Valuing Private Equity Investments Strip by Strip

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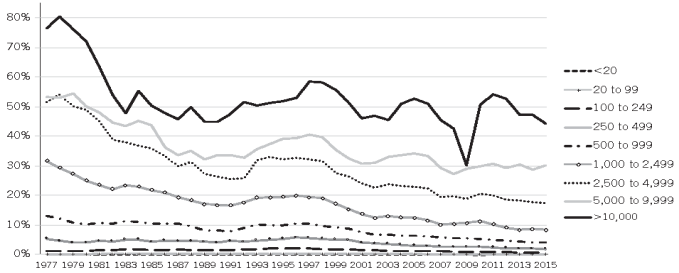
NBER Long-Term Asset Management

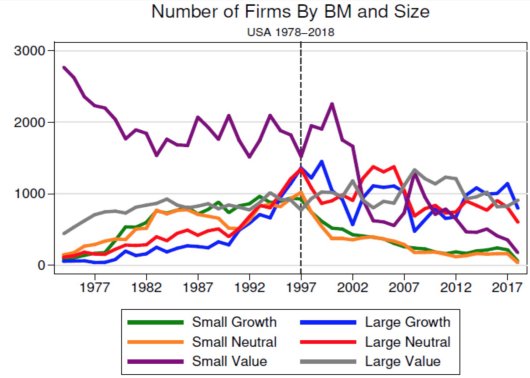
Private Universe is Expanding Relative to Public Stocks...

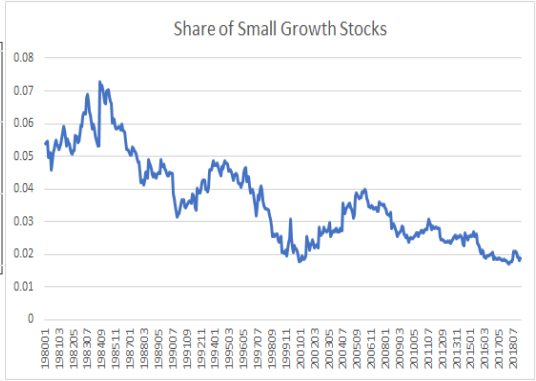
Figure 4 **Firm size, industry, and listing propensity**



Source: The Center for Research in Security Prices (CRSP), Compustat, and the U.S. Census Bureau's Longitudinal Business Database.
Notes: Listed firms include U.S. firms in CRSP and Compustat on the NYSE, AMEX, and Nasdaq that we can assign to an employee size group. Investment companies, mutual funds, REITs, and other collective investment vehicles are excluded. The percentage of firms that are listed in each employee size group equals listed firms/total firms, where total firms includes public and private firms. The sample period is from 1977 to 2015.

Which Complicates the Assessment of Factor Exposure





Measurement Problems in Assessment of Risk and Return in Private Equity

1. **The shift in capital towards private markets, especially for small growth firms, means that we need to know how risk is priced in private markets**

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2. **Existing approaches to private equity valuation have not taken into account the multivariate nature of risk nor the temporal composition of risk**
   - Standard approaches:
     - TVPI (no discounting, no risk)
     - IRR (no risk)
     - PME (beta = 1)
     - GPME (beta constant)
   - Limitations to all approaches: only one aggregate source of risk
     - If a bad assumption in equities (CAPM) likely also the case in PE
     - Especially in “alternative” categories like Real Estate Funds

Measurement Problems in Assessment of Risk and Return in Private Equity

1. Shift to private markets
2. Literature has struggled with cross-section and term structure of risk

3. **To address these limitations, we draw from asset pricing literature emphasizing rich cross-section of factors and term structure of risk**
   - Multifactor models: Fama and French (2016, 2018) and Hou, Xue, and Zhang (2015, 2017, 2018)
   - Term structure: van Binsbergen, Brandt, and Koijen (2012), van Binsbergen and Koijen (2017)
     - Term structure of risk potentially upward or downward sloping, depending on factor
     - Term structure of strips on other factors not known or traded

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4. **Problem: Observe cashflows, not returns**

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4. Problem: Observe cashflows, not returns

5. **Our Solution:**
   - Estimate exposures of PE fund cash flows to cash flows on bond and cross-section of stock strips
   - Use asset pricing model to price these strips

Delivers PE factor exposure, expected return, risk-adjusted profit, NAV

Key Takeaways

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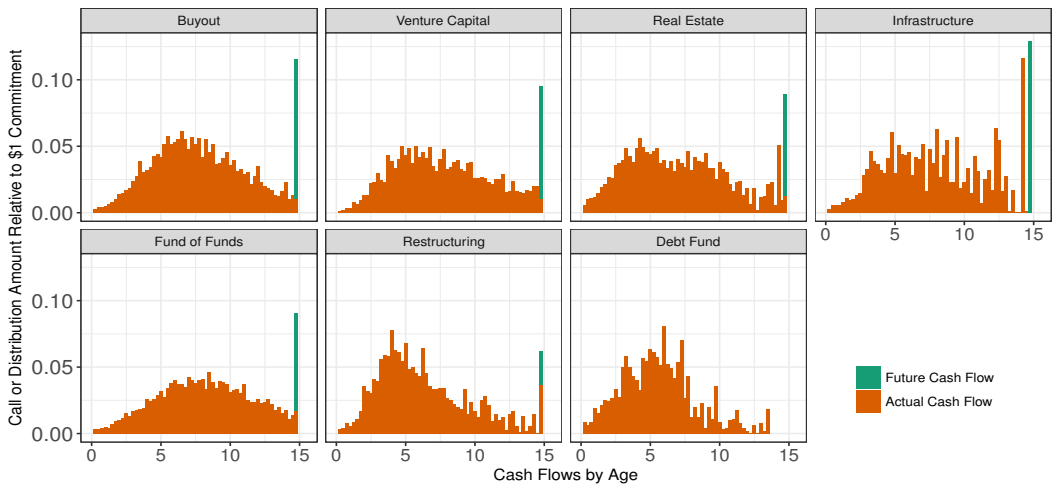
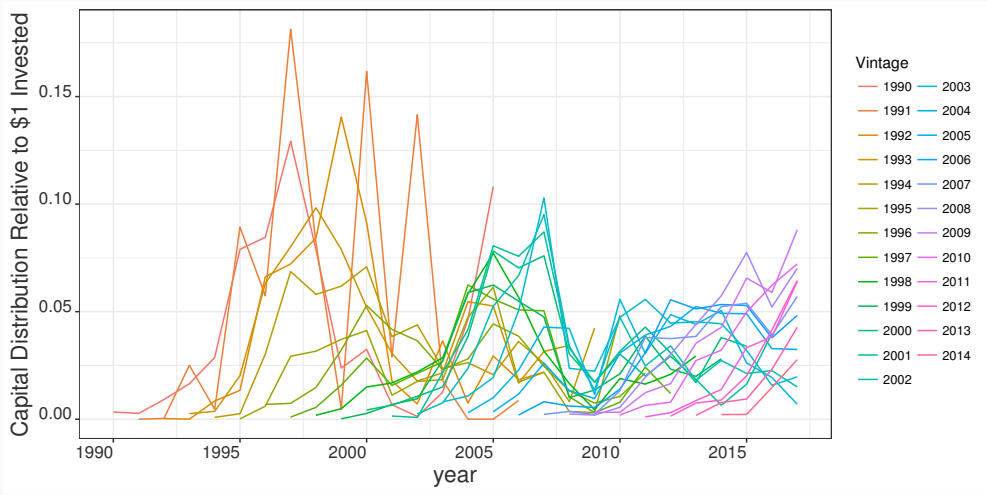
2. Risk exposure is more equity-like early in fund life
3. Decreasing realized profits and expected returns in more recent vintages
4. **Substantial small and growth factor exposure, suggesting these factor loadings are prominent in PE**

Want to Understand Cash-Flow Profiles of Private Equity Funds



Cash-Flow Variation Across Horizon and Vintage — Buyout

Venture Capital | Real Estate | Infrastructure



Break out Factor Exposure Strip by Strip to Match PE Cash Distributions to LPs

- Construct $\boldsymbol{F}_{t+h}$ cash flows on replicating portfolio:
  - Zero coupon bond, pays out \$1 in horizon $t + h$
  - Dividend strips: pay one risky cash flow at $t + h$

- h-period stochastic discount factor chains one-period SDFs:

$$M_{t+h}^{h} = \prod_{k=1}^{h} M_{t+k}$$

- Defining prices of these strips:

$$\boldsymbol{P}_{t,h} = \mathbb{E}_t[M_{t+h}^{h} \boldsymbol{F}_{t+h}]$$

Estimate Factor Exposure Strip-by-Strip

- Three-factor model fitting PE fund cash flows, in quarterly strips, against factor cash flows:

$$X^i_{t+h} = \beta^i_{t,h} \boldsymbol{F}_{t+h} + e^i_{t+h}$$

Factors are:

- Buyout: bond, stock, small
- VC: bond, growth, small
- Real Estate: bond, stock, REIT
- Infrastructure: bond, stock, infra

Shrinkage Estimators Measure Factor Exposure

Structure of exposure (Buyout):

$$\begin{aligned} X^{i\in c}_{t+h} &= \beta^{b}_{t,h} + \beta^{mkt}_{t,h} F^{m}_{t+h} + \beta^{stock}_{t,h} F^{stock}_{t+h} + e^{i}_{t+h} \\ &= a^{1}_{t} b^{1}_{h} + a^{2}_{t} b^{2}_{h} F^{m}_{t+h} + a^{3}_{t} b^{3}_{h} F^{stock}_{t+h} + e^{i}_{t+h} \end{aligned}$$

Allow b_h to vary for each horizon (quarter)
a_t varies for each tercile of the P/D distribution of vintage year
Two estimation techniques:

1. OLS
2. Lasso:

$$\hat{\beta}_{lasso} = \arg \min_{\beta \in \mathbf{R}^{189}} \|X^{i}_{t+h} - \beta^{i}_{t,h} \boldsymbol{F}_{t+h}\|_2^2 + \lambda \mathbf{1}\{\beta > 0\}, \quad \lambda = \infty$$

We connect Strip Prices + Fund Exposures = Replicating Portfolio

- Define scaled long-positions in each factor that are budget feasible, where

$$\boldsymbol{q}^i_{t,h} = \frac{\beta^i_{t,h}}{\sum_{h=1}^{H} \beta^i_{t,h} \boldsymbol{P}_{t,h}} \quad \Rightarrow \quad \sum_{h=1}^{H} \boldsymbol{q}^i_{t,h} \boldsymbol{P}_{t,h} = 1.$$

  where $\boldsymbol{P}_{t,h}$ comes from an asset pricing model

- Null: present discounted value of fund cash distributions is 1:

$$\mathbb{E}_t \left[\sum_{h=1}^{H} M^h_{t+h} X^i_{t+h} \right] = \mathbb{E}_t \left[\sum_{h=1}^{H} M^h_{t+h} \boldsymbol{q}^i_{t,h} \boldsymbol{F}_{t+h} \right] = \sum_{h=1}^{H} \boldsymbol{q}^i_{t,h} \boldsymbol{P}_{t,h} = 1$$

Estimation Enables Novel Understanding of PE Asset Pricing

- Use model to understand expected returns:

$$\mathbb{E}_t\left[R^i\right] = \sum_{h=1}^{H}\sum_{k=1}^{K} \boldsymbol{w}^i_{t,h}(k)\mathbb{E}_t\left[\boldsymbol{R}_{t+h}(k)\right]$$

- Profit corrects for risk, but may include premium for illiquidity:

$$\begin{aligned} v^i_{t+h} &= X^i_{t+h} - \boldsymbol{q}^i_{t,h}\boldsymbol{F}_{t+h} \\ RAP^i_t &= \sum_{h=1}^{H} P^{\$}_{t,h} v^i_{t+h} \end{aligned}$$

Asset Pricing Model

- State variables follow Gaussian first-order VAR:

$$\boldsymbol{z}_t = \boldsymbol{\Psi}\boldsymbol{z}_{t-1} + \boldsymbol{\Sigma}^{\frac{1}{2}}\varepsilon_t, \quad \varepsilon_t \sim i.i.d.\mathcal{N}(0, I)$$

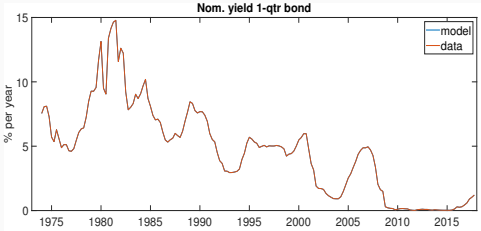
- Bond variables: nominal short rate, realized inflation, 5-year - 1-month Treasury spread

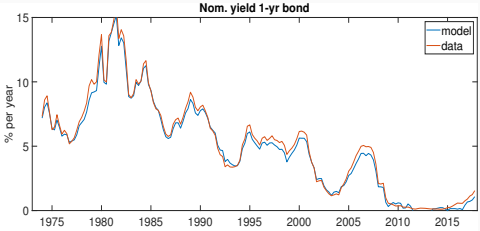
- Stock variables: log price-dividend, log real dividend growth for: CRSP, NAREIT real estate, listed infra, small, growth

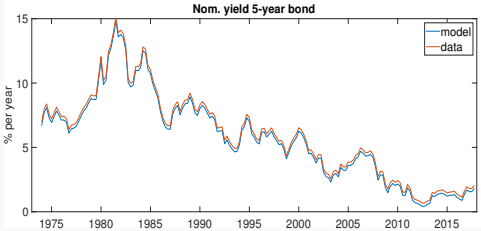
- SDF:

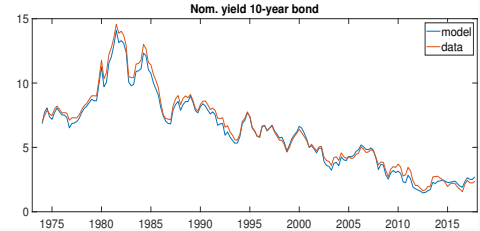
$$m^{\$}_{t+1} \quad = \quad -y^{\$}_t(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1}$$

Model Matches Time-Series of Bond Yields

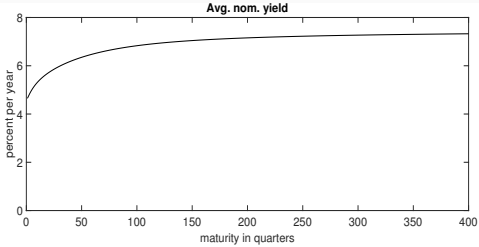


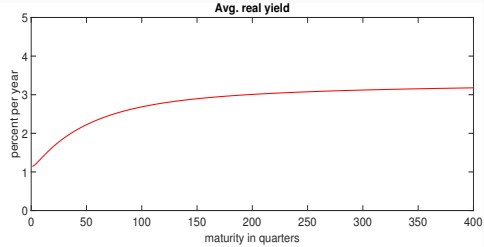


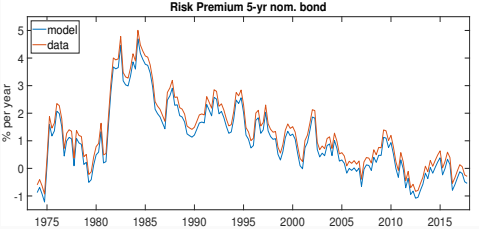


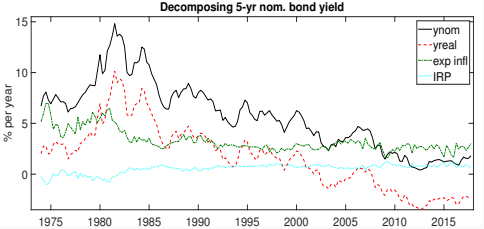


Also Matches underlying Components of Bond Yield: Real + Nominal

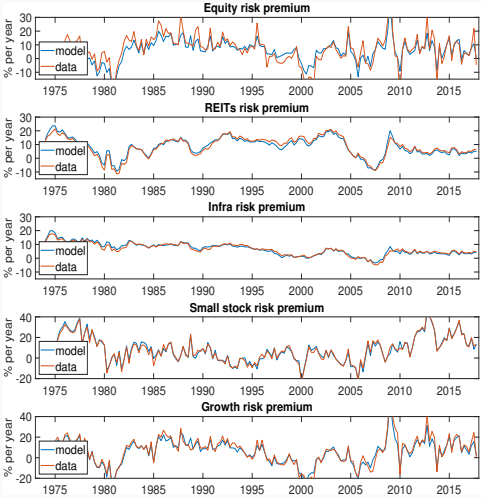


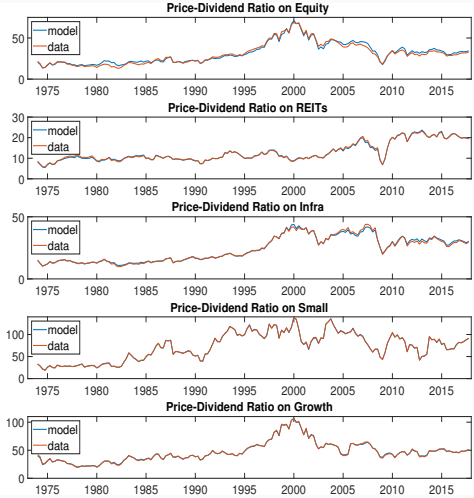




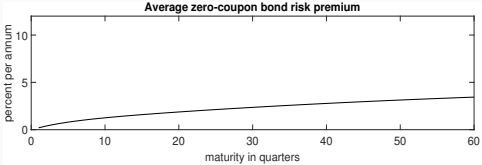


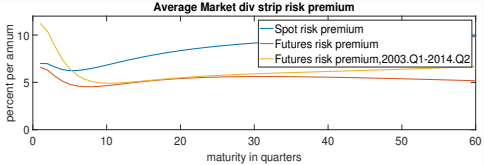
Fits Equity Risk *Premia* as well as Stock Price *Levels*

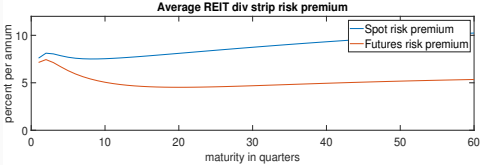


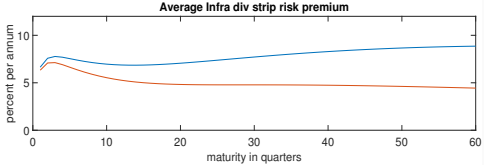


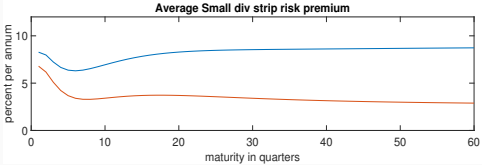
Rich Patterns in Temporal Pricing of Risk

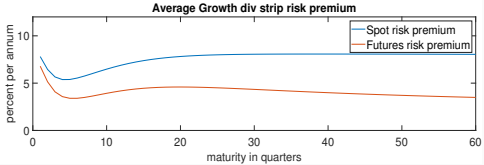




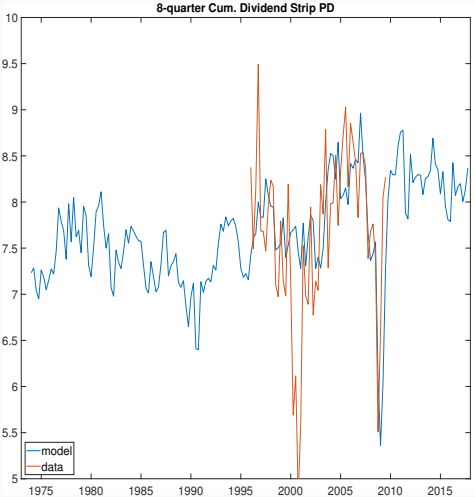


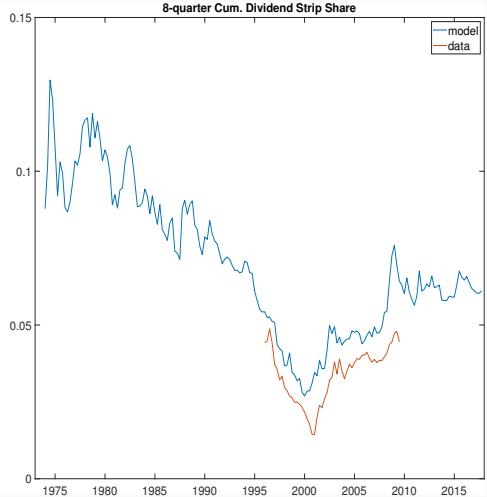




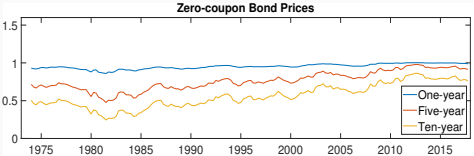


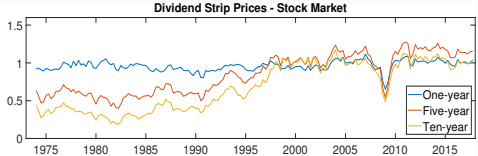
Imputed Dividend Strip Model Matches Data when Available

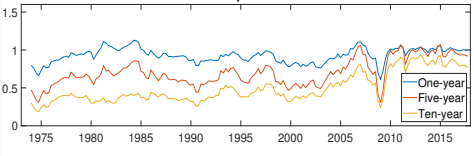


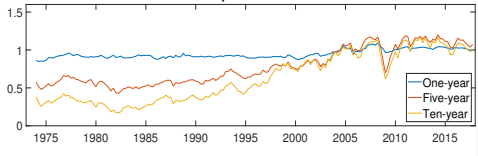


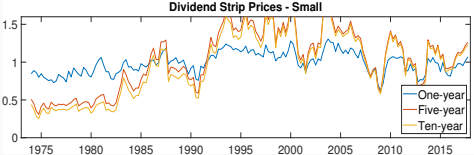
Outcome of Model: Bond + Dividend Strip Prices

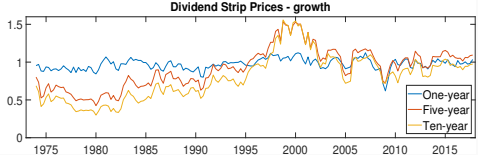




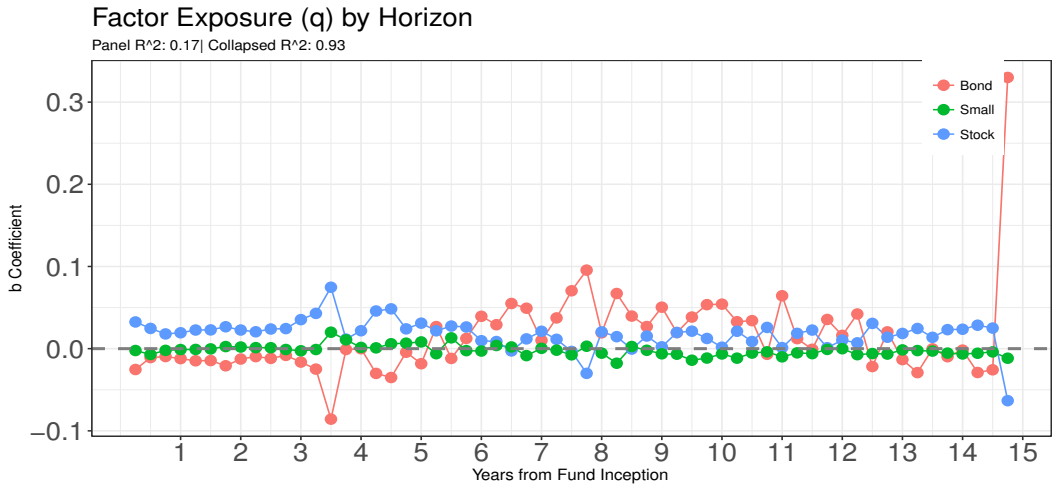






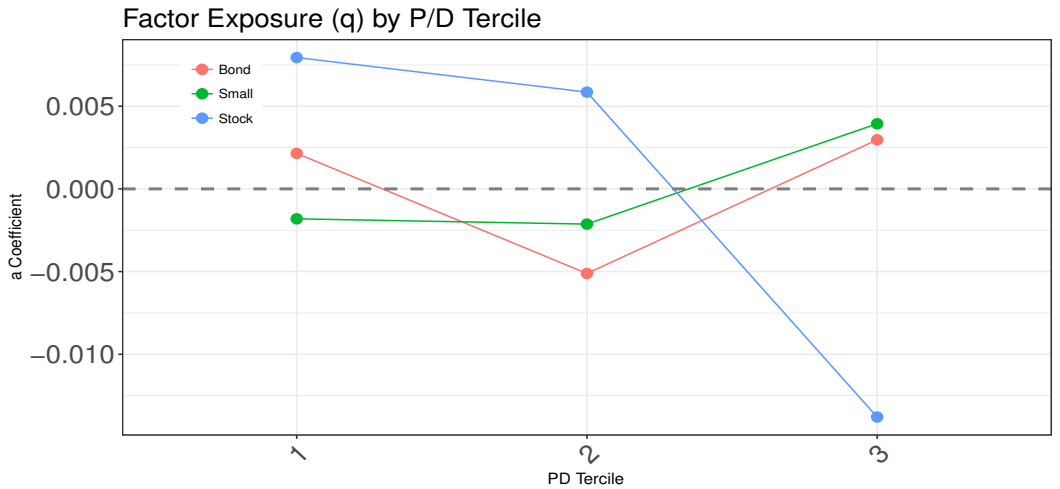


Factor Exposure in PE Funds by Horizon — Buyout



Factor Exposure in PE Funds by P/D Ratio — Buyout

Venture Capital | Real Estate | Infrastructure



Factor Exposure in PE Funds by Horizon — Buyout, Lasso

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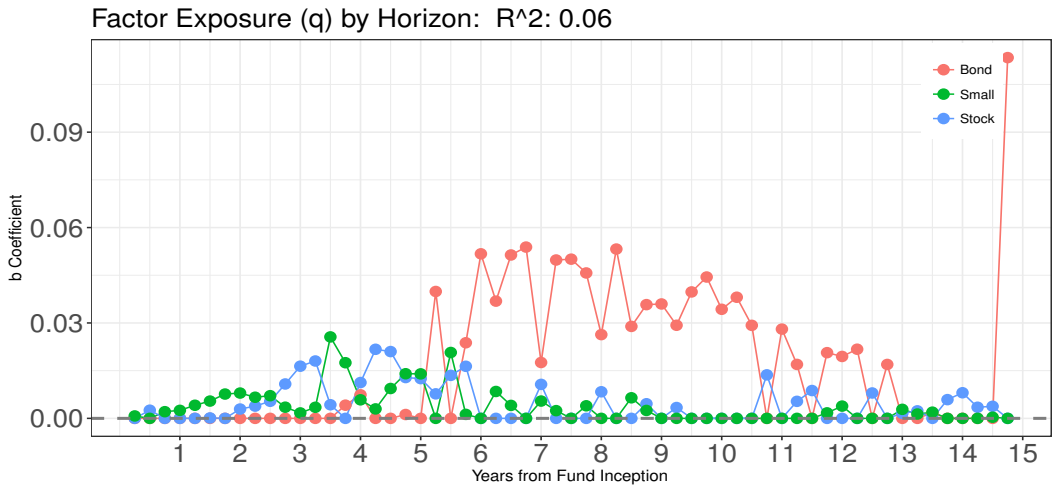
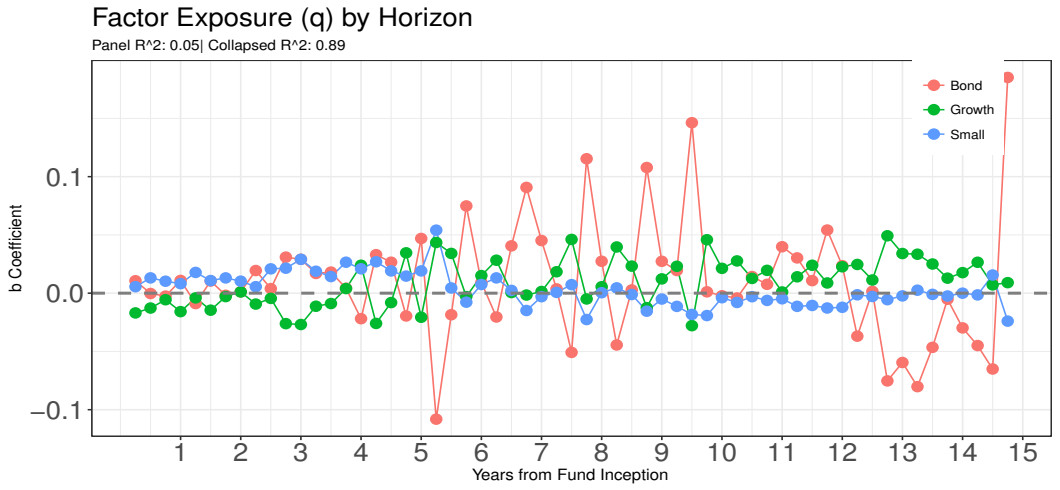
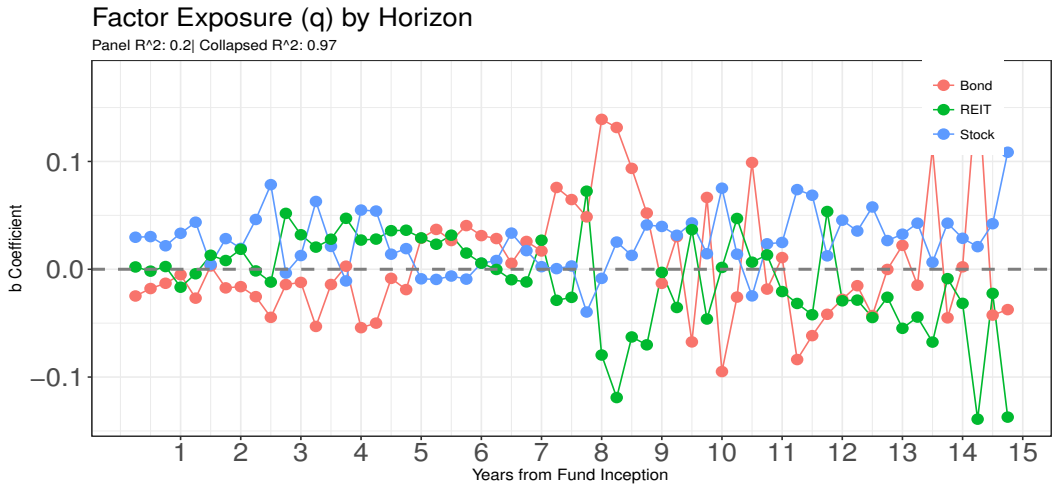


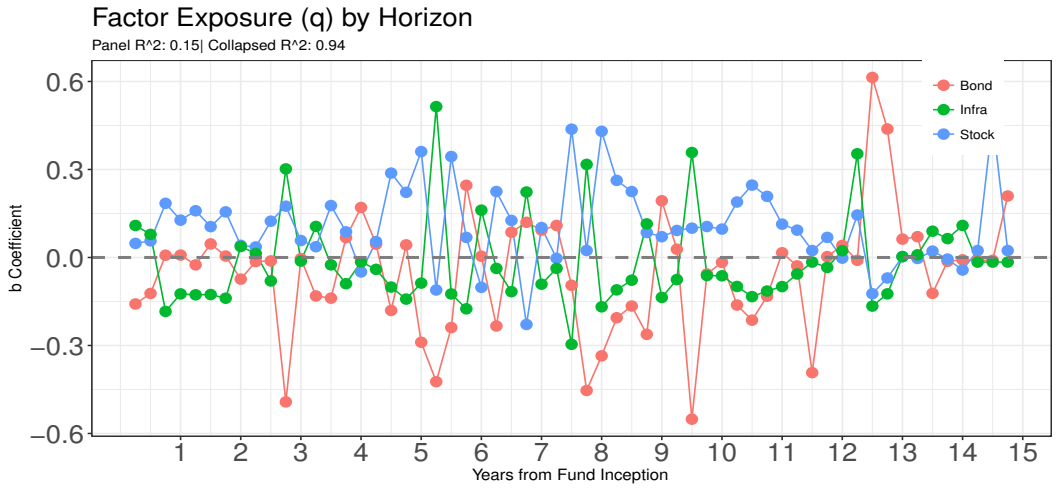
Factor Exposure in PE Funds by Horizon — VC



Factor Exposure in PE Funds by Horizon — Real Estate



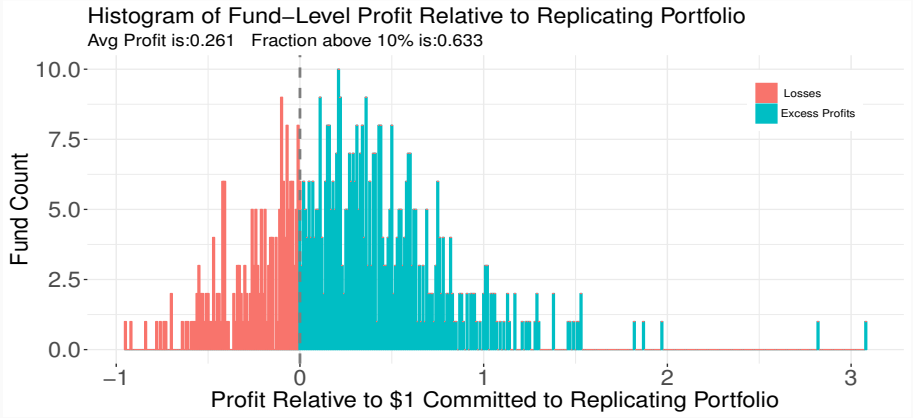
Factor Exposure in PE Funds by Horizon — Infrastructure



PE Fund Risk-Adjusted Profits — Buyout

Venture Capital | Real Estate | Infrastructure

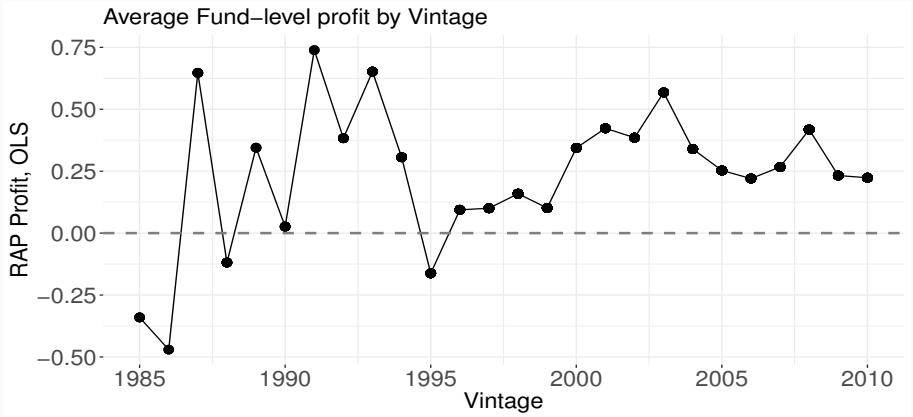
Buyout - Lasso | Venture Capital - Lasso | Real Estate - Lasso | Infrastructure - Lasso



PE Fund Risk-Adjusted Profits — Buyout

Venture Capital | Real Estate | Infrastructure | Burgiss - Venture Capital

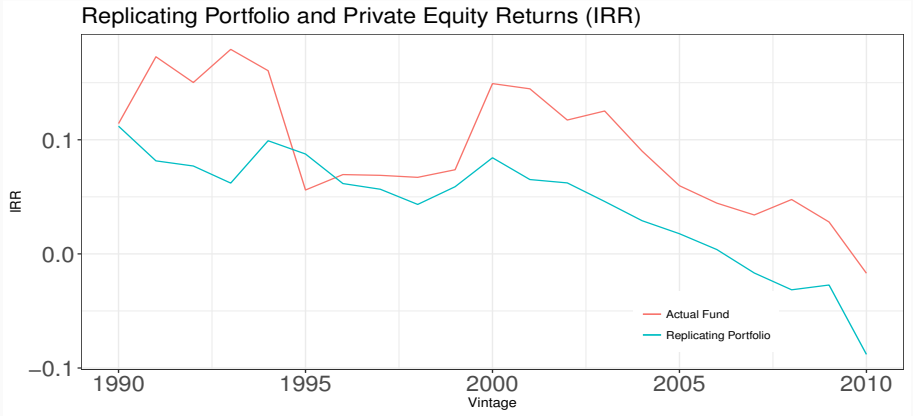
Buyout - Lasso | Venture Capital - Lasso | Real Estate - Lasso | Infrastructure - Lasso



Aggregated Replicating Portfolios Match Fund Cash Flows Each Vintage — Buyout

Venture Capital | Real Estate | Infrastructure

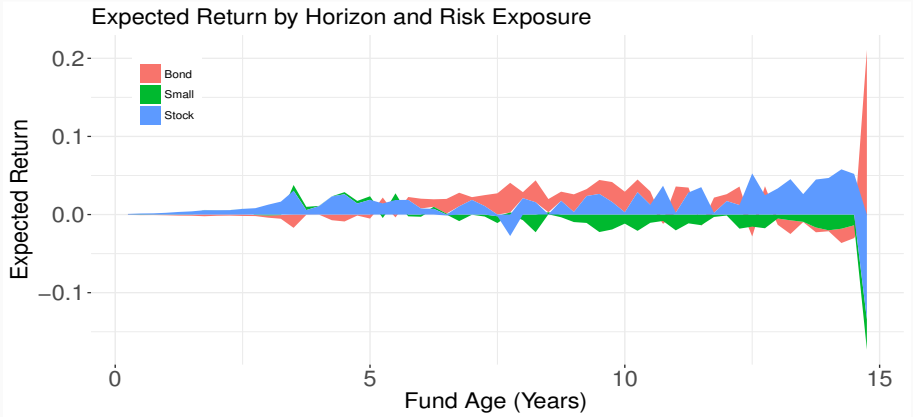
Buyout - Lasso | Venture Capital - Lasso | Real Estate - Lasso | Infrastructure - Lasso



Private Equity Fund Expected Return — Buyout

Venture Capital | Real Estate | Infrastructure

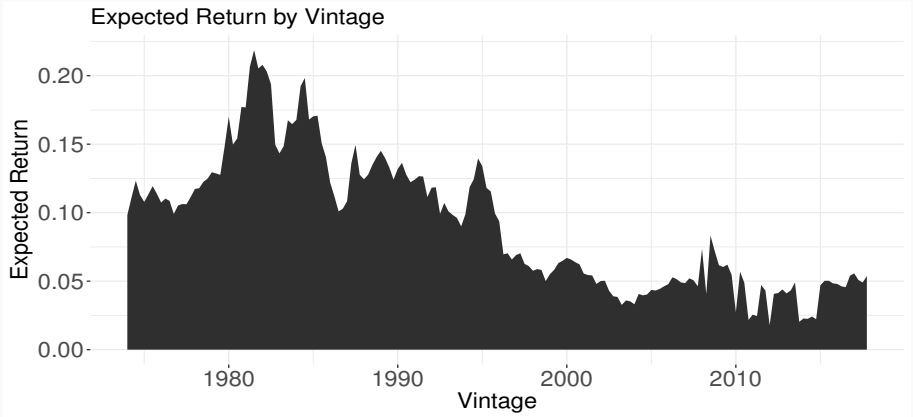
Buyout | Venture Capital | Real Estate | Infrastructure



PE Expected Return — Buyout

Venture Capital | Real Estate | Infrastructure | Duration | Betas

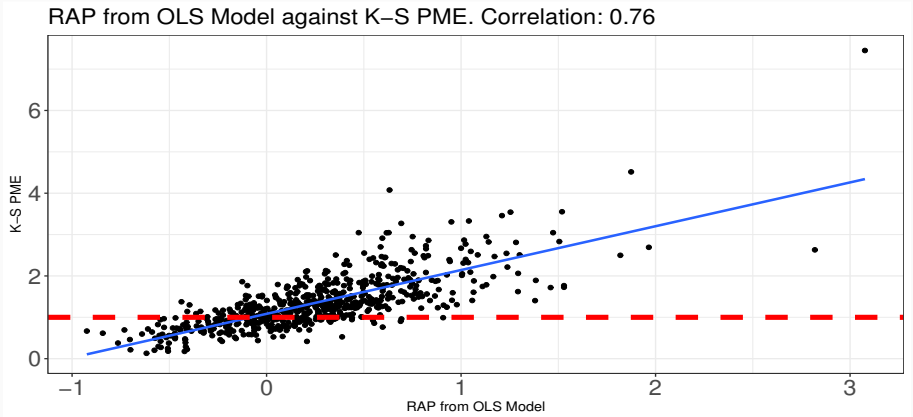
Buyout - Lasso | Venture Capital - Lasso | Real Estate - Lasso | Infrastructure - Lasso



PE Comparison with PME — Buyout

Venture Capital | Real Estate | Infrastructure

Buyout, Lasso | Venture Capital, Lasso | Real Estate, Lasso | Infrastructure, Lasso



Takeaways

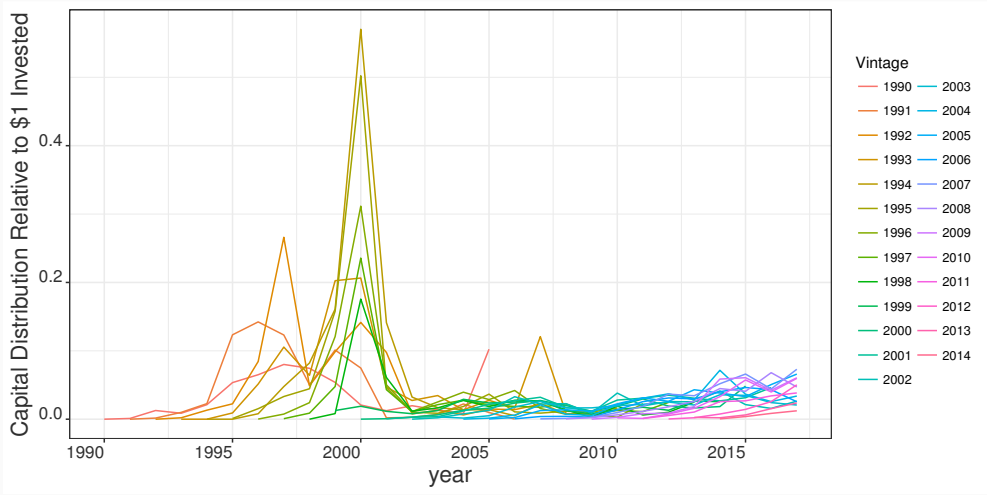
1. Develop methodology to value and understand risk/return characteristics when only cash flows, not returns, are available

2. Find PE funds take asset-specific specific exposure. Small, growth, real estate, infra exposure has migrated to PE

3. Risk-adjusted profit (and compensation for illiquidity), as well as expected return on replicating portfolios, declining over time

Cash-Flow Variation Across Horizon and Vintage — Venture Capital

Back



Cash-Flow Variation Across Horizon and Vintage — Real Estate

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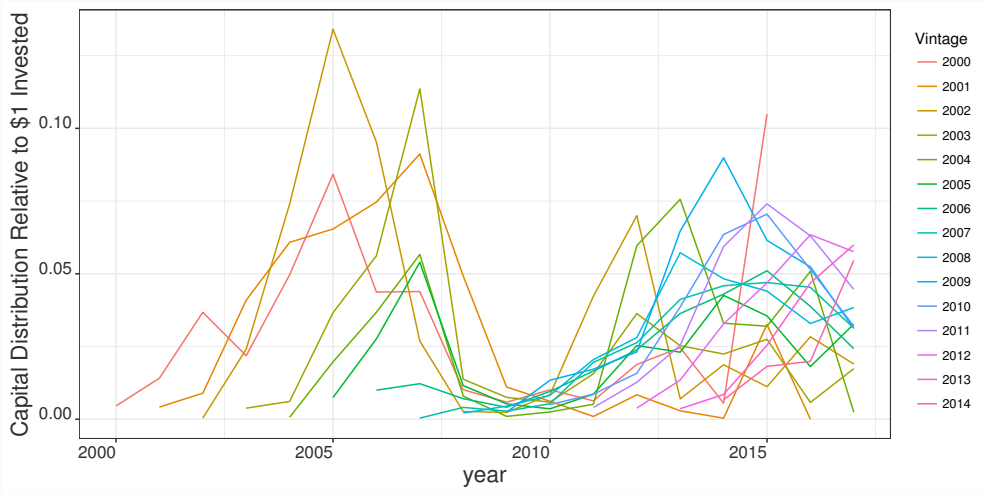

Cash-Flow Variation Across Horizon and Vintage — Infrastructure

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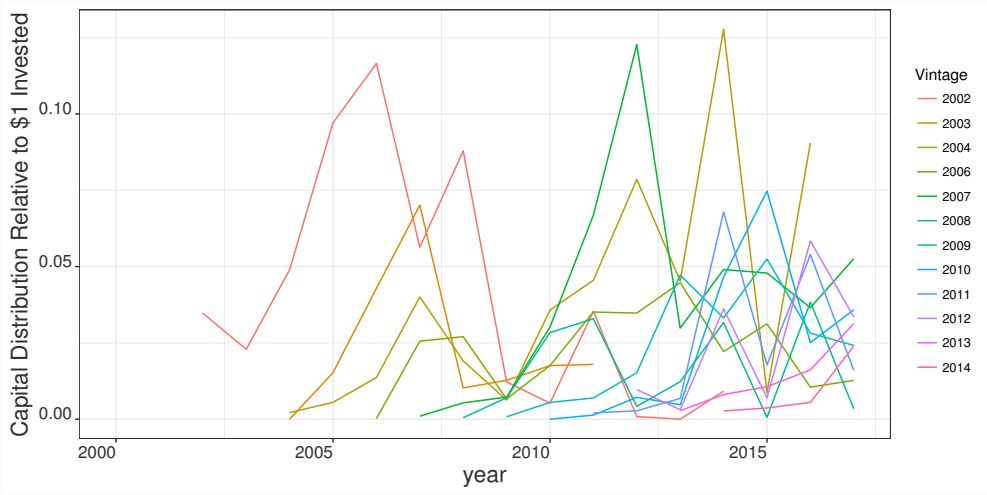

Factor Exposure in PE Funds by P/D Ratio — VC

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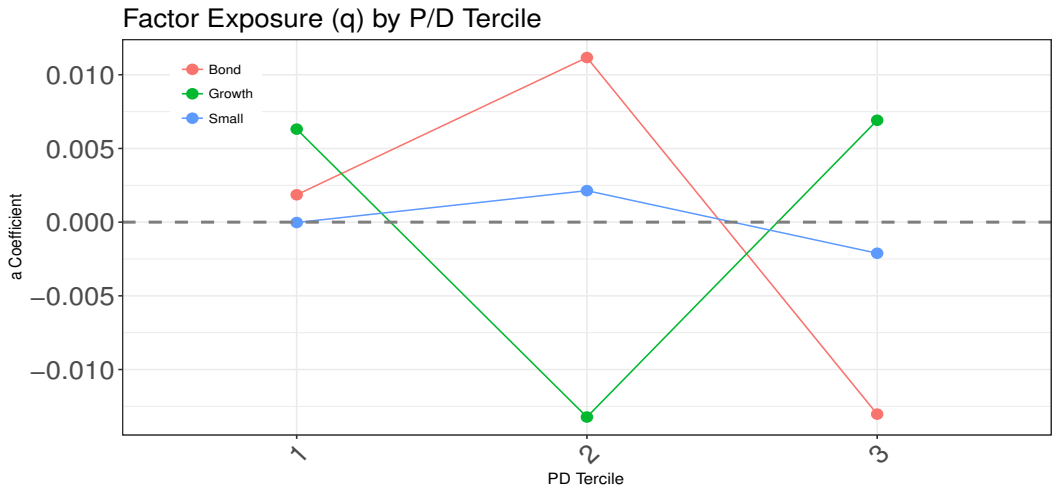

Factor Exposure in PE Funds by P/D Ratio — Real Estate

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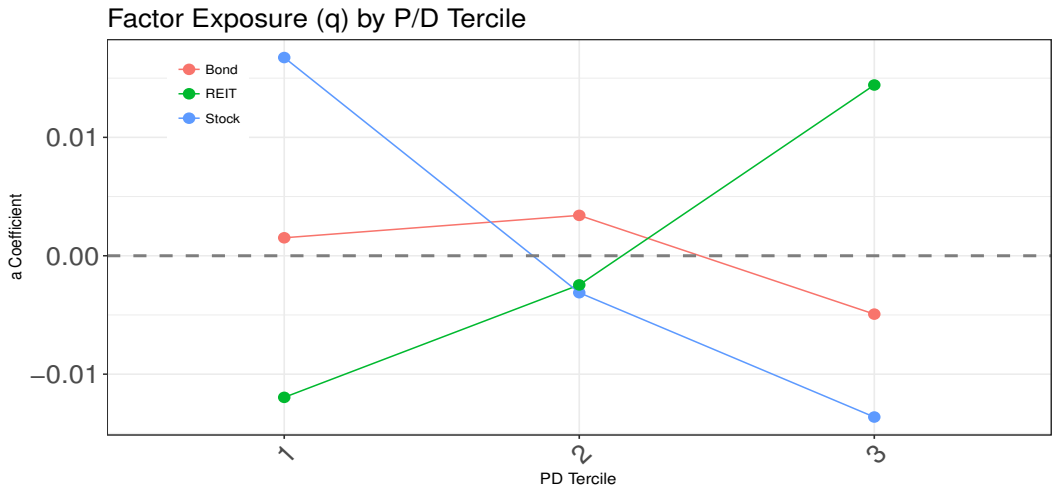

Factor Exposure in PE Funds by P/D Ratio — Infrastructure

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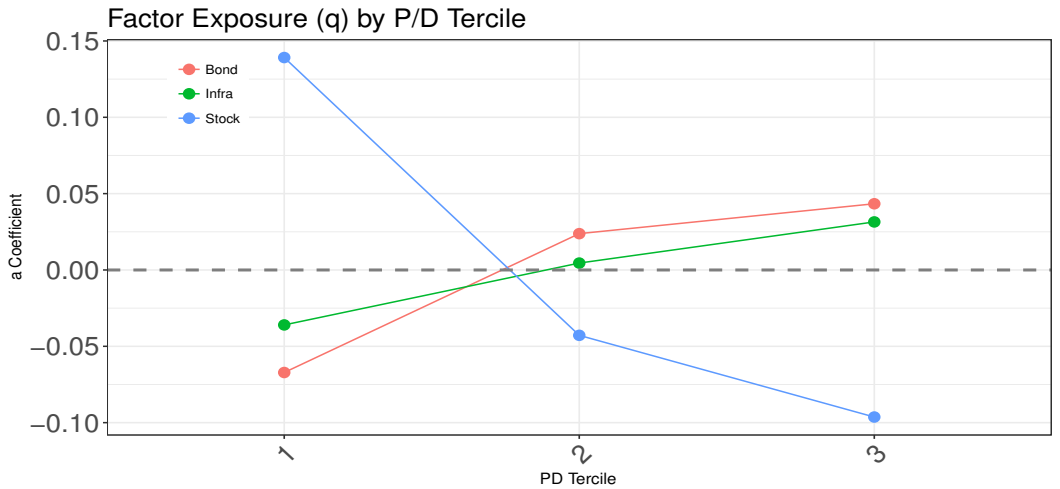

Factor Exposure in PE Funds by Horizon — VC, Lasso

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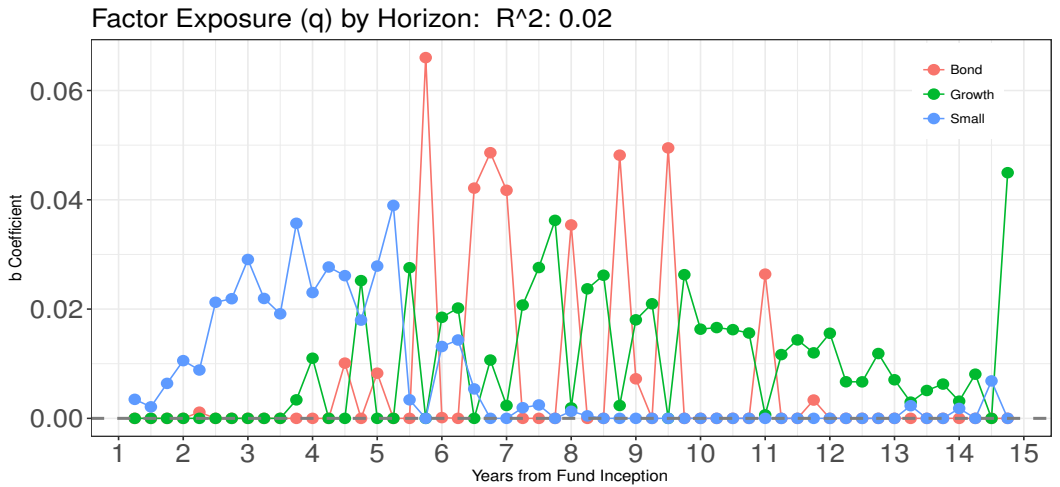

Factor Exposure in PE Funds by Horizon — Real Estate, Lasso

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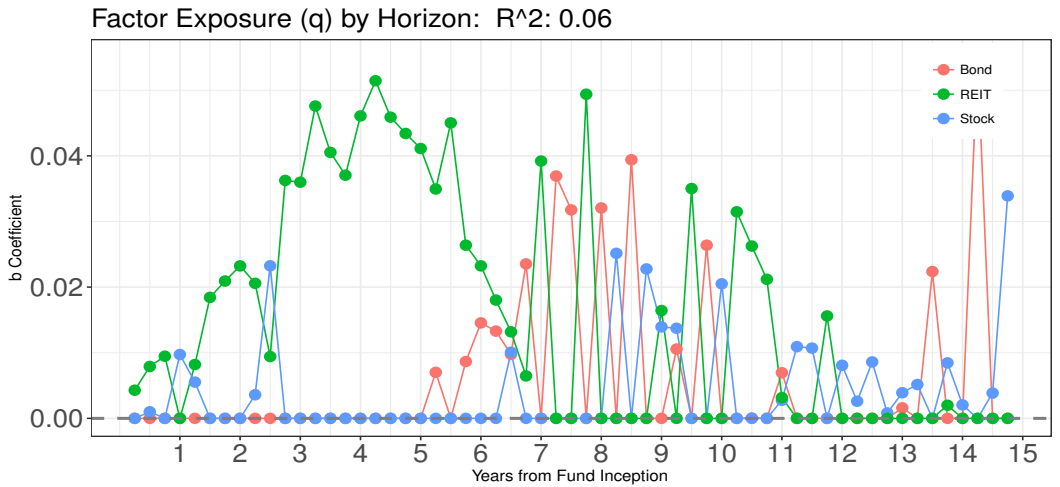

Factor Exposure in PE Funds by Horizon — Infrastructure, Lasso

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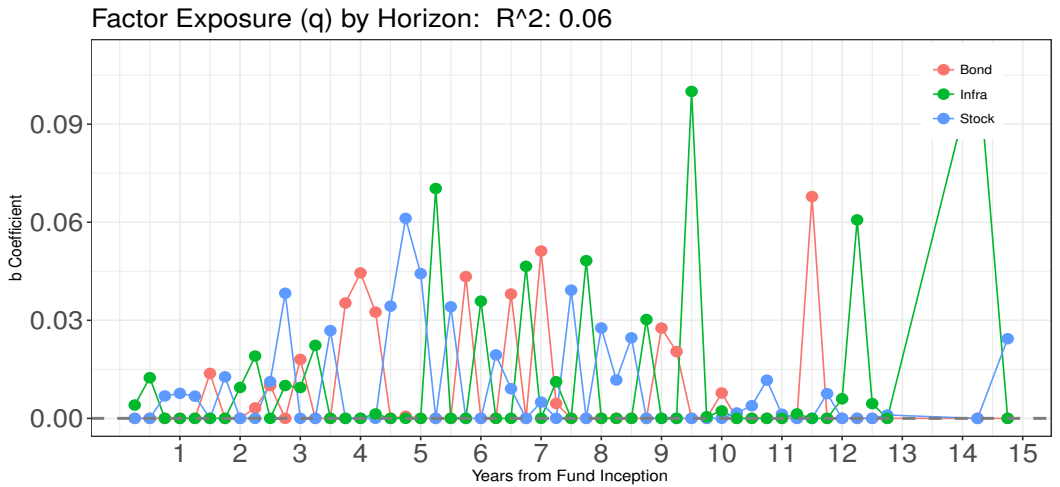
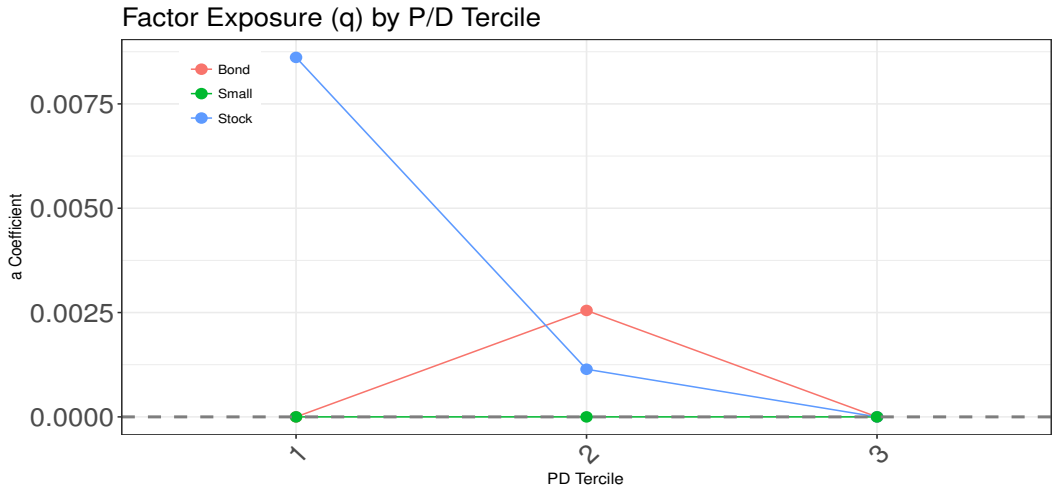
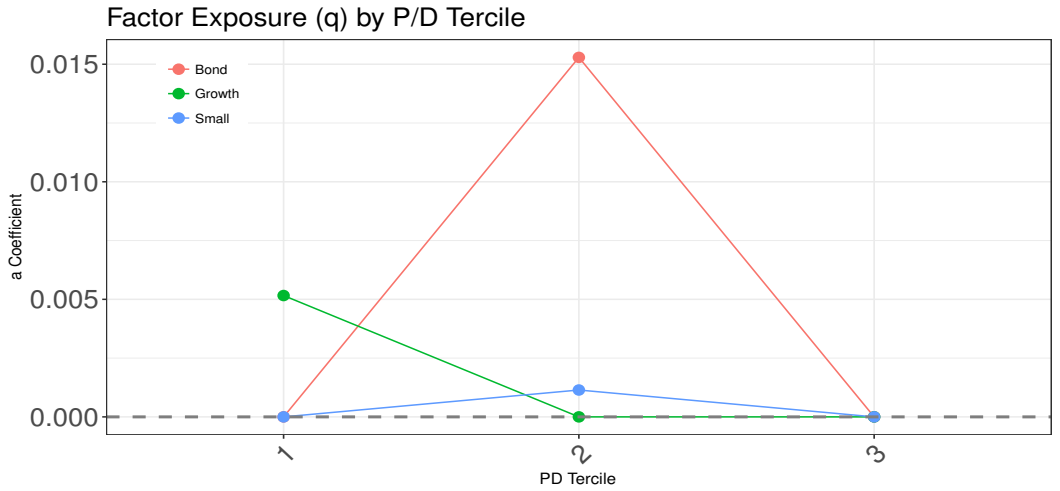


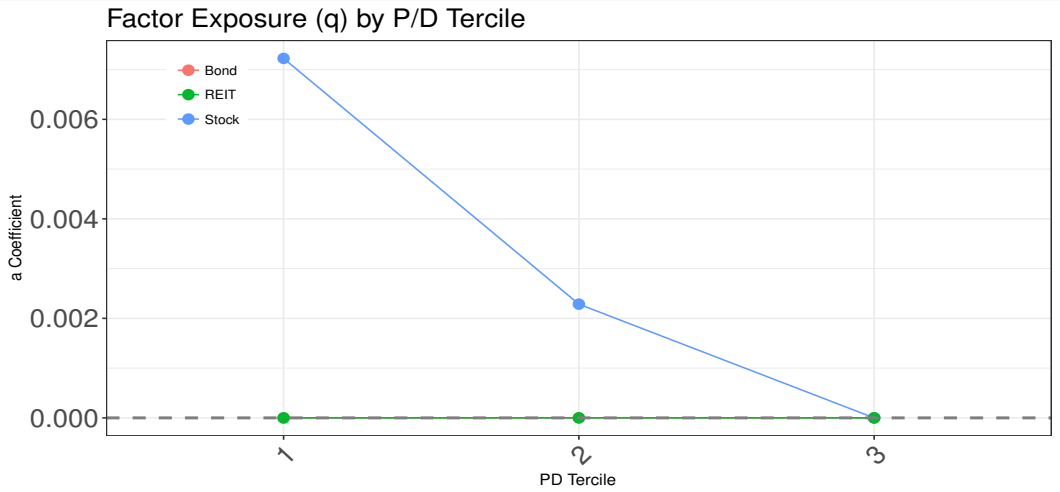
Factor Exposure in PE Funds by Horizon — Buyout, Lasso



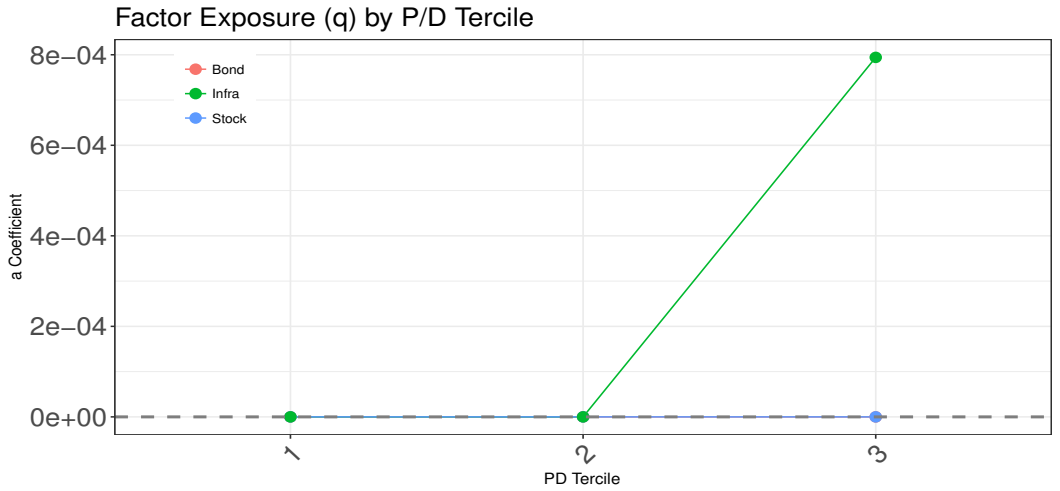
Factor Exposure in PE Funds by Horizon — VC, Lasso



Factor Exposure in PE Funds by Horizon — Real Estate, Lasso

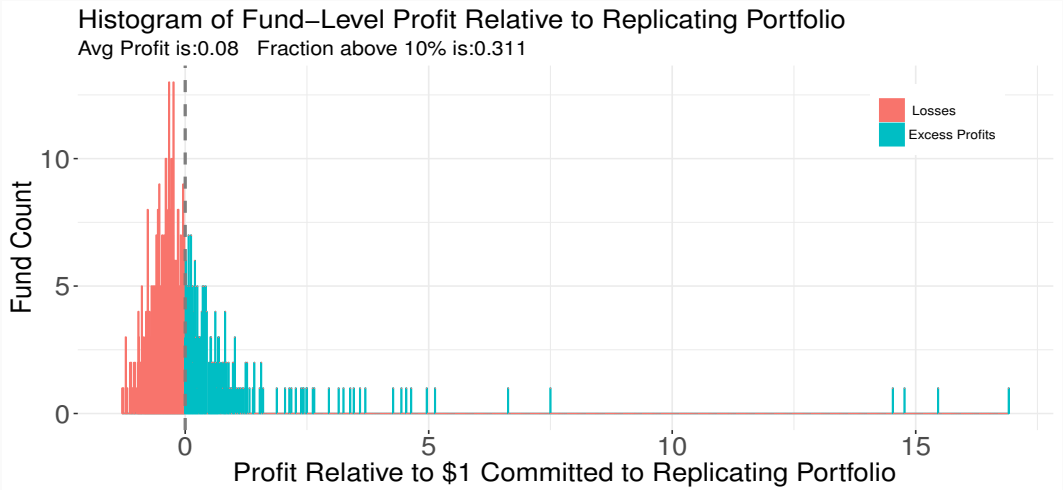


Factor Exposure in PE Funds by Horizon — Infrastructure, Lasso



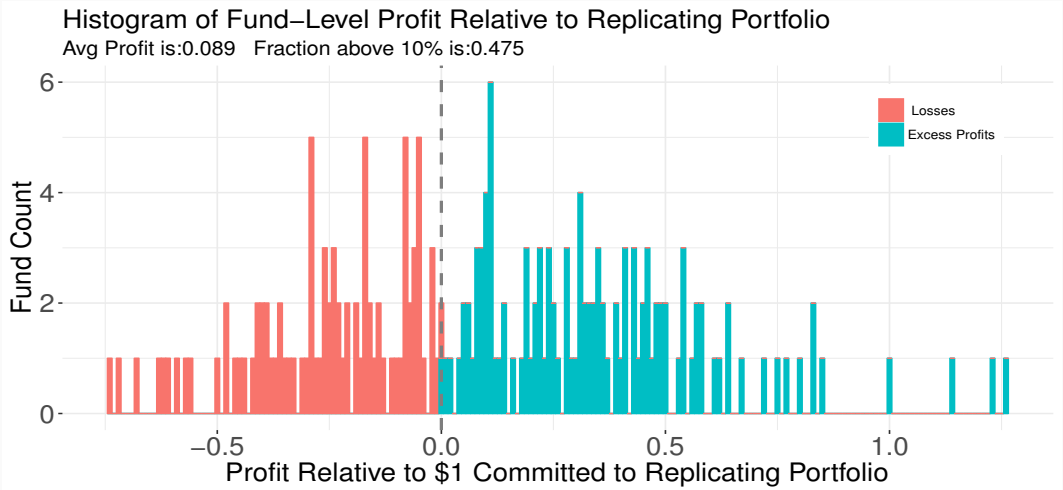
PE Fund Risk-Adjusted Profits — VC

Back



PE Fund Risk-Adjusted Profits — Real Estate

Back



PE Fund Risk-Adjusted Profits — Infrastructure

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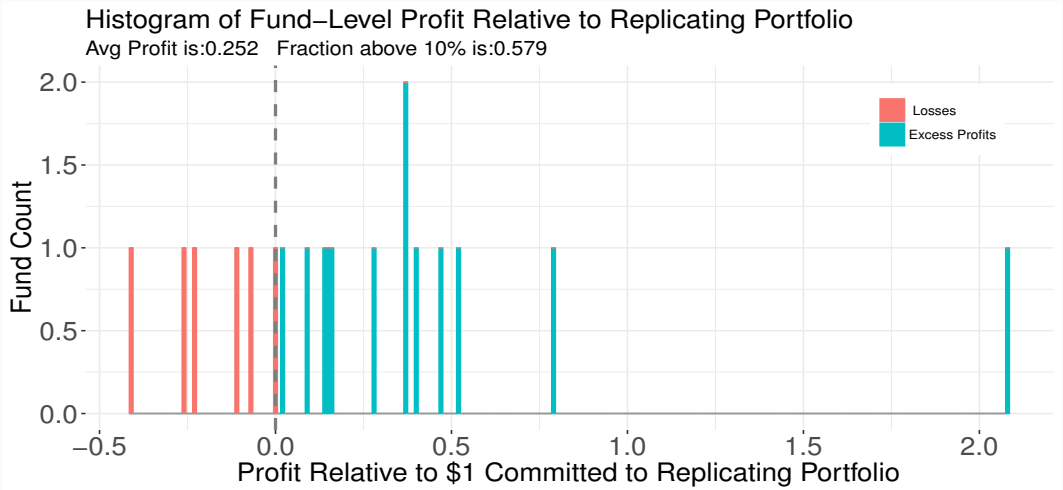
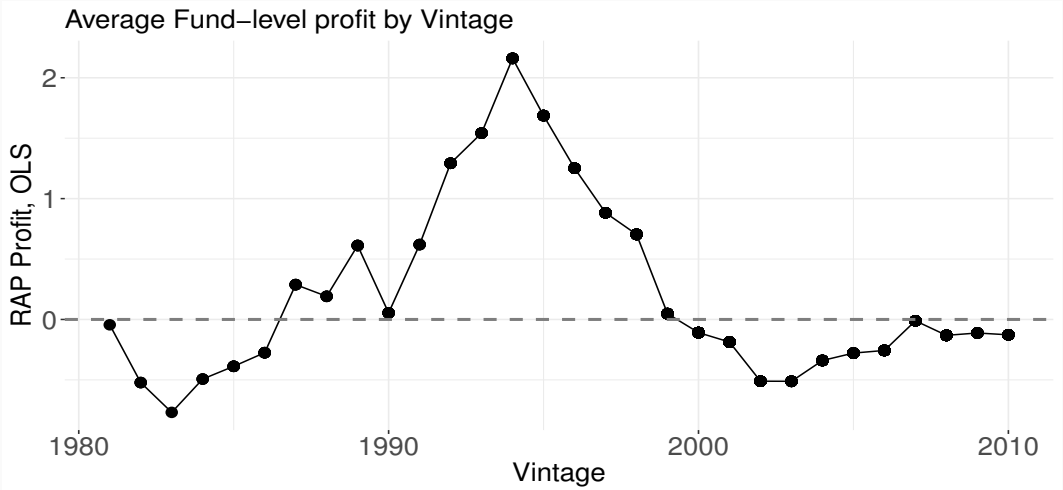


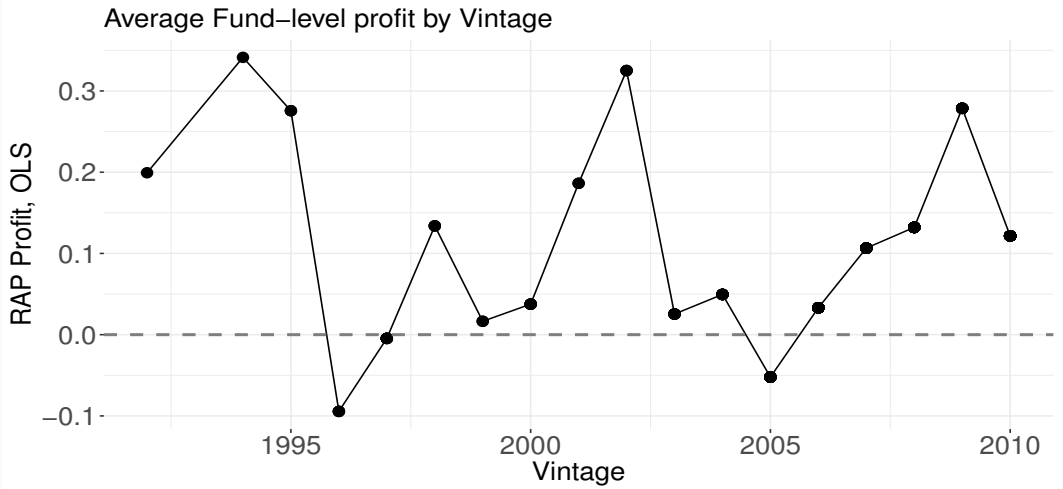
PE Fund Risk-Adjusted Profits — VC

Back



PE Fund Risk-Adjusted Profits — Real Estate

Back



PE Fund Risk-Adjusted Profits — Infrastructure

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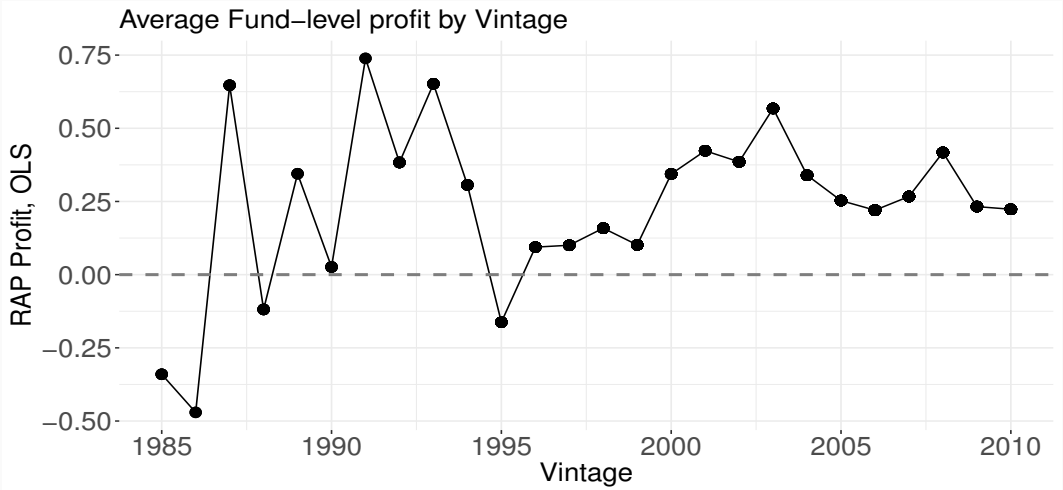
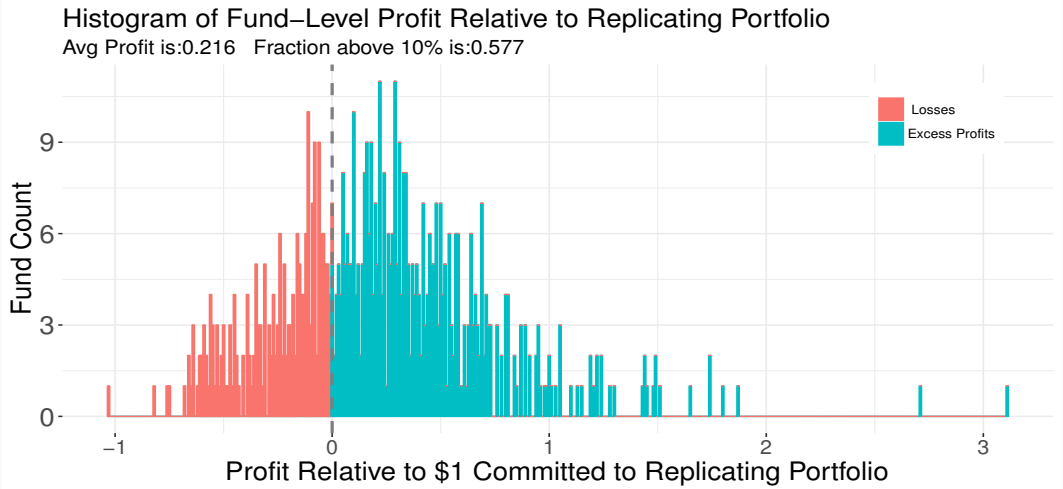


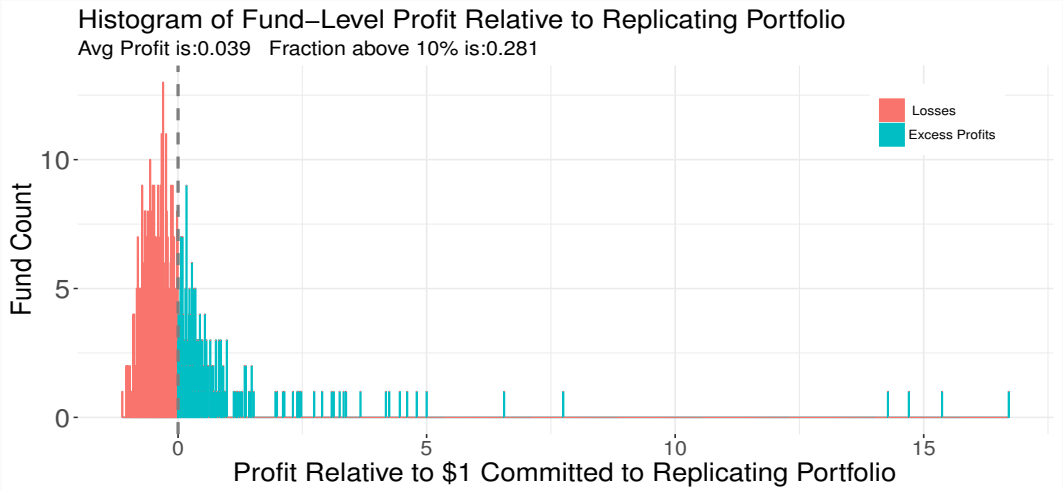
PE Fund Risk-Adjusted Profits — Buyout, Lasso

Back



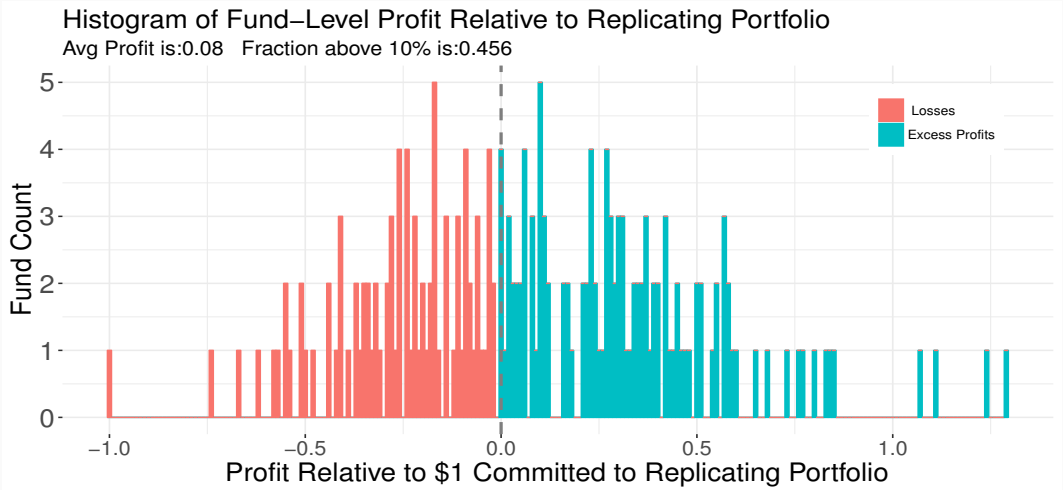
PE Fund Risk-Adjusted Profits — VC, Lasso

Back



PE Fund Risk-Adjusted Profits — Real Estate, Lasso

Back

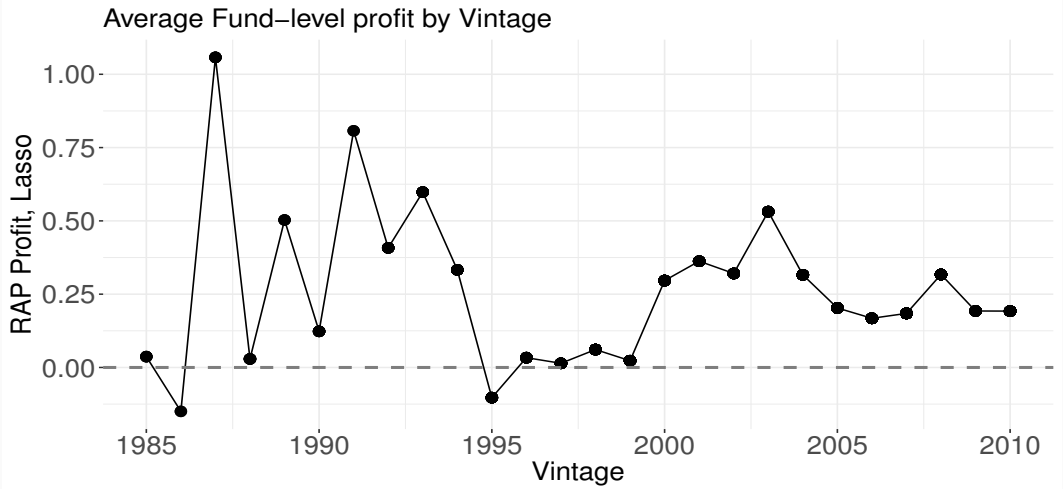


PE Fund Risk-Adjusted Profits — Infrastructure, Lasso

Back

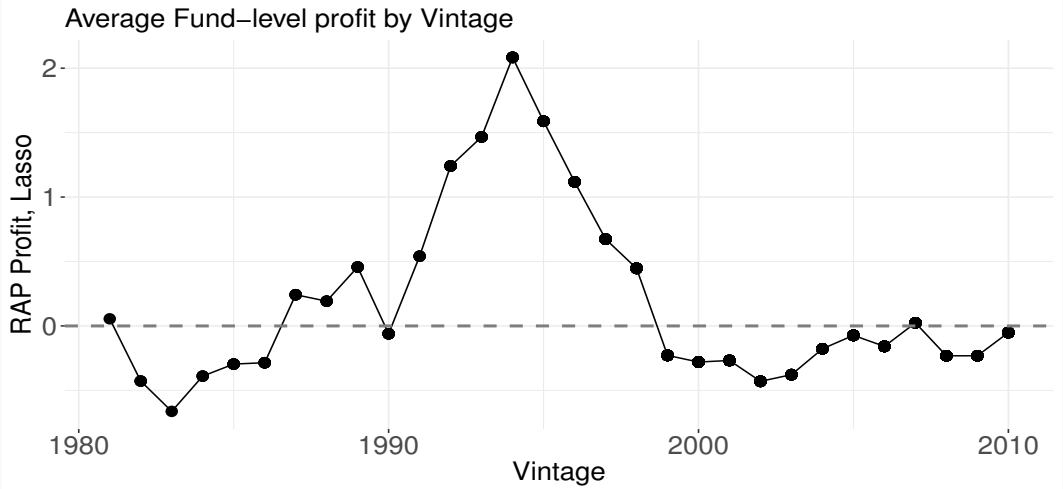
PE Fund Risk-Adjusted Profits — Buyout, Lasso

Back



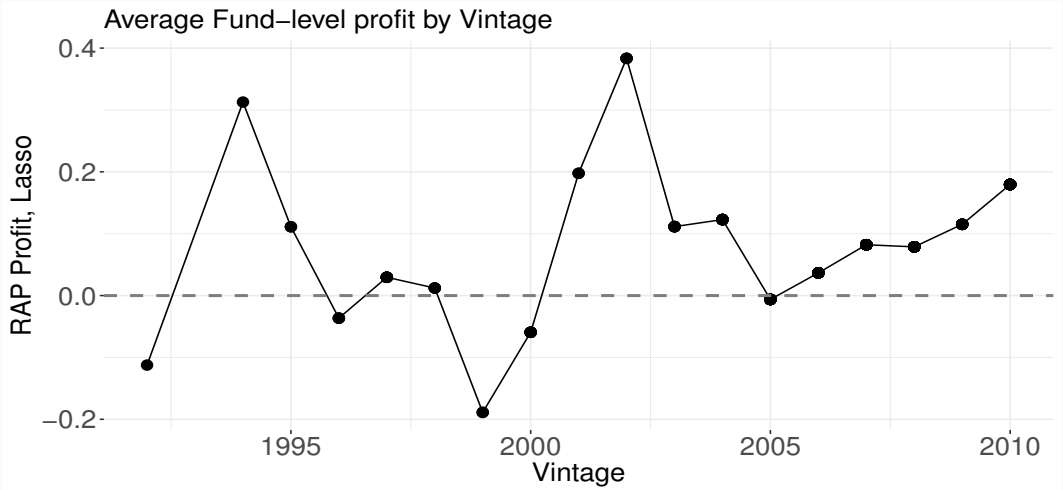
PE Fund Risk-Adjusted Profits — VC, Lasso

Back



PE Fund Risk-Adjusted Profits — Real, Lasso Estate

Back

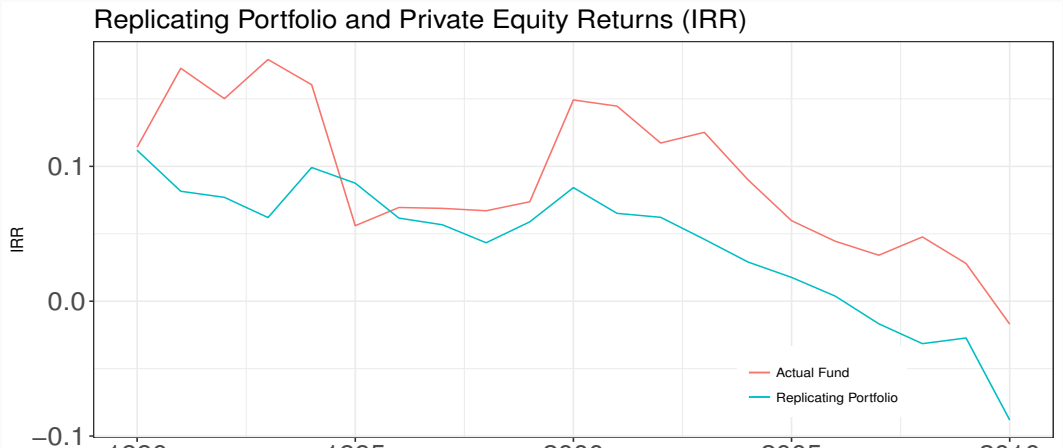


PE Fund Risk-Adjusted Profits — Infrastructure, Lasso

Back

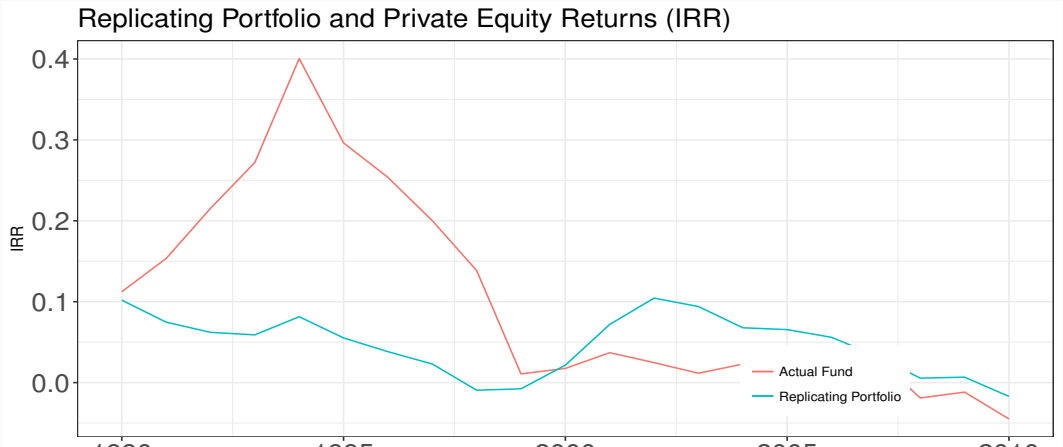
Aggregated Replicating Portfolios Match Fund Cash Flows Each Vintage — Buyout, Lasso

Back



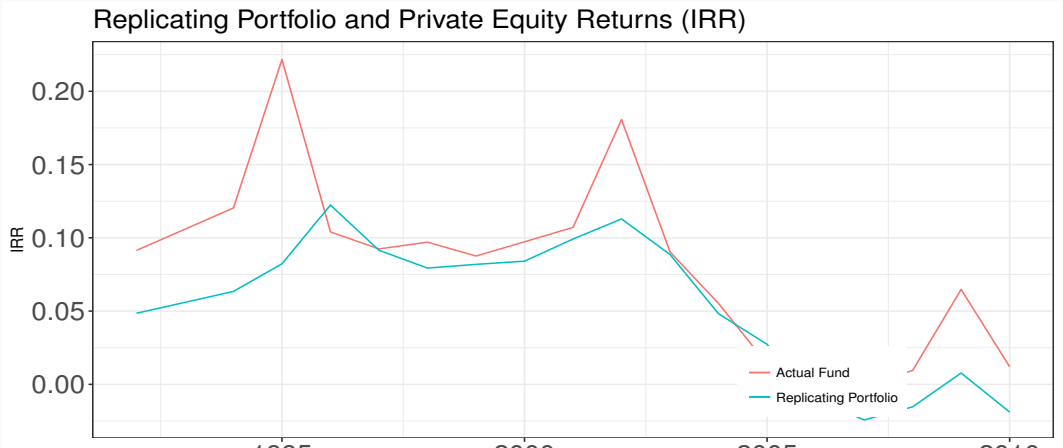
Aggregated Replicating Portfolios Match Fund Cash Flows Each Vintage — VC, Lasso

Back



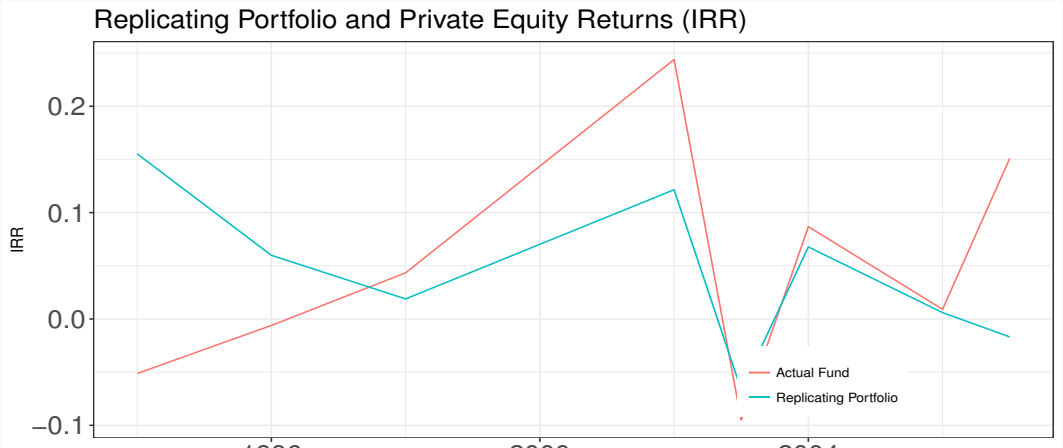
Aggregated Replicating Portfolios Match Fund Cash Flows Each Vintage — Real Estate, Lasso

Back



Aggregated Replicating Portfolios Match Fund Cash Flows Each Vintage — Infrastructure, Lasso

Back



Private Equity Fund Expected Return — Buyout, Lasso

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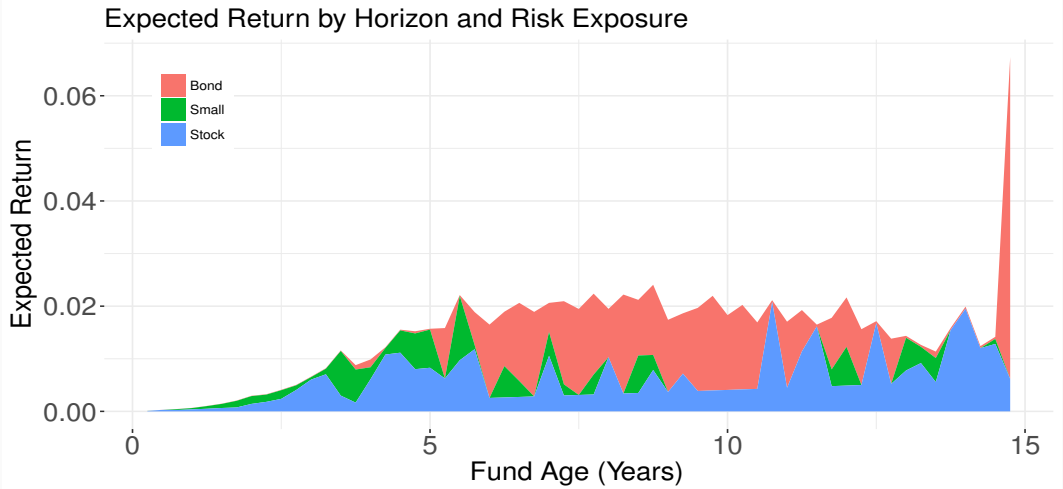
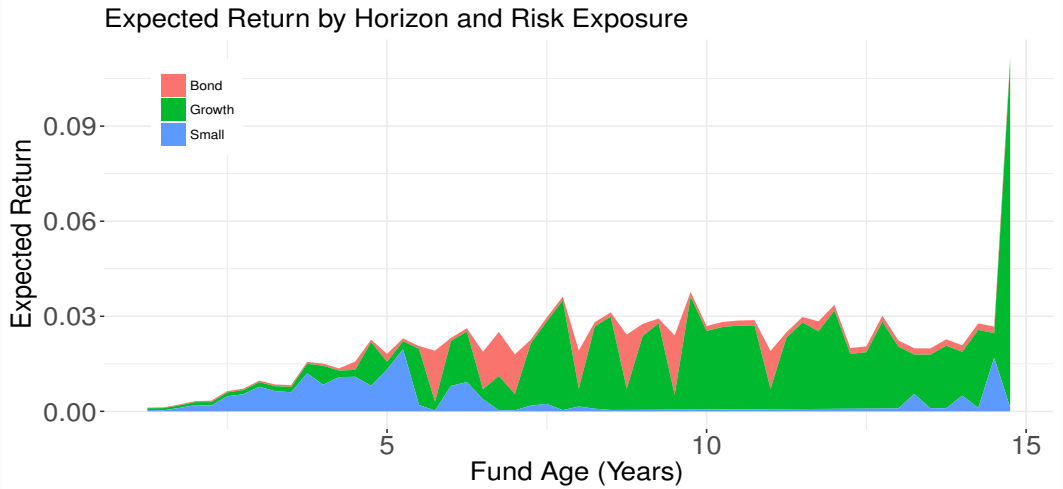


Private Equity Fund Expected Return — VC, Lasso

Back



Private Equity Fund Expected Return — Real Estate, Lasso

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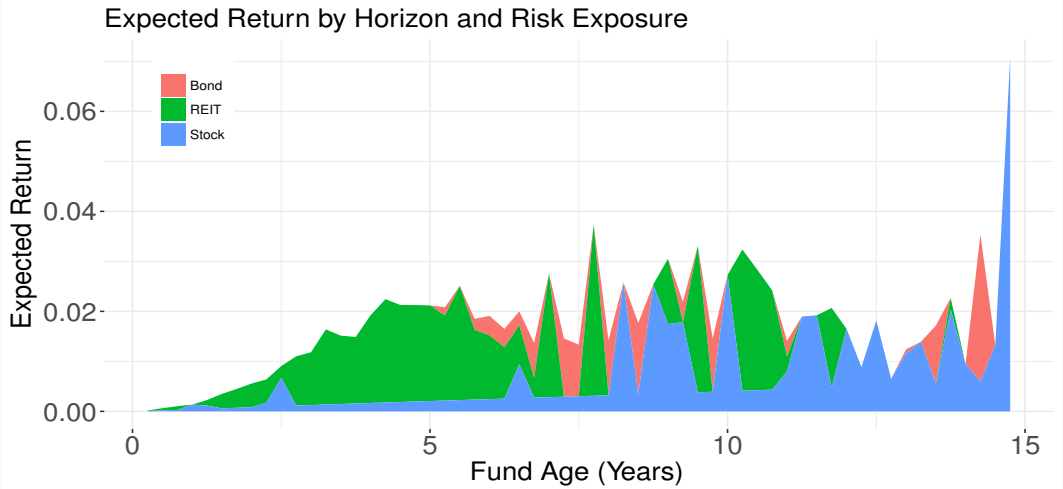

Private Equity Fund Expected Return — Infrastructure, Lasso

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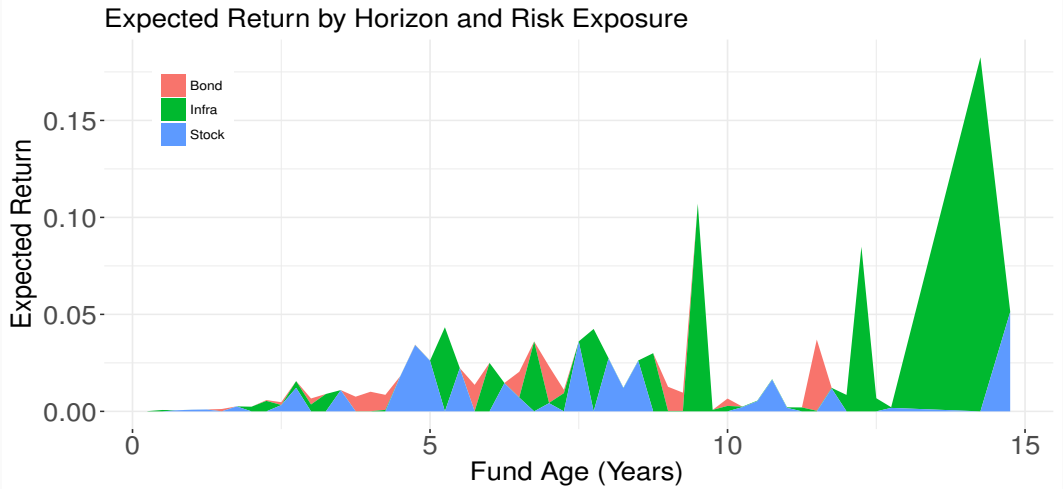
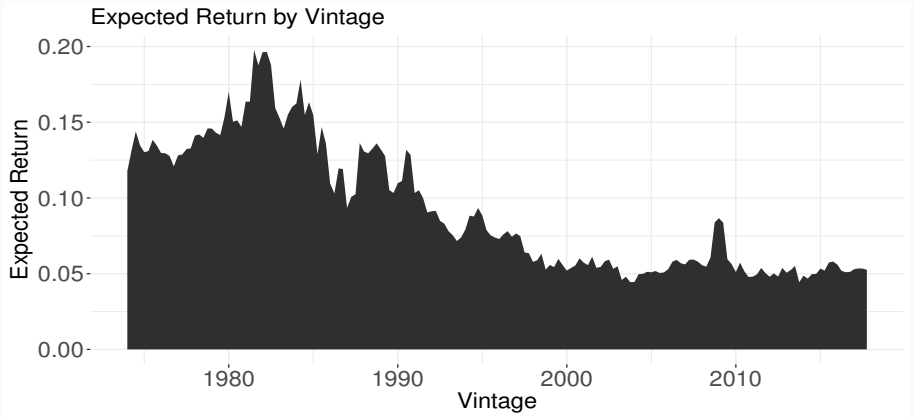


PE Expected Return — Buyout, Lasso

Back



PE Expected Return — VC, Lasso

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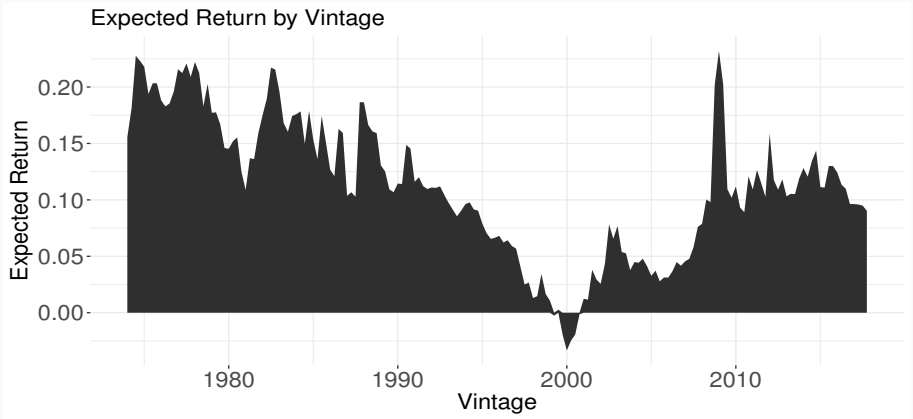
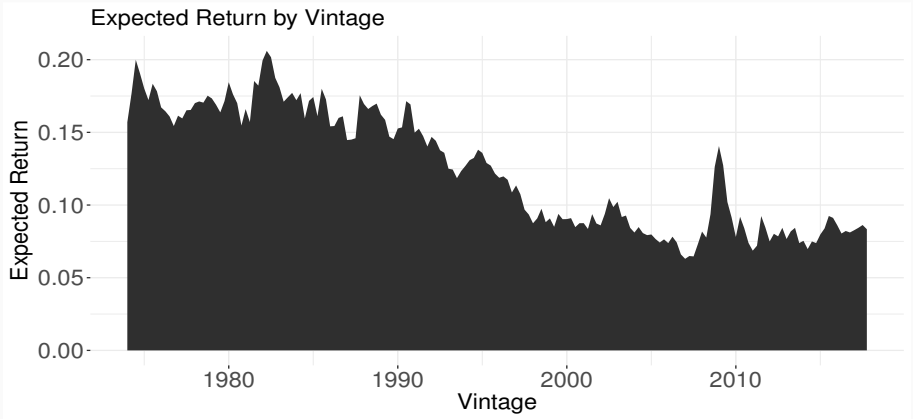


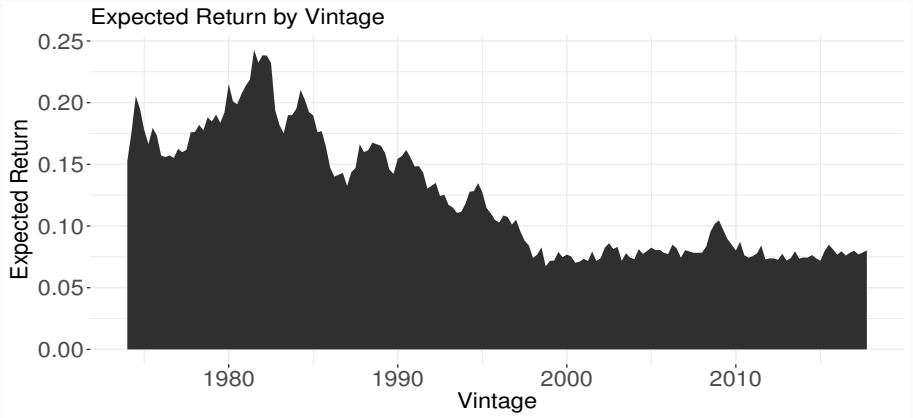
PE Expected Return — Real Estate, Lasso

Back



PE Expected Return — Infrastructure, Lasso

Back



PE Comparison with PME — VC

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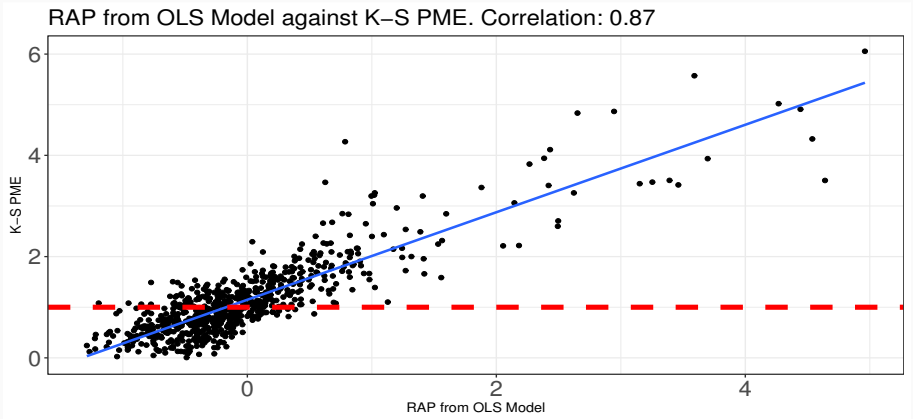
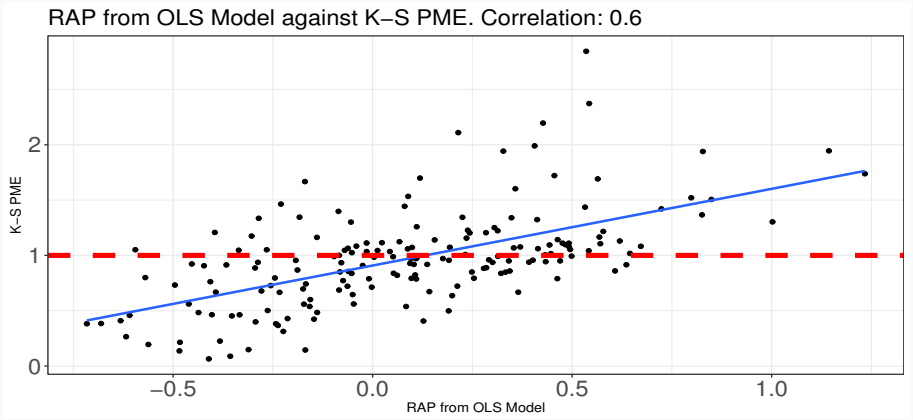


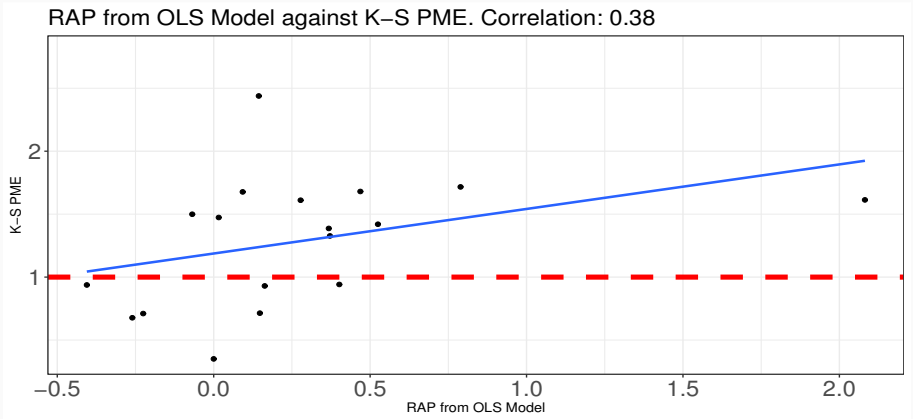
PE Comparison with PME — Real Estate

Back



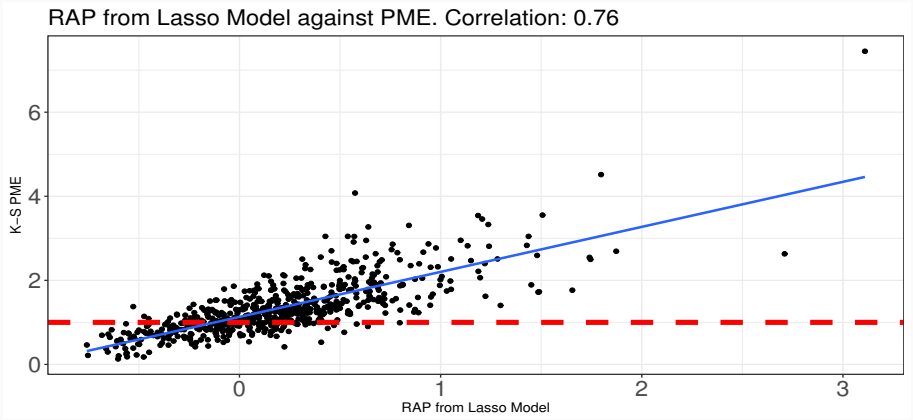
PE Comparison with PME — Infrastructure

Back



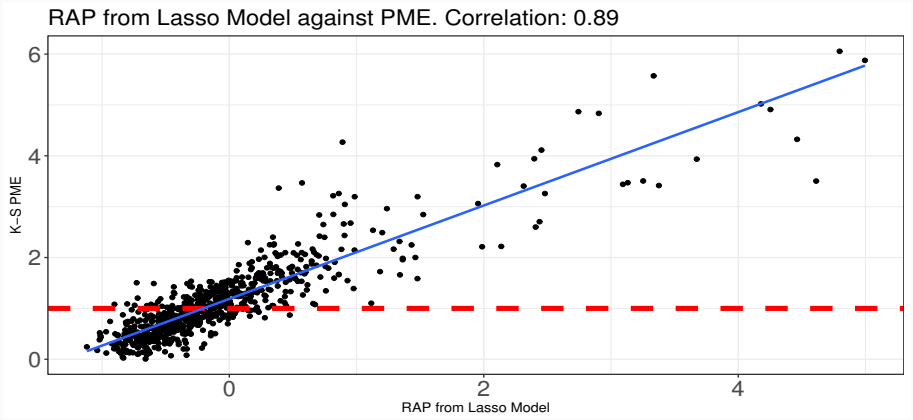
PE Comparison with PME — Buyout, Lasso

Back



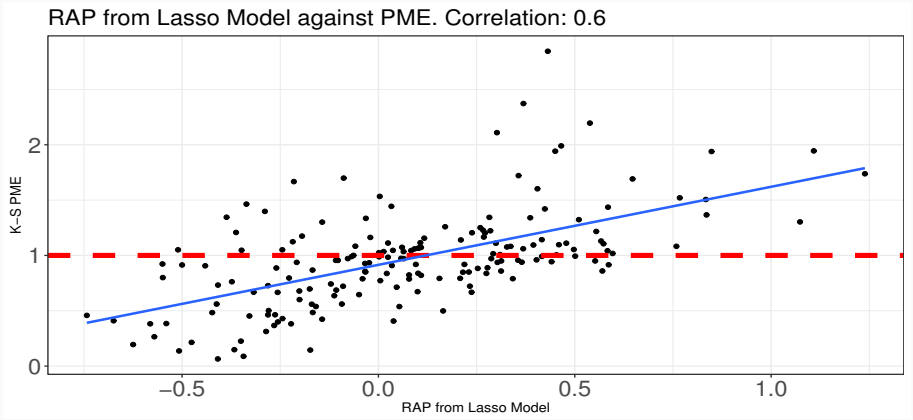
PE Comparison with PME — VC, Lasso

Back



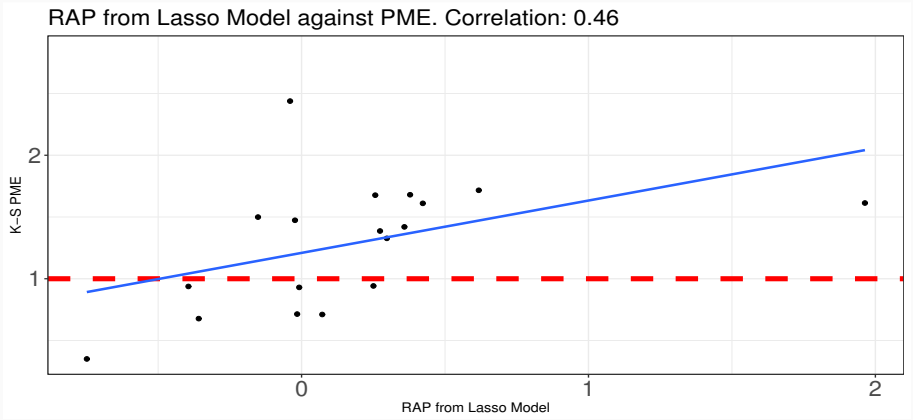
PE Comparison with PME — Real, Lasso Estate

Back



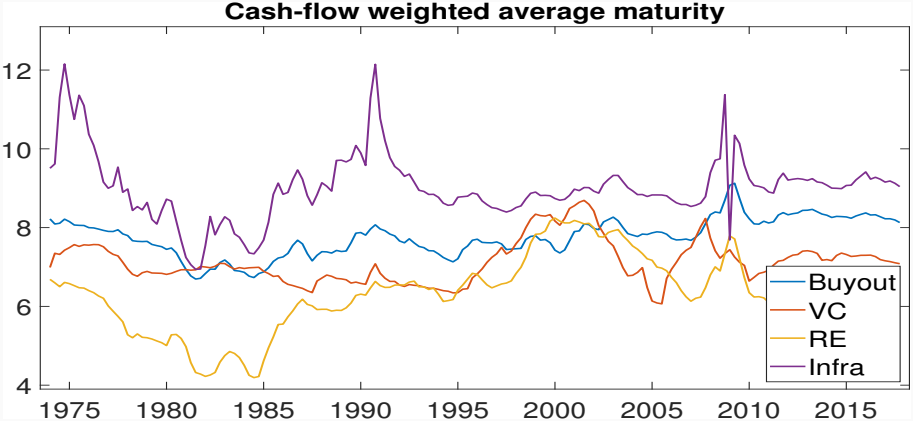
PE Comparison with PME — Infrastructure, Lasso

Back



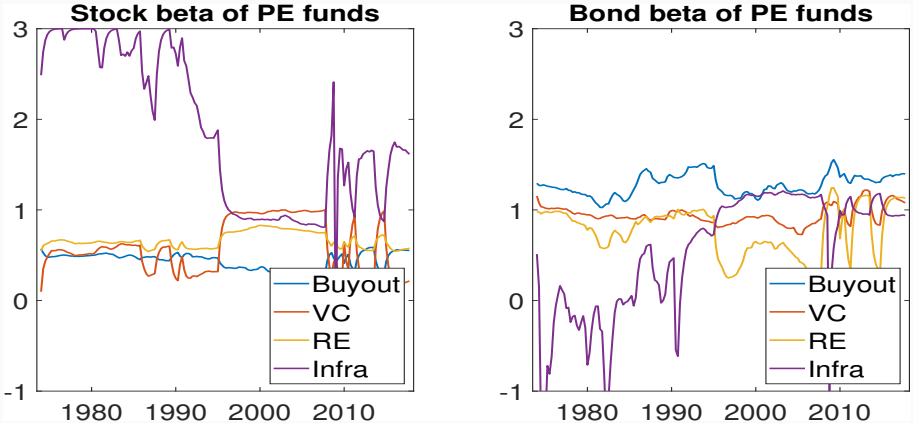
PE Duration

Back



Implied Stock and Bond Betas

Back



PE Fund Risk-Adjusted Profits — VC, Burgiss

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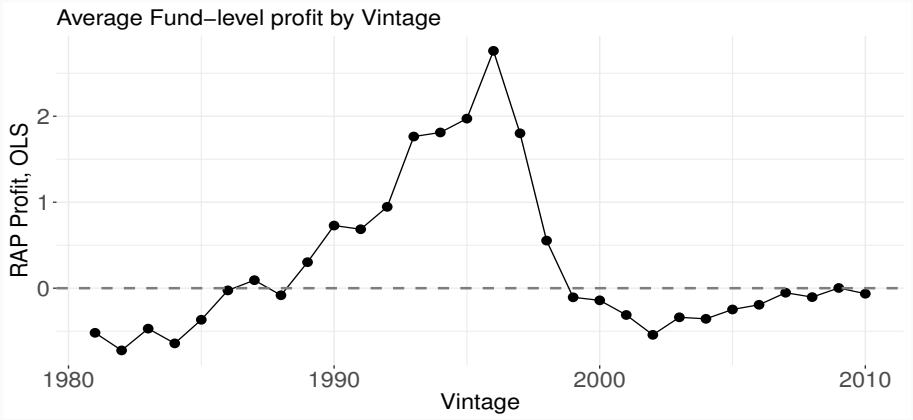
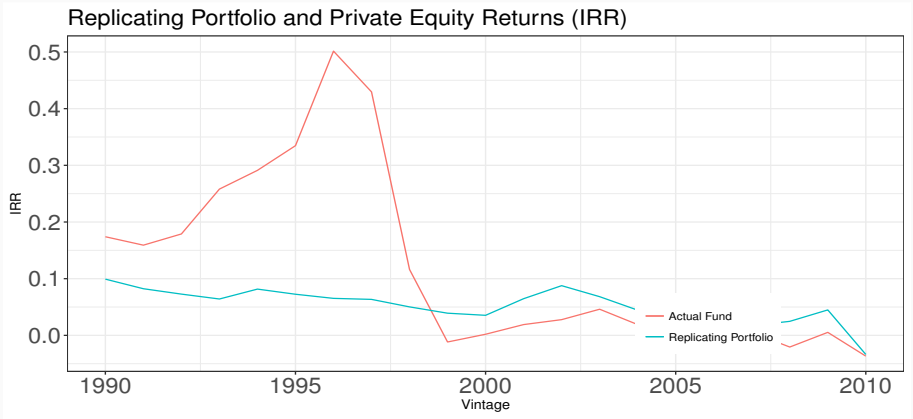


PE Fund Risk-Adjusted Profits — VC, Burgiss

Back



PE Fund Risk-Adjusted Profits — Buyout, Burgiss

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