Define $Z_s = \left(\int_i e^{\frac{\theta_s}{1-\theta_s} z_{is}}\right)^{\frac{1-\theta_s}{\theta_s}}$. Aggregating across all firms in the industry implies (after taking the power θ_s leads to:

$$1 = Z_s \left(\frac{\phi_s}{Y_s}\right) \left(\frac{\alpha_s}{\lambda}\right)^{\alpha_s} \left(\frac{\beta_s}{\mu}\right)^{\beta_s} \prod_{u=1}^{S} \left(\frac{\gamma_{su} Y_u}{\phi_u}\right)^{\gamma_{su}}$$

Define $\mathbf{log}(\boldsymbol{Z}) = (\log(Z_s))_{s\in[1,S]}$, $\mathbf{log}(\boldsymbol{Y}) = (\log(Y_s))_{s\in[1,S]}$, $\mathbf{log}(\boldsymbol{\phi}) = (\log(\phi_s))_{s\in[1,S]}$. The previous equations implies across industries s imply the following matrix equation:

$$(I-\Gamma)\,\mathbf{log}(\boldsymbol{Y}) = (I-\Gamma)\,\mathbf{log}(\boldsymbol{\phi}) + \mathbf{log}(\boldsymbol{Z}) + \boldsymbol{\alpha} \circ \mathbf{log}(\boldsymbol{\alpha}) + \boldsymbol{\beta} \circ \mathbf{log}(\boldsymbol{\beta}) - \boldsymbol{\alpha}\log(\lambda) - \boldsymbol{\beta}\log(\mu)$$

Remember that: $\lambda K = \alpha^*$ and $\mu L = (1-\alpha^*)$. Therefore, define $\boldsymbol{\alpha}\,\mathbf{log}(\boldsymbol{\alpha}) = (\alpha_s \log(\alpha_s))_{s\in[1,S]}$ and $\boldsymbol{\beta}\,\mathbf{log}(\boldsymbol{\beta}) = (\beta_s \log(\beta_s))_{s\in[1,S]}$:

$$(I-\Gamma)\,\mathbf{log}(\boldsymbol{Y}) = (I-\Gamma)\,\mathbf{log}(\boldsymbol{\phi}) + \mathbf{log}(\boldsymbol{Z}) + \boldsymbol{\alpha}\log(K) + (1-\boldsymbol{\alpha})\log(L) + \text{cst},$$

where cst depends on the model's parameters and is independent of (K, L). Remember that total output is such that:

$$\log(Y) = \phi'\,\mathbf{log}(\boldsymbol{Y})$$

The last two equations imply that total output in the efficient economy is given by:

$$\log(Y^*(K,L)) = \text{cst} + \boldsymbol{\phi}'(I-\Gamma)^{-1}\,\mathbf{log}(\boldsymbol{Z}) + \alpha^*\log(K) + (1-\alpha^*)\log(L),$$

where cst does not depend on (K, L). Therefore, as in our baseline case, we define aggregate TFP: as $Y = \text{TFP} \times K^{\alpha^*} L^{1-\alpha^*}$, which corresponds to the output loss experienced in the actal economy relative to the efficient economy with the same amount of aggregate capital and labor than the actual economy.

We start with the following expression:

$$\log TFP = -\alpha^* \log\frac{K}{Y} - (1-\alpha^*)\log\frac{L}{Y}$$

We start with the labor term. Knowing that $wL_s = \alpha_s\theta_s P_s Q_s$ and that $P_s Q_s$ are fixed fractions of Y, we obtain that $\frac{L}{Y}$ is proportional fo $\frac{1}{w}$, hence, given our final derivation for log output:

$$\log\frac{L}{Y} = -\frac{\boldsymbol{\phi'}\left(I-\Gamma'\right)^{-1}\log(\mathbf{A})}{1-\alpha^*} + cst$$

We note ϕ_s^* the s^{th} element of $(I-\Gamma)^{-1}\boldsymbol{\phi}$, as the linkage-adjusted industry share. Then:

$$\Delta\log\frac{L}{Y} = -\sum_s \left(\frac{\alpha_s\phi_s^*}{1-\alpha^*}\right)\left(-\Delta\mu_\tau(s) + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha_s\Delta\sigma_\tau^2(s) - 2\Delta\sigma_{z\tau}(s)\right)\right)$$

We now compute the second-term. Start with the fact that:

$$\frac{Y}{K} = \sum_{s=1}^{S} \frac{K_s}{K} \frac{Y_s}{K_s}$$

We need to calculate K_s. Note that:

$$p_{is} q_{is} = \frac{P_s Q_s}{J_s} \frac{e^{\frac{\theta_s}{1-\theta_s} z_i}}{(1+\tau_i)^{\frac{\alpha_s \theta_s}{1-\theta_s}}}$$

so that capital demand is given by:

$$k_{is} = \theta_s \alpha_s \frac{P_s Q_s}{R J_s} \frac{e^{\frac{\theta_s}{1-\theta_s} z_i}}{(1+\tau_i)^{1+\frac{\alpha_s \theta_s}{1-\theta_s}}}$$

so that the industry level capital stock is:

$$K_s = \theta_s \alpha_s \frac{P_s Q_s}{R J_s} \underbrace{\int \frac{e^{\frac{\theta_s}{1-\theta_s} z_i}}{(1+\tau_i)^{1+\frac{\alpha_s \theta_s}{1-\theta_s}}} di}_{I_s}$$

Using the fact that $\boldsymbol{P} \circ \boldsymbol{Q} = \left((I - \aleph)^{-1} \boldsymbol{\phi} \right) Y$, we obtain

$$\frac{K_s}{Y} = \eta_s \frac{I_s}{J_s}$$

where $\eta_s = \boldsymbol{\alpha} \circ \boldsymbol{\theta} \circ \left((I - \aleph)^{-1} \boldsymbol{\phi} \right)$ is a function of parameters ($\boldsymbol{\alpha}$, $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$). Hence:

$$\log \frac{K}{Y} = \log \left(\sum_s \eta_s \frac{I_s}{J_s} \right)$$

Define $\delta_s = \Delta \log \left(\frac{I_s}{J_s} \right)$ and K^1, Y^1 (resp. K^0, Y^0) the capital stock and output in the economy where all firms receive the treatment (resp. where no firms receive the treatment). To the second-order:

$$\begin{aligned}
\log \frac{K^1}{Y^1} &= \log \left(\sum_s \eta_s \frac{I_s^0}{J_s^0} e^{\delta_s} \right) \\
&\approx \log \left(\sum_s \eta_s \frac{I_s^0}{J_s^0} \left(1 + \delta_s + \frac{\delta_s^2}{2} \right) \right) \\
&\approx \log \frac{K^0}{Y^0} + \log \left(1 + \sum_s \kappa_s \left(\delta_s + \frac{\delta_s^2}{2} \right) \right) \\
&\approx \log \frac{K^0}{Y^0} + \sum_s \kappa_s \delta_s + \frac{1}{2} \left(\sum_s \kappa_s \delta_s^2 - \left(\sum_s \kappa_s \delta_s \right)^2 \right)
\end{aligned}$$

where $\kappa_s = \frac{K_s^0}{K^0}$ is the capital share of each industry in the original economy.

A second order Taylor expansion leads to:

$$\delta_s = -\Delta \mu_\tau(s) + \frac{1}{2} \left(\left(1 + 2 \frac{\alpha_s \theta_s}{1-\theta_s} \right) \Delta \sigma_\tau^2(s) - 2 \frac{\theta_s}{1-\theta_s} \Delta \sigma_{z\tau}(s) \right),$$

which we can use to find how the ratio of capital K to output Y varies in our aggregate counterfactual:

$$\Delta \log \frac{K}{Y} = \sum_s \kappa_s - \Delta\mu_\tau(s) + \frac{1}{2}\left(\left(1 + 2\frac{\alpha_s\theta_s}{1-\theta_s}\right)\Delta\sigma_\tau^2(s) - 2\frac{\theta_s}{1-\theta_s}\Delta\sigma_{z\tau}(s)\right) + \frac{1}{2}\text{var}_{\kappa_s}\left(\mu(s)\right)$$

This leads to the TFP formula:

$$\begin{aligned}\Delta \log TFP = &-\alpha^* \sum_s \kappa_s \left(-\Delta\mu_\tau(s) + \frac{1}{2}\left(\left(1 + 2\frac{\alpha_s\theta_s}{1-\theta_s}\right)\Delta\sigma_\tau^2(s) - 2\frac{\theta_s}{1-\theta_s}\Delta\sigma_{z\tau}(s)\right)\right) - \frac{\alpha^*}{2}\text{var}_{\kappa_s}\left(\mu(s)\right) \\ &+ \sum_s \alpha_s\phi_s^* \left(-\Delta\mu_\tau(s) + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha_s\Delta\sigma_\tau^2(s) - 2\Delta\sigma_{z\tau}(s)\right)\right)\end{aligned}$$

We can re-organize this last equation into:

$$\begin{aligned}\Delta \log TFP = &-\frac{\alpha^*}{2}\sum_s \kappa_s \left(1 + \frac{\alpha_s\theta_s}{1-\theta_s}\right)\Delta\sigma_\tau^2(s) - \frac{\alpha^*}{2}\text{var}_{\kappa_s}\left(\mu(s)\right) \\ &+ \sum_s \left(\alpha_s\phi_s^* - \alpha^*\kappa_s\right)\left(-\Delta\mu_\tau(s) + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha_s\Delta\sigma_\tau^2(s) - 2\Delta\sigma_{z\tau}(s)\right)\right)\end{aligned}$$

A.8 Proof of Proposition 7

We consider here S heterogeneous industries with M_s firms operating in industry s. The setup is similar to Section 6, except that there is no input-output linkages and that the final good market produces by combining industry outputs according to a CES production function:

$$Y = \left(\sum_{k=1}^{S} \chi_s Y_s^{\psi}\right)^{\frac{1}{\psi}}, \text{ with } \sum_{s=1}^{S} \chi_s = 1$$

Each firm within industry s has a Cobb-Douglas production function with the same factor shares, however the price-elasticity of demand is allowed to vary by industry

$$y_{it} = e^{z_{it}} k_{it}^{\alpha} l_{it}^{1-\alpha}, \qquad Y_s = \left(\int_i y_{is}^{\theta_s} di\right)^{\frac{1}{\theta_s}}$$

Profit maximization in the final good market gives the demand for industry s output:

$$\frac{P_s}{P} = \chi_s \left(\frac{Y_s}{Y}\right)^{\psi-1}$$

Similarly, profit maximization in industry s gives the demand for firm i in industry s:

$$\frac{p_{is}}{P_s} = \left(\frac{y_{is}}{Y_s}\right)^{\theta_s - 1}$$

Labor demand for firm i comes from:

$$\max_{l_{is}}\{p_{is}y_{is} - wl_{is}\} = \max_{l_{is}}\left(\chi_s P\left(\frac{Y_s}{Y}\right)^{\psi-1} Y_s^{1-\theta_s}\right) y_{is}^{\theta_s} - wl_{is}$$

$$\implies l_{is} = \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{1}{1-(1-\alpha)\theta_s}} \left(\chi_s P \left(\frac{Y_s}{Y}\right)^{\psi-1} Y_s^{1-\theta_s}\right)^{\frac{1}{1-(1-\alpha)\theta_s}} e^{\frac{\theta_s}{1-(1-\alpha)\theta_s} z_{is}} k_{is}^{\frac{\alpha\theta_s}{1-(1-\alpha)\theta_s}}$$

we have, for each firm in industry s: $(1-\alpha)\theta_s p_{is} y_{is} = w l_{is}$. Replacing above yields:

$$p_{is} y_{is} = \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\theta_s}{1-(1-\alpha)\theta_s}} \left(\chi_s P \left(\frac{Y_s}{Y}\right)^{\psi-1} Y_s^{1-\theta_s}\right)^{\frac{1}{1-(1-\alpha)\theta_s}} e^{\frac{\theta_s}{1-(1-\alpha)\theta_s} z_{is}} k_{is}^{\frac{\alpha\theta_s}{1-(1-\alpha)\theta_s}} \tag{15}$$

The first-order condition for firm i capital is simply: $(1+\tau_{is})R = \alpha\theta_s \frac{p_{is} y_{is}}{k_{is}}$. Combining with the labor first-order condition, we obtain:

$$k_{is} = \left(\chi_s \left(\frac{Y}{Y_s}\right)^{1-\psi} Y_s^{1-\theta_s}\right)^{\frac{1}{1-\theta_s}} \left(\frac{\alpha\theta_s}{(1+\tau_{is})R}\right)^{\frac{1-(1-\alpha)\theta_s}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\theta_s}{1-\theta_s}} e^{\frac{\theta_s}{1-\theta_s} z_{is}}$$

Firm i output can be written as:

$$p_{is} y_{is} = \left(\chi_s \left(\frac{Y}{Y_s}\right)^{1-\psi} Y_s^{1-\theta_s}\right)^{\frac{1}{1-\theta_s}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\theta_s}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\theta_s}{1-\theta_s}} \left(\frac{e^{z_{is}}}{(1+\tau_{is})^{\alpha}}\right)^{\frac{\theta_s}{1-\theta_s}}$$

We can combine the demand equation for firm i and industry s output to write firm i production in the following way:

$$y_{is} = \left(\chi_s \left(\frac{Y}{Y_s}\right)^{1-\psi} Y_s^{1-\theta_s}\right)^{\frac{1}{1-\theta_s}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)}{1-\theta_s}} \left(\frac{e^{z_{is}}}{(1+\tau_{is})^{\alpha}}\right)^{\frac{1}{1-\theta_s}}$$

Aggregating within the industry, and since $Y_s = (\int_i y_{is}^{\theta_s})^{\frac{1}{\theta_s}}$:

$$\left(\frac{Y_s}{Y}\right)^{(1-\psi)\frac{\theta_s}{1-\theta_s}} = \chi_s^{\frac{\theta_s}{1-\theta_s}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\theta_s}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\theta_s}{1-\theta_s}} \underbrace{\left(\int_i \left(\frac{e^{z_{is}}}{(1+\tau_{is})^{\alpha}}\right)^{\frac{\theta_s}{1-\theta_s}} di\right)}_{I_s}$$

Since $Y = \left(\sum_{s=1}^{S} \chi_s Y_s^{\psi}\right)^{\frac{1}{\psi}}$, we can sum across industries s to pin down the wage:

$$1 = \sum_{s=1}^{S} \chi_s^{\frac{1}{1-\psi}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\psi}{1-\psi}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\psi}{1-\psi}} I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}$$

$$\implies w = \left[\sum_{s=1}^{S} \chi_s^{\frac{1}{1-\psi}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\psi}{1-\psi}} \left((1-\alpha)\theta_s\right)^{\frac{(1-\alpha)\psi}{1-\psi}} I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}\right]^{\frac{1-\psi}{(1-\alpha)\psi}}$$

Aggregate labor market clearing yields the following:

$$(1-\alpha)\theta_s p_{is} y_{is} = w l_{is} \implies L_s = \frac{(1-\alpha)\theta_s}{w} P_s Y_s$$

$$\bar{L}\left(\frac{w}{\bar{w}}\right)^{1+\epsilon} = \sum_{s=1}^{S}\left(\frac{(1-\alpha)\theta_s}{\bar{w}}\right)P_sY_s = Y\sum_{s=1}^{S}\left(\frac{(1-\alpha)\theta_s}{\bar{w}}\right)\chi_s\left(\frac{Y_s}{Y}\right)^{\psi}$$

Therefore:

$$Y = \frac{\frac{\bar{L}}{\bar{w}^{1+\epsilon}}\left[\sum_{s=1}^{S}\chi_s^{\frac{1}{1-\psi}}\left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\psi}{1-\psi}}\left((1-\alpha)\theta_s\right)^{\frac{(1-\alpha)\psi}{1-\psi}}I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}\right]^{1+\frac{(1-\psi)(1+\epsilon)}{(1-\alpha)\psi}}}{\sum_{s=1}^{S}\left(\frac{(1-\alpha)\theta_s}{\bar{w}}\right)\chi_s^{\frac{1}{1-\psi}}\left((1-\alpha)\theta_s\right)^{(1-\alpha)\frac{\psi}{1-\psi}}\left(\frac{\alpha\theta_s}{R}\right)^{\alpha\frac{\psi}{1-\psi}}I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}}$$

Let $I_s(1)$ (resp. $I_s(0)$) be the value of I_s when $\Theta^s = \Theta_1^s$ (resp. Θ_0^s). Let $\Delta_s = \log(I_s(1)) - \log(I_s(0))$. Note w_0 the wage in the original economy. We start with the effect of treatment on the log numerator:

$$\Delta_1 = \log\left[\sum_{s=1}^{S}\chi_s^{\frac{1}{1-\psi}}\left((1-\alpha)\theta_s\right)^{(1-\alpha)\frac{\psi}{1-\psi}}\left(\frac{\alpha\theta_s}{R}\right)^{\alpha\frac{\psi}{1-\psi}}I_s(1)^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}\right]$$

$$-\log\left[\sum_{s=1}^{S}\chi_s^{\frac{1}{1-\psi}}\left((1-\alpha)\theta_s\right)^{(1-\alpha)\frac{\psi}{1-\psi}}\left(\frac{\alpha\theta_s}{R}\right)^{\alpha\frac{\psi}{1-\psi}}I_s(0)^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}\right]$$

$$= \log\left(\frac{\sum_{s=1}^{S}\chi_s^{\frac{1}{1-\psi}}\left(\frac{(1-\alpha)\theta_s}{w_0}\right)^{(1-\alpha)\frac{\psi}{1-\psi}}\left(\frac{\alpha\theta_s}{R}\right)^{\alpha\frac{\psi}{1-\psi}}I_s(1)^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}}{\sum_{s=1}^{S}\chi_s^{\frac{1}{1-\psi}}\left(\frac{(1-\alpha)\theta_s}{w_0}\right)^{(1-\alpha)\frac{\psi}{1-\psi}}\left(\frac{\alpha\theta_s}{R}\right)^{\alpha\frac{\psi}{1-\psi}}I_s(0)^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}}\right)$$

$$= \log\left(\frac{\sum_{s=1}^{S}\chi_s^{\frac{1}{1-\psi}}\left(\frac{(1-\alpha)\theta_s}{w_0}\right)^{(1-\alpha)\frac{\psi}{1-\psi}}\left(\frac{\alpha\theta_s}{R}\right)^{\alpha\frac{\psi}{1-\psi}}I_s(0)^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}e^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}\Delta_s}}{\sum_{s=1}^{S}\chi_s^{\frac{1}{1-\psi}}\left(\frac{(1-\alpha)\theta_s}{w_0}\right)^{(1-\alpha)\frac{\psi}{1-\psi}}\left(\frac{\alpha\theta_s}{R}\right)^{\alpha\frac{\psi}{1-\psi}}I_s(0)^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}}\right)$$

Recall that

$$\chi_s\left(\frac{Y_s^0}{Y^0}\right)^{\psi} = \frac{P_s^0Y_s^0}{P^0Y^0} = \chi_s^{\frac{1}{1-\psi}}\left(\frac{(1-\alpha)\theta_s}{w_0}\right)^{(1-\alpha)\frac{\psi}{1-\psi}}\left(\frac{\alpha\theta_s}{R}\right)^{\alpha\frac{\psi}{1-\psi}}I_s(0)^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}$$

$$\implies \Delta_1 = \log\left(\frac{\sum_{s=1}^{S}\frac{P_s^0Y_s^0}{P^0Y^0}e^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}\Delta_s}}{\sum_{s=1}^{S}\frac{P_s^0Y_s^0}{P^0Y^0}}\right) = \log\left(\sum_{s=1}^{S}\frac{P_s^0Y_s^0}{P^0Y^0}e^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}\Delta_s}\right)$$

The treatment effect on each industry is weighted by its share of the total output in the initial economy. Let $\gamma_s = \frac{P_s^0Y_s^0}{P^0Y^0}$, and $u_s = \frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}\Delta_s$. The Taylor expansion gives:

$$\Delta_1 = \log\left(\sum_{s=1}^{S}\gamma_s e^{u_s}\right) \approx \log\left(\sum_{s=1}^{S}\gamma_s\left(1+u_s+\frac{u_s^2}{2}\right)\right) \approx \sum_{s=1}^{S}\gamma_s u_s + \frac{1}{2}\sum_{s=1}^{S}\gamma_s u_s^2 - \frac{1}{2}\left(\sum_{s=1}^{S}\gamma_s u_s\right)^2$$

Replace u_s with its approximation value:

$$u_s \approx \frac{\psi}{1-\psi}\left[-\alpha\Delta\mu_\tau(s) + \frac{1}{2}\left(\frac{\theta_s}{1-\theta_s}\right)\left(-2\alpha\Delta\sigma_{z\tau}(s) + \alpha^2\Delta\sigma_\tau^2(s)\right)\right] \tag{16}$$

$$\implies \Delta_1 \approx \sum_{s=1}^{S} \frac{\gamma_s \alpha\psi}{1-\psi}\left[-\Delta\mu_\tau(s) + \frac{1}{2}\left(\frac{\theta_s}{1-\theta_s}\right)\left(-2\Delta\sigma_{z\tau}(s) + \alpha\Delta\sigma_\tau^2(s)\right)\right]$$

$$+\frac{1}{2}\left(\frac{\psi}{1-\psi}\right)^2 \alpha^2 \left(\sum_{s=1}^{S}\gamma_s\left(\Delta\mu_\tau(s)\right)^2 - \frac{1}{2}\left(\sum_{s=1}^{S}\gamma_s\Delta\mu_\tau(s)\right)^2\right)$$

For the effect of treatment on log denominator (Δ_2), we have:

$$\Delta_2 = \log\left(\sum_{s=1}^{S}\left(\frac{(1-\alpha)\theta_s}{\bar{w}}\right)\chi_s^{\frac{1}{1-\psi}}\left((1-\alpha)\theta_s\right)^{\frac{(1-\alpha)\psi}{1-\psi}}\left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\psi}{1-\psi}} I_s(1)^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}\right)$$

$$-\log\left(\sum_{s=1}^{S}\left(\frac{(1-\alpha)\theta_s}{\bar{w}}\right)\chi_s^{\frac{1}{1-\psi}}\left((1-\alpha)\theta_s\right)^{\frac{(1-\alpha)\psi}{1-\psi}}\left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\psi}{1-\psi}} I_s(0)^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}\right)$$

$$\Delta_2 = \log\left(\frac{\sum_{s=1}^{S}\theta_s\frac{P_s^0Y_s^0}{P^0Y^0}e^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}\Delta_s}}{\sum_{s=1}^{S}\theta_s\frac{P_s^0Y_s^0}{P^0Y^0}}\right) = \log\left(\sum_{s=1}^{S}\theta_s\gamma_s e^{u_s}\right) - \log\left(\sum_{s=1}^{S}\theta_s\gamma_s\right)$$

$$\approx \log\left(\sum_{s=1}^{S}\theta_s\gamma_s\left(1+u_s+\frac{u_s^2}{2}\right)\right) - \log\left(\sum_{s=1}^{S}\theta_s\gamma_s\right) = \log\left(1+\frac{\sum_{s=1}^{S}\theta_s\gamma_s u_s + \frac{1}{2}\sum_{s=1}^{S}\theta_s\gamma_s u_s^2}{\sum_{s=1}^{S}\theta_s\gamma_s}\right)$$

$$\Delta_2 \approx \frac{\sum_{s=1}^{S}\theta_s\gamma_s u_s + \frac{1}{2}\sum_{s=1}^{S}\theta_s\gamma_s u_s^2}{\sum_{s=1}^{S}\theta_s\gamma_s} - \frac{1}{2}\left(\frac{\sum_{s=1}^{S}\theta_s\gamma_s u_s}{\sum_{s=1}^{S}\theta_s\gamma_s}\right)^2$$

Replacing u_s with equation (16) gives:

$$\Delta_2 \approx \frac{\alpha\psi}{(1-\psi)}\sum_{s=1}^{S}\iota_s\left[-\Delta\mu_\tau(s) + \frac{1}{2}\left(\frac{\theta_s}{1-\theta_s}\right)\left(-2\Delta\sigma_{z\tau}(s) + \alpha\Delta\sigma_\tau^2(s)\right)\right]$$

$$+\frac{1}{2}\left(\frac{\psi\alpha}{1-\psi}\right)^2\left(\sum_{s=1}^{S}\iota_s\left(\Delta\mu_\tau(s)\right)^2 - \left(\sum_{s=1}^{S}\iota_s\Delta\mu_\tau(s)\right)^2\right)$$

after noting $\iota_s = \frac{\theta_s\gamma_s}{\sum_{s=1}^{S}\theta_s\gamma_s}$. The overall effect of the reform on aggregate output is then given by:

$$\Delta\log(Y) = \left(1 + \frac{(1-\psi)(1+\epsilon)}{(1-\alpha)\psi}\right)\Delta_1 - \Delta_2$$

We inject the two expressions for Δ_1 and Δ_2 and obtain:

$$\begin{aligned}\Delta \log(Y) = &\frac{(1+\epsilon)\alpha}{1-\alpha}\sum_{s=1}^{S}\gamma_s\left[-\Delta\mu_\tau(s)+\frac{1}{2}\left(\frac{\theta_s}{1-\theta_s}\right)\left(-2\Delta\sigma_{z\tau}(s)+\alpha\Delta\sigma_\tau^2(s)\right)\right]\\&+\frac{1}{2}\left(\frac{\alpha\psi}{1-\psi}\right)\left(\frac{\alpha}{1-\alpha}\right)(1+\epsilon)Var_{\gamma_s}\left(\Delta\mu_\tau(s)\right)\\&+\frac{\alpha\psi}{1-\psi}\sum_{s=1}^{S}(\gamma_s-\iota_s)\left[-\Delta\mu_\tau(s)+\frac{1}{2}\left(\frac{\theta_s}{1-\theta_s}\right)\left(-2\Delta\sigma_{z\tau}(s)+\alpha\Delta\sigma_\tau^2(s)\right)\right]\\&+\frac{1}{2}\left(\frac{\psi\alpha}{1-\psi}\right)^2\left[\text{var}_{\gamma_s}\left(\Delta\mu_\tau(s)\right)-\text{var}_{\iota_s}\left(\Delta\mu_\tau(s)\right)\right]\end{aligned}$$

where $\text{var}^\star_\omega\left(\Delta\mu_\tau(s)\right)=\sum_s\omega_s\left(\Delta\mu_\tau(s)\right)^2-\left(\sum_s\omega_s\Delta\mu_\tau(s)\right)^2$.

A.9 Proof of Proposition 8

In this Appendix, we derive approximate aggregation formulas in the case where (1) intermediate input producers follow a CES technology as in Equation (12) and (2) production is organized as in Section 2.

A.9.1 Equilibrium Formulas & Notations

Given the firm-level production function, firm output is given by: $p_iy_i=Y^{1-\theta}e^{\theta z_i}[\alpha k_i^\rho+(1-\alpha)l_i^\rho]^{\frac{\theta}{\rho}}$. We combine this equation with the definition of the capital wedge τ_i and the static FOC in labor. We obtain:

$$\begin{cases}k_i=Ye^{\frac{\theta}{1-\theta}z_i}\left(\frac{\alpha\theta}{R(1+\tau_i)}\right)^{\frac{1}{1-\rho}}\left[\alpha\left(\frac{\alpha\theta}{R(1+\tau_i)}\right)^{\frac{\rho}{1-\rho}}+(1-\alpha)\left(\frac{(1-\alpha)\,\theta}{w}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{\theta-\rho}{\rho(1-\theta)}}\\l_i=Ye^{\frac{\theta}{1-\theta}z_i}\left(\frac{(1-\alpha)\theta}{w}\right)^{\frac{1}{1-\rho}}\left[\alpha\left(\frac{\alpha\theta}{R(1+\tau_i)}\right)^{\frac{\rho}{1-\rho}}+(1-\alpha)\left(\frac{(1-\alpha)\,\theta}{w}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{\theta-\rho}{\rho(1-\theta)}}\end{cases},$$

and optimal firm-level output is simply:

$$p_iy_i=e^{z_i\frac{\theta}{1-\theta}}Y\left[\alpha\left(\frac{\alpha\theta}{R(1+\tau_i)}\right)^{\frac{\rho}{1-\rho}}+(1-\alpha)\left(\frac{(1-\alpha)\,\theta}{w}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}\frac{\theta}{1-\theta}},$$

Note μ and σ_τ^2 the mean and variance of $\log(1+\tau)$. Furthermore, note:

$$\begin{aligned}a&=(1-\alpha)\left(\frac{(1-\alpha)\,\theta}{w^*}\right)^{\frac{\rho}{1-\rho}}\\b&=\alpha\left(\frac{\alpha\theta}{Re^\mu}\right)^{\frac{\rho}{1-\rho}}\end{aligned}$$

where w^* is the wage prevailing in a fictitious economy where $\log(1+\tau_i)=\mu$ for all firms. We note $\delta_w=\log(w/w^*)$ and $\delta_i=\log(1+\tau_i)-\mu$ the deviation of the equilibrium from this hypothetical equilibrium

with homogeneous distiortion.

The general equilibrium of this economy (w,Y) is defined by the labor and product market clearing conditions:

$$\begin{cases} 1 = \int_{z,\tau} e^{z_i \frac{\theta}{1-\theta}} \left[a e^{-\frac{\rho}{1-\rho}\delta_w} + b e^{-\frac{\rho}{1-\rho}\delta_i} \right]^{\frac{1-\rho}{\rho}\frac{\theta}{1-\theta}} di \\ \bar{L} \left(\frac{w}{\bar{w}} \right)^{\epsilon} = Y \left(\frac{(1-\alpha)\theta}{w} \right)^{\frac{1}{1-\rho}} \int_i e^{\frac{\theta z_i}{1-\theta}} \left[a e^{-\frac{\rho}{1-\rho}\delta_w} + b e^{-\frac{\rho}{1-\rho}\delta_i} \right]^{\frac{\theta-\rho}{\rho(1-\theta)}} di \end{cases}$$

which define w and Y as functions of the moments of the distributions of log capital wedges and log productivity.

Note that, by definition of a, b, we have that:

$$\begin{cases} 1 = (a+b)^{\frac{1-\rho}{\rho}\frac{\theta}{1-\theta}} \int_{z,\tau} e^{z_i \frac{\theta}{1-\theta}} di \\ \bar{L} \left(\frac{w^*}{\bar{w}} \right)^{\epsilon} = Y^* (a+b)^{\frac{\theta-\rho}{\rho(1-\theta)}} \left(\frac{(1-\alpha)\theta}{w^*} \right)^{\frac{1}{1-\rho}} \int_i e^{\frac{\theta z_i}{1-\theta}} di \end{cases}$$

A.9.2 Approximation

We want to compute the change in log output due to the aggregation of the reform. Index with 0 the moments of $\log(1+\tau_i)$ without the policy, and with 1 the moments of log distortions when the policy is applied to all firms. Then, we decompose the change in log output into:

$$\begin{aligned} \Delta log Y &= \log Y(\{\log(1+\tau_i^1)\}_i) - \log Y(\{\log(1+\tau_i^0)\}_i) \\ &= \underbrace{\log Y(\{\log(1+\tau_i^1)\}_i) - \log Y(\mu_1)}_{H_1^Y} \\ &+ \underbrace{\log Y(\mu_1) - \log Y(\mu_0)}_{M^Y} \\ &- \underbrace{\left(\log Y(\mu_0) - \log Y(\{\log(1+\tau_i^0)\}_i) \right)}_{H_0^Y} \end{aligned}$$

where H_i^Y is the pure effect of heterogeneous distortions, starting from an economy where all log wedges are equal to μ_i. M^Y is the pure effect of changing the mean distortion, assuming no heterogeneity. We use the same decomposition to compute $\Delta \log w$.

In the following, we make two separate assumptions. First, we assume that in both treated and untreated economies, that log productivity and log wedges experience small deviation from their means. This allows to compute H_0^Y and H_1^Y. Second, we assume that the experiment has a small impact on the mean distortion, i.e. that $\Delta\mu = \mu_1 - \mu_0$ is small. This second assumption allows us to compute M^Y. We use these two assumptions to expand the equilibrium formulas up to the second order.

First, we note that a second order Taylor expansion in z, combined with the product market market equilibrium leads to:

$$a + b = 1 - \frac{1}{2} \frac{\rho}{1-\rho} \frac{\theta}{1-\theta} \sigma_z^2$$

so that terms in $a+b$ multiplied by first- or second-order terms are equal to 1.

The pure effect on the mean wedge is given by:

$$M^w = -\frac{b}{a}\Delta\mu + \frac{b^2}{2a}\frac{\rho}{1-\rho}(\Delta\mu)^2$$
$$M^Y = \left(\epsilon + \frac{1}{1-\rho}\right)\left(-\frac{b}{a}\Delta\mu + \frac{b^2}{2a}\frac{\rho}{1-\rho}(\Delta\mu)^2\right)$$

The pure effect of wedge heterogeneity is given by (omitting the $0,1$ index):

$$H^w = \frac{b}{2a}\left(\frac{\rho}{1-\rho}a + \frac{\theta}{1-\theta}b\right)\sigma_\tau^2 - \frac{b}{a}\frac{\theta}{1-\theta}\sigma_{z\tau}$$
$$H^Y = \frac{1}{2(1-\theta)}\left(b^2\frac{\rho(\theta-\rho)}{(1-\rho)^2}a + \left(\epsilon + \frac{1}{1-\rho}\right)\left(\frac{\rho-\theta}{1-\theta}b + \theta\frac{b}{a}\right)\right)\sigma_\tau^2 - \left(\epsilon + \frac{1}{1-\rho}\right)\frac{b}{a}\frac{\theta}{1-\theta}\sigma_{z\tau}$$

where a and b are given by the above formulas. In particular, a depends on μ, the expectation of log wedges and b depends on w^*, the wage prevailing in an economy where the average log wedge is μ but the dispersion is zero.

To compute the change in output (the same applies to wage), we need to compute $H_1^Y - H_0^Y$, which involves two sets of coefficients (a_1, b_1) (which depend on μ_1 and w_1^*)and (a_0, b_0) (which depend on μ_0 and w_0^*).

We can write that:

$$a_1 = a_0\left(1 + \frac{\rho}{1-\rho}\frac{b_0}{a_0}\Delta\mu + O(2)\right)$$
$$b_1 = b_0\left(1 - \frac{\rho}{1-\rho}\Delta\mu + O(2)\right)$$

which shows that the difference between the two coefficients is of order one. Given that H^w and H^Y multiply these coefficients by terms of order 2, we obtain directly that $H_1^w = H_0^w$ and $H_1^Y = H_0^Y$.

Hence, noting $a = a_0$ and $b = b_0$, the formula for $\Delta \log Y$ can be shown to be:

$$\begin{aligned}\Delta \log Y = &-\frac{b}{a}\left(\epsilon + \frac{1}{1-\rho}\right)\Delta\mu \\ &+ \frac{1}{2(1-\theta)}\left(b^2\frac{\rho(\theta-\rho)}{(1-\rho)^2} + \left(\epsilon + \frac{1}{1-\rho}\right)\left(\frac{\rho-\theta}{1-\theta}b + \theta\frac{b}{a}\right)\right)\Delta\sigma_\tau^2 \\ &- \left(\epsilon + \frac{1}{1-\rho}\right)\frac{b}{a}\frac{\theta}{1-\theta}\Delta\sigma_{z\tau} \\ &+ \left(\epsilon + \frac{1}{1-\rho}\right)\frac{b}{2a^2}\frac{\rho}{1-\rho}(\Delta\mu)^2\end{aligned}$$

while the formula for $\Delta \log w$ is:

$$\begin{aligned}\Delta \log w = &-\frac{b}{a}\Delta\mu \\ &+ \frac{b}{2a}\left(\frac{\rho}{1-\rho}a + \frac{\theta}{1-\rho}b\right)\Delta\sigma_\tau^2 \\ &- \frac{b}{a}\frac{\theta}{1-\theta}\Delta\sigma_{z\tau} \\ &+ \frac{b}{2a^2}\frac{\rho}{1-\rho}(\Delta\mu)^2\end{aligned}$$

with:

$$\begin{aligned}a &= (1-\alpha)\left(\frac{(1-\alpha)\,\theta}{w_0^*}\right)^{\frac{\rho}{1-\rho}} \\ b &= \alpha\left(\frac{\alpha\theta}{Re^{\mu_0}}\right)^{\frac{\rho}{1-\rho}}\end{aligned}$$

A.9.3 Measurement

In order to use the above formulas, we face two challenges. First, we need to retrieve a and b from the data. Second, we need to measure $\Delta\mu$ and $\Delta\sigma_\tau$. Let us tackle these two problems in turn.

Measuring a and b

To retrieve a and b, we start from the formula of the labor share:

$$s_i^L = \theta\frac{ae^{-\frac{\rho}{1-\rho}\delta_w}}{ae^{-\frac{\rho}{1-\rho}\delta_w} + be^{-\frac{\rho}{1-\rho}\delta_i}}$$

We expand this formula to the second order and take the expectation of s_i^L, which can be directly measured in the data:

$$a = \frac{\widehat{s_L}}{\theta}\left(1 + \left(\frac{1}{2} + a\right)b\left(\frac{\rho}{1-\rho}\right)^2\sigma_\tau^2\right)$$

where $\widehat{s_L}$ is the average labor share. Hence, $a = \frac{\widehat{s_L}}{\theta} + O(2)$. Given that a is multiplied by terms of order 1 or 2, the difference between a and $\frac{\widehat{s_L}}{\theta}$ is negligible at the second order of approximation.

We can then use the product equilibrium condition to see that:

$$a + b = 1 - \frac{1}{2}\frac{\theta}{1-\theta}\frac{\rho}{1-\rho}\sigma_z^2$$

hence, for the same reason as above, we can approximate b with $1 - \frac{\widehat{s_L}}{\theta}$.

Measuring $\Delta\mu$ and $\Delta\sigma_\tau$

We now turn to the estimation of $\Delta\mu$ and $\Delta\sigma_\tau$. Using the full notation, we have that:

$$\Delta\mu = E\log(1+\tau(\boldsymbol{z_i};\boldsymbol{\Theta_1},w_1,Y_1)) - E\log(1+\tau(\boldsymbol{z_i};\boldsymbol{\Theta_0},w_0,Y_0))$$
$$\Delta\sigma_\tau = \mathrm{Var}\log(1+\tau(\boldsymbol{z_i};\boldsymbol{\Theta_1},w_1,Y_1)) - \mathrm{Var}\log(1+\tau(\boldsymbol{z_i};\boldsymbol{\Theta_0},w_0,Y_0))$$

where $\boldsymbol{z_i}$ is the entire past history of productivity shocks of firm i.

For now, we assume that we can observe the distribution of log wedges in the data (we return to measurement of wedges with CES technology below). Even then, the problem is that we do not directly observe these moments in the data. Assuming the experiment is small, the moments we observe are:

$$\widehat{\Delta\mu} = E\log(1+\tau(\boldsymbol{z_i};\boldsymbol{\Theta_1},w_0,Y_0)) - E\log(1+\tau_i(\boldsymbol{z_i};\boldsymbol{\Theta_0},w_0,Y_0))$$
$$\widehat{\Delta\sigma_\tau} = \mathrm{Var}\log(1+\tau_i(\boldsymbol{z_i};\boldsymbol{\Theta_1},w_0,Y_0)) - \mathrm{Var}\log(1+\tau_i(\boldsymbol{z_i};\boldsymbol{\Theta_0},w_0,Y_0))$$

or put differently: the data only show us the effect of the treatment on moments in *partial equilibrium*. To deal with this problem, we need to put structure on how wedges are affected by macro variables.

First, we show that these moments do *not* depend on aggregate demand Y (like in the Cobb-Douglas case):

Lemma 2. *If the production function is homogeneous of degree 1, then the distribution of wedges does not depend on output Y.*

$$\tau(\boldsymbol{z_i};\boldsymbol{\Theta},w_0,Y) = \tau(\boldsymbol{z_i};\boldsymbol{\Theta},w_0,Y_0)\ \text{, for all } Y$$

Proof. This property uses the degree one homogeneity of the production function. First, show that EBITDA can be rescaled by total output Y:

$$\begin{aligned}
\pi(z,k;w,Y) &= \max_l \left(Y^{1-\theta}e^{\theta}z\left(F(k,l)\right)^{\theta} - wl\right)\\
&= Y\max_l\left(e^{\theta}z\left(F\left(\frac{k}{Y},\frac{l}{Y}\right)\right)^{\theta} - w\frac{l}{Y}\right)\\
&= Y\max_{l'}\left(e^{\theta}z\left(F\left(\frac{k}{Y},l'\right)\right)^{\theta} - wl'\right)\\
&= Y\pi\left(z,\frac{k}{Y};w,Y_0\right)
\end{aligned}$$

The rest of the proof then follows the logic of the proof of proposition 2. □

However, the distribution of wedges and productivity may depend on the aggregate wage w. We now put structure on this dependence. The following Lemma does this:

Lemma 3. *At the second order approximation, the following equations must hold:*

$$\begin{aligned}\Delta\mu &= \widehat{\Delta\mu} + \beta\Delta\log\frac{w_1}{w_0} + \gamma\Delta\left(\log\frac{w_1}{w_0}\right)^2\\ \Delta\sigma^2 &= \widehat{\Delta\sigma^2}\\ \Delta\sigma_{z\tau} &= \widehat{\Delta\sigma_{z\tau}}\end{aligned}$$

β and γ corresponds to the coefficient estimates of a regression of treated firms*'s log-capital wedges on exogenous sources of variation of log wages*
The two second order moments do not depend on the level of wages in the new aggregate counterfactual economy.

The coefficients β and γ can be estimated if one observes a cross-section of economies (for instance, cities) where the policy experiment is implemented and there are variations in wages across local labor markets, which are exogenous to the policy experiments (e.g., exogenous shifters in local labor supply across markets): in this case, β and γ can simply be obtained by regressing, in a cross-section of economies, the log-capital wedge of treated firms on the exogenous variations in log local wage.

Proof. To prove the above result, we first need to log-linearize wedges with respect to w and z_i up to the second order and write:

$$\begin{aligned}\log(1+\tau(z_i,\Theta,w)) = \log(1+\tau(z_i,\Theta,w_0)) &+ \beta(z_i,\Theta,w_0)\log\left(\frac{w}{w_0}\right)\\ &+ \frac{1}{2}\gamma(z_i,\Theta,w_0)\left(\log\left(\frac{w}{w_0}\right)\right)^2 + o(2)\end{aligned}$$

The coefficients β and γ depend on the model generating the frictions which is generically described by Θ. We describe below how to estimate them from the data.

We then Taylor-expand both coefficients β and γ with respect to z_i around 0 (remember that $\mathbb{E}z_i = 0$). First order is enough:

$$\begin{aligned}\beta(z_i,\Theta,w_0) &= \beta(0,\Theta,w_0) + \beta_1(0,\Theta,w_0)z_i + o(1)\\ \gamma(z_i,\Theta,w_0) &= \gamma(0,\Theta,w_0) + \gamma_1(0,\Theta,w_0)z_i + o(1)\end{aligned}$$

which leads to the following expression for wedges:

$$\begin{aligned}\log(1+\tau(z_i,\Theta,w)) = \log(1+\tau(z_i,\Theta,w_0)) &+ \beta(0,\Theta,w_0)\log\left(\frac{w}{w_0}\right)\\ &+ \beta_1(0,\Theta,w_0)z_i\log\left(\frac{w}{w_0}\right)\\ &+ \frac{1}{2}\gamma(0,\Theta,w_0)\left(\log\left(\frac{w}{w_0}\right)\right)^2 + o(2)\end{aligned}$$

So we are now ready to compute the mean and variances of the wedge distribution. Let us start with the mean:

$$\begin{aligned}\mu(\Theta, w) &= \mathbb{E}\log(1+\tau(z_i,\Theta,w)) \\ &= \mu(\Theta,w_0) + \underbrace{\beta(0,\Theta,w_0)}_{\equiv\beta(\Theta,w_0))}\log\left(\frac{w}{w_0}\right) + \frac{1}{2}\underbrace{\gamma(0,\Theta,w_0)}_{\equiv\gamma(\Theta,w_0))}\left(\log\left(\frac{w}{w_0}\right)\right)^2\end{aligned}$$

which shows that the mean wedge is a function of $\log w$ and its square.

For the variance, we further need to expand:

$$\log(1+\tau(z_i,\Theta,w_0) = \log(1+\tau(0,\Theta,w_0)) + T_1(0,\Theta,w_0)z_i + o(1)$$

so that the variance writes:

$$\begin{aligned}\sigma^2_\tau(\Theta,w) &= \text{var}\left(\log(1+\tau(z_i,\Theta,w))\right) \\ &= \sigma^2_\tau(\Theta,w_0) \\ &+ 2\underbrace{\text{cov}\left(T_1(0,\Theta,w_0)z_i, \beta_1(0,\Theta,w_0)z_i\log\left(\frac{w}{w_0}\right)\right)}_{T_1(0,\Theta,w_0)\beta_1(0,\Theta,w_0)\sigma^2_z\log\left(\frac{w}{w_0}\right)=o(2)} \\ &+ \underbrace{\text{var}\left(\beta_1(0,\Theta,w_0)z_i\log\left(\frac{w}{w_0}\right)\right)}_{=o(2)} \\ &= \sigma^2_\tau(\Theta,w_0) + o(2)\end{aligned}$$

which ensures that the variance of wedges does not depend on $\log w$ in our second order approximation.

Near identical algebra shows that $\sigma_{z\tau}(\Theta,w)$ is also independent from $\log w$ in our second order approximation.

We now compute the empirical moments as a function of observed sufficient statistics on wedges. Let us start again with the mean:

$$\begin{aligned}\Delta\mu &= \mu(\Theta_1,w_1) - \mu(\Theta_0,w_0) \\ &= (\mu(\Theta_1,w_1) - \mu(\Theta_1,w_0)) + \underbrace{(\mu(\Theta_1,w_0) - \mu(\Theta_0,w_0))}_{\widehat{\Delta\mu}} \\ &= \underbrace{\beta(\Theta_1,w_0))}_{\equiv\beta}\log\left(\frac{w_1}{w_0}\right) + \frac{1}{2}\underbrace{\gamma(\Theta_1,w_0))}_{\equiv\gamma}\left(\log\left(\frac{w_1}{w_0}\right)\right)^2 + \widehat{\Delta\mu}\end{aligned}$$

The coefficients β and γ can be estimated from a regression of treated firms' log-capital wedges on the log local wage. As an illustration, consider the case where a tax experiment is implemented in a large cross-section of cities. At the same time, there may be exogenous shifters in local labor supply driven by aggregate variations in industry-level output: for instance, Oberfield and Raval (2014) construct a Bartik instrument interacting industry growth at the national level and initial industry shares in local labor

market; they use this instrument as a source of exogenous variations in local wages. Under the identifying assumption that this instrument is orthogonal to the policy experiment, then a regression of treated firms log-capital wedges on instrumented local wages. Obviously, estimating β and γ represents an empirical challenge. However, we show below through simulations that the precision losses from omitting β and γ from the aggregation formula may be limited (Figure A.1 and A.2).

Assuming we know β and γ, we use the above formula for $\Delta\mu$ jointly with the previously derived formula on $\log w$:

$$\log\left(\frac{w}{w_0}\right) = \alpha_1\Delta\mu + \alpha_2(\Delta\mu)^2 + \underbrace{\alpha_3\Delta\sigma_\tau^2 + \alpha_4\Delta\sigma_{\tau,z}}_{X}$$

where the terms in X are not affected by the level of wages as we have just shown. Then:

$$\Delta\mu = \widehat{\Delta\mu} + \beta(\alpha_1\Delta\mu + \alpha_2(\Delta\mu)^2 + X) + \gamma(\alpha_1)^2(\Delta\mu)^2$$

Rearranging:

$$(\Delta\mu)^2\left(\beta\alpha_2 + \gamma(\alpha_1)^2\right) + \Delta\mu\left(B\alpha_1 - 1\right) + \beta X + \widehat{\Delta\mu} = 0$$

Now we let $\Delta\mu = \kappa_1\widehat{\Delta\mu} + \kappa_2\left(\widehat{\Delta\mu}\right)^2 + \kappa_3 X$ in the equation above. We obtain, by identification:

$$\begin{aligned}
\kappa_1 &= \frac{1}{1-\beta\alpha_1} = \frac{a}{a+b\beta} \\
\kappa_2 &= \frac{\beta\alpha_2 + \gamma(\alpha_1)^2}{(1-\beta\alpha_1)^3} = \frac{a\beta b}{(a+\beta b)^3}\left(\frac{b\gamma}{\beta} + \frac{1}{2}\frac{\rho}{1-\rho}\right) \\
\kappa_3 &= \frac{\beta}{1-\beta\alpha_1} = \frac{a\beta}{a+b\beta}
\end{aligned}$$

so that:

$$\Delta\mu = \frac{a}{a+b\beta}\widehat{\Delta\mu} + \frac{a\beta b}{(a+\beta b)^3}\left(\frac{b\gamma}{\beta} + \frac{1}{2}\frac{\rho}{1-\rho}\right)\left(\widehat{\Delta\mu}\right)^2 + \frac{a\beta}{a+b\beta}(\alpha_3\Delta\sigma_\tau^2 + \alpha_4\Delta\sigma_{\tau,z})$$

Note that observed changed in variance is equal to the theoretical one given our second order approximation:

$$\begin{aligned}
\Delta\sigma_\tau^2 &= \sigma_\tau^2(\Theta_1, w_1) - \sigma_\tau^2(\Theta_0, w_0) \\
&= \underbrace{\left(\sigma_\tau^2(\Theta_1, w_0) - \sigma_\tau^2(\Theta_0, w_0)\right)}_{\widehat{\Delta\sigma_\tau^2}}
\end{aligned}$$

Obviously, the same algebra works for $\Delta\sigma_{z\tau}$. □

Given Lemma 2 and 3, we know how to compute $\Delta\mu, \Delta\sigma_\tau^2$, and $\Delta\sigma_{z\tau}$ as a function of the observed sufficient statistics $\widehat{\Delta\mu}$, $\widehat{\Delta\sigma_\tau^2}$, and $\widehat{\Delta\sigma_{z\tau}}$. We plug these into the formula of $\Delta\log Y$ and obtain:

$$\Delta \log Y = A\widehat{\Delta\mu} + B\widehat{\Delta\sigma_\tau^2} + C\widehat{\Delta\sigma_{\tau z}} + D(\widehat{\Delta\mu})^2$$

where:

$$\begin{aligned}
A &= -\frac{b}{a+\beta b}\left(\epsilon + \frac{1}{1-\rho}\right) \\
B &= \frac{b^2}{2}\frac{\rho(\theta-\rho)}{(1-\theta)(1-\rho)^2} + \frac{1}{2}\left(\epsilon + \frac{1}{1-\rho}\right)\frac{b}{a+b\beta}\left(\frac{\rho}{1-\rho}a + \frac{\theta}{1-\theta}b\right) \\
C &= -\frac{b}{a+b\beta}\left(\epsilon + \frac{1}{1-\rho}\right)\frac{\theta}{1-\theta} \\
D &= \frac{1}{2}\frac{b}{(a+b\beta)^3}\left(\epsilon + \frac{1}{1-\rho}\right)\left(a\frac{\rho}{1-\rho} - 2b^2\gamma\right)
\end{aligned}$$

A.9.4 Measuring wedges at the firm level

The last step consists of measuring wedges at the firm level in order to recover the three moments $\widehat{\Delta\mu}$, $\widehat{\Delta\sigma_{\tau z}}$ and $\widehat{\Delta\sigma_\tau^2}$. We start from the fact that firm-level labor and capital shares are given by:

$$\begin{aligned}
s_i^L &= \theta\frac{B}{A_i + B} \\
s_i^K &= \frac{\theta}{(1+\tau_i)}\frac{A_i}{A_i + B}
\end{aligned}$$

where $B = (1-\alpha)\left(\frac{(1-\alpha)\theta}{w}\right)^{\frac{\rho}{1-\rho}}$ and $A_i = \alpha\left(\frac{\alpha\theta}{R(1+\tau_i)}\right)^{\frac{\rho}{1-\rho}}$.

Combining these two equations, we obtain that:

$$\log(1+\tau_i) = \log\left(\frac{\theta - s_i^L}{s_i^K}\right)$$

which gives a simple way to compute the distortion at the firm level in the CES case. It converges to the formula used in the Cobb-Douglas case ($\frac{py}{k}$) wen ρ goes to zero. The downside of this formula is however that it requires to know the cost of capital R in order to compute the capital share, as well as the mark-up (θ).

A.9.5 Pulling it All Together

We combine all the above insights in the following summary:

1. We compute the following three sufficient statistics from the small-scale experiment:

$$\widehat{\Delta\mu} = E\left(\log\left(\frac{\theta - s_i^L}{s_i^K}\right)|T_i = 1\right) - E\left(\log\left(\frac{\theta - s_i^L}{s_i^K}\right)|T_i = 0\right)$$
$$\widehat{\Delta\sigma_\tau^2} = \text{Var}\left(\log\left(\frac{\theta - s_i^L}{s_i^K}\right)|T_i = 1\right) - E\left(\log\left(\frac{\theta - s_i^L}{s_i^K}\right)|T_i = 0\right)$$
$$\widehat{\Delta\sigma_{\tau z}} = \text{Cov}\left(\log\left(\frac{\theta - s_i^L}{s_i^K}\right)|T_i = 1\right) - E\left(\log\left(\frac{\theta - s_i^L}{s_i^K}\right)|T_i = 0\right)$$

   where s_i^L is the labor share at the firm level and s_i^K is the capital share.

2. Compute the following two aggregation sufficient statistics β and γ by running the regression, for firm i in economy c:

$$\log\left(\frac{\theta - s_{ic}^L}{s_{ic}^K}\right) = \text{cst} + \beta \log w_c + \gamma(\log w_c)^2 + u_i c$$

   running this regression requires firm level data on labor and capital shares in a cross-section of economies (sectors, cities) and an exogenous source of variation for w.

3. Compute the additional parameter $a = \frac{s^L}{\theta}$ where s^L is the average labor share in the cross-section. Note $b = 1 - a$.

4. Then, the aggregate effect of the experiment would be given by:

$$\Delta \log Y = A\widehat{\Delta\mu} + B\widehat{\Delta\sigma_\tau^2} + C\widehat{\Delta\sigma_{\tau z}} + D(\widehat{\Delta\mu})^2$$

   where A, B, C and D are known functions of a, β, γ, as well as θ, ϵ and ρ.

The formulas for A, B, C, D are given by:

$$A = -\frac{b}{a + \beta b}\left(\epsilon + \frac{1}{1-\rho}\right)$$
$$B = \frac{b^2}{2}\frac{\rho(\theta - \rho)}{(1-\theta)(1-\rho)^2} + \frac{1}{2}\left(\epsilon + \frac{1}{1-\rho}\right)\frac{b}{a + b\beta}\left(\frac{\rho}{1-\rho}a + \frac{\theta}{1-\theta}b\right)$$
$$C = -\frac{b}{a + b\beta}\left(\epsilon + \frac{1}{1-\rho}\right)\frac{\theta}{1-\theta}$$
$$D = \frac{1}{2}\frac{b}{(a + b\beta)^3}\left(\epsilon + \frac{1}{1-\rho}\right)\left(a\frac{\rho}{1-\rho} - 2b^2\gamma\right)$$

The first order version of the overall formula is much simpler, with:

$$\Delta \log Y = -\frac{\theta - s^L}{\beta\theta + s^L(1-\beta)}\left(\epsilon + \frac{1}{1-\rho}\right)\Delta\widehat{E\log\left(\frac{\theta - s_i^L}{s_i^K}\right)},$$

which shows quite clearly that the CES case requires three types of adjustment. First, under CES, distortions are affected by the wage w prevailing in the economy: this is why *beta* appears in this previous

formula. Second, under CES production, GE effects themselves are more dampening than under Cobb-Douglas if $\rho < 0$, because labor cannot be substituted with capital as easily. This is why we see $\frac{1}{1-\rho}$ in the previous formula instead of 1. Third, wedges cannot be computed using the sales to capital ratio as in the Cobb-Douglas case. Instead, we use input shares to measure distortions.

Further, note that the GE effect is negligible when:

$$\beta \ll \frac{s_L}{\theta - s_L} \approx \frac{.7}{.8 - .7} = 7$$

. We finally turn to a simple simulation exercise to quantify the magnitude of β and γ in simple simulated experiments.

A.9.6 Simulation

In this section, we calibrate a simple version of our model to assess our CES formula. In particular, we are interested in the importance of the β and γ adjustments in the aggregation formula (which measure how log-capital wedges for treated firms respond to variations in the wage w prevailing in labor markets).

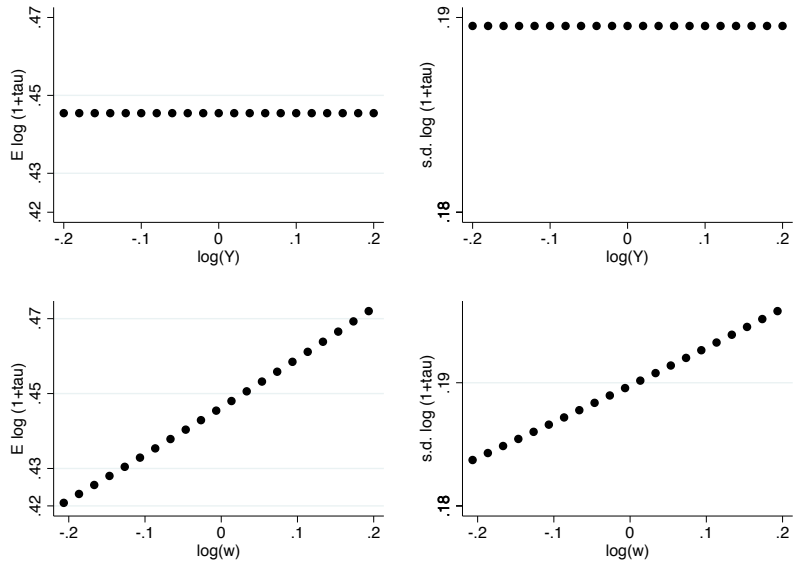
We start from a version of our baseline model with no tax. Firms have CES production technology $y = e^z \left(\alpha k^\rho + (1-\alpha) l^\rho\right)^{1/\rho}$. There are quadratic adjustment costs $c\frac{I^2}{K}$. Firms cannot issue any equity ($e \geq 0$), and can only borrow up to ζ times their next period capital stock k'. The market structure corresponds to the baseline model described in Section 2. We simulate the model in equilibrium via iteration. We take (Y, w) as given, solve the Bellman problem, simulate a large number of firms in steady-state, and then compute aggregate output Y^s and labor demand L^d. We iterate until $Y \approx Y^s$ and $L^d \approx L^s(w)$ with some pre-determined precision.

We use a standard calibration: the capital share is $\alpha = .3$; θ is set to .85; the labor supply elasticity is $\epsilon = .5$; the rate of physical obsolescence is $\delta = .06$; the safe rate of return is $r = .03$; z_{it} follows an AR(1) process with persistence $\rho = .5$ and volatility $\sigma_z = .6$. Since we want to explore the effect of imperfect capital-labor substitution, we vary ρ from -1.55 to -.05 (near Cobb-Douglas case). An elasticity of substitution of .5 corresponds to $\rho = -1$.

We start by simulating an economy with $\zeta = .2$ for all firms and compute the general equilibrium (w_0, Y_0). As a first pass to understand the effect of CES production function on aggregation, we investigate how, in this economy, the distribution of log-capital wedges depend, in partial equilibrium on the wage w and total output Y. We simply re-solve numerically for the optimal policies of firms assuming a different wage w or a different aggregate output Y – but holding all other parameters in the firm's optimization problem constant. Figure A.1 shows the resulting distribution of log-capital wedges. First, note that the distribution of the log-capital wedges does not vary with Y, as we showed above analytically. This distribution does, however, vary with w. The elasticities implied by this graph are fairly constant (the log-log line is straight). Additionally, these elasticities are small: in this simulation, $\beta = .1$ and $\gamma = .025$; as an illustration, given the formula for B, we see that the effect of β on B is negligible if $\beta \ll \frac{s_L}{\theta - s_L} \approx 7$. As a result, neglecting β and γ in this simulated example should have a negligible effect on the aggregation formula.

To confirm this intuition, we now consider an actual policy experiment in this simulated economy: the collateral constraint parameter, ζ, goes from .2 to .4. In the original economy (w_0, Y_0), we first solve numerically for the optimal policies of "treated" firms – i.e firms for which $\zeta = .4$: we simulate a sample of such firms and use it to measure the effect of the treatment on the sufficient statistics of Proposition

Figure A.1: Effect of Changing w and Y on $E\log(1+\tau_i)$

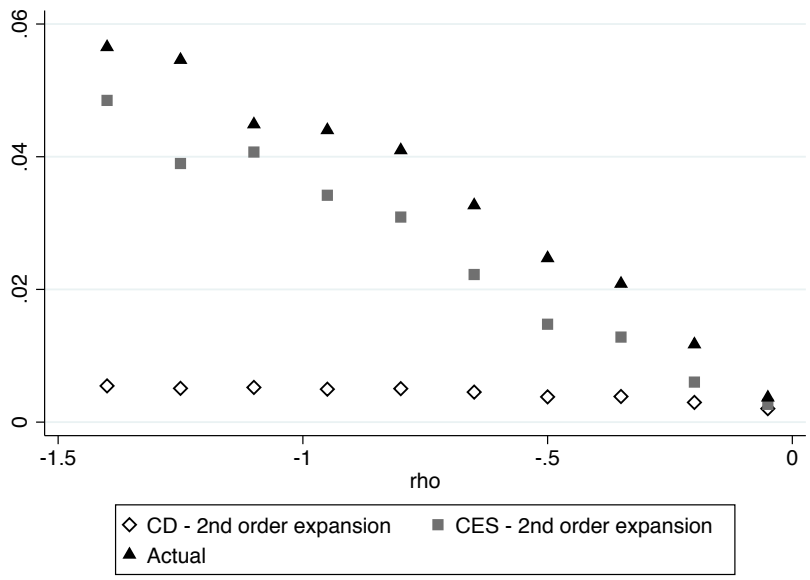


Source: Authors' simulations. Note: see text for details on the calibration. This figure shows how the distribution of log-capital wedges respond, in partial equilibrium, to changes in $\log(w)$ and $\log(Y)$. The simulations are normalized so that $Y = w = 1$ corresponds to the general equilibrium of this economy.

5.6: $\widehat{\Delta\mu}, \widehat{\Delta\sigma_\tau^2}, \widehat{\Delta\sigma_{\tau z}}$. We then compute aggregate output in the economy where all firms have $\zeta = .4$ (our counterfactual economy) using three different methodology: (1) we compute the actual output by re-computing numerically the general equilibrium of the model when all firms have $\zeta = .4$ (dark triangles in Figure A.2) (2) we use our CES aggregation formula in Proposition 5.6, but assuming that $\beta = \gamma = 0$ – i.e. we do not account for the general equilibrium feedback that changes in w will change the estimated sufficient statistics measure in the initial economy (grey squares in Figure A.2) (3) we use our baseline Cobb-Douglas aggregation formula, Equation 8 (white diamonds in Figure A.2).

We report the results in Figure A.2. Our second-order CES formula that neglects β and γ underestimates the effect on total output of raising ζ from .2 to .4 for all firms by less than 1 percentage point. This provides an idea of the precision loss, in this simulation, of neglecting 3rd order terms and β and γ (the feedback of equilibrium on estimated sufficient statistics). Using the Cobb-Douglas formulas leads to significant under-estimation of these aggregate effects: when the data generating process has $\rho = -1$ – a conventional value of substitution elasticity – the Cobb Douglas formulas underestimate the effect of the generalization of the policy on aggregate output by about 5 percentage points. As expected, as ρ approaches 0 and we get closer to the Cobb-Douglas case, all three values for the aggregate counterfactuals converge to the same value.

Figure A.2: Effect on Output of Changing ζ from .2 to .4



Source: Authors' simulations. Note: see text for details on the calibration. We compute changes in aggregate output resulting from ζ going from .2 to .4 for all firms in the economy. The dark triangles correspond to the actual change. The grey squares use the estimated change using our CES aggregation formula in Proposition 5.6 but assuming that $\beta = \gamma = 0$. The white diamonds use our baseline Cobb-Douglas aggregation formula, Equation 8.

B Additional Results

B.1 Effect of Treatment on Productivity

Though we focus on policy experiments that purely affect wedges and not productivity, our framework can easily be extended to accommodate such productivity experiments. In this extension, we start from the baseline model but allow the treatment to affect the distribution of firm log productivities z. Like in the rest of the paper, we assume that z can be measured by the econometrician. Such an extension could for instance apply to firm-level interventions designed to increase productivity via improved management practices Bloom et al. (2013a) or R&D subsidies (Bloom et al., 2013b). The following proposition describes the aggregation procedure for such policy experiments:

Proposition 9. *Assume the economy is described by the model of Section 2. Furthermore, assume the empirical treatment affects the joint distribution of distortions τ_i and productivities z_i. Then, the results of proposition 2 apply and the procedure to compute the aggregate counterfactual is as follows:*

1. *Estimate the following sufficient statistics resulting from the treatment:*

$$\widehat{\Delta\mu_\tau} = \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=1\right) - \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=0\right)$$
$$\widehat{\Delta\sigma^2_\tau} = \text{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=1\right) - \text{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=0\right)$$
$$\widehat{\Delta\sigma_{z\tau}} = \text{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i=1\right) - \text{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i=0\right)$$
$$\widehat{\Delta\mu_z} = \mathbb{E}\left(z_{it}|T_i=1\right) - \mathbb{E}\left(z_{it}|T_i=0\right)$$
$$\widehat{\Delta\sigma^2_z} = \text{Var}\left(z_{it}|T_i=1\right) - \text{Var}\left(z_{it}|T_i=0\right)$$

2. *We can combine these sufficient statistics to estimate the following aggregation formulas:*

$$\Delta\log Y = \underbrace{\frac{1+\epsilon}{1-\alpha}\left(\widehat{\Delta\mu_z} + \frac{1}{2}\frac{\theta}{1-\theta}\widehat{\Delta\sigma^2_z}\right)}_{\textit{productivity effect}} + \underbrace{\frac{\alpha(1+\epsilon)}{1-\alpha}\left(-\widehat{\Delta\mu_\tau} + \frac{1}{2}\frac{\theta}{1-\theta}\left(\alpha\widehat{\Delta\sigma^2_\tau} - 2\widehat{\Delta\sigma_{\tau z}}\right)\right)}_{\textit{equation (9)}}$$

$$\Delta\log(TFP) = \underbrace{\widehat{\Delta\mu_z} + \frac{1}{2}\frac{\theta}{1-\theta}\widehat{\Delta\sigma^2_z}}_{\textit{productivity effect}} - \underbrace{\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\widehat{\Delta\sigma^2_\tau}}_{\textit{equation (8)}}$$

The intuition for these extended formulas is straightforward. A new "productivity" term $\widehat{\Delta\mu_z} + \frac{1}{2}\frac{\theta}{1-\theta}\widehat{\Delta\sigma^2_z}$ is added to capture the direct effect of the treatment on firm productivity. This new term has two components: the effect of the treatment on mean log productivity plus the standard variance correction, accounting for the concavity of the log function. This variance correction is necessary here because our sufficient statistics concern the distribution of log productivities, not the distribution of productivities. Our formulas could be written in terms of productivity levels, but in practice estimating logs yields more robust estimates. The productivity term enters the TFP aggregate directly, without GE effects since effects on productivity are not crowded out. It also enters the output formula, multiplied by a coefficient $\frac{1+\epsilon}{1-\alpha}$, which captures how quantities of labor and capital respond to aggregate productivity changes.

Proposition 9 also highlights the importance of accounting for distortions even if the treatment is only designed to affect productivity. Even if the treatment does not directly affect distortions (think, for instance, of an improvement in management practices or process innovation), it may *interact* with existing distortions (real or financial) and affect the distributions of τ_i's among treated firms. The above formulas make clear that the effect of the treatment on log TFP is not the only sufficient statistic to look at in order to obtain the correct aggregate counterfactual.

B.2 Decreasing returns to scale

In this appendix, we extend our baseline model to allow for decreasing technological returns to scale (e.g., span of control): $y_{it} = e^{z_{it}}(k_{it}^\alpha l_{it}^{1-\alpha})^\nu$ where $\nu < 1$. The aggregation formulas are then similar to those in our baseline case, with minor modifications:

Proposition 10 (Decreasing Returns to Scale)**.** *With decreasing technological returns to scale ν and under the homogeneity assumptions of Proposition 2, the joint-distribution of z and τ does not depend on (w, Y).*

Additionally, the aggregation formulas become:

$$\Delta \log(Y) = \frac{\alpha\nu\,(1+\epsilon)}{(1-\alpha)\nu + (1+\epsilon)(1-\nu)} \left(-\widehat{\Delta\mu_\tau} + \frac{1}{2}\frac{\nu\theta}{1-\nu\theta}\left(\alpha\widehat{\Delta\sigma_\tau^2} - 2\widehat{\Delta\sigma_{z\tau}}\right)\right)$$
$$\Delta \log(TFP) = -\frac{\alpha\nu}{2}\left(1 + \frac{\alpha\nu\theta}{1-\nu\theta}\right)\widehat{\Delta\sigma_\tau^2},$$

where $\widehat{\Delta\mu_\tau}$, $\widehat{\Delta\sigma_\tau^2}$, $\widehat{\Delta\sigma_{z\tau}}$ are the same treatment effects defined in Section 3.4.

Proof. See Appendix C.1. □

The modifications introduced by decreasing returns to scale are marginal. Proposition 10 makes clear that our approach also applies to models of perfect competition ($\theta = 1$) and decreasing returns to scale such as Hopenhayn (2014) or Midrigan and Xu (2014). It also makes clear that the modifications induced by decreasing returns to scale $\nu < 1$ will quantitatively be small, since ν is typically estimated close to 1. For instance, assuming $\alpha = .3$, $\epsilon = 1.5$ and $\nu = .95$, we find a pre-multiplying factor on output of .57 when decreasing returns to scale are accounted for vs. .64 in our baseline case.

B.3 Non-parametric Formulas

Here, we explore here the effect of relaxing the assumption of small variations in distortions and productivity. It turns out that simple formulas, similar to (8-9) can be developed. These formulas rely more heavily on the Cobb-Douglas nature of production, but present the advantage that they do not require the estimation of firm-level TFP shocks z. They rely on slightly different sufficient statistics than the sales to capital ratio.

Proposition 11.
Consider the baseline framework of Section 2. Assume that the assumptions in Proposition 2 hold.

Define the following treatment effects for labor and capital:

$$\begin{cases} \widehat{\Delta l} = \log\left(\mathbb{E}[l_{it}|T_i = 1)\right) - \log\left(\mathbb{E}[l_{it}|T_i = 0]\right) \\ \widehat{\Delta k} = \log\left(\mathbb{E}[k_{it}|T_i = 1]\right) - \log\left(\mathbb{E}[k_{it}|T_i = 0]\right) \end{cases}$$

Then, the effect of generalizing the treatment to all firms in the economy on aggregate output and TFP is given by the following formulas:

$$\begin{cases} \Delta \log Y = \dfrac{(1+\epsilon)(1-\theta)}{(1-\alpha)\theta} \times \widehat{\Delta l} \\ \Delta \log TFP = \left(\dfrac{1}{\theta} - (1-\alpha)\right) \times \widehat{\Delta l} - \alpha \times \widehat{\Delta k} \end{cases}$$

Proof. See Appendix C.2. □

The formulas in Proposition 11 are intuitive and simply leverage the fact that reduction in distortions due to the treatment translates into changes in input use. The appeal of these formulas is that they do not require assumptions about the joint-distribution of productivity and wedges, or equivalently, assumption about the size of variations of productivity and wedges. However, these formulas may be unpractical from

an empirical standpoint: with log-normally distributed labor and capital, estimating treatment effects in levels is likely to be inconsistent.

B.4 Exogenous Labor distortions

In this appendix, we assume that firms also face distortions in labor markets. We assume that these distortions are exogenous (e.g, a firm-specific payroll taxes). We follow our treatment of log-capital wedges and define log-labor wedges as wedges between marginal productivities and the market wage w:

$$(1+\eta_{it})w = (1-\alpha)\theta\frac{py_{it}}{l_{it}}$$

However, in contrast with the log-capital wedges we consider in our main analysis, we assume here that the distribution of η is exogenous. This formulation leads to the following proposition:

Proposition 12. *Assume firm-specific, exogenous labor wedges. Furthermore, assume for the sake of clarity, that thee distribution of these labor wedges is unaffected by the experiment: the experiment only affects capital distortions.*[21]

Then, the results of proposition 2 apply and the procedure to compute the aggregate counterfactual is as follows:

1. *Estimate the following sufficient statistics resulting from the treatment:*

$$\widehat{\Delta\mu_\tau} = \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=1\right) - \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=0\right)$$
$$\widehat{\Delta\sigma^2_\tau} = \text{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=1\right) - \text{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=0\right)$$
$$\widehat{\Delta\sigma_{z\tau}} = \text{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i=1\right) - \text{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i=0\right)$$
$$\widehat{\Delta\sigma_{\eta\tau}} = \text{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), \log\left(\frac{p_{it}y_{it}}{l_{it}}\right)|T_i=1\right) - \text{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), \log\left(\frac{p_{it}y_{it}}{l_{it}}\right)|T_i=0\right)$$

2. *The effect of generalizing the treatment to all firms in the economy on aggregate output and TFP is given by the following formulas:*

$$\Delta\log Y = \underbrace{-\frac{\theta}{1-\theta}\left(1+\alpha(1+\epsilon)\right)\widehat{\Delta\sigma_{\eta\tau}}}_{\textit{interaction effect}} + \underbrace{\frac{\alpha(1+\epsilon)}{1-\alpha}\left(-\widehat{\Delta\mu_\tau} + \frac{1}{2}\frac{\theta}{1-\theta}\left(\alpha\widehat{\Delta\sigma^2_\tau} - 2\widehat{\Delta\sigma_{\tau z}}\right)\right)}_{\textit{equation (9)}}$$
$$\Delta\log TFP = \underbrace{-\frac{\alpha(1-\alpha)\theta}{1-\theta}\widehat{\Delta\sigma_{\eta\tau}}}_{\textit{interaction effect}} + \underbrace{-\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\widehat{\Delta\sigma^2_\tau}}_{\textit{equation (8)}}$$

Proof. See Appendix C.3. □

The above proposition simply states that the baseline formulas (8-9) need to be adjusted with a corrective term. This corrective term involves an additional sufficient statistic: the effect of the treatment on the covariance between labor and capital wedges. This is because of labor-capital complementarity: if the

[21] Our results can easily be extended to settings where the policy experiment affects the distribution of labor wedges.

treatment increases capital distortions disproportionately in firms in which labor is heavily distorted, both output and productivity will be further reduced. This formula also makes clear that, even if the policy experiment is not supposed to affect labor distortions directly, labor distortions, if they exist, affect how the generalization of the policy change aggregate outcomes since labor and capital are complement in production.

B.5 Labor distortions generated by binding minimum wages

In this appendix, we focus on one particular type of distortion: a binding minimum wage on low-skill labor. To investigate the effect of such distortions, we allow firms to hire both skilled and unskilled labor:

$$y_{it} = k_{it}^{\alpha} l_{s,it}^{\beta} l_{u,it}^{1-\alpha-\beta}$$

where l_{it}^{s} is the quantity of skilled labor, and l_{it}^{u} is the quantity of unskilled labor. Unskilled labor is subject to a binding minimum wage $\underline{w}$, but skilled labor is not, so that static labor optimization yields:

$$\begin{aligned} \beta\theta \frac{py_{it}}{l_{s,it}} &= w \\ (1-\alpha-\beta)\theta \frac{py_{it}}{l_{u,it}} &= \underline{w}_{it} w^{\zeta} \end{aligned}$$

where w is the market clearing skilled wage. We allow the minimum wage to vary across firms (for instance due to collective agreements) and be indexed on the skilled wage with elasticity ζ.

In this economy, we can easily extend our baseline formulas:

Proposition 13. *Assume firms use capital, skilled and unskilled labor for production with a unit elasticity of substitution:*

$$y_{it} = k_{it}^{\alpha} (l_{s,it})^{\beta} (l_{u,it})^{1-\alpha-\beta}$$

Assume the market for skilled labor clears, but firms face a firm-specific binding minimum wage indexed on skilled wage: $\underline{w}_{it} w^{\zeta}$.
Then, the scale invariance property of proposition 2 apply. Estimating the aggregate counterfactual requires the following steps:

1. *Estimate the following sufficient statistics resulting from the treatment:*

$$\begin{aligned} \widehat{\Delta\mu_{\tau}} &= \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=1\right) - \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=0\right) \\ \widehat{\Delta\sigma_{\tau}^2} &= \mathrm{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=1\right) - \mathrm{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=0\right) \\ \widehat{\Delta\sigma_{z\tau}} &= \mathrm{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i=1\right) - \mathrm{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i=0\right) \\ \widehat{\Delta\sigma_{\underline{w}\tau}} &= \mathrm{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), \underline{w}_{it}|T_i=1\right) - \mathrm{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), \underline{w}_{it}|T_i=0\right) \end{aligned}$$

2. *Inject these sufficient statistics into the following aggregation formulas:*

$$\Delta \log TFP = \underbrace{-\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\widehat{\Delta\sigma_\tau^2}}_{\text{equation (8)}} + \underbrace{\frac{\theta\alpha(1-\alpha-\beta)}{1-\theta}\left[\widehat{\Delta\mu_\tau} - \frac{1}{2}\frac{\theta\alpha}{1-\theta}\widehat{\Delta\sigma_\tau^2} - \frac{\theta}{1-\theta}\widehat{\Delta\sigma_{z\tau}} - \left(1+\frac{\theta(1-\alpha-\beta)}{1-\theta}\right)\widehat{\Delta\sigma_{\underline{w}\tau}}\right]}_{\text{correcting for minimum wage}}$$

$$\Delta \log Y = \frac{1-\alpha}{\beta+\zeta(1-\alpha-\beta)}\underbrace{\frac{\alpha(1+\epsilon)}{1-\alpha}\left(-\widehat{\Delta\mu_\tau} + \frac{1}{2}\frac{\theta}{1-\theta}\left(\alpha\widehat{\Delta\sigma_\tau^2} - 2\widehat{\Delta\sigma_{z\tau}}\right)\right)}_{\text{equation (9)}} - \underbrace{\frac{\alpha(1+\epsilon)}{\beta+\zeta(1-\alpha-\beta)}\frac{\theta(1-\alpha-\beta)}{1-\theta}\widehat{\Delta\sigma_{\underline{w}\tau}}}_{\text{correcting for minimum wage}}$$

Proof. See Appendix C.4. □

The introduction of a minimum wage for unskilled workers adds a few corrective terms to our baseline, which obviously disappear when there is no unskilled labor ($1-\alpha-\beta=0$). These terms all require the estimation of an additional sufficient statistic: the covariance between log capital wedge and firm-level minimum wage. This statistic can also be measured indirectly as the covariance between capital and unskilled labor wedges ($\frac{p_{it}y_{it}}{l_{u,it}}$). Output is reduced if the treatment increases capital distortions more in companies already facing a larger minimum wage. The baseline effect of the treatment on output is also dampened by the indexation of the minimum wage on aggregate wages. More indexation (i.e. a larger ζ) implies a smaller effect of the treatment on aggregate output.

C Proofs of Appendix Results

C.1 Proof of Proposition 10

We first show that Proposition 2 still holds with decreasing returns to scale.

With monopolistic competition and decreasing returns to scale, for a firm i with a stock of capital k_{it}, operating profits after optimizing labor demand are given by:

$$\begin{aligned} p_i y_i - w l_i &= (1-(1-\alpha)\nu\theta)\left(\frac{(1-\alpha)\nu\theta}{w}\right)^{\frac{(1-\alpha)\nu\theta}{1-(1-\alpha)\nu\theta}} Y^{\frac{1-\theta}{1-(1-\alpha)\nu\theta}} e^{z_i \frac{\theta}{1-(1-\alpha)\nu\theta}} k_i^{\frac{\alpha\nu\theta}{1-(1-\alpha)\nu\theta}} \\ &= S \times (1-(1-\alpha)\nu\theta)\left((1-\alpha)\nu\theta\right)^{\frac{(1-\alpha)\nu\theta}{1-(1-\alpha)\nu\theta}} e^{z_i \frac{\theta}{1-(1-\alpha)\nu\theta}} \left(\frac{k_i}{S}\right)^{\frac{\alpha\nu\theta}{1-(1-\alpha)\nu\theta}} \end{aligned}$$

where $S = \frac{Y^{\frac{1-\theta(1-\alpha\nu)}{1-(1-\alpha)\nu\theta}}}{w^{\frac{\nu\theta}{1-(1-\alpha)\nu\theta}}}$.

It follows directly from the proof of Proposition 2 that in this economy, and under the assumptions of Proposition 2, the ergodic joint distribution of capital wedges and productivity is independent of (w, Y) and depend only on the parameters Θ. Let $F(z, \tau; \Theta)$ denote this distribution as before.

With decreasing returns to scale ν, profit maximization for firm i in industry s as a function of a capital wedge τ_{is} leads to:

$$\begin{cases} k_i \propto \left(\frac{1}{w}\right)^{\frac{\nu(1-\alpha)\theta}{1-\nu\theta}} Y^{\frac{1-\theta}{1-\nu\theta}} e^{z_i \frac{\theta}{1-\nu\theta}} \left(\frac{1}{(1+\tau_i)}\right)^{\frac{1-(1-\alpha)\nu\theta}{1-\nu\theta}} \\ l_i \propto \left(\frac{1}{w}\right)^{\frac{1-\alpha\nu\theta}{1-\theta}} Y^{\frac{1-\theta}{1-\nu\theta}} e^{z_i \frac{\theta}{1-\nu\theta}} \left(\frac{1}{(1+\tau_i)}\right)^{\frac{\alpha\nu\theta}{1-\nu\theta}} \end{cases}$$

Firm i output at the optimum is given by:

$$p_i y_i \propto Y^{\frac{1-\theta}{1-\nu\theta}} \left(\frac{1}{w}\right)^{\frac{(1-\alpha)\nu\theta}{1-\nu\theta}} \left(\frac{1}{1+\tau_i}\right)^{\frac{\alpha\nu\theta}{1-\nu\theta}} e^{z_i \frac{\theta}{1-\nu\theta}} \tag{17}$$

Omiting the i subscripts, equilibrium on the product market implies that:

$$w \propto Y^{-\frac{1-\nu}{(1-\alpha)\nu}} \left(\int_{z,\tau} \frac{e^{z\frac{\theta}{1-\nu\theta}}}{(1+\tau)^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z,\tau;\Theta)\right)^{\frac{1-\nu\theta}{(1-\alpha)\nu\theta}} \tag{18}$$

Equilibrium on the labor market implies that $Y \propto w^{1+\epsilon}$

Combining these two equations provides the following expression for aggregate output:

$$Y \propto \left(\int_{(z,\tau)} \frac{e^{z\frac{\theta}{1-\nu\theta}}}{(1+\tau)^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z,\tau;\Theta)\right)^{\frac{(1+\epsilon)(1-\nu\theta)}{\theta((1-\alpha)\nu+(1+\epsilon)(1-\nu))}}$$

which then leads to the expression in the proposition after Taylor expansion.

Finally, aggregate TFP admits a simple expression:

$$TFP = \frac{Y}{K^{\alpha\nu} L^{(1-\alpha)\nu}}$$
$$= \left(\int_{z,\tau} \frac{z^{\frac{\theta}{1-\nu\theta}}}{(1+\tau)^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z,\tau;\Theta) \right)^{\frac{1-(1-\alpha)\nu\theta}{\theta}} \left(\int_{z,\tau} \frac{z^{\frac{\theta}{1-\nu\theta}}}{(1+\tau)^{\frac{1-(1-\alpha)\nu\theta}{1-\nu\theta}}} dF(z,\tau;\Theta) \right)^{-\alpha\nu}$$

which then leads to the formula in the proposition after straightforward Taylor expansion.

C.2 Proof of Proposition 11

Optimal labor demand as a function of firm-level capital wedge is:

$$l_i \propto \left(\frac{1}{w}\right)^{\frac{1-\alpha\theta}{1-\theta}} Y e^{\frac{\theta}{1-\theta} z_i} \left(\frac{1}{(1+\tau_i)}\right)^{\frac{\alpha\theta}{1-\theta}}$$

The following sufficient statistic can be computed for both the treatment and control group:

$$\mathbb{E}[l_i | T_i = T] \propto \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{\frac{\alpha\theta}{1-\theta}}} dF(z,\tau,\Theta_T; w, Y) = \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{\frac{\alpha\theta}{1-\theta}}} dF(z,\tau,\Theta_T)$$

where $T \in \{0,1\}$ and where the second equality comes from Proposition 2 – the joint distribution of log-capital wedges and productivity is independent of (w, Y).

We now introduce the log difference in mean employment:

$$\widehat{\Delta l} = \log\left(\mathbb{E}[l_i | T_i = 1]\right) - \log\left(\mathbb{E}[l_i | T_i = 0]\right)$$
$$= \log \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{\frac{\alpha\theta}{1-\theta}}} dF(z,\tau,\Theta_1) - \log \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{\frac{\alpha\theta}{1-\theta}}} dF(z,\tau,\Theta_0)$$

Given the output equation (3), it follows directly that

$$\Delta \log Y = \frac{(1+\epsilon)(1-\theta)}{(1-\alpha)\theta} \widehat{\Delta l}$$

We now compute TFP, which requires calculating the capital stock. Similarly, optimal capital demand implies that:

$$k_i \propto \left(\frac{1}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}} Y e^{\frac{\theta}{1-\theta} z_i} \left(\frac{1}{1+\tau_i}\right)^{\frac{1-(1-\alpha)\theta}{1-\theta}}$$

Like for employment, we use this to compute the new capital sufficient statistic:

$$\widehat{\Delta k} = \log\left(\mathbb{E}[k_i | T_i = 1]\right) - \log\left(\mathbb{E}[k_i | T_i = 0]\right)$$
$$= \log \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{1+\frac{\alpha\theta}{1-\theta}}} dF(z,\tau,\Theta_1) - \log \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{1+\frac{\alpha\theta}{1-\theta}}} dF(z,\tau,\Theta_0)$$

Given the TFP formula (4), $\widehat{\Delta l}$ and $\widehat{\Delta k}$ can be straightforwardly combined into the formula given in the proposition.

C.3 Proof of Proposition 12

Noting η_{it} the wedge between the marginal product of labor and the wage, and assuming that capital and labor wedges, as well as productivity, has small deviation around their means, we obtain that:

$$\begin{aligned} c^{\frac{\theta}{1-\theta}} &= I_Y \\ c^{\frac{\theta}{1-\theta}} wL &= (1-\alpha)\theta Y I_L \\ c^{\frac{\theta}{1-\theta}} RK &= \alpha\theta Y I_K, \end{aligned}$$

where:

$$\begin{aligned} I_Y &= \int e^{\frac{\theta}{1-\theta}z_i} \left(\frac{1}{1+\tau_i}\right)^{\frac{\alpha\theta}{1-\theta}} \left(\frac{1}{1+\eta_i}\right)^{\frac{(1-\alpha)\theta}{1-\theta}} di \\ I_L &= \int e^{\frac{\theta}{1-\theta}z_i} \left(\frac{1}{1+\tau_i}\right)^{\frac{\alpha\theta}{1-\theta}} \left(\frac{1}{1+\eta_i}\right)^{\frac{(1-\alpha)\theta}{1-\theta}+1} di \\ I_K &= \int e^{\frac{\theta}{1-\theta}z_i} \left(\frac{1}{1+\tau_i}\right)^{\frac{\alpha\theta}{1-\theta}+1} \left(\frac{1}{1+\eta_i}\right)^{\frac{(1-\alpha)\theta}{1-\theta}} di \end{aligned}$$

and $c = \left(\frac{R}{\alpha\theta}\right)^{\alpha} \left(\frac{w}{(1-\alpha)\theta}\right)^{(1-\alpha)}$

Using a second order approximation or a log-normal assumption on z_i, τ_i and η_i, we obtain the following formulas for TFP and output:

$$\begin{aligned} \log TFP &= \left(\mu_z + \frac{1}{2}\frac{\theta}{1-\theta}\sigma_z^2\right) - \frac{\alpha}{2}\left(\sigma_\tau^2 + \sigma_\eta^2\right) - \frac{1}{2}\frac{\theta}{1-\theta}\text{var}(\Delta) \\ \log Y &= -\left(\mu_\eta + \frac{\sigma_\eta^2}{2}\right) + \frac{1+\epsilon}{1-\alpha}\left(-\mu_z + E\Delta + \frac{1}{2}\frac{\theta}{1-\theta}\text{var}(z-\Delta)\right) + \frac{\theta}{1-\theta}\text{cov}\left(z-\Delta, \eta\right) \end{aligned}$$

where we note $\mu_\eta = E\log(1+\eta)$, $\sigma_\eta^2 = \text{var}\log(1+\eta)$ and $\Delta = \alpha\log(1+\tau) + (1-\alpha)\log(1+\eta)$.

We then assume that the mean and variances of z and η are unaffected by the experiment, which gives the results in the proposition.

C.4 Proof of Proposition 13

Scale invariance of capital wedges

Omitting the i, t subscripts, operating profits are given by:

$$\pi = \max_{l_s, l_u} \left(Y^{1-\theta} e^{\theta z} l_s^{\beta\theta} l_u^{(1-\alpha-\beta)\theta} - w l_s - \underline{w} w^{\zeta} \right)$$

which rewrites:

$$\pi = (1-\theta(1-\alpha)) Y^{\frac{1-\theta}{1-\theta(1-\alpha)}} e^{\frac{\theta z}{1-\theta(1-\alpha)}} \left(\left(\frac{\theta\beta}{w} \right)^{\beta} \left(\frac{\theta(1-\alpha-\beta)}{\underline{w} w^{\zeta}} \right)^{1-\alpha-\beta} \right)^{\frac{\theta}{1-\theta(1-\alpha)}} k^{\frac{\theta\alpha}{1-\theta(1-\alpha)}}$$

From the above formula, it appears easily that the firm problem given Y, w can be written as a firm problem given $1, 1$ scaled by:

$$S = \frac{Y}{w^{\frac{\theta(\beta+\zeta(1-\beta-\alpha))}{1-\theta}}}$$

so that the distribution of capital wedges is unaffected by the equilibrium. Note that the scaling parameter is the same as in proposition 2 when $\zeta = 0$.

Equilibrium formulas

We use the following notations:

$$\begin{aligned}
\chi &= \frac{\theta}{1-\theta} \left(\beta + \zeta(1-\alpha-\beta) \right) \\
u_{it} &= \frac{\theta}{1-\theta} \left(z_{it} - \alpha \log(1+\tau_{it}) - (1-\alpha-\beta) \log \underline{w}_{it} \right) \\
v_{it} &= u_{it} - \log(1+\tau_{it}) \\
\omega_{it} &= u_{it} - \log \underline{w}_{it}
\end{aligned}$$

Then, the three market clearing conditions write:

$$\begin{aligned}
w^{\chi} &\propto Eu \\
K &\propto \frac{1}{w^{\chi}} Ev \\
wL^s Y & \\
w^{\zeta} L^u &\propto YE\omega
\end{aligned}$$

After some manipulation, we get the expression for TFP:

$$\Delta \log TFP = \underbrace{-\frac{\alpha}{2} \left(1 + \frac{\alpha\theta}{1-\theta} \right) \sigma_{\tau}^2}_{\text{equation (8)}} + \underbrace{\frac{\theta\alpha(1-\alpha-\beta)}{1-\theta} \left[\mu_{\tau} - \frac{1}{2} \frac{\theta\alpha}{1-\theta} \sigma_{\tau}^2 - \frac{\theta}{1-\theta} \sigma_{z\tau} - \left(1 + \frac{\theta(1-\alpha-\beta)}{1-\theta} \right) \sigma_{\underline{w}\tau} \right]}_{\text{correcting for minimum wage}}$$

omitting the terms in $\mu_{\underline{w}}$, $\sigma^2_{\underline{w}}$, μ_z and σ_z^2. These terms are assumed to be unaffected by the experiment.

The formula for log output is even simpler:

$$\log Y = \frac{\alpha(1+\epsilon)}{\beta+\zeta(1-\alpha-\beta)}\left(\underbrace{-\mu_\tau + \frac{1}{2}\frac{\theta}{1-\theta}\left(\alpha\sigma_\tau^2 - 2\sigma_{\tau z}\right)}_{\text{equation (9)}} - \underbrace{\frac{\theta(1-\alpha-\beta)}{1-\theta}\sigma_{\underline{w}\tau}}_{\text{correcting for minimum wage}}\right)$$