# Monetary Independence and Rollover Crises *

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**Abstract**

This paper shows that the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis. We study a sovereign default model with self-fulfilling rollover crises, foreign currency debt, and nominal rigidities. When the government lacks monetary independence, lenders anticipate that the government would face a severe recession in the event of a liquidity crisis, and are therefore more prone to run on government bonds. In a quantitative application, we find that the lack of monetary autonomy played a central role in making Spain vulnerable to a rollover crisis during 2011-2012. Finally, we argue that a lender of last resort can go a long way towards reducing the costs of giving up monetary independence.

**Keywords:** Sovereign Debt Crises, Rollover Risk, Monetary unions.
**JEL Codes:** E4, E5, F34, G15

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# 1 Introduction

A prominent concern of policy makers during the Eurozone crisis was the risk of a rollover crisis. The fear was that an adverse shift in market expectations would restrict governments' ability to roll over their debt, creating liquidity problems that would feed back into investors' expectations and ultimately lead governments to default. At the same time, the premise was that the lack of monetary independence was aggravating the sovereign debt crisis in Southern Europe and, as a result, there was an increased risk of a breakup of the monetary union.[1]

The goal of this paper is to investigate how the lack of monetary independence affects the vulnerability to a rollover crisis. In particular, does a country become more vulnerable after joining a monetary union? To tackle this question, we propose a theory of rollover crises and the interplay with the macroeconomic stabilization role of monetary policy.Our main theoretical result is that the lack of monetary independence increases the vulnerability to a rollover crisis. The key insight is that lenders' pessimism can trigger a demand driven recession, making the option to default more attractive and, in turn, validating the initial pessimism. A government that has monetary independence can alleviate the recession in the context of a rollover crisis, making investors less prone to run in the first place. Quantitative simulations show that while an economy that possess monetary independence is almost immune to a rollover crisis, it can become significantly vulnerable once it joins a monetary union. Moreover, we show that a lender of last resort can significantly mitigate the welfare costs from joining a monetary union.

The environment we consider is a version of the canonical model of sovereign default à la Eaton and Gersovitz, 1981 that incorporates the possibility of rollover crises, as in Cole and Kehoe (2000). The government issues debt before deciding on whether to repay or to default. When investors expect the government to default, the government is shut-down from credit markets and is forced to repay the maturing debt exclusively out of its tax revenues. When the maturing debt is large enough, investors expectations are validated and a self-fulfilling rollover crisis arises. We depart from this setup by considering nominal rigidities, which creates a scope for a stabilization role for monetary policy. External debt is denominated in real terms, or equivalently in foreign currency, eliminating the possibility of inflating away the debt. The model features tradable and non-tradable goods and downward nominal wage rigidity, as in Schmitt-Grohé and Uribe (2016). In this environment, a shock leading to a contraction in aggregate demand reduces the price of non-tradables in equilibrium, generating a decline in labor demand. When the decline in wages necessary to clear the market is too large, involuntary unemployment arises. A government with an independent monetary policy can

[1] On September 6th, 2012, Mario Draghi, the president of the European Central Bank, expressed that "the assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a 'bad equilibrium,' namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios." Preceding these remarks, Draghi famously pledged to do "whatever it takes to preserve the euro."

use the nominal exchange rate as a shock absorber, by altering real wages, and eliminate inefficient fluctuations in unemployment (Friedman, 1953).

Our main theoretical result is that the inability to conduct monetary policy stabilization increases the vulnerability to a rollover crisis. To understand the mechanisms in the model, consider what happens when a government is trying to roll-over its debt and investors suddenly panic and refuse to lend to the government. As the government is shut-down from credit markets, it needs to raise tax revenues and cut down on expenditures in order to service the maturing debt. In the presence of nominal rigidities and constraints on monetary policy, this situation has macroeconomic implications. The fiscal contraction generates a decline in aggregate demand, which leads to involuntary unemployment and makes repayment less attractive for the government. If the increase in unemployment is sufficiently large, the government finds optimal to default, which in turn validates the initial panic by investors and generates a self-fulfilling rollover crisis. Interestingly, for this pessimistic equilibrium to emerge, unemployment does not have to be realized in equilibrium. In fact, it is the off-equilibrium outcome of a large recession that pushes the government to default and triggers the rollover crisis. On the other hand, if the government can use monetary policy, the fiscal contraction does not have macroeconomic implications and the government's willingness to repay is relatively less affected when it is shut-down from credit markets. As a result, a panic is less likely to be triggered in the first place under monetary independence.

We also show that our main theoretical insight applies along several extensions of the baseline model. Among others, we show that the same results apply when the source of nominal rigidity is on prices rather than wages, when there are costs from depreciating the exchange rate, when the government follows a fixed exchange rate regime and finally when debt is denominated in domestic currency.

We then proceed to conduct a quantitative analysis to analyze the interplay between monetary policy and rollover crises. We start by considering a calibration of the model under monetary independence, in particular a flexible exchange rate regime. In this regime, the government implements the full-employment allocations by depreciating the currency, in line with the traditional argument for flexible exchange rates. (The government, however, cannot alter the value of the debt since it is denominated in foreign currency.) In a calibrated version of the model with flexible exchange rate, we find that rollover crises play a modest role. In fact, less than $1\%$ of default episodes are driven by rollover crises. Almost all defaults occur because of fundamental factors.

After we provide a theoretical characterization of the interplay between monetary policy and rollover crises, we proceed with a quantitative investigation. We start by considering a calibration of the model under monetary independence, in particular a flexible exchange rate regime. In this regime, the government implements the full-employment allocations by depreciating the currency, in line with the traditional argument for flexible exchange rates. (The government, however, cannot alter the value of the debt since it is denominated in foreign currency.) In a calibrated version of the model with flexible exchange rate, we find that rollover crises play a modest role. In fact, less than $1\%$ of default episodes are driven by rollover crises. Almost all defaults occur because of fundamental factors.

We then examine the effects of giving up monetary independence. One can think of a small open economy that has a fixed exchange rate regime, or equivalently, a single small economy within a monetary union in which wages (and the denomination of debt) are set in the currency of the union, and the conduct of monetary policy is exogenous to the single small economy. Keeping the same parameter values for the calibration of the flexible exchange rate regime, we find that the economy faces a significantly larger fraction of defaults due to rollover crisis, which can reach about 20% compared to less than 1% in the flexible exchange rate regime. The large quantitative difference also remains if we recalibrate the fixed exchange rate regime to match the same targets for debt and spreads as for the flexible exchange rate regime. Our quantitative findings therefore suggest that joining a monetary union entails significant cost in terms of a higher exposure to rollover crises.

Using the calibrated model for the fixed exchange rate regime, we then simulate the path of the Spanish economy starting at the time of the adoption of the euro. We find that the economy hits the "crisis zone" around 2012, in line with the turmoil in sovereign debt markets that occurred at the time. As a counterfactual, we then consider what the outcome would have been if Spain had exited the Eurozone in 2012. According to our model, the government would have remained immune to a rollover crisis, thanks to the ability to use monetary policy for macroeconomic stabilization. The goal of this exercise, however, is not to argue that being part of a monetary union is undesirable, but to uncover an important cost and at the same time help shed light on policies. In particular, we find that a lender of last resort goes a long way towards reducing the costs of joining a monetary union.

**Related literature.** Our paper contributes to a vast literature on monetary unions, pioneered by the seminal work of Mundell (1961). The traditional view is that the benefits of joining a monetary union are given by larger international trade, fostered by lower transaction costs. A more modern view, stressed by Alesina and Barro (2002), has emphasized the reduction in the inflationary bias generated by the time inconsistency problem of monetary policy from the seminal work of Barro and Gordon (1983). The main theme in the literature is that these benefits have to be traded-off against the losses from inefficient macroeconomic fluctuations due to nominal rigidities and the lack of monetary independence. A comprehensive discussion of these issues, which took center stage since the formation of the Eurozone, is provided in Alesina, Barro, and Tenreyro (2002), Silva and Tenreyro (2010) and De Grauwe (2018).

Our paper adds a new dimension to the costs from joining a monetary union: a higher exposure to rollover crises. Our welfare analysis shows that the new cost that we uncovered is substantial and suggests that an adequate evaluation of the overall net benefits should consider these costs. In this respect, our results shed some light on the Outright Monetary Transactions facility established by the European Central Bank (ECB) to purchase distressed country debt, following Mario Draghi's July 2012 speech pledging to do "whatever it takes to preserve the euro." Indeed, the paper shows that a lender of last resort can substantially reduce the costs for a country to remain in a monetary union.

Our paper also belongs to the literature on rollover crises in sovereign debt markets, starting with Sachs (1984), Alesina, Prati, and Tabellini (1990), and Cole and Kehoe (2000). Our formulation follows Cole-Kehoe, which has become the workhorse model in the quantitative sovereign default literature in the tradition of Aguiar and Gopinath (2006) and Arellano (2008). Examples include Chatterjee and Eyigungor (2012), Bocola and Dovis (2016), Roch and Uhlig (2018), Conesa and Kehoe (2017), and Aguiar, Chatterjee, Cole, and Stangebye (2016). Different from these contributions, we consider an economy with production and nominal rigidities, and establish how the exchange rate regime is central for the exposure to rollover crises. With a flexible exchange rate regime, we find the exposure to a rollover crisis to be minimal, which is in line with Chatterjee and Eyigungor (2012), who showed that in a canonical endowment economy model with long-term debt calibrated to the data, the presence of rollover crises has a negligible effect on debt and spreads. By contrast, we show that with a fixed exchange rate regime, the multiplicity region expands significantly, and the government is heavily exposed to a rollover crisis. [2]

The paper that is perhaps most closely related to ours is Aguiar, Amador, Farhi, and Gopinath (2013), who address the question of whether the government's ability to inflate away its debt reduces its exposure to rollover crises, an argument notably raised by De Grauwe (2013) and Krugman (2011) who made the observation that Spain and Portugal had higher levels of sovereign spreads compared to the UK, despite having lower levels of debt. Aguiar et al. consider an endowment economy with domestic currency debt and show that when commitment to low inflation is weak, an independent monetary policy can actually increase the vulnerability to a rollover crisis, contrary to De Grauwe and Krugman's argument. Our paper also studies how monetary policy matters for the exposure to a rollover crisis but considers instead a model with nominal rigidities and foreign currency debt. Our results show that the lack of monetary autonomy does increase the vulnerability to a rollover crisis and provides a new perspective that ascribes a role for monetary policy to deal with rollover crises,

[2] With one-year maturity, as in Cole and Kehoe (2000), the exposure to a rollover crisis is typically large because the government has to roll over a large amount of debt relative to output every period. While Cole and Kehoe (1996, 2000) were motivated by the 1994 Mexican crisis with maturity of less than a year, the typical maturity for sovereign bonds, is much larger, averaging around six years for the Eurozone. Conesa and Kehoe (2017) and Bocola and Dovis (2016) obtain a more moderate role for rollover crises, but implicitly rely on a minimum subsistence level for consumption, which they set to about 70 percent of income, in addition to calibrating to values of government debt close to 100 percent of GDP. Bocola and Dovis (2016) still find a limited role for rollover crisis even around the time in which the ECB's promise to buy government bonds effectively lead to a sharp reduction in sovereign spreads—notably this occurred without the ECB actually purchasing any bonds.

even when debt is denominated in foreign currency.[3]

A related literature studies sovereign debt crises, but in the tradition of Calvo (1988), in which the government lacks commitment to debt issuances. If investors expect high inflation, the government borrows at high rate and it finds optimal to inflate ex post, validating the initial expectations. In that model, the fact that debt is denominated in domestic currency and that the government can inflate away the debt is at the core of the fragility problem. [4] Our analysis abstract from these features to highlight a new channel by which monetary policy can actually help reduce a fragility problem that arises due to rollover crises.

Our paper is also related to an emerging literature, which incorporates nominal rigidities into the workhorse sovereign default model. Na, Schmitt-Grohé, Uribe, and Yue (2018) study a sovereign default model with downward nominal wage rigidity and show that it can account for the joint occurrence of large nominal devaluations and defaults, a phenomenon they dub the "twin Ds." Bianchi, Ottonello, and Presno (2018b) examine the trade-off between the expansionary effects of government spending and the increase in sovereign risk. Arellano, Mihalache, and Bai (2018) study the comovements of sovereign spreads with domestic nominal rates and inflation. The contribution of our paper to this literature is to consider an economy with rollover crises and examine how monetary policy affects the vulnerability to these episodes.

**Layout.** Section 2 presents the model, and Section 3 presents the theoretical analysis. Section 4 presents the quantitative analysis, and Section 5 concludes. The proofs are listed in Appendix A.

# 2 Model

We study a small open economy (SOE) model of endogenous sovereign default subject to rollover crises. The SOE is populated by households, firms, and a government. In the international financial markets, risk-neutral lenders buy the long-term government bonds of the SOE denominated in foreign currency. A single tradable good can be traded without any frictions, and as a result, the law of one

[3] Aguiar, Amador, Farhi, and Gopinath (2015) consider a setup similar to Aguiar, Amador, Farhi, and Gopinath (2013) but with multiple countries and a union-wide monetary policy. They show that for a country with high level of debt, it is preferable to join a monetary union with a mixed of high and low debt countries as a way to balance the costs from inflationary bias and the reduction in the vulnerability to rollover crises by inflating away the debt ex post. Although we focus on a single country, a likely implication of our analysis is that an optimal currency area would feature countries with similar debt positions, more in line with Mundell's criteria for optimal currency area. Other recent papers addressing issues of debt crises with a focus on the Eurozone are Gourinchas, Martin, and Messer (2017), De Ferra and Romei (2018), Broner, Erce, Martin, and Ventura (2014).

[4] A large literature on multiple equilibria follows this tradition: Corsetti and Dedola (2016) study the role of central bank backstop policies; Farhi and Maggiori (2017) study the implications for the international monetary system; Bacchetta, Perazzi, and Van Wincoop (2018) study conventional and unconventional monetary policy, building on a dynamic version of Calvo by Lorenzoni and Werning (2013). The role of inflation as partial default also plays a key role in recent work by Araujo, Leon, and Santos (2013), Du and Schreger (2016), Bassetto and Galli (2017), Nuño and Thomas (2017), Camous and Cooper (2018), and Hur, Kondo, and Perri (2018).

price holds. In addition, a non-tradable good in the SOE is produced using labor, and downward nominal wage rigidity creates the possibility of involuntary unemployment. We describe next the decision problems of households, firms, lenders, and the government.

## 2.1 Households

There is a unit measure of households with preferences over consumption given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, U(c_t), \tag{1}$$

with

$$c_t = C(c_t^T, c_t^N) = [\omega (c_t^T)^{-\mu} + (1-\omega)(c_t^N)^{-\mu}]^{-1/\mu}, \quad \omega \in (0,1), \ \mu > -1.$$

The utility function $U(c)$ is of the constant relative risk aversion (CRRA) form, where $c$ is a composite of tradable ($c^T$) and non-tradable goods ($c^N$), with constant elasticity of substitution (CES) equal to $1/(1+\mu)$.

Each period, households receive $y_t^T$ units of tradable endowment, which is stochastic and follows a stationary first-order Markov process. Assuming a constant unit price of tradable goods in terms of foreign currency, the value of the tradable good endowment in domestic currency is given by $e_t y^T$, where $e_t$ denotes the exchange rate measured as domestic currency per foreign currency (an increase in $e_t$ denotes a depreciation of the domestic currency). Households also receive firms' profits, which we denote by $\phi_t^N$, and labor income, $W_t h_t$, where $W$ is the wage expressed in domestic currency and $h$ is the amount of hours worked. Households inelastically supply $\overline{h}$ hours of work to the labor markets, but due to the presence of downward wage rigidity, they will work a strictly lower amount of hours when wage rigidity is binding. As we will discuss below, when wage rigidity is binding, the actual hours worked will be determined by firms' labor demand given prices and wages.

As is standard in the sovereign debt literature, we assume that households do not have direct access to external credit markets, although the government can borrow abroad and distribute the net proceedings to the households using lump-sum taxes or transfers. The households' budget constraint, expressed in domestic currency, is therefore given by

$$e_t c_t^T + P_t^N c_t^N = e_t y_t^T + \phi_t^N + W_t h_t - T_t, \tag{2}$$

where $P_t^N$ denotes the price of non-tradables in domestic currency, and $T_t$ denotes lump-sum taxes/transfers in units of domestic currency.

The households' problem consists of choosing $c_t^T$ and $c_t^N$ to maximize (1) given the sequence of prices for non-tradables $\{P_t^N\}$, labor income $\{W_t h_t\}$, profits $\{\phi_t^N\}$, and taxes $\{T_t\}$. The static optimality condition equates the relative price of non-tradables to the marginal rate of substitution between tradables and non-tradables.

$$\frac{P_t^N}{e_t} = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{1+\mu}. \tag{3}$$

Because preferences are homothetic, as a result of the assumptions of the CRRA utility function and the CES consumption aggregator, the relative consumption of tradable to non-tradable consumption is only a function of the relative price.

## 2.2 Firms

Firms operate a production function $y^N = F(h)$ where $y_t^N$ denotes the output of non-tradable goods, and $h_t$ denotes employment, the sole input. The production function $F(\cdot)$ is a differentiable, increasing, and concave function. In particular, we will consider a homogeneous production function $F(h) = h^\alpha$ where $\alpha \in (0, 1]$.

Firms operate in perfectly competitive markets, and each period they maximize profits that are given by

$$\phi_t^N = \max_{h_t} P_t^N F(h_t) - W_t h_t. \tag{4}$$

The optimal choice of labor employment $h_t$ equates the value of the marginal product of labor to the nominal wage:

$$P_t^N F'(h_t) = W_t. \tag{5}$$

Given the price of non-tradables, a higher wage leads to lower employment. Likewise, given the wage, a lower price of non-tradables leads to lower employment. As we will see below, how the price of non-tradables reacts in general equilibrium will have important implications for debt crises.

### 2.2.1 Downward Nominal Wage Rigidity

We model downward nominal wage rigidity, following Schmitt-Grohé and Uribe (2016). For an economy that is outside a currency union, we assume, in particular, that wages in domestic currency cannot fall below $\overline{W}$.[5]

$$W_t \geq \overline{W} \tag{6}$$

for all $t$.

The parameter $\overline{W}$ determines the severity of the wage rigidity.[6] There are two cases. If the nom-

[5]There is a large empirical evidence on downward wage rigidity. In particular, a recent literature has used micro level data to highlight the important role that this friction played in the European crisis (e.g. Faia and Pezone (2018); Ronchi and Di Mauro (2017)).

[6]In Schmitt-Grohé and Uribe (2016), $\overline{W}$ depends on the previous period wage and a parameter that controls the speed of wage adjustment. For numerical tractability, we take $\overline{W}$ as an exogenous (constant) value, as in Bianchi et al. (2018b). Notice also that allowing for indexation to CPI inflation would not affect our theoretical mechanism because a nominal exchange rate depreciation in a state with unemployment would lead to a real exchange rate depreciation, and the price of non-tradables in domestic currency would rise by more than the increase in wages due to indexation.

inal wage that clears the labor market is higher than $\overline{W}$, the economy is at full employment and (6) is not binding. If, however, the nominal wage that would clear the market is below $\overline{W}$, the economy experiences involuntary unemployment. In this case, the amount of employment in equilibrium is determined by the amount of labor demand (5), and households work strictly less than their endowment of hours. Formally, wages and employment need to satisfy the following slackness condition:

$$\left(W_t - \overline{W}\right)\left(\overline{h} - h_t\right) = 0. \tag{7}$$

For an economy within a currency union, wages are set in foreign currency (the currency of the union) and the lower bound is therefore also in foreign currency. As we will see, in a fixed exchange rate, the wage becomes effectively rigid in foreign currency.

### 2.3 Government

The government can issue a non-contingent long-term bond and can default at any point in time. As in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), a bond issued in period $t$ promises an infinite stream of coupons that decrease at an exogenous constant rate $1-\delta$.[7] In particular, a bond issued in period $t$ promises to pay $\delta(1-\delta)^{j-1}$ units of the tradable good (or foreign currency) in period $t+j$, for all $j \geq 1$. Hence, debt dynamics can be represented by the following law of motion:

$$b_{t+1} = (1-\delta)b_t + i_t, \tag{8}$$

where $b_t$ is the stock of bonds due at the beginning of period $t$, and $i_t$ is the stock of new bonds issued in period $t$.

Debt contracts cannot be enforced. If the government chooses to default, it faces two punishments. First, the government switches to financial autarky and cannot borrow for a stochastic number of periods. Second, there is a utility loss $\kappa(y_t^T)$, assumed to be increasing in tradable income. We think of this utility loss as capturing various default costs related to reputation, sanctions, or the misallocation of resources.[8]

The government's budget constraint in a period starting with good credit standing is

$$\delta e_t b_t(1-d_t) = T_t + e_t q_t i_t(1-d_t), \tag{9}$$

[7] We take maturity as a primitive. There is an active literature studying maturity choices in sovereign default models (Arellano and Ramanarayanan, 2012; Bocola and Dovis, 2016; Sanchez, Sapriza, and Yurdagul, 2018).

[8] Utility losses from default in sovereign debt models are also used by Bianchi, Hatchondo, and Martinez (2018a) and Roch and Uhlig (2018), among others. An alternative often used is the output costs from default. If the utility function is log over the composite consumption, and output losses from default are proportional to the composite consumption in default, the losses from default would be identical across the two specifications. In any case, as it will become clear below, what will be important for our mechanism is the difference in the values of repayment when investors lend and when they refuse to lend, but not the explicit form of default cost.

where $d_t = 0(1)$ if the government repays (defaults) and $q(\cdot)$ denotes the price schedule, which we will characterize below. The budget constraint indicates that repayment of outstanding debt obligations is made by collecting lump-sum taxes and issuing new debt.[9]

The timing within each period follows Cole and Kehoe (2000). At the beginning of each period, the government has outstanding debt liabilities $b_t$ and could be in good or bad credit standing. If the government is in good credit standing, it chooses new debt issuances at the price schedule offered by investors. At the end of each period, the government decides whether to default or repay the initial debt outstanding. The difference with respect to Eaton and Gersovitz (1981) that will give rise to multiplicity is that here the government does not have the ability to commit to repaying within the period.[10] As we will see, negative beliefs about the decision of the government to default can become self-fulfilling.

**Monetary regimes.** Regarding the policy for exchange rates, we will consider two regimes: a flexible exchange rate and a fixed exchange rate. In the flexible exchange rate regime, the government will choose the optimal exchange rate at all dates without commitment. In the fixed exchange rate regime, we assume that the government fixes the exchange rate to an exogenous level $e = \bar{e}$ at all times. One can also think about a fixed exchange rate as the policy of a single economy that enters a monetary union and gives up its currency, in which case wages would be directly denominated in the foreign currency. These cases are equivalent because the government is unable to alter the value of the currency is vis-à-vis the rest of the world.

## 2.4 International Lenders

Sovereign bonds are traded with atomistic, risk-neutral foreign lenders. In addition to investing through the defaultable bonds, lenders have access to a one-period risk-free security paying a net interest rate $r$. By a no-arbitrage condition, equilibrium bond prices when the government repays are then given by

$$q_t = \frac{1}{1+r}\mathbb{E}_t[(1-d_{t+1})(\delta + (1-\delta)q_{t+1})]. \tag{10}$$

[9] We consider only lump-sum taxes and transfers and abstract from fiscal instruments such as specific taxes on consumption or payroll subsidies that could mimic a nominal depreciation, as studied in Farhi, Gopinath, and Itskhoki (2013) and Schmitt-Grohé and Uribe (2016). As long as there are some limits (either political or economic) to the use of these fiscal instruments that prevent the government from reaching full employment, our main results will continue to hold. As we will explain in the calibration section, we will calibrate the model to match the observed increase in unemployment, and so implicitly this captures that the government could in practice be using these policies to some extent.

[10] A different source of multiplicity following Calvo (1988) arises if the government has to issue a fixed amount of debt revenues. In this case, the fact that bond prices decrease with debt generates a Laffer curve, which leads, directly through the budget constraint, to a high debt/spreads equilibrium and a low debt/spreads equilibrium. Lorenzoni and Werning (2013) explore this kind of multiplicity in a dynamic context with fiscal rules and long-term maturity and show how this gives rise to "slow moving debt crises" (see also Ayres, Navarro, Nicolini, and Teles, 2016).

Equation (10) indicates that in equilibrium, an investor has to be indifferent between investing in a risk-free security and buying a government bond at price $q_t$, bearing the risk of default. In case of repayment next period, the payoff is given by the coupon $\delta$ plus the market value $q_{t+1}$ of the nonmaturing fraction of the bonds. Since we assume no recovery, the bond price is zero in the event of default.

## 2.5 Equilibrium

In equilibrium, the market for non-tradable goods clears:

$$c_t^N = F(h_t). \tag{11}$$

In addition, using the households' and government budget constraint (2) and the definition of the firms' profits and market clearing condition (11), we obtain the resource constraint for tradable goods in the economy:

$$c_t^T = y_t^T + (1 - d_t)[\delta b_t - q_t(b_{t+1} - (1 - \delta)b_t)]. \tag{12}$$

Before proceeding to study a Markov equilibrium in which the government chooses policies optimally without commitment, let us examine equilibrium for given government policies. A competitive equilibrium given government policies in our economy is defined as follows:

**Definition 1** (Competitive Equilibrium)**.** Given an initial debt $b_0$, an initial credit standing, government policies $\{T_t, b_{t+1}, d_t, e_t\}_{t=0}^{\infty}$ , and an exogenous process for the tradable endowment $\{y_t^T\}_{t=0}^{\infty}$ and for reentry after default, a *competitive equilibrium* is a sequence of allocations $\{c_t^T, c_t^N, h_t\}_{t=0}^{\infty}$ and prices $\{P_t^N, W_t, q_t\}_{t=0}^{\infty}$ such that:

1. Households and firms solve their optimization problems.
2. Government policies satisfy the government budget constraint (9).
3. The bond pricing equation (10) holds.
4. The market for non-tradable goods clears (11), and the resource constraint for tradables (12) holds.
5. The labor market satisfies conditions (6), (7), and $h \leq \overline{h}$.

**Employment, Consumption, and Wages** Using market clearing for non-tradable goods (11), together with the optimality conditions for households (3) and firms (5), we can obtain a useful (partial) characterization of equilibrium in a system of these three static equations and three variables

$(c_t^T, h_t, w_t)$, where $w_t \equiv W_t/e_t$ denotes the wage denominated in tradable goods, assuming a constant unitary price of the tradable good in terms of foreign currency. Using this system of equations, we can then derive in every period a real equilibrium wage solely as a function of $(c_t^T, h_t)$.

**Lemma 1.** *In any equilibrium, the real wage in terms of tradable goods is a function of tradable consumption and employment,*

$$\mathcal{W}(c_t^T, h_t) \equiv \frac{1-\omega}{\omega}\left(\frac{c_t^T}{F(h_t)}\right)^{1+\mu} F'(h_t). \tag{13}$$

*Moreover, $\mathcal{W}(c_t^T, h_t)$ is increasing with respect to $c_t^T$ and decreasing with respect to $h_t$.*

One implication of Lemma 1, which will be importance once we turn to the determination of the entire dynamic equilibrium, is that an increase in the amount of tradable consumption is associated with a higher equilibrium wage. This occurs because higher tradable consumption is associated in equilibrium with a higher relative price of non-tradables, which in turn leads to a larger demand for labor and an increase in the real wage for a given level of employment. In addition, an increase in employment is associated in equilibrium with a lower real wage, to be consistent with firms' labor demand.

In equilibrium, we then have that downward nominal wage rigidity can be expressed as

$$\mathcal{W}(c_t^T, h_t)e_t \geq \overline{W}. \tag{14}$$

According to this lemma, we then have that if (14) is binding, a reduction in the amount of tradable consumption is associated with low employment in equilibrium. This result has important implications for the general equilibrium effects in the full dynamic system. If a shock reduces the demand for total consumption, we must have that for a given level of non-tradable output, the price of non-tradables needs to decline so that households switch consumption from tradables toward non-tradables and the market for non-tradable goods clears. Absent wage rigidity, we would have that the wage falls, and the only implication for the real economy is the reduction in tradable consumption. However, if wages are downwardly rigid, the decline in the relative price of non-tradables will lead to a decline in employment.

Based on Lemma 1, we can also analogously construct an equilibrium employment that is a function of $c_t^T$ and $\overline{w_t} \equiv \overline{W}/e_t$.

**Lemma 2.** *In any equilibrium, employment is given by*

$$\mathcal{H}(c_t^T, \overline{w_t}) = \begin{cases} \left[\frac{1-\omega}{\omega}\left(\frac{\alpha}{\overline{w_t}}\right)\right]^{\frac{1}{1+\alpha\mu}} \left(c_t^T\right)^{\frac{1+\mu}{1+\alpha\mu}} & \mathit{if}\, c_t^T \leq \overline{c}_{\overline{w_t}}^T, \\ \overline{h} & \mathit{if}\, c_t^T > \overline{c}_{\overline{w_t}}^T \end{cases}, \tag{15}$$

*where*

$$\overline{c}^T_{\overline{w}_t} = \left[\frac{\omega}{1-\omega}\left(\frac{\overline{w}_t}{\alpha}\right)\right]^{\frac{1}{1+\mu}} \left(\overline{h}\right)^{\frac{1+\alpha\mu}{1+\mu}} .$$

This condition implies that when the wage rigidity is binding and there is unemployment, the government will realize that repaying debt and cutting back on consumption will create more unemployment. We will see below how the implied increase in the cost of repayment affects the vulnerability to a rollover crisis.

## 2.6 Recursive Government Problem

We consider the optimal policy of a benevolent government with no commitment, which chooses consumption and external borrowing to maximize households' welfare, subject to the implementability conditions. We focus on the Markov equilibria.

Every period in which the government enters with access to financial markets, it evaluates the lifetime utility of households if debt contracts are honored against the lifetime utility of households if they are repudiated. We use $\mathbf{s} = (y^T, \zeta)$ to denote the vector of exogenous states in every period. The variable $\zeta$ is a sunspot variable to index for the possibility of multiplicity of equilibria, as in Cole and Kehoe (2000), which we will describe below. Different from the equilibrium according to the timing in Eaton and Gersovitz (1981), the possibility of a rollover crisis implies that the bond price is a function of the initial debt position and the sunspot, in addition to the debt choice and current income shock.

Regarding the policy for exchange rates, we will start with the case in which the government is under a fixed exchange rate regime. That is, the exchange rate is fixed at an exogenous level $e = \overline{e}$ for every period. We can define a *real wage rigidity* constraint as $w \geq \overline{w}$ where $\overline{w} \equiv \overline{W}/\overline{e}$ and $w \equiv W/e$. We can then rewrite (14) as $\mathcal{W}(c^T, h) \geq \overline{w}$. Later on, we will study the case in which we allow the government to depreciate its currency. As should be clear from (14), an exchange rate depreciation will be able to undo the wage rigidity, and this will be the optimal policy for the government.

The government problem with access to financial markets can be formulated in recursive form as follows:

$$V(b, \mathbf{s}) = \max_{d \in \{0,1\}} \{(1-d)V_R(b, \mathbf{s}) + dV_D(y^T)\}, \tag{16}$$

where $V_R(b, \mathbf{s})$ and $V_D(y^T)$ denote, respectively, the values of repayment and default.

The value of repayment is given by the following Bellman equation:

$$V_R(b, \mathbf{s}) = \max_{b', c^T, h \leq \bar{h}} \left\{ u(c^T, F(h)) + \beta \mathbb{E} V(b', \mathbf{s}') \right\} \tag{17}$$

subject to

$$c^T = y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta) b)$$

$$\mathcal{W}(c^T, h) \geq \overline{w},$$

where $q(b', b, \mathbf{s})$ denotes the debt price schedule, taken as given by the government, and $\mathcal{W}$ is defined in (13).[11] Meanwhile, the value of default is given by

$$V_D(y^T) = \max_{c^T, h \leq \bar{h}} \left\{ u\left(c^T, F(h)\right) - \kappa(y^T) + \beta \mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi) V_D(y^{T\prime})\right] \right\} \tag{18}$$

subject to

$$c^T = y^T$$

$$\mathcal{W}(c^T, h) \geq \overline{w},$$

where $\psi \in [0,1]$ is the probability of reentering financial markets after a default.

Let $\left\{ \hat{d}(b, \mathbf{s}), \hat{c}^T(b, \mathbf{s}), \hat{b}(b, \mathbf{s}), \hat{h}(b, \mathbf{s}) \right\}$ be the optimal policy rules associated with the government problem. A Markov-perfect equilibrium is then defined as follows.

**Definition 2** (Markov-perfect equilibrium)**.** A Markov-perfect equilibrium is defined by value functions $\{V(b, \mathbf{s}), V_R(b, \mathbf{s}), V_D(y^T)\}$, policy functions $\{\hat{d}(b, \mathbf{s}), \hat{c}^T(b, \mathbf{s}), \hat{b}(b, \mathbf{s}), \hat{h}(b, \mathbf{s})\}$, and a bond price schedule $q(b', b, \mathbf{s})$ such that

1. Given the bond price schedule, policy functions solve problems (16), (17), and (18),

2. The debt price schedule satisfies

$$q(b', b, \mathbf{s}) = \begin{cases} \frac{1}{1+r} \mathbb{E}[(1-d')(\delta + (1-\delta) q(b'', b', \mathbf{s}'))] & \text{if } \hat{d}(b, \mathbf{s}) = 0, \\ 0 & \text{if } \hat{d}(b, \mathbf{s}) = 1 \end{cases},$$

   where

$$\begin{aligned} b'' &= \hat{b}(b', \mathbf{s}') \\ d' &= d(b', \mathbf{s}'). \end{aligned}$$

[11] An equivalent representation uses equilibrium employment (15) rather than the explicit wage rigidity constraint.

For the economy with a flexible exchange rate, the only difference would be that the government chooses $e$ in addition to the prices and allocations that are chosen under the fixed exchange rate regime.

## 2.7 Multiplicity of Equilibrium

As in Cole and Kehoe (2000), the government is subject to self-fulfilling rollover crises. Let us define the debt price schedule, assuming there will be no default and the break-even condition of lenders is satisfied. We will call this the *fundamental* debt price schedule:

$$\tilde{q}(b', y^T) \equiv \frac{1}{1+r}\mathbb{E}[(1-d')(\delta + (1-\delta)q(b'', b', \mathrm{s}'))], \tag{19}$$

where $b'' = \hat{b}(b', \mathrm{s}')$ and $d' = d(b', \mathrm{s}')$. This debt price schedule does not depend on the sunspot nor on the current amount of debt held by the government. Using this price schedule, we can construct the value of repayment when international lenders believe that the government will honor its debt commitments at the end of the period and therefore extend credit to the government. This value is as follows:

$$V_R^+(b, y^T) = \max_{b', c^T, h \leq \bar{h}} \left\{u(c^T, F(h)) + \beta\mathbb{E}V(b', \mathrm{s}')\right\} \tag{20}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b + \tilde{q}(b', y^T)[b' - (1-\delta)b]$$

$$\mathcal{W}(c^T, h) \geq \overline{w}.$$

Denote by $\hat{b}^+(b, y^T)$ the solution to the previous problem. Divide the state space where the government finds it optimal to issue strictly positive amounts of debt:

$$\mathcal{B} \equiv \left\{(b, y^T) \in \mathbb{R} \times \mathbb{R}_+ : \quad \hat{b}^+(b, y^T) > (1-\delta)b\right\}.$$

Consider now the case in which investors are unwilling to lend to the government. Denote by $V_R^-$ the value function in this case, when the government decides to repay. If $(b, y^T) \notin \mathcal{B}$, we have that $V_R^-(b, y^T) = V_R^+(b, y^T)$, as the government is not issuing debt even when investors are willing to lend to the government. Then, if $(b, y^T) \in \mathcal{B}$, the value is given by

$$V_R^-(b, y^T) = \max_{c^T, h \leq \bar{h}} \left\{ u(c^T, F(h)) + \beta \mathbb{E} V((1-\delta)b, \mathbf{s}') \right\} \tag{21}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b$$

$$\mathcal{W}(c^T, h) \geq \overline{w}.$$

Lemma 3 states that the value of repayment when lenders refuse to roll over government bonds is never greater than the value when lenders are willing to roll over. This must be the case since the government can always choose not to borrow when lenders are willing to roll over.[12]

**Lemma 3.** *For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level $b \in \mathbb{R}$, we have that $V_R^+(b, y^T) \geq V_R^-(b, y^T)$.*

An important implication of the fact that tradable consumption is lower when the government does not have access to borrowing, is that the wage rigidity constraint wil become binding for lower levels of debt, in line with Lemma 1. As a result, the presence of wage rigidity will have a stronger effect $V_R^-$ than $V_R^+$ and lead to an increase the gap between these two values. As we will see below, this will have important implications for the occurrence of self-fulfilling rollover crises.

**Three zones.** Let us separate the state space $(b, y^T)$ into three zones: the safe zone, default zone, and crisis zone. The *safe zone* will denote those states in which the government finds it optimal to repay its debt even if international lenders are not willing to issue more debt to the government. That is,

$$\mathcal{S} \equiv \left\{ (b, y^T) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^-(b, y^T) \right\}.$$

The *default zone* defines those states in which the government finds it optimal to default even if international lenders are willing to lend at the fundamental debt price schedule. That is,

$$\mathcal{D} \equiv \left\{ (b, y^T) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) > V_R^+(b, y^T) \right\}.$$

Finally, the *crisis zone* will correspond to those states in which the government finds it optimal to repay if investors are willing to lend at the fundamental debt price schedule, but finds it optimal to

[12] One element implicit here is that if the government were to try to repurchase debt when investors are unwilling to lend, the price of bonds would rise to the fundamental price, and hence the budget constraint when $(b, y^T) \in \mathcal{B}$ would be $c^T = y^T - \delta b$, as reflected in (21). See Aguiar and Amador (2013) and Bocola and Dovis (2016) for an elaboration of this point.

default if investors are not willing to lend. That is,

$$\mathcal{C} \equiv \left\{(b, y^T) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_R^+(b, y^T) > V_D(y^T) > V_R^-(b, y^T)\right\}.$$

In this zone, the outcome is undetermined and depends on investors' beliefs. If investors believe the government will repay, the government will find it optimal to repay whereas if they believe that the government will default, the government will default. To select an equilibrium, we will use a sunspot $\zeta \in \{0, 1\}$. If $\zeta = 0$, we will say there is a "good sunspot", in which case the equilibrium with repayment is selected. If $\zeta = 1$, we will say there is a "bad sunspot", in which case the equilibrium with default is selected. We assume that $\zeta$ follows an *i.i.d.* process with probability $\pi$ of selecting $\zeta = 1$.

Following these definitions, the optimal binary default decision and the optimal debt price schedule will satisfy

$$d(b, \mathbf{s}) = \begin{cases} 1 & \text{if } (b, y^T) \in \mathcal{D} \\ 1 & \text{if } (b, y^T) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ 0 & \text{if } (b, y^T) \in \mathcal{C} \quad \& \quad \zeta = 0 \\ 0 & \text{if } (b, y^T) \in \mathcal{S} \end{cases} \tag{22}$$

$$q(b', b, \mathbf{s}) = \begin{cases} 0 & \text{if } (b, y^T) \in \mathcal{D} \\ 0 & \text{if } (b, y^T) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ \tilde{q}(b', y^T) & \textit{in every other case} \end{cases}. \tag{23}$$

# 3 Theoretical Analysis

In this section, we provide an analytical characterization of how monetary policy and downward nominal wage rigidity affect the government's vulnerability to a rollover crisis. The central point we will show is that fixing the exchange rate leaves a government more vulnerable to a rollover crisis. Simply put, the crisis zone will be larger for an economy with a fixed exchange rate.

## 3.1 Flexible Exchange Rate Regime

The flexible exchange rate regime allows the government to pick any nominal exchange rate every period. The following proposition characterizes the optimal exchange rate policy.

**Proposition 1** (Optimal Exchange Rate Policy)**.** *Under a flexible exchange rate regime, the government chooses an exchange rate that delivers full employment in all states.*

This proposition establishes that the government finds it optimal to choose an exchange rate that delivers full employment, a result that can be seen from the value functions (17) and (18). If there was unemployment in the economy, the government could always relax the wage constraint by sufficiently depreciating the nominal exchange rate without bearing any other costs. This basic result is of course in line with the traditional benefit of having a flexible exchange rate in the presence of nominal rigidities, going back to Friedman (1953) and Mundell (1961). One difference here is that to ensure full employment, the government needs to depreciate the currency not only on-equilibrium but also off-equilibrium.

It is worth pointing out that while we do not explicitly model why the government would fix the exchange rate or join a monetary (and therefore depart from the optimal exchange rate policy), doing so, in practice, offers a number of well-studied benefits. The gains could arise, for example, from lower inflationary bias (Alesina and Barro, 2002, Barro and Gordon, 1983) or from improvements in trade due to lower volatility and transaction costs (Mundell, 1961, Frankel and Rose, 2002). Following a large literature on monetary unions, we do not model explicitly these gains. We will show, however, that the main theoretical results to be presented below will continue to hold in the presence of an exogenous specified costs from altering the nominal exchange rate.[13]

## 3.2 Uncovering the Role of Rigidities and Monetary Policy

In this section, we study how wage rigidity and the exchange rate regime shape default decisions and the exposure to a rollover crisis. As a starting point, we consider the impact of a temporary change in the degree of wage rigidity. To fix ideas, we think about a current tightening of the wage rigidity. Since $\overline{w} = \frac{\overline{W}}{e}$ and the , the tigthening that can arise either because of an increase in $\overline{W}$, or a more appreciated exchange rate, or some combination of the two.[14] We will later study the consequences of permanent changes in the exchange rate regime, but studying a temporary change in wage rigidity is useful because it allows us to isolate current changes in monetary policy while leaving constant future policies. Moreover, one implication of assuming that the change in the exchange rate regime is only for one period is that the *fundamental* price schedule remains the same. This is because continuation values do not change, and hence future default decisions also remain unchanged.

Denote the current value functions with the one-period change in wage rigidity $\overline{w}$ as $\tilde{V}_D(y^T;\overline{w})$, $\tilde{V}_R^+(b, y^T;\overline{w})$, $\tilde{V}_R^-(b, y^T;\overline{w})$. We maintain the same notation for continuation values $V$ and the bond price $q$, which correspond to the value functions and bond price schedule of the Markov equilbrium we defined above. To fix ideas, one can think about this continuation values being associated with the economy

[13]It remains a topic for future research to explicitly model the gains from joining a monetary union studied in the literature and the new cost identified in this paper.

[14]Alternatively, one can think of a decrease in the price of tradables in foreign currency $P^*$, which we have assumed constant and normalized to one for simplicity. In this case, by the law of one price we have $\overline{w} = \frac{\overline{W}}{eP^*}$ and a reduction in $P^*$ would have the same tightening effects.

with flexible wage case. What is important in this comparative static exercise is that we change the current wage rigidity ($\overline{w}_t$) leaving the $t+1, t+2....$ rigidity constant, and so our results apply for any arbitrary continuation Markov equilibrium, including the one under flexible exchange rate. The problem the government faces when there is a temporary change in wage rigidity can then be expressed as:

$$\tilde{V}_D(y^T;\overline{w}) = \max_{c^T, h\leq\bar{h}} \left\{u\left(c^T,h\right) - \kappa(y^T) + \beta\mathbb{E}\left[\psi V(0,\mathbf{s}') + (1-\psi)V_D(y^{T\prime})\right]\right\} \tag{24}$$

$$\text{subject to}$$

$$c^T = y^T,$$

$$\mathcal{W}(c^T,h) \geq \overline{w},$$

$$\tilde{V}_R^+(b,y^T;\overline{w}) = \max_{b',c^T,h\leq\bar{h}} \left\{u(c^T,F(h)) + \beta\mathbb{E}V(b',\mathbf{s}')\right\} \tag{25}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b + \tilde{q}(b',y^T)(b'-(1-\delta)b),$$

$$\mathcal{W}(c^T,h) \geq \overline{w},$$

and

$$\tilde{V}_R^-(b,y^T;\overline{w}) = \max_{c^T,F(h)\leq\bar{h}} \left\{u(c^T,h) + \beta\mathbb{E}V((1-\delta)b,\mathbf{s}')\right\} \tag{26}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b,$$

$$\mathcal{W}(c^T,h) \geq \overline{w}.$$

In order to show how the three zones (safe, default, and crisis) are affected by the nominal rigidity, we first present some useful properties.

**Lemma 4.** *The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect to debt $b$.*

**Lemma 5** (Debt Thresholds)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exist levels of debt $\bar{b}^+,\bar{b}^- \in \mathbb{R}_+$ such that $\tilde{V}_D(y^T) = V_R^+\left(\bar{b}^+,y^T\right)$ and $\tilde{V}_D(y^T) = V_R^-\left(\bar{b}^-,y^T\right)$. Furthermore, $\bar{b}^+ \geq \bar{b}^-$.*

The previous lemmas help us to construct the three zones into intervals conditional on a given level of tradable endowment. Lemma 4 says that the repayment value functions are strictly decreasing with respect to current debt. Using this result and the fact that the value of default is independent of debt, Lemma 5 states that the threshold at which the government is indifferent between repaying and defaulting depends on whether investors are willing to lend or not. In particular, the amount of debt in which the government is indifferent between repaying or defaulting when investors are willing to lend is lower than the amount of debt in which the government is indifferent between repaying or defaulting when investors refuse to lend.

Using these results, we can construct a safe region, a crisis region and a default region for every level of income:

$$\tilde{S}(y^T) \equiv \left(-\infty, \bar{b}^-\right], \qquad \tilde{C}(y^T) \equiv \left(\bar{b}^-, \bar{b}^+\right], \qquad \text{and} \qquad \tilde{D}(y^T) \equiv \left(\bar{b}^+, \infty\right).$$

Next we study now how these regions expand or contract as the real wage rigidity increases.[15]

**Proposition 2** (Default Region Expansion)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{D}(y^T; \overline{w}_1) \subseteq \tilde{D}(y^T; \overline{w}_2)$.*

**Proposition 3** (Safe Region Contraction)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{S}(y^T; \overline{w}_2) \subseteq \tilde{S}(y^T; \overline{w}_1)$.*

Proposition 2 tells us that if the government defaults for a given $\overline{w}$, it will also default for a higher wage rigidity. Likewise, Proposition 3 tells us that if the government is in the safe zone for a given $\overline{w}$, it will remain in the safe zone for a looser wage rigidity. The essence of these two propositions is that the value of repayment is decreasing in $\overline{w}$, whereas the value of default does not respond to $\overline{w}$ provided that wage rigidity is not too tight.

Movements in the crisis region are not as straightforward as they are in the other two regions. If the safe region shrinks, the crisis region increases and the economy arrives at the crisis region with lower levels of debt. At the same time, if the default region expands, then the crisis region decreases. Nevertheless, we are able to provide a sharp result that establishes the conditions under which the crisis region expands with higher rigidity.

**Proposition 4** (Crisis Region Expansion)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists $\overline{w}_C \in \mathbb{R}_+$ such that if $\overline{w}_1, \overline{w}_2 < \overline{w}_C$ and $\overline{w}_1 < \overline{w}_2$, then $\tilde{C}(y^T; \overline{w}_1) \subseteq \tilde{C}(y^T; \overline{w}_2)$. Moreover, there exists $\overline{w}_S \in \mathbb{R}_+$ such that if $\overline{w}_2 > \overline{w}_S$, then $\tilde{C}(y^T; \overline{w}_1) \subset \tilde{C}(y^T; \overline{w}_1)$*

Proposition 4 establishes that starting from a low $\overline{w}$, an increase in the real wage rigidity makes the crisis region weakly increase. Key for this result is that starting from full employment, a small increase in wage rigidity first affects the safe zone, thereby increasing the crisis region, and only after a sufficiently large increase does the default region start to increase. Moreover, we are able to show that for a sufficiently high increase in rigidity up to some level, the crisis region increases unequivocally.

[15]Different from the "zones" constructed above which are in the $(b, y^T)$ space, the "regions" fix the level of tradable endowment.

## 3.3 Graphical Illustration

Following the theoretical analysis above, this section provides a graphical illustration of how wage rigidity tends to increase the vulnerability to a rollover crisis. To construct the following figures, we use the calibrated version of our model, which we will explain in the quantitative section.

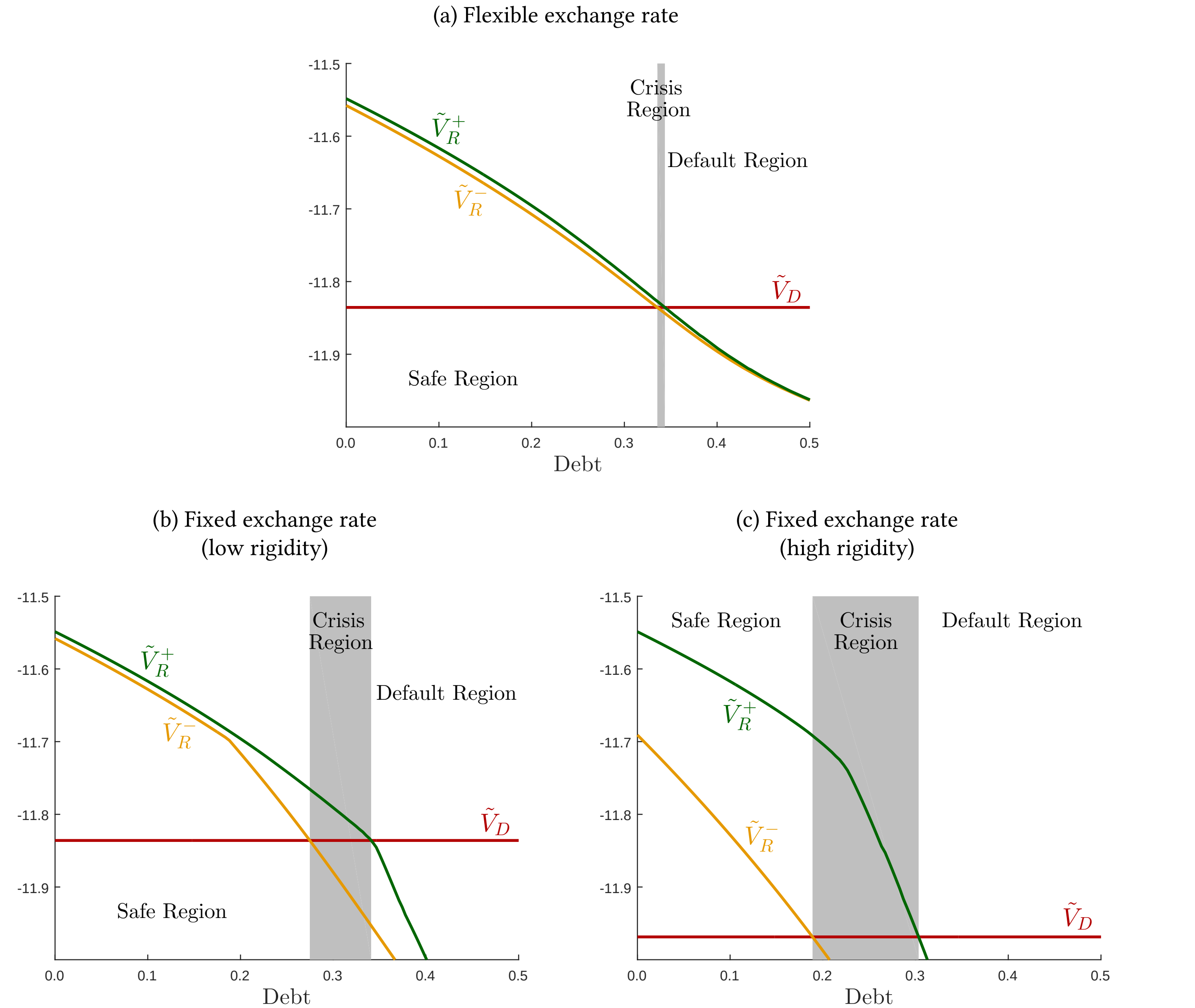


Figure 1: Value Functions and Crisis Regions

Notes: The income shock in the three panels is set to $-4.3\%$ below the mean, which is the average income shock before a default episode in the flexible exchange regime. Panel (a) uses parameter values from the calibrated flexible exchange rate economy. Panels (b) and (c) use the same parameters with the exception of the current level of wage rigidity $\overline{w}$. In panel (a), $\overline{w}$ is set to its highest value where full employment is achieved under a good sunspot. This is 1.33 times the real wage in the flexible exchange rate regime. Panel (c) increases the wage rigidity to 1.66 times higher than the wage in the flexible exchange rate regime.

In Figure 1 we present the values $\left\{\tilde{V}_D, \tilde{V}_R^+, \tilde{V}_R^-\right\}$ for different levels of debt. We fix the tradable endowment to the average value in default episodes in our simulation exercise for the flexible exchange rate regime (technically, the element in the grid that is closest to this point). This level is 4.3% below average. To facilitate the reading of the figures, we normalize debt by average GDP. Unless we specify otherwise, all numbers reported will be expressed in this way. Notice that in Figure 1, the actual value function $V$, as defined in (16), is given by the upper envelope of $\tilde{V}_D$ and $\tilde{V}_R^+$ in the case of the good sunspot and by the upper envelope of $\tilde{V}_D$ and $\tilde{V}_R^-$ in the case of the bad sunspot. Panel (a) presents the values for the flexible exchange rate regime. It should be understood that when we refer to the flexible exchange rate, we mean the exchange rate policy that delivers the full employment case in all states. For the case of a fixed exchange rate, it will be useful to consider two values for $\overline{w}$. Panel (b) corresponds to the fixed exchange rate regime with "low" wage rigidity and panel (c) to a fixed exchange rate regime with "high" wage rigidity.

Using these value functions, it is straightforward to represent graphically the safe region, crisis region, and default region in Figure 1. The crisis region (i.e., the levels of debt in which a default would occur if there is a bad sunspot) appears shaded in the figure. The safe region (i.e., the levels of debt in which the government repays regardless of whether lenders extend credit or not) is to the left of the crisis region. The default region (i.e., the levels of debt in which the government defaults regardless of whether lenders extend credit or not) is to the right. It is apparent from these figures that vulnerability to a rollover crisis is higher in a fixed exchange rate regime than in a flexible one for both degrees of wage rigidity.

**Crisis region for flexible exchange rate.** Let us describe now how we arrive at the crisis region in the flexible exchange regime in panel (a) of Figure 1. The value of default $\tilde{V}_D$ is a constant because it does not depend on the amount of debt the government owes.[16] The value of repayment when rollover is feasible, $\tilde{V}_R^+$, and when no borrowing is allowed, $\tilde{V}_R^-$, is decreasing in debt in both cases because this means that the government owes more to international lenders and the resource constraint becomes tighter. The value function $\tilde{V}_R^+$ is uniformly above $\tilde{V}_R^-$. Moreover, the difference between these two values is higher for low levels of debt (when the government wants to issue more debt), and the values become identical for very high levels of debt (when the government does not issue debt even when it has access to financial markets).

At the debt level in which the curves $\tilde{V}_R^+$ and $\tilde{V}_D$ intersect, the government is indifferent between repaying when having access to credit markets and defaulting. For debt positions higher than this level, the government defaults regardless of the international lenders' beliefs. This is what we define as the default region. On the other hand, at the debt level in which the curves $\tilde{V}_R^-$ and $\tilde{V}_D$ intersect, the government is indifferent between defaulting and repaying when unable to roll over the debt. For

[16] In the case of flexible wages, $\tilde{V} = V$. However, we keep the notation with $\tilde{V}$ to make it more uniform with the fixed exchange rate regime.

debt positions lower than this level, the value of repayment is higher than the value of default, and the government repays its debt. This is what we define as the safe region, levels of debt in which the government repays even if investors are pessimistic. In between these two regions, there is an interval of debt positions in which the government will only default if international lenders are unwilling to roll over the debt. This is what we define as the crisis region: it is the range of debt levels in which the government only defaults because of pessimistic beliefs. This region, which appears shaded in panel (a) of Figure 1, is less than 1% of debt in terms of average GDP: the range is between 33.5% and 34.4%. The region in which the government is vulnerable to a rollover crisis is small for a flexible exchange rate regime.

**Crisis region for fixed exchange rate.** Panels (b) and (c) of Figure 1 consider the one-period fixed exchange rate regime. As described above, we consider a situation in which there is a fixed exchange rate regime for only the current period and the flexible regime prevails from next period onward. The impact of the fixed exchange rate regime depends, of course, on the level of nominal wages and the level of the exchange rate—in particular, a sufficient variable is $\overline{w}$, the lower bound on wages in foreign currency. We consider two values for this real wage rigidity $\overline{w}$. In panel (b), we consider the highest value of $\overline{w}$ such that the default region remains unchanged relative to the flexible wage. This case allows us to consider a situation in which only the left threshold of the crisis region changes while the right threshold remains the same.[17] One can also see that $\tilde{V}^D$ is at exactly the same level as in the flexible regime because the wage rigidity constraint is not binding for this income shock. In panel (c) we consider a higher degree of wage rigidity, in which case we also see a reduction in $\tilde{V}^D$ because the wage rigidity is also binding under default. In this case, we also see a decline in $\tilde{V}_R^+$ in the crisis region.

Panels (b) and (c) reveal that there is a bigger gap between $\tilde{V}_R^+$ and $\tilde{V}_R^-$ with a fixed exchange rate regime compared to the flexible exchange rate regime. In other words, both values drop, but $\tilde{V}_R^-$ is reduced by much more than $V_R^+$. Key for this result is the behavior of unemployment, as we will explain below. The consequence of the increase in the gap between these two curves is the increase in vulnerability to a rollover crisis, in line with Proposition 4.[18] In panel (b), the range of the crisis region is about $7\%$ of GDP and goes from $27.1\%$ to $34.4\%$. In panel (c), the crisis region increases to more than $12\%$ percentage points of GDP and represents more than a third of the average debt-GDP. Moreover, the economy enters a rollover crisis with a level of debt that is $14$ percentage points of GDP lower than the level it takes under a flexible exchange rate regime.

[17] To obtain the value of $\overline{w}$ that delivers this, we have to infer the highest value of $\overline{w}$ such that there is no unemployment when the economy faces a good sunspot. Intuitively, an increase in $\overline{w}$ beyond this point would reduce the value from repaying and lead the government to default when investors are willing to lend, which would imply an increase in the default region.

[18] Notice that the gap between $\tilde{V}_R^+$ and $\tilde{V}_R^-$ is not affected by the form of the current default cost because the government is not defaulting.

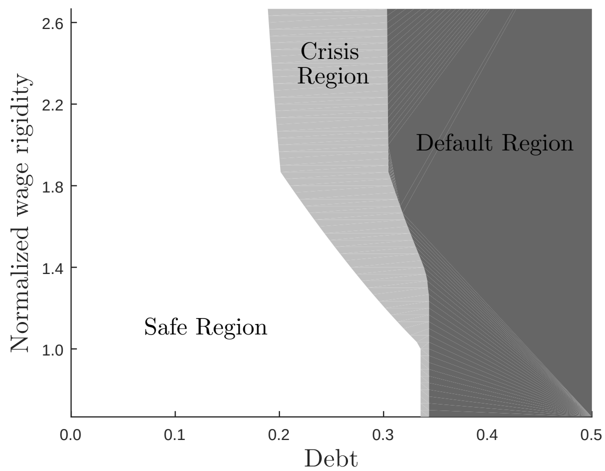


Figure 2: Safe, Crisis, and Default Regions under Different Wage Rigidities

We have illustrated so far how the exchange rate regime shapes the crisis region for two values of $\overline{w}$. In Figure 2 we show how the safe, crisis, and default regions change for a whole range of $\overline{w}$, keeping the income level the same as before. The value of $\overline{w}$ is normalized by the highest $\overline{w}$ that is consistent with no changes in the three zones.[19] In this way, a value lower than unity in Figure 2 will correspond effectively to the flexible exchange rate regime. As soon as wage rigidity rises above one, by construction, it becomes binding and the crisis region starts to expand. For low values of wage rigidity, the intersection between $\tilde{V}_D$ and $V_R^+$ remains unaffected, and hence the crisis region expands at the expense of the safe region without changes in the default region. Once $\overline{w}$ reaches 1.33, which is the value used in panel (b) in Figure 1, the value of default starts to expand as well and it does so at the expense of the crisis region. However, we can see in Figure 2 that the crisis region continues to expand significantly because the safe region contracts by an amount greater than the default region expansion.

**Crisis zone.** We have shown the safe, crisis, and default regions for a given level of the tradable endowment. To have a more complete picture, we consider a whole range of combinations of debt and income. We show in Figure 3 the three zones in the $(b, y^T)$ state space. For any given level of debt, the economy is in the default zone for a sufficiently low level of tradable endowment. As we increase the tradable endowment, the economy arrives in the crisis zone at some point. Finally, increasing it even further makes the economy reach the safe zone.

Again, we can clearly see how vulnerability to a rollover crisis is lower in a flexible exchange rate regime compared to a fixed regime, and this occurs for all income levels.[20] An implication of this, as

[19] This level can be computed by first obtaining $b^-$ such that $V_R^-(b^-, y^T) = V_D(y^T)$ and then finding $\overline{w}$ such that $\tilde{V}_R^+(b^-, y^T; \overline{w}) = V^D(y^T)$.

[20] Along the horizontal y-axis with $y^T = 0.957$ for all panels in Figure 3, we recover exactly the same thresholds that separate the three regions in Figure 1. Notice also that for any income level different from $y^T = 0.957$, the default region will change in panel (b).

we will see below, is that the economy with a fixed exchange rate regime will spend more time in the crisis region than will the flexible exchange rate regime. As a result, the economy with a fixed exchange rate will experience more defaults due to rollover crises. [21]

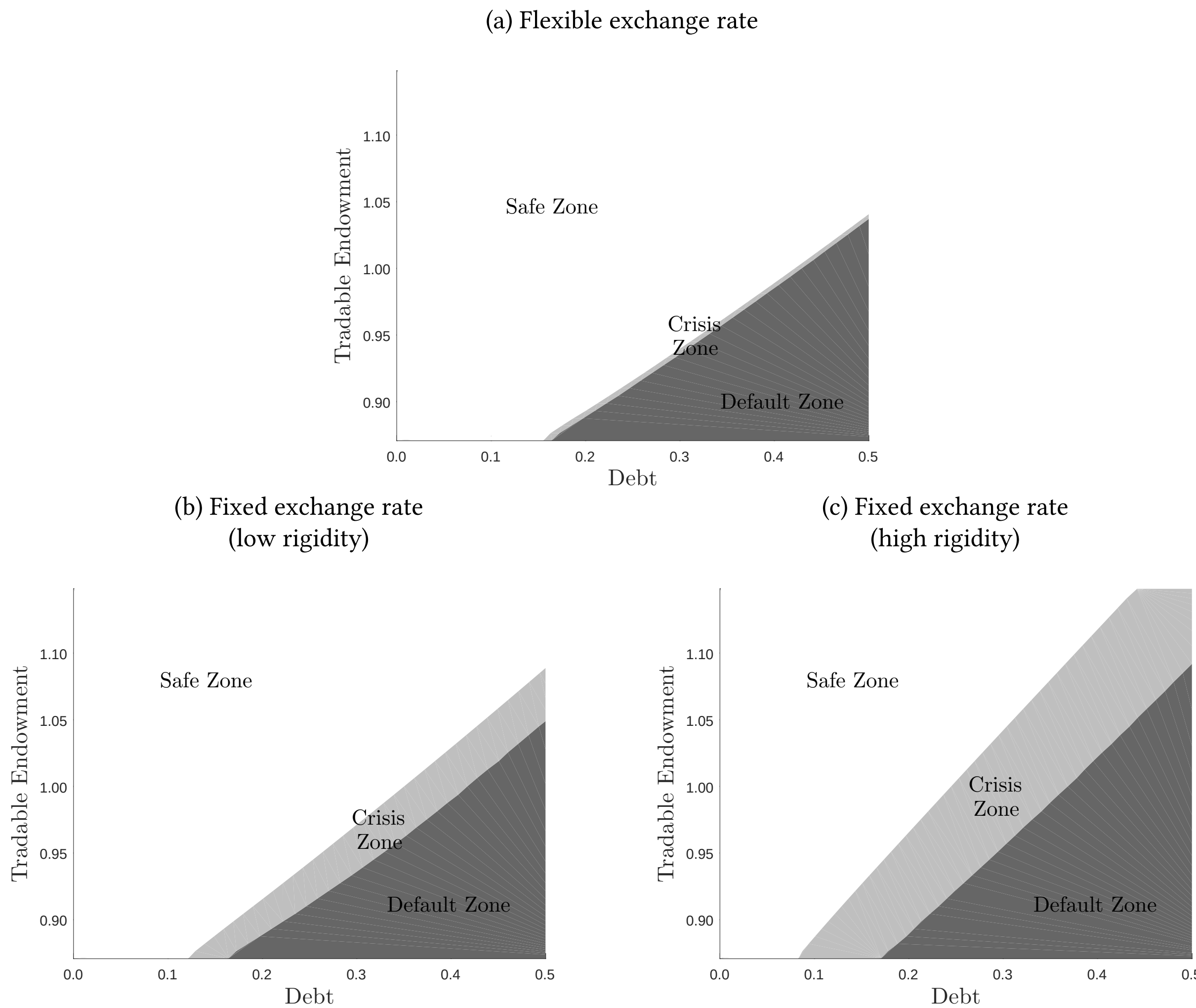


Figure 3: Safe, Crisis, and Default Zones under Different Wage Rigidities

## 3.4 Inspecting the Mechanism

We have established that an economy in a fixed exchange rate regime faces a large crisis region and hence a greater exposure to a rollover crisis. This section delves deeper into this result and underscores the importance of the response of unemployment for incentives of the government to repay.

[21] Notice that in Cole and Kehoe (2000), with no government impatience relative to international lenders ($\beta R = 1$), the government always eventually escapes the crisis region if bad sunspots do not trigger default. This is not the case in our model, given that we will consider $\beta R < 1$. Even in the case of $\beta R = 1$, our results from a larger crisis region for a fixed exchange rate regime suggest that the government will take more time to exit the crisis region with a fixed exchange rate regime.

Figure 4 shows the behavior of unemployment associated with the two different levels of wage rigidities considered earlier. For each panel, there are three lines: $u_D$ denotes the unemployment rate if the government chooses to default: $u_R^+$ is the unemployment rate if the government chooses to repay when investors lend, and $u_R^-$ is the unemployment rate if the government chooses to repay when investors refuse to lend. The on-equilibrium unemployment rate depends on which region the debt level is in. In the crisis region, which again appears shaded in the figures, unemployment rate can take two values, $u_D$ or $u_R^+$, depending on the realization of the sunspot. In the safe region, the on-equilibrium unemployment rate is $u_R^+$, while in the default region it is $u_D$.

When the government repays, unemployment is increasing in the current amount of debt both when the government can access the debt market and when it cannot. This is because a higher debt level reduces aggregate demand, which in turn generates a decline in the price of non-tradables in terms of foreign currency.[22] Under a fixed exchange rate, the wage rigidity in terms of domestic currency becomes a wage rigidity in foreign currency. Because of the downward rigidity in wages, the decline in the price of non-tradables leads to a rise in unemployment. When the government defaults, the unemployment rate is, of course, constant (and zero for the low-rigidity case). Following the way we set $\overline{w}$ in the low-rigidity case, unemployment only starts to become strictly positive at levels of debt in which the government, under a flexible regime, would be indifferent between defaulting and repaying when investors are willing to lend. Interestingly, this increase in unemployment that we see in the fixed regime is innocuous since the government would default anyway.

When investors refuse to lend, unemployment starts rising earlier (i.e., for lower levels of debt), and it is always higher than when investors are willing to lend—conditional, of course, on the government repaying in both cases. The reason is that when investors refuse to roll over the debt, the government is forced to raise tax revenues, which generate a decline in aggregate demand. In turn, this leads to deflationary pressures on the price of non-tradable goods, which cause a decline in labor demand. Because wages are downwardly rigid, the rollover crisis generates involuntary unemployment.

It is interesting to realize that in panel (a) of Figure 4, no unemployment equilibrium arises on the equilibrium path. In other words, what leads to default is the desire to avoid the large unemployment that would emerge if the government were to repay when it is cut off from the credit markets. In panel (b), because the wage rigidity is tighter, we do observe unemployment on the equilibrium path depending on the initial debt and the realization of the sunspot. Still, the large levels of unemployment that we observe in the case that investors refuse to lend are not observed in equilibrium. If the government finds it optimal to default in this scenario, unemployment falls to $u^D$, whereas if the government finds it optimal to repay, unemployment falls to $u_R^+$ because the run would not occur in

[22]To understand this, recall that the price of tradables in terms of domestic currency is constant in a fixed exchange rate regime because the price of tradables in foreign currency is constant. A decline in aggregate demand therefore requires a decline in the price of non-tradables to clear the market for non-tradables.

equilibrium.

This increase in unemployment that emerges from fluctuations in labor demand from the non-tradable sector is at the heart of the mechanism to generate a larger exposure to a rollover crisis. It is useful to point out that having production in the tradable sector would not affect the differences in employment when investors lend vis-à-vis when investors refuse to lend. The level of the exchange rate would affect employment in the tradable sector, but this would be independent of investors' beliefs. The key idea is that for tradable goods, the relevant demand is the international one. On the other hand, in the non-tradable sector, the availability of domestic resources is critical to determine the domestic price of tradables and firms' labor demand.

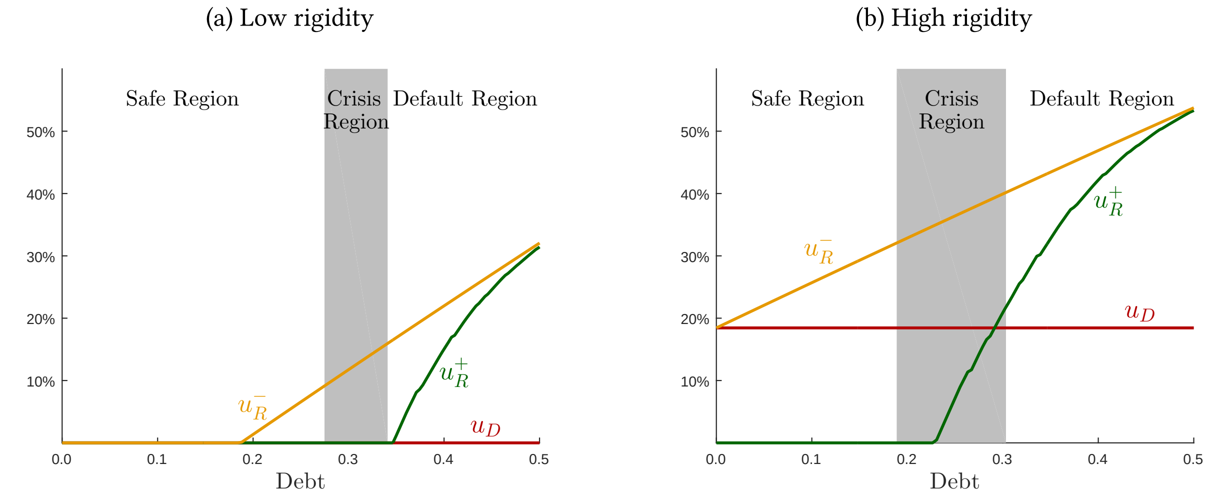


Figure 4: Unemployment Rates with Fixed Exchange Rate Regime

Notes: The unemployment rate if the government chooses to default is denoted by $u_D$. When the government repays, the unemployment rate is denoted by $u_R^+$ in the good sunspot and by $u_R^-$ in the bad sunspot.

These differences in unemployment that arise depending on whether investors are willing to lend or not translate into differences in the value functions. Figure 5 shows how $\tilde{V}_R^+$ and $\tilde{V}_R^-$ change when we introduce rigidities. These are the same value functions from Figure 1, but now we put them together to better appreciate the differences and mark the thresholds at which unemployment emerges. Consider first the repayment value functions under a flexible exchange rate regime, which are denoted with dashed lines. We can see that the gap between the two is very small: there is zero unemployment regardless of whether investors lend or not. Moreover, the gap is relatively wider at very low levels of debt (because the government wants to issue more debt). However, at those levels of debt, the government has a value of repayment that is far larger than the value of default, and hence this gap between $\tilde{V}^+$ and $\tilde{V}^-$ is innocuous. As debt increases and we approach the value of default, the gap becomes smaller (because the government does not want to issue as much debt). The outcome is a narrow crisis region.

Figure 5 shows that when the exchange rate is fixed, all value functions drop relative to the flexible

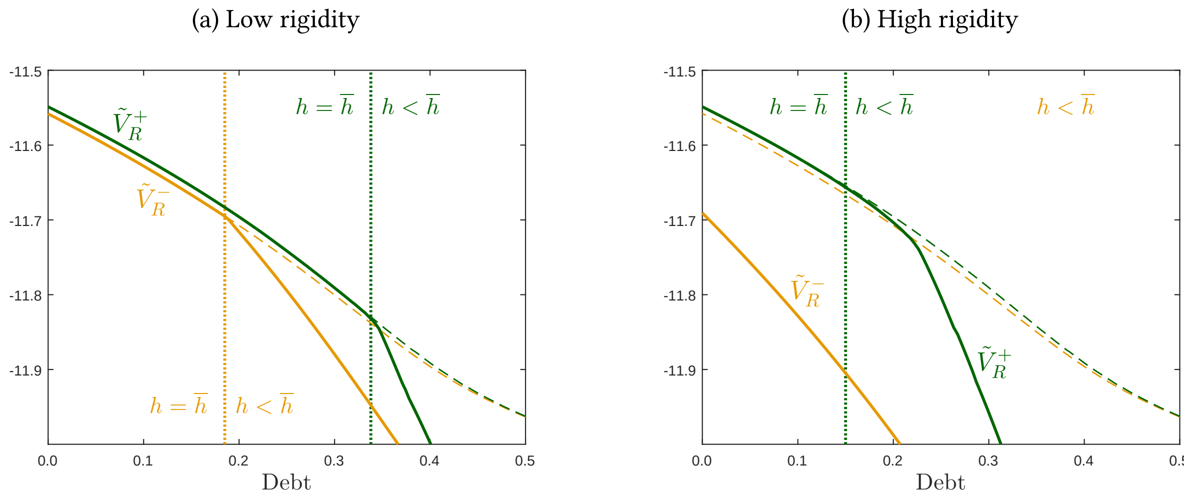


Figure 5: Values of Repayment in Flexible vs. Fixed Exchange Rate Regime.

Notes: Dashed lines correspond to the flexible exchange rate regime and straight lines correspond to the fixed exchange rate regime. Green (dark) lines correspond to $\tilde{V}_R^+$, and yellow (light) lines correspond to $\tilde{V}_R^-$ .

case, and there is a strict decline at the debt threshold in which unemployment emerges. Most importantly, however, is that $\tilde{V}^-$ is reduced by more than $\tilde{V}^+$, and hence there is a bigger gap between the two compared to the flexible exchange rate. This arises because of the substantially different unemployment levels that arise depending on whether investors lend or not. Moreover, the widening of the gap occurs precisely at debt levels at which lenders' beliefs matter for the repayment decision. The outcome is a wide crisis region.

## 3.5 Extensions, Generalizations and a Simple Example

The main theoretical results that we have presented so far can be extended and generalized in a relatively straightforward manner. While the full details are presented in the Addendum, we discuss here the main elements of each of these extensions.[23] In addition, we also present in Section 3.5.1 a simple example with deterministic income.

The same results can be obtained in a model with sticky prices instead of sticky wages. Consider a situation in which investors become pessimistic and the government raises taxes to service the debt. With sticky wages, we showed that the resulting decline in aggregate demand leads to a decline in the price of non-tradables, which generates a decline in employment and makes repayment more costly. With sticky prices, firms respond to the cut in demand by reducing production, which in turn generates lower labor demand, lower wages and lower employment. . In both cases, repayment becomes very costly when investors turn pessimistic and this precipitates a rollover crisis. Appendix A shows how all the propositions extend to the case of price stickiness and show that with linear

[23]We leave entirely for the appendix the case with arbitrary maturity structure and elastic labor supply.

production functions, results are identical.

The same results can also be obtained when there are costs from exchange rate fluctuations, so that a fixed exchange rate regime is not necessarily a dominated regime. In our baseline model, a higher exchange rate unambiguously increase the utility flow at any particular state given that it reduces unemployment and does not involve any cost (see Proposition 1). We consider two specifications of costs from exchange rate fluctuations (see Appendix B.1). In one specification, we consider a quadratic cost of departing from a target exchange rate $\bar{e}$.These costs could come from redistributive effects or monetary distortions but we prefer not to take a stance on the source of these costs. In this situation, the government will trade-off the benefits from higher employment with the costs of exchange rate fluctuations. The higher are the costs from depreciating, the more similar will be the economies under flexible and fixed exchange rate. Regardless of how large are the costs, however, an economy under flexible exchange rate displays a smaller crisis zone, and all our theoretical results continue to hold.

In the second specification, we consider a version of the model in which the costs from exchange rate fluctuations arise from the expectation of future depreciations, rather than from the current one. Lacking commitment to an exchange rate policy, the government always finds optimal to depreciate the currency enough to deliver full employment, generating an inflationary bias. An economy that fixes the exchange rate or enters a monetary union is able to avoid this inflationary bias and doing so could be desirable if these costs are sufficiently large. Still, the economy under a flexible exchange rate will feature a lower exposure to rollover crises as in our baseline model. These two specifications are useful because they highlight that our main result is not altered by the fact that our baseline model abstracts from modelling the reasons why the government implement a fixed exchange rate.

We also consider an inflation targeting regime. In particular, we focus on a regime in which the government keeps constant the price of the composite consumption good. When investors turn pessimistic, the government can depreciate the currency to alleviate unemployment but there is a limit given by achieving the inflation target. A negative shock to aggregate demand in this economy leads to deflationary pressures and a reduction in the price of non-tradables. As the government depreciates the domestic currency, real wages fall, which stimulates labor demand. The price of tradable goods rises at the same and so this puts a limit to the ability to stabilize employment while fulfilling the inflation target. Appendix C shows how the same propositions as in our baseline model hold under inflation targeting. The general lesson is that presence of monetary policy constraints, either in the form of a fixed exchange rate or inflation targeting, can make an economy more vulnerable to rollover crises.

Finally, we also consider a model with debt denominated in domestic currency. In the baseline model, the only differences between a flexible exchange rate regime and fixed exchange rate regime is that in the former, the government can use monetary policy to stabilize macroeconomic fluctuations. We made this assumption partly to better highlight the new channel regarding the role of monetary policy to reduce the vulnerability to rollover crises. In principle, however, an economy that is away from a monetary union can also issue debt in domestic currency, which opens the possibility to inflating away the debt. We argue that the main insight of the paper remains when we allow for this possibility. In Appendix F we consider a version of the model where a nominal depreciation allows to simultaneously affect the real value of the debt and affect the level of employment.[24] In this economy, depreciating the currency allows for an increase in the amount of consumption by effectively diluting the real value of foreign lenders' debt. Importantly, this allows for an increase in aggregate demand and through the mechanism highlighted above reduce unemployment and make repayment less costly in the event of investors's panic. The possibility of depreciating the currency, therefore, reduces again the incentives of investors to run, reducing the exposure to a rollover crisis.

### 3.5.1 Simple Example

In this section, we consider a simple version of the model with one-period debt in which (i) income is deterministic, (ii) $\beta R = 1$, and (iii) permanent exclusion after default. These assumptions imply that if the government is in the crisis zone, it will eventually exit. The key additional insight that will emerge is that a government under a fixed exchange rate will exit more slowly the crisis zone.

In contrast with the theoretical results presented above, we now allow for a permanent change in wage rigidity. That is, rather than changing only the current wage rigidity keeping future rigidities constant, we change $\overline{w}$ over all periods. Following the same steps as above, we proceed to analyze how the default thresholds $B^+$ and $B^-$ change with the rigidity. Thanks to the simplifying assumptions in this example, the thresholds are now straightforward to characterize. In particular,the value functions of default can be computed as:

$$V_D(\overline{w}) = \frac{1}{1-\beta}[u\left(\overline{y}^T, \mathcal{H}\left(\overline{y}^T, \overline{w}\right)\right) - \kappa] \tag{27}$$

$$V_R^-(b;\overline{w}) = u\left(\overline{y}^T - b, \mathcal{H}\left(\overline{y}^T - b, \overline{w}\right)\right) + \frac{\beta}{1-\beta}u(\overline{y}^T, \mathcal{H}\left(\overline{y}^T, \overline{w}\right)) \tag{28}$$

$$V_R^+(b;\overline{w}) = \begin{cases} \frac{1}{1-\beta}u\left(\overline{y}^T - \frac{r}{1+r}b, \mathcal{H}\left(\overline{y}^T - \frac{r}{1+r}b, \overline{w}\right)\right), \ \forall b \leq \bar{B}^+ \\ \max_{b'} u\left(\overline{y}^T - b + qb', \mathcal{H}\left(\overline{y}^T - b + qb', \overline{w}\right)\right) + \beta EV(b';\overline{w}) \ \ \forall b \geq \bar{B}^+ \end{cases} \tag{29}$$

with $\mathcal{H}$ defined as Lemma 2. The simple formulation of $V_D$ in 27 follows from the fact that there is permanent exclusion, making the value of default determined fully by exogenous parameters. Regarding the computation of $V_R^-$, once the government pays the entire stock of debt—debt is restricted to one-period—from tomorrow onward, it consumes the annuity value of the income. The latter follows from the fact that $\beta R = 1$ and income being deterministic. Similarly, if the economy is in the safe zone, the government consumes every period the annuity value of the income minus the interest

[24] Aguiar et al. (2013) and Aguiar et al. (2015) study the exposure to rollover crises in a model with nominal debt but without nominal rigidities.

payments, hence the first line of the expression for $V_R^+(b;\overline{w})$. On the other hand, if the economy is not in the safe zone, the government will not find optimal to keep the debt constant as doing so would expose the government to a rollover crisis.

Figure 6 compares the incentives to save for the fixed and flexible exchange rate economies by showing the policy functions for debt (the left and the right panel present the flex and fixed case respectively). The dashed line in these figures denote the $45^o$ line. The crisis region is again larger for the fixed exchange rate although now both appear larger because debt has one period maturity in this simple example.[25] In addition, the default region always expands now because with the permanent change in wage rigidity, the continuation value in repayment is reduced. When the economy is in the safe zone, debt is kept constant. Because $\beta R = 1$, the government finds optimal to keep debt and consumption constant over time. When the economy is in the crisis region, the government finds optimal to reduce $b'$, as illustrated by the fact that the policies are below the $45^o$ line (as in Cole and Kehoe (2000)). The reason why the government finds optimal to save its way out of the crisis zone is to avoid the default costs that carry the realization of a bad sunspot while in the crisis zone. Essentially, the government chooses a constant consumption profile while in the crisis zone and the level of consumption that guides the speed at which it exits the crisis zone. The debt policy functions are therefore piecewise flat with discontinuities a the points in which the government decides to take one further period to exit the crisis zone. The further away from the crisis zone, the longer it takes to exit.

Comparing panels (a) and (b) of Figure 6 show that under a fixed exchange rate, the government features more jumps in the policy function which are an indication that the government exits more slowly the crisis zone. One may have thought that since the crisis zone is larger under fixed, the government will reduce the debt more aggressively to exit the crisis zone. Why is this not the case? The government realizes that if it were to save more to speed up the exit, it will generate a recession. To avoid the recession, the government reduces borrowing at a slower pace and remains more exposed to a rollover crisis. In other words, the government gambles for redemption.

In Figure 7, we further examine how the speed at which the government exists the crisis zone changes with wage rigidity. Fixing an initial value of debt at $b =$, we show how the number of periods it takes to exit increase with wage rigidity.

[25] The advantage of modelling one-period debt is that the thresholds are straightforward to compute, as shown above, but the results we emphasize regarding the difference between flex and fixed would be essentially the same with long-term debt.

(a) End-of period debt under flexible exchange rate

(b) End-of period debt under fixed exchange rate

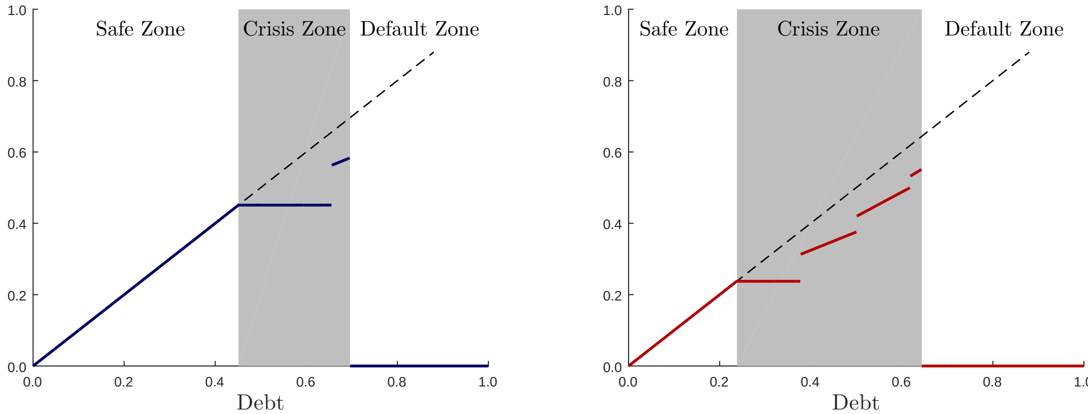


Figure 6: Policy Function for Debt

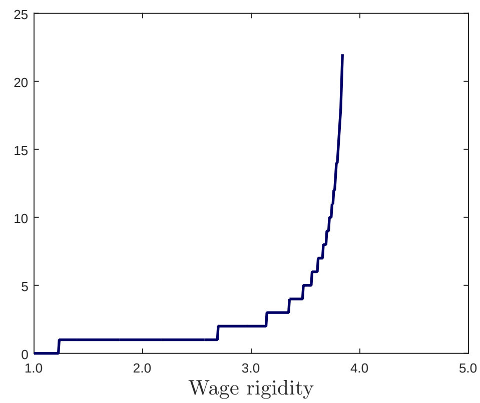


Figure 7: Time to Safety

Note: The figure depicts the number of periods it takes to the government to exit the crisis zone if no bad sunspot is triggered.

# 4 Quantitative Analysis

The goal of this section is to quantitatively assess how exposed is an economy to a rollover crises and how this depends on the exchange rate regime. While we already argued that the crisis zone is substantially smaller under a flexible exchange rate regime, we will investigate here the differences in exposure over a sample of long-run simulations. We will also perform a counterfactual experiment applied to the crisis in Spain as well as a welfare analysis to determine who significant are the costs from monetary independence and the potential gains from a lender of last resort.

## 4.1 Calibration

We calibrate the model at an annual frequency using Spain as a case study.[26]

**Functional forms.** We use a CRRA utility function,

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \text{with} \quad \sigma > 0.$$

We parameterize the default utility cost as follows:

$$\kappa(y^T) = \max\left\{0, \kappa_0 + \kappa_1 \ln\left(y^T\right)\right\}.$$

As shown in Arellano (2008) and Chatterjee and Eyigungor (2012), a non-linear specification of the cost of default is important to allow the model to match the levels of debt and spreads in the data. In particular, we follow Bianchi et al. (2018a) in specifying this default cost function in terms of utility.

The tradable endowment process follows a lognormal AR(1) process,

$$\ln(y_t^T) = \rho \ln(y_{t-1}^T) + \sigma_y \varepsilon_t,$$

where $|\rho| < 1$ and the shock $\varepsilon_t$ is $i.i.d.$ and normally distributed, $\varepsilon \sim N(0,1)$. To estimate the tradable endowment stochastic process, we use the value-added series in the manufacturing and agricultural sectors in Spain. After we log-quadratically detrend the series, we estimate a persistence parameter of $\rho = 0.777$ and a standard deviation of $\sigma_y = 2.9\%$.

**Model Parameters.** Table 1 shows all the baseline calibration values for the parameters of the model. A first subset of parameters is specified directly. These are parameters that can be calibrated straight from the data or are relatively standard in the literature. We then pick a second subset of parameters to match key moments in the data under two different regimes: flexible and fixed exchange rates.

We start with the first subset of parameters. First, we specify the parameters governing preferences and technology, which will take standard values in the literature. The coefficient of risk aversion will be set to $\sigma = 2$. Meanwhile, the elasticity of substitution between tradable and non-tradable goods is set to $\frac{1}{1+\mu} = 0.5$, which is in the range of empirical estimates. The share of tradable goods in the consumption aggregator is set to $\omega = 0.197$, so it matches the ratio between tradable good and total

[26] The model is solved numerically using value function iteration with interpolation. Linear interpolation is used for the endowment and debt levels. We use 25 grid-points for the tradable endowment grid and 99 grid-points for debt. To compute expectations, we use 105 quadrature points for the endowment realizations.

Table 1: Parameter Values

| Parameter | Value | | Description |
|---|---|---|---|
| $\overline{h}$ | 1.000 | | Normalization |
| $\sigma$ | 2.000 | | Standard risk aversion |
| $\omega$ | 0.197 | | Share of tradables |
| $\mu$ | 1.000 | | Unitary elasticity of substitution between T-NT |
| $\rho$ | 0.777 | | Output persistence |
| $\sigma_y$ | 0.029 | | Standard deviation of tradable output shock |
| $\alpha$ | 0.750 | | Labor share in non-tradable sector |
| $r$ | 0.020 | | German 6-year government bond yield |
| $\delta$ | 0.141 | | Spanish bond maturity 6 years |
| $\psi$ | 0.240 | | Reentry to financial markets probability |
| $\pi$ | 0.030 | | Sunspot probability |
| Calibration | Flexible | Fixed | Target |
| $\beta$ | 0.914 | 0.908 | Average external debt-GDP ratio 29.05% |
| $\kappa_0$ | 0.101 | 0.315 | Average spread 2.01% |
| $\kappa_1$ | 0.759 | 3.273 | Standard deviation interest rate spread 1.42% |
| $\overline{w}$ | - | 2.493 | $\Delta$ unemployment rate 2.00% |

output, which averages around 20% for Spain in the period considered.[27] Regarding the labor share in non-tradable production, we set $\alpha = 0.75$, an estimate from Uribe (1997) for the non-tradable sector. Last, we normalize the inelastic labor supply of households to $\overline{h} = 1$.

Next, we set the parameters from financial markets. We set the international risk-free interest rate to $r = 2\%$, which is the average annual gross yield on German 6-year government bonds over the period 2000 to 2015. We calculate a maturity parameter of $\delta = 0.141$ to reproduce an average bond duration of 6 years, in line with Spanish data.[28] We set the reentry to financial markets probability after default to $\psi = 0.24$ to capture an average autarky spell of 4 years, in line with Gelos, Sahay, and Sandleris (2011). Finally, we need to set the sunspot probability, which is a more difficult parameter to calibrate. In the literature, the probability of drawing bad sunspot is usually set to a relatively low value (e.g., Chatterjee and Eyigungor, 2012, study a range between [0,0.10]). Our baseline value is 3%, but we examine a wide range as well.

For the second subset of parameters $\{\beta, \kappa_0, \kappa_1, \overline{w}\}$, we will set these parameters so that the mo-

[27]In a nonstochastic version of the model with a mean value of debt $\bar{b}$ and average employment $\bar{h}$, the value of $\omega$ can be pinned down from $\frac{y^T}{y^T + \frac{1-\omega}{\omega}\left(\frac{y^T + r\bar{b}}{F(\bar{h})}\right)^{\mu+1}} = 20\%$.

[28]The Macaulay duration of a bond with price $q$ and our coupon structure is given by

$$D = \sum_{t=1}^{\infty} t \frac{\delta}{q} \left(\frac{1-\delta}{1+i_b}\right)^t = \frac{1+i_b}{\delta + i_b},$$

where the constant per-period yield $i_b$ is determined by $q = \sum_{t=1}^{\infty} \delta (\frac{1-\delta}{1+i_b})^t$.

ments in the model match the counterparts in the data. Since we have two different exchange rate regimes, we have two sets of parameters. The difference in the two calibrations is that $\overline{w}$ is set to zero for the flexible exchange rate regime, whereas this value has to be calibrated for the fixed exchange rate regime. In particular, we calibrate $\overline{w}$ in the fixed exchange rate regime to be consistent with the increase in unemployment during episodes of high sovereign spreads. In the data for Spain, the increase in unemployment relative to the HP-filtered trend was $2\%$ in 2011, the year prior to the EU and ECB's intervention.[29] With a value of $\overline{w} = 2.493$ and given the rest of the calibrated parameters, the average increase in unemployment in the year prior to default is $2\%$ in the model, matching the empirical counterpart.[30]

For both regimes, we calibrate the parameters $\beta$, $\kappa_0$, and $\kappa_1$ to match three moments from the data, and we follow Hatchondo, Martinez, and Sosa-Padilla (2016) in considering the moments in the years following 2008 to concentrate on the period around the crisis. The three moments targeted are the average debt-GDP ratio, and the average and standard deviation of spreads. For the average debt-GDP ratio, we target an average external debt of 29%. For the average and the volatility of spreads, we target 2.0 and 1.4, respectively.[31] The resulting values for these parameters appear in Table 1.

## 4.2 Simulation Results: Exposure to Rollover Crises

We now conduct simulations to investigate how the exchange rate regime determines what type of default, fundamental or rollover crisis, is more likely.

We start from the flexible exchange rate economy. In this economy, out of 100 default episodes, the share of defaults due to a rollover crisis is only 0.92. In line with this result, only 0.77% of the time, the economy is in the crisis zone and therefore vulnerable to a rollover crisis. To examine how the degree of wage rigidity matters for the exposure to a rollover crisis, we vary the wage rigidity parameter $\overline{w}$ while keeping the same calibrated parameters for the flexible exchange rate economy.[32] In Figure **??** (panel a) we present the fraction of defaults that are explained by rollover crises as a function of $\overline{w}$. We can see here that the tighter is the wage rigidity, the larger the fraction of defaults that are explained by non-fundamentals. In line with this result, Panel (b) of Figure **??** also shows that the fraction of time the economy spends in the crisis region becomes larger with the degree of rigidity.

It is worth highlighting that in these simulations, we also obtain that the average debt level falls

[29] We use a smoothing parameter of 100 for the HP filtering. If we use a log-quadratic filter, we obtain a value closer to $3\%$.

[30] As we mentioned in footnote 9, governments have available fiscal instruments like subsidies on wages to stimulate employment. In terms of our model, this would imply that the wage rigidity would be governed by $\overline{w}$ net of these subsidies. Our approach to calibrate $\overline{w}$, therefore, incorporates implicitly these effects.

[31] The debt level in the model is computed as the present value of future payment obligations discounted at the risk-free rate $r$. Given our coupon structure, we thus have that the debt level is $\frac{\delta}{1-(1-\delta)/(1+r)} b_t$.

[32] Different from our analysis in the comparative statics exercise of Section 3, the change in $\overline{w}$ is now permanent, and therefore the bond price schedule is affected.

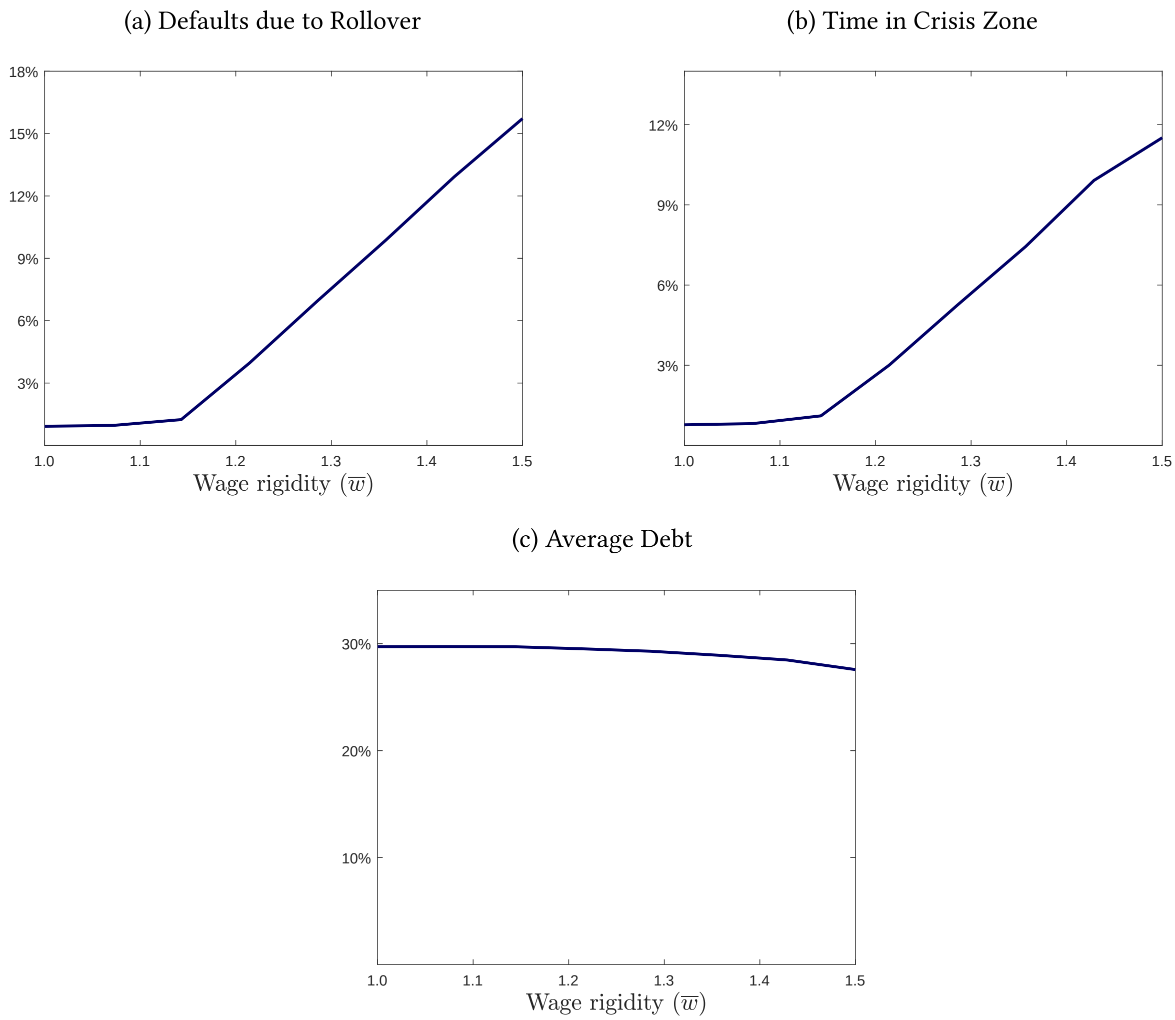


Figure 8: Role of Wage Rigidity

with $\overline{w}$ (panel c). Two reasons explain this. First, the government faces borrowing terms that are more adverse, given that there are higher incentives to default in the future for both fundamental and non-fundamental reasons. Second, the government also attempts to stay further away from the crisis zone by reducing debt. Despite this attempt, the fact that the crisis region expands significantly implies that the government ends up being more heavily exposed to a rollover crisis.

When we vary the degree of wage rigidity, the long-run moments to which we calibrate the flexible exchange rate economy also change. In particular, as mentioned above, the economy under fixed ends up borrowing less than the economy under flex. To take these changes into account, we complement the results above which keep all parameters constant, by recalibrating the economy to hit the same targets: mean debt , mean spreads, and volatility of spreads while calibrating the value of $\bar{w}$ to match the increase in unemployment, as described in Section 4.1.

**Sunspot Probability.** The fraction of defaults that are the outcome of a rollover crisis depends on two factors. One factor is the probability of a bad sunspot (i.e., the probability of selecting the bad equilibrium whenever the economy is conditional on being in the crisis zone). The second factor is the probability of ending up in the crisis zone in the first place, which is an endogenous outcome that depends critically on borrowing decisions and on the monetary policy regime. Next, we increase the probability of selecting the bad equilibrium while keeping the rest of the parameter values for fixed and flexible exchange rate regimes at their respective baseline values.

Table 2 shows how increasing the likelihood of a bad sunspot increases the fraction of defaults due to a rollover crisis for the two economies, and in particular for the economy under a fixed exchange rate regime. Specifically, when the probability of a bad sunspot is $20\%$, up from $3\%$ in the baseline, about 1/5th of all defaults are for non-fundamental reasons. Moreover, one can see that the fraction of time spent in the crisis region decreases as the government reduces its exposure, but this duration is not enough to offset the higher likelihood of a bad sunspot.

Table 2: Sensitivity to Sunspot Probability

| Sunspot probability | $\pi = 3\%$ | | $\pi = 10\%$ | | $\pi = 20\%$ | |
|---|---|---|---|---|---|---|
| (percentage %) | Flexible | Fixed | Flexible | Fixed | Flexible | Fixed |
| Average spread | 2.46 | 1.43 | 2.45 | 1.47 | 2.46 | 1.53 |
| Average debt-income | 29.73 | 31.33 | 29.58 | 29.29 | 29.37 | 28.53 |
| Spread volatility | 1.33 | 1.60 | 1.30 | 1.72 | 1.31 | 1.75 |
| Unemployment increase | 0.00 | 1.83 | 0.00 | 1.80 | 0.00 | 1.35 |
| Fraction of time in crisis region | 0.77 | 2.59 | 0.68 | 1.93 | 0.58 | 1.41 |
| Fraction of defaults due to rollover crisis | 0.92 | 6.53 | 3.70 | 11.80 | 6.20 | 19.80 |

Notes: All parameter values correspond to the benchmark calibrations for fixed and flexible exchange rate regimes. The benchmark calibration uses $\pi = 3\%$.

### 4.3 Welfare

We tackle here two important welfare considerations: (i) What is the welfare cost of the lack of monetary independence? (ii) What are the welfare costs of rollover crises?

Our first result is that the possibility of rollover crisis substantially increases the welfare costs of giving up monetary independence. We examine for all initial states, how much are household willing to give up of the composite consumption good to move for *one period* to a flexible exchange rate. Thee are two steps involved. The first step is to compute the value for the government of being able to vary the exchange rate today in a state in which the government is participating in financial markets:

$$V_0(b, s) = \max[V_0^D(s), V_0^R(b, s)] \tag{30}$$

where

$$V_0^R(b,\mathbf{s}) = \max_{c^T,b'} u(c^T,\bar{h}^\alpha) + \beta E V_{FIXED}(b'(b,s),s) \tag{31}$$
$$c + b = y^T + q(b',b,\mathbf{s})(b' - (1-\delta)b)$$

and

$$V_0^D(s) = u(y^T,\bar{h}^\alpha) + \beta\psi V_{FIXED}(0,s') + \beta(1-\psi)V_{FIXED}^D(y^{T\prime}) \tag{32}$$

The second step is to compare the value $V_0(b,s)$ with the value for the government in a fixed exchange rate from the Markov equilibrium. Using these two values, we compute the welfare gain $\theta_0^{flex}(b,s)$ as given by

$$V_0(b,\mathbf{s}) = (1-d_0(b,\mathbf{s}))[(1+\theta_0^{flex}(b,\mathbf{s}))^{1-\sigma}u(c_0^T(b,\mathbf{s}),c_0^N(b,\mathbf{s})) + \beta\mathbb{E}V^A(b_0'(b,\mathbf{s}),\mathbf{s})] +$$
$$d_0(b,\mathbf{s})[(1+\theta_0^{flex}(b,\mathbf{s}))^{1-\sigma}u(c_0^T(b,\mathbf{s}),c_0^N(b,\mathbf{s})) + \beta\psi V(0,\mathbf{s}') + \beta(1-\psi)V^D(y^{T\prime})] \tag{33}$$

where $d_0, c_0^T, c_0^N, b_0'$ correspond to the optimal policies from (30)-(32).

Figure 9 shows the welfare cost of belonging to a monetary union for a range values of debt and for a given endowment shock. For reference, we show the safe region, crisis region and default region for the economy under a fixed exchange rate, and the welfare gains are presented for the good sunspot, $\zeta = 0$ and the bad sunspot $\zeta = 1$. Starting from the left, if debt is very low, there is no unemployment and no cost from having a fixed exchange rate. As debt approaches 0.2, unemployment emerges, and there is a positive welfare cost. Under the good sunspot, the welfare cost increases continuously until debt reaches about 0.3, at which point the government chooses to default under a fixed exchange rate and this helps to mitigate the effects from the wage rigidities. Here, the welfare costs from a fixed exchange rate become decreasing in the level of debt because the value function is independent of debt under fixed but it is still decreasing under flex. Importantly, while the economy under fixed exchange rate features no unemployment, there is still a welfare cost from a fixed exchange rate because it is precisely the lack of flexibility that triggers the government default and the economy suffers from the default costs. For debt levels higher than 0.35, the government under flexible exchange rate also chooses to default and there are no costs from rigidity. Under the bad sunspot, the welfare costs increases discretely once the debt enters the crisis zone. This occurs because the lack of exchange rate flexibility prompts the government to default if investors refuse to rollover the government bonds.

The next welfare consideration that we tackle is on the welfare cost of rollover crises. We interpret these costs as the potential gains from having a lender of last resort from the perspective of the small open economy. As is well understood, a third party with deep pockets can eliminate the coordination problem behind a rollover crisis. The basic argument is that by purchasing a sufficiently large amount of government bonds, either in the primary market or in the secondary market, this can induce the

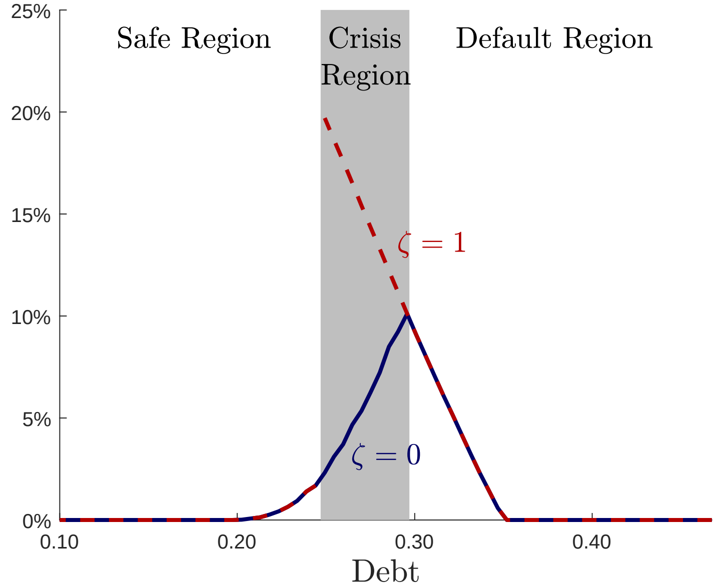


Figure 9: Welfare gains from one-period flexible exchange rate

The regions highlighted in the future correspond to the economy with fixed exchange rate.

government to repay and therefore make investors willing to lend to the government. [33]

We ask how much households would be willing to pay in terms of consumption to permanently eliminate the possibility of a rollover crisis. To compute these welfare costs, we take the fixed and flexible exchange rate economies with their respective calibrations, and solve for the Markov equilibrium after setting the sunspot probability to zero. For each economy, we compute the welfare gains in terms of consumption equivalence as

$$\theta^{roll-over}(b, \mathbf{s}) = \left( \frac{V(b, \mathbf{s})^{NoSunspot}}{V(b, \mathbf{s})} \right)^{1/(1-\sigma)} - 1 \tag{34}$$

for every initial state.[34] Under a fixed exchange rate regime, the gains from having a lender of last resort can reach about 1.5% of permanent consumption and average 0.5%. Having access to a lender of last resort allows both for an improvement in the borrowing terms, and to a reduction in default costs. For the flexible exchange rate, however, the unconditional welfare gains from having a lender of last resort are negligible, in line with the minimal exposure to rollover crises.

It is worth highlighting that a successful implementation of lender of last resort hinges on the ability to correctly identify whether a default is being driven by fundamentals or by self-fulfilling beliefs. Moral hazard concerns would naturally emerge when the government and investors expect

[33] See Roch and Uhlig (2018) and Bocola and Dovis (2016) for an analysis of lender of last resort in the context of the Outright Monetary Transactions (OMT) program by the ECB.

[34] Equation (34) uses homotheticity of the utility function and transforms default costs into consumption equivalence.

interventions in defaults driven by fundamentals. [35] Therefore, in a scenario in which the lender of last resort does not observe the source of the default, a trade-off is likely to emerge between the benefits from offsetting the coordination problem and the moral hazard effects.[36] Our analysis shows that while economies that lack monetary independence are likely to strongly benefit from a lender of last resort, this is less valuable for a flexible exchange rate regime, since defaults are almost exclusively driven by fundamental reasons.

Overall, this welfare analysis provides important lessons. First, in the presence of rollover crises, the lack of monetary independence can become very costly. In particular, governments can become severely exposed to a rollover crisis and costly defaults because of the lack of monetary independence. Second, a lender of last resort can help ease the costs for an economy of giving up monetary independence.

### 4.4 The Path to Spain's 2012 Rollover Crisis

In this section, we use the model to shed light on the Spanish experience after giving up the peseta and adopting the euro. There are two main points we wish to emphasize. First, the model predicts that Spain is in the crisis zone in 2012 while exiting the Eurozone would make the economy safe from a rollover crisis. Second, the bulk of the welfare losses from lacking monetary independence can be mitigated by access to a lender of last resort.

The exercise is as follows. Starting at Spain's external debt-GDP ratio in 2000, we feed the sequence of shocks to tradable output and simulate the model under a fixed exchange rate regime. From 2000 until 2011, the economy remains in the safe zone (and hence the sunspot realization is irrelevant). As a matter of fact, the model predicts that the economy is in the crisis zone in 2012, and a negative sunspot would trigger a rollover crisis and a default. While Spain did not actually default in 2012, a €$100$ billion assistance package by the European Union was channeled through the European Financial Stability Fund and the European Stability Mechanism. In addition, the announcement of the ECB's OMT bond purchasing program following the "whatever it takes" speech by Mario Draghi appeared to have a calming effect in financial markets.

Figure 10 summarize the results of the exercise. Panel (a) shows the tradable output we feed into the model and the one-period ahead probability of falling into the crisis zone. To compute this probability, we use the end-of period level of debt, and compute the probability of receiving an income shock in the following period that would take the economy to the crisis zone. Panels (b) and (c) of Figure 10 show the dynamics of debt and spread in these simulations. In early 2000, the government

[35] As argued by Aguiar et al. (2015), an alternative to a lender of last resort would be some form of fiscal union in which the government receives transfers from other countries but this is more likely to be plagued with moral hazard and other problems.

[36] See Bianchi (2016) for a quantitative analysis of the trade-off between the moral hazard effects from bailouts and the stabilization benefits in the context of firms' borrowing.

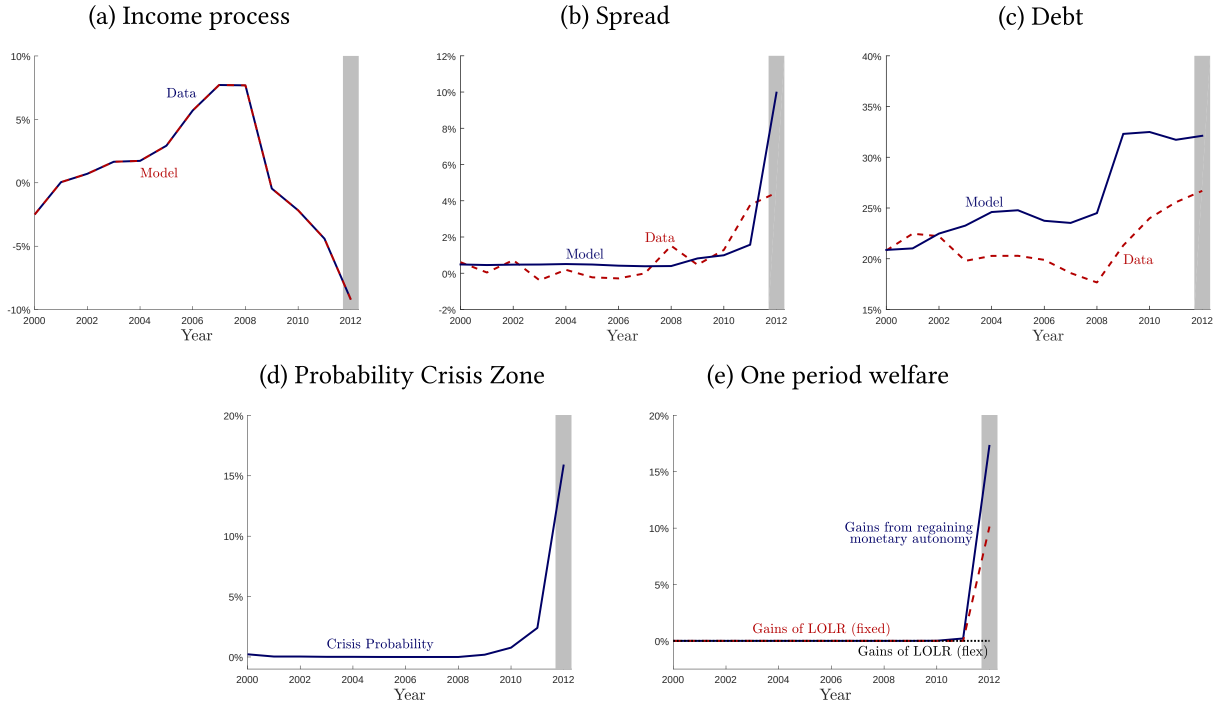


Figure 10: Path to Spain's 2012 Rollover Crisis

Notes: Welfare gains in panel (e) correspond to policies that are in place for one period, reverting to the baseline Markov equilibrium. Crisis probability denotes the probability that the economy would be in a crisis region in the following period given the current choices of debt and initial states. The tradable endowment shock was obtained using a log-quadratic filter to the Spanish tradable output from 1995 to 2017. Debt levels in the data correspond to Spain's external debt-GDP ratio. The shaded region denotes that the economy is in the crisis zone.

increases its debt, and this is driven by the low initial debt (recall that the calibrated mean external debt is close to $30\%$) and by relatively good income shocks that allow for favorable borrowing terms. These dynamics are fairly similar to those in the data, except that the model overpredicts the initial increase. One can also see that the model is able to replicate the low and stable spreads before 2008 in the data. Finally, the evolution of the probability of being in the crisis zone in panel (d) of Figure 10 reveals interesting dynamics. After the debt accumulation that occurs initially and the negative income shocks that pile up after 2008, the economy's probability of a rollover crisis becomes more significant. By 2012, the year in which the ECB intervened, the economy becomes significantly exposed to a rollover crisis, with a $20\%$ probability of being in the crisis zone.

The final block of the exercise is a series of policy counterfactuals. Building on the analysis of Section 4.3, we first consider what are the costs from the lack of monetary independence. More precisely, throughout the simulations, we ask how much are households in Spain willing to pay in terms of current consumption to recover monetary independence for one period. As the blue line in panel (e) shows, the gains are about zero until 2010, there is modest increase in 2011, and strikingly the welfare cost reaches about 17 percent in 2012. As it turns out, close to 60% of these costs are due to the presence of rollover crises. That is, if there was no possibility of a rollover crisis, the increase in welfare from regaining monetary independence would be reduced by 60%—the remaining 40% would be a direct reduction in unemployment. To illustrate this, we compute the gains from eliminating a rollover crisis throughout the simulation for the fixed exchange regime (see the red line in panel (??)). As the figure shows, the gains are zero from 2000-2011 since the economy is in the safe zone but these gains reach 10% in 2012 as the economy arrives to the crisis zone. For comparison, one can see in the figure that the gains from eliminating a rollover crisis remain zero if the government could regain monetary autonomy. In other words, the bulk of the welfare losses from the lack of monetary independence arise because it exposes the government to a rollover crisis and costly default. By the same token, a lender of last resort would help to ease significantly the costs from giving up monetary autonomy.

An implication of our analysis is that if Spain had exit the monetary union in 2012, it would not have been subject to a rollover crisis.[37] We wish to make a few remarks about this counterfactual experiment. First, we are keeping all parameters, except monetary policy, constant when we analyze the implications of exiting the Eurozone. We are therefore abstracting from any structural changes that Spain that could experience upon exiting a monetary union. To the extent that these structural changes would symmetrically affect ${V_R}^+$ and ${V_R}^-$, we expect that the large gap between these two values that arise because of the inability to depreciate the currency would remain intact, and hence these structural changes should not significantly alter the crisis region. Second, we do not suggest that Spain would have been better off by exiting the monetary union. There are indeed many benefits from being in a monetary union that we are not modeling. Our goal is to point out an additional cost of remaining in a monetary union, which arises from the higher exposure to rollover crises.

# 5 Conclusion

This paper showed that the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis and points to a new cost from joining a monetary union. When a government lacks monetary autonomy, a run on government bonds can lead to a large recession in the presence of nominal rigidities. In turn, anticipating that the government will find it more costly to repay, investors become more prone to run and the crisis becomes self-fulfilling. In a calibrated version of the model, we have found that an economy with a flexible exchange rate is relatively immune to a rollover crisis. On the other hand, a substantial defaults under a fixed exchange rate regime are driven by rollover crises.

[37] While the welfare results of Figure 10 correspond to a situation in which the Spain regains monetary autonomy for one period, the same result holds if there was a permanent exit from the Euro. In both cases, we continue to assume that debt remains denominated in foreign currency, a natural assumption since a currency redenomination would be akin to a default. However, one would expect that after some time from regaining monetary autonomy, Spain would start issuing debt in its own currency. This would introduce another channel by which monetary policy affects the vulnerability to a rollover crisis, which is by inflating away the debt. We abstract implicitly from this channel.

Our analysis provides a new perspective on discussions about whether the lack of monetary autonomy in Southern European countries made them more vulnerable to a rollover crisis. According to a popular view, the fact that their debt was not denominated in domestic currency contributed to their vulnerability by preventing them from inflating away the debt. We argue instead that monetary policy has a role in preventing rollover crises that goes beyond the ability to inflate away the debt. By the same token, our analysis suggests that a lender of last resort contributes to ease the costs from giving up monetary independence and could be highly desirable for the stability of a monetary union.

Extending beyond our current analysis, several avenues remain for future work. In terms of debt management, our model suggests that economies with more rigid labor markets or a less flexible monetary policy should seek longer debt maturities. Another interesting avenue is to provide a more explicit modelling of the benefits from joining a monetary union and quantify the relevant trade-offs involved. Finally, one could also extend the analysis to consider general equilibrium within a monetary union.

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# A Proofs

## Proof of Lemma 1

*In any equilibrium, the real wage in terms of tradable goods is a function of tradable consumption and employment:*

$$\mathcal{W}(c_t^T, h_t) \equiv \frac{1-\omega}{\omega} \left( \frac{c_t^T}{F(h_t)} \right)^{1+\mu} F'(h_t).$$

*Moreover, $\mathcal{W}(c_t^T, h_t)$ is increasing with respect to $c_t^T$ and decreasing with respect to $h_t$.*

*Proof.* Using the firm's first order condition (5) and the equilibrium relative price, the equilibrium real wages in terms of tradable goods can be defined as a function of tradable consumption goods and employment:

$$\mathcal{W}(c_t^T, h_t) = p_t^N F'(h_t) = \frac{1-\omega}{\omega} \left( \frac{c_t^T}{F(h_t)} \right)^{1+\mu} F'(h_t).$$

Using this, we can find that

$$\begin{aligned} \frac{\partial \mathcal{W}_t}{\partial c_t^T} &= \frac{(1+\mu) p_t^N F'(h_t)}{c_t^T} \qquad \text{and} \\ \frac{\partial \mathcal{W}_t}{\partial h_t} &= -(1+\mu) p_t^N F'(h_t) \left( \frac{F'(h_t)}{F(h_t)} + \left( \frac{1}{1+\mu} \right) \frac{-F''(h_t)}{F'(h_t)} \right). \end{aligned}$$

Because $F(\cdot)$ is a nonnegative, strictly increasing, and decreasing returns to scale function, we know that $F$, $F' > 0$, and $F'' < 0$. Therefore, we can conclude that $\frac{\partial \mathcal{W}_t}{\partial c_t^T} > 0$ and $\frac{\partial \mathcal{W}_t}{\partial h_t} < 0$. □

## Proof of Lemma 2

*Under a fixed exchange rate regime, the employment function is a piecewise linear function:*

$$\mathcal{H}(c^T) = \begin{cases} \left[ \left( \frac{1-\omega}{\omega} \right) \left( \frac{\alpha}{\overline{w}} \right) \right]^{\frac{1}{1+\alpha\mu}} \left( c^T \right)^{\frac{1+\mu}{1+\alpha\mu}} & \textit{if } c^T \leq \overline{c}_{\overline{w}}^T \\ \overline{h} & \textit{if } c^T > \overline{c}_{\overline{w}}^T \end{cases},$$

*where*

$$\overline{c}_{\overline{w}}^T = \left[ \left( \frac{\omega}{1-\omega} \right) \left( \frac{\overline{w}}{\alpha} \right) \right]^{\frac{1}{1+\mu}} \overline{h}^{\,\frac{1+\alpha\mu}{1+\mu}}.$$

*Proof.* When the real wage rigidity is binding,

$$\overline{w} = \mathcal{W}(c^T, h) = \frac{1-\omega}{\omega}\left(\frac{c^T}{F(h)}\right)^{1+\mu} F'(h) = \frac{1-\omega}{\omega}\left(\frac{\left(c^T\right)^{1+\mu}}{F(h)^{\mu}}\right)\left(\frac{F'(h)}{F(h)}\right).$$

Using the property that $F(\cdot)$ is a homogeneous function of degree $\alpha \in [0,1]$, we then know that $F'(h)$ is homogeneous of degree $\alpha - 1$. Moreover, because it is a unidimensional function, we can also assert that $F(h) = h^{\alpha}$ and $F'(h) = h^{\alpha-1}$. Finally, we also can say that $hF'(h) = \alpha F(h)$. Hence, we can say that

$$\overline{w} = \frac{1-\omega}{\omega}\left(\frac{\alpha\left(c^T\right)^{1+\mu}}{h^{1+\alpha\mu}}\right).$$

Hence, solving for $h$, we get

$$h_{\overline{w}}(c^T) = \left[\left(\frac{1-\omega}{\omega}\right)\left(\frac{\alpha}{\overline{w}}\right)\right]^{\frac{1}{1+\alpha\mu}} \left(c^T\right)^{\frac{1+\mu}{1+\alpha\mu}}.$$

Moreover, this labor function is increasing with respect to the consumption of nontradables. Knowing that labor cannot go above the household's labor endowment of $\overline{h}$, we can compute the consumption of tradables threshold in which employment reaches this cap. Hence, we solve for this level:

$$\overline{c}^T_{\overline{w}} = \left[\left(\frac{\omega}{1-\omega}\right)\left(\frac{\overline{w}}{\alpha}\right)\right]^{\frac{1}{1+\mu}} \overline{h}^{\frac{1+\alpha\mu}{1+\mu}}.$$

In levels of tradable consumption above this threshold, the supply of labor in the economy will be in full employment. □

## Proof of Lemma 3

*For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level $b \in \mathbb{R}$, we have that $V_R^+(b, y^T) \geq V_R^-(b, y^T)$.*

*Proof.* Realize that problem (21) is a particular case of (20). That is,

$$\begin{aligned}
V_R^+(b, y^T) &= \max_{b', h \leq \overline{h}} \left\{u(y^T - \delta b + \tilde{q}(b', y^T)\left(b' - (1-\delta)b\right), h) + \beta\mathbb{E}\left[V(b', \mathbf{s}')\right]\right\} \\
&\geq \max_{h \leq \overline{h}} \left\{u(y^T - \delta b, h) + \beta\mathbb{E}\left[V((1-\delta)b, \mathbf{s}')\right]\right\} \\
&= V_R^-(b, y^T),
\end{aligned}$$

where both problems satisfy the same labor and wage constraints. □

## Proof of Proposition 1

*Under a flexible exchange rate regime, the government chooses an exchange rate that delivers full employment in all states.*

*Proof.* The value of repayment when the government can choose the exchange rate is given by the following Bellman equation:

$$V_R(b, \mathbf{s}) = \max_{b', c^T, h \leq \overline{h}, e} \left\{ u(c^T, F(h)) + \beta \mathbb{E} V(b', \mathbf{s}') \right\} \tag{35}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta)b)$$

$$\mathcal{W}(c^T, h)e \geq \overline{W}$$

Meanwhile, the value of default when the government can choose the exchange rate is given by the following Bellman equation:

$$V_D(y^T) = \max_{c^T, h \leq \overline{h}, e} \left\{ u\left(c^T, F(h)\right) - \kappa(y^T) + \beta \mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi) V_D(y^{T\prime})\right] \right\} \tag{36}$$

$$\text{subject to}$$

$$c^T = y^T$$

$$\mathcal{W}(c^T, h)e \geq \overline{W}$$

It is immediate from (35) and (36) that an increase in $e$ relaxes the wage rigidity constraint without tightening any other constraint. Fully relaxing the wage rigidity constraint allows the government to achieve full employment.

□

## Proof of Lemma 4

*The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect to debt $b$.*

*Proof.* Suppose two different debt values $b_1, b_2 \in \mathbb{R}$ such that $b_1 > b_2$. First, analyze the tradable resource constraint of $V_R^-$ for these two values of debt:

$$c_1^T = y^T - \delta b_1 < y^T - \delta b_2 = c_2^T.$$

Likewise, do the same for the $V_R^+$ tradable resource constraint:

$$\begin{aligned} c_1^T &= y^T + \tilde{q}(b', y^T)b' - \left(\delta + \tilde{q}(b', y^T)(1-\delta)\right) b_1 \\ &< y^T + \tilde{q}(b', y^T)b' - \left(\delta + \tilde{q}(b', y^T)(1-\delta)\right) b_2 = c_2^T. \end{aligned}$$

In other words, the budget constraint is tighter when debt is higher. Furthermore, for both problems, the wage rigidity constraint will imply that

$$\mathcal{W}(c_1^T, h_1) \leq \mathcal{W}(c_2^T, h_2),$$

where $h_1 \leq h_2$. Therefore, we can conclude that $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing. □

## Proof of Lemma 5

*For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exist levels of debt $\bar{b}^+, \bar{b}^- \in \mathbb{R}$ such that $\tilde{V}_D(y^T) = V_R^+\left(\bar{b}^+, y^T\right)$ and $\tilde{V}_D(y^T) = V_R^-\left(\bar{b}^-, y^T\right)$. Furthermore, it also satisfies $\bar{b}^+ \geq \bar{b}^-$.*

*Proof.* First, realize that for every level of tradable endowment $y^T \in \mathbb{R}_+$, if $b = 0$ then $V_D(y^T) \leq \tilde{V}_R^-(0, y^T) \leq \tilde{V}_R^+(0, y^T)$. Now, pick a level of debt outrageously high $b >> 0$, then $\tilde{V}_D(y^T) > \tilde{V}_R^+(b, y^T) \geq \tilde{V}_R^-(b, y^T)$. Because $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are continuous functions, then there exist levels of debt $\bar{b}^+, \bar{b}^- \in \mathbb{R}$ such that $\tilde{V}_D(y^T) = \tilde{V}_R^+\left(\bar{b}^+, y^T\right)$ and $\tilde{V}_D(y^T) = \tilde{V}_R^-\left(\bar{b}^-, y^T\right)$. Acknowledge that for every level of endowment $y^T \in \mathbb{R}_+$

$$\tilde{V}_R^-\left(\bar{b}^-, y^T\right) = \tilde{V}_D(y^T) = \tilde{V}_R^+\left(\bar{b}^+, y^T\right) \geq \tilde{V}_R^-\left(\bar{b}^+, y^T\right).$$

Using that $V_R^-$ is decreasing, we can conclude that $\bar{b}^+ \geq \bar{b}^-$. □

## Auxiliary Lemmas for Propositions 2, 3, 4

**Lemma A1** (Default Real Wage Rigidity Neutrality)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists a real wage rigidity $\overline{w}_D \in \mathbb{R}_+$ such that for $\overline{w}_1, \overline{w}_2 \leq \overline{w}_D$ the value of default is the same, $\tilde{V}_D\left(y^T; \overline{w}_1\right) = \tilde{V}_D\left(y^T; \overline{w}_2\right)$.*

*Proof.* Pick an arbitrary level of tradable endowment $y^T \in \mathbb{R}_+$. Using the real wage function, define the real wage rigidity $\overline{w}_D \in \mathbb{R}_+$ under full employment as

$$\overline{w}_D \equiv \mathcal{W}(y^T, \overline{h}) = \frac{1-\omega}{\omega}\left(\frac{y^T}{F(\overline{h})}\right)^{1+\mu} F'(\overline{h}).$$

This level $\overline{w}_D$ is the highest level of wage rigidity where full employment can be achieved. This means that if we pick two arbitrary real wage rigidities such that $\overline{w}_1, \overline{w}_2 \leq \overline{w}_D$, then the default state is in full employment because the real wage constraint is not binding. Therefore, the optimal allocations of full employment are achieved and are the same, $\tilde{V}_D\left(y^T;\overline{w}_1\right)=\tilde{V}_D\left(y^T;\overline{w}_2\right)$. $\square$

**Lemma A2** (Repayment Real Wages Ordering)**.** *For every level of tradable endowment $y^T\in\mathbb{R}_+$ and level of debt $b\in\mathbb{R}_+$, the repayment real wages in equilibrium under full employment is higher than in the real wages when there is a rollover crisis and full employment is achieved.*

*Proof.* Pick an arbitrary level of tradable endowment $y^T\in\mathbb{R}_+$ and debt level $b\in\mathbb{R}_+$. Using Proposition 1, we can guarantee that full employment is always achieved. Call $\hat{b}$ the optimal solution for the problem $\tilde{V}_R^+$. Define the consumption level of tradables $\hat{c}_R^+$ and $\hat{c}_R^-$ for the problems $\tilde{V}_R^+$ and $\tilde{V}_R^-$, respectively. These are

$$\hat{c}_R^+=y^T-\delta b+\tilde{q}(b,y^T)\left(\hat{b}'-(1-\delta)b\right)\qquad\text{and}$$

$$\hat{c}_R^-=\begin{cases}y^T-\delta b+\tilde{q}(b,y^T)\left(\hat{b}-(1-\delta)b\right) & \text{if }\hat{b}<(1-\delta)b\\ y^T-\delta b & \text{if }\hat{b}\geq(1-\delta)b\end{cases}.$$

In other words, by construction we know that $\hat{c}_R^+\geq\hat{c}_R^-$. Using Lemma 1, we can conclude that $\mathcal{W}\left(\hat{c}_R^+,\overline{h}\right)\geq\mathcal{W}\left(\hat{c}_R^-,\overline{h}\right)$. $\square$

**Lemma A3** (Default Region Neutrality)**.** *For every level of tradable endowment $y^T\in\mathbb{R}_+$, there exists a real wage rigidity $\overline{w}_C\in\mathbb{R}_+$ such that, for any $\overline{w}_1,\overline{w}_2\leq\overline{w}_C$, the default region is unchanged $\tilde{\mathcal{D}}\left(y^T;\overline{w}_1\right)=\tilde{\mathcal{D}}\left(y^T;\overline{w}_2\right)$.*

*Proof.* Pick an arbitrary level of tradable endowment $y^T\in\mathbb{R}_+$. Set for a moment $\overline{w}=0$ in (24) and (25) and call $\overline{b}\in\mathbb{R}_+$ the level of debt that matches $\tilde{V}_R^+\left(\overline{b},y^T;0\right)=\tilde{V}_D\left(y^T;0\right)$. Also, call $\hat{b}\in\mathbb{R}_+$ the optimal level of debt that solves (25). Now, define the real wage floor $\overline{w}_R^+\in\mathbb{R}_+$ such that

$$\overline{w}_R^+=\mathcal{W}\left(\hat{c}_T^+,\overline{h}\right)=\left(\frac{1-\omega}{\omega}\right)\left(\frac{y^T-\delta\overline{b}+\tilde{q}\left(\hat{b},y^T\right)\left(\hat{b}-(1-\delta)\overline{b}\right)}{F\left(\overline{h}\right)}\right)^{1+\mu}F'\left(\overline{h}\right).$$

Using Lemma A3, call $\overline{w}_C\equiv\min\left\{\overline{w}_D,\overline{w}_R^+\right\}$ and pick two arbitrary real wage rigidities $\overline{w}_1,\overline{w}_2\leq\overline{w}_C$. Using Lemma 5, call the thresholds $\overline{b}_1^+,\overline{b}_2^+\in\mathbb{R}_+$ for the problems under real wage floors $\overline{w}_1$ and $\overline{w}_2$. Acknowledge that with these real wage floors, full employment is achieved. Then it follows that

$$\tilde{V}_R^+\left(\overline{b}_1,y^T;\overline{w}_1\right)=\tilde{V}_D\left(y^T;\overline{w}_1\right)=\tilde{V}_D\left(y^T;\overline{w}_2\right)=\tilde{V}_R^+\left(\overline{b}_2,y^T;\overline{w}_2\right).$$

This implies that $\overline{b}_1=\overline{b}_2$, leading to the conclusion that $\tilde{\mathcal{D}}\left(y^T;\overline{w}_1\right)=\tilde{\mathcal{D}}\left(y^T;\overline{w}_2\right)$. $\square$

**Lemma A4** (Safe Region Neutrality)**.** *For every level of tradable endowment $y^T\in\mathbb{R}_+$, there exists a real wage rigidity $\overline{w}_S\in\mathbb{R}_+$ such that, for any $\overline{w}_1,\overline{w}_2\leq\overline{w}_S$, the default region is unchanged $\tilde{\mathcal{S}}\left(y^T;\overline{w}_1\right)=\tilde{\mathcal{S}}\left(y^T;\overline{w}_2\right)$.*

*Proof.* Pick an arbitrary level of tradable endowment $y^T \in \mathbb{R}_+$. Set for a moment $\overline{w} = 0$ in (24) and (26) and call $\bar{b} \in \mathbb{R}_+$ the level of debt that matches $\tilde{V}_R^-\left(\bar{b}, y^T; 0\right) = \tilde{V}_D\left(y^T; 0\right)$. Now define the real wage floor $\overline{w}_R^- \in \mathbb{R}_+$ such that

$$\overline{w}_R^- = \mathcal{W}\left(\hat{c}_T^-, \overline{h}\right) = \left(\frac{1-\omega}{\omega}\right)\left(\frac{y^T - \delta\bar{b}}{F\left(\overline{h}\right)}\right)^{1+\mu} F'\left(\overline{h}\right).$$

Using Lemma A3, call $\overline{w}_S \equiv \min\left\{\overline{w}_D, \overline{w}_R^-\right\}$ and pick two arbitrary real wage rigidities $\overline{w}_1, \overline{w}_2 \leq \overline{w}_S$. Using Lemma 5, call the thresholds $\overline{b}_1^-, \overline{b}_2^- \in \mathbb{R}_+$ for the problems under real wage floors $\overline{w}_1$ and $\overline{w}_2$, respectively. Acknowledge that with these real wage floors, full employment is achieved. Then it follows that

$$\tilde{V}_R^-\left(\bar{b}_1, y^T; \overline{w}_1\right) = \tilde{V}_D\left(y^T; \overline{w}_1\right) = \tilde{V}_D\left(y^T; \overline{w}_2\right) = \tilde{V}_R^-\left(\bar{b}_2, y^T; \overline{w}_2\right).$$

This implies that $\bar{b}_1 = \bar{b}_2$, leading to the conclusion that $\tilde{\mathcal{S}}\left(y^T; \overline{w}_1\right) = \tilde{\mathcal{S}}\left(y^T; \overline{w}_2\right)$. □

**Lemma A5** (Safe Region Strict Contraction)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, for any $\overline{w}_1, \overline{w}_2 \in \mathbb{R}_+$ such that $\overline{w}_2 > \overline{w}_S$ and $\overline{w}_1 < \overline{w}_2$, then $\tilde{\mathcal{S}}\left(y^T; \overline{w}_2\right) \subset \tilde{\mathcal{S}}\left(y^T; \overline{w}_1\right)$.*

*Proof.* Pick an arbitrary level of tradable endowment $y^T \in \mathbb{R}_+$ and real wage rigidities $\overline{w}_1, \overline{w}_2 \in \mathbb{R}_+$ such that $\overline{w}_2 > \overline{w}_S$ and $\overline{w}_1 < \overline{w}_2$. Using Lemma 5, call the thresholds $\overline{b}_1^-, \overline{b}_2^- \in \mathbb{R}_+$ for the problems under real wage rigidities $\overline{w}_1$ and $\overline{w}_2$, respectively. Call $h_1, h_2 \in \mathbb{R}_+$ the labor in the economy under real wage rigidities $\overline{w}_1$ and $\overline{w}_2$, respectively. Using Lemma 1, it follows that $h_2 < h_1 \leq \overline{h}$. Hence, it follows that $\tilde{V}_R^-\left(\overline{b}_2^-, y^T; \overline{w}_2\right) < \tilde{V}_R^-\left(\overline{b}_2^-, y^T; \overline{w}_1\right)$. Thus,

$$\tilde{V}_R^-\left(\overline{b}_2^-, y^T; \overline{w}_1\right) > \tilde{V}_R^-\left(\overline{b}_2^-, y^T; \overline{w}_2\right) = \tilde{V}_D\left(y^T; \overline{w}_2\right) = \tilde{V}_D\left(y^T; \overline{w}_1\right) = \tilde{V}_R^-\left(\overline{b}_1^-; \overline{w}_1\right).$$

Using Lemma 4, we arrive at $\overline{b}_1^- > \overline{b}_2^-$. Finally, this tells us that $\tilde{\mathcal{S}}\left(y^T; \overline{w}_2\right) \subset \tilde{\mathcal{S}}\left(y^T; \overline{w}_1\right)$. □

## Proof of Proposition 2

*For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{D}(y^T; \overline{w}_1) \subseteq \tilde{D}(y^T; \overline{w}_2)$.*

*Proof.* Pick arbitrary levels of tradable endowment $y^T \in \mathbb{R}_+$, and real wage rigidities $\overline{w}_1, \overline{w}_2 \leq \overline{w}_D$ where $\overline{w}_1 > \overline{w}_2$. Using Lemma A1, we know that there exists $\overline{w}_D \in \mathbb{R}_+$ such that $\tilde{V}_D\left(y^T; \overline{w}_1\right) = \tilde{V}_D\left(y^T; \overline{w}_2\right)$. Using Lemma 5, define $\overline{b}_1^+$ and $\overline{b}_2^+$ as the debt thresholds that limit the default region under real wage rigidites $\overline{w}_1$ and $\overline{w}_2$, respectively. Acknowledging that a higher real wage rigidity makes the problem of repayment when new debt contracts are allowed more constrained, we know that $\tilde{V}_R^+ \left(b, y^T; \overline{w}_2\right) \geq \tilde{V}_R^+ \left(b, y^T; \overline{w}_1\right)$ for any amount of debt $b \in \mathbb{R}$. Thus,

$$\tilde{V}_R^+ \left(\overline{b}_1^+, y^T; \overline{w}_1\right) = \tilde{V}_D \left(y^T, ; \overline{w}_1\right) = \tilde{V}_D \left(y^T, ; \overline{w}_2\right) = \tilde{V}_R^+ \left(\overline{b}_2^+, y^T; \overline{w}_2\right) \geq \tilde{V}_R^+ \left(\overline{b}_2^+, y^T; \overline{w}_1\right).$$

Using Lemma 4, it follows that $\overline{b}_2^+ \geq \overline{b}_1^+$. This implies that $\tilde{D}(y^T; \overline{w}_2) \subseteq \tilde{D}(y^T; \overline{w}_1)$. □

## Proof of Proposition 3

*For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{S}(y^T; \overline{w}_2) \subseteq \tilde{S}(y^T; \overline{w}_1)$.*

*Proof.* Pick arbitrary levels of tradable endowment $y^T \in \mathbb{R}_+$ and real wage rigidities $\overline{w}_1, \overline{w}_2 \leq \overline{w}_D$ where $\overline{w}_1 > \overline{w}_2$. Using Lemma A1, we know that there exists $\overline{w}_D \in \mathbb{R}_+$ such that $\tilde{V}_D \left(y^T; \overline{w}_1\right) = \tilde{V}_D \left(y^T; \overline{w}_2\right)$. Using Lemma 5, define $\overline{b}_1^-$ and $\overline{b}_2^-$ as the debt thresholds that limit the safe region under real wage rigidites $\overline{w}_1$ and $\overline{w}_2$, respectively. Acknowledging that a higher real wage rigidity makes the value of repayment under no borrowing more likely to bind and result in possibly more unemployment, we know that $V_R^- \left(b, y^T; \overline{w}_2\right) \geq V_R^- \left(b, y^T; \overline{w}_1\right)$ for any amount of debt $b \in \mathbb{R}$. Thus,

$$\tilde{V}_R^- \left(\overline{b}_1^-, y^T; \overline{w}_1\right) = \tilde{V}_D \left(y^T; \overline{w}_1\right) = \tilde{V}_D \left(y^T; \overline{w}_2\right) = \tilde{V}_R^- \left(\overline{b}_2^-, y^T; \overline{w}_2\right) \geq \tilde{V}_R^- \left(\overline{b}_2^-, y^T; \overline{w}_1\right).$$

Using Lemma 4, it follows that $\overline{b}_2^- \geq \overline{b}_1^-$. This implies that $\tilde{S}(y^T; \overline{w}_1) \subseteq \tilde{S}(y^T; \overline{w}_2)$. □

## Proof of Proposition 4

*For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists $\overline{w}_C \in \mathbb{R}_+$ such that if $\overline{w}_1, \overline{w}_2 < \overline{w}_C$ and $\overline{w}_1 < \overline{w}_2$, then $\tilde{C}(y^T; \overline{w}_1) \subseteq \tilde{C}(y^T; \overline{w}_2)$. Moreover, there exists $\overline{w}_S \in \mathbb{R}_+$ such that if $\overline{w}_2 > \overline{w}_S$ then $\tilde{C}(y^T; \overline{w}_1) \subset \tilde{C}(y^T; \overline{w}_2)$.*

*Proof.* Pick an arbitrary level of tradable endowment $y^T \in \mathbb{R}_+$. Using Lemma A3, there exists $\overline{w}_C \in \mathbb{R}_+$ such that if $\overline{w}_1, \overline{w}_2 < \overline{w}_C$ and $\overline{w}_1 < \overline{w}_2$, then $\tilde{\mathcal{D}}\left(y^T; \overline{w}_2\right) = \tilde{\mathcal{D}}\left(y^T; \overline{w}_1\right)$. Using Lemma A4, there exists $\overline{w}_S$ such that if $\overline{w}_1, \overline{w}_2 < \overline{w}_S$ and $\overline{w}_1 < \overline{w}_2$, then $\tilde{\mathcal{S}}\left(y^T; \overline{w}_2\right) = \tilde{\mathcal{S}}\left(y^T; \overline{w}_1\right)$. Using Lemma **??**, we can arrive at the conclusion that $\tilde{C}(y^T; \overline{w}_1) \subseteq \tilde{C}(y^T; \overline{w}_2)$. Furthermore, if $\overline{w}_2 > \overline{w}_S$, then $\tilde{C}(y^T; \overline{w}_1) \subset \tilde{C}(y^T; \overline{w}_2)$. □

# Online Addendum to "Monetary Independence and Rollover Crises"

By Javier Bianchi and Jorge Mondragon

In this appendix, we show that our main theoretical results hold in various extensions from our baseline model. In Section A we consider sticky prices. In Section B, we consider a version of the model in which there are costs from exchange rate fluctuations, providing a rationale for adopting a fixed exchange rate or joining a monetary union. In Section C, we consider an inflation targeting regime, that provides an "intermediate" regime between a flexible exchange rate regime that seeks to achieve full employment at every period, and a fixed exchange rate regime. In Section D, we show how the results can be extended in a model with a rich maturity structure. In Section E, we consider an elastic labor supply.

## A Sticky Prices

In this section, we explore a price rigidity as an alternative to wage rigidity. We assume that wages and the price of tradables are flexible, and that the nominal nontradable price in the economy cannot go lower than a threshold $\overline{P} > 0$.[38] Using the disequilibrium formulation of Barro and Grossman (1971), we have that firms supply $h^s = F^{-1}(h)$ whenever $p^N > \overline{P}$.

The constraint in the economy can be described as $P^N \geq \overline{P}$. Using the optimality condition of the household and the nontradable market clearing condition, we can construct the real nontradable price function as

$$\mathcal{P}(c^T, h) \equiv \frac{P^N}{e} = \frac{1-\omega}{\omega}\left(\frac{c^T}{F(h)}\right)^{1+\mu}$$

**Lemma A6.** *The real nontradable price function is increasing in consumption of tradables and decreasing in labor.*

*Proof.* Taking the first derivatives with respect of consumption of tradables and labor

$$\frac{\partial \mathcal{P}}{\partial c^T} = (1+\mu)\frac{\mathcal{P}(c^T, h)}{c^T} > 0 \qquad \& \qquad \frac{\partial \mathcal{P}}{\partial h} = -(1+\mu)F'(h)\frac{\mathcal{P}(c^T, h)}{F(h)} < 0.$$

Hence, the real nontradable price function is increasing in consumption of tradables and decreasing in labor. □

We will first define the government problem and the bond pricing under this new environment. The problem of the government either to default or repay debt can be described as

$$V(b, \mathbf{s}) = \max_{d \in \{0,1\}} \left\{ dV_D(y^T) + (1-d)V_R(b, \mathbf{s}) \right\}.$$

[38] We could also assume that the price rigidity goes in both directions, but we model the asymmetry to have a more direct comparison with the model with downward wage rigidity. The approach of a downward price rigidity is commonly attributed to "social norms" and is followed for example by FCaballero and Farhi (2017).

In this way, the maximization problem in default can be described as

$$
\begin{aligned}
V_D\left(y^T\right) = \max_{e,c^T,h\leq\bar{h}} & \left\{u\left(c^T, F\left(h\right)\right) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[\psi V\left(0, \mathbf{s}'\right) + (1-\psi)V_D\left(y^{T\prime}\right)\right]\right\} \\
\text{s.t. } & c^T = y^T \\
& \overline{P} \leq e\mathcal{P}(c^T, h)
\end{aligned}
$$

The value of repayment transforms to

$$
\begin{aligned}
V_R\left(b, \mathbf{s}\right) = \max_{e,b',c^T,h\leq\bar{h}} & \left\{u\left(c^T, F\left(h\right)\right) + \beta\mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\
\text{s.t. } & c^T = y^T - \delta b + q\left(b', b, \mathbf{s}\right)\left(b' - (1-\delta)b\right) \\
& \overline{P} \leq e\mathcal{P}(c^T, h)
\end{aligned}
$$

**Proposition A1** (Nominal Rigidities Equivalence)**.** *Suppose the production function $F(h)$ is linear, then if $\left\{V, V_D, V_R, q, \hat{b}\right\}$ is the solution of the Markov Recursive Equilibrium from Definition 2 with a downward nominal wage rigidity $\overline{W} \in \mathbb{R}_{=}$, then it also an equilibrium for the environment in this section when the downward nominal nontradable price rigidity satisfies $\overline{P} \equiv \overline{W}/F'(h)$.*

*Proof.* By assuming that the production function is linear, then $F'(h) = \phi$ is a constant that does not depend in labor. Realize that the only difference between the models are the downward nominal rigidities, so inspecting the downward nominal wage rigidity

$$
\overline{W} \leq e\mathcal{W}(c^T, h) = e\frac{1-\omega}{\omega}\left(\frac{c^T}{F(h)}\right)^{1+\mu} F'(h) = e\mathcal{P}(c^T, h)\phi.
$$

By defining the downward nominal nontradable price rigidity $\overline{P} \equiv \overline{W}/\phi$, we conclude that both rigidities are the same. Hence, the solutions are the same. □

The value of repayment can be studied as before under two different scenarios: when rollover debt is allowed and when it is not. Let us start by analyzing the problem when new debt contracts can be issued and hence rollover debt is allowed. Under this scenario, the bond pricing that satisfies the no-arbitrage condition by the part of the international lenders is applied. Then, the value of repayment when rollover is allowed transforms to

$$
\begin{aligned}
V_R^+\left(b, y^T\right) = \max_{e,b',c^T,h\leq\bar{h}} & \left\{u\left(c^T, F\left(h\right)\right) + \beta\mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\
\text{s.t. } & c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b \\
& \overline{P} \leq e\mathcal{P}(c^T, h)
\end{aligned}
\tag{A.1}
$$

Call $\hat{b}\left(b, y^T\right)$ the optimal solution of new portfolio debt maturities that solve the previous problem. As before, call the state space in which it is optimal for the government to increase debt issuances as

$$
\mathcal{B} = \left\{\left(b, y^T\right) \in \mathbb{R}^\infty \times \mathbb{R}_+ : \quad \hat{b}\left(b, y^T\right) > (1-\delta)b\right\}.
$$

As before, the value of repayment when rollover is not allowed can be divided into two cases. When $\left(b, y^T\right) \notin \mathcal{B}$, the government finds optimal to reduce debt issuances. In this case, we can say that $V_R^-\left(b, y^T\right) = V_R^+\left(b, y^T\right)$ because the government is buying back its debt. Neverthless, if $\left(b, y^T\right) \in \mathcal{B}$, then the government wants to increase its debt issuances. International lenders set a price of $\tilde{q} = 0$ representing their reluctancy to rollover debt. In this way, the value of repayment when new debt contracts are forbidden can be expressed as

$$V_R^- \left(b, y^T\right) = \max_{e, c^T, h \leq \bar{h}} \left\{u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V\left((1-\delta)b, \mathbf{s}'\right)\right]\right\} \tag{A.2}$$
$$\text{s.t. } c^T = y^T - \delta b$$
$$\overline{P} \leq e\mathcal{P}(c^T, h)$$

The following Lemma follows the same steps as the one stated before following the fact that $V_R^-$ is a particular case of $V_R^+$ maximization problem.

**Lemma A7.** *For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level $b$, we have that $V_R^+\left(b, y^T\right) \geq V_R^-\left(b, y^T\right)$.*

Now, let us define the safe zone, default zone, and repayment zone contingent to the portfolio of different debt maturities as

$$\begin{aligned}
\mathcal{S} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^-\left(b, y^T\right)\right\} \\
\mathcal{D} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) > V_R^+\left(b, y^T\right)\right\} \\
\mathcal{C} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^+\left(b, y^T\right) \quad \& \quad V_D(y^T) > V_R^-\left(b, y^T\right)\right\}.
\end{aligned}$$

Using these zones, the bond pricing following the no-arbitrage condition for each maturity structure, can be represented with the following recursion

$$\tilde{q}(b', y^T) = \frac{1}{1+r}\mathbb{E}\left[\left(1 - d(b', \mathbf{s}')\right)\left(\delta + (1-\delta)q\left(\hat{b}\left(b', \mathbf{s}'\right), b', \mathbf{s}'\right)\right)\right].$$

Finally, using the zones and the multiplicity of equilibria, the overall bond pricing can be described as

$$q\left(b', b, \mathbf{s}\right) = \begin{cases} 0 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ \tilde{q}(b', y^T) & \textit{in every other case} \end{cases}$$

and the optimal default decision as

$$d\left(b, \mathbf{s}\right) = \begin{cases} 1 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \& \quad \zeta = 0 \\ 1 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{S} \end{cases}$$

The following proposition follows the same steps as the one stated before.

**Proposition A2** (Optimal Exchange Rate Policy)**.** *Under a flexible exchange rate regime, the government chooses an en exchange rate that delivers full employment in all states*

Let us focus now in the flexible exchange rate regime and solve the model. Call the flexible exchange rate regime solutions $\left\{V^{flex}, V_D^{flex}, \tilde{q}^{flex}\right\}$ and let us study the one period fixed exchange rate regime shocks. To do this, let me define the downward real nontradable price rigidity $\overline{p} \equiv \overline{P}/\overline{e}$ under a fixed exchange rate regime. The value of default will transform to

$$
\begin{aligned}
\tilde{V}_D\left(y^T\right) = \max_{c^T, h \le \bar{h}} & \left\{u\left(c^T, F\left(h\right)\right) - \kappa\left(y^T\right) + \beta \mathbb{E}\left[\psi V^{flex}\left(\mathbf{0}, \mathbf{s}'\right) + (1-\psi) V_D^{flex}\left(y^{T\prime}\right)\right]\right\} \\
\text{s.t. } & c^T = y^T \\
& \overline{p} \le \mathcal{P}(c^T, h)
\end{aligned}
$$

Also, the value of repayment when rollover debt is allowed

$$
\begin{aligned}
\tilde{V}_R^+\left(b, y^T\right) = \max_{b', c^T, h \le \bar{h}} & \left\{u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V^{flex}\left(b', \mathbf{s}'\right)\right]\right\} \\
\text{s.t. } & c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b \\
& \overline{p} \le \mathcal{P}(c^T, h)
\end{aligned}
$$

Finally, the value of repayment when new debt contracts of any maturity are forbidden then

$$
\begin{aligned}
\tilde{V}_R^-\left(b, y^T\right) = \max_{c^T, h \le \bar{h}} & \left\{u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V^{flex}\left((1-\delta)b, \mathbf{s}'\right)\right]\right\} \\
\text{s.t. } & c^T = y^T - \delta b \\
& \overline{p} \le \mathcal{P}(c^T, h)
\end{aligned}
$$

The following lemmas and propositions follow the same steps stated in the section before.

**Lemma A8.** *The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect the debt level $b$*

**Lemma A9** (Debt Thresholds)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists levels of debt that currently matures $\overline{b}^+, \overline{b}^- \in \mathbb{R}$, such that $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^+\left(\overline{b}^+, y^T\right)$ and $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^-\left(\overline{b}^-, y^T\right)$. Furthermore, $\overline{b}^+ \ge \overline{b}^-$.*

Now call the regions as

$$
\tilde{S}\left(y^T\right) \equiv \left(-\infty, \overline{b}^-\right], \qquad \tilde{C}\left(y^T\right) \equiv \left(\overline{b}^-, \overline{b}^+\right], \qquad \text{and} \qquad \tilde{D}\left(y^T\right) \equiv \left(\overline{b}^+, \infty\right).
$$

The following propositions follow the same steps stated in the section before.

**Proposition A3** (Default Region Expansion)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists $\overline{p}_D \in \mathbb{R}_+$ such that if $\overline{p}_1 < \overline{p}_2 \le \overline{p}_D$, then $\tilde{\mathcal{D}}\left(y^T; \overline{p}_1\right) \subseteq \tilde{\mathcal{D}}\left(y^T; \overline{p}_2\right)$.*

**Proposition A4** (Safe Region Contraction)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists $\overline{p}_D \in \mathbb{R}_+$ such that if $\overline{p}_1 < \overline{p}_2 \le \overline{p}_D$, then $\tilde{\mathcal{S}}\left(y^T; \overline{p}_2\right) \subseteq \tilde{\mathcal{S}}\left(y^T; \overline{p}_1\right)$.*

**Proposition A5** (Crisis Region Expansion)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists $\overline{p}_C \in \mathbb{R}_+$ such that if $\overline{p}_1 < \overline{p}_2 \le \overline{p}_C$, then $\tilde{\mathcal{C}}\left(y^T; \overline{p}_1\right) \subseteq \tilde{\mathcal{C}}\left(y^T; \overline{p}_2\right)$. Moreover, there exists $\overline{p}_S \in \mathbb{R}_+$ such that if $\overline{p}_2 > \overline{p}_S$, then $\tilde{\mathcal{C}}\left(y^T; \overline{p}_1\right) \subset \tilde{\mathcal{C}}\left(y^T; \overline{p}_2\right)$.*

Figure 11 compares the crisis region under rigid wages and rigid prices, and shows that these two forms of rigidities yield very similar implications.

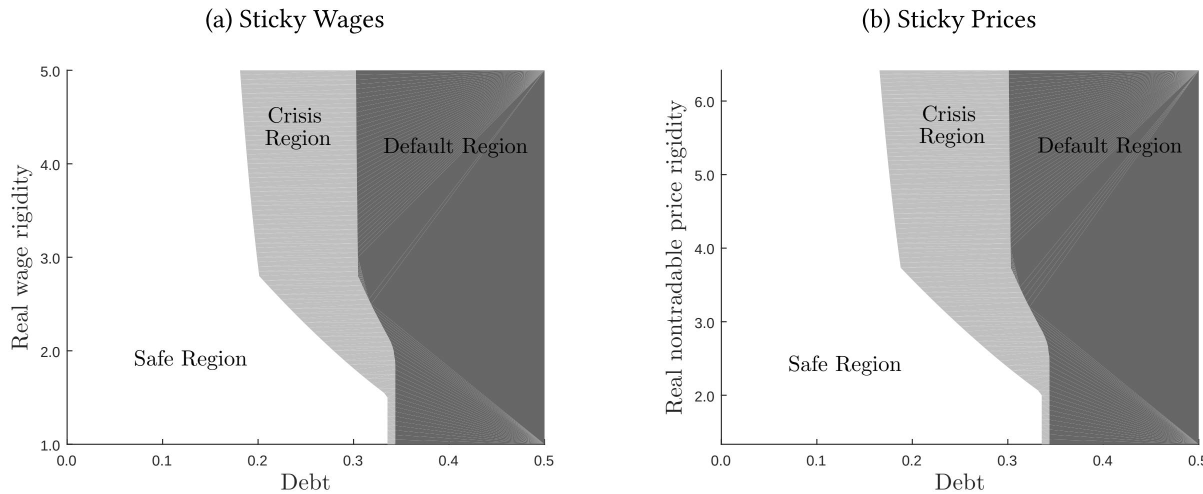


Figure 11: Regions change under different downward nominal rigidities

Notes: The tradable endowment is fixed to its long-run level. The grid for the real nontradable price rigidity is set from its level in default corresponding to the real wage rigidity grid.

# B Devaluation Costs

In this section, we explore a version of the model in which the government can choose the exchange rate every period, but there is a cost associated with exchange rate fluctuations. We consider two variants: in one version there are costs from current depreciations costs and in another version, there are costs from future expected depreciations. Both versions provide a rationale from joining a monetary union or fixing the exchange rate.

## B.1 Costs from Current Depreciations

We assume that exchange rate devaluations above a "natural" level $\overline{e} > 0$, incur in a penalty $\Phi\left(e - \overline{e}\right) \geq 0$ that satisfies $\Phi(0) = \Phi'(0) = 0$ and $\Phi'(\cdot) > 0$. Using the downward nominal rigidity constraint and the properties of the devaluation utility cost, the optimal exchange rate can be expressed as

$$e(c^T) = \overline{e} \cdot \max\left\{ \frac{\overline{w}}{\mathcal{W}\left(c^T, \overline{h}\right)}, 1 \right\} \tag{B.1}$$

We will first define the government problem and the bond pricing under this new environment. The problem of the government either to default or repay debt can be described as

$$V(b, \mathbf{s}) = \max_{d \in \{0,1\}} \left\{ dV_D(y^T) + (1-d)V_R\left(b, \mathbf{s}\right) \right\}.$$

In this way, the maximization problem in default can be described as

$$\begin{aligned} V_D\left(y^T\right) = \max_{e, c^T} & \left\{ u\left(c^T, F\left(\overline{h}\right)\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta \mathbb{E}\left[\psi V\left(0, \mathbf{s}'\right) + (1-\psi) V_D\left(y^{T\prime}\right)\right] \right\} \\ \text{s.t. } & c^T = y^T \end{aligned}$$

The value of repayment transforms to

$$V_R(b, \mathbf{s}) = \max_{e,b',c^T} \left\{ u\left(c^T, F\left(\overline{h}\right)\right) - \kappa\left(y^T\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta \mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right] \right\}$$
$$\text{s.t. } c^T = y^T - \delta b + q\left(b', b, \mathbf{s}\right)\left(b' - (1-\delta)b\right)$$

The value of repayment can be studied as before under two different scenarios: when rollover debt is allowed and when it is not. Let us start by analyzing the problem when new debt contracts can be issued and hence rollover debt is allowed. Under this scenario, the bond pricing that satisfies the no-arbitrage condition by the part of the international lenders is applied. Then, the value of repayment when rollover is allowed transforms to

$$V_R^+\left(b, y^T\right) = \max_{e,b',c^T} \left\{ u\left(c^T, F\left(\overline{h}\right)\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta \mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right] \right\} \tag{B.2}$$
$$\text{s.t. } c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b$$

Call $\hat{b}\left(b, y^T\right)$ the optimal solution of new portfolio debt maturities that solve the previous problem. As before, call the state space in which it is optimal for the government to increase debt issuances as

$$\mathcal{B} = \left\{ \left(b, y^T\right) \in \mathbb{R}^\infty \times \mathbb{R}_+ : \quad \hat{b}\left(b, y^T\right) > (1-\delta)b \right\}.$$

As before, the value of repayment when rollover is not allowed can be divided into two cases. When $\left(b, y^T\right) \notin \mathcal{B}$, the government finds optimal to reduce debt issuances. In this case, we can say that $V_R^-\left(b, y^T\right) = V_R^+\left(b, y^T\right)$ because the government is buying back its debt. Neverthless, if $\left(b, y^T\right) \in \mathcal{B}$, then the government wants to increase its debt issuances. International lenders set a price of $\tilde{q} = 0$ representing their reluctancy to rollover debt. In this way, the value of repayment when new debt contracts are forbidden can be expressed as

$$V_R^-\left(b, y^T\right) = \max_{e,c^T} \left\{ u\left(c^T, F\left(\overline{h}\right)\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta \mathbb{E}\left[V\left((1-\delta)b, \mathbf{s}'\right)\right] \right\} \tag{B.3}$$
$$\text{s.t. } c^T = y^T - \delta b$$

The following Lemma follows the same steps as the one stated before following the fact that $V_R^-$ is a particular case of $V_R^+$ maximization problem.

**Lemma B10.** *For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level $b$, we have that $V_R^+\left(b, y^T\right) \geq V_R^-\left(b, y^T\right)$.*

Now, let us define the safe zone, default zone, and repayment zone contingent to the portfolio of different debt maturities as

$$\begin{aligned}
\mathcal{S} &\equiv \left\{ \left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^-\left(b, y^T\right) \right\} \\
\mathcal{D} &\equiv \left\{ \left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) > V_R^+\left(b, y^T\right) \right\} \\
\mathcal{C} &\equiv \left\{ \left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^+\left(b, y^T\right) \quad \& \quad V_D(y^T) > V_R^-\left(b, y^T\right) \right\}.
\end{aligned}$$

Using these zones, the bond pricing following the no-arbitrage condition for each maturity structure, can be represented with the following recursion

$$\tilde{q}(b', y^T) = \frac{1}{1+r} \mathbb{E}\left[ \left(1 - d(b', \mathbf{s}')\right) \left(\delta + (1-\delta) q\left(\hat{b}\left(b', \mathbf{s}'\right), b', \mathbf{s}'\right)\right) \right].$$

Finally, using the zones and the multiplicity of equilibria, the overall bond pricing can be described as

$$q\left(b', b, \mathbf{s}\right) = \begin{cases} 0 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ \tilde{q}(b', y^T) & \textit{in every other case} \end{cases}$$

and the optimal default decision as

$$d\left(b, \mathbf{s}\right) = \begin{cases} 1 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \& \quad \zeta = 0 \\ 1 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{S} \end{cases}$$

The following proposition relies that under a flexible exchange rate, the downward nominal rigidity can be ignored. In addition, when there are no devaluation costs, then the objective functions coincide in both environments. Hence the maximization problems are the same.

**Proposition B6** (No Devaluation Costs)**.** *Let* $\left\{V^{flex}, V_D^{flex}, q^{flex}, \hat{b}^{flex}\right\}$ *be a recursive equilibrium under a flexible exchange rate regime from Section 3.2, then it is also an equilibrium in this environment when there are no devaluation costs* $\Phi(\cdot) = 0$.

Let us focus now in the no devaluation costs environment. Call the no devaluation costs environment solutions $\left\{V^{flex}, V_D^{flex}, \left\{\tilde{q}_n^{flex}\right\}_{n=1}^{\infty}\right\}$ and let us study the one period devaluation cost shock. The value of default will transform to

$$\begin{aligned} \tilde{V}_D\left(y^T\right) = \max_{c^T} & \left\{u\left(c^T, F\left(\overline{h}\right)\right) - \kappa\left(y^T\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta\mathbb{E}\left[\psi V^{flex}\left(\mathbf{0}, \mathbf{s}'\right) + (1-\psi)V_D^{flex}\left(y^{T\prime}\right)\right]\right\} \\ & \text{s.t. } c^T = y^T \end{aligned}$$

Also, the value of repayment when rollover debt is allowed

$$\begin{aligned} \tilde{V}_R^+\left(b, y^T\right) = \max_{b', c^T} & \left\{u\left(c^T, F\left(\overline{h}\right)\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta\mathbb{E}\left[V^{flex}\left(b', \mathbf{s}'\right)\right]\right\} \\ & \text{s.t. } c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b \end{aligned}$$

Finally, the value of repayment when new debt contracts of any maturity are forbidden then

$$\begin{aligned} \tilde{V}_R^-\left(b, y^T\right) = \max_{c^T} & \left\{u\left(c^T, F\left(\overline{h}\right)\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta\mathbb{E}\left[V^{flex}\left((1-\delta)b, \mathbf{s}'\right)\right]\right\} \\ & \text{s.t. } c^T = y^T - \delta b \end{aligned}$$

The following lemmas and propositions follow the same steps stated in the section before.

**Lemma B11.** *The value functions* $\tilde{V}_R^+$ *and* $\tilde{V}_R^-$ *are decreasing with respect the debt level* $b$

**Lemma B12** (Debt Thresholds)**.** *For every level of tradable endowment* $y^T \in \mathbb{R}$*, there exists levels of debt that currently matures* $\overline{b}^+, \overline{b}^- \in \mathbb{R}$*, such that* $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^+\left(\overline{b}^+, y^T\right)$ *and* $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^-\left(\overline{b}^-, y^T\right)$*. Furthermore,* $\overline{b}^+ \geq \overline{b}^-$.

Now call the regions as

$$\tilde{S}\left(y^T\right) \equiv \left(-\infty, \overline{b}^-\right], \qquad \tilde{C}\left(y^T\right) \equiv \left(\overline{b}^-, \overline{b}^+\right], \qquad \text{and} \qquad \tilde{D}\left(y^T\right) \equiv \left(\overline{b}^+, \infty\right).$$

**Lemma B13** (Devaluation Costs Ordering). *For every level of tradable endowment $y^T \in \mathbb{R}_+$ and level of debt $b \in \mathbb{R}$, the devaluation and its penalty needed when borrowing is not allowed is at least as high than when rollover is allowed.*

*Proof.* Using Lemma A2, it follows that $\mathcal{W}\left(\hat{c}_R^+, \overline{h}\right) \geq \mathcal{W}\left(\hat{c}_R^-, \overline{h}\right)$. This implies by (B.1) that $e\left(\hat{c}_R^-\right) \geq e\left(\hat{c}_R^+\right)$. Moreover, because of the properties of the devaluation penalty $\Phi_R^- \geq \Phi_R^+$. $\square$

The following propositions follow the same steps stated in the section before. We use Lemma B13 to argue that the devaluation incurred in the no borrowing scenario is deeper than the one rollover is allowed. Using the properties of the devaluation utility cost, a deeper the devaluation implies the higher utility loss.

**Proposition B7** (Default Region Expansion). *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{\mathcal{D}}\left(y^T; \overline{w}_1\right) \subseteq \tilde{\mathcal{D}}\left(y^T; \overline{w}_2\right)$.*

**Proposition B8** (Safe Region Contraction). *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{\mathcal{S}}\left(y^T; \overline{w}_2\right) \subseteq \tilde{\mathcal{S}}\left(y^T; \overline{w}_1\right)$.*

**Proposition B9** (Crisis Region Expansion). *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists $\overline{w}_C \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_C$, then $\tilde{\mathcal{C}}\left(y^T; \overline{w}_1\right) \subseteq \tilde{\mathcal{C}}\left(y^T; \overline{w}_2\right)$. Moreover, there exists $\overline{w}_S \in \mathbb{R}_+$ such that if $\overline{w}_2 > \overline{w}_S$, then $\tilde{\mathcal{C}}\left(y^T; \overline{w}_1\right) \subset \tilde{\mathcal{C}}\left(y^T; \overline{w}_2\right)$.*

## B.2 Costs from Future Depreciations

In this section, we consider a version of the model in which the costs from depreciating the exchange rate arise from the expectation of *future* depreciations, rather than from the current one. We assume that every period, there is cost incurred today whenever there is positive expected depreciation. Following the section above, suppose an exogenous long-run level of exchange rate $\overline{e} > 0$, and that there is penalty $\Phi = (\mathbb{E}e_{t+1} - \overline{e}) \geq 0$ associated with an expected exchange rate above this level. We assume the following increasing convex function as the expected devaluation penalty, $\Phi\left(\mathbb{E}e_{t+1} - \overline{e}\right) = \phi\left(\mathbb{E}e_{t+1} - \overline{e}\right)^2$.

Using the downward nominal wage rigidity constraint, the equilibrium exchange rate policy can be described as a function of tradable consumption and labor

$$e(c^T, h) = \max\left\{\frac{\overline{W}}{\mathcal{W}\left(c^T, h\right)}, \overline{e}\right\}.$$

In this setup, under a discretionary optimal policy, the government will choose to deliver the devaluation that is necessary to achieve full employment ex post. This devaluation will be excessive from an ex ante point of view, in the spirit of Barro and Gordon (1983). A government would like to promise a lower depreciation in the future but ex post, the optimal policy is always to achieve full employment by depreciating the exchange rate. In a Markov equilibrium, the government will take into consideration how its current choices affect future exchange rate policies because this will affect the current devaluation costs.

Let us denote the equilibrium devaluation penalty in repayment and default scenarios as

$$\Phi_R\left(b, \mathbf{s}\right) \equiv \phi\left(e\left(\hat{c}\left(b, \mathbf{s}\right), \overline{h}\right) - \overline{e}\right)^2 \qquad \text{and} \qquad \Phi_D\left(y^T\right) \equiv \phi\left(e\left(y^T, \overline{h}\right) - \overline{e}\right)^2,$$

where the equilibrium consumption in repayment is defined as

$$\hat{c}\left(b, \mathbf{s}\right) = y^T - \delta b + q\left(\hat{b}\left(b, \mathbf{s}\right), b, \mathbf{s}\right)\left(\hat{b}\left(b, \mathbf{s}\right) - (1-\delta)b\right).$$

Firstly, the maximization problem of the government between to default or repay debt remains unchanged as

$$V\left(b, \mathbf{s}\right) = \max\left\{V_R\left(b, \mathbf{s}\right), V_D\left(y^T\right)\right\}.$$

Nevertheless, the value of default and repayment will incorporate the costs of devaluating for future periods. The maximization problem in default can be written as

$$\begin{aligned} V_D\left(y^T\right) = \max_{c^T} &\left\{u\left(c^T, F(\overline{h})\right) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[\psi\left(V\left(0, \mathbf{s}'\right) - \Phi_R\left(0, \mathbf{s}'\right)\right) + (1-\psi)\left(V_D\left(y^{T\prime}\right) - \Phi_D\left(y^{T\prime}\right)\right)\right]\right\} \\ \text{s.t.} \quad & c^T = y^T \end{aligned}$$

Meanwhile the value of repayment can be written as

$$\begin{aligned} V_R\left(b, \mathbf{s}\right) = \max_{b', c^T} &\left\{u\left(c^T, F(\overline{h})\right) + \beta\mathbb{E}\left[V\left(b', \mathbf{s}'\right) - \Phi_R\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t.} \quad & c^T - q\left(b', b, \mathbf{s}\right)\left(b' - (1-\delta)b\right) = y^T - \delta b \end{aligned}$$

The value of repayment can be studied as before under two different scenarios: when rollover debt is allowed and when it is not. Let us start by analyzing the problem when new debt contracts can be issued and hence rollover debt is allowed. Under this scenario the bond pricing that satisfies the no-arbitrage condition by the part of the international lenders is applied. Then, the value of repayment when rollover is allowed transforms to

$$\begin{aligned} V_R^+\left(b, y^T\right) = \max_{b', c^T} &\left\{u\left(c^T, F\left(\overline{h}\right)\right) + \beta\mathbb{E}\left[V\left(b', \mathbf{s}'\right) - \Phi_R\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b \end{aligned}$$

Call $\hat{b}\left(b, y^T\right)$ the optimal solution that solve the previous problem. As before, call the state space in which it is optimal for the government to increase debt issuances as

$$\mathcal{B} \equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \qquad \hat{b}(b, y^T) > (1-\delta)b\right\}.$$

As before, the value of repayment when rollover is not allowed can be divided into two cases. When $(b, y^T) \notin \mathcal{B}$, the government finds optimal to reduce debt issuances. In this case, we can say that $V_R^-(b, y^T) = V_R^+(b, y^T)$ because the government is buying back its debt. Nevertheless, if $(b, y^T) \in \mathcal{B}$, then the government wants to increase its debt issuances. International lenders set a price to debt of zero representing their reluctancy to rollover debt. In this way, the value of repayment when new debt contracts are forbidden can be expressed as

$$\begin{aligned} V_R^-\left(b, y^T\right) = \max_{c^T} &\left\{u\left(c^T, F\left(\overline{h}\right)\right) + \beta\mathbb{E}\left[V\left((1-\delta)b, \mathbf{s}'\right) - \Phi_R\left((1-\delta)b, \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T = y^T - \delta b \end{aligned}$$

Table 3 shows how an economy with flexible exchange rate but that incurs this expected devaluation costs remain relatively immune to a rollover crisis as in the baseline model.

Table 3: Sensitivity to Expected Devaluation Costs

| Devaluation penalty $\phi$ | Benchmark | 0.20 | 0.40 | 0.60 | 0.80 |
|---|---|---|---|---|---|
| Average spread | 2.40 | 2.01 | 1.37 | 1.51 | 1.43 |
| Average debt-income | 29.61 | 28.52 | 25.58 | 26.14 | 25.04 |
| Spread volatility | 1.29 | 1.11 | 0.95 | 1.00 | 1.09 |
| Unemployment increase | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Fraction of time in crisis region | 0.80 | 0.62 | 0.45 | 0.46 | 0.37 |
| Fraction of defaults due to rollover crisis | 0.96 | 1.28 | 1.16 | 0.68 | 0.92 |

Notes: All parameter values correspond to the benchmark calibrations for flexible exchange rate regimes. The benchmark calibration uses $\phi = 0.0$, this is there are no penalties from expected depreciations.

# C Inflation Targeting

In this section, we present a version of the model in which the government adopts an inflation targeting regime. The goal is to consider a regime that falls in the middle between a fully flexible exchange rate regime that achieves full employment at every period and a fixed exchange rate that fully stabilizes the nominal exchange rate. In this intermediate regime, the economy has monetary autonomy, yet, inflation targeting act as a monetary policy constraint that limits the ability to achieve full employment.

In line with the inflation targeting regime, we assume there is a long-run aggregate consumption price level $\overline{P} > 0$. Using the final consumption aggregator, we can construct the final consumption price aggregator as,

$$P\left(P^T, P^N\right) \equiv \left(\omega^{\frac{1}{1+\mu}} \left(P^T\right)^{\frac{\mu}{1+\mu}} + (1-\omega)^{\frac{1}{1+\mu}} \left(P^N\right)^{\frac{\mu}{1+\mu}}\right)^{\frac{1+\mu}{\mu}}.$$

Hence, the inflation target condition that must be satisfied can be expressed as $P\left(P^T, P^N\right) = \overline{P}$. Define the real aggregate price function as

$$\mathcal{P}\left(c^T, h\right) \equiv \frac{1}{\omega}\left(\frac{c^T}{c\left(c^T, F(h)\right)}\right)^{1+\mu}.$$

**Lemma C14.** *The inflation targeting condition yields an exchange rate policy* $e = \overline{P}/\mathcal{P}\left(c^T, h\right)$.

*Proof.* The real aggregate price function is increasing in consumption of tradable goods and decreasing in labor. Furthermore, realise that the final consumption price aggregator with the law of one price and the optimality conditions from the households and firms can be rewritten as

$$\begin{aligned}
P\left(e, P^N\right) &= e\left(\omega^{\frac{1}{1+\mu}} + (1-\omega)^{\frac{1}{1+\mu}}\left(\frac{P^N}{e}\right)^{\frac{\mu}{1+\mu}}\right)^{\frac{1+\mu}{\mu}} \\
&= e\left(\omega^{\frac{1}{1+\mu}} + (1-\omega)^{\frac{1}{1+\mu}}\left(\frac{1-\omega}{\omega}\left(\frac{c^T}{F(h)}\right)^{1+\mu}\right)^{\frac{\mu}{1+\mu}}\right)^{\frac{1+\mu}{\mu}} \\
&= \frac{e}{\omega}\left(\frac{c^T}{c\left(c^T, F(h)\right)}\right)^{1+\mu} \\
&= e\mathcal{P}\left(c^T, h\right).
\end{aligned}$$

Hence, the exchange rate policy follows $e = \overline{P}/\mathcal{P}\left(c^T, h\right)$. □

Define the aggregate price real wage function as $\mathcal{F}\left(c^T, h\right) \equiv \mathcal{W}\left(c^T, h\right)/\mathcal{P}\left(c^T, h\right)$. In this way the downward nominal wage rigidity using the exchange rate that follows the inflation targeting condition transforms to $\overline{W} \leq \overline{P}\mathcal{F}\left(c^T, h\right)$.

**Lemma C15.** *The aggregate price real wage function is increasing in consumption of tradables and decreasing in labor.*

*Proof.* Realise that the aggregate price real wage function can be rewritten as

$$\mathcal{F}\left(c^T, h\right) = \frac{\mathcal{W}\left(c^T, h\right)}{\mathcal{P}\left(c^T, h\right)} = (1-\omega)\left(\frac{c\left(c^T, F(h)\right)}{F(h)}\right)^{1+\mu} F'(h).$$

Taking the partial derivatives fro the aggregate price real wage function we find

$$\frac{\partial \mathcal{F}}{\partial c^T} = \omega(1-\omega)(1+\mu)\left(\frac{c\left(c^T, F(h)\right)}{F(h)}\right)^{1+\mu}\left(\frac{c\left(c^T, F(h)\right)}{c^T}\right)^{\mu}\left(\frac{F'(h)}{c^T}\right) > 0 \qquad \text{and}$$

$$\frac{\partial \mathcal{F}}{\partial h} = \omega(1-\omega)\left(\frac{c\left(c^T, F(h)\right)}{F(h)}\right)^{1+\mu} F'(h)\left[\frac{F''(h)}{F'(h)} - (1+\mu)\left(\frac{c\left(c^T, F(h)\right)}{c^T}\right)^{\mu}\frac{F'(h)}{F(h)}\right] < 0,$$

because $F(\cdot)$ is concave, thus $F''(\cdot) < 0$. In other words, the aggregate price real wage function is increasing in consumption of tradables and decreasing in labor. □

We will first define the government problem and the bond pricing under this new environment. The problem of the government either to default or repay debt can be described as

$$V(b, \mathbf{s}) = \max_{d \in \{0,1\}}\left\{dV_D(y^T) + (1-d)V_R(b, \mathbf{s})\right\}.$$

In this way, the maximization problem in default can be described as

$$\begin{aligned}
V_D\left(y^T\right) = \max_{c^T, h \leq \overline{h}} &\left\{u\left(c^T, F\left(\overline{h}\right)\right) + \beta\mathbb{E}\left[\psi V\left(0, \mathbf{s}'\right) + (1-\psi)V_D\left(y^{T\prime}\right)\right]\right\} \\
\text{s.t. } & c^T = y^T \\
& \overline{W} \leq \overline{P}\mathcal{F}\left(c^T, h\right)
\end{aligned}$$

The value of repayment transforms to

$$
\begin{aligned}
V_R\left(b,\mathbf{s}\right) = \max_{b',c^T,h\leq\overline{h}} & \left\{u\left(c^T,F\left(\overline{h}\right)\right) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[V\left(b',\mathbf{s}'\right)\right]\right\} \\
\text{s.t. } & c^T = y^T - \delta b + q\left(b',b,\mathbf{s}\right)\left(b' - (1-\delta)b\right) \\
& \overline{W} \leq \overline{P}\mathcal{F}\left(c^T,h\right)
\end{aligned}
$$

The value of repayment can be studied as before under two different scenarios: when rollover debt is allowed and when it is not. Let us start by analyzing the problem when new debt contracts can be issued and hence rollover debt is allowed. Under this scenario, the bond pricing that satisfies the no-arbitrage condition by the part of the international lenders is applied. Then, the value of repayment when rollover is allowed transforms to

$$
\begin{aligned}
V_R^+\left(b,y^T\right) = \max_{b',c^T,h\leq\overline{h}} & \left\{u\left(c^T,F\left(h\right)\right) + \beta\mathbb{E}\left[V\left(b',\mathbf{s}'\right)\right]\right\} \\
\text{s.t. } & c^T - \tilde{q}\left(b',y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b \\
& \overline{W} \leq \overline{P}\mathcal{F}\left(c^T,h\right)
\end{aligned} \tag{C.1}
$$

Call $\hat{b}\left(b,y^T\right)$ the optimal solution of new portfolio debt maturities that solve the previous problem. As before, call the state space in which it is optimal for the government to increase debt issuances as

$$
\mathcal{B} = \left\{\left(b,y^T\right) \in \mathbb{R}^\infty \times \mathbb{R}_+ : \quad \hat{b}\left(b,y^T\right) > (1-\delta)b\right\}.
$$

As before, the value of repayment when rollover is not allowed can be divided into two cases. When $\left(b,y^T\right) \notin \mathcal{B}$, the government finds optimal to reduce debt issuances. In this case, we can say that $V_R^-\left(b,y^T\right) = V_R^+\left(b,y^T\right)$ because the government is buying back its debt. Neverthless, if $\left(b,y^T\right) \in \mathcal{B}$, then the government wants to increase its debt issuances. International lenders set a price of $\tilde{q} = 0$ representing their reluctancy to rollover debt. In this way, the value of repayment when new debt contracts are forbidden can be expressed as

$$
\begin{aligned}
V_R^-\left(b,y^T\right) = \max_{c^T,h\leq\overline{h}} & \left\{u\left(c^T,F\left(h\right)\right) + \beta\mathbb{E}\left[V\left((1-\delta)b,\mathbf{s}'\right)\right]\right\} \\
\text{s.t. } & c^T = y^T - \delta b \\
& \overline{W} \leq \overline{P}\mathcal{F}\left(c^T,h\right)
\end{aligned} \tag{C.2}
$$

The following Lemma follows the same steps as the one stated before following the fact that $V_R^-$ is a particular case of $V_R^+$ maximization problem.

**Lemma C16.** *For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level $b$, we have that $V_R^+\left(b,y^T\right) \geq V_R^-\left(b,y^T\right)$.*

Now, let us define the safe zone, default zone, and repayment zone contingent to the portfolio of different debt maturities as

$$
\begin{aligned}
\mathcal{S} &\equiv \left\{\left(b,y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^-\left(b,y^T\right)\right\} \\
\mathcal{D} &\equiv \left\{\left(b,y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) > V_R^+\left(b,y^T\right)\right\} \\
\mathcal{C} &\equiv \left\{\left(b,y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^+\left(b,y^T\right) \quad \& \quad V_D(y^T) > V_R^-\left(b,y^T\right)\right\}.
\end{aligned}
$$

Using these zones, the bond pricing following the no-arbitrage condition for each maturity structure, can be represented with the following recursion

$$\tilde{q}(b', y^T) = \frac{1}{1+r}\mathbb{E}\left[\left(1 - d(b', \mathbf{s}')\right)\left(\delta + (1-\delta)q\left(\hat{b}\left(b', \mathbf{s}'\right), b', \mathbf{s}'\right)\right)\right].$$

Finally, using the zones and the multiplicity of equilibria, the overall bond pricing can be described as

$$q\left(b', b, \mathbf{s}\right) = \begin{cases} 0 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ \tilde{q}(b', y^T) & \textit{in every other case} \end{cases}$$

and the optimal default decision as

$$d\left(b, \mathbf{s}\right) = \begin{cases} 1 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \& \quad \zeta = 0 \\ 1 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{S} \end{cases}$$

Define now the real aggreage wage rigidity as $\overline{w} \equiv \overline{W}/\overline{P}$ and let us focus now in the now rigidity environment $\overline{w} = 0$. Call the solutions of it $\left\{V^{flex}, V_D^{flex}, \tilde{q}^{flex}\right\}$ and let us study the one period rigidity shock. The value of default will transform to

$$\begin{aligned} \tilde{V}_D\left(y^T\right) = \max_{c^T, h \leq \overline{h}} & \left\{u\left(c^T, F\left(h\right)\right) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[\psi V^{flex}\left(\mathbf{0}, \mathbf{s}'\right) + (1-\psi)V_D^{flex}\left(y^{T\prime}\right)\right]\right\} \\ \text{s.t. } & c^T = y^T \\ & \overline{w} \leq \mathcal{F}(c^T, h) \end{aligned}$$

Also, the value of repayment when rollover debt is allowed

$$\begin{aligned} \tilde{V}_R^+\left(b, y^T\right) = \max_{b', c^T, h \leq \overline{h}} & \left\{u\left(c^T, F\left(h\right)\right) + \beta\mathbb{E}\left[V^{flex}\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b \\ & \overline{w} \leq \mathcal{F}(c^T, h) \end{aligned}$$

Finally, the value of repayment when new debt contracts of any maturity are forbidden then

$$\begin{aligned} \tilde{V}_R^-\left(b, y^T\right) = \max_{c^T, h \leq \overline{h}} & \left\{u\left(c^T, F\left(h\right)\right) + \beta\mathbb{E}\left[V^{flex}\left((1-\delta)b, \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T = y^T - \delta b \\ & \overline{w} \leq \mathcal{F}(c^T, h) \end{aligned}$$

The following lemmas and propositions follow the same steps stated in the section before.

**Lemma C17.** *The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect the debt level $b$*

**Lemma C18** (Debt Thresholds). *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists levels of debt $\overline{b}^+, \overline{b}^- \in \mathbb{R}$, such that $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^+\left(\overline{b}^+, y^T\right)$ and $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^-\left(\overline{b}^-, y^T\right)$. Furthermore, $\overline{b}^+ \geq \overline{b}^-$.*

Now call the regions as

$$\tilde{S}\left(y^T\right) \equiv \left(-\infty, \bar{b}^-\right], \qquad \tilde{C}\left(y^T\right) \equiv \left(\bar{b}^-, \bar{b}^+\right], \qquad \text{and} \qquad \tilde{D}\left(y^T\right) \equiv \left(\bar{b}^+, \infty\right).$$

The following propositions follow the same steps stated in the section before.

**Proposition C10** (Default Region Expansion)**.** *For every level of tradable endowment* $y^T \in \mathbb{R}$*, there exists* $\overline{w}_D \in \mathbb{R}_+$ *such that if* $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$*, then* $\tilde{\mathcal{D}}\left(y^T; \overline{w}_1\right) \subseteq \tilde{\mathcal{D}}\left(y^T; \overline{w}_2\right)$*.*

**Proposition C11** (Safe Region Contraction)**.** *For every level of tradable endowment* $y^T \in \mathbb{R}$*, there exists* $\overline{w}_D \in \mathbb{R}_+$ *such that if* $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$*, then* $\tilde{\mathcal{S}}\left(y^T; \overline{w}_2\right) \subseteq \tilde{\mathcal{S}}\left(y^T; \overline{w}_1\right)$*.*

**Proposition C12** (Crisis Region Expansion)**.** *For every level of tradable endowment* $y^T \in \mathbb{R}$*, there exists* $\overline{p}_C \in \mathbb{R}_+$ *such that if* $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_C$*, then* $\tilde{\mathcal{C}}\left(y^T; \overline{w}_1\right) \subseteq \tilde{\mathcal{C}}\left(y^T; \overline{w}_2\right)$*. Moreover, there exists* $\overline{w}_S \in \mathbb{R}_+$ *such that if* $\overline{w}_2 > \overline{w}_S$*, then* $\tilde{\mathcal{C}}\left(y^T; \overline{w}_1\right) \subset \tilde{\mathcal{C}}\left(y^T; \overline{w}_2\right)$*.*

# D Maturity Choice

In this section we expand our baseline model to show that our theoretical results hold when the government chooses a portfolio of bonds with different maturities.

Define the set of different debt maturities in the beginning of the period $t$ as $\mathbf{b}_t \equiv \{b_{t,n}\}_{n=0}^{\infty}$, where $b_{t,n}$ represent the amount of government's debt due in $n$ periods ahead. In this sense, when the government has access to international markets, it chooses a new portfolio of debt with different maturities $\mathbf{b}_{t+1}$. Then, the budget constraint of the government in period $t$ can be described as

$$c_t^T = y_t^T - b_t + \sum_{n=1}^{\infty} q_{t,n} \left(b_{t+1,n-1} - b_{t,n}\right),$$

where $q_{t,n}$ is the price of the bond in period $t$ that matures in $n$ periods ahead.

We will first define the government problem and the bond pricing under this new environment. The problem of the government either to default or repay debt can be described as

$$V(\mathbf{b}, \mathbf{s}) = \max_{d \in \{0,1\}} \left\{ dV_D(y^T) + (1-d)V_R\left(\mathbf{b}, \mathbf{s}\right) \right\}.$$

We will assume that if the government defaults, then it will default in all its portfolio of different debt maturities. In addition to this, we will assume that when the government reenters international financial markets, then its portfolio resets to zero for all different debt maturities. In this way, the maximization problem in default can be described as

$$\begin{aligned} V_D\left(y^T\right) = \max_{e, c^T, h \leq \bar{h}} & \left\{ u\left(c^T, h\right) - \kappa\left(y^T\right) + \beta \mathbb{E}\left[\psi V\left(\mathbf{0}, \mathbf{s}'\right) + (1-\psi) V_D\left(y^{T\prime}\right)\right] \right\} \\ \text{s.t. } & c^T = y^T \\ & \overline{W} \leq e \mathcal{W}\left(c^T, h\right) \end{aligned}$$

where $\mathbf{0}$ is an infinite vector with zeros for all entries. The value of repayment transforms to

$$\begin{aligned} V_R\left(\mathbf{b}, \mathbf{s}\right) = \max_{e, \mathbf{b}', c^T, h \leq \bar{h}} & \left\{ u\left(c^T, h\right) + \beta \mathbb{E}\left[V\left(\mathbf{b}', \mathbf{s}'\right)\right] \right\} \\ \text{s.t. } & c^T = y^T - b_0 + \sum_{n=1}^{\infty} q_n\left(\mathbf{b}', \mathbf{b}, \mathbf{s}\right)\left(b'_{n-1} - b_n\right) \\ & \overline{W} \leq e \mathcal{W}\left(c^T, h\right) \end{aligned}$$

where $q_n\left(\mathbf{b}', \mathbf{b}, \mathbf{s}\right)$ is the bond pricing that is contingent to not only the future portfolio of debt maturities $\mathbf{b}'$, but also the current portfolio of debt maturities $\mathbf{b}$ and the sunspot $\zeta$.

The value of repayment can be studied under two different scenarios: when rollover debt is allowed and when it is not. Let us start by analyzing the problem when new debt contracts can be issued and hence rollover debt is allowed. Under this scenario, the bond pricing that satisfies the no-arbitrage condition by the part of the international lenders is applied. Like before, call this fundamental bond pricing as $q_n\left(\mathbf{b}', y^T\right)$ for every single bond with maturity $n$. Then, the value of repayment when rollover is allowed transforms to

$$V_R^+\left(\mathbf{b}, y^T\right) = \max_{e,\mathbf{b}',c^T,h\leq\overline{h}} \left\{u\left(c^T, h\right) + \beta\mathbb{E}\left[V\left(\mathbf{b}', \mathbf{s}'\right)\right]\right\} \tag{D.1}$$

$$\text{s.t. } c^T = y^T - b_0 + \sum_{n=1}^{\infty} \tilde{q}_n\left(\mathbf{b}', y^T\right)\left(b'_{n-1} - b_n\right)$$

$$\overline{W} \leq e\mathcal{W}\left(c^T, h\right)$$

Call $\hat{\mathbf{b}}\left(\mathbf{b}, y^T\right)$ the optimal solution of new portfolio debt maturities that solve the previous problem. Call the state space in which it is optimal for the government to create new debt contracts that incur in a positive flux of resources from international lenders as

$$\mathcal{B} = \left\{\left(\mathbf{b}, y^T\right) \in \mathbb{R}^\infty \times \mathbb{R}_+ : \quad \sum_{n=1}^{\infty} \tilde{q}_n\left(\hat{\mathbf{b}}\left(\mathbf{b}, y^T\right), y^T\right)\left(\hat{b}_{n-1}\left(\mathbf{b}, y^T\right) - b_n\right) > 0\right\}.$$

If $\left(\mathbf{b}, y^T\right) \in \mathcal{B}$, we can say that the government finds optimal in that state to overall incur in more debt with international lenders. In other words, the government increases net debt issuances.

The value of repayment when rollover is not allowed can be divided into two cases. When $\left(\mathbf{b}, y^T\right) \notin \mathcal{B}$, the government finds optimal to reduce net debt issuances changing its portolfio of different debt maturities. In this case, we can say that $V_R^-\left(\mathbf{b}, y^T\right) = V_R^+\left(\mathbf{b}, y^T\right)$ because the international lenders are being repaid instead of asked for more net debt. Neverthless, if $\left(\mathbf{b}, y^T\right) \in \mathcal{B}$, then the government wants to increase its net debt issuances. International lenders set a price of $\tilde{q}_n = 0$ for all different maturities $n$ representing their reluctancy to rollover debt. In this way, the value of repayment when new debt contracts of any maturity are forbidden can be expressed as

$$V_R^-\left(\mathbf{b}, y^T\right) = \max_{e,c^T,h\leq\overline{h}} \left\{u\left(c^T, h\right) + \beta\mathbb{E}\left[V\left(\{b_n\}_{n=1}^{\infty}, \mathbf{s}'\right)\right]\right\} \tag{D.2}$$

$$\text{s.t. } c^T = y^T - b_0$$

$$\overline{W} \leq e\mathcal{W}\left(c^T, h\right)$$

The following lemma follows the same steps as the one stated before with the difference that now the option of choice is not a single debt level, but a portfolio of different debt maturities.

**Lemma D19.** *For every tradable endowment $y^T \in \mathbb{R}_+$ and debt portfolio $\boldsymbol{b} = \{b_n\}_{n=0}^{\infty} \in \mathbb{R}^\infty$, we have that $V_R^+\left(\boldsymbol{b}, y^T\right) \geq V_R^-\left(\boldsymbol{b}, y^T\right)$.*

Now, let us define the safe zone, default zone, and repayment zone contingent to the portfolio of different debt maturities as

$$\begin{aligned}
\mathcal{S}(\mathbf{b}) &\equiv \left\{y^T \in \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^-\left(\mathbf{b}, y^T\right)\right\} \\
\mathcal{D}(\mathbf{b}) &\equiv \left\{y^T \in \mathbb{R}_+ : \quad V_D(y^T) > V_R^+\left(\mathbf{b}, y^T\right)\right\} \\
\mathcal{C}(\mathbf{b}) &\equiv \left\{y^T \in \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^+\left(\mathbf{b}, y^T\right) \quad \& \quad V_D(y^T) > V_R^-\left(\mathbf{b}, y^T\right)\right\}.
\end{aligned}$$

Using these zones, the probability of defaulting for each forward period using the optimal portfolio of different debt maturities can be defined as

$$p_n(\mathbf{b}', y^T) \equiv (1-\pi)\left[\int_{\mathcal{S}(\mathbf{b}')\cup\mathcal{C}(\mathbf{b}')} p_{n-1}\left(\hat{\mathbf{b}}\left(\mathbf{b}', y^{T\prime}\right), y^{T\prime}\right) dF(\mathbf{b}'', y^{T\prime}) dF(y^{T\prime}|y^T)\right]$$
$$+\pi\left[\int_{\mathcal{S}(\mathbf{b}')} p_{n-1}\left(\hat{\mathbf{b}}\left(\mathbf{b}', y^{T\prime}\right), y^{T\prime}\right) dF(y^{T\prime}|y^T)\right],$$

for every forward period $n \in \mathbb{N}$ and where $p_0 = 1$. Taking these probabilities, the bond pricing following the no-arbitrage condition for each maturity structure, can be represented with the following recursion

$$\tilde{q}_n(\mathbf{b}', y^T) = \left(\frac{1}{1+r}\right)^n p_n\left(\mathbf{b}', y^T\right), \qquad \text{for every } n \in \mathbb{N}.$$

Finally, using the zones and the multiplicity of equilibria, the overall bond pricing can be described as

$$q_n\left(\mathbf{b}', \mathbf{b}, \mathbf{s}\right) = \begin{cases} 0 & \text{if } y^T \in \mathcal{D}\left(\mathbf{b}\right) \\ 0 & \text{if } y^T \in \mathcal{C}\left(\mathbf{b}\right) \quad \& \quad \zeta = 1 \\ \tilde{q}_n(\mathbf{b}', y^T) & \textit{in every other case} \end{cases}$$

and the optimal default decision as

$$d\left(\mathbf{b}, \mathbf{s}\right) = \begin{cases} 1 & \text{if } y^T \in \mathcal{D}\left(\mathbf{b}\right) \\ 0 & \text{if } y^T \in \mathcal{C}\left(\mathbf{b}\right) \quad \& \quad \zeta = 0 \\ 1 & \text{if } y^T \in \mathcal{C}\left(\mathbf{b}\right) \quad \& \quad \zeta = 1 \\ 0 & \text{if } y^T \in \mathcal{S}\left(\mathbf{b}\right) \end{cases}$$

The following proposition follows the same steps as the one stated before.

**Proposition D13** (Optimal Exchange Rate Policy)**.** *Under a flexible exchange rate regime, the government chooses an en exchange rate that delivers full employment in all states*

Let us focus now in the flexible exchange rate regime and solve the model. Call the flexible exchange rate regime solutions $\left\{V^{flex}, V_D^{flex}, \left\{\tilde{q}_n^{flex}\right\}_{n=1}^{\infty}\right\}$ and let us study the one period fixed exchange rate regime shocks. The value of default will transform to

$$\tilde{V}_D\left(y^T\right) = \max_{c^T, h \le \bar{h}} \left\{u\left(c^T, F(h)\right) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[\psi V^{flex}\left(\mathbf{0}, \mathbf{s}'\right) + (1-\psi)V_D^{flex}\left(y^{T\prime}\right)\right]\right\}$$
$$\text{s.t. } c^T = y^T$$
$$\overline{w} \le \mathcal{W}\left(c^T, h\right)$$

Also, the value of repayment when rollover debt is allowed

$$\tilde{V}_R^{+}\left(\mathbf{b}, y^T\right) = \max_{\mathbf{b}', c^T, h \le \bar{h}} \left\{u\left(c^T, F(h)\right) + \beta\mathbb{E}\left[V^{flex}\left(\mathbf{b}', \mathbf{s}'\right)\right]\right\}$$
$$\text{s.t. } c^T = y^T - b_0 + \sum_{n=1}^{\infty} \tilde{q}_n^{flex}\left(\mathbf{b}', y^T\right)\left(b'_{n-1} - b_n\right)$$
$$\overline{w} \le \mathcal{W}(c^T, h)$$

Finally, the value of repayment when new debt contracts of any maturity are forbidden then

$$\tilde{V}_R^-\left(\mathbf{b}, y^T\right) = \max_{\mathbf{b}', c^T, h \leq \bar{h}} \left\{ u\left(c^T, F(h)\right) + \beta \mathbb{E}\left[V^{flex}\left(\{b_n\}_{n=1}^{\infty}, \mathbf{s}'\right)\right]\right\}$$
$$\text{s.t. } c^T = y^T - b_0$$
$$\overline{w} \leq \mathcal{W}(c^T, h)$$

For convenience, define as the portfolio of current not matured debt as $\mathbf{b}_{-0} = \{b_n\}_{n=1}^{\infty}$. The following lemmas and propositions follow the same steps stated in the section before with the difference that besides fixing a tradable endowment $y^T \in \mathbb{R}_+$, we also fix a portfolio of current not matured debt $\mathbf{b}_{-0} \in \mathbb{R}^{\infty}$. The main important level of debt to study is the current debt matured in the period $b_0 \in \mathbb{R}$. We focus in this amount of debt and define the regions studied before.

**Lemma D20.** *The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect to debt that currently matures $b_0$*

**Lemma D21** (Debt Thresholds)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$ and portfolio of current not matured debt $\boldsymbol{b}_{-0} \in \mathbb{R}^{\infty}$, there exists levels of debt that currently matures $\overline{b}_0^+, \overline{b}_0^- \in \mathbb{R}$, such that $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^+\left(\left\{\overline{b}_0^+, \boldsymbol{b}_{-0}\right\}, y^T\right)$ and $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^-\left(\left\{\overline{b}_0^-, \boldsymbol{b}_{-0}\right\}, y^T\right)$. Furthermore, $\overline{b}_0^+ \geq \overline{b}_0^-$.*

Now call the regions as

$$\tilde{S}\left(\mathbf{b}_{-0}, y^T\right) \equiv \left(-\infty, \overline{b}_0^-\right], \qquad \tilde{C}\left(\mathbf{b}_{-0}, y^T\right) \equiv \left(\overline{b}_0^-, \overline{b}_0^+\right], \qquad \text{and} \qquad \tilde{D}\left(\mathbf{b}_{-0}, y^T\right) \equiv \left(\overline{b}_0^+, \infty\right).$$

**Proposition D14** (Default Region Expansion)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$ and portfolio of current not matured debt $\boldsymbol{b}_{-0} \in \mathbb{R}^{\infty}$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{\mathcal{D}}\left(\boldsymbol{b}_{-0}, y^T; \overline{w}_1\right) \subseteq \tilde{\mathcal{D}}\left(\boldsymbol{b}_{-0}, y^T; \overline{w}_2\right)$.*

**Proposition D15** (Safe Region Contraction)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$ and portfolio of current not matured debt $\boldsymbol{b}_{-0} \in \mathbb{R}^{\infty}$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{\mathcal{S}}\left(\boldsymbol{b}_{-0}, y^T; \overline{w}_2\right) \subseteq \tilde{\mathcal{S}}\left(\boldsymbol{b}_{-0}, y^T; \overline{w}_1\right)$.*

**Proposition D16** (Crisis Region Expansion)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$ and portfolio of current not matured debt $\boldsymbol{b}_{-0} \in \mathbb{R}^{\infty}$, there exists $\overline{w}_C \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_C$, then $\tilde{\mathcal{C}}\left(\boldsymbol{b}_{-0}, y^T; \overline{w}_1\right) \subseteq \tilde{\mathcal{C}}\left(\boldsymbol{b}_{-0}, y^T; \overline{w}_2\right)$. Moreover, there exists $\overline{w}_S \in \mathbb{R}_+$ such that if $\overline{w}_2 > \overline{w}_S$, then $\tilde{\mathcal{C}}\left(\boldsymbol{b}_{-0}, y^T; \overline{w}_1\right) \subset \tilde{\mathcal{C}}\left(\boldsymbol{b}_{-0}, y^T; \overline{w}_2\right)$.*

# E Elastic Labor Supply

In this section, we expand the baseline model to allow for an elastic supply of labor. While the amount of hours will continue to be demand determined when the wage rigidity is binding, the amount of hours will not be fixed at $\bar{w}$ when the wage rigidity is slack because households will adjust their labor supply. In this setup, the household problem is to solve

$$\max_{h_t, c_t^N, c_t^T} \left\{ \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \right] \right\}$$
$$\text{s.t. } P_t^T c_t^T + P_t^N c_t^N = P_t^T y_t^T + W_t h_t + \phi_t + T_t$$
$$c_t = \left( \omega \left( c_t^T \right)^{-\mu} + (1-\omega) \left( c_t^N \right)^{-\mu} \right)^{-\frac{1}{\mu}}$$

The first order conditions are

$$\omega \left( \frac{c_t}{c_t^T} \right)^{1+\mu} U_c(t) = \lambda_t P_t^T \qquad (1-\omega) \left( \frac{c_t}{c_t^N} \right)^{1+\mu} U_c(t) = \lambda_t P_t^N \qquad -U_h(t) = \lambda_t W_t$$

Let us drop the time subscript and define the real wages and nontradable prices as $w = W/P^T$ and $p^N = P^N/P^T$. Joining the FOC conditions,

$$\frac{1-\omega}{\omega} \left( \frac{c^T}{c^N} \right)^{1+\mu} = p^N \qquad \& \qquad \frac{w}{p^N} = \frac{1}{1-\omega} \left( \frac{-U_h}{U_c} \right) \left( \frac{c^N}{c} \right)^{1+\mu}$$

Recall the FOC condition of the firm and the market clearing of nontradables goods $p^N F'(h) = w$ and $c_N = F(h)$, respectively. Hence,

$$1 = (1-\omega) F'(h) \left( \frac{U_c}{-U_h} \right) \left( \frac{c}{F(h)} \right)^{1+\mu}.$$

**Assumption E1.** *The production function and utility functions can be described respectively as*

$$F(h) = h^{\alpha} \qquad \textit{and} \qquad U(c,h) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{h^{1+\nu}}{1+\nu}.$$

It will be useful to establish the following lemma, which is analogous to Lemma 1 in the main text.

**Lemma E22.** *Under flexible exchange rate regime, the real wage function is increasing with respect of consumption of tradables and optimal labor is increasing in consumption of tradables;* $\frac{dh}{dc^T} > 0$ *and* $\frac{\partial \mathcal{W}}{\partial c^T} > 0$*, respectively.*

*Proof.* Define the function $\mathcal{F}(c_T, h)$ from joining the FOCs from the households and firms as

$$\mathcal{F}(c^T, h) \equiv h^{\alpha(1-\sigma)-(1+\nu)} \left( \omega \left( \frac{c^T}{h^{\alpha}} \right)^{-\mu} + (1-\omega) \right)^{\frac{1+\mu-\sigma}{-\mu}} = \frac{\chi}{\alpha(1-\omega)}$$

Therefore,

$$\frac{dh}{dc^T} = \frac{\partial \mathcal{F}/\partial c^T}{-\partial \mathcal{F}/\partial h} = \frac{h}{c^T}\left(\frac{1+\mu}{\alpha(1+\mu)+((1+\nu)-\alpha(1-\sigma))\left(1+\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right)}\right)$$

In other words

$$\frac{c^T}{h}\frac{dh}{dc^T} = \frac{1+\mu}{\alpha(1+\mu)+((1+\nu)-\alpha(1-\sigma))\left(1+\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right)}$$

Because, $\left(\frac{1-\omega}{\omega}\right)\left(\frac{c_T}{h^\alpha}\right)^\mu > 0$, then

$$\begin{aligned}0 &< \alpha(\sigma+\mu)+(1+\nu)\\ &= \alpha(1+\mu)+(1+\nu)-\alpha(1-\sigma)\\ &< \alpha(1+\mu)+((1+\nu)-\alpha(1-\sigma))\left(1+\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right)\end{aligned}$$

In other words, $\frac{dh}{dc^T} > 0$.

Now, also realize that

$$\mathcal{W}(c^T,h) = \alpha\frac{1-\omega}{\omega}\left(\frac{c^T}{F(h)}\right)^{1+\mu}F'(h) = \left(\frac{1-\omega}{\omega}\right)\frac{\left(c^T\right)^{1+\mu}}{h^{1+\alpha\mu}}$$

So,

$$\frac{\partial \mathcal{W}}{\partial c^T} = \frac{\alpha(1+\mu)}{h}\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)\left(1-\left(\frac{1+\alpha\mu}{1+\mu}\right)\frac{c^T}{h}\frac{dh}{dc^T}\right)$$

Therefore,

$$\begin{aligned}1-\left(\frac{1+\alpha\mu}{1+\mu}\right)\frac{c^T}{h}\frac{dh}{dc^T} &= 1-\frac{1+\alpha\mu}{\alpha(1+\mu)+((1+\nu)-\alpha(1-\sigma))\left(1+\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right)}\\ &= \frac{((1+\nu)-\alpha(1-\sigma))\left(1+\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right)-(1-\alpha)}{\alpha(1+\mu)+((1+\nu)-\alpha(1-\sigma))\left(1+\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right)}\end{aligned}$$

Because, $\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu > 0$, then

$$\begin{aligned}0 &< \alpha\sigma+\nu\\ &= \sigma+\nu-(1-\alpha)\sigma\\ &= \sigma+\nu+(1-\alpha)(1-\sigma)-(1-\alpha)\\ &= ((1+\nu)-\alpha(1-\sigma))-(1-\alpha)\\ &< ((1+\nu)-\alpha(1-\sigma))\left(1+\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right)-(1-\alpha).\end{aligned}$$

In other words, $\frac{\partial \mathcal{W}}{\partial c^T} > 0$.

Finally we can conclude that that when the downward nominal wage rigidity is not binding, then the real wage function is increasing with respect of consumption of tradables and optimal labor is increasing in consumption of tradables. □

To ease the formulation, define the equilibrium labor function in terms of consumption of tradables as $\hat{h}\left(c^T\right)$ that satisfies the followin first order equation,

$$1 = \left(\alpha + \frac{((1+\nu) - \alpha(1-\sigma))}{1+\mu}\right)\left(1 + \left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{\hat{h}\left(c^T\right)^{\alpha}}\right)^{\mu}\right)\left(\frac{\hat{h}'\left(c^T\right)}{\hat{h}\left(c^T\right)}\right)c^T.$$

Acknowledge that from Lemma E22 we know that this function satisfies $\hat{h}'(\cdot) > 0$.

Now, we will define the government problem and the bond pricing under this new environment. The problem of the government either to default or repay debt can be described as

$$V(b, \mathbf{s}) = \max_{d \in \{0,1\}} \left\{ dV_D(y^T) + (1-d)V_R(b, \mathbf{s}) \right\}.$$

In this way, the maximization problem in default can be described as

$$\begin{aligned} V_D\left(y^T\right) = \max_{e, c^T, h \leq \hat{h}(c^T)} & \left\{ u\left(c^T, F(h)\right) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[\psi V\left(0, \mathbf{s}'\right) + (1-\psi)V_D\left(y^{T\prime}\right)\right]\right\} \\ \text{s.t. } & c^T = y^T \\ & \overline{W} \leq e\mathcal{W}(c^T, h) \end{aligned}$$

The value of repayment transforms to

$$\begin{aligned} V_R(b, \mathbf{s}) = \max_{e, b', c^T, h \leq \hat{h}(c^T)} & \left\{ u\left(c^T, F(h)\right) + \beta\mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T = y^T - \delta b + q\left(b', b, \mathbf{s}\right)\left(b' - (1-\delta)b\right) \\ & \overline{W} \leq e\mathcal{W}(c^T, h) \end{aligned}$$

The value of repayment can be studied as before under two different scenarios: when rollover debt is allowed and when it is not. Let us start by analyzing the problem when new debt contracts can be issued and hence rollover debt is allowed. Under this scenario, the bond pricing that satisfies the no-arbitrage condition by the part of the international lenders is applied. Then, the value of repayment when rollover is allowed transforms to

$$\begin{aligned} V_R^+\left(b, y^T\right) = \max_{e, b', c^T, h \leq \hat{h}(c^T)} & \left\{ u\left(c^T, F(h)\right) + \beta\mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b \\ & \overline{W} \leq e\mathcal{W}(c^T, h) \end{aligned} \tag{E.1}$$

Call $\hat{b}\left(b, y^T\right)$ the optimal solution of new portfolio debt maturities that solve the previous problem. As before, call the state space in which it is optimal for the government to increase debt issuances as

$$\mathcal{B} = \left\{ \left(b, y^T\right) \in \mathbb{R}^{\infty} \times \mathbb{R}_+ : \quad \hat{b}\left(b, y^T\right) > (1-\delta)b \right\}.$$

As before, the value of repayment when rollover is not allowed can be divided into two cases. When $\left(b, y^T\right) \notin$

$\mathcal{B}$, the government finds optimal to reduce debt issuances. In this case, we can say that $V_R^-\left(b, y^T\right) = V_R^+\left(b, y^T\right)$ because the government is buying back its debt. Neverthless, if $\left(b, y^T\right) \in \mathcal{B}$, then the government wants to increase its debt issuances. International lenders set a price of $\tilde{q} = 0$ representing their reluctancy to rollover debt. In this way, the value of repayment when new debt contracts are forbidden can be expressed as

$$V_R^-\left(b, y^T\right) = \max_{e, c^T, h \leq \hat{h}(c^T)} \left\{u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V\left((1-\delta)b, \mathbf{s}'\right)\right]\right\} \tag{E.2}$$
$$\text{s.t. } c^T = y^T - \delta b$$
$$\overline{W} \leq e\mathcal{W}(c^T, h)$$

The following Lemma follows the same steps as the one stated before following the fact that $V_R^-$ is a particular case of $V_R^+$ maximization problem.

**Lemma E23.** *For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level $b$, we have that $V_R^+\left(b, y^T\right) \geq V_R^-\left(b, y^T\right)$.*

Now, let us define the safe zone, default zone, and repayment zone contingent to the portfolio of different debt maturities as

$$\begin{aligned}
\mathcal{S} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^-\left(b, y^T\right)\right\} \\
\mathcal{D} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) > V_R^+\left(b, y^T\right)\right\} \\
\mathcal{C} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^+\left(b, y^T\right) \quad \& \quad V_D(y^T) > V_R^-\left(b, y^T\right)\right\}.
\end{aligned}$$

Using these zones, the bond pricing following the no-arbitrage condition for each maturity structure, can be represented with the following recursion

$$\tilde{q}(b', y^T) = \frac{1}{1+r}\mathbb{E}\left[\left(1 - d(b', \mathbf{s}'\right)\left(\delta + (1-\delta)q\left(\hat{b}\left(b', \mathbf{s}'\right), b', \mathbf{s}'\right)\right)\right].$$

Finally, using the zones and the multiplicity of equilibria, the overall bond pricing can be described as

$$q\left(b', b, \mathbf{s}\right) = \begin{cases} 0 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ \tilde{q}(b', y^T) & \textit{in every other case} \end{cases}$$

and the optimal default decision as

$$d\left(b, \mathbf{s}\right) = \begin{cases} 1 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \& \quad \zeta = 0 \\ 1 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{S} \end{cases}$$

Let us focus now in the flexible exchange rate regime and solve the model. Call the flexible exchange rate regime solutions $\left\{V^{flex}, V_D^{flex}, \tilde{q}^{flex}\right\}$ and let us study the one period fixed exchange rate regime shocks. The value of default will transform to

$$\tilde{V}_D\left(y^T\right) = \max_{c^T, h \leq \hat{h}(c^T)} \left\{ u\left(c^T, F\left(h\right)\right) - \kappa\left(y^T\right) + \beta \mathbb{E}\left[\psi V^{flex}\left(0, \mathbf{s}'\right) + (1-\psi) V_D^{flex}\left(y^{T\prime}\right)\right]\right\}$$
$$\text{s.t. } c^T = y^T$$
$$\overline{w} \leq \mathcal{W}(c^T, h)$$

Also, the value of repayment when rollover debt is allowed

$$\tilde{V}_R^+\left(b, y^T\right) = \max_{b', c^T, h \leq \hat{h}(c^T)} \left\{ u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V^{flex}\left(b', \mathbf{s}'\right)\right]\right\}$$
$$\text{s.t. } c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b$$
$$\overline{w} \leq \mathcal{W}(c^T, h)$$

Finally, the value of repayment when new debt contracts of any maturity are forbidden then

$$\tilde{V}_R^-\left(b, y^T\right) = \max_{c^T, h \leq \hat{h}(c^T)} \left\{ u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V^{flex}\left((1-\delta)b, \mathbf{s}'\right)\right]\right\}$$
$$\text{s.t. } c^T = y^T - \delta b$$
$$\overline{w} \leq \mathcal{W}(c^T, h)$$

The following lemmas and propositions follow the same steps stated in the section before.

**Lemma E24.** *The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect the debt level $b$*

**Lemma E25** (Debt Thresholds)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists levels of debt that currently matures $\overline{b}^+, \overline{b}^- \in \mathbb{R}$, such that $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^+\left(\overline{b}^+, y^T\right)$ and $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^-\left(\overline{b}^-, y^T\right)$. Furthermore, $\overline{b}^+ \geq \overline{b}^-$.*

Now call the regions as

$$\tilde{S}\left(y^T\right) \equiv \left(-\infty, \overline{b}^-\right], \qquad \tilde{C}\left(y^T\right) \equiv \left(\overline{b}^-, \overline{b}^+\right], \qquad \text{and} \qquad \tilde{D}\left(y^T\right) \equiv \left(\overline{b}^+, \infty\right).$$

The following propositions follow the same steps stated in the section before.

**Proposition E17** (Default Region Expansion)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{\mathcal{D}}\left(y^T; \overline{w}_1\right) \subseteq \tilde{\mathcal{D}}\left(y^T; \overline{w}_2\right)$.*

**Proposition E18** (Safe Region Contraction)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{\mathcal{S}}\left(y^T; \overline{w}_2\right) \subseteq \tilde{\mathcal{S}}\left(y^T; \overline{w}_1\right)$.*

**Proposition E19** (Crisis Region Expansion)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists $\overline{w}_C \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_C$, then $\tilde{\mathcal{C}}\left(y^T; \overline{w}_1\right) \subseteq \tilde{\mathcal{C}}\left(y^T; \overline{w}_2\right)$. Moreover, there exists $\overline{w}_S \in \mathbb{R}_+$ such that if $\overline{w}_2 > \overline{w}_S$, then $\tilde{\mathcal{C}}\left(y^T; \overline{w}_1\right) \subset \tilde{\mathcal{C}}\left(y^T; \overline{w}_2\right)$.*

# F Nominal Debt

In this section, we start from the simplified version of the model from Section 3.5.1 that has deterministic income, $\beta R = 1$, permanent exclusion after default and one-period debt. In addition, we consider a cost from depreciating the currency as in section ??. The resource constraint for tradables is given by

$$c^T = y^T - \frac{b}{e} + q\frac{b'}{e} \tag{F.1}$$

Notice in this equation how an increase in $e$ reduces real payments to foreigners and increase tradable consumption.

Denote $\mathcal{E}(b)$ the optimal excahange rate as a function of the level of debt $e' = \mathcal{E}(b')$ .[39] If investors are pessimistic, the value of repayment for the government is

$$V_R^-(b) = \max_{e,b'} u\left(y^T - \frac{b}{e}, \mathcal{H}\left(y^T - \frac{b}{e}, \frac{W}{e}\right)\right) - \psi(e/\bar{e}) + \beta V_R^+(0). \tag{F.2}$$

The value of default is

$$V^D = \max_e \frac{u(y^T, \mathcal{H}\left(y^T, \frac{W}{e}\right)) - \kappa - \psi(e/\bar{e})}{1-\beta} \tag{F.3}$$

Inspecting (F.2), one can observe how the increase in $e$ not only raises employment through the reduction in the real wage rigidity constraint but also through the increase in tradable consumption.

[39]From investors' side, arbitrage implies that the fundamental bond price must satisfy

$$q(b')(1+r) = \frac{e}{\mathcal{E}(b')}.$$