The views expressed here are those of the authors and do not necessarily represent the views of others in the Federal Reserve System.
Long-run trends in macroeconomics

- Long-run steady state of the economy changes.
  - Example: productivity and growth slowdown in 1970s
- Stochastic trends are widely used in empirical work.

**Trend inflation:** $\pi_t^*$

- Perceived inflation target of central bank
- Important for inflation forecasting

**Equilibrium real interest rate:** $i_t^*$

- Real interest rate consistent with output at potential and inflation at target, neutral rate for monetary policy
- Determined by fundamentals: productivity growth, demographics, price of capital goods, etc.
Macro trends and interest rates

- Interest rates are extremely persistent.
  - Longstanding challenge for financial econometrics/yield curve literature

- Theory says that interest rate trend driven by macro trends:

\[ i_t^* = r_t^* + \pi_t^* \]

- Do we see this in the data?
Secular decline in U.S. long-term interest rates
Secular decline in U.S. long-term interest rates
Long-run trends in finance models

- No-arbitrage yield curve models assume **stationary** interest rates and don’t allow for (macro) trends.

  “The level of nominal interest rates is surely a stationary variable in a fundamental sense: we have observations near 6% as far back as ancient Babylon, and it is about 6% again today.” (Cochrane, 2005)

- Long-run expectations are stable because of mean reversion.

- Swings in long-term interest rates are attributed mainly to term premium.

- Models do poorly in forecasting (don’t beat random walk).

⇒ **Disconnect between macroeconomics and finance**
This paper

Question
How much do macro trends matter for the yield curve?

What we do

- Quantify importance of macro trends for yield curve dynamics
- Model-free stylized facts: use various proxies for $\pi_t^*, r_t^*$ and $i_t^*$, link to yield curve, cointegration, predict excess bond returns
- Dynamic term structure model with time-varying $i_t^*$

What we find

- Quantitatively important role for both macro trends
- Yields revert to $i_t^*$, not to constant mean
- Accounting for trends changes estimated bond risk premia and improves out-of-sample yield forecasts
Literature

Inflation trend and interest rates

Equilibrium real interest rate

Interest-rate forecasting
Diebold and Li (2006), Christensen et al. (2011), Dijk et al. (2014)

Time series models for interest rates with both $\pi^*_t$ and $r^*_t$
Outline

Introduction

Trends: concepts and estimates

Macro trends and yields: stylized facts

Dynamic term structure model with shifting $i_t^*$

Conclusion
Outline

Introduction

Trends: concepts and estimates

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Dynamic term structure model with shifting $i^*_t$

Conclusion
The equilibrium nominal interest rate: $i_t^*$

- Decomposition of long-term interest rates:

$$y_t^{(n)} = \frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j} + TP_t^{(n)} = i_t^* + \frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}^c + TP_t^{(n)},$$

- Equilibrium/long-run mean/Beveridge-Nelson trend in nominal short rate:

$$i_t^* \equiv \lim_{j \to \infty} E_t i_{t+j}$$

- Fisher equation: $i_t = r_t + E_t \pi_{t+1} \Rightarrow i_t^* = r_t^* + \pi_t^*$

- Macro trends shift the level of expectations and yields, but are changes in $i_t^*$ quantitatively important for yields and risk premia?
Trend inflation: $\pi^*_t$

- Inflation target of the central bank (in policy rule/obj.fn.)
  - Most recent DSGE models allow for time-varying $\pi^*_t$

- Trend component of inflation (e.g., UCSV model)
  - To forecast $\pi_t$ need nowcast and $\pi^*_t$ (Faust and Wright, 2013)

- Our preferred estimate of $\pi^*_t$: Perceived Target Rate (PTR) from FRB/US model
  - Long-run expectations of PCE inflation
  - Survey of Professional Forecasters since 1979
  - Model-based (Kozicki and Tinsley, 2001) before 1979
  - Stable at two percent since 2000
  - Widely used in research studies
  - Generally in line with other estimates of $\pi^*_t$
Equilibrium real interest rate: $r_t^*$

- Several different definitions of $r_t^*$: equilibrium/neutral/natural rate
  - For example, neutral rate: real short rate at which monetary policy is neither expansionary nor contractionary

- We focus on long-run trend in real rate:

$$r_t^* = \lim_{h \to \infty} E_t r_{t+h}$$

- Growing literature estimates and analyzes $r_t^*$

- Difficult to pin down the level of $r_t^*$

- But estimates point to a substantial decline over past 20 years
  - Slower productivity growth, aging population, secular stagnation, ...
Estimates of $r_t^*$

- Existing estimates of long-run trend in real rate
  - Johannsen and Mertens (2016, 2018), JM
  - Del Negro et al. (2017), DGGT

- Existing estimates of neutral real rate from macro models
  - Laubach and Williams (2003, 2016), LW
  - Holston, Laubach and Williams (2017), HLW
  - Kiley (2015)
  ⇒ Consistent with long-run trend because $r_t^*$ is martingale

- Our own estimates of long-run trend
  - Univariate unobserved components model
  - State space model with nominal rates and inflation (and PTR)
  - SSM with macro trend proxies – long moving averages of GDP growth and labor hours growth (Lunsford and West, 2017)
  - Simple exponentially-weighted moving average ($\alpha = 0.98$)
Estimates of $r_t^*$

Smoothed: LW, Kiley, JM, DGGT, our own three models. Filtered: LW, HLW, Kiley
Real-time: JM, DGGT, our own three models, moving average
Outline

Introduction

Trends: concepts and estimates

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Dynamic term structure model with shifting $i_t^*$

Conclusion
Data

- Sample period: 1971:Q4 to 2018:Q1
- Interest rates: 3m, 6m T-bill rates and 1-15y zero-coupon Treasury yields from Gürkaynak, Sack and Wright (2007)
- PTR estimate of $\pi_t^*$
- Filtered, real-time and moving-average estimates of $r_t^*$
Macro-finance trends and persistence in yields

Theoretical prediction
Persistence in yields driven by underlying macro trend \( i_t^* = \pi_t^* + r_t^* \)

- \( y_t^{(n)} \) and \( i_t^* \) cointegrated
- \( y_t^{(n)} \) and \( \pi_t^* \) not cointegrated because of \( r_t^* \)

Empirical investigation
Is the variation in macro trends large enough to matter?
Interest rate cycles

- Ten-year yield, demeaned
- Difference between yield and i*
- Cointegration residual between yield and i*
Cointegration (DOLS) regressions for ten-year yield

<table>
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<th>Yield</th>
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<td></td>
<td>1.65</td>
<td>1.53</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.11)</td>
<td>(0.07)</td>
<td></td>
</tr>
<tr>
<td>$r^*_t$</td>
<td></td>
<td></td>
<td>1.76</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(0.16)</td>
<td></td>
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<tr>
<td>$i^*_t$</td>
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<td></td>
<td>1.67</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.06)</td>
</tr>
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<td>$R^2$</td>
<td></td>
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<td>0.96</td>
<td>0.95</td>
</tr>
<tr>
<td>SD</td>
<td></td>
<td>2.94</td>
<td>1.31</td>
<td>0.70</td>
</tr>
<tr>
<td>$\hat{\rho}$</td>
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<td>0.65</td>
<td>0.64</td>
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<tr>
<td>Half-life</td>
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<td>5.6</td>
<td>1.6</td>
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<td>-2.60</td>
<td>-5.32***</td>
<td>-5.37***</td>
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<td>0.72</td>
<td>0.71</td>
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<tr>
<td>Johansen</td>
<td>$r = 0$</td>
<td>13.34</td>
<td>46.83***</td>
<td>30.69***</td>
</tr>
<tr>
<td>Johansen</td>
<td>$r = 1$</td>
<td>1.29</td>
<td>11.57</td>
<td>0.73</td>
</tr>
<tr>
<td>ECM $\hat{\alpha}$</td>
<td>-0.11</td>
<td>-0.44</td>
<td>-0.45</td>
<td>(0.03)</td>
</tr>
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</table>
Cointegration (DOLS) regressions of yields on $i_t^*$
Predictive regressions for bond returns

- ECM suggests that trend $i_t^*$ determines future evolution of bond yields.

- What matters for investors are excess bond returns:
  \[ r_{x_t,t+h}^{(n)} = p_{t+h}^{(n-h)} - p_t^{(n)} - y_t^{(h)} \]

- We use one-quarter holding period ($h = 1$) and predict average $\overline{r}_{x_t,t+1}$

- Do $\pi_t^*$, $r_t^*$, $i_t^*$ have incremental predictive power beyond what’s in the yield curve at time $t$?
  1. Test spanning hypothesis using bootstrap (Bauer and Hamilton, 2017)
  2. Compare predictive power of principal components (PCs) of yields and PCs of detrended yields
### Predictive regressions for excess bond returns

<table>
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<td>0.98</td>
<td>1.39</td>
<td>2.38</td>
<td>2.04</td>
<td>2.47</td>
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<tr>
<td></td>
<td>(0.17)</td>
<td>(0.26)</td>
<td>(0.39)</td>
<td>(0.67)</td>
<td>(0.56)</td>
<td>(0.61)</td>
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<tr>
<td>PC2</td>
<td>0.43</td>
<td>0.47</td>
<td>0.43</td>
<td>0.67</td>
<td>0.68</td>
<td>0.70</td>
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<tr>
<td></td>
<td>(0.17)</td>
<td>(0.17)</td>
<td>(0.17)</td>
<td>(0.15)</td>
<td>(0.15)</td>
<td>(0.15)</td>
</tr>
<tr>
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<td>-1.79</td>
<td>-1.92</td>
<td>-0.92</td>
<td>-0.90</td>
<td>-0.86</td>
</tr>
<tr>
<td></td>
<td>(1.34)</td>
<td>(1.27)</td>
<td>(1.22)</td>
<td>(1.39)</td>
<td>(1.43)</td>
<td>(1.35)</td>
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<tr>
<td>$\pi_t^*$</td>
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<td>-2.21</td>
<td>-4.40</td>
<td>-3.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(0.47)</td>
<td>(1.10)</td>
<td>(0.92)</td>
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<tr>
<td>$r_t^*$</td>
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<td>[0.00]</td>
<td>[0.00]</td>
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<td></td>
<td>-1.19</td>
<td>-3.89</td>
<td>-2.70</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(0.59)</td>
<td>(1.47)</td>
<td>(1.04)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.14]</td>
<td>[0.07]</td>
<td>[0.04]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_t^*$</td>
<td></td>
<td></td>
<td></td>
<td>-4.50</td>
<td>(1.05)</td>
<td>[0.00]</td>
</tr>
</tbody>
</table>

$R^2$       | 0.09  | 0.16  | 0.18  | 0.21  | 0.20  | 0.21  |

Memo: $r^*$ filtered real-time mov. avg. real-time
Predictive regressions with detrended yields (residuals)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
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<tbody>
<tr>
<td>PC1</td>
<td>0.08</td>
<td>0.98</td>
<td>1.25</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.25)</td>
<td>(0.51)</td>
<td>(0.50)</td>
</tr>
<tr>
<td>PC2</td>
<td>0.43</td>
<td>0.48</td>
<td>0.76</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.17)</td>
<td>(0.16)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>PC3</td>
<td>-2.37</td>
<td>-1.77</td>
<td>-0.79</td>
<td>-0.73</td>
</tr>
<tr>
<td></td>
<td>(1.34)</td>
<td>(1.26)</td>
<td>(1.38)</td>
<td>(1.33)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.09</td>
<td>0.15</td>
<td>0.18</td>
<td>0.20</td>
</tr>
</tbody>
</table>

(1) PCs of yields
(2) PCs of yields detrended by $\pi_t^*$
(3) PCs of yields detrended by $\pi_t^*$ and $r_t^*$ (real-time)
(4) PCs of yields detrended by $i_t^*$
Outline

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Conclusion
Specification of no-arbitrage model

- Four state variables: $i_t^*$ and $P_t = WY_t$ (3 linear combinations of yields)
- Short rate: $i_t = \delta_0 + \delta_1' P_t$
- Risk-neutral dynamics (stationary; Joslin-Singleton-Zhu normalization):
  \[ P_t = \mu^Q + \Phi^Q P_{t-1} + u_t^Q \]
- Yields are affine: $Y_t = A + BP_t$
- Real-world dynamics (our main innovation):
  \[
  P_t = \bar{P} + \gamma i_t^* + \tilde{P}_t \quad \text{one common trend}
  \]
  \[
  i_t^* = i_{t-1}^* + \xi_t \quad \text{trend}
  \]
  \[
  \tilde{P}_t = \Phi \tilde{P}_{t-1} + w_t \quad \text{cycles}
  \]

- In a nutshell: model with unspanned shifting endpoint (in the Joslin-Priebsch-Singleton sense); unit root under $P$-measure but not under $Q$
Estimation

Problem

- Estimation simple in principle (Kalman filter, MLE), but estimation of long-run trends with limited samples is difficult (Watson, 1986)
- Estimates of $i^*_t$ not sufficiently stable and robust, as others have found for $\pi^*_t$, $r^*_t$

Solution: Two ways to add more information

1. Data: pin down $i^*_t$ with external proxy
   → Observed Shifting Endpoint (OSE) Model

2. Prior: impose tight prior on variance of $\Delta i^*_t$ to have smooth path of $i^*_t$, estimate with MCMC
   → Estimated Shifting Endpoint (ESE) Model
Model-based estimate of $i_t$
Ten-year yield – trend and cycle
Loadings/regressions of yields on $i_t^*$
Model matches stylized facts: predicting excess returns

<table>
<thead>
<tr>
<th></th>
<th>$R^2$ PCs only</th>
<th>$R^2$ with $i_t^*$</th>
<th>$\Delta R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.09</td>
<td>0.21</td>
<td>0.12</td>
</tr>
<tr>
<td>$FE$ model</td>
<td>0.09</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>[0.04, 0.17]</td>
<td>[0.05, 0.17]</td>
<td>[0.00, 0.04]</td>
</tr>
<tr>
<td>$OSE$ model</td>
<td>0.10</td>
<td>0.19</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>[0.04, 0.18]</td>
<td>[0.13, 0.26]</td>
<td>[0.02, 0.18]</td>
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<tr>
<td>$ESE$ model</td>
<td>0.07</td>
<td>0.15</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>[0.02, 0.13]</td>
<td>[0.05, 0.27]</td>
<td>[0.00, 0.20]</td>
</tr>
</tbody>
</table>

- We simulate 10,000 short samples and run excess-return regressions with and without $i_t^*$, report mean and 95% intervals of $R^2$ in sim. data
- Our shifting-endpoint models are able to capture substantial predictive gains.
Term premium in long-term yields

- Crucial question in macroeconomics and finance: What are the underlying drivers of changes in long-term interest rates?

- Two components: expectations and term premium

\[ y_t^{(n)} = \frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j} + TP_t^{(n)} \]

- Does accounting for movements in \(i_t^*\) make a difference for estimates of the term premium?

- Compare our models to restricted special case with Fixed Endpoint (\(FE\), \(i_t^* = i^*\))
  - This is the stationary DTSM of Joslin, Singleton, Zhu (2011)
Term premium: stationary FE model
Term premium: OSE model
Term premium: ESE model
# Out-of-sample forecasts for 10-year yield

## Root-mean-squared forecast errors and Diebold-Mariano $p$-values

<table>
<thead>
<tr>
<th>Horizon $h$ (quarters):</th>
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<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
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<tr>
<td>Random walk ($RW$)</td>
<td>1.33</td>
<td>1.85</td>
<td>2.52</td>
<td>2.60</td>
<td>2.88</td>
</tr>
<tr>
<td>Fixed endpoint ($FE$)</td>
<td>1.42</td>
<td>2.25</td>
<td>3.28</td>
<td>3.72</td>
<td>4.19</td>
</tr>
<tr>
<td>Observed shifting endpoint ($OSE$)</td>
<td>1.17</td>
<td>1.76</td>
<td>2.37</td>
<td>2.39</td>
<td>2.60</td>
</tr>
<tr>
<td>$p$-value: $OSE \geq RW$</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>$p$-value: $OSE \geq FE$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.03</td>
<td>0.05</td>
</tr>
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First OOS forecast in 1976:Q3 (five years of data)
Out-of-sample forecasts: models vs. Blue Chip survey

Root-mean-squared forecast errors and Diebold-Mariano \( p \)-values

<table>
<thead>
<tr>
<th>Horizon in years</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
<tr>
<td>Blue Chip (( BC ))</td>
<td>1.06</td>
<td>1.39</td>
<td>1.59</td>
<td>1.79</td>
<td>1.99</td>
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<tr>
<td>Random walk (( RW ))</td>
<td>0.85</td>
<td>1.08</td>
<td>1.21</td>
<td>1.37</td>
<td>1.56</td>
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<tr>
<td>Fixed endpoint (( FE ))</td>
<td>1.53</td>
<td>2.08</td>
<td>2.52</td>
<td>2.96</td>
<td>3.34</td>
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<tr>
<td>Observed shifting endpoint (( OSE ))</td>
<td>0.87</td>
<td>0.95</td>
<td>1.04</td>
<td>1.18</td>
<td>1.37</td>
</tr>
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</table>

| \( p \)-value: \( OSE \geq BC \) | 0.10 | 0.08 | 0.15 | 0.18 | 0.20 |
| \( p \)-value: \( OSE \geq RW \) | 0.58 | 0.05 | 0.01 | 0.04 | 0.08 |
| \( p \)-value: \( OSE \geq FE \) | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

- Forecasts are for 10-year yield
Conclusion

- Long-run trends important in macroeconomics but largely ignored in finance
- Theory predicts that macro trends $\pi_t^*$ and $r_t^*$ are reflected in nominal yield curve
- Macro trends indeed quantitatively important for interest rates
  - Strong evidence for cointegration: trends account for persistence in interest rates
  - Trends change dynamics of estimated bond risk premia (expected excess returns)
- Novel term structure model with time-varying $i_t^*$
  - Consistent with stylized facts
  - New and different estimates of the term premium
  - More accurate out-of-sample yield forecasts
- Trends are a crucial macro-finance link