

# Job-Finding and Job-Losing: A Comprehensive Model of Heterogeneous Individual Labor-Market Dynamics \*

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## Abstract

We track the path that a worker follows after losing a job. Initially, the typical job-loser spends some time out of the labor force and in job search. Only a month or two later, in normal times, the worker lands a job. But the job is frequently brief. Over the next few months, the worker finds a good match that becomes a long-term job. Short-term jobs tend to precede long-term ones. Short-term employment shares some of the characteristics of unemployment and some of the characteristics of employment. We show that this pattern of moving among working, searching for a job, and being out of the labor force is concentrated in a segment of the working-age population. In other segments, individuals are insulated from disturbances to their activities in the labor market. Some work continuously while others are always out of the labor market. We develop a model that incorporates heterogeneity across and within these segments.

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<i>Month</i>	<i>Out of labor force</i>	<i>Unemployment</i>	<i>Short-term job</i>	<i>Long-term job</i>
0	0	100	0	0
1	26	39	17	18
2	31	21	18	29
3	32	15	18	35
4	32	13	17	38
$\infty$	29	10	18	44

Table 1: Probabilities of Future States Facing an Unemployed Individual

Table 1 shows a basic finding of this paper. We consider four labor-market states: (1) out of labor force, (2) searching for a job (unemployed), (3) holding a short-term job, and (4) holding a long-term job. We describe the probability distribution of the future labor-market states of an individual who is unemployed in month zero. The rows in the table show the distribution perceived in month zero across the various states over the four succeeding months and in the longer run. The findings are derived from data for a period of normal tightness in the labor market, with national unemployment at its long-term average.

In month 0, the individual is unemployed. The unsurprising feature of the table is that the probability of being out of the labor force or employed in a long-term job rises month by month, as the probability of unemployment declines. What is a surprise is that the probability of holding a short-term job jumps up immediately in month 1 up to or even above its later level. There are no lags in that probability, unlike the others.

Short-term jobs are easy to find under normal conditions in the US labor market. While holding a short-term job, the worker has a chance of moving up to a long-term job. We find that short-term jobholding is akin to unemployment—it is another step on the way to a long-term job. The same force that causes a downward movement in the probability of unemployment offsets the pro-employment trend that is visible in the growth of the probability of long-term employment. Our model shows that it is twice as likely for the first three months of the post-unemployment period to have a short job precede a long job than the other way around. Short jobs are stepping stones to long-term jobs.

The two main conclusions of the paper are:

- Short-term jobs are partly a substitute for unemployment—they are a natural extension of the search process.
- The time that a job-loser spends out of the labor force or searching for jobs is small compared to the time spent in short-term jobs.

<i>Labor market state</i>	<i>Non-polar types</i>			<i>All Non-Polar Types</i>	<i>All E</i>	<i>All N</i>	<i>Total model</i>
	<i>Type 1</i>	<i>Type 2</i>	<i>Type 3</i>				
Out of labor force	6.7	19.0	59.6	29.3	0.0	100.0	24.2
Unemployed	4.9	34.8	5.8	9.8	0.0	0.0	3.7
Work in short-term job	26.9	12.0	8.7	17.5	0.0	0.0	6.6
Work in long-term job	61.5	34.2	25.8	43.5	100.0	0.0	65.5
Weight in total	0.16	0.07	0.16	0.39	0.50	0.12	1.00

Table 2: Distribution of the Working-Age Population by Segment

In normal times, frequent transitions among working, searching for a job, and being out of the labor force are concentrated in a set of three types among the working-age population. Two other types are insulated from disturbances to their activities in the labor market. Some work continuously while others are always out of the labor market. Table 2 shows the distribution over labor market states for the working-age population by type implied by our model.

Individuals of type 1, comprising 16 percent of the working-age population, spend most of their time working. Among that type, in the typical month, 61.5 percent are working in long-term jobs and 26.9 percent in short-term jobs. Only 6.7 percent are out of the labor force and 4.9 percent are not working but are searching for work. Those of type 2 work about half as much and are particularly likely to be unemployed, 34.8 percent of that type. Those of type 3, another 16 percent of the working age population, are likely to be out of the labor force—59.6 percent of that type. Half of the working age population belongs to type all-E and they all work in long-term jobs. The last type, all-N, is always out of the labor force and accounts for 12 percent of the working-age population. Overall, the fraction of time spent in short-term jobs is almost twice the fraction of time spent searching for work—6.6 percent against 3.7 percent.

The conclusions illustrated in these tables derive from a detailed dynamic model estimated from the US Current Population Survey. The model has two dimensions of heterogeneity. The first is based on observables. We build four models based on age and gender. The age groups are *young*, 16 through 24 years old, and *prime age*, 25 through 54 years.

We also take account of latent heterogeneity that is not based on direct observation. We define a type of individual by a vector of parameters for the personal dynamic programs of the members of the type. These dynamic programs are generalizations of those considered in the search-and-matching literature in the Diamond-Mortensen-Pissarides tradition. The parameters include the flow values of time spent out of the labor force and time spent searching, the wages of the two kinds of jobs, the probabilities of occurrence of favorable

events such as locating and accepting a short- or long-term job, and the probability of occurrence of unfavorable events such as losing a job.

We distinguish between a person’s *state* and that person’s *activity*. The state is not directly observed while the activity is observed and recorded in the CPS. The most important distinction along this line is for employment, where we recognize two states, *employed in short-term job* and *employed in long-term job*, but only one corresponding observed activity, *employed*. The challenges to estimation of the models’ parameters are to deal with heterogeneity as expressed in our types and to deal with the hidden states. Both finite-type heterogeneity and hidden states are powerful ways to describe the complicated patterns of time dependence that are present in the activity histories in the CPS. Although we describe the evolution of the states of an individual of a given type in terms of a first-order Markov process, the evolution of the observed activities is described by a probability mixture of Markov processes, which we show is far from first-order.

Respondents contribute four months of data after entering the CPS and another four months a year later. Thus the survey covers a 16-month period in their labor-market experiences. It is well suited for studying dynamic issues over that time span, but not to studying life-cycle issues, where long panel data with much less frequent interviews are better suited. The CPS better serves our objectives than would quarterly and annual administrative data on earnings, because the CPS records key monthly information about labor-market activities for a large representative cross-section of the population.

We distill the data for each demographic group into a vector of frequencies of activity paths. Our model uses all of the information in the CPS, particularly the relations between activities observed over a year or more apart. The object we study is the distribution of paths in the labor market as captured in the CPS. The survey records labor-market activity (employed, E, unemployed, U, and out of labor force, N) in four consecutive months, then records activity in the same months a year later. The information for a given respondent can be written as a sequence of 8 letters—for example, UUEE-NNNU. There are  $3^8 = 6561$  different paths, so the distribution is a vector of length 6561, summing to one.

The distribution of activities over the 8 months recorded in the CPS offers a vastly richer description of labor-market dynamic outcomes than the one-month transition matrixes that have been studied historically. A recent literature has explored improvements over the traditional first-order Markov model of the labor-force transitions reported in the CPS. Kudlyak and Lange (2018) find that employment in the months immediately preceding a month dramatically raises the conditional probability of a move from out of the labor force to employment made in that month into the following month. This finding implies that studying transitions in a month, especially into employment from out of the labor force, is

mistaken—the conditioning information from employment in earlier months has large effects. Hall and Schulhofer-Wohl (2018), following Krueger, Cramer and Cho (2014), look at the issue in the reverse way, by considering more than just one month ahead, conditional on the current month. This approach generates very different longer-run transition rates than would occur from repeated application of one-month transition probabilities. The distribution of paths in the labor market as captured in the CPS is a sufficient statistic for our statistical model. In particular, any linear regression involving observed activities can be calculated from our distillation into the frequency distribution across the 6561 paths.

In each of our four models for the demographic groups, we recognize five types. Two of our five types are highly stable. One type always holds a job. The second is always out of the labor force. We call these the *polar* types. The other three, non-polar types describe the labor-market histories in terms of transition probabilities among the four states. These types are designed to fit the observed individual dynamics in the CPS, which we attribute to the non-polar types, because the polar types do not change their activities—they have no dynamics. The non-polar types contribute to the incidence of all-E and all-N activity paths—the observed frequencies of those two paths are the sums of the polar types and a fraction of the non-polar types.

This paper is entirely about personal dynamics and not about aggregate dynamics. We estimate the model with data from a quiescent period of moderate unemployment. Although individuals experience dynamic change, the sum across millions of them changes very little each month. In future work, we plan to use the same tools to study aggregate changes, with emphasis on the explosion of unemployment following the financial crisis in late 2008.

The paper is also not offered as a contribution to the longstanding literature on decomposing observed duration dependence into components arising from heterogeneity and true duration effects. Our model includes one important type of true duration dependence through its distinction between short-term and long-term jobs. But its main emphasis is on the importance of heterogeneity.

## 1 Economic Model

Our model is a considerable extension of the existing DMP class of models. The model is structured as a probability mixture of personal dynamic programs. Random events govern the individual’s choices. Each month, the individual chooses from among a set of available options, picking the one with the highest Bellman value. The transition matrix is the result of those choices. It is an economic object, not a predetermined set of transition probabilities. In our application, there are 6 Bellman values and 6 Bellman equations. We solve the system for

the Bellman values. Based on the solved values, we can determine the choice the individual makes at each choice point. Thus the Bellman values determine a transition matrix among the four states. The Appendix discusses some of the details related to these calculations.

The discussion in this section applies to a single type. All of the parameters in the model depend on the type,  $\theta$ . To simplify the notation, we do not always include the subscript  $\theta$ .

The model has two non-work states,  $s = 1$  and  $s = 2$ , called *non-work* and *activated non-work*. The work states are  $s = 3$  and  $s = 4$ , called *short-term job* and *long-term job*. The probability of landing a job while in activated non-work is higher than from non-activated non-work. Our concept of activation captures some of the behavior recorded in the CPS among people close to the margin of participation in the labor market. For example, those recorded as out of the labor force but desiring to work and available to work are much more likely to find work in the ensuing month than those who are not available (see Hall and Schulhofer-Wohl (2018)). Becoming available through some random change in personal or family circumstances is similar to our concept of activation. An activated individual is more likely to start a new job in the ensuing month in our model. We assume that activated people who choose to search are counted as unemployed in the CPS.

The states are *partially* hidden because when a worker is not employed we know that she is in  $s = 1$  or  $s = 2$ , while an employed worker is in  $s = 3$  or  $s = 4$ . An unemployed individual or one out of the labor force may be in  $s = 1$  or  $s = 2$ .

In the model, individuals face random events—the occurrence of favorable opportunities and of adverse shocks. The main logic of the choices open to the individual is as follows. Whenever an individual is presented with an opportunity to move to a higher state, the individual can always choose any of the lower states. Whenever a shock forces a move to a lower state, the individual needs to move to that state or can choose any of the lower states. Thus individuals always keep in mind that exiting the labor force or quitting a job may be the best available alternative.

While in a non-working state,  $s = 1$  or  $s = 2$ , an individual can be either out of the labor force or searching, with the flow values  $z$  and  $z + b$ , respectively. With  $s = 1$ , the two job finding rates, called  $\psi_1$  and  $\phi_1$ , are low. Conditional on finding a job from state 1, most of the transition chances are for short-term jobs, with  $s = 3$ , and there is a small probability  $\gamma_2$  of going straight into a long-term job, with  $s = 4$ . With  $s = 2$ , the job finding rates  $\psi_2$  and  $\phi_2$  are higher. Conditional on finding a job from state 2, the fraction of activated jobseekers who draw a long-term job is  $\gamma$ . While in the activated group of non-workers, there is a probability  $\eta$  of dropping down to state 1. An inactive individual may experience activation with probability  $\rho$ .

While in a short-term job, a worker earns  $w_3$ . The low-tier job terminates into non-work with probability  $\delta_3$ . When that happens, there is a probability  $\kappa_2$  that the worker drops to state 2 rather than to state 1. Conditional on not separating into non-work, there is a probability  $\mu$  that the worker advances to a long-term job.

While in a long-term job, a worker earns  $w_4$ , which we normalize at 1, so the values in the model are in wage units, using the wage of the long-term job to define the unit. The long-term job terminates into non-work with a small probability  $\delta_4$ . When that happens, there is a probability  $\kappa$  that the worker drops to state 2 rather than to state 1. This generates a flow of experienced, successful workers into the activated non-work. Conditional on not separating into non-work, there is a probability  $\nu$  that the worker is demoted to the short-term job. This captures endogenous layoffs—the “slippery job ladder”.

The timing of the events is as follows. At the beginning of a period an individual gets the flow value of the state and activity that he is in. The chance to transition to another state arrives. The individual chooses which state to transition to with the highest state being the one for which the chance has arrived. Furthermore, for either non-employed state, an individual chooses between two activities: unemployment or out of the labor force. The two non-work states allow to potentially observe two non-work activities for a worker type—unemployment and out of labor force—depending on the state.

The Bellman values are denoted:

- $X_1$ : Value of remaining out of the labor force in state 1
- $S_1$ : Value of searching in state 1
- $X_2$ : Value of remaining out of the labor force in state 2
- $S_2$ : Value of searching in state 2
- $V_3$ : Value of holding short-term job in state 3
- $V_4$ : Value of holding long-term job in state 4

The Bellman equations are, for those out of the labor force with  $s = 1$ ,

$$X_1 = z + \frac{1}{1+r} [(1 - \psi_1 - \rho) \max(X_1, S_1) + \rho \max(X_1, S_1, X_2, S_2) + \psi_1 [(1 - \gamma_2) \max(X_1, S_1, X_2, S_2, V_3) + \gamma_2 \max(X_1, S_1, X_2, S_2, V_3, V_4)]], \quad (1)$$

for searchers with  $s = 1$ ,

$$S_1 = z + b + \frac{1}{1+r} [(1 - \phi_1 - \rho) \max(X_1, S_1) + \rho \max(X_1, S_1, X_2, S_2) + \phi_1 [(1 - \gamma_2) \max(X_1, S_1, X_2, S_2, V_3) + \gamma_2 \max(X_1, S_1, X_2, S_2, V_3, V_4)]], \quad (2)$$

for those out of the labor force with  $s = 2$ ,

$$X_2 = z + \frac{1}{1+r} [\eta \max(X_1, S_1) + (1 - \eta - \psi_2) \max(X_1, S_1, X_2, S_2) + \psi_2 [(1 - \gamma) \max(X_1, S_1, X_2, S_2, V_3) + \gamma \max(X_1, S_1, X_2, S_2, V_3, V_4)]], \quad (3)$$

for searchers with  $s = 2$ ,

$$S_2 = z + b + \frac{1}{1+r} [\eta \max(X_1, S_1) + (1 - \eta - \phi_2) \max(X_1, S_1, X_2, S_2) + \phi_2 [(1 - \gamma) \max(X_1, S_1, X_2, S_2, V_3) + \gamma \max(X_1, S_1, X_2, S_2, V_3, V_4)]], \quad (4)$$

for workers holding a short-term job with  $s = 3$ , and

$$V_3 = w_3 + \frac{1}{1+r} [\delta_3 [(1 - \kappa_2) \max(X_1, S_1) + \kappa_2 \max(X_1, S_1, X_2, S_2)] + (1 - \delta_3) [(1 - \mu) \max(X_1, S_1, X_2, S_2, V_3) + \mu \max(X_1, S_1, X_2, S_2, V_3, V_4)]], \quad (5)$$

for workers holding a short-term job with  $s = 4$

$$V_4 = w_4 + \frac{1}{1+r} [\delta_4 [(1 - \kappa) \max(X_1, S_1) + \kappa \max(X_1, S_1, X_2, S_2)] + (1 - \delta_4) [\nu \max(X_1, S_1, X_2, S_2, V_3) + (1 - \nu) \max(X_1, S_1, X_2, S_2, V_3, V_4)]]. \quad (6)$$

## 1.1 The transition matrix across states

The model determines the transition matrix across states based on the Bellman values. A choice involves finding the maximum value of a particular set of values. To describe the construction of the transition matrix, we define the choice indicator operator  $\succ$  as the choice among those listed in curly braces following the  $\succ$ . The operator has the value 1 if the value on the left is the one chosen from the set on the right and zero otherwise. For example,  $4 \succ \{1, 2, 3, 4\}$  has the value 1 if the individual chooses state 4 out of the set of possible states  $\{1, 2, 3, 4\}$ . The choice operator has priority over multiplication, so  $\psi_1 4 \succ \{1, 2, 3, 4\}$  means  $\psi_1 \times (4 \succ \{1, 2, 3, 4\})$ .

An individual makes two different kinds of choices. The first is the choice between labor market states. When a shock hits, the individual compares the Bellman values in different states and chooses a state to transition to. A transition from one state to another occurs if the corresponding shock hits and the particular state is chosen from the set of possible states triggered by the shock. An entry in the transition matrix thus contains the probability of the shock multiplied by the choice indicator for the state.

The second choice is between unemployment and out of labor force, while in the non-work states 1 or 2. This choice determines the probabilities with which certain shocks hit. For example, if the person chooses unemployment in state 2, the job finding rate is  $\phi_2$ , while



	Non-work	Activated non-work	Work in short-term job	Work in long-term job	
	1	2	3	4	
Non-work	1	$\frac{1-\rho-\lambda_1+}{\rho_1 \succ (1,2)+}$ $\lambda_1(1-\gamma_2)1 \succ (1,2,3)+$ $\lambda_1\gamma_21 \succ (1,2,3,4)$	$\rho \times 2 \succ (1,2)+$ $\lambda_1(1-\gamma_2) \times 2 \succ (1,2,3)+$ $\lambda_1\gamma_2 \times 2 \succ (1,2,3,4)$	$\lambda_1(1-\gamma_2) \times 3 \succ (1,2,3)+$ $\lambda_1\gamma_2 \times 3 \succ (1,2,3,4)$	$\lambda_1\gamma_2 \times 4 \succ (1,2,3,4)$
Activated non-work	2	$\frac{\eta+}{1-\eta-\lambda_2 \times 1 \succ (1,2)+}$ $\lambda_2(1-\gamma) \times 1 \succ (1,2,3)+$ $\lambda_2\gamma \times 1 \succ (1,2,3,4)$	$1-\eta-\lambda_2 \times 2 \succ (1,2)+$ $\lambda_2(1-\gamma) \times 2 \succ (1,2,3)+$ $\lambda_2\gamma \times 2 \succ (1,2,3,4)$	$\lambda_2(1-\gamma) \times 3 \succ (1,2,3)+$ $\lambda_2\gamma \times 3 \succ (1,2,3,4)$	$\lambda_2\gamma \times 4 \succ (1,2,3,4)$
Work in short-term job	3	$\frac{\delta_3(1-\kappa_2)+}{\delta_3\kappa_2 \times 1 \succ (1,2)+}$ $(1-\delta_3)(1-\mu) \times 1 \succ (1,2,3)+$ $(1-\delta_3)\mu \times 1 \succ (1,2,3,4)$	$\delta_3\kappa_2 \times 2 \succ (1,2)+$ $(1-\delta_3)(1-\mu) \times 2 \succ (1,2,3)+$ $(1-\delta_3)\mu \times 2 \succ (1,2,3,4)$	$(1-\delta_3)(1-\mu) \times 3 \succ (1,2,3)+$ $(1-\delta_3)\mu \times 3 \succ (1,2,3,4)$	$(1-\delta_3)\mu \times 4 \succ (1,2,3,4)$
Work in long-term job	4	$\frac{\delta_4(1-\kappa)+}{\delta_4\kappa \times 1 \succ (1,2)+}$ $(1-\delta_4)\nu \times 1 \succ (1,2,3)+$ $(1-\delta_4)(1-\nu) \times 1 \succ (1,2,3,4)$	$\delta_4\kappa \times 2 \succ (1,2)+$ $(1-\delta_4)\nu \times 2 \succ (1,2,3)+$ $(1-\delta_4)(1-\nu) \times 2 \succ (1,2,3,4)$	$(1-\delta_4)\nu \times 3 \succ (1,2,3)+$ $(1-\delta_4)(1-\nu) \times 3 \succ (1,2,3,4)$	$(1-\delta_4)\nu \times 4 \succ (1,2,3,4)$

Table 3: Deriving the Transition Probabilities from the Bellman Values

a choice of out-of-labor-force in state 2, implies that rate is  $\psi_2$ . We use the same operator notation when the choice is among states, as above, and when the choice is between pairs of activities, as in  $X_1 \succ \{X1, S1\}$  to query whether the individual chooses to be out of the labor force in state 1. To make the transition table more compact, we use the following notation for the probabilities of shocks that depend on the choice of activity:  $\lambda_1$  denotes the job finding rate from state 1:  $\lambda_1 = \psi_1 X_1 \succ \{X1, S1\} + \phi_1 S_1 \succ \{X1, S1\}$ , and  $\lambda_2$  denotes the job finding rate from state 2,  $\lambda_2 = \psi_2 X_2 \succ \{X2, S2\} + \phi_2 S_2 \succ \{X2, S2\}$ .

Table 3 lays out the transition matrix of a non-polar type.

The transition matrix of the all-E polar type is

$$\pi_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and the transition matrix of the all-N polar type is

$$\pi_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

## 1.2 The activity probability vector of a type

The function describing the mapping of the partially hidden state variable  $s$  to the observed activities  $a$  is:

- If  $(X_1 \succ \{X_1, S_1\}$  and  $s = 1)$  or  $(X_2 \succ \{X_2, S_2\}$  and  $s = 2)$ ,  $a = N$
- If  $(S_1 \succ \{X_1, S_1\}$  and  $s = 1)$  or  $(S_2 \succ \{X_2, S_2\}$  and  $s = 2)$ ,  $a = U$
- If  $s = 3, 4$ ,  $a = E$

The transition matrix, and its associated vector of ergodic probabilities of a type, assign a probability to each of the  $4^8 = 65,536$  state paths. Were it not for the 8-month gap separating a respondent’s first and second appearance in the CPS, the probability would be the product of the ergodic probability of the state in the first month and the transition probabilities for the following 7 transitions. The Appendix describes how the calculation accounts for the 8-month gap.

We obtain the probability of an activity path by adding together the probabilities of all the state paths that map into a given activity path. For example, the state paths 1112-3311, 1112-3411, 1112-4311, and 1112-4411, all map into NNNU-EENN, so the probability of the activity path is the sum of the probabilities of the four state paths. The adding-up process generates the vector of  $3^8 = 6561$  activity-path probabilities.

## 2 Model Solution

The Bellman system for a particular type, equation (1) through equation (6), describes the stationary state as

$$V = f + \frac{1}{1+r} P \odot M(V). \quad (7)$$

Here  $\odot$  generates the matrix of element-by-element multiplications. For each type,  $V$  is a vector of Bellman values,  $f$  is a vector of flow values corresponding to  $V$ ,  $r$  is the discount rate applied to the succeeding month,  $P$  is a matrix of probabilities that appear in the Bellman equations that are functions of parameters, and  $M(V)$  is a matrix of functions of the Bellman values containing  $\max()$  functions over selected Bellman values to describe the individual’s choices governed by those values. The flow values may be subject to inequality constraints.

The parameter space for a type comprises the vector of flow values  $f$  and the probabilities  $P$ . Each type also has an  $M(V)$  that describes the structure of the type’s decision problem. We index types by the integer  $\theta$ , but frequently suppress it from the notation when discussing a single type, as in this section. Each type described by  $\{f, P, \text{ and } M(\cdot)\}$  implies a set of Bellman values and conditional choice of actions.

We define a *region* as a subspace of the parameter space within which every  $\max()$  over a given set of Bellman values dictates the same choice for all the parameters in the subspace. A region implies a set of inequalities in the Bellman values. Within region  $R$ , the fairly complicated object  $M(V)$  becomes the much simpler  $M(V) = M_R V$ , where  $M_R$  is a square matrix of functions of the probability parameters.  $M_R$  is the same everywhere in the region.

Then the Bellman system becomes

$$V = f + \frac{1}{1+r} P \odot M_R V. \quad (8)$$

The probability parameters can vary within a region, and the same vector of probability parameters can inhabit more than one region, coupled with different flow-value parameters  $f$ .

The Bellman system in equation (8) is homogeneous of degree one in the flow values and Bellman values. To pin down the values, we normalize a wage in a long-term job to be 1. We then interpret the other values in terms of wage units.

Most of our work with the model involves a specialization of the region that we believe is reasonable. In it, people all have the same monotonic mapping of states into activities:

- In state 1 (inactive without job) choose to be out of the labor force.
- In state 2 (active without job) choose to search for a job.
- In state 3 (short-term job available) choose to work in the short-term job.
- In state 4 (long-term job available) choose to work in the long-term job.

We call this the *benchmark region*. This outcome will occur if the Bellman values satisfy  $V_4 > V_3 > S_2 > X_1 > S_1$  and  $S_2 > X_2$ . By considering the benchmark region, we are confining the parameters to a space such that the solution to the Bellman system satisfies these inequalities. It turns out that this space is quite rich—we can set up types with fairly different parameter values so that the linear combination of five types comes close to fitting the observed distribution of frequencies. And all of the types have this monotonic mapping of states into activities.

This approach requires that any proposed type have Bellman values that satisfy the monotonic mapping. The Appendix describes a procedure to check that the mapping holds and to calculate bounds on some of the parameters.

With the monotonic mapping, the Bellman equations are, for those out of the labor force with  $s = 1$ ,

$$X_1 = z + \frac{1}{1+r} [(1 - \psi_1 - \rho)X_1 + \rho S_2 + \psi_1[(1 - \gamma_2)V_3 + \gamma_2 V_4]], \quad (9)$$

for searchers with  $s = 1$ ,

$$S_1 = z + b + \frac{1}{1+r} [(1 - \phi_1 - \rho)X_1 + \rho S_2 + \phi_1[(1 - \gamma_2)V_3 + \gamma_2 V_4]], \quad (10)$$

	<i>From state</i>	<i>To state</i>			
		1	2	3	4
1	Inactive non-work	$1-\psi_1-\rho$	$\rho$	$\psi_1(1-\gamma_2)$	$\psi_1\gamma_2$
2	Active non-work	$\eta$	$1-\eta-\phi_2$	$(1-\gamma)\phi_2$	$\gamma\phi_2$
3	Short-term job	$\delta_3(1-\kappa_2)$	$\delta_3\kappa_2$	$(1-\delta_3)(1-\mu)$	$(1-\delta_3)\mu$
4	Long-term job	$(1-\kappa)\delta_4$	$\kappa\delta_4$	$\nu(1-\delta_4)$	$(1-\nu)(1-\delta_4)$

Table 4: Parameters of the Non-Polar Transition Matrixes

for those out of the labor force with  $s = 2$ ,

$$X_2 = z + \frac{1}{1+r} \{ \eta X_1 + (1 - \eta - \psi_2) S_2 + \psi_2 [(1 - \gamma) V_3 + \gamma V_4] \}, \quad (11)$$

for searchers with  $s = 2$ ,

$$S_2 = z + b + \frac{1}{1+r} \{ \eta X_1 + (1 - \eta - \phi_2) S_2 + \phi_2 [(1 - \gamma) V_3 + \gamma V_4] \}, \quad (12)$$

for workers holding a short-term job with  $s = 3$ , and

$$V_3 = w_3 + \frac{1}{1+r} [ \delta_3 ((1 - \kappa_2) X_1 + \kappa_2 S_2) + (1 - \delta_3) ((1 - \mu) V_3 + \mu V_4) ], \quad (13)$$

for workers holding a short-term job with  $s = 4$

$$V_4 = w_4 + \frac{1}{1+r} [ \delta_4 ((1 - \kappa) X_1 + \kappa S_2) + (1 - \delta_4) (\nu V_3 + (1 - \nu) V_4) ]. \quad (14)$$

The Bellman system, the set of inequalities in the Bellman values, and the flow-value normalization, define a region as a polytope in the space of  $\{f, V\}$  vectors.

Table 4 shows the non-polar transition matrix in this region. The transition matrix together with the mapping between states and activities is used to construct activity paths for a type. In Section 3, we describe our estimation procedure for the parameters for each types and the type-specific weights in the population.

## 2.1 Flow values

The flow values from the Bellman system do not enter the parametrization of the transition matrix and thus they are not estimated from the data. But the flow values define the Bellman values that are used to make a choice when the shock hits. We thus need to make sure that there exist a set of flow values that define the Bellman values consistent with inequalities that describe the benchmark region which is captured by the transition matrix described in Table 4.

The flow values that satisfy the regions Bellman-value inequalities form a fairly large set. To find the bounds on the flow parameters in  $f$  that constitute the edges of the set in that dimension, we run linear programs to maximize and minimize each of those flow values, taking the Bellman equations as equality constraints along with the inequalities in the Bellman values. If there are no feasible values of the flow values that satisfy the constraints, so there is no solution to the linear program, then the proposed definition of the region is not feasible.

We find that a relatively wide range of flow values is consistent with the inequalities that describe the region. Consequently, the solution to the Bellman system described above that is consistent with a transition matrix described in Table 4 is far from unique. Rather, there is a set of Bellman values consistent with these transitions, which is defined by vector of flow values  $f$ .

## 2.2 Parameters off the equilibrium path

For a given region definition, it is possible that some of the probability parameters do not appear in the actual transition probabilities that embody the choices that an individual makes, but are influential in determining that the individual did not make a particular choice.

For example, our model includes a type who chooses to be out of the labor force and never chooses to search in state 1. The Bellman value for searching in state 1,  $S_1$ , will only appear in one place in the Bellman system—in the equation for  $S_1$ —because  $S_1$  will never be chosen as a continuation value. The Bellman value for  $S_1$  applies to the choice not taken.

The equation for  $S_1$  involves a parameter  $\phi_1$  that is the probability that somebody who chose to search in state 1 would find a job. This probability does not appear in the type's transition matrix. But it does influence the choice not to search in state 1. Given a solution to the Bellman system, the Bellman equation for  $S_1$  imposes a linear relation between  $S_1$  and  $\phi_1$ , given the solved values of the rest of the Bellman values. Any point on this line that satisfies the inequality that the value for the not-chosen option falls short of the value of the chosen option is admissible. The bounds on the flow values and the Bellman values derived earlier induce bounds on  $\phi_1$ . If the bounds do not include a value in  $[0, 1]$ , we conclude that the Bellman system has no solution because there is no feasible value of  $\phi_1$  that can satisfy the Bellman value of the choice not taken.

In the region we emphasize,  $S_1$  and  $X_2$  do not appear on the right-hand side of any equation in the Bellman system because the individual never chooses to search in state 1 or be out of the labor force in state 2. We solve the Bellman system for  $X_1$ ,  $S_2$ ,  $V_3$ , and  $V_4$ ,

ignoring the equations for  $S_1$  and  $X_2$ , and then check whether there exists the range of values of  $\phi_1$  and  $\psi_2$  such that equations for  $S_1$  and  $X_2$  hold. We find the minimum and maximum values of  $\phi_1$  and  $\psi_2$  that are consistent with the constraints.

More generally, we could fully saturate the Bellman system and the transition matrix in Table 3 by allowing different parameters for  $\eta$ ,  $\rho$ ,  $\gamma$  and  $\gamma_2$  depending on the activity—out of labor force or unemployment—in states 1 and states 2. In the benchmark region, as with  $\phi_1$  and  $\psi_2$ , the parameters that appear only in the equations for Bellman values that are not chosen do not enter the transition matrix in Table 4 but describe the off-equilibrium influence in the region. As long as there exists a value for such parameters that satisfies the inequality constraints on Bellman values that describe the region, the region is well defined. By setting the same values for  $\eta$ ,  $\rho$ ,  $\gamma$  and  $\gamma_2$  in the Bellman equations for both out of labor force and unemployment in states 1 and 2 and finding the solution, we guarantee that the definition of the region is feasible.

## 3 Statistical Method

### 3.1 Probabilities

Our model accounts for latent heterogeneity by hypothesizing a finite set of types in the working-age population. Each type  $\theta$  has a distribution of its activity paths,  $\tilde{M}_\theta$ , a vector of 6561 probabilities. The distribution of types in the population is  $\omega_\theta$ . The probability distribution within the population implied by the model is the mixture, with weights  $\omega_\theta$ , of these distributions,

$$\tilde{M} = \sum_{\theta} \omega_{\theta} \tilde{M}_{\theta}. \quad (15)$$

As mentioned above, we distinguish between (1) the partially observed *states* that describe the evolution of an individual worker’s experience over the 8 months recorded in the CPS, and (2) the observed *activities* recorded in the CPS. A 4-state Markov process governs the states. An individual’s activity—E, U, or N—is a function of the individual’s state.

The Markov process implies a probability defined on the  $4^8 = 65,536$  possible paths of the states. We then add up the probabilities of all the states that map to a given activity to find the probability distribution across the 6561 activities.

For each type  $\theta$  and each path  $j$ , we compute the probability  $\tilde{M}_{\theta,j}$  of each path. We start with the type’s ergodic distribution and account for the 8 months of unobserved activities between month 4 and month 5 of the observed activities.

Each type’s transition matrix across states  $s$  is first-order Markov among the 4 partly hidden states. A key idea in the model is that the transition probabilities among states,

$\pi_{\theta,s,s'}$ , are determined by choices made by the individual based on the Bellman values of the type- $\theta$  individual’s dynamic program. The driving forces of transitions are the random arrival of new opportunities and of adverse shocks. An employed person chooses whether to continue in the current job, search for a new better job, which may be immediately successful, or may take one or more months, or exit the labor market. A searcher may encounter a new job, or continue searching, or exit the labor market. A person out of the labor market may become a searcher, again with either immediate success or entry to unemployment, or may choose to remain out of the market.

The parameters of the model comprise vectors of probability parameters for the non-polar types plus the vector  $\omega$  of mixing probabilities. All of these parameters are constrained to be non-negative. The probability parameters are also constrained to be no greater than one, and the mixing weights are constrained to sum to one.

### 3.2 Estimation and sampling distribution of the estimates

Estimation involves finding the values of the parameters that imply probabilities  $\tilde{M}_j$  that best fit the observed frequencies in the CPS data,  $M_j$ . Here  $j$  indexes the frequency vector over the  $3^8$  activity paths. The natural starting point for measuring the distance is the likelihood function. We first consider the hypothesis that the only random variation in the model arises from the finite sample. To use the log likelihood as a measure of distance, we take its negative:

$$D = -R \sum_j M_j \log \tilde{M}_j. \tag{16}$$

$R$  is the number of respondents. It turns out that our sample sizes  $R$  are so large—more than a million for the prime-age respondents—that the actual discrepancies between the two vectors cannot plausibly arise from sampling alone. We consider an augmented likelihood containing discrepancies  $\epsilon_j$  between the model embedded in  $\tilde{M}_j$  and the true model generating the observed frequencies  $M_j$ . We assume that the observed values of the parameters of the model that minimize the distance are additively separable in the discrepancies:

$$\beta = \bar{\beta} + f(\epsilon). \tag{17}$$

Here  $\bar{\beta}$  is the true value of the vector of 41 parameters (probability parameters of three non-polar types and five values of the mixing weights  $\omega$ ). We assume that the expected value of the discrepancy effect  $f(\epsilon)$  is zero. To recover the distribution of  $f(\epsilon)$ , and thus the sampling distribution of the parameter estimates, we use a bootstrap technique—we re-estimate the parameters repeatedly after resampling the frequency vectors and the fitted probabilities from the model. Although bootstraps in survey data usually resample the data

by respondent, that approach would give standard errors of essentially zero, given the size of our samples.

Thus our estimation procedure is to use constrained minimization of the likelihood distance,  $D$ , to find the vector of parameters  $\beta$ . The standard deviations of 50 bootstrap replications with resampled data from the vector of frequencies  $M$  are taken as the standard errors of the parameter estimates.

### 3.3 Application

We carry out estimation for each of the four demographic groups. We consider five types in each group. The first three types have vectors of 12 parameters. We also estimate the mixing parameters,  $\omega_\theta$  that reveal the importance of the types. There are in effect four values of  $\omega_\theta$ , given that they sum to one.

As discussed earlier, more than half the prime-age male respondents were employed in all 8 months covered in the CPS and more than 10 percent had all 8 months out of the labor force. These findings identify an important type of heterogeneity. Accordingly, we hypothesize two types, one with probability 1 for the EEEE-EEEE activity path and zeros for all other paths, and the second with probability 1 for the NNNN-NNNN path. These two types are named all-E and all-N and are numbered 4 and 5. .

We validate an estimated type by checking if the bounds on the flow-values,  $b$ ,  $w_3$ , and  $z$ , all contain acceptable values and that the implied ranges of values of the parameters  $\phi_1$  (hypothetical job-finding rate for non-activated jobseekers in state 1) and  $\psi_2$  (hypothetical rate for activated out-of-labor-force individuals) are reasonable. These two parameters are identified only by their off-equilibrium role—they do not appear in the transition rates.

The coefficient  $\omega_4$ , giving the mixing parameter for the all-E type, is one source of probability for the EEEE-EEEE activity path. The first three types may, and often do in our results, contribute some probability of an all-E realization, even though realizations with some Us and Ns also occur for those types. And, of course, the same point applies to the overall probability of all Ns.

## 4 Data

We use data from the Current Population Survey. Each respondent contributes a path of labor-market activities. We consider frequency distributions for four demographic groups: Women aged 16 through 24, women aged 25 through 54, and men in those age groups.

Our data are for the years 2014 through 2017. On average, conditions in the labor market, notably the unemployment rate, were close to long-run averages during those years, along a



<i>Age</i>	<i>Gender</i>	
	<i>Female</i>	<i>Male</i>
Young, 16 to 24	339,039	344,935
Prime, 25 to 54	1,255,294	1,164,770

Table 5: Number of Respondents in the Four Demographic Groups

	<i>First appearance in survey</i>				<i>Second appearance in survey</i>			
	1	2	3	4	5	6	7	8
Observation number								
Months from entry to survey	0	1	2	3	12	13	14	15
Fraction Employed	0.823	0.818	0.817	0.818	0.806	0.809	0.809	0.809
Fraction unemployed	0.029	0.028	0.029	0.026	0.024	0.023	0.022	0.023
Fraction out of labor force	0.148	0.154	0.154	0.156	0.169	0.168	0.169	0.168

Table 6: Distribution of Population across the Three Activities, by Months in CPS

downward trend toward a somewhat tighter market. Thus we believe our findings describe normal conditions. In years of high unemployment, such as 2010, the main difference would be substantially lower job-finding rates.

Table 5 gives the numbers of respondents in the data. We include all individuals in the CPS with reported labor-market activities for all 8 months.

Hall and Schulhofer-Wohl (2018) discuss the problem of attrition in the CPS and document its incidence. We include the respondents who have complete activity histories, so there could be some bias from our implicit assumption that the included respondents are typical of the population.

Table 6 shows the distribution of the population across the three activities, by length of time the individual has been in the CPS. In principle, the distributions should be the same for each duration. In fact, the table confirms an issue in the CPS called *rotation group bias*—people tend to be classified more as employed and unemployed and less as out of the labor force when they enter the survey. It is as if continuing to participate in the CPS drives people out of the labor force. We do not think this problem has any material adverse effect on our work.

## 5 Results

### 5.1 Parameter estimates

Table 7 and Table 8 show the estimated values of the parameters of the model for women and men, respectively, along with bootstrap standard errors in parentheses. In general, the results suggest that it is feasible to estimate the 12 probability parameters for each of the three non-polar types, plus the 5 values of the mixing weights,  $\omega_\theta$ . Parameters that vary considerably across the types within demographic groups are  $\eta$ , the downgrade rate from state 2 to state 1,  $\phi_2$ , the jobfinding rate in state 2, and  $\mu$ , the rate of advance from state 3, short-term employment, to long-term employment.

The estimated separation rate from long-term jobs,  $\delta_4$ , is substantially lower than the separation rate from short-term jobs,  $\delta_3$ , in all four demographic groups. This finding is powerful support for the hypothesis that our model has successfully captured the distinction between interim jobs and more permanent jobs. The parameter  $\nu$  describes another source of flow out of long-term jobs.

The model has 12 separate flow rates, along with 4 more that are controlled by the principle that the sum of the rows of the transition matrix sum to 1. Hence it is difficult to think through the implications of the model from its transition rates. In the next sections of the paper, we put the model through a wide variety of demonstrations of its implications.

### 5.2 Ergodic distributions

There is substantial heterogeneity across the three non-polar types. But across all four demographic groups, the type 1s are similar, and the same holds for type 2 and type 3. Here, and elsewhere in the paper, we order the non-polar types by their ergodic contributions to long-term employment—the ergodic probability for long-term employment multiplied by the type’s mixing weight.

By definition, type 1 spends the most time in long-term jobs among the three types—these people have the highest ergodic probability of a long-term job (see Table 9 and Table 10). For young and prime-age women, the long-run probability of long-term job is about 60 percent. For young men the probability is 53 percent and for prime-age men it is 66 percent. Among the three types, those of type 2 spends the most time in unemployment. For example, for young women the long-run probability is 21 percent, for prime-age women—37 percent, for young men—34 percent and for prime-age men—37 percent. Finally, type 3 spends the most time out of the labor force—between 55 and 65 percent, depending on the group.

While type 1 spends the most time in long-term employment, they also spend the most time in short-term employment. Their time in unemployment is brief compared to type 2.

<i>Parameter</i>	<i>Description</i>	<i>Young women 16-24</i>			<i>Prime-age women 25-54</i>		
		<i>Type 1</i>	<i>Type 2</i>	<i>Type 3</i>	<i>Type 1</i>	<i>Type 2</i>	<i>Type 3</i>
$\rho$	Probability of upgrading from inactive to active	0.091 (0.109)	0.157 (0.114)	0.034 (0.035)	0.088 (0.091)	0.395 (0.040)	0.039 (0.069)
$\eta$	Prob of downgrading from active to inactive	0.140 (0.107)	0.339 (0.073)	0.581 (0.084)	0.110 (0.109)	0.170 (0.090)	0.459 (0.030)
$\varphi_2$	Jobfinding rate from activated search	0.528 (0.100)	0.120 (0.119)	0.303 (0.118)	0.567 (0.112)	0.117 (0.073)	0.174 (0.068)
$\psi_1$	Jobfinding rate from non-activated OLF	0.645 (0.105)	0.035 (0.157)	0.121 (0.085)	0.651 (0.132)	0.115 (0.080)	0.072 (0.096)
$\delta_4$	Separation rate from long-term jobs	0.057 (0.083)	0.060 (0.123)	0.040 (0.118)	0.046 (0.085)	0.082 (0.139)	0.047 (0.135)
$\delta_3$	Separation rate from short-term jobs	0.232 (0.072)	0.379 (0.123)	0.530 (0.085)	0.140 (0.122)	0.239 (0.101)	0.569 (0.112)
$\gamma_2$	Splits jobfinding while OLF into short- and long-term jobs	0.398 (0.063)	0.334 (0.101)	0.237 (0.118)	0.424 (0.026)	0.485 (0.104)	0.296 (0.138)
$\gamma$	Splits jobfinding from activated search into short- and long-term jobs	0.532 (0.032)	0.515 (0.047)	0.715 (0.123)	0.507 (0.053)	0.468 (0.006)	0.543 (0.167)
$\kappa_2$	Splits seps from short-term jobs between active and inactive	0.183 (0.116)	0.526 (0.016)	0.040 (0.105)	0.307 (0.090)	0.704 (0.021)	0.104 (0.078)
$\kappa$	Splits seps from long-term jobs between active and inactive	0.464 (0.089)	0.530 (0.106)	0.188 (0.087)	0.380 (0.102)	0.639 (0.081)	0.292 (0.070)
$\nu$	Prob of dropping from long-term to short-term job	0.199 (0.125)	0.094 (0.106)	0.141 (0.077)	0.271 (0.102)	0.265 (0.066)	0.102 (0.098)
$\mu$	Prob of upgrading from short-term to long-term job	0.554 (0.026)	0.495 (0.128)	0.268 (0.060)	0.613 (0.040)	0.602 (0.069)	0.546 (0.069)
$\omega_1$ to $\omega_3$	Type weights	0.178 (0.078)	0.147 (0.110)	0.298 (0.171)	0.150 (0.125)	0.041 (0.069)	0.151 (0.176)
$\omega_4$ and $\omega_5$	Type weights	0.185 (0.178)	0.192 (0.182)		0.518 (0.311)	0.141 (0.200)	

Table 7: Parameter Values for Women

<i>Parameter</i>	<i>Description</i>	<i>Young men 16-24</i>			<i>Prime-age men 25-54</i>		
		<i>Type 1</i>	<i>Type 2</i>	<i>Type 3</i>	<i>Type 1</i>	<i>Type 2</i>	<i>Type 3</i>
$\theta$	Probability of upgrading from inactive to active	0.100 (0.081)	0.279 (0.093)	0.046 (0.049)	0.058 (0.111)	0.505 (0.053)	0.062 (0.139)
$\eta$	Prob of downgrading from active to inactive	0.132 (0.097)	0.249 (0.080)	0.601 (0.077)	0.063 (0.108)	0.120 (0.118)	0.412 (0.035)
$\varphi_2$	Jobfinding rate from activated search	0.510 (0.066)	0.114 (0.069)	0.214 (0.064)	0.601 (0.134)	0.142 (0.059)	0.138 (0.025)
$\psi_1$	Jobfinding rate from non-activated OLF	0.586 (0.091)	0.038 (0.112)	0.092 (0.045)	0.726 (0.133)	0.116 (0.084)	0.077 (0.074)
$\delta_4$	Separation rate from long-term jobs	0.085 (0.071)	0.042 (0.123)	0.048 (0.146)	0.031 (0.114)	0.030 (0.136)	0.030 (0.060)
$\delta_3$	Separation rate from short-term jobs	0.205 (0.117)	0.658 (0.143)	0.506 (0.130)	0.152 (0.093)	0.421 (0.114)	0.584 (0.090)
$\gamma_2$	Splits jobfinding while OLF into short- and long-term jobs	0.452 (0.036)	0.280 (0.041)	0.178 (0.089)	0.477 (0.022)	0.492 (0.157)	0.233 (0.160)
$\gamma$	Splits jobfinding from activated search into short-and long-term jobs	0.539 (0.047)	0.473 (0.021)	0.554 (0.029)	0.475 (0.022)	0.364 (0.012)	0.545 (0.145)
$\kappa_2$	Splits seps from short-term jobs between active and inactive	0.254 (0.102)	0.737 (0.092)	0.057 (0.084)	0.493 (0.127)	0.852 (0.073)	0.136 (0.074)
$\kappa$	Splits seps from long-term jobs between active and inactive	0.507 (0.103)	0.562 (0.078)	0.244 (0.078)	0.465 (0.108)	0.569 (0.144)	0.472 (0.028)
$\nu$	Prob of dropping from long-term to short-term job	0.281 (0.058)	0.060 (0.105)	0.091 (0.117)	0.217 (0.115)	0.128 (0.050)	0.070 (0.104)
$\mu$	Prob of upgrading from short-term to long-term job	0.571 (0.022)	0.394 (0.105)	0.154 (0.078)	0.621 (0.053)	0.568 (0.139)	0.426 (0.071)
$\omega_1$ to $\omega_3$	Type weights	0.183 (0.079)	0.114 (0.078)	0.331 (0.168)	0.162 (0.103)	0.051 (0.137)	0.068 (0.126)
$\omega_4$ and $\omega_5$	Type weights	0.217 (0.183)	0.155 (0.146)		0.661 (0.349)	0.058 (0.123)	

Table 8: Parameter Values for Men

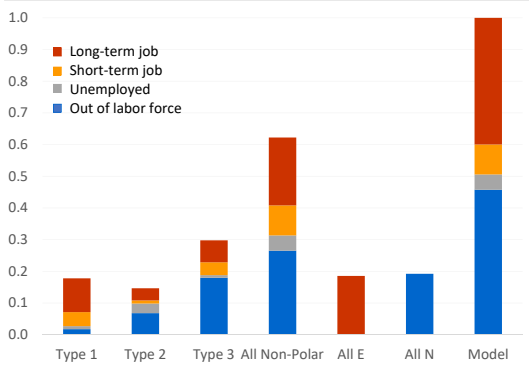
Activity	Labor Market State	Non-Polar Types			All Non-Polar	All E	All N	Model	Data
		Type 1	Type 2	Type 3					
Young Women									
N	Non-Work	9.9	46.4	60.3	42.6	0	100	45.7	45.8
U	Active Non-Work	5.3	20.7	2.9	7.8	0	0	4.8	4.8
E	Work in Short-Term Job	24.9	6.7	13.6	15.2	0	0	9.5	49.4
	Work in Long-Term Job	59.9	26.2	23.2	34.4	100	0	40.0	
Unemployment Rate		5.9	38.6	7.2	13.5	0	0	8.9	8.9
Employment to population ratio		84.8	32.9	36.9	49.6	100	0	49.4	49.4
Labor Force Participation Rate		90.1	53.6	39.7	57.4	100	0	54.3	54.2
Weights among Non-Polar Types		29	24	48	100				
Weights in the population		18	15	30	62	19	19	100	
Prime-Age Women									
N	Non-Work	6.7	16.4	62.7	32.6	0	100	25.2	25.2
U	Active Non-Work	4.3	37.2	5.1	8.5	0	0	2.9	2.8
E	Work in Short-Term Job	28.9	15.5	7.5	17.8	0	0	6.1	72.0
	Work in Long-Term Job	60.1	30.9	24.7	41.0	100	0	65.8	
Unemployment Rate		4.6	44.5	13.7	12.7	0	0	3.9	3.8
Employment to population ratio		89.0	46.4	32.2	58.8	100	0	71.9	72.0
Labor Force Participation Rate		93.3	83.6	37.3	67.4	100	0	74.8	74.8
Weights among Non-Polar Types		44	12	44	100				
Weights in the population		15	4	15	34	52	14	100	

Table 9: Ergodic Distribution Across States and Labor Market Activities, for Women

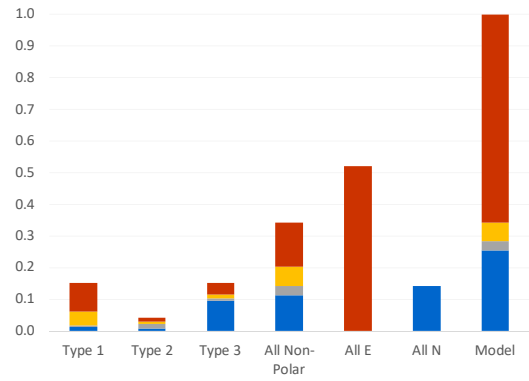
But comparison with type 3s unemployment depends on the group—young or prime-age. Among young women and men, type 1 spends more time in unemployment than type 3. Among prime-age women and men, type 1 spends less time in unemployment than type 3.

Figure 1 shows the ergodic distributions implied by our results, weighted by their respective mixing weights. The height of each bar gives the weight and the distribution within the bar is the ergodic distribution. For young women, the out-of-labor-force state, shown in blue, is an important part of the level and dispersion across types. A small fraction of type-1 young women are typically OLF. Even though this type accounts for a fairly small fraction of the population, it is the largest of the non-polar types in terms of long-term employment, shown in red. The more numerous type 3, in contrast, spends more than half of their time out of the labor force. Relative to other demographic groups, the all-E type is quite small for this group.

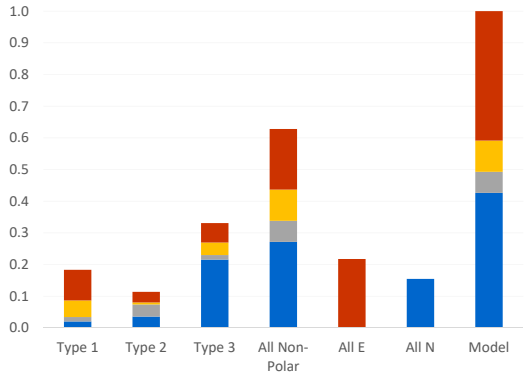
Figure 2 shows the relative roles of the non-polar types combined, on the one hand, and the polar types, on the other hand, for four demographic groups. For each group, the bar on the left refers to the population that is out of the labor force and the bar on the right refer to the employed population. The upper part of the bar, colored blue or green, describes the fraction attributable to the non-polar types and the bottom part to the all-N or all-E types.



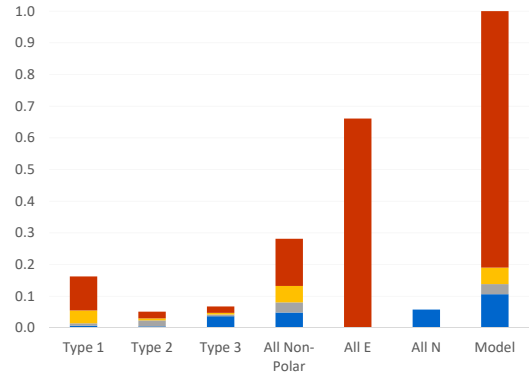
(a) Young women



(b) Prime-age women



(c) Young men



(d) Prime-age men

Figure 1: Contributions by State, Type, and Demographic Group

Activity	Labor Market State	Non-Polar Types			All Non-Polar	All E	All N	Model	Data
		Type 1	Type 2	Type 3					
<i>Young Men</i>									
N	Non-Work	11.1	31.4	65.2	43.2	0	100	42.6	42.7
U	Active Non-Work	7.6	33.6	4.3	10.6	0	0	6.6	6.6
E	Work in Short-Term Job	28.6	5.7	11.9	15.7	0	0	9.9	50.7
	Work in Long-Term Job	52.7	29.3	18.6	30.5	100	0	40.9	
Unemployment Rate		8.5	49.0	12.5	18.7	0	0	11.6	11.6
Employment to population ratio		81.4	35.0	30.5	46.2	100	0	50.7	50.7
Labor Force Participation Rate		88.9	68.6	34.8	56.8	100	0	57.4	57.3
Weights among Non-Polar Types		29	18	53	100				
Weights in the population		18	11	33	63	22	15	100	
<i>Prime-Age Men</i>									
N	Non-Work	4.2	9.2	54.5	17.2	0	100	10.6	10.6
U	Active Non-Work	4.6	37.0	8.0	11.4	0	0	3.2	3.2
E	Work in Short-Term Job	25.0	12.1	7.5	18.5	0	0	5.2	86.2
	Work in Long-Term Job	66.1	41.7	29.9	52.9	100	0	81.0	
Unemployment Rate		4.9	40.8	17.6	13.7	0	0	3.6	3.5
Employment to population ratio		91.1	53.8	37.5	71.4	100	0	86.2	86.2
Labor Force Participation Rate		95.8	90.8	45.5	82.8	100	0	89.4	89.4
Weights among Non-Polar Types		58	18	24	100				
Weights in the population		16	5	7	28	66	6	100	

Table 10: Ergodic Distribution Across States and Labor Market Activities, for Men

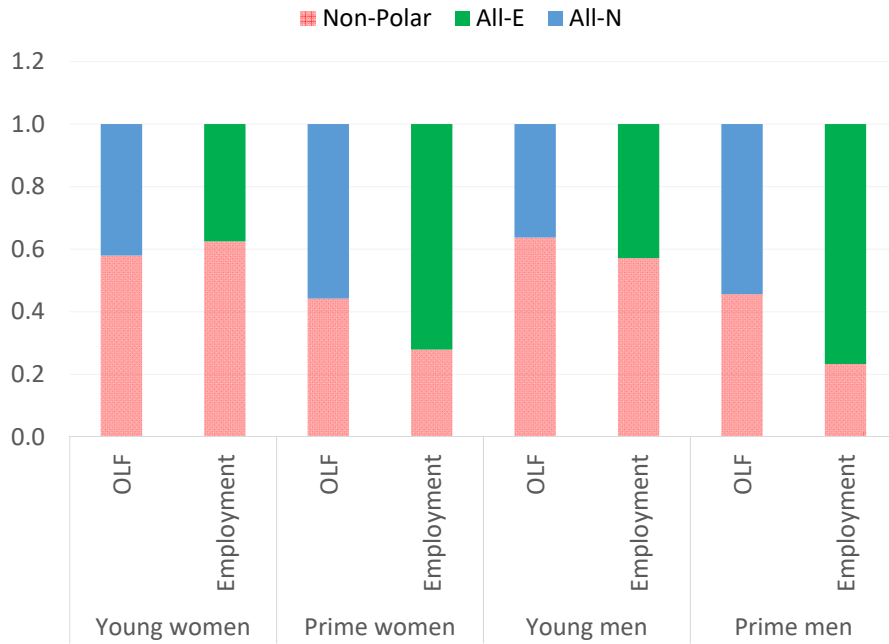


Figure 2: Composition of Activities by Type

Young people are similar in their division of time. The all-E fraction for young men is somewhat higher than for young women and the all-N fraction somewhat lower. Prime-age people are also similar for women and men. They are more likely than the young groups to belong to the stable polar types than the less stable polar types.

### 5.3 Individual labor-market dynamics

Figure 3 and Figure 4 show the path of states following unemployment and out of labor force, respectively, for three types of prime-age men. The Appendix contains results for other demographic groups.

The dynamics following unemployment and out of labor force are very similar. Type 1 converges to the highest ergodic probability of long-term and lowest probabilities of unemployment and out of labor force among the three types. The convergence is relatively fast. Type 1 also converges to the highest ergodic probability of short-term jobs, initially overshooting its ergodic probability.

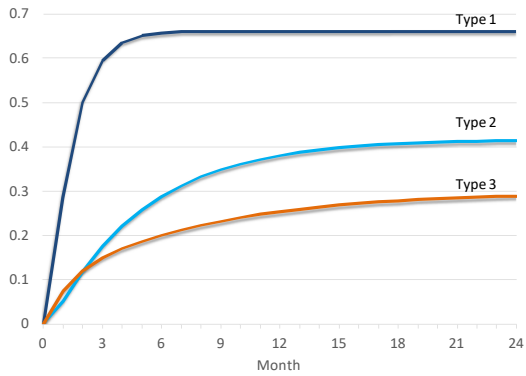
Figure 5 and Figure 6 show the path of states following short- and long-term job, respectively, for three types of prime-age men.

### 5.4 Further information about the roles of short-term and long-term jobs

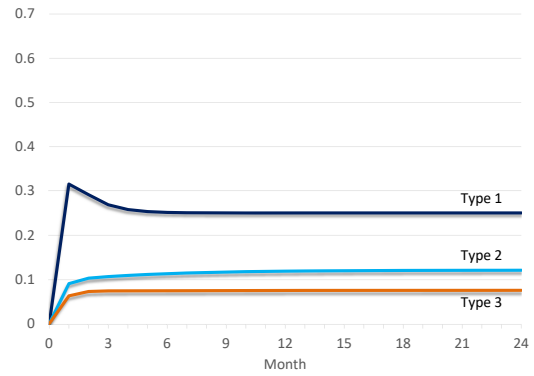
Figure 7 describes probabilities related to the acquisition of a long-term job starting from unemployment in month 0, for the three types of prime-age men. The blue bars show the probability distribution of the time to find the first long-term job in the succeeding 12 months. The other bars break down that distribution into three mutually exclusive cases. The orange bar shows the probability that the path to the earliest long-term job involves at least one short-term job. The gray bar shows the probability that the path involves only unemployment, and the purple bar shows the probability that the path involves a combination of unemployment and out-of-labor force. The three bars sum to the blue bar. The results for the other demographic groups are in the Appendix.

About 30 percent of type 1, the most prevalent type and the one with the highest propensity to hold long-term jobs, finds long-term employment in the month following unemployment. The share declines quickly in the following months. It drops to about 10 percent in the fourth month and to about 3 percent in the sixth month. At least half of the unemployed who do not find a long-term job the following month, go through short-term employment before finding a long-term job. As the time passes, this share quickly climbs up. A long-term

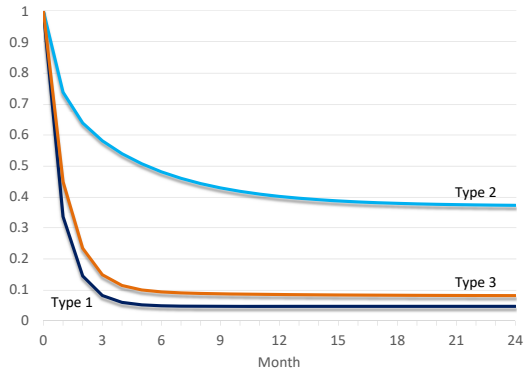




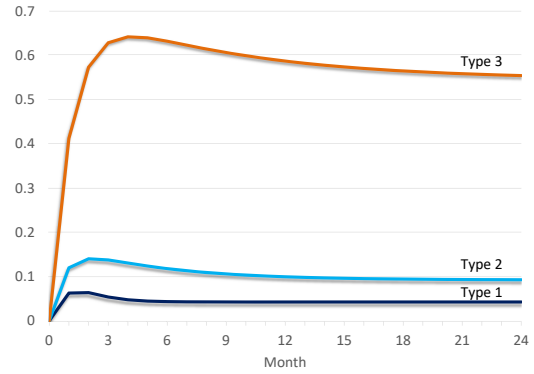
(a) To long-term job



(b) To short-term job

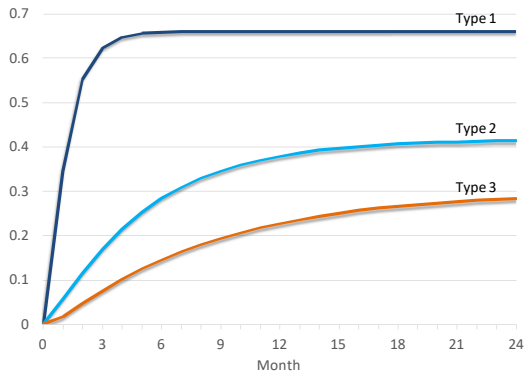


(c) To unemployment

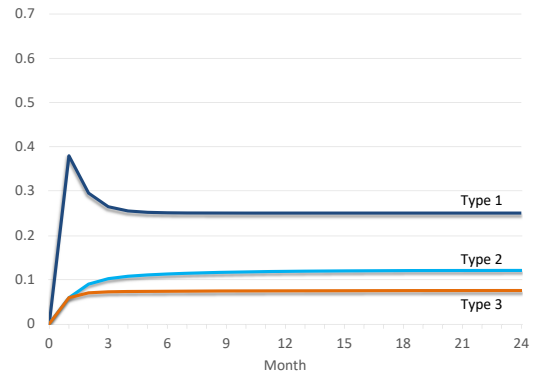


(d) To out of labor force

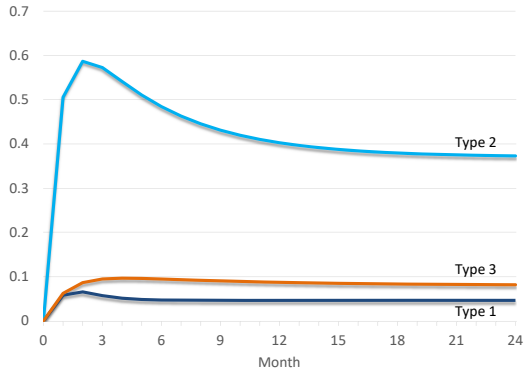
Figure 3: Paths of States Following Unemployment, by Type for Prime-Age Men



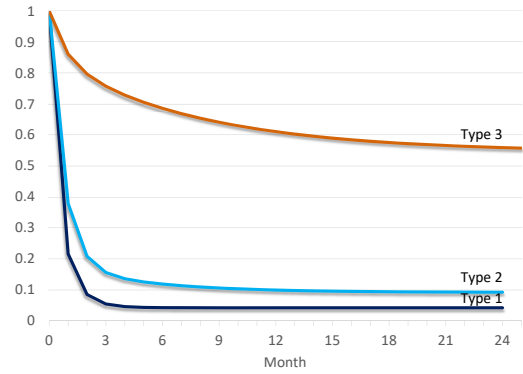
(a) To long-term job



(b) To short-term job

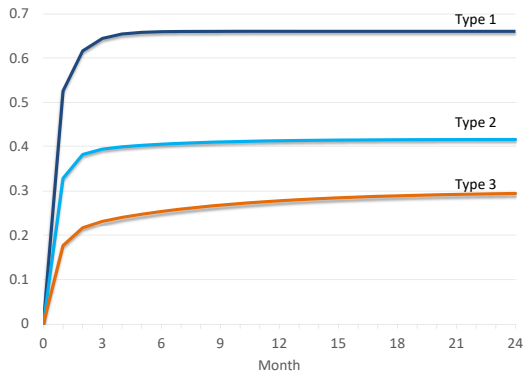


(c) To unemployment

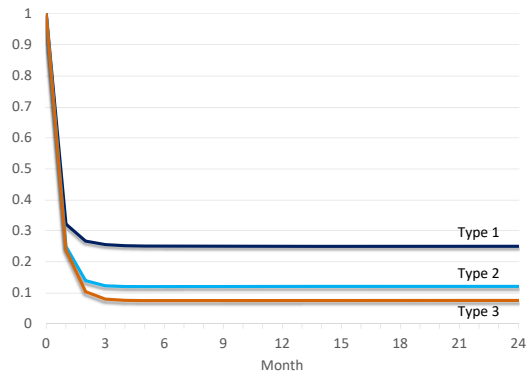


(d) To out of labor force

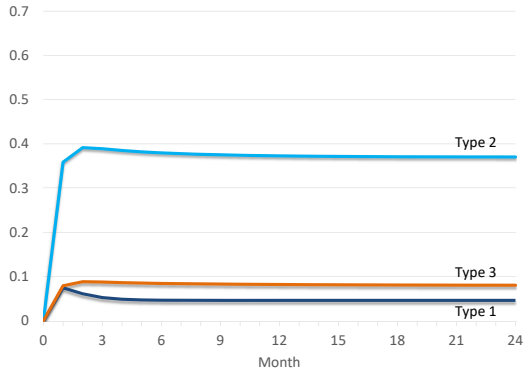
Figure 4: Paths of States Following Out of Labor Force, by Type for Prime-Age Men



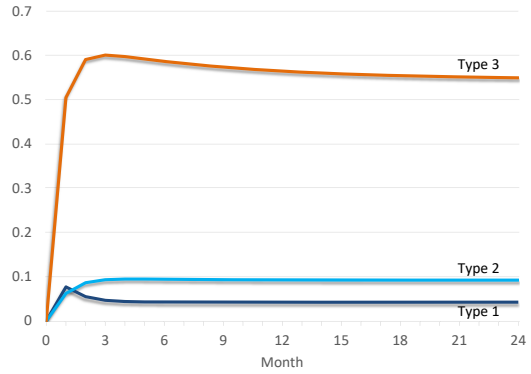
(a) To long-term job



(b) To short-term job

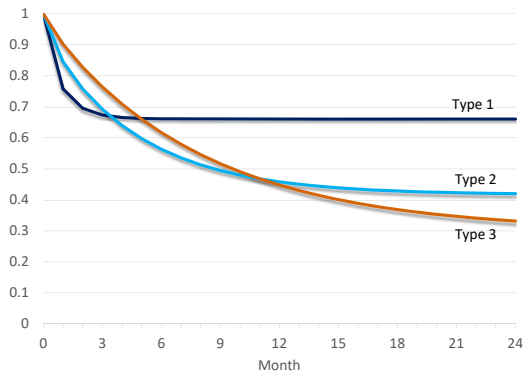


(c) To unemployment

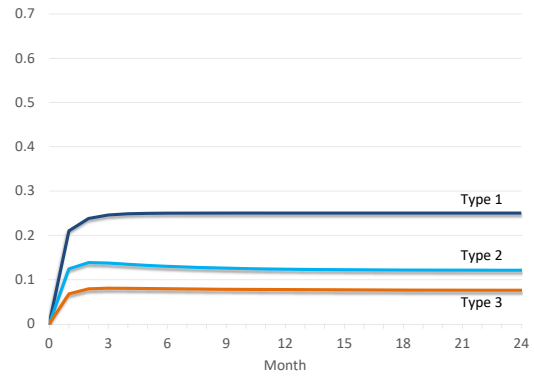


(d) To out of labor force

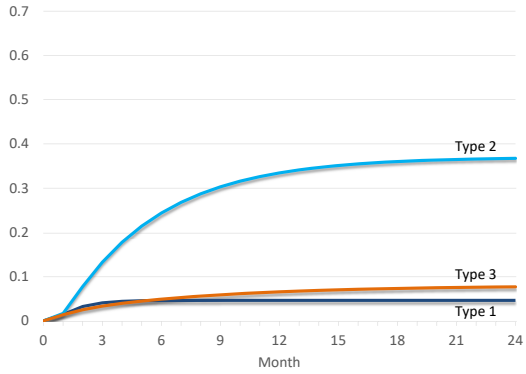
Figure 5: Paths of States Following Short-Term Job, by Type for Prime-Age Men



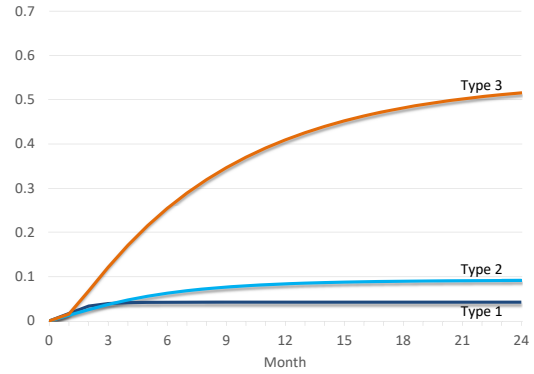
(a) To long-term job



(b) To short-term job



(c) To unemployment



(d) To out of labor force

Figure 6: Paths of States Following Long-Term Job, by Type for Prime-Age Men

job found after six months has a probability close to one of being preceded by a short-term job.

For type 2, the probability of finding long term-employment from unemployment is only a fraction of the probability of type 1. Five or six percent of type 2 find long-term employment in the month following unemployment. As times goes, the share declines only slightly. About half of them go through short-term employment before setting into a long-term job. But in contrast to Type 1, as time passes, the fraction that goes through short-term employment does not increase and remains stable even after a year.

Type 3 spends most of the time out of labor force among the three types. The job-finding rate in the month immediately following unemployment differs between young and prime-age groups. The figures in the Appendix show that for young groups, the probability of finding long term-employment for type 3 is 1.5 times than for type 2. However, if they do not immediately find long-term employment, the probability drops to that of type 2 or lower. For prime-age groups, the job-finding probability is lower than that of Type 2. Approximately half of them go through short-term employment before settling into a long-term job. This fraction remains stable over the entire year.

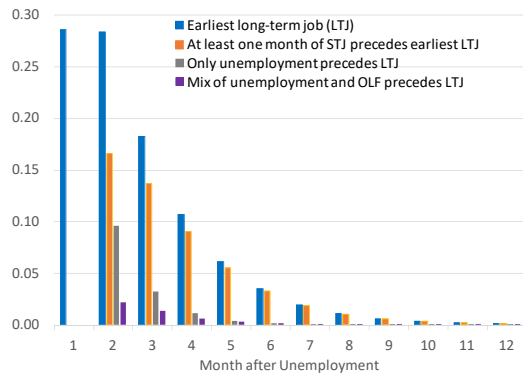
It appears that Type 1 has a strong drive toward employment. They spends little time in non-employment and would rather take short-term employment while looking for a long-term job. In contrast, Types 2 and 3 spend considerable amount of time in unemployment and out of labor force. When they find themselves non-employed, they take time to find employment, either short- or long-term.

Figure 19 in the Appendix shows the path from out-of-labor-force to long-term job, for three types of prime-age men. The path is similar to the one that starts from unemployment.

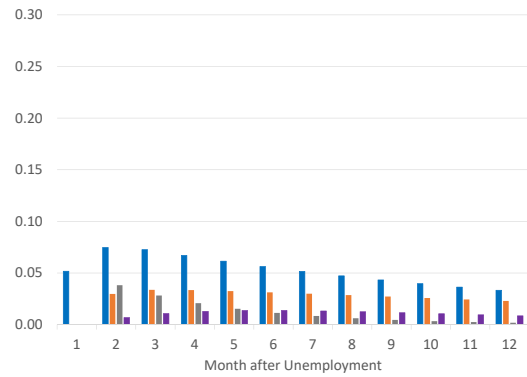
Figure 8 shows a path from short-term to long-term job for the type-1 prime-age men, the type that accounts for 58 percent of the weight accorded the polar types in this group. More than half of this type finds a long-term job in the very next month. Paths for the other types appear in the Appendix.

## 5.5 Results from the CPS-like panel

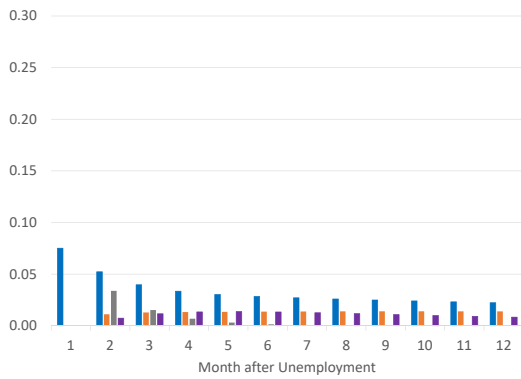
Table 11 shows the distribution of months of unemployment, according to the model, in the 8-month panel that mimics the CPS panel structure (with an 8-month gap between month 4 and 5). Most respondents have zero months—75 percent of young men to 89 percent of prime-age women and men in the full model. All unemployment in the model comes from the non-polar types. Among the non-polar types, 68 percent have zero month of unemployment among prime-age women, 66 percent among young women, 62 percent among prime-age men, and 59 percent among young men. Among the non-polar types, 3.5 percent of young



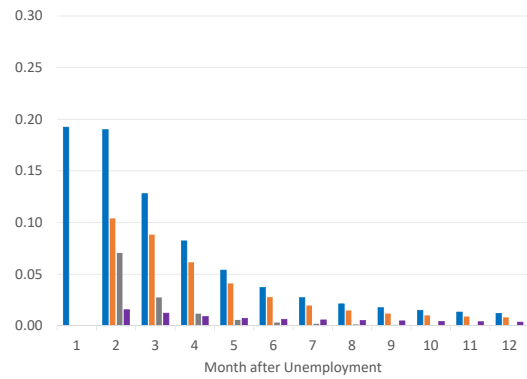
(a) Type 1 (58 percent)



(b) Type 2 (18 percent)



(c) Type 3 (24 percent)



(d) All Non-Polar Types

Figure 7: Path to Long-Term Job from Unemployment in Month 0, by Type for Prime-Age Men

Note: Colored bars sum to the blue bar. Type's weight in the pool of Non-Polar Types is in parenthesis.

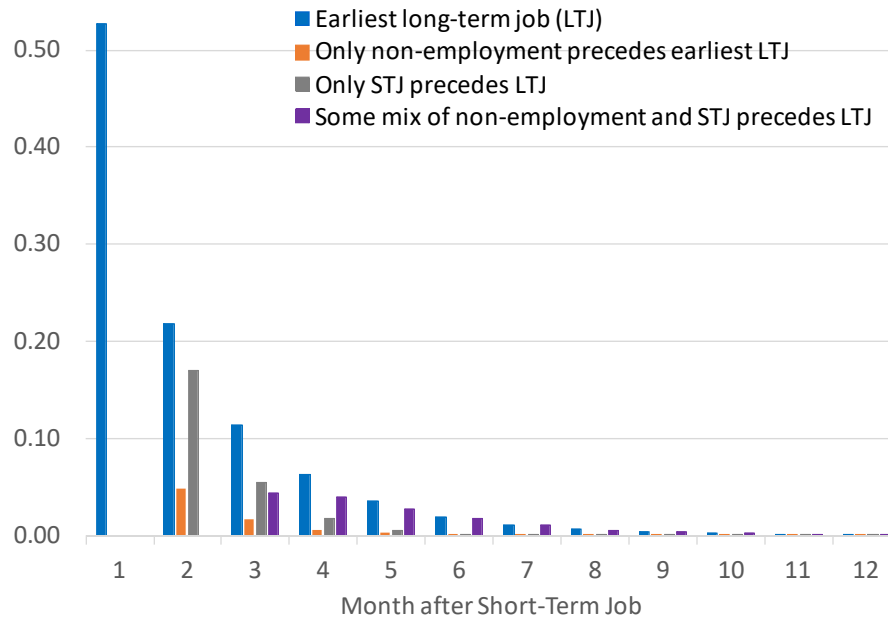


Figure 8: Path to Long-Term Job from Short-Term Job in Month 0, for Type-1 Prime-Age Men

women have 4 or more months of unemployment and this share goes up to 8.4 percent among prime-age men. Note that this is unemployment while in the CPS-like panel: some people might have additional months of unemployment during the 8-month gap.

There is considerable heterogeneity in terms of the months spent unemployed across the three non-polar types. Type 1 spends a few months in unemployment while a large fractions of type 2 for young groups and type 3 for prime-age groups have 4 or more months of unemployment. The share of type 1 with 4 months of unemployment or more varies between 0.8 percent for young women and 4.2 percent for prime-age men. The share of type 3 with 4 months of unemployment is more than 40 percent for prime-age women and men.

Table 12 studies the success rate of the unemployed in a given month in gaining employment in later months. We again construct an 8-month panel in the model which mimics the CPS panel structure. The one-month success rate is fairly high, at about 25 percent. But further progress is much slower. In fact, within a type, the probability remains almost unchanged to observe a person employed 12 or 15 months later. The increase in the probability in the full model during this horizon results from the mixture of types with different probabilities of success—a good example of duration dependence arising from heterogeneity.

Table 13 displays the probability that a worker employed in month 7 of the CPS-like 8-month panel will lose that job, broken down by the number of months of non-work in months 1 through 6. The probability of job loss is much higher among the non-polar types

<i>Months of unemployment</i>	<i>Type 1</i>	<i>Type 2</i>	<i>Type 3</i>	<i>Non-Polar Types</i>	<i>Full Model</i>	<i>Type 1</i>	<i>Type 2</i>	<i>Type 3</i>	<i>Non-Polar Types</i>	<i>Full Model</i>
	<i>Young Women</i>					<i>Prime-Age Women</i>				
0	71.5	30.6	80.5	66.2	79.0	79.3	76.4	15.6	73.2	67.8
1	18.6	22.9	16.5	18.6	11.6	11.3	15.9	12.4	17.0	16.0
2	6.9	19.1	2.7	7.7	4.8	4.8	5.5	15.1	6.6	7.1
3	2.2	13.4	0.3	4.0	2.5	2.4	1.7	16.6	2.3	3.7
4+	0.8	13.9	0.0	3.5	2.2	2.3	0.5	40.3	0.9	5.4
	<i>Young Men</i>					<i>Prime-Age Men</i>				
0	61.8	17.1	73.0	59.6	74.7	75.2	74.8	20.0	63.4	62.1
1	22.9	14.3	20.6	20.1	12.6	12.0	16.6	10.5	19.5	16.2
2	10.0	17.2	5.1	8.7	5.5	5.4	6.0	13.0	10.0	8.3
3	3.7	17.5	1.0	4.8	3.0	3.2	1.9	15.6	4.6	5.1
4+	1.5	33.9	0.2	6.7	4.2	4.2	0.6	40.9	2.6	8.4

Table 11: Distribution of Months of Unemployment in the CPS-like 8-month panel

<i>Months after unemployment</i>	<i>Type 1</i>	<i>Type 2</i>	<i>Type3</i>	<i>Non-Polar</i>	<i>Full Model</i>	<i>Type 1</i>	<i>Type 2</i>	<i>Type3</i>	<i>Non-Polar</i>	<i>Full Model</i>
	<i>Young Women</i>					<i>Prime-Age Women</i>				
1	0.528	0.120	0.303	0.324	0.232	0.567	0.117	0.174	0.340	0.231
12	0.848	0.298	0.369	0.489	0.418	0.890	0.447	0.305	0.579	0.506
15	0.848	0.310	0.369	0.492	0.425	0.890	0.457	0.312	0.583	0.513
	<i>Young Men</i>					<i>Prime-Age Men</i>				
1	0.510	0.114	0.214	0.282	0.219	0.601	0.142	0.138	0.406	0.250
12	0.814	0.302	0.297	0.449	0.408	0.911	0.499	0.328	0.696	0.567
15	0.814	0.318	0.300	0.453	0.418	0.911	0.517	0.343	0.703	0.580

Table 12: Success Rates in Becoming Employed after Being Unemployed



<i>Months of Non-Employment in a CPS-like 8 month panel</i>	<i>Type 1</i>	<i>Type 2</i>	<i>Type3</i>	<i>Non-Polar</i>	<i>Full Model</i>	<i>Type 1</i>	<i>Type 2</i>	<i>Type3</i>	<i>Non-Polar</i>	<i>Full Model</i>
	<i>Young Women</i>					<i>Prime-Age Women</i>				
0	0.101	0.103	0.149	0.125	0.040	0.074	0.130	0.113	0.098	0.013
3	0.119	0.118	0.199	0.157	0.163	0.081	0.134	0.144	0.115	0.117
6	0.153	0.239	0.390	0.286	0.348	0.097	0.165	0.395	0.237	0.355
	<i>Young Men</i>					<i>Prime-Age Men</i>				
0	0.124	0.090	0.132	0.122	0.036	0.062	0.093	0.081	0.072	0.010
3	0.131	0.121	0.200	0.165	0.160	0.071	0.114	0.117	0.090	0.098
6	0.146	0.396	0.402	0.326	0.398	0.094	0.270	0.421	0.205	0.368

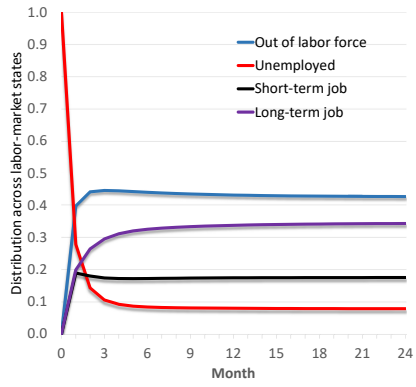
Table 13: Probability of Job Loss in Month 7 by Number of Months of Non-Work in Earlier Months in the CPS-like 8-month panel

than in the full model because these are the only types that can experience job loss in the model. Among the non-polar types, it varies from 7.2 percent if the worker had worked in every prior month in the prime-age men sample to 32.6 percent if the worker spent the previous 6 months not working in the young men sample.

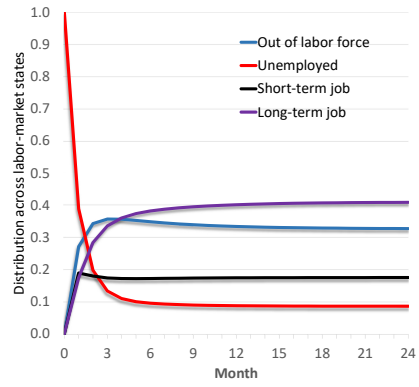
The types vary by how much the probability of the job loss increases with the number of past months of non-employment. For type 1, the probability of job loss increases only modestly. For example, among prime-age men, it increases from 6.2 percent if the worker had worked in every prior month to 9.4 percent if the worker spent the previous 6 months not working. For type 3, it increases considerably. Among prime-age men, it increases from 8.1 percent if the worker had worked in every prior month to 42.1 percent if the worker spent the previous 6 months not working.

## 5.6 The typical paths from unemployment among the non-polar types

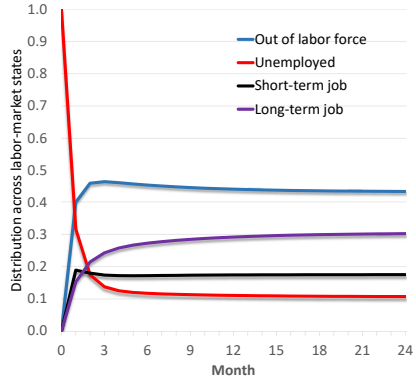
Figure 9 shows the paths of states following unemployment, by demographic group, for the non-polar types. The paths are the probability distributions of future states, conditional on being unemployed in month zero. The distribution across states converges over time, fairly rapidly, to the ergodic distribution. Convergence is faster for young people than for the prime-age groups. Convergence is immediate in the case of short-term jobs. The decline in unemployment is rapid at first, but slows down after a couple of months. The restoration of normal fractions out of the labor force follows the same pattern, in the opposite direction from unemployment. Movement into long-term jobs is rapid at first, but that fraction of the population is still rising after six months. The path from unemployment to long-term jobs involves delays resulting from taking short-term jobs which have high separation rates.



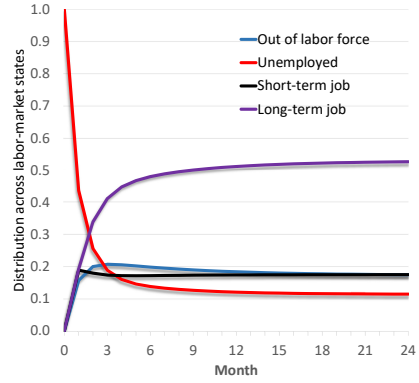
(a) Young women



(b) Prime-age women



(c) Young men



(d) Prime-age men

Figure 9: Paths of States Following Unemployment, by Demographic Group

As we noted earlier, the path of the probability of holding a short-term job is remarkably flat. It shares none of the gradual dynamics of the probabilities of the other three states. Figure 9 shows that this property holds for all four demographic groups. The property shows that short-term employment is a way-station in the labor market. An unemployed person is quite likely to find a short-term job by the next month. Thus the probability conditional on being unemployed in month zero of holding a short-term job in month 1 is substantial. As the probability of being unemployed declines rapidly in later months, the flow from unemployment falls in proportion, but it is replaced by the rising flow from separations out of long-term jobs. The replacement is essentially complete, so the probability of holding a short-term job is flat across months at close to its ergodic level.

## 5.7 How estimation distinguishes short-term from long-term jobs

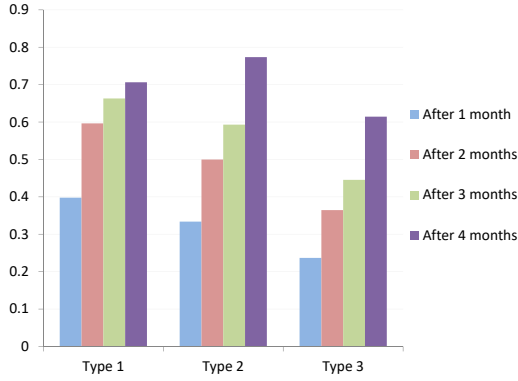
Our estimation method uses all the information about a respondent's activities over the 8 months included in the CPS, but not other information that would distinguish short-term from long-term jobs directly. The key feature of the data that leads to the distinction in the model is the following: Consider a person who is unemployed now, and consider the probability that the person will be employed in month  $\tau$  from now. In the CPS, that probability rises from month 1 to month 2 by a substantial proportion, rises from month 2 to month 3 by a smaller proportion, and so on. Let  $p_\tau$  be the probability, as of month zero, of employment  $\tau$  months in the future. Let  $a_\tau$  be the rate of adjustment of the probability toward its ergodic value  $p^*$ :

$$a_\tau = 1 - \frac{p^* - p_\tau}{p^* - p_{\tau-1}}. \quad (18)$$

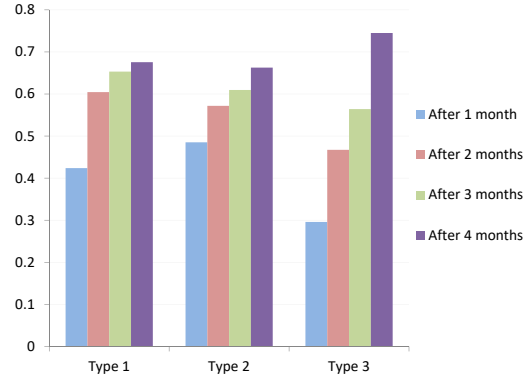
In all four demographic groups, the adjustment rate  $a_\tau$  declines as  $\tau$  rises.

Our model replicates this behavior, in the sense that the weighted average of the three types' probability paths matches the path with declining adjustment rates. An alternative model different in one critical respect fails badly, because its adjustment rates are close to constant. The difference is that the exit-rates parameters from jobs are the same for short-term jobs and long-term jobs:  $\delta_3 = \delta_4 = 0.15$ . Thus the model fits considerably better from having two kinds of jobs, differing in longevity.

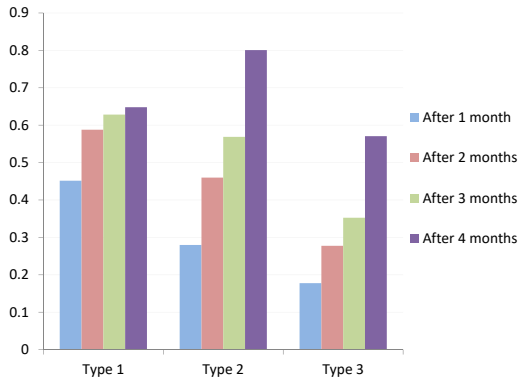
The model does not assign individual jobs to the short-term or long-term categories. But it does imply a probability that a particular job in a particular situation is long-term rather than short-term. The situation is described by the activities of the respondent in the 8 observed months. Figure 10 shows the probability that an observed job is long-term, conditional on those 8 activities, for a respondent with type 1 to 3 characteristics, who was out of the labor force in the first month she was in the survey. We calculate the probability



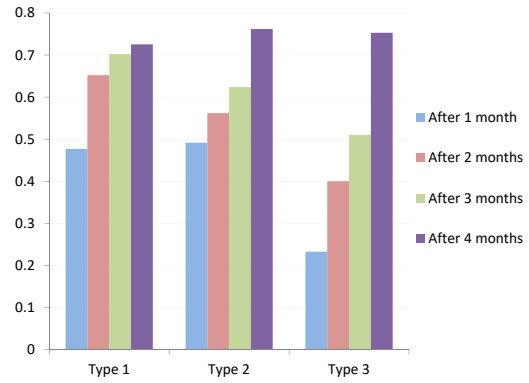
(a) Young women



(b) Prime-age women



(c) Young men



(d) Prime-age men

Figure 10: Fraction of Jobs that are Long-term, Starting from Out-of-labor-force

by going through the  $4^7$  distinct state paths emanating from state 1 in month 1 and adding up separately, for each month after month 1, the probabilities of the paths that have an S or an L in that month. We arrive at probabilities, for each month, for the two states. The probability that a given job is long-term is the ratio of the L probability to the sum of the S and L probabilities.

The influence of the original month out of the labor force dissipates over about 4 months, after which the mix of long-term and short-term jobs is quite close to its ergodic value. Recall that people with type 1 to 3 characteristics tend to move in and out of the labor force, so this fairly speedy dissipation of the influence is not a surprise. Young people have considerably lower long-term employment probabilities than those of prime age. Type 1 young people tend to have higher long-term job probabilities, except at four months out. Among prime-

	<i>Young women</i>	<i>Prime- age women</i>	<i>Young men</i>	<i>Prime- age men</i>
Percent probability that short-term job precedes long-term job	7.5	12.6	6.6	17.1
Percent probability that long-term job precedes short-term job	5.8	6.6	4.9	6.4

Table 14: Probabilities that short-term jobs precede long-term jobs, and probabilities of the reverse

age workers, long-term jobs are fairly common even just two months after being out of the labor force in types 1 and 2.

## 5.8 Evidence that short-term jobs precede long-term jobs

Here we measure the extent to which short-term jobs precede long-term ones, as suggested by the hypothesis that short-term jobholding is a variety of job search. We consider the three months following a month of unemployment. We say that a short-term job (labeled S) precedes a long-term job (L) if the three months are SSL, SLL, USL, NSL, SUL, SNL, SLU, or SLN. The list of sequences defining L preceding S is this list with the Ss and Ls reversed. In other sequences, such as SLS, neither L nor S precedes. Our model assigns a probability based on its transition probabilities to each of these sequences. Table 14 shows the sums of the probabilities of the types weighted by the mixing weights,  $\omega_\theta$ . In all four demographic groups, the probability of S preceding L is above the probability for L preceding S in all groups and the ratio is close to or above two in the two prime-age groups. We conclude that the model is consistent with the hypothesis that holding short-term jobs functions, in part, as a form of job search.

## 6 Economic Values

In this section, we derive information about the Bellman values of the types. As discussed in Section 2.1, the construction of the transition matrix in the benchmark region in Table 4 does not require specifying the flow values of employment or non-employment. Rather, the transition matrix is consistent with any triplet of flow values  $\{b, w_3, z\}$  such that the corresponding Bellman values satisfy the definition of the benchmark region which underlies the transition matrix, which is  $X_1 \leq S_2 \leq V_3 \leq V_4$ . Similarly, the transition matrix does not

Parameter	Description	Type 1		Type 2		Type 3		Type 1		Type 2		Type 3	
		min	max	min	max	min	max	min	max	min	max	min	max
		<i>Young Women</i>						<i>Prime-Age Women</i>					
z	Flow value of non-employment	-1.00	0.96	-1.00	0.98	-1.00	0.97	-1.00	0.97	-1.00	0.98	-1.00	0.98
b	Flow value of searching	-0.04	1.00	-1.00	0.37	-1.00	0.64	-0.01	1.00	-0.01	0.75	-1.00	0.56
w3	Wage in short-term job	0.00	0.95	0.00	0.95	0.00	0.95	0.00	0.95	0.00	0.95	0.00	0.95
$\phi_1$	Jobfinding rate from non-activated search	0.00	0.71	0.00	0.19	0.00	0.92	0.00	0.68	0.00	0.12	0.00	0.32
$\psi_2$	Jobfinding rate from activated OLF	0.00	1.00	0.00	0.16	0.00	0.42	0.00	1.00	0.00	0.26	0.00	0.25
		<i>Young Men</i>						<i>Prime-Age Men</i>					
z	Flow value of non-employment	-1.00	0.96	-1.00	0.98	-1.00	0.96	-1.00	0.96	-1.00	0.98	-1.00	0.98
b	Flow value of searching	-0.01	1.00	-1.00	0.30	-1.00	0.52	0.00	1.00	-0.14	0.76	-1.00	0.62
w3	Wage in short-term job	0.00	0.95	0.00	0.95	0.00	0.95	0.00	0.95	0.00	0.95	0.00	0.95
$\phi_1$	Jobfinding rate from non-activated search	0.00	0.61	0.00	0.19	0.00	0.67	0.00	0.60	0.00	0.13	0.00	0.32
$\psi_2$	Jobfinding rate from activated OLF	0.00	1.00	0.00	0.15	0.00	0.29	0.00	1.00	0.00	0.29	0.00	0.21

Table 15: Flow Values and Off-Equilibrium Parameters

require specifying the job-finding rates from off the equilibrium path,  $\phi_1$  and  $\psi_2$ . Rather, the transition matrix is consistent with any  $\phi_1$  and  $\psi_2$  such that the corresponding Bellman values satisfy  $X_1 \geq S_1$  and  $S_2 \geq X_2$ . But we need these parameters if we want to construct a full set of six Bellman values of an individual dynamic program.

To find the minimum and maximum flow values and job-finding rates from off the equilibrium path, we solve the constrained optimization problem that takes into account the linear and non-linear constraints that define the benchmark region,  $X_1 \leq S_2 \leq V_3 \leq V_4, X_1 \geq S_1, S_2 \geq X_2$ . We impose additional feasibility constraints on the flow values as follows:  $-1 \leq b \leq 1, 0 \leq w_3 \leq 0.95, -1 \leq z \leq 1$ , given our normalization  $w_4 = 1$ .

Table 15 shows the minimum and maximum values for  $b, w_3, z, \phi_1$  and  $\psi_2$  for each type and each demographic group. For each type there is a range of parameter values consistent with our benchmark region. For example, any short-term job wage  $w_3$  from the feasible set  $[0, 0.95]$  is admissible in the region.

The set of Bellman values consistent with the transition matrix is not unique. Each triplet  $\{b, w_3, z\}$  together with  $\phi_1$  and  $\psi_2$  defines a set of six Bellman values in the region— $X_1, S_2, V_3, V_4$  for activities actually chosen and for the rejected values  $X_2$  and  $S_1$  off the equilibrium path. We present six sets of Bellman values for each type, each corresponding to the minimum and maximum values of  $b, w_3$ , and  $z$ . Table 16 lists the Bellman values for the types of young women. Values for other demographic groups are in the Appendix. The Bellman values are centered around their respective ergodic means; that is, within each type they are stated as the deviations from the average of the Bellman values of the type weighted by the ergodic probabilities of the states for that type.

Value/ para- meter	Description	Type 1						Type 2						Type 3					
		z		b		w3		z		b		w3		z		b		w3	
		min	max	min	max	min	max	min	max	min	max	min	max	min	max	min	max	min	max
<i>Young Women</i>																			
X1	Value of OLF when inactive non-employment	-2.39	-0.04	-0.89	-2.18	-2.08	-2.07	-4.65	-0.03	-4.56	-4.72	-3.85	-1.11	-2.89	-0.04	-2.53	-2.97	-2.36	-0.74
S1	Value of search when active non-employment	-2.98	-0.06	-1.15	-2.39	-2.57	-2.57	-4.97	-0.03	-5.27	-4.72	-4.12	-1.18	-3.43	-0.06	-3.37	-2.97	-2.87	-0.91
X2	Value of OLF when inactive non-employment	-3.00	-0.06	-1.13	-2.27	-2.47	-2.60	-3.94	-0.03	-4.69	-3.27	-3.29	-0.93	-2.13	-0.04	-2.29	-1.53	-1.77	-0.59
S2	Value of search when active non-employment	-1.88	-0.04	-0.89	-1.10	-1.74	-1.68	-3.44	-0.03	-4.48	-2.50	-2.90	-0.80	-1.27	-0.04	-1.78	-0.21	-1.14	-0.39
V3	Value of short-term job	-0.22	-0.04	-0.89	-0.41	-0.91	-0.07	3.63	-0.03	3.97	3.98	2.35	0.94	0.54	-0.04	0.39	0.91	-0.25	0.20
V4	Value of long-term job	0.65	0.03	0.60	0.63	0.88	0.52	10.01	0.08	10.59	9.32	8.50	2.36	7.33	0.13	6.55	7.19	6.41	1.86
z	Flow value of non-employment	-1.00	0.96	0.25	-0.83	-0.74	-0.73	-1.00	0.98	-0.92	-1.00	-0.73	0.53	-1.00	0.97	-0.76	-1.00	-0.75	0.50
b	Flow value of searching	0.62	0.00	-0.04	1.00	0.35	0.51	-0.28	-0.01	-1.00	0.37	-0.25	-0.06	-0.41	-0.03	-1.00	0.64	-0.52	-0.16
w3	Wage in short-term job	0.78	0.95	0.00	0.60	0.00	0.95	0.51	0.95	0.67	0.95	0.00	0.95	0.60	0.95	0.55	0.95	0.00	0.95
$\varphi 1$	Jobfinding rate from non-activated search	0.17	0.27	0.26	0.09	0.19	0.19	0.03	0.09	0.06	0.00	0.03	0.03	0.09	0.32	0.16	0.00	0.12	0.11
$\varphi 2$	Jobfinding rate from activated OLF	0.29	0.21	0.18	0.39	0.31	0.32	0.05	0.01	0.02	0.08	0.04	0.05	0.11	0.02	0.07	0.18	0.10	0.10

Table 16: Centered Bellman Values for Young Women

The value of a long-term job exceeds the value of a short-term job, which in turn exceeds the value of searching, which in turn exceeds the value of being out of labor force. In state 1, non-active non-employment, the value of out of labor force exceeds the value of searching. In state 2, active non-employment, the value of search exceeds the value of out of labor force. These inequalities define the benchmark region that we study in this paper.

Second, within each type, the parameters describing the equilibrium path (those of the transition matrix) and the ergodic distribution across the four states are constant within the region. Thus, the differences among the sets of the Bellman equations are entirely due to the differences in the flow values and the job finding rates off the equilibrium path. The dispersion is negatively related to the flow value of non-employment,  $z$ .

The centered Bellman value of a long-term job is always positive, while the centered Bellman value of a short-term job might be positive or negative within a type, depending on the flow values. This depends on the un-centered Bellman values, which are governed by the flow values, as well as on the ergodic distribution across the states, which is the same for all sets of parameters within a type. For example, for type 1 young women, all six triplets of flow values in the table produce a negative centered Bellman value of a short-term job. For type 2 young women, five triplets of flow values in the table produce a positive centered Bellman value of short-term job; the centered Bellman value of short-term job turns negative when the flow value of non-employment is at its maximum.

## 7 Evaluating the Success of the Model in Capturing Key Features of the Data

In this section, we describe a variety of views into the actual frequencies of activity paths in the CPS and the success of the model in matching them. We study the differences between

statistics computed from the actual frequencies and the corresponding probabilities of those statistics implied by our model.

We start in Figure 11 with our results on the re-employment process. For each of the four demographic groups, we show the probability of employment by the number of months following a month when an individual is unemployed. All of the curves are concave, and start at zero by construction. These curves describe the quite rapid initial progress into employment (both short-term and long-term), which our model attributes to the ease of finding short-term jobs. By month 3, the process has gone most of the way to its asymptote of around 50 percent. The re-employment process comes nowhere near the overall ergodic employment rate for the population, because the selection of individuals who are unemployed in the first place implies, in the model, that the individual has a non-polar type, and, in the data, that the individual does not come from a part of the population with low likelihoods of unemployment.

The match of model to data is outstanding for the prime-age groups and for the months before the eight-month break in the CPS schedule for the young groups, and is not too bad for the young. The discrepancies between model and data arise essentially entirely from the specification discrepancies we mentioned in the statistics section—absent them, probabilities calculated from the model match the data precisely, because they are derived from a representation of the data that includes all of its relevant properties.

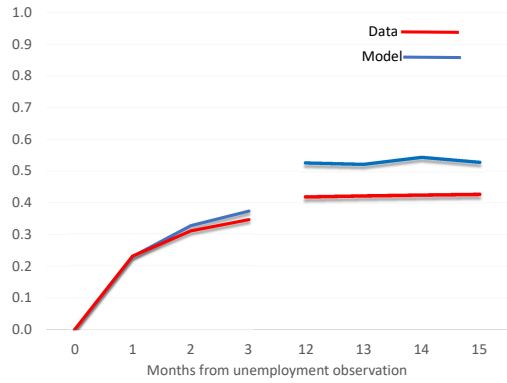
Figure 12 shows the tracking of the model to the data for all three observed activities, for prime-age women. The model understates unemployment somewhat, starting at two months, and correspondingly overstates out-of-the-labor-force, given that the match is so good for employment.

Figure 13 shows the distribution of months of unemployment in the data. The preponderance of respondents had zero months. But between about 10 percent (for prime-age men) and 25 percent (for young men) had some unemployment. Note that this is unemployment while in the survey. Some people entered the survey after already spending considerable time unemployed. Our model replicates the frequency distribution in the data well.

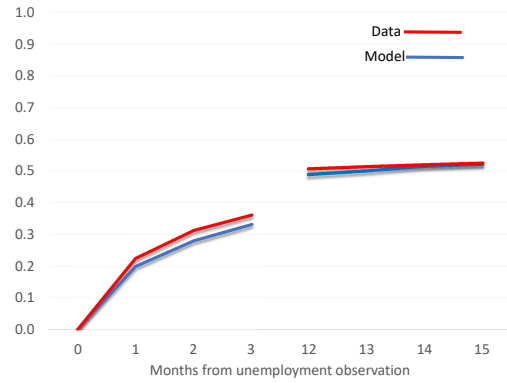
Figure 14 studies the success rate of the unemployed in a given month in terms of being employed in later months. The one-month success rate is fairly high, at about 25 percent. But further progress is much slower. Even 7 months later, only 59 percent are at work. This is another illustration of the failing of the traditional assumption of uniform job-finding rates among the unemployed.

Figure 15 displays the probability that a worker in month 7 of the CPS will lose that job in month 8, broken down by the number of months of work in months 1 through 6. The job loss probability is just over one percent if the worker had worked in every prior month, but

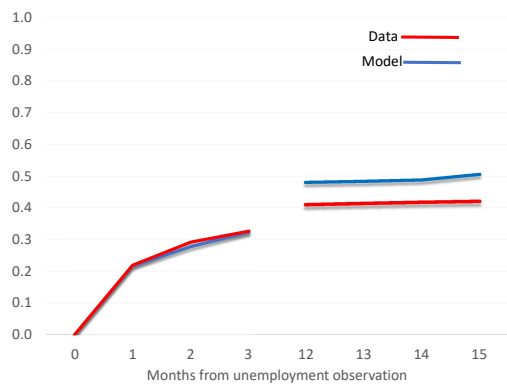




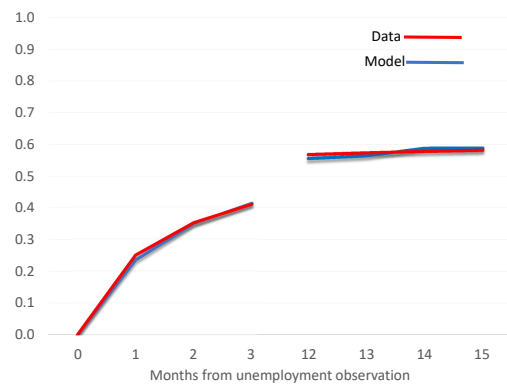
(a) Young women



(b) Prime-age women

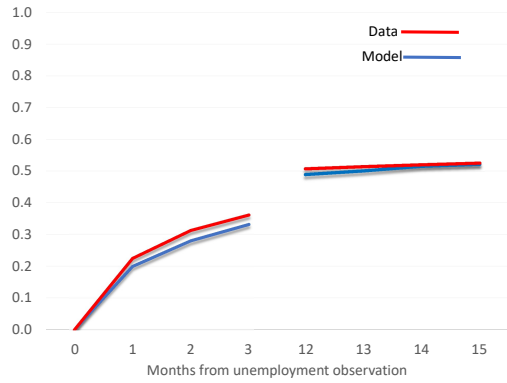


(c) Young men

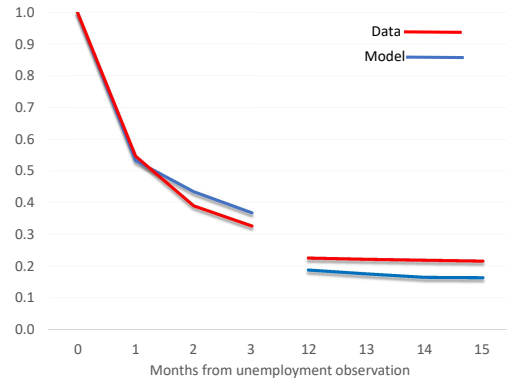


(d) Prime-age men

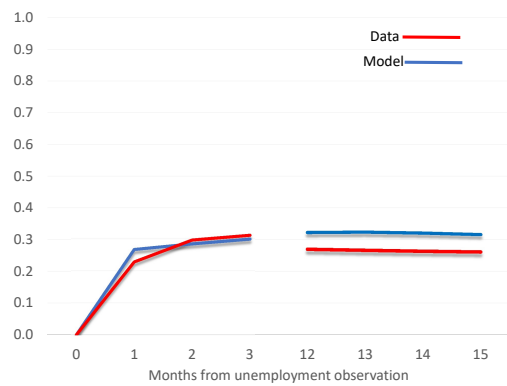
Figure 11: Paths of Employment Following Unemployment, by Demographic Group



(a) Employment



(b) Unemployment

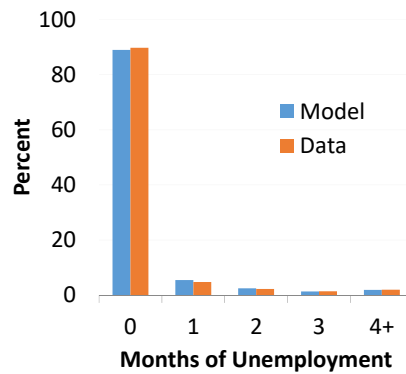


(c) OLF

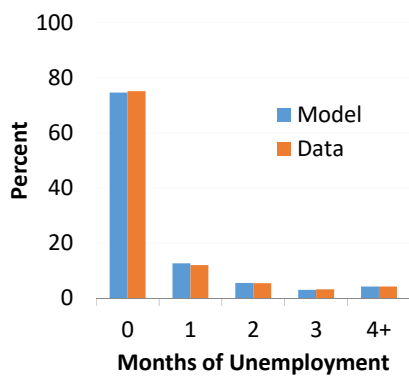
Figure 12: Paths of Three Activities Following Unemployment, for Prime-Age Women



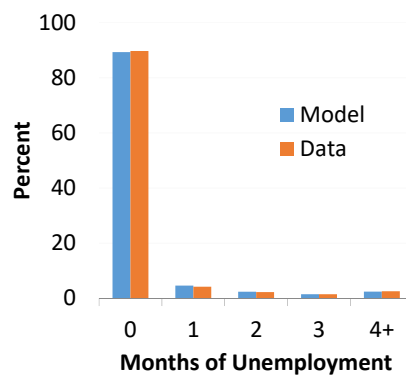
(a) Young women



(b) Prime-age women

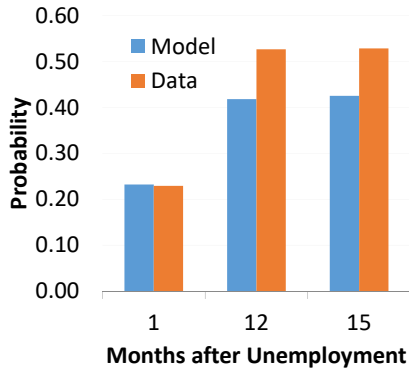


(c) Young men

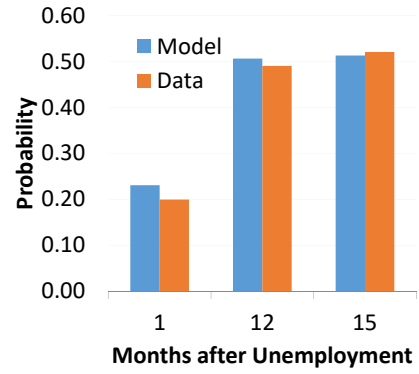


(d) Prime-age men

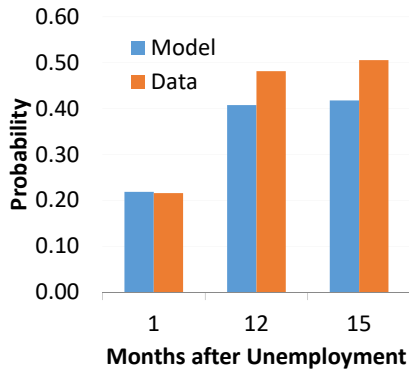
Figure 13: Number of Months of Unemployment during the 8 Months of the CPS



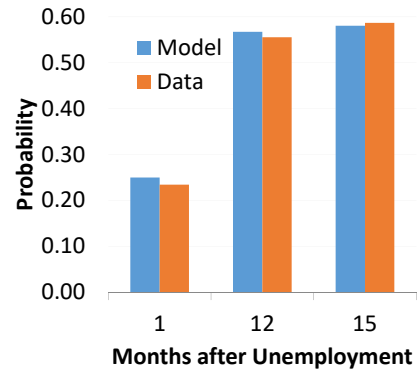
(a) Young women



(b) Prime-age women



(c) Young men



(d) Prime-age men

Figure 14: Success Rates in Becoming Employed after Being Unemployed

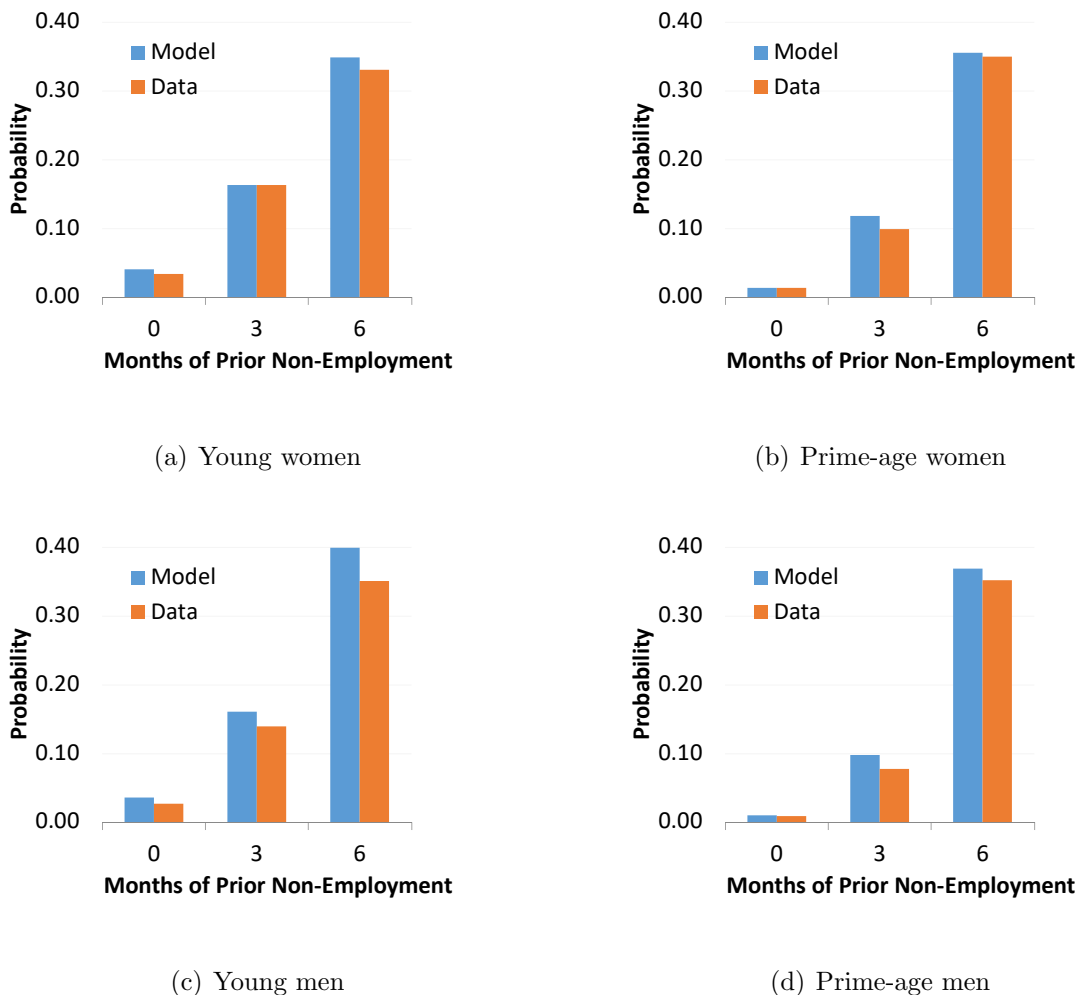


Figure 15: Probability of Job Loss in Month 7 by Number of Months of Non-Work in Earlier Months

reaches a stunning 40 percent if the job was brand new in month 7 and the worker spent the previous 6 months not working. Our model replicates this property of labor-market dynamics.

## 8 Comparison of the Model to One with a Single First-Order Markov Process in the Activities

Table 17 and Table 18 compare the one-month and 15-month transition rates in the data, in our model and in one that is based on a single first-order Markov structure in the observed activities for women and men, respectively. We compute the one-month transition rates from

	<i>Young Women</i>			<i>Prime-Age Women</i>		
	<i>To activity, one month later</i>			<i>To activity, one month later</i>		
	E	U	N	E	U	N
From activity	<i>Data</i>			<i>Data</i>		
E	0.908	0.017	0.075	0.971	0.008	0.021
U	0.244	0.415	0.341	0.224	0.541	0.235
N	0.078	0.040	0.882	0.054	0.026	0.920
	<i>Full model</i>			<i>Full model</i>		
E	0.906	0.019	0.076	0.971	0.009	0.020
U	0.232	0.425	0.343	0.231	0.536	0.233
N	0.077	0.040	0.882	0.056	0.029	0.915
	<i>First-order model</i>			<i>First-order model</i>		
E	0.908	0.017	0.075	0.971	0.008	0.021
U	0.244	0.415	0.341	0.224	0.541	0.235
N	0.078	0.040	0.882	0.054	0.026	0.920
	<i>To activity, 15 months later</i>			<i>To activity, 15 months later</i>		
	E	U	N	E	U	N
	<i>Data</i>			<i>Data</i>		
E	0.796	0.026	0.178	0.922	0.018	0.060
U	0.529	0.102	0.369	0.521	0.164	0.315
N	0.258	0.053	0.688	0.158	0.025	0.816
	<i>Full model</i>			<i>Full model</i>		
E	0.757	0.041	0.202	0.923	0.021	0.056
U	0.425	0.148	0.427	0.513	0.218	0.269
N	0.217	0.045	0.737	0.160	0.031	0.809
	<i>First-order model</i>			<i>First-order model</i>		
E	0.530	0.044	0.426	0.778	0.024	0.198
U	0.496	0.046	0.458	0.637	0.031	0.332
N	0.476	0.047	0.477	0.530	0.036	0.434

Table 17: Properties of the Model Compared to One Based on a First-Order Markov Process, for Women

the average across the 6 monthly transitions for the respondents. We raise that transition matrix to the 15th power to display its implications for the longest transition measured in the CPS.

Our model fits the one-month transition rates almost perfectly. The first-order model fits the one-month transition rates perfectly by construction.

On the other hand, the first-order model fits the 15-month transitions poorly, because the first-order assumption does not hold. It understates the persistence of employment, unemployment, and out-of-the-labor-force. The understatement is especially notable for the non-employment activities for prime-age workers. This issue is discussed in Krueger et al. (2014) and Hall and Schulhofer-Wohl (2018). Correspondingly, it overstates transitions out of the states. For example, for prime-age men, the probability of transition from out of labor force to employment in the data is 0.174 while in the first-order model it is 0.702. Similarly, for prime-age men, the probability of transition from employment to out of labor force in the data is 0.027 while in the first-order model it is 0.093.

	<i>Young Men</i>			<i>Prime-Age Men</i>		
	<i>To activity, one month later</i>			<i>To activity, one month later</i>		
	E	U	N	E	U	N
From activity	<i>Data</i>			<i>Data</i>		
E	0.909	0.023	0.068	0.981	0.009	0.010
U	0.228	0.471	0.301	0.253	0.583	0.164
N	0.078	0.050	0.872	0.075	0.044	0.881
	<i>Full model</i>			<i>Full model</i>		
E	0.906	0.025	0.069	0.981	0.009	0.010
U	0.219	0.480	0.301	0.250	0.594	0.156
N	0.078	0.051	0.871	0.079	0.048	0.873
	<i>First-order model</i>			<i>First-order model</i>		
E	0.909	0.023	0.068	0.981	0.009	0.010
U	0.228	0.471	0.301	0.253	0.583	0.164
N	0.078	0.050	0.872	0.075	0.044	0.881
	<i>To activity, 15 months later</i>			<i>To activity, 15 months later</i>		
	E	U	N	E	U	N
	<i>Data</i>			<i>Data</i>		
E	0.822	0.036	0.142	0.954	0.019	0.027
U	0.506	0.192	0.303	0.587	0.200	0.214
N	0.266	0.061	0.673	0.174	0.037	0.788
	<i>Full model</i>			<i>Full model</i>		
E	0.769	0.054	0.177	0.954	0.021	0.024
U	0.418	0.227	0.355	0.580	0.255	0.165
N	0.210	0.056	0.734	0.195	0.050	0.756
	<i>First-order model</i>			<i>First-order model</i>		
E	0.542	0.061	0.397	0.879	0.028	0.093
U	0.509	0.063	0.428	0.792	0.036	0.171
N	0.489	0.065	0.447	0.702	0.045	0.252

Table 18: Properties of the Model Compared to One Based on a First-Order Markov Process, for Men

Our model is much more successful in accounting for movements between three activities and the persistence of the activities. The success of our model is especially notable for prime-age groups. There are two main aspects of our model that deviate from the first-order assumption. First, our model contains a mixture of types. Second, for the non-polar types, even though the states follow Markov processes, the observed activities do not. This is because the states are partially hidden—there are 4 states and 3 activities. This heterogeneity in our model allows to replicate the transition probabilities over long horizons in the data—higher persistence of the activities and lower transition rates among activities than the first-order model predicts.

## 9 The Model’s Treatment of Frequent Changes between Unemployment and Out of Labor Force

In the CPS, the path from non-employment to employment often involves cycling between unemployment and out of labor force. An approach in the existing literature to frequent changes between unemployment and out of the labor force tries to correct what it regards as classification errors in the CPS. A recent practice in the literature is to treat transition reversals between unemployment and OLF as classification error and to re-code such transitions into one of the two continuous non-employment statuses. Elsby, Hobijn and Şahin (2015) call this “DeNUNification” because it replaces a NUN sequence with NNN.

Our model, in contrast, accepts the CPS data as a true record of behavior. Among the three non-polar types, our model attributes frequent changes between unemployment and OLF to types 2 and 3. These types have much lower ergodic probability in employment as compared to type 1. Consequently, the model finds that frequent circling between unemployment and out of labor force in the data is associated with lower employment rates. Kudlyak and Lange (2018) make a related point by showing that individuals with labor market histories of NUN have five times higher probability to find a job than those with the three consecutive months of N.

This is not to say that the CPS survey is free of classification error. Instead, our view is that frequent changes between reported labor force statuses contain information about respondent’s types.

How do we reconcile our view with the results of the Re-Interview Survey conducted by the BLS during 1977 through 1982? In the re-interview, the labor force status of a fraction of respondents was reclassified based on interviews conducted one week after the original interview (Abowd and Zellner (1985), Poterba and Summers (1986), and Jones and Riddell (1999)). As a result of the the re-interview, 99 percent of respondents who initially were



reported as being employed were retained in that classification and 99 percent of respondents who were initially reported as being OLF were continued as OLF—see Table 6 in Abowd and Zellner. Only 90 percent of the respondents who initially reported as being unemployed were retained as unemployed. 2.3 percent of them were re-classified as employed and 7.7 percent as out of the labor force. The results of the re-interview survey provide the main support for the misclassification-error hypothesis.

Through the lens of our model, the individuals who can be easily reclassified from one labor market status to another upon re-interview are those whose values of search or no search are very similar. The individuals who remain in the original classification upon the re-interview are those for whom the distinction between values of search and non-search are sufficiently dissimilar. We believe that retaining the original coding of the interview result is superior to fixed arbitrary rules such as recoding NUN as NNN.

## 10 Related Literature

Rust (1994) discusses identification and estimation in the class of models that we consider for each of our types, but does not deal with heterogeneity modeled as mixtures of types. Shibata (2015) proposes a hidden state Markov model of individual labor market dynamics. He finds a high-order overall transition matrix among hidden states and lets the observed distribution across observed statuses be a linear function of the hidden states. Shibata appeals to an identification theorem in Allman, Matias and Rhodes (2009). He offers meagre interpretation of his results—his main point is that that his more general setup nests the standard model based on one-month transitions, and that a test of the standard model against the more general model overwhelmingly rejects the standard model in favor of more subtle dynamics. See also Feng and Hu (2013) for a related model. Alvarez, Borovičková and Shimer (2017) focus on the long-standing problem of separating the effects of heterogeneity from those of duration dependence in a model of the jobfinding hazard. They characterize heterogeneity with in a continuous distribution of job-seeker types. Ahn and Hamilton (2019) consider two types in a model describing the critical role of unobserved heterogeneity.

Hall (1995) offers a simple model of the job-finding process that includes brief interim jobs as part of the explanation of the persistence of unemployment. Pries (2004) builds a formal model that includes interim jobs. Cole and Rogerson (1999) points out that observed high job-finding rates and relatively low separation rates are inconsistent with the persistence of unemployment in the specification following Mortensen and Pissarides (1994). Farber (1994) finds high worker separation hazard at monthly frequency. Hyatt and Spletzer (2015) and Hyatt and Spletzer (2013) document the high incidence of very brief jobs. Böheim and

Taylor (2002) show that jobs following a spell of unemployment last for fewer months than those following an earlier job.

Morchio (2017) examines long-run issues in individual earnings dynamics. He finds evidence that unemployment summed over workers' lifetimes is concentrated in a relatively small fraction of workers. Using the NLSY79, he finds that two-thirds of prime-age unemployment in a cohort is accounted for by 10 percent of workers. He finds that time spent in unemployment when a worker is young is a powerful predictor of time spent in unemployment during prime-age. He argues that *ex ante* heterogeneity is required to explain the facts.

Our work is related to a huge literature on the experiences of workers following displacement, defined as the loss of a job with at least several years of tenure. Jacobson, LaLonde and Sullivan (1993) is an important early contribution. Fallick (1996), Kletzer (1998), and Farber (2015) summarize others. Davis and von Wachter (2011) uses high-quality administrative data on post-displacement earnings and discusses the integration of this line of research with the DMP modeling tradition. Jarosch (2015) shows how the anticipation of likely job loss influences careers. Gagliarducci (2005) investigates repeated temporary jobs in Italian data. Eliason and Storrie (2006) use Swedish data on matched workers and employers to study long-run post-displacement experiences. Chan and Huff Stevens (2001) measure employment-rate depression following displacement among older workers.

## 11 Concluding Remarks

Our model makes sense out of the 16-month spans of individual paths of labor-market activity recorded in the Current Population Survey. Some people, presumably those with substantial match-specific job capital, are employed in all eight observations. Others, with consistently better opportunities at home rather than in the job market, are out of the labor force in all eight observations. We account for these two types, but most of our modeling effort goes into accounting for people who move around, sometimes out of the labor force, sometimes searching for work, and sometimes working. We portray them as pursuing personal dynamic programs that respond to random events in their lives.

An important part of our model is its distinction between short-term and long-term jobs. We use only the realizations of job duration that are recorded in the CPS to make this distinction. We show that short-term employment is poised between search that occurs while jobless, which declines rapidly after an individual is found to be searching, and long-term jobholding, which rises during the period following unemployment. Short-term employment

remains flat every month following unemployment. We also show that what we identify as short-term jobs tend to precede long-term jobs.

All of our focus in this paper is on personal dynamics. Our estimation relies on data for a boring period in US labor-market history, when the market was neither too cold nor too hot. A next step in this investigation is to look at a period of sharp aggregate change in the labor market, starting in 2008.

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# Appendixes

## A General Model

Our model comprises a variety of types of individuals. For each type, we specify labor-market activity in terms of a personal dynamic program. Opportunities become available to an individual and shocks occur. Individuals make forward-looking decisions in this stochastic environment.

For each type, we let  $V$  be a vector of Bellman values,  $f$  be a vector of flow values corresponding to  $V$ ,  $r$  be the discount rate applied to the succeeding month,  $P$  be a matrix of probabilities that appear in the Bellman equations that are functions of parameters, and  $M(V)$  be a matrix of functions of the Bellman values containing  $\max(\cdot)$  functions over selected Bellman values to describe the individual's choices governed by those values. The flow values may be subject to inequality constraints.

The system of Bellman equations describing the stationary state is

$$V = f + \frac{1}{1+r} P \odot M(V). \quad (19)$$

Here  $\odot$  is the matrix of element-by-element multiplications. For example, a Bellman equation for the value  $S$  involving the decision to search or be out of the labor force might be

$$S = z + b + \frac{1}{1+r} [(1 - \phi) \max(X, S) + \phi_1 \max(X, S, V)], \quad (20)$$

where  $z + b$  is the flow value received while searching,  $X$  is the Bellman value for those out of the labor force,  $\phi$  is the job-finding rate, and  $V$  is the Bellman value for holding a job.

The parameter space for a type comprises the vector of flow values  $f$  and the probabilities  $P$ . Each type also has an  $M(V)$  that describes the structure of the type's decision problem. We index types by the integer  $\theta$ , but frequently suppress it from the notation when discussing a single type, as in this section. We will show that each type described by  $\{f, P, \text{ and } M(\cdot)\}$  implies a unique set of Bellman values and conditional choice of actions.

We define a *region* as a subspace of the parameter space within which every  $\max(\cdot)$  over a given set of Bellman values dictates the same choice for all the parameters in the space. For example, in the equation above, if  $\max(X, S) = X$  for one parameter vector in a region, then  $\max(X, S) = X$  for every parameter vector in the region.

A region implies a set of inequalities in the Bellman values. In the example above, the  $\max(X, S) = X$  implies  $X \geq S$ .

Within region  $R$ , the fairly complicated object  $M(V)$  becomes the much simpler  $M(V) = M_R V$ , where  $M_R$  is a square matrix of functions of the probability parameters.  $M_R$  is the

same everywhere in the region. Then the Bellman system becomes

$$V = f + \frac{1}{1+r} P \odot M_R V. \quad (21)$$

The Bellman system is homogeneous of degree one in the flow values and Bellman values. To pin down the values, we normalize one of the flow values to be 1. If the normalization is a wage, we then interpret the other values in terms of wage units.

The Bellman system, the set of inequalities in the Bellman values, and the flow-value normalization, define a region as a convex polytope in the space of  $\{f, V\}$  vectors. To find the corresponding bounds on the flow parameters in  $f$ , we run linear programs to maximize and minimize each of those parameters, taking the Bellman equations as equality constraints along with the inequalities in the Bellman values. If the constraints are inconsistent, so there is no solution to the linear program, the probability parameters are inconsistent with the proposed definition of the region. If there is a solution, and its values for the flow parameters satisfy the constraints on those flow—we conclude that the linear the specification is valid, in the sense that there is a region

Here we assume that all of the probability parameters are known from estimation.

The probability parameters can vary within a region, and the same vector of probability parameters can inhabit more than one region, coupled with different flow-value parameters  $f$ .

For a given region definition, it is possible that some of the probability parameters do not appear in the actual transition probabilities that embody the choices that an individual makes, but are influential in determining that the individual did not make a particular choice. For example, our model includes a type who chooses to be out of the labor force in state 1. There is a parameter that is the probability that somebody who chose to search would find a job, but that probability does not appear in the type's transition matrix. But it does influence the choice not to search. In this situation, there is a Bellman equation with a Bellman value on the left-hand side that does not appear elsewhere in the Bellman system. The equation also contains the probability parameter that does not appear in the transition equation. This Bellman value applies to the choice not taken. It describes a line in the space defined by that Bellman value and the transition probability parameter. Any point on this line that satisfies the inequality that the value for the not-chosen option falls short of the value of the chosen option is admissible.

In the example above, there is a Bellman equation for being out of the labor force:

$$X = z + \frac{1}{1+r} [(1 - \psi) \max(X, S) + \psi \max(X, S, V)]. \quad (22)$$

Suppose we specify a region such that  $S = \max(X, S)$  and  $V = \max(X, S, V)$ . The Bellman value for  $X$  will only appear in one place in the Bellman system—here in the equation for



$X$ — because  $X$  will never be chosen as a continuation value. Within the region, the Bellman equation for  $X$  is

$$X = z + \frac{1}{1+r}[(1-\psi)S + \psi V]. \quad (23)$$

The equation for  $X$  also involves the job finding rate  $\psi$  which is specific to being out of the labor force and thus is not present anywhere else in the Bellman system and is not a part of the probability parameters of the model,  $P$ . Given a solution to the system, this Bellman equation imposes a linear relation between  $X$  and  $\psi$ , given the solved values of  $S$  and  $V$ . The bounds on  $z$  and the Bellman values derived earlier induce bounds on  $\psi$ . If the bounds do not include a value in  $[0, 1]$ , we conclude that the type is not in the proposed region.

We solve the Bellman system for  $S$  and  $V$ , ignoring the equation for  $X$ , and then check whether there exists the range of values of  $\psi$  such that  $X \leq S$  holds. We find the minimum and maximum values of  $\psi$  that are consistent with the constraints.

## B Details on Calculating the Probability of a State Path

We calculate the probability of a path  $S = [s_1, \dots, s_8]$  as follows: Let  $\pi_\theta$  be the transition matrix over states for type  $\theta$ , and  $P_\theta$  be the associated ergodic distribution vector, with elements  $P_{\theta,s}$ . The probability of path  $S$  for type  $\theta$  is the product of  $P_{\theta,s_1}$ , the stationary probability of the specified state in month 1,  $B_{\theta,s}$ , the product of the transition probabilities of the three specified transitions, from month 1 to month 2, month 2 to month 3, and month 3 to month 4,  $C_{\theta,s}$ , the probability of the transition from a specified activity in month 4 to the specified activity in month 5 via the 8 unspecified activities in the 8 months when a respondent does not provide data, and  $D_{\theta,s}$ , the product of the specified transition probabilities from month 5 to month 6, month 6 to month 7, and month 7 to month 8.

The compound probabilities are:

$$B_{\theta,s} = \prod_{t=1}^3 \pi_{\theta,s_t,s_{t+1}} \quad (24)$$

and

$$C_{\theta,s} = \pi_{\theta,s_4,:} \pi_\theta^7 \pi_{\theta,:,s_5}. \quad (25)$$

where  $\pi_{\theta,s_4,:}$  is the row of the transition matrix corresponding to the specified state in month 4, and  $\pi_{\theta,:,s_5}$  is the column of the transition matrix corresponding to the specified state in month 5.  $C_{\theta,s}$  is the probability of being in state  $s_5$  conditional on having been in state  $s_4$

and not conditional on the intervening activities hidden from the CPS. Finally,

$$D_{\theta,s} = \prod_{t=5}^7 \pi_{\theta,s_t,s_{t+1}} \quad (26)$$

Thus

$$\tilde{M}_n = \sum_{\theta} \omega_s P_{\theta,s_1} B_{\theta,n} C_{\theta,n} D_{\theta,n}. \quad (27)$$

## C Complete Description of the Full Economic Model

The model has two non-work states,  $s = 1$  and  $s = 2$ , called *non-work* and *activated non-work*. The work states are  $s = 3, 4$ , called *short-term job* and *long-term job*. The states are *partially* hidden because an unemployed individual or one out of the labor force may be in  $s = 1$  or  $s = 2$ , while an employed worker is in  $s = 3$  or  $s = 4$ .

While in a non-working state,  $s = 1$  or  $s = 2$ , an individual can be either out of the labor force or a searcher, with the flow values  $z$  and  $z + b$ , respectively. With  $s = 1$ , the two job finding rates, called  $\psi_1$  and  $\phi_1$ , are low and all jobseeking transitions are to short-term jobs, with  $s = 3$ . On the other hand, with  $s = 2$ , the job finding rates  $\psi_2$  and  $\phi_2$  are higher, and there is a positive probability of going straight into a long-term job, with  $s = 4$ . The fraction of activated jobseekers who draw a long-term job is  $\gamma$ . While in the activated group of non-workers, there is a probability  $\eta$  of transiting to state 1. An inactive individual may experience activation with probability  $\rho$ .

While in a short-term job, a worker earns  $w_3$ . The short-term job expires with probability  $\delta_3$  and the worker moves to state 1. While in the low job tier, there is probability  $\mu$  that the worker transitions to the top tier job.

While in the top job tier, a worker earns  $w_4$ . The high-tier job expires with probability  $\delta_4$ . Should a job expire, there is a probability  $\kappa$  that the worker drops to state 2 rather than to state 1. This generates a flow of experienced, successful workers into the favored group of non-workers. While in the top job tier, there is probability  $\nu$  that the worker is demoted to the low tier job. This captures endogenous layoffs—the “slippery job ladder”.

The timing of the events is as follows. At the beginning of a period an individual gets the flow value of the state and activity that he is in. The chance to transition to another state arrives. If the chance is for the higher than the current state, the individual can chose to transition to the higher state or any of the lower states. If the chance is for the lower than the current state, the individual must transition to the new state or choose a lower state. Furthermore, for any non-employed state, an individual chooses between two activities: unemployment or out of the labor force.

The Bellman equations are, for those out of the labor force with  $s = 1$ ,

$$X_1 = z + \frac{1}{1+r} [(1 - \psi_1 - \rho) \max(X_1, S_1) + \rho \max(X_1, S_1, X_2, S_2) + \psi_1 \max(X_1, S_1, X_2, S_2, V_3)], \quad (28)$$

for searchers with  $s = 1$ ,

$$S_1 = z + b + \frac{1}{1+r} [(1 - \phi_1) \max(X_1, S_1) + \phi_1 \max(X_1, S_1, X_2, S_2, V_3)], \quad (29)$$

for those OLF with  $s = 2$ ,

$$\begin{aligned} X_2 = z + \frac{1}{1+r} \{ & \eta \max(X_1, S_1) + (1 - \eta - \psi_2) \max(X_1, S_1, X_2, S_2) \\ & + \psi_2 [(1 - \gamma) \max(X_1, S_1, X_2, S_2, V_3) + \gamma \max(X_1, S_1, X_2, S_2, V_3, V_4)] \}, \end{aligned} \quad (30)$$

for searchers with  $s = 2$ ,

$$\begin{aligned} S_2 = z + b + \frac{1}{1+r} \{ & \eta \max(X_1, S_1) + (1 - \eta - \phi_2) \max(X_1, S_1, X_2, S_2) \\ & + \phi_2 [(1 - \gamma) \max(X_1, S_1, X_2, S_2, V_3) + \gamma \max(X_1, S_1, X_2, S_2, V_3, V_4)] \}, \end{aligned} \quad (31)$$

and for workers with  $s = 3$  and  $s = 4$ , respectively,

$$\begin{aligned} V_3 = w_3 + \frac{1}{1+r} [ & \delta_3 \max(X_1, S_1) + (1 - \delta_3 - \mu) \max(X_1, S_1, X_2, S_2, V_3) \\ & + \mu \max(X_1, S_1, X_2, S_2, V_3, V_4)], \end{aligned} \quad (32)$$

$$\begin{aligned} V_4 = w_4 + \frac{1}{1+r} [ & \delta_4 ((1 - \kappa) \max(X_1, S_1) + \kappa \max(X_1, S_1, X_2, S_2)) + \\ & \nu \max(X_1, S_1, X_2, S_2, V_3) + (1 - \delta_4 - \nu) \max(X_1, S_1, X_2, S_2, V_3, V_4)]. \end{aligned} \quad (33)$$

These equations are linear in the unknown Bellman values, except for the max functions. Replacement of those functions with inequalities converts the system to a linear program, for which highly efficient solution methods exist. For a given max function—for example,  $\max(X_1, S_1)$  in the first equation—we define the value of the max expression as a new variable, say  $y$ , and observe that  $y$  must not be less than any of its arguments:

$$y \geq X_1 \text{ and } y \geq S_1. \quad (34)$$

We then rewrite the Bellman equation system with the  $y$ -variables in place of the max functions, making it fully linear. We include all of the new inequalities, and find the minimum of the sum of the  $y$  variables. This collection is a linear program. The objective function reaches its minimum value when each inequality binds at the maximum value within each max function's arguments. The values of the variables then constitute a solution to the original Bellman system.

## D Transitions

This section describes the way the model determines the transition matrix. The determination is based on the Bellman values. The choices involve finding the maximum value of particular sets of those values. We describe the choices in the first 6 subsections. In the next subsection, we consider the issue of how choices are made when two or more Bellman values tie for best choice. The last subsection describes the mapping of the partly hidden states  $s$  to the observed activities  $a$ .

In the model, individuals face random events—the occurrence of favorable opportunities and of adverse shocks. The main logic of the choices open to the individual is as follows. Whenever an individual is presented with an opportunity to move to a higher state, the individual can always choose any of the lower states. Whenever a shock forces a move to a lower state, the individual needs to move to that state or can choose any of the lower states. Thus individuals always keep in mind that exiting the labor force or quitting a job may be the best available alternative.

### D.1 From state 1, inactive non-work

An inactive non-searching individual may, with probability  $\psi_1$ , experience an opportunity for taking a lower-level job next month. The individual may then choose to take the lower-level job, or may not take the job and activate but not search, or may activate and search, or search without taking the job and without activating, or continue non-search without activating. This choice is summarized by  $\max(X_1, S_1, X_2, S_2, V_3)$ . If the job opportunity does not arise, the individual chooses searching if its value,  $S_1$  exceeds the value of not searching,  $X_1$ . This choice is summarized by  $\max(X_1, S_1)$ .

An inactive non-worker's situation may change and result in activation, with probability  $\rho$ .

The benefit of becoming a searcher in the inactive non-work state occurs if searching results in a higher job-finding rate:  $\phi_1 > \psi_1$ , or if there is a positive flow value of searching,  $b$ .

### D.2 From state 2, active non-work

An active but non-searching individual may, with probability  $\eta$ , become inactive, in which case the individual chooses searching if its value,  $S_1$  exceeds the value of being out of the labor force,  $X_1$ , corresponding to  $\max(X_1, S_1)$  in the Bellman equation. This event corresponds to some alteration in personal circumstances favoring home rather than market activities.

Further, the non-searching individual in the active non-work state may, with probability  $\psi_2$ , experience an opportunity for taking a job next month. In that case, with probability  $\gamma$ , an opportunity may develop to take a high-level job. Then the individual may choose to work in the higher-level job, work in a low-level job, become inactive and out of the labor force, inactive and searching, active and out of the labor force, or active and searching, corresponding to the  $\max(X_1, S_1, X_2, S_2, V_3, V_4)$  in the Bellman equation. Otherwise, with probability  $1-\gamma$ , there is an opportunity for a low-level job. In that case, the choices correspond to the  $\max(X_1, S_1, X_2, S_2, V_3)$  in the Bellman equation. If neither a job opportunity arises nor a move into the inactive non-work state occurs, she still has a choice, with probability  $1 - \eta - \psi_2$ , between being inactive out-of-labor-force with value  $X_1$ , inactive search with value  $S_1$ , active out-of-labor-force with value  $X_2$ , or active search with value  $S_2$ , corresponding to  $\max(X_1, S_1, X_2, S_2)$  in the Bellman equation.

Similarly, an active searching individual may, with probability  $\phi_2$ , experience an opportunity for taking a job next month. In that case, with probability  $\gamma$ , an opportunity may develop to take a high-level job. Then the individual may choose to work in the higher-level job, work in a low-level job, become inactive and out of the labor force, inactive and searching, active and out of the labor force, or active and searching, corresponding to the  $\max(X_1, S_1, X_2, S_2, V_3, V_4)$  in the Bellman equation. Otherwise, with probability  $1-\gamma$ , the opportunity is for a low-level job. Then the individual has the same choices except for the higher-level employment, corresponding to the  $\max(X_1, S_1, X_2, S_2, V_3)$  in the Bellman equation. If neither a job opportunity arise nor a move into an inactive non-work state occurs, she still has a choice, with probability  $1 - \eta - \phi_2$ , between being inactive out-of-labor-force with value  $X_1$ , inactive search with value  $S_1$ , active out-of-labor-force with value  $X_2$ , or active search with value  $S_2$ , corresponding to the  $\max(X_1, S_1, X_2, S_2)$  in the Bellman equation.

The benefit of becoming a searcher versus non-searcher in the active non-work state occurs if searching results in a higher job-finding rate:  $\phi_2 > \psi_2$  or if there is a positive flow value of searching,  $b$ .

### D.3 From state 3, lower-level employment

An individual in a lower-level job may, with probability  $\mu$ , receive an opportunity for upper-level employment, in which case, she has a choice among all 6 of the Bellman values, corresponding to the  $\max(X_1, S_1, X_2, S_2, V_3, V_4)$  in the Bellman equation. With probability  $\delta_3$ , the individual loses the lower-level job and separates into inactive non-work. She then chooses between inactive search or non-search, corresponding to the  $\max(X_1, S_1)$  in the Bellman equation. If neither a higher-level job opportunity arrives nor a separation into an inactive non-work state occurs, she still has a choice, with probability  $1 - \delta_3 - \mu$ , between working

at a low-level job, being inactive out-of-labor-force with value  $X_1$ , inactive search with value  $S_1$ , active out-of-labor-force with value  $X_2$ , or active search with value  $S_2$ , corresponding to the  $\max(X_1, S_1, X_2, S_2, V_3)$  in the Bellman equation.

#### D.4 From state 4, upper-level employment

An individual in an upper-level job may, with probability  $\nu$ , suffer a demotion to a lower-level job, so she faces a choice among that Bellman value,  $V_3$ , and the four non-employment Bellman values, corresponding to the  $\max(X_1, S_1, X_2, S_2, V_3)$  in the Bellman equation. With probability  $\delta_4$ , the individual suffers a demotion that eliminates both the upper-level and lower-level employment opportunities. In that case, with probability  $\kappa$ , the individual drops to state 1 with only a choice between inactive out-of-the-labor-market and inactive search, corresponding to the  $\max(X_1, S_1)$  in the Bellman equation. With probability  $1 - \kappa$  the choice is among all 4 non-employment Bellman values, corresponding to the  $\max(X_1, S_1, X_2, S_2)$  in the Bellman equation. With probability  $1 - \delta_4 - \nu$ , the individual may choose to remain employed at the high-level job, or to be in any other state or activity, corresponding to the  $\max(X_1, S_1, X_2, S_2, V_3, V_4)$  in the Bellman equation.

#### D.5 Resolution of ties among Bellman values

Ties in Bellman values may occur by coincidence, but are most common in cases where the logic of the model creates them. Our interest in segments of the population who are close to the boundary of participation in the labor market requires that we deal carefully with ties. The resolution of ties has consequences for the distribution of the population among the three activities in our model. This arises in the construction of the transition matrix among the partly hidden states.

There are 15 pairs of Bellman values that may be equal. The strategy is to check each pair and assign a potential resolution in case the pair shares the max value for a max in a Bellman equation. To describe this aspect of the model, we define the operator  $\succ$  as the ruling choice for a particular tie.

1.  $X_1 = S_1$ . The resolution is in favor of inactive out-of-labor-force, corresponding to  $X_1$ , if  $z \geq 1$  and in favor of inactive search, corresponding to  $S_1$ , otherwise. We write this as  $X_1 \succ \{X_1, S_1 \text{ and } z \geq 1\}$ . The flow value of non-search,  $z$ , is at least as high as the wage at the upper-level job. Although if  $b > 0$ , individuals will enter the labor market, it is to harvest benefits, not to take a job. We presume that unmodeled restrictions on benefits would eliminate that option.
2.  $X_2 \succ \{X_1, X_2\}$  (arbitrary)

3.  $X_1 \succ \{X_1, S_2 \text{ and } z \geq 1\}$  Assign to OLF. When  $z \geq 1$ , there is no benefit to search, because the highest wage is 1, but finding that higher-level job involves costly effort.
4.  $X_1 \succ \{X_1, V_3 \text{ and } z \geq 1\}$  Same as 3.
5.  $X_1 \succ \{X_1, V_4 \text{ and } z \geq 1\}$  Same as 3.
6.  $X_2 \succ \{S_1, X_2 \text{ and } z \geq 1\}$  Same as 3.
7.  $S_2 \succ \{S_1, S_2\}$  (arbitrary)
8.  $V_3 \succ \{S_1, V_3 \text{ and } b > 0\}$  (UI rules require taking a job because the positive flow value of searching,  $b > 0$ , otherwise may induce the person to decline the job and remain a searcher to harvest the benefits )
9.  $V_4 \succ \{S_1, V_4 \text{ and } b > 0\}$  Same as 8.
10.  $X_2 \succ \{X_2, S_2 \text{ and } z \geq 1\}$  Same as 3.
11.  $X_2 \succ \{X_2, V_3 \text{ and } z \geq 1\}$  Same as 3.
12.  $X_2 \succ \{X_2, V_4 \text{ and } z \geq 1\}$  Same as 3.
13.  $V_3 \succ \{S_2, V_3 \text{ and } b > 0\}$  Same as 8.
14.  $V_4 \succ \{S_2, V_4 \text{ and } b > 0\}$  Same as 8.
15.  $V_4 \succ \{V_3, V_4\}$  We generally expect  $V_4 > V_3$ , because the state 4 job is higher level; if not, resolve the tie as if that job is preferred.

Now we consider cases involving more than a pair of Bellmans. They can be constructed by chaining the pairwise conclusions. For example,

$$X_1 \succ \{X_1, S_1, X_2, S_2\} = X_1 \succ \{X_1, S_1\} \text{ and } X_1 \succ \{X_1, X_2\} \text{ and } X_1 \succ \{X_1, S_2\} \quad (35)$$

The same approach gives  $S_1 \succ \{X_1, S_1, X_2, S_2\}$  and its other two relatives.

The other cases involving more than a pair of Bellmans involve  $X_1, S_1, X_2, S_2, V_3$  and  $X_1, S_1, X_2, S_2, V_3, V_4$  which we handle the same way.

## D.6 Building the transition matrix

To better understand the transition probabilities across states, consider, for example, a transition from state 4 to state 1. There are four different ways for an individual to make such a transition. First, the individual receives a separation shock  $\delta_4(1-\kappa)$  and must separate into state 1. Second, the individual receives a separation shock into active non-work  $\delta_4\kappa$  but the values of non-search or search in state 1 are higher than the ones in state 2. Third, the individual receives a demotion shock  $\nu$  and the values of search or non-search in state 1 are higher than the ones in states 2 or 3. Finally, the individual receives no shock but the values of search or non-search in state 1 are higher than the ones in states 2, 3, or 4.

To describe the construction of the transition matrix, we extend the discussion of  $\succ$  to include indicating when a Bellman value controls a choice because it strictly exceeds all the other available choices. Then we insert the choice operator into the appropriate positions in the transition matrix, multiplied by the corresponding probabilities. For example,

$$\pi_{3,3} = (1 - \mu - \delta_3)V_3 \succ \{X_1, S_1, X_2, S_3, V_3\} + \mu V_4 \succ \{X_1, S_1, X_2, S_2, V_3, V_4\} \quad (36)$$

## D.7 Mapping the partially hidden state variable $s$ to the observed activities $a$

The function describing the mapping is:

- If  $(X_1 \succ \{X_1, S_1\})$  and  $s = 1$  or  $(X_2 \succ \{X_2, S_2\})$  and  $s = 2$ ,  $a = N$
- If  $(S_1 \succ \{X_1, S_1\})$  and  $s = 1$  or  $(S_2 \succ \{X_2, S_2\})$  and  $s = 2$ ,  $a = U$
- if  $s = 3, 4$ ,  $a = E$

## E Validating a Type

For the non-polar types, with probabilities distributed across most of the  $3^8$  possible activity histories, there are reasonably fat regions in the space of the flow values  $\{b, w_3, z\}$  within which individuals choose the same mapping of states into activities. For example, a salient case maps those in state 1 out of the labor force, in state 2 to unemployment (search), and in states 3 and 4 to employment. Within a region, the probability parameters can vary, but each one always apply to the same Bellman value. In other words, the max functions disappear from the system of Bellman equations. Further, the transition matrix involves the same parametric probabilities within the region.

At the edge of a region, where one of the max functions in the Bellman system switches to a different winning argument, the transition matrix and corresponding probability induced on



activities change discontinuously. Thus standard hill-climbing methods cannot work. The distribution among paths corresponding to a particular type is not a continuous function and certainly not a continuously differentiable function of the parameters. But within a region, we can estimate the set of probability parameters relevant to the region by standard minimum distance principles. The distance is a continuously differentiable function of the parameters, so we can use a Newton-type algorithm to estimate them.

After carrying out the estimation, we validate it by confirming that the mapping of states into activities is the one assumed in estimation.

To deal with this issue, we note that the discontinuities occur at the point where the winning Bellman value switches within the expressions such as  $\max(X_1, S_1, X_2, S_2)$  in the Bellman equations. We carry out a separate analysis within various regions of the parameter space. The model has four max functions:

1.  $\max(X_1, S_1)$  with 2 choices
2.  $\max(X_1, S_1, X_2, S_2, V_3)$  with 5 choices
3.  $\max(X_1, S_1, X_2, S_2)$  with 4 choices
4.  $\max(X_1, S_1, X_2, S_2, V_3, V_4)$  with 6 choices

All told, because of overlaps, there are much fewer than the non-binding upper bound,  $2 \times 5 \times 4 \times 6 = 240$  regions.

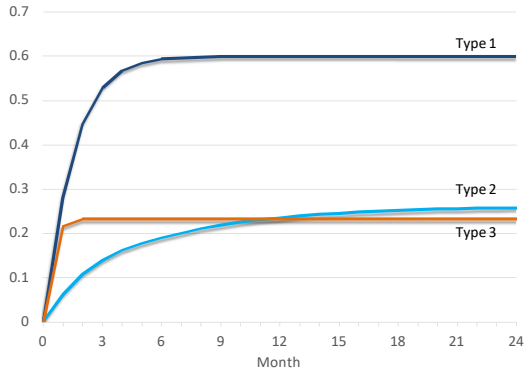
A salient set of choices would be  $X_1, S_2, V_3$ , and  $V_4$ . This set of choices define what we call the base type. It describes an individual close to the margin, who is out of the labor force in the least promising situation, chooses to search if activated, takes a lower-level job if it is the only type available, and takes the upper-level job if it is available.

After removing redundant inequalities, those defining the base type are

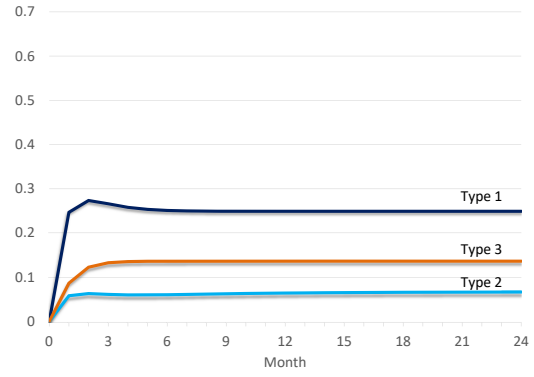
1.  $X_1 > S_1$
2.  $S_2 > X_1$  and  $S_2 > X_2$
3.  $V_3 > S_2$
4.  $V_4 > V_3$

The second phase is to find the intersection of (1) the set of Bellman values implied by some vector of the three flow-value parameters,  $x = [b, w_3, z]$  and (2) the set of Bellman values satisfying the inequalities implied by the base type. We use the linear program

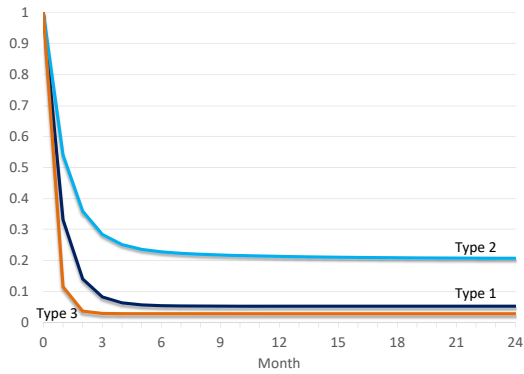
$$\max qx \text{ subject to } x = \mathbb{B}\mathbb{V} \text{ and } \mathbb{C}\mathbb{V} > 0, \tag{37}$$



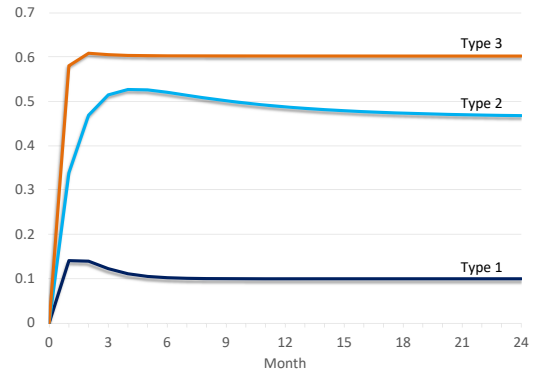
(a) To long-term job



(b) To short-term job



(c) To unemployment

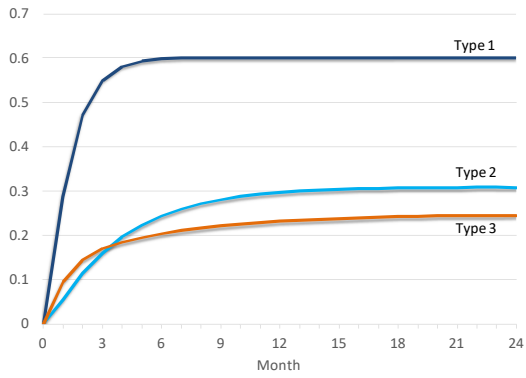


(d) To out of labor force

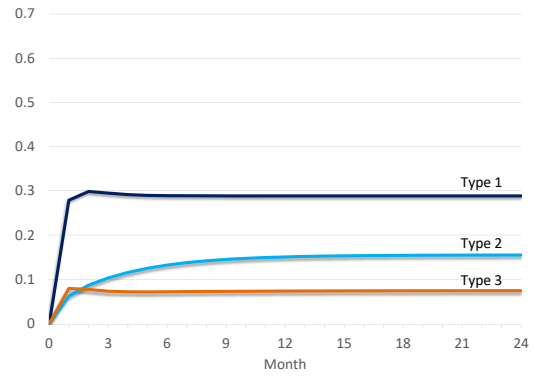
Figure 16: Paths of States Following Unemployment, by Type, for Young Women

where  $q$  is a vector of weights (such as a row of an identity matrix),  $\mathbb{V}$  is the vector of Bellman values,  $x = \mathbb{B}\mathbb{V}$  expresses the Bellman system, and  $\mathbb{C}\mathbb{V} > 0$  expresses the inequalities defining the base case. Solving the linear program for a variety of weighting vectors will illustrate the alternative values of  $b$ ,  $w_3$ , and  $z$  that could account for the base vector. Further, a linear program is an easy way to determine if the intersection is empty, meaning that no base type can be constructed for the given set of choices.

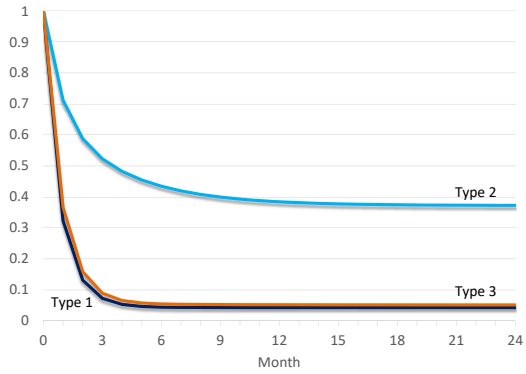
## F Additional results



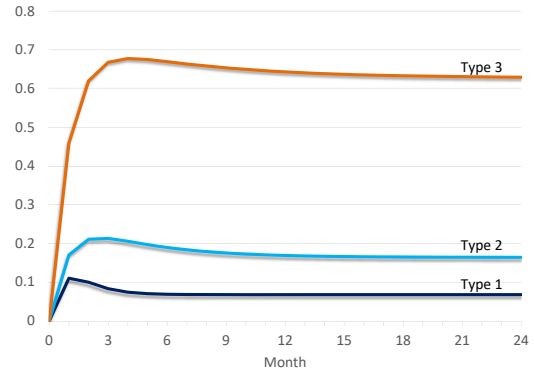
(a) To long-term job



(b) To short-term job

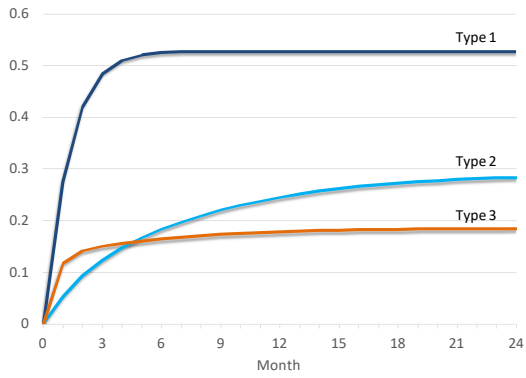


(c) To unemployment

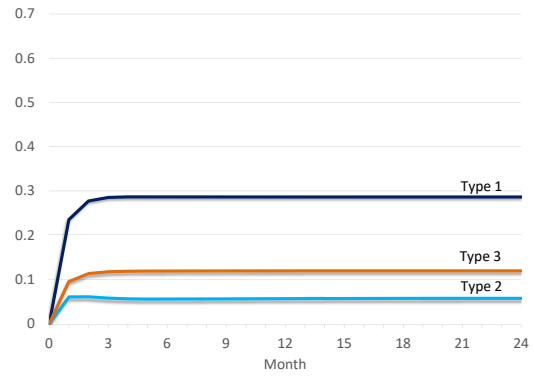


(d) To out of labor force

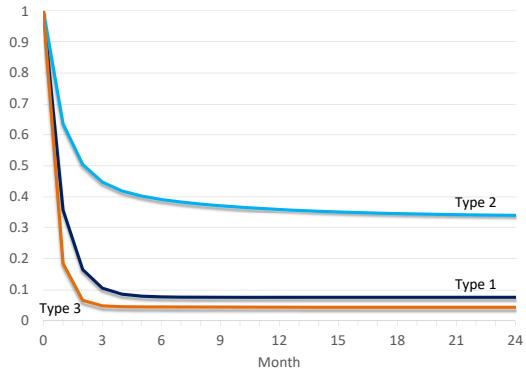
Figure 17: Paths of States Following Unemployment, by Type for Prime-Age Women



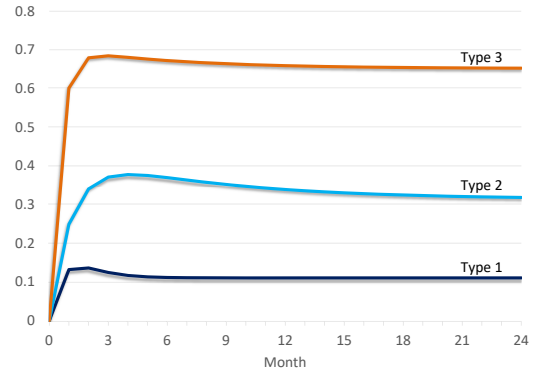
(a) To long-term job



(b) To short-term job

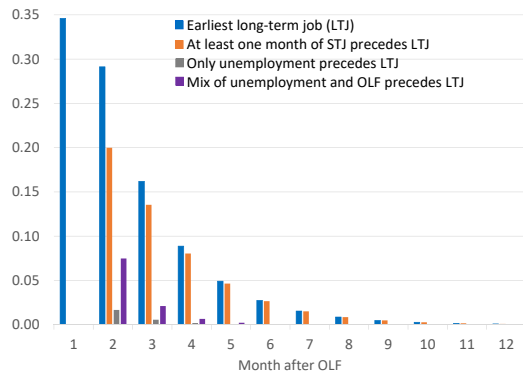


(c) To unemployment

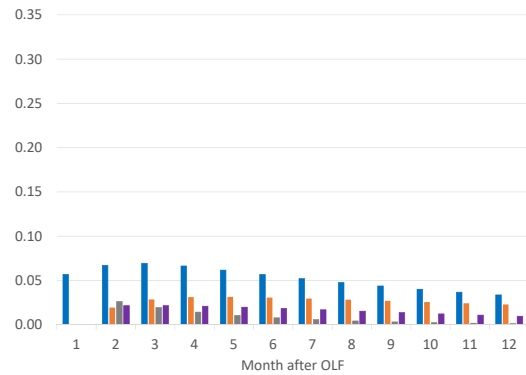


(d) To out of labor force

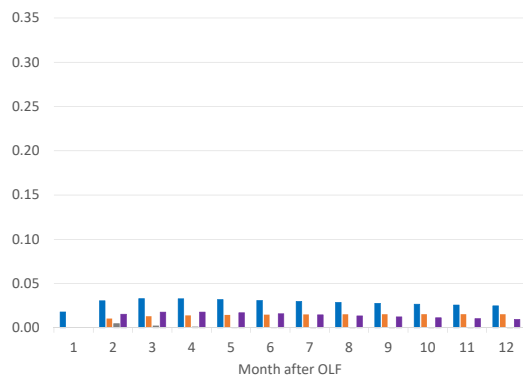
Figure 18: Paths of States Following Unemployment, by Type for Young Men



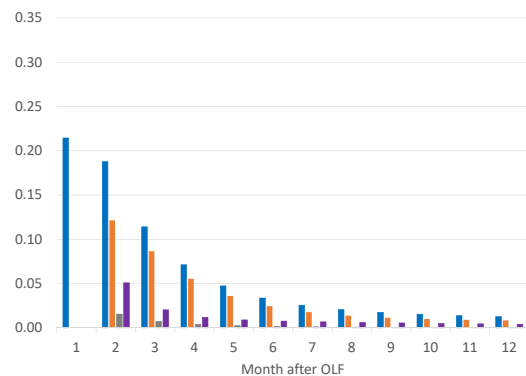
(a) Type 1 (58 percent)



(b) Type 2 (18 percent)



(c) Type 3 (24 percent)



(d) All Non-Polar Types

Figure 19: Path to Long-Term Job from Out of Labor Force in Month 0, by Type for Prime-Age Men

Note: Colored bars sum to the blue bar. Type's weight in the pool of Non-Polar Types is in parenthesis.

Value/ parameter	Description	Type 1						Type 2						Type 3					
		z		b		w3		z		b		w3		z		b		w3	
		min	max	min	max	min	max	min	max	min	max	min	max	min	max	min	max	min	max
<i>Young Women</i>																			
X1	Value of OLF when inactive non-employment	-2.39	-0.04	-0.89	-2.18	-2.08	-2.07	-4.65	-0.03	-4.56	-4.72	-3.85	-1.11	-2.89	-0.04	-2.53	-2.97	-2.36	-0.74
S1	Value of search when active non-employment	-2.98	-0.06	-1.15	-2.39	-2.57	-2.57	-4.97	-0.03	-5.27	-4.72	-4.12	-1.18	-3.43	-0.06	-3.37	-2.97	-2.87	-0.91
X2	Value of OLF when inactive non-employment	-3.00	-0.06	-1.13	-2.27	-2.47	-2.60	-3.94	-0.03	-4.69	-3.27	-3.29	-0.93	-2.13	-0.04	-2.29	-1.53	-1.77	-0.59
S2	Value of search when active non-employment	-1.88	-0.04	-0.89	-1.10	-1.74	-1.68	-3.44	-0.03	-4.48	-2.50	-2.90	-0.80	-1.27	-0.04	-1.78	-0.21	-1.14	-0.39
V3	Value of short-term job	-0.22	-0.04	-0.89	-0.41	-0.91	-0.07	3.63	-0.03	3.97	3.98	2.35	0.94	0.54	-0.04	0.39	0.91	-0.25	0.20
V4	Value of long-term job	0.65	0.03	0.60	0.63	0.88	0.52	10.01	0.08	10.59	9.32	8.50	2.36	7.33	0.13	6.55	7.19	6.41	1.86
z	Flow value of non-employment	-1.00	0.96	0.25	-0.83	-0.74	-0.73	-1.00	0.98	-0.92	-1.00	-0.73	0.53	-1.00	0.97	-0.76	-1.00	-0.75	0.50
b	Flow value of searching	0.62	0.00	-0.04	1.00	0.35	0.51	-0.28	-0.01	-1.00	0.37	-0.25	-0.06	-0.41	-0.03	-1.00	0.64	-0.52	-0.16
w3	Wage in short-term job	0.78	0.95	0.00	0.60	0.00	0.95	0.51	0.95	0.67	0.95	0.00	0.95	0.60	0.95	0.55	0.95	0.00	0.95
$\phi 1$	Jobfinding rate from non-activated search	0.17	0.27	0.26	0.09	0.19	0.19	0.03	0.09	0.06	0.00	0.03	0.03	0.09	0.32	0.16	0.00	0.12	0.11
$\psi 2$	Jobfinding rate from activated OLF	0.29	0.21	0.18	0.39	0.31	0.32	0.05	0.01	0.02	0.08	0.04	0.05	0.11	0.02	0.07	0.18	0.10	0.10
<i>Prime-Age Women</i>																			
X1	Value of OLF when inactive non-employment	-2.42	-0.04	-0.72	-2.18	-2.00	-2.25	-3.45	-0.02	-3.77	-3.53	-2.69	-1.24	-3.67	-0.03	-3.51	-3.76	-3.07	-0.89
S1	Value of search when active non-employment	-3.06	-0.05	-0.92	-2.40	-2.45	-2.87	-3.72	-0.02	-4.24	-3.53	-2.88	-1.32	-4.06	-0.04	-4.31	-3.76	-3.39	-0.98
X2	Value of OLF when inactive non-employment	-3.07	-0.05	-0.90	-2.26	-2.45	-2.80	-3.59	-0.02	-4.24	-3.16	-2.78	-1.25	-2.99	-0.03	-3.69	-2.24	-2.52	-0.73
S2	Value of search when active non-employment	-2.00	-0.04	-0.72	-1.06	-1.60	-1.82	-3.07	-0.02	-3.77	-2.42	-2.38	-1.04	-2.38	-0.03	-3.41	-1.28	-2.04	-0.58
V3	Value of short-term job	-0.23	-0.04	-0.72	-0.36	-0.68	0.05	2.51	-0.02	3.36	2.46	1.48	0.96	1.39	-0.03	1.43	1.82	0.57	0.40
V4	Value of long-term job	0.53	0.02	0.48	0.49	0.66	0.36	4.27	0.04	4.85	3.55	3.54	1.43	9.39	0.10	9.17	9.26	8.04	2.25
z	Flow value of non-employment	-1.00	0.97	0.30	-0.85	-0.73	-0.81	-1.00	0.98	-1.00	-1.00	-0.73	0.31	-1.00	0.98	-0.91	-1.00	-0.73	0.52
b	Flow value of searching	0.50	0.00	-0.01	1.00	0.40	0.51	0.25	0.00	-1.01	0.75	0.20	0.13	-0.23	-0.01	-1.00	0.56	-0.22	-0.06
w3	Wage in short-term job	0.56	0.95	0.00	0.43	0.00	0.95	0.51	0.95	0.95	0.95	0.00	0.95	0.52	0.95	0.61	0.95	0.00	0.95
$\phi 1$	Jobfinding rate from non-activated search	0.20	0.26	0.28	0.09	0.20	0.18	0.04	0.05	0.06	0.00	0.04	0.03	0.05	0.15	0.10	0.00	0.05	0.05
$\psi 2$	Jobfinding rate from activated OLF	0.30	0.23	0.25	0.39	0.29	0.33	0.07	0.06	0.06	0.12	0.08	0.08	0.07	0.02	0.03	0.12	0.07	0.07
<i>Young Men</i>																			
X1	Value of OLF when inactive non-employment	-2.33	-0.03	-0.66	-2.05	-1.80	-2.14	-5.51	-0.06	-5.89	-5.31	-4.69	-1.32	-2.89	-0.04	-2.63	-2.96	-2.41	-0.68
S1	Value of search when active non-employment	-2.70	-0.04	-0.87	-2.22	-2.24	-2.76	-5.80	-0.07	-6.58	-5.31	-4.95	-1.38	-3.31	-0.06	-3.42	-2.96	-2.78	-0.79
X2	Value of OLF when inactive non-employment	-2.50	-0.04	-0.85	-2.04	-2.19	-2.76	-4.98	-0.06	-6.06	-4.20	-4.27	-1.17	-2.33	-0.05	-2.63	-1.81	-1.95	-0.56
S2	Value of search when active non-employment	-1.37	-0.03	-0.66	-0.90	-1.46	-1.82	-4.57	-0.06	-5.88	-3.57	-3.94	-1.06	-1.67	-0.04	-2.30	-0.76	-1.45	-0.41
V3	Value of short-term job	0.15	-0.03	-0.66	-0.03	-0.55	0.19	-0.88	-0.06	-0.65	-0.21	-1.42	-0.13	0.64	-0.04	0.55	1.10	-0.26	0.21
V4	Value of long-term job	0.61	0.03	0.59	0.57	0.89	0.61	11.32	0.15	13.18	9.82	9.82	2.65	10.13	0.19	9.42	9.87	8.95	2.33
z	Flow value of non-employment	-1.00	0.96	0.24	-0.78	-0.73	-0.84	-1.00	0.98	-1.00	-1.00	-0.73	0.52	-1.00	0.96	-0.82	-1.00	-0.75	0.54
b	Flow value of searching	0.90	0.00	-0.01	1.00	0.34	0.41	-0.28	-0.01	-1.00	0.30	-0.26	-0.05	-0.34	-0.02	-1.00	0.52	-0.37	-0.10
w3	Wage in short-term job	0.93	0.95	0.00	0.76	0.00	0.95	0.52	0.95	0.94	0.95	0.00	0.95	0.57	0.95	0.55	0.95	0.00	0.95
$\phi 1$	Jobfinding rate from non-activated search	0.11	0.25	0.25	0.07	0.18	0.18	0.04	0.09	0.07	0.00	0.04	0.03	0.08	0.26	0.14	0.00	0.09	0.08
$\psi 2$	Jobfinding rate from activated OLF	0.38	0.21	0.21	0.39	0.27	0.28	0.04	0.01	0.01	0.07	0.04	0.05	0.08	0.01	0.04	0.13	0.07	0.07
<i>Prime-Age Men</i>																			
X1	Value of OLF when inactive non-employment	-2.35	-0.04	-0.04	-2.20	-1.12	-2.16	-4.96	-0.04	-5.44	-4.48	-1.02	-1.98	-5.51	-0.05	-5.56	-5.50	-4.70	-1.43
S1	Value of search when active non-employment	-3.00	-0.05	-0.05	-2.46	-1.35	-2.78	-5.32	-0.05	-6.03	-4.48	-1.05	-2.09	-5.91	-0.06	-6.39	-5.50	-5.04	-1.53
X2	Value of OLF when inactive non-employment	-3.06	-0.06	-0.06	-2.37	-1.36	-2.93	-5.16	-0.05	-5.96	-4.14	-1.04	-2.00	-4.85	-0.06	-5.87	-3.91	-4.16	-1.25
S2	Value of search when active non-employment	-2.01	-0.04	-0.04	-1.17	-0.95	-1.79	-4.57	-0.04	-5.44	-3.34	-0.94	-1.73	-4.23	-0.05	-5.56	-2.94	-3.65	-1.09
V3	Value of short-term job	-0.61	-0.04	-0.04	-0.72	-0.86	-0.07	0.22	-0.04	0.74	0.50	-0.84	0.22	-0.39	-0.05	-0.20	0.08	-0.99	-0.02
V4	Value of long-term job	0.52	0.02	0.02	0.50	0.47	0.29	5.10	0.06	5.82	3.81	1.30	1.91	11.27	0.13	11.68	10.79	9.79	2.90
z	Flow value of non-employment	-1.00	0.96	0.96	-0.88	0.02	-0.81	-1.00	0.98	-1.00	-1.00	0.48	0.19	-1.00	0.98	-0.99	-1.00	-0.73	0.49
b	Flow value of searching	0.53	0.00	0.00	1.00	0.24	0.54	0.19	0.00	-0.14	0.76	0.07	0.14	-0.19	-0.01	-1.00	0.62	-0.20	-0.04
w3	Wage in short-term job	0.31	0.95	0.95	0.18	0.00	0.95	0.52	0.95	0.95	0.95	0.00	0.95	0.52	0.95	0.68	0.95	0.00	0.95
$\phi 1$	Jobfinding rate from non-activated search	0.21	0.24	0.24	0.11	0.20	0.21	0.04	0.05	0.06	0.00	0.03	0.03	0.05	0.15	0.10	0.00	0.05	0.05
$\psi 2$	Jobfinding rate from activated OLF	0.33	0.28	0.28	0.41	0.35	0.29	0.08	0.07	0.06	0.13	0.11	0.09	0.06	0.02	0.03	0.10	0.06	0.06

Table 19: Centered Bellman Values