Disentangling Global Value Chains*

Alonso de Gortari
Princeton University and Dartmouth College

October 2018

Abstract

The patterns of production underlying the recent rise of global value chains (GVCs) have become increasingly complex. NAFTA supply chains, for example, are now deeply integrated: Using Mexican customs data, I find that exports to the U.S. use a much higher share of American inputs than exports to other countries. However, the conventional framework used to measure GVCs ignores this heterogeneity since it assumes that all output uses the same input mix. I develop a new framework that combines input-output data with additional information on supply chain linkages in order to construct GVCs reflecting the use of inputs observed in the latter. Improving measurement matters quantitatively since it affects both value-added trade measures and counterfactual experiments: I show that incorporating Mexican customs data raises the estimated share of U.S. value in U.S. imported Mexican manufactures from 17% to 30% and doubles the U.S. welfare cost of a NAFTA trade war.

*Contact me at alonso.degortari@princeton.edu. This paper is based on my 2017 job market paper of the same title and the 2018 paper "Bounding the Gains from Trade." I am extremely grateful to my advisors Pol Antràs, Elhanan Helpman, and Marc Melitz for their mentorship and guidance. I especially thank Kirill Borusyak, Guillermo Noguera, Zhi Wang, Kei-Mu Yi, and numerous participants at seminars and conferences for very helpful comments. I gratefully acknowledge the hospitality of Banco de México, where part of this paper was written. Finally, I thank Gurobi and Odyssey for making this project possible. All errors are my own.

1 Introduction

While this paper was written, the North American Free Trade Agreement (NAFTA) got renegotiated for the first time since its inception in 1994, the United Kingdom discussed its potential exit from the European Customs Union, and the seeds of a possible full-blown trade war between the United States and China were sown. What are the potential costs of these economic shocks? How do they ripple across country borders? Are the shocks amplified in this age of globalization where over two-thirds of world trade is in intermediate inputs? Does a higher tariff on imported Mexican vehicles hurt the American worker more or less depending on the share of American value built into these cars?

Addressing these questions requires developing a more accurate and systematic understanding of the nature of the global value chains (GVCs, henceforth) underlying world trade than has so far been achieved. Indeed, measuring GVCs requires taking a stand on how to use multi-country input-output data to trace value across different stages of production. The conventional approach does this by assuming that all output, within each country-industry, is built with the same input mix. This assumption is sharply at odds with the evidence on supply chain linkages based on richer micro-level datasets showing that, in reality, the use of inputs depends on the downstream use of output. For example, while the conventional approach assumes a common input mix in all Mexican vehicle production, figure 1 uses Mexican customs microdata to show that the U.S. accounts for a colossal 74% of the foreign inputs embedded in vehicles sold to U.S. consumers but for only 18% of the inputs of those sold to German consumers.[1]

I show that accurately measuring GVCs matters because it affects the quantitative exercises carried out by both academics and policymakers to study global trade in a world of highly fragmented production. In particular, I show that taking the heterogeneity in the use of inputs into account is crucial for quantifying the implications of economic shocks and the extent of globalization through value-added trade measures.

My main theoretical contribution is to develop a new measurement framework that leverages both input-output data and other sources of information, such as that in figure 1, to better measure GVC flows. The motivation is that while input-output data contains no information on supply chain linkages — i.e. which inputs are used for which output — researchers often have access to small but rich micro-level datasets providing some supply chain information about some country-industries. Hence, while existing microdata is often too limited for measuring GVCs directly, combining available microdata with input-output datasets yields more accurate GVCs than the conventional GVCs based solely on the latter.

My main empirical result is that measuring GVCs while incorporating Mexican customs data roughly doubles both the share of U.S. value in U.S. imported Mexican manufactures and the U.S. welfare cost of a NAFTA trade war. These results are in line with Yi's (2003) landmark study arguing that deep vertical specialization magnifies the effects of economic shocks and showcase how conventionally measured GVC flows miss crucial elements present in today's highly fragmented supply chains. More generally, any

[1] Ultimately, the challenge surrounding GVC measurement is about aggregation and would (mostly) disappear in firm- or product-level input-output datasets. However, current datasets are so highly aggregated that this is a major issue for both academic and policy work. For example, the widely-used WIOD features only 19 manufacturing industries. To put this into perspective, this means that 6 trillion dollars of U.S. manufacturing output is divided into only 19 categories. This issue is unlikely to disappear anytime soon. First, many countries do not construct firm-to-firm datasets. Second, building a multi-country firm-level input-output database requires merging firm-level data across countries and faces considerable political and legal roadblocks.

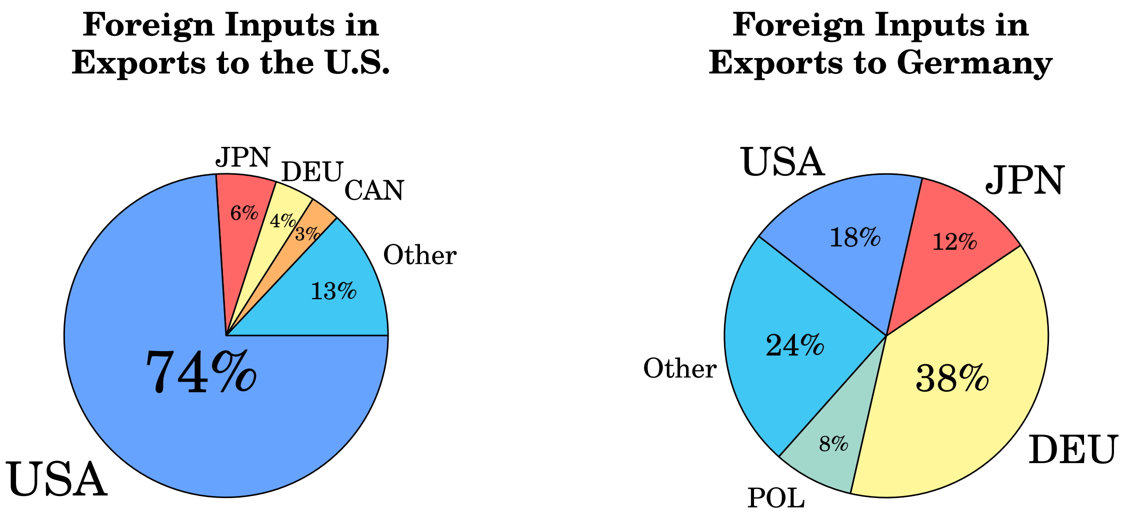


Figure 1: Distribution of Foreign Inputs Used in Mexican Final Good Motor Vehicle Exports to the U.S. and Germany: The shares are constructed using Mexican customs shipment-level data for 2014; details are discussed in section 2.4.1. In contrast to these charts, the conventional approach for measuring GVCs assumes common input distributions across destinations.

question studied by the GVC literature can be revisited with this new measurement framework while incorporating whatever additional information is both relevant and available in each context.

I kick off in section 2 by developing a general GVC theory that can accommodate, with further assumptions, how different classes of microfounded models behave in equilibrium and what implications they have on GVC flows. This general theory is useful for two reasons. First, because it formalizes the connection between the literature on counterfactuals based on microfounded theories of production and the literature on value-added trade based on equilibrium theories of production — two literatures that have evolved mostly independently and in parallel. Second, because it formalizes the key insight underlying this paper: That any input-output dataset is consistent with many different GVC networks. In particular, the general GVC theory is useful for comparing how different equilibrium theories of production construct GVCs from input-output data. For example, most trade models incorporate intermediate inputs by assuming that technology features *roundabout production* in which all of a country-industry's output is produced with the same input mix.[2] While microfoundations differ substantially, all roundabout models imply that GVCs should be constructed recursively from input-output data using first-order Markov chains.

I argue in favor of models featuring *specialized inputs* — models in which goods sold to different countries and industries are built with different input mixes. Specialized inputs models weaken the proportionality assumptions built into roundabout production models and instead imply that GVCs be constructed using higher-order Markov chains.[3] In other words, while roundabout models construct a unique GVC network out of any given input-output dataset, specialized inputs models are consistent with many GVC networks. Importantly, the latter can incorporate the heterogeneity in figure 1 while the former cannot.

[2]Roundabout models come in many varieties, some examples include Krugman and Venables (1995), Eaton and Kortum (2002), Balistreri et al. (2011), di Giovanni and Levchenko (2013), Bems (2014), Caliendo and Parro (2015), Ossa (2015), Allen et al. (2017).

[3]In equilibrium, specialized inputs can be thought of as a generalization of input-output analysis in which the expenditure shares are conditional on both the purchasing country-industry and the subsequent supply chain through which inputs flow.

The case for specialized inputs is supported by both the anecdotal and empirical evidence on modern supply chains in which input suppliers customize their goods to be compatible with specific downstream uses and in which firms make complex decisions when deciding where to locate each stage of their supply chain. For example, the lithium battery supplier in Apple's famously long iPod supply chain manufactures it exactly to the size of the metal frame while the screen supplier ensures that the touch, color, and dimming capabilities are in line with Apple's iOS software (Linden et al. 2011). Today, this form of input specialization is ubiquitous (Rauch 1999, Nunn 2007, Antràs and Staiger 2012, Antràs and Chor 2013) and implies that the use of inputs varies depending on the use of output since firms exporting to different countries and industries have different supply chains.[4] As figure 1 illustrates, Mexican vehicle manufacturers exporting to the U.S. rely heavily on U.S. supply chains while those exporting to Germany do not.

I then argue in section 3 that the distinction between roundabout production and specialized inputs matters because different GVC networks lead to different quantitative counterfactual predictions. For the sake of clarity, and at the cost of generality, I illustrate this with the simplest possible microfoundation for specialized inputs — basically, an extension of the perfect competition Armington model where each country-industry produces a specific variety for each market. Since this model features specialized inputs, many parameterizations fit the input-output data and this matters quantitatively because the welfare gains from trade depend on the expenditure share on domestic inputs used in the production of domestically-sold goods. In other words, mapping the model to different GVC networks delivers different counterfactual estimates following any economic shock — even though all parameterizations replicate the same data in the benchmark equilibrium. In particular, roundabout production is a knife-edge parameterization in which the welfare gains depend on the aggregate domestic expenditure share as in Arkolakis et al. (2012).[5]

I quantify the potential mismeasurement by constructing bounds on counterfactual estimates using the specialized inputs model — i.e. the GVC networks that minimize/maximize the gains from trade. For example, the autarky bounds in the 2014 World Input-Output Database (WIOD) with a trade elasticity of 5 are wide and increasing in trade openness: the U.S. gains (relatively closed) lie between 2.6-4.0% but the Taiwan gains (relatively open) lie between 12-129%. Intuitively, the lower (upper) bounds correspond to GVCs in which many (few) domestic inputs are used to produce domestically-sold goods. Meanwhile, the knife-edge roundabout model where all output uses the same input mix predicts gains of 3.5% and 18%.[6]

Analogously to counterfactuals, section 4 shows that measures of globalization — i.e. measures quan-

[4]Various recent studies suggest that the use of inputs, within country-industries, depend on the downstream use of output. For example, within-industry exports vary across destinations due to quality (Bastos and Silva 2010), trade regime (Dean et al. 2011), and credit constraints (Manova and Yu 2016). Likewise, the use of imports varies across firm size (Gopinath and Neiman 2014, Blaum et al. 2017a, 2017b, Antràs et al. 2017), multinational activity (Hanson et al. 2005), firm capital intensity (Schott 2004), and the quality of output (Fieler et al. 2017). Further, recent research has made explicit connections between imports and exports through quality linkages (Bastos et al. 2018), trade participation (Manova and Zhang 2012), and rules-of-origin (Conconi et al. 2018). Finally, production processes vary also in terms of the intensity of labor inputs. Processing trade firms export lower-cost labor assembly goods (De La Cruz et al. 2011, Koopman et al. 2012) while firms exporting to richer countries hire higher-skilled workers (Brambilla et al. 2012, Brambilla and Porto 2016). Thus, value-added shares also differ depending on the use of output.

[5]The sufficiency of input-output data for both measuring GVCs and quantifying the effects of economic shocks is intimately linked to the roundabout assumptions. In the more general case of specialized inputs, however, this sufficiency no longer holds.

[6]While these exercises rely on a specific class of microfoundations, I conjecture that richer models yield similar qualitative implications. Other specialized inputs microfoundations include Yi (2003), Yi (2010), Costinot et al. (2012), Antràs and Chor (2013), Fally and Hillberry (2016), Johnson and Moxnes (2016), Blanchard et al. (2017), Antràs and de Gortari (2017), and Oberfield (2018).

tifying the fragmentation of production such as value-added trade (Hummels et al. 2001, Johnson and Noguera 2012, Koopman et al. 2014) or average downstreamness (Antràs et al. 2012) — also depend on the GVC network. In particular, while the literature defines these measures directly with input-output analysis (Leontief 1941), I define them broadly using the general GVC theory. This is useful because the former are only consistent with the equilibrium of roundabout production models whereas the general theory can be used to derive the correct measures for other equilibrium theories such as specialized inputs.

I quantify the potential mismeasurement by constructing approximate bounds on measures of globalization using the specialized inputs model. In particular, I argue that value-added trade might be severely mismeasured and thus may be misguiding trade policy. For example, while counterfactual exercises typically dominate academic debates, policy debates like the NAFTA renegotiation are often based on measures of supply chain integration such as how much U.S. value-added returns home through Mexican imports.[7] Higher shares are typically interpreted as proxying higher costs of disruption — i.e. restricting Mexican imports will ripple back and hurt the U.S. more when it provides more value to these supply chains — and conventional (roundabout) estimates put the share of U.S. value-added in Mexican manufacturing imports at about 17%. In contrast, I show that the same 2014 WIOD data is consistent with bounds as low as 6% and as high as 47%. In other words, the data is consistent with both little and highly integrated Mexican-American supply chains.[8] In a second exercise, I revisit Johnson and Noguera (2012) — who found that the U.S.-China value-added deficit is smaller than the gross deficit — and show that specialized inputs are actually consistent with both a value-added surplus and a much larger deficit.

In sum, the key message of sections 2, 3, and 4, is that many GVC networks are consistent with any input-output dataset and that constructing bounds based on specialized inputs is useful for determining the potential mismeasurement trickling over from the GVC flows to quantitative counterfactual estimates and measures of globalization. Since all GVC networks exhaust the information contained in the input-output data, the latter can shed no further light on which estimates are most accurate.

Finally, section 5 improves measurement by incorporating new sources of information. Specifically, I use the latter to discipline a set of targets in the objective function of a quadratic program that searches over all GVCs consistent with a given input-output dataset. This approach thus constructs the best informed guess of the true GVC network while exploiting more information than that contained in input-output data and is useful when the additional information is insufficient for measuring GVC flows directly. For example, since Mexican customs data contains no information on domestic transactions, GVCs cannot be directly measured because there is not enough information to convert the foreign input expenditure shares in figure 1 into overall input expenditure shares.[9] In such cases, researchers can still improve measurement by taking a stand on how to map the additional information into expenditure shares with auxiliary assumptions. However, in general, these expenditure shares will not aggregate up perfectly to the input-output data — because they rely on some imperfect assumptions — and this is where the optimization

[7] For example, U.S. Secretary of Commerce Wilbur Ross argued in the Washington Post (September 21, 2017) that disrupting Mexican-American supply chains was not worrisome since Mexican imports contained 'only' 16% of U.S. value-added (in 2011).

[8] In contrast to figure 1, computing value-added trade requires tracing where value is created along all stages of production.

[9] Directly measuring GVC segments requires very rich data such as datasets covering the universe of country-level firm-to-firm transactions. However, these are quite rare. Belgian data is one exception (see Tintelnot et al. 2017, Kikkawa et al. 2017).

problem becomes useful. The latter takes these shares as targets and reallocates flows in order to construct the GVC network that is closest to the researcher's targets among all the GVCs consistent with a given input-output dataset. In sum, while these GVCs ultimately still depend on some assumptions, they are closer to the true GVCs underlying input-output data since they weaken the roundabout GVCs' strong (theoretical) assumptions by using additional (empirical) information.

Incorporating Mexican customs data reveals that Mexican-American supply chains are more integrated and disrupting them is more costly than previously thought. Specifically, I map the customs data to the optimization targets by taking the stand that Mexico only does processing trade — i.e. that exports use only imported inputs. The GVCs based on this best-informed guess then imply that 30% of the value in U.S. imported Mexican manufactures is U.S. value-added and not 17% as given by the roundabout GVCs. In addition, the U.S. welfare cost of a NAFTA trade war is about twice as high as implied by the latter.

This GVC framework is easily adaptable and can incorporate additional information in a practical manner. While large datasets on supply chain linkages are rarely available, researchers often have access to partial snippets of the overall supply chains underlying global trade that are extremely informative about how intermediate inputs are used. My application focuses on Mexico since I have access to Mexican microdata, but the tools can be readily applied to study any other aspect of global production networks with other datasets. Moreover, the type of information brought in can be tailored to the specific question being asked. For example, while Chinese customs data might not be of immediate relevance for studying a NAFTA trade war, it might be of paramount importance when considering a U.S.-China trade war.

From a history of science standpoint, this paper is inspired by Samuelson (1952) who asked how to measure bilateral trade flows in the presence of only aggregate export data. This paper takes the same idea to the next iteration: How to measure GVC flows in the presence of only bilateral input-output data? From a philosophy of science standpoint, this paper is inspired by Popper (1959) and argues for a falsifiable approach to GVC measurement. That is, instead of imposing the theoretically-based roundabout approach outright, I argue in favor of studying GVCs under initially broad sets of plausibly accurate GVCs obtained through specialized inputs and to then refine these estimates as more information becomes available.

The paper's structure is as follows. Section 2 provides the GVC framework used to compare the equilibrium theories of roundabout production and specialized inputs in the three next sections. Section 3 studies counterfactuals, section 4 studies measures of globalization, and section 5 studies GVC measurement. The appendix provides additional results and details on numerical implementation. All code is posted online.

2 The Hunt for GVCs: The Measurement Challenge

This section provides the GVC framework used throughout the paper to discuss counterfactuals, measures of globalization, and measurement in a GVC world. I proceed in four steps. First, I describe the data contained in multi-country input-output datasets. Second, I develop a general theory that provides notation and a unifying framework for comparing specific theories of production — this will also prove useful for deriving explicitly the connection between the literature on structural models and counterfactuals and the literature on measures of globalization. Third, I use the general GVC theory to discuss three specific theories of production: two widely used theories given by a world of 'only trade in final goods' and a world of roundabout production and a third more modern theory with specialized inputs. Fourth, and finally, I provide empirical evidence in favor of specialized inputs using Mexican customs shipment-level data and U.S. domestic input-output tables.

2.1 Multi-Country Input-Output Data

Let $\mathcal{J}$ denote both the set and number of countries and $\mathcal{K}$ the set and number of industries. I define $\mathcal{S} = \mathcal{J} \times \mathcal{K}$ as the set and number of country-industries, with a generic element $s \in \mathcal{S}$ being a country-industry denoted as $s = \{j, k\}$ with $j \in \mathcal{J}$ and $k \in \mathcal{K}$. Multi-country input-output datasets typically contain data on bilateral intermediate input flows across two country-industry pairs, with $X(s', s)$ the dollar value of intermediate inputs sold from country-industry s' to country-industry s, and final good flows between a country-industry and consumers, with $F(s', j)$ the dollar value of final goods sold from country-industry s' to consumers in country j. These are the basic building blocks from which all other aggregate moments are built. For example, the gross output and gross domestic product of country-industry s' equal

$$GO(s') = \sum_{s \in \mathcal{S}} X(s', s) + \sum_{j \in \mathcal{J}} F(s', j), \qquad GDP(s') = GO(s') - \sum_{s \in \mathcal{S}} X(s, s').$$

There are currently various sources of multi-country input-output datasets such as those produced by the World Input-Output Database Project (WIOD), the Global Trade Analysis Project (GTAP), the Institute for Developing Economies (IDE-JETRO), the Eora Global Supply Chain Database (Eora MRIO), and the OECD Inter-Country Input-Output Tables (ICIO). Each dataset has its own advantages and limitations and the analysis in this paper can be readily applied to each. I focus throughout on the WIOD — the most widely used dataset by the international trade literature — which is available in its 2016 release for $\mathcal{J} = 44$ countries, $\mathcal{K} = 56$ industries (19 in manufacturing), and for the years 2000-2014 (see Timmer et al. 2015).

2.2 A General GVC Theory

GVC flows constitute the key building blocks of this theory. Define $\mathcal{G}(\cdot)$ as the dollar value of goods flowing from an initial country-industry down through a specific ordered set of country-industries all the way to final consumption. To fix ideas, suppose there is a single industry (i.e., $\mathcal{S} = \mathcal{J}$). Take three countries $j, j', j'' \in \mathcal{J}$. Then $\mathcal{G}(j', j)$ denotes the dollar value of final goods sold from j' to j while $\mathcal{G}(j'', j', j)$ is the dollar value of intermediate inputs sold from j'' to j' which j' uses as inputs for the final goods sold to j.

More generally, intermediate inputs may be traded at a stage of production that is $N \in \mathbb{N}$ stages upstream relative to the production of final consumption goods. I write a generic truncated GVC flow as $\mathcal{G}^N(j^N, j^{N-1}, \ldots, j^1, j)$ where the superscript N on $\mathcal{G}^N(\cdot)$ indicates the dimension of this function, i.e. N is the number of nodes previous to final consumption that are specified. Every node corresponds to a country $j^n \in \mathcal{J}\ \forall n$ and the n is only meant to indicate the node at which country j^n is located. The flow $\mathcal{G}^N(j^N, j^{N-1}, \ldots, j^1, j)$ thus indicates the dollar value of inputs from j^N sold to j^{N-1}, that j^{N-1} uses to produce new inputs sold to j^{N-2}, so on and so forth, until the goods arrive at j^1 and are put into final goods shipped and sold to consumers in j. Since using apostrophes is cumbersome with large N, in general I will use the notation $\mathcal{G}^1\left(j^1, j\right)$ instead of $\mathcal{G}\left(j', j\right)$ and likewise $\mathcal{G}^2\left(j^2, j^1, j\right)$ instead of $\mathcal{G}\left(j'', j', j\right)$.

The extension to a multi-industry world is immediate. GVCs can be defined generically as follows.

Definition 2.1. For any length $N \in \mathbb{N}$, $\mathcal{G}^N : \mathcal{S}^N \times \mathcal{J} \to \mathbb{R}^+$ is the function describing truncated GVC flows leading to final consumption in countries in $\mathcal{J}$ through a sequence of N upstream stages of production given by an element of $\mathcal{S}^N = \prod_{n=1}^N \mathcal{S}$.

A generic GVC is $\mathcal{G}^N\left(s^N, \ldots, s^1, j\right)$ and, as before, I refer to the elements of a country-industry pair as $s^n = \{j^n, k^n\}$ with $j^n \in \mathcal{J}$ the country and $k^n \in \mathcal{K}$ the industry of $s^n \in \mathcal{S}$, where the n is only meant to indicate the node of $\mathcal{G}^N(\cdot)$ at which s^n is located. For example: a flow of length $N = 1$ could be $\mathcal{G}^1\left(s^1, j\right) = \mathcal{G}^1\left(\{\text{Mexico,cars}\}, \text{U.S.}\right)$, the sales of Mexican cars to U.S. consumers, while a flow of length $N = 2$ could be $\mathcal{G}^2\left(s^2, s^1, j\right) = \mathcal{G}^2\left(\{\text{U.S.,steel}\}, \{\text{Mexico,cars}\}, \text{U.S.}\right)$, the sales of U.S. steel in the form of intermediate inputs that are used exclusively by the Mexican car industry to produce final goods sold to U.S. consumers. Analogously for any $N \in \mathbb{N}$ and any sequence of production in $\mathcal{S}^N$ that produces a final good eventually sold to consumers in some country in $\mathcal{J}$.

The measurement challenge embedded in this GVC theory is that the word *truncated* appears in definition 2.1. Specifically, $\mathcal{G}^N(\cdot)$ is a truncated GVC because it only specifies the flow through N stages of production even though its most upstream stage, s^N, also uses inputs and the full chain of production is characterized by a (potentially) infinite number of stages. Since $\mathcal{G}^N(\cdot)$ is unobserved in the data, the challenge is to develop a theory of production — i.e. a reasonable set of assumptions — that links GVC flows across different stages of production. That is, take an arbitrary $\mathcal{G}^N\left(s^N, s^{N-1}, \ldots, s^1, j\right)$. Since this tells how many inputs are sold from s^N to the sequence $s^{N-1} \to \cdots \to s^1 \to j$ then there has to be some relation with the flow $\mathcal{G}^{N-1}\left(s^{N-1}, \ldots, s^1, j\right)$ of inputs that s^{N-1} itself sells to this production sequence.

In its most general form, the only restriction I impose is that flows across different stages of production must satisfy

$$\sum_{s^N \in \mathcal{S}} \mathcal{G}^N\left(s^N, s^{N-1}, \ldots, s^1, j\right) \leqslant \mathcal{G}^{N-1}\left(s^{N-1}, \ldots, s^1, j\right). \tag{1}$$

That is, the right-hand side denotes the value of intermediate inputs sold by s^{N-1} to be used through the sequence in $\mathcal{G}^{N-1}\left(s^{N-1}, \ldots, s^1, j\right)$. The left-hand side denotes the total value of intermediate inputs, across all sources $s^N \in \mathcal{S}$, sold to s^{N-1} and used down this same sequence of production. Imposing equation (1) thus implies that the total value of inputs purchased by s^{N-1} for a specific downstream sequence of production need be less or equal than the value of the output that s^{N-1} itself produces for that sequence.

This theory is general and can encompass most production processes. It relies only on the key restriction that the value of output not fall as goods flow down the value chain. Whenever the value of output increases, thus implying equation (1) holds with strict inequality, I say that value was added at the $N-1$th stage of production to the inputs purchased from stage N. For example, this theory assumes that

$$\sum_{s^2 \in \mathcal{S}} \mathcal{G}^2\left(s^2, \{\text{Mexico,cars}\}, \text{U.S.}\right) \leqslant \mathcal{G}^1\left(\{\text{Mexico,cars}\}, \text{U.S.}\right).$$

The right-hand side indicates the dollar value of Mexican cars sold to U.S. consumers and corresponds to a truncated GVC flow because the Mexican car industry uses intermediate inputs produced further upstream to produce these cars. Meanwhile, $\mathcal{G}^2(\{\text{U.S.,steel}\},\{\text{Mexico,cars}\},\text{U.S.})$ is the dollar value of U.S. steel bought as inputs directly in order to produce these exports, so that the summation across all possible input sources $s^2 \in \mathcal{S}$ yields aggregate input sales to the downstream sequence on the right-hand side. The inequality holds strictly if the Mexican car industry adds domestic value-added directly into the intermediate inputs purchased from the previous stage of production.

I refer to equation (1) as the *GVC challenge* which can only be solved by taking a stand on how to trace value across stages of production. That is, on how $\mathcal{G}^N\left(s^N, s^{N-1}, \ldots, s^1, j\right)$ and $\mathcal{G}^{N-1}\left(s^{N-1}, \ldots, s^1, j\right)$ relate to each other across all stages and sequences of production.[10]

2.2.1 Relation to Multi-Country Input-Output Data

GVC flows are not observed directly in input-output data. Rather, the data contains only some (non-exhaustive) information about the true GVCs. Disentangling GVCs from the data then requires using an equilibrium theory of production in order to fill in with assumptions whatever information is not available.

I now describe the information that is available in input-output data. The first thing to note is that the data provides precise information about the last stage of production. Hence, the simplest GVC flows, those with $N = 1$, are observed and final good flows can be defined in terms of GVCs as

$$F\left(s', j\right) = \mathcal{G}^1\left(s', j\right). \tag{2}$$

This mapping is the basic building block from which all theories of intermediate input trade will build upon since this is the only part of the supply chain that is observed directly in input-output data.

Second, bilateral intermediate input flows are much more complicated since they aggregate the dollar value of inputs traded across two country-industries across all stages of the supply chain. The relation between these aggregate flows and GVC flows is given by

$$X\left(s', s\right) = \sum_{N=2}^{\infty} \sum_{s^{N-2} \in \mathcal{S}} \cdots \sum_{s^1 \in \mathcal{S}} \sum_{j \in \mathcal{J}} \mathcal{G}^N\left(s', s, s^{N-2}, \ldots, s^1, j\right). \tag{3}$$

The flow $\mathcal{G}^N(\cdot)$ is the input value from s' sold to s at the Nth stage of production and used through the downstream sequence $s^{N-2} \to \cdots \to s^1 \to j$. Summing up across $s^{N-2} \in \mathcal{S}$, ..., $s^1 \in \mathcal{S}$, $j \in \mathcal{J}$ thus delivers the aggregate input value from s' sold to s at the Nth stage of production used across all downstream sequences of production. The first summation across $N \geqslant 2$ then sums up the input value traded across all stages of production. This aggregate value thus equals the input flows reported in input-output data.

Hence, while input-output data provide precise information on $X(s', s)$ and $F(s', j)$, disentangling the GVC flows $\mathcal{G}^N\left(s^N, \ldots, s^1, j\right)$ across all upstream production stages $N \geqslant 2$ requires further assumptions since a lot of the information is potentially lost in the aggregation into bilateral input flows in (3).

[10] Two further comments about the interpretation of $\mathcal{G}^N(\cdot)$. First, GVCs can be interpreted directly as firm-level supply chains by fixing $\mathcal{K}$ as the set of firms instead of the set of industries. Second, though the paper is written in terms of a static production world where all goods are produced simultaneously, this theory can also accommodate dynamic models since s can be interpreted as a country-industry-time triple in which inputs of past periods flow down the value chain to be used as inputs in future periods.

2.3 Two Old Solutions, and One New One

I discuss three possible solutions to the GVC challenge — each given by a set of assumptions corresponding to a specific equilibrium theory of production. Importantly, constructing a GVC network out of input-output data only requires specifying how GVCs behave in equilibrium and not on how such equilibrium was achieved. Hence, while microfoundations vary substantially, for the purposes of GVC measurement the specific microfoundation can be ignored and all that matters is how the theory implies that value be traced across stages of production in equilibrium. Of course, computing counterfactuals does require unpacking a microfoundation and this will be done in the next section.

2.3.1 The 'Only Trade in Final Goods' Solution

The simplest solution to disentangling GVCs in (1) is to assume that GVC linkages are non-existent. Starting from the observed GVCs $\mathcal{G}^1\left(s^1, j\right)$, assume that the mapping into previous stages at $N \geqslant 2$ is given by

$$\mathcal{G}^N\left(s^N, s^{N-1}, \ldots, s^1, j\right) = 0, \tag{4}$$

across any production sequence. Substituting into the definition of intermediate input flows in equation (3) implies that

$$\Rightarrow X\left(s', s\right) = 0,$$

across all country-industry pairs s' and s since inputs are not used at any stage of production. Hence, these GVCs imply that final output is entirely domestic value-added created at the most downstream stage of production

$$\Rightarrow GO\left(s'\right) = GDP\left(s'\right) = \sum_{j \in \mathcal{J}} \mathcal{G}^1\left(s', j\right) = \sum_{j \in \mathcal{J}} F\left(s', j\right).$$

These assumptions are extreme and at odds with today's global economy since over two-thirds of world trade is in intermediate inputs. Indeed, most datasets report $X\left(s', s\right) > 0$ for the majority of country-industry pairs so that this GVC characterization cannot be squared with current data. However, these restrictions were still widely imposed even a few decades ago. For example, both the modern version of the Armington model developed by Anderson (1979) and the classical Ricardian model of Dornbusch et al. (1977) assume that intermediate inputs play no role so that both microfoundations can be characterized, in equilibrium, by these GVC flows. While this paradigm is less prevalent today, it is an useful starting point for showing how to map more complex theories of international trade into the above GVC framework.

2.3.2 The Roundabout Solution

Disentangling GVCs in the presence of intermediate input trade is much more complex since, in principle, many theories of production can solve the GVC challenge in (1). This observation motivates this paper since, so far, both the literature on counterfactuals and the literature on measures of globalization have largely focused on the solution in which every single dollar of output within each country-industry is produced using the exact same input mix. Formally, this implies solving the mapping by assuming the existence of a set of technical coefficients $a(s'|s)$ denoting the expenditure share on inputs from s' used by s to produce output at any stage of production and for any sequence of production. Starting from the observed GVCs, $\mathcal{G}^1(s^1, j)$, the mapping into previous stages of production at $N \geqslant 2$ is given by

$$\mathcal{G}^N\left(s^N, s^{N-1}, \ldots, s^1, j\right) = a\left(s^N \middle| s^{N-1}\right) \mathcal{G}^{N-1}\left(s^{N-1}, \ldots, s^1, j\right). \tag{5}$$

Rearranging, any GVC flow is thus characterized entirely by final good flows and the technical coefficients

$$\Rightarrow \mathcal{G}^N\left(s^N, s^{N-1}, \ldots, s^1, j\right) = \prod_{n=2}^{N} a\left(s^n \middle| s^{n-1}\right) F\left(s^1, j\right). \tag{6}$$

Substituting into the relation between GVC flows and intermediate input flows in (3), the following holds

$$\Rightarrow X\left(s'', s'\right) = a\left(s'' \middle| s'\right) \left(\sum_{s \in \mathcal{S}} X\left(s', s\right) + \sum_{j \in \mathcal{J}} F\left(s', j\right) \right).$$

In other words, since $X(s', s)$ and $F(s', j)$ are observed in input-output data, this theory of production can only be squared with the data if the technical coefficients are given by

$$\Rightarrow a\left(s' | s\right) = \frac{X\left(s', s\right)}{GO\left(s\right)}. \tag{7}$$

The expenditure by s on inputs from s' is simply given by aggregate value of inputs purchased from s' relative to gross output. Since gross output is typically larger than aggregate intermediate input purchases, this implies that every dollar of output of s has a share of domestic value-added given by

$$\Rightarrow \beta\left(s\right) = 1 - \sum_{s' \in \mathcal{S}} a\left(s' | s\right) = \frac{GDP\left(s\right)}{GO\left(s\right)}. \tag{8}$$

The roundabout solution is currently the most popular approach for incorporating intermediate inputs into structural models of international trade (see footnote 2).[11] In particular, it is so highly tractable that the measurement problem regarding how to disentangle GVCs is completely eliminated as long as one has input-output data at hand. Roundabout production implies that any GVC flow in (6) is characterized by final good flows and the technical coefficients, but since the latter are characterized by input-output data as well in (7), then any GVC flow is fully and uniquely characterized by input-output data.

In other words, any roundabout microstructure has GVCs that can be characterized, in equilibrium, by the mapping in (6) and is thus equivalent to input-output analysis (Leontief 1941) — a measurement frame-

[11] It is also enormously influential beyond trade. Roundabout production has been widely used ever since Samuelson (1951) provided the key insight that input-output analysis is consistent with the equilibrium of a constant returns to scale production economy. For example, it has been used in the macroeconomics literature following the seminal input-output models of Domar (1961), Hulten (1978), and Long and Plosser (1983) to study business cycles (Basu 1995), growth (Jones 2011), misallocation (Jones 2013, Bigio and La'O 2016, Caliendo et al. 2017), aggregate fluctuations (Acemoglu et al. 2012, Carvalho and Gabaix 2013, Carvalho 2014, di Giovanni et al. 2014, Baqaee 2014, Baqaee and Farhi 2017), and development accounting (Bartelme and Gorodnichenko 2015, Cuñat and Zymek 2017). As the GVC literature, these papers can be extended to specialized inputs.

work fully characterized by input-output data and with no degrees of freedom.[12] Importantly, though, while input-output analysis is defined directly as a set of input and value-added shares given by (7) and (8), I derived these input shares from first principles in the sense that I imposed assumptions on the mapping of GVCs across different stages of the value chain in (5) and then derived the input shares as an implication. This latter approach is more useful since now I can impose different assumptions on to the general GVC theory and compare the implications regarding how to disentangle GVCs across theories.

2.3.3 The Specialized Inputs Solution

The specialized inputs solution generalizes the roundabout solution and assumes that the use of inputs depends on the destination of output and the use of output, both in terms of whether goods are sold as final goods or intermediate inputs and to which industry they are sold to as inputs in the latter case. The GVC flow of inputs used directly for the production of final goods is thus given by

$$\mathcal{G}^2\left(s^2, s^1, j\right) = a_F\left(s^2 \left| s^1, j\right.\right) F\left(s^1, j\right), \tag{9}$$

where $a_F\left(s'' | s', j\right)$ is the share of inputs from country-industry s'' used in the final goods produced by country-industry s' that are sold to consumers in market j. Analogously, the use of intermediate inputs in the production of new intermediate inputs is given by

$$\mathcal{G}^N\left(s^N, s^{N-1}, \dots, s^1, j\right) = a_X\left(s^N \left| s^{N-1}, s^{N-2}\right.\right) \mathcal{G}^{N-1}\left(s^{N-1}, \dots, s^1, j\right), \ \forall N \geqslant 3, \tag{10}$$

where $a_X\left(s'' | s', s\right)$ is the share of inputs from country-industry s'' used in the production of intermediate inputs by country-industry s' sold to country-industry s. Note that while the intermediate input shares depend on the destination and use of inputs, they are common across all stages of production. That is, the input mix used to produce inputs in s' and sold to s is the same in all production stages $N \geqslant 2$.

In this context, value-added shares also depend on the destination and use of output and are given by

$$\beta_F\left(s', j\right) = 1 - \sum_{s'' \in \mathcal{S}} a_F\left(s'' \left| s', j\right.\right) \geqslant 0, \qquad \beta_X\left(s', s\right) = 1 - \sum_{s'' \in \mathcal{S}} a_X\left(s'' \left| s', s\right.\right) \geqslant 0.$$

These shares have to be greater or equal than zero given the assumption in (1) that the dollar value of output never falls as goods flow along the value chain. Further, at least one of these shares has to be strictly positive since GDP in every country-industry s' in the data is positive.

Relative to the roundabout solution, now it is not possible to characterize input shares directly using the input-output data. Rather, this GVC solution is richer since there are many different sets of input shares that perfectly fit the data (i.e. many GVC networks replicate the same bilateral trade, gross output, and gross domestic product flows). To see this, substitute in the specialized inputs solution in (9) and (10) into

[12] Input-output analysis is typically described using matrix algebra. Imposing the GVC mapping (5) on the definition of bilateral intermediate input flows in (3) and using matrix algebra implies that

$$\Rightarrow \mathbf{X} = \mathbf{a}\mathbf{F} + \mathbf{a}^2\mathbf{F} + \cdots = \mathbf{a}\left[\mathbb{I} - \mathbf{a}\right]^{-1}\mathbf{F},$$

where $\mathbf{GO} = \left[\mathbb{I} - \mathbf{a}\right]^{-1}\mathbf{F}$ is gross output and $\left[\mathbb{I} - \mathbf{a}\right]^{-1}$ is known as the Leontief inverse matrix.

the set of linear constraints relating GVC flows to the observed input-output data in (3) to obtain

$$X\left(s'', s'\right) = \sum_{N=2}^{\infty} \sum_{s^{N} \in s''} \sum_{s^{N-1} \in s'} \sum_{s^{N-2} \in \mathcal{S}} \cdots \sum_{s^{1} \in \mathcal{S}} \sum_{j \in \mathcal{J}} \left[\prod_{n=3}^{N} a_X\left(s^{n} \left| s^{n-1}, s^{n-2}\right.\right) \right] a_F\left(s^{2} \left| s^{1}, j\right.\right) F\left(s^{1}, j\right). \quad (11)$$

Equation (11) is tedious but straightforward and sums up the inputs sold by s'' to s' across all stages and chains of production. Conditional on N, the first two stages of the sequence are $s^{N} = s''$ and $s^{N-1} = s'$ (I abuse notation slightly by indicating two summations over single-valued sets). The subsequent summations sum up the use of inputs across all downstream sequences of production $s^{N-2} \in \mathcal{S}, \ldots, s^{1} \in \mathcal{S}, j \in \mathcal{J}$, while the summation over $N \geqslant 2$ sums up the exchange of inputs across all production stages.

Fortunately, the recursive structure of the specialized inputs solution assumed in (10) implies that the mapping between input shares and input-output data in (11) can be rewritten much more succinctly as

$$X\left(s'', s'\right) = \sum_{s \in \mathcal{S}} a_X\left(s'' \left| s', s\right.\right) X\left(s', s\right) + \sum_{j \in \mathcal{J}} a_F\left(s'' \left| s', j\right.\right) F\left(s', j\right). \quad (12)$$

In words, the right-hand side sums up all the intermediate inputs from s'' used by s' to produce further downstream inputs sold to all $s \in \mathcal{S}$ and final goods sold to all $j \in \mathcal{J}$. Since this is the total value of inputs sold from s'' to s', it has to equal the observed flow $X\left(s'', s'\right)$.

Since all of the information in input-output data is contained in $X\left(s', s\right)$ and $F\left(s', j\right)$, any set of input shares $a_X\left(s'' | s', s\right)$ and $a_F\left(s'' | s, j\right)$ satisfying (12) for all bilateral pairs characterize a system of GVC flows that perfectly fit the observable data. Crucially, fitting the data requires imposing $\mathcal{S} \times \mathcal{S}$ restrictions but the specialized inputs GVC network depends on $\mathcal{S} \times \mathcal{S} \times (\mathcal{S} + \mathcal{J})$ input shares. These degrees of freedom imply that there are many different GVC networks that replicate the same observable data. In particular, the roundabout solution is the knife-edge case in which the use of inputs is independent of the use output. That is, when $a_X\left(s'' | s', s\right) = a_F\left(s'' | s', j\right) = a\left(s'' | s'\right)\ \forall s \in \mathcal{S}$ and $\forall j \in \mathcal{J}$ then (12) implies (7).

2.3.4 Taking Stock

Of the three discussed solutions to the GVC challenge in (1), each subsequent theory is more general than the previous and all three are useful for understanding the aggregation issues present in input-output data.

First, a few decades ago, bilateral trade data did not distinguish between intermediate input and final good trade and so, in practice, the data was silent regarding whether the 'only trade in final goods' solution was potentially accurate or not. Current input-output datasets, however, show that the majority of world trade is in intermediate inputs and so are now disaggregate enough to be able to reject this GVC theory. Second, the roundabout solution incorporates intermediate input flows albeit in a highly simplified manner. In particular, this theory is the knife-edge case that fits the data perfectly in a unique way, but it also implicitly implies assuming that further disaggregating the data would yield no additional insights or information. Third, and more generally, the specialized inputs solution fits the data perfectly in many ways and thus implicitly assumes there is important information hidden by the aggregation present in input-output datasets. The rest of the paper is concerned with using the specialized inputs solution to understand the implications of such aggregation in currently available input-output datasets.

As a final comment, note that there are many other potential ways of disentangling the GVC challenge in (1). For example, a richer form of input specialization could depend on input shares $a_X(s'''|s'', s', s)$ where the input mix used in s'' for exports to s' is tailored according to the further downstream production stage at s. More formally, this corresponds to building GVCs recursively using third-order Markov chains while the above specialized inputs and roundabout solutions correspond to the special cases of second-order and first-order Markov chains.[13] Alternatively, one could move beyond recursive GVCs and assume instead finite GVCs with output at some stage $N > 1$ consisting entirely of domestic value-added. I focus on the specialized inputs solution since it is, in my view, the most natural and tractable way of generalizing the roundabout solution in order to account for the patterns observed in figure 1. But the reader should keep in mind that this GVC framework can be used to study many other solutions in future research.

2.4 Evidence for Specialized Inputs

The empirical evidence in favor of specialized inputs has been steadily accumulating over the last couple of years (see footnote 4). On the intermediate input side, Manova and Zhang (2012) found that large Chinese firms export to more countries and use inputs from more source countries than small firms while Bastos et al. (2018) showed that Portuguese firms selling to richer countries export higher quality products built with higher quality inputs. On the value-added side, Brambilla et al. (2012) and Brambilla and Porto (2016) discovered that Argentinian firms exporting to richer countries hire relatively more skilled workers and pay higher wages while Koopman et al. (2012) and Kee and Tang (2016) established that Chinese processing trade firms use less domestic value-added than non-processing trade exporting firms. These facts imply that both the use of intermediate inputs and value-added varies at the country-industry level depending on the use of output in a variety of settings. I now provide further evidence for Mexico and the U.S.

2.4.1 Evidence from Firm-Level Data

The case for specialized inputs is supported by Mexican customs data. Specifically, I use the universe of import/export shipments in 2014 to show that the use of inputs varies in exports to different markets. I proceed in three steps. First, for each firm I construct its aggregate input purchases from and exports to each country. Second, I assume that all output within each firm is produced using the same input mix and obtain the dollar value of imports from each country used in the exports to each country at the firm-level.[14]. Third, I take all of the firms within a manufacturing industry and compute the aggregate value of imports from a given source used in the exports to a given destination. This delivers the distribution of foreign inputs used in exports to each destination market — which should be common across markets if the roundabout solution were accurate at the industry-level.[15]

[13] A previous version of this paper, de Gortari (2017), shows how to disentangle GVCs using Markov chains of any order.

[14] This assumption is strong in multi-product firms where different goods likely use different inputs. However, imposing a common input mix within the firm is weaker than imposing it within industries; Ludema et al. (2018) take the same approach.

[15] Customs data does not contain domestic purchases so value-added shares cannot be measured at the firm-level and this analysis also rests on assuming common value-added shares across firms within an industry. Imposing the roundabout solution at the industry-level also assumes this and so, in this respect, this analysis is just as restrictive as the conventional approach.

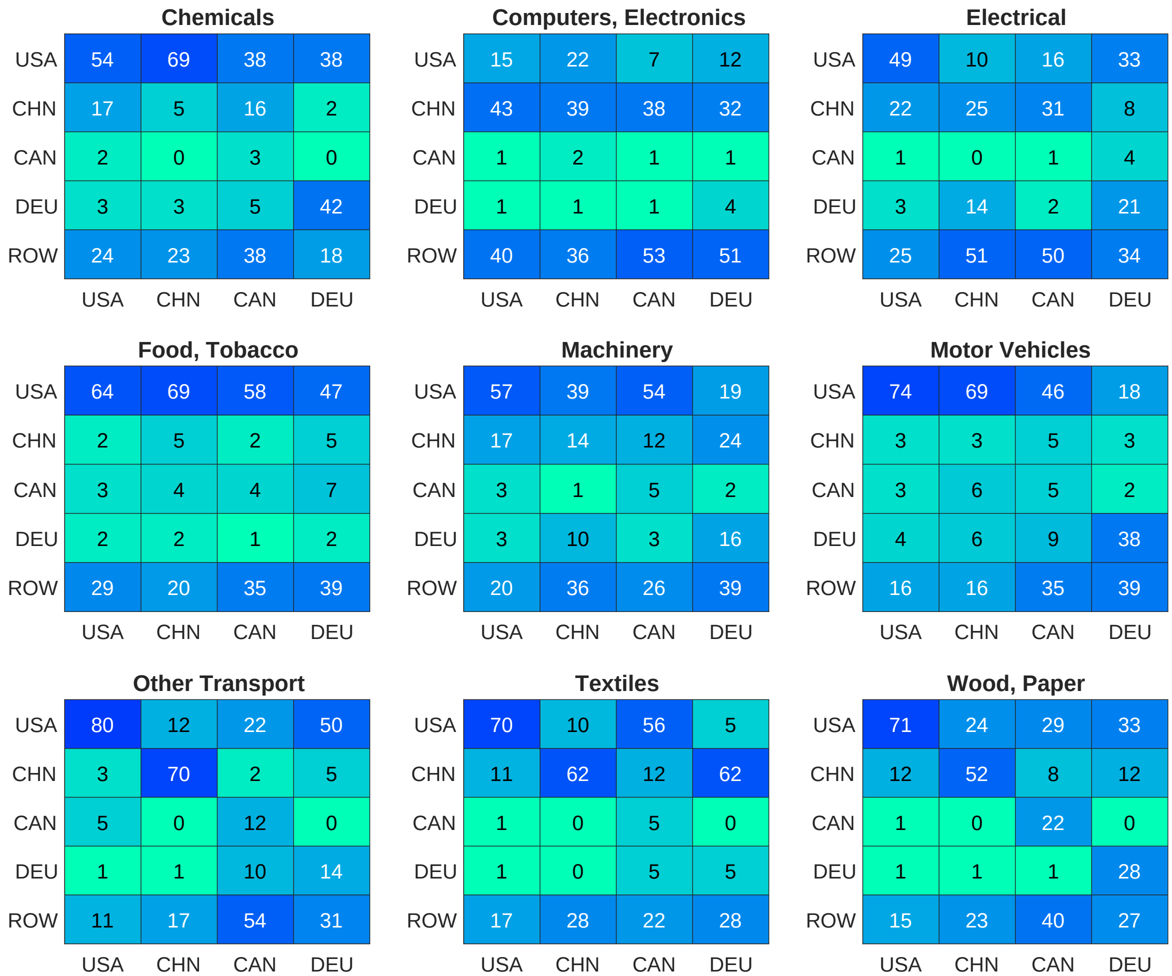


Figure 2: Foreign Input Shares in Mexican Manufacturing Exports Across Destinations: Each chart presents the share of foreign inputs sourced from Mexico's four main trade partners and a rest of world remainder (y-axis) used in Mexico's manufacturing exports to each of its four main trade partners (x-axis). That is, cells across rows within each column sum up to 100%. The shares are constructed using Mexican customs shipment-level data and these nine manufacturing industries account for 95% of Mexico's final good manufacturing exports. In contrast, assuming the roundabout solution at the industry-level implies common input distributions across export destinations.

Figure 2 confirms the prevalence of specialized inputs in Mexican manufacturing final good exports at the level of aggregation consistent with typical multi-country datasets.[16] Specifically, each column in each chart plots the share of foreign inputs sourced from Mexico's four main trade partners — the U.S., China, Canada, and Germany — and a rest of world remainder used in exports to each of these markets. In other words, the cells across a column represent the distribution of foreign inputs used to produce a specific type of manufacturing exports and add up to 100%. For example, motor vehicles is Mexico's main export

[16]The latter is an important point since one could define different firms as different manufacturing industries and then the distribution of inputs used in exports to different destinations would be common by construction since I have assumed a common use of inputs within the firm. However, the charts in figure 2 are presented at the relevant level of aggregation since, for example, manufacturing flows in the WIOD are available for only 19 aggregate manufacturing industries. Going forward, while multi-country datasets are likely to become more disaggregate over time it is unlikely that these datasets become available at a disaggregate enough level to be consistent with the roundabout solution at the industry-level anytime soon (see footnote 1).

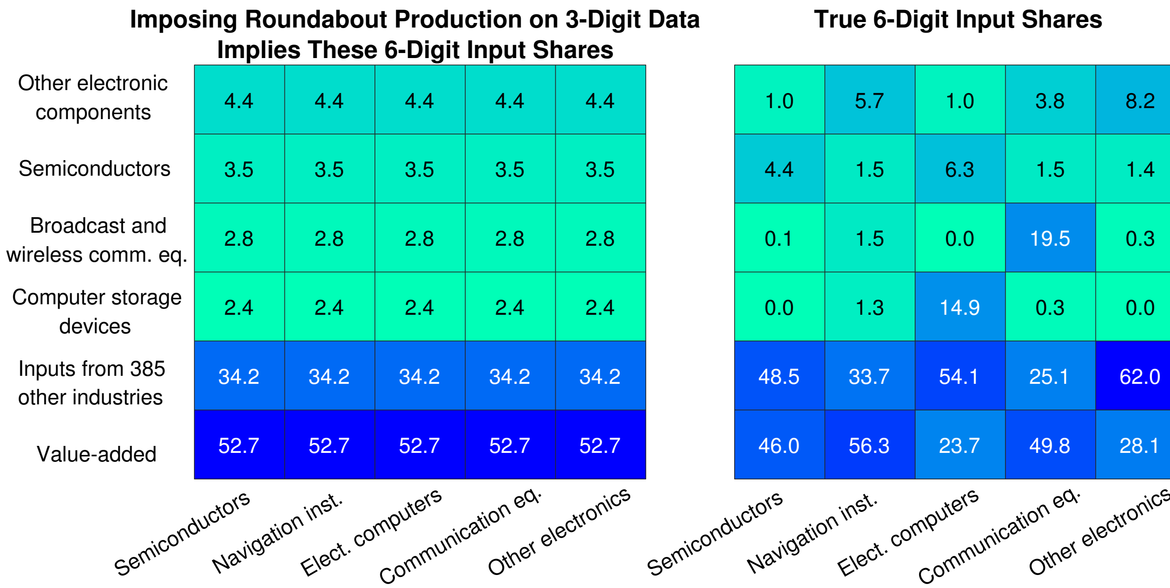


Figure 3: Implied and True Input and Value-Added Shares Within the Computer and Electronics Industry: Each chart presents the expenditure share on the top four input suppliers, the other 385 input suppliers, and value-added (y-axis) in the production of the top five subindustries (x-axis). The left chart plots shares implied by imposing the roundabout solution on the aggregate 3-digit industry computer and electronics while the right panel plots the true shares using the disaggregate 6-digit data. Data is from 2007 U.S. input-output tables from the Bureau of Economic Analysis.

industry and the corresponding chart shows that the use of inputs in exports to the U.S. and Germany differ substantially (i.e. these are the distributions in figure 1).

Overall, figure 2 shows substantial heterogeneity in input shares in sales to different destinations and reveals interesting patterns. In particular, the U.S. tends to have an outsized role as input supplier in the exports that return to its own market — thus confirming the widely-available anecdotal evidence that Mexico-U.S. trade is based heavily on goods that cross the border back and forth. Sections 4 and 5 will show this translates into a high share of U.S. content in U.S.-bound exports through richer empirical analysis that traces where value is created across all stages of the value chain.

2.4.2 Evidence from Disaggregate Domestic Input-Output Tables

The case for specialized inputs is also supported by domestic input-output tables. Specifically, the U.S. Bureau of Economic Analysis reports data for the year 2007 at a level of disaggregation of both 389 and 71 industrial categories (roughly 6- and 3-digit NAICS codes). This data is useful because it can help study whether the use of inputs — at the industry-level — varies depending on the industry to which output is sold to. I conduct the following thought experiment: Compare the input shares of the 6-digit industries bundled into single 3-digit industries. If only 6-digit industries with common input mixes are bundled together then there is no aggregation issue. If not, then the roundabout solution at the 3-digit is misspecified.

Figure 3 illustrates the aggregation issue in the 3-digit computers and electronics industry. The latter is composed of twenty 6-digit industries — the five largest are semiconductors, navigation instruments, electronic computers, communication equipment, and other electronics — while its four largest input suppliers are other electronic components, semiconductors, broadcast and wireless communication equipment, and computer storage devices. Figure 3's left panel shows that imposing the roundabout solution at the 3-digit implies all 6-digit subindustries use the same input and value-added mix. Figure 3's right panel, however, uses the disaggregate 6-digit data to show that input shares vary substantially within each subindustry. For example, computer storage devices are used intensively in electronic computers (14.9% of output value) but only marginally in other subindustries; in contrast, the left chart assumes a common 2.4% share.[17]

Appendix section A.1 shows similar patterns hold across all U.S. manufacturing industries in that there is substantial heterogeneity in input shares across sales to different industries. This exercise is informative about multi-country tables since the latter are typically available at an industrial classification level similar to the 3-digit NAICS. Hence, while this exercise cannot be done with multi-country tables, it is likely that the issue of industrial aggregation is just as prevalent as implied by the U.S. domestic tables.[18]

3 GVCs and Counterfactuals

A first strand of the GVC literature is concerned with understanding the implications of economic shocks, such as changes in trade barriers, on international trade. In particular, in an influential contribution, Arkolakis et al. (2012) (ACR henceforth) argued that, with some assumptions in hand, the welfare gains from trade — across a variety of microfoundations — rely only on the domestic expenditure share and thus depend only on data and a trade elasticity. Though their benchmark analysis is carried out in a world of 'only trade in final goods,' they also prove that their results extend to roundabout production.

This section shows that, in more complex equilibrium theories of production than that of roundabout production, the quantitative implications of economic shocks differ depending on how GVCs are constructed using input-output data. Conceptually, the starting point is section 2's solutions to the GVC challenge in (1) since these theories determine how to build the benchmark equilibrium's GVC network. The next step is to go deeper and unpack the microfoundation underlying these equilibrium theories of production in order to pin down how the GVC network changes following any economic shock.

I proceed in four steps. First, I develop the simplest microfoundation for specialized inputs through a variant of the Armington model. Second, I extend the ACR insights and show that the gains from trade depend on the change in a set of domestic expenditure shares — though here the relevant shares are the expenditures on domestic inputs used for the production of domestically-sold goods. Since any input-output dataset is consistent with many GVC networks delivering different values for the latter, this implies that any counterfactual exercise is consistent with a range of numerical values. Third, I show formally that the aggregate domestic expenditure share is not the relevant sufficient statistic in a world of specialized inputs because it fails to capture how changes in trade barriers ripple through GVC linkages. Fourth, and finally, I show how to construct bounds on any counterfactual exercise based on the class of models consistent with the above sufficient statistics formulas and illustrate this empirically with the 2014 WIOD.

[17] Note that what matters is the relative difference in shares across columns; the shares in levels are low since there are 389 6-digit industries. Also note that, in contrast to figure 2, these are shares of output value and thus include a row for value-added.

[18] The issue of aggregation in input-output data motivated an important literature in the 1950's with several papers developing conditions under which aggregation is innocuous. The outlook on whether they might hold in practice was grim, though. In the words of Hatanaka (1952) and McManus (1956), "There is very little chance that they will be fulfilled by any model".

3.1 Armington Meets Specialized Inputs

I extend the Armington model with roundabout production (such as in Costinot and Rodríguez-Clare 2014) to specialized inputs. There are $\mathcal{J}$ countries and $\mathcal{K}$ industries, with each country-industry $s \in \mathcal{J} \times \mathcal{K}$ producing $\mathcal{J}$ differentiated varieties — each tailored to a specific market. The roundabout model is the special case in which each country-industry produces the same differentiated variety for all markets.

The model is based on five main assumptions: (i) both intermediate inputs and final goods are produced with the same technology, (ii) production is specialized in terms of destination country but not destination industry, (iii) production features constant returns to scale with an upper-tier Cobb-Douglas production function across labor and intermediate inputs from each industry and a lower-tier constant elasticity of substitution (CES) composite of inputs across source countries, (iv) market structure is perfect competition, (v) the only source of value-added in country j is equipped labor $L(j)$ and commands a wage $w(j)$.

3.1.1 Production

Formally, assumptions (iii) and (iv) imply the model can be described directly in terms of unit prices, the dual, with the price of a unit of goods from s' sold to j given by the marginal cost

$$p(s',j) = w(j')^{\beta(s',j)} \prod_{k'' \in \mathcal{K}} \left(\sum_{s'' \in \mathcal{J} \times k''} \alpha(s''|s',j) \left(p(s'',j')\,\tau(s'',j')\right)^{1-\sigma(k'')} \right)^{\frac{\gamma(k''|s',j)}{1-\sigma(k'')}}, \tag{13}$$

where notation is such that country-industry pairs are summarized by $s'' = \{j'', k''\}$ and $s' = \{j', k'\}$. The upper-tier Cobb-Douglas is characterized by $\beta(s',j)$, the value-added share, and $\gamma(k''|s',j)$, the expenditure share on industry k'' inputs, with $\beta(s',j) + \sum_{k'' \in \mathcal{K}} \gamma(k''|s',j) = 1$. The lower-tier CES composite is characterized by two parameters. First, an elasticity $\sigma(k'') \geqslant 1$ governing the substitutability of industry k'' inputs purchased across sources $j'' \in \mathcal{J}$ — i.e. the industry k'' composite combines inputs across sources as indexed by $s'' \in \mathcal{J} \times k''$. Second, a set of exogenous input shifters $\alpha(s''|s',j)$ governing the relative expenditure on industry k'' inputs from each source $j'' \in \mathcal{J}$ satisfying $\sum_{s'' \in \mathcal{J} \times k''} \alpha(s''|s',j) = 1$ $\forall k'' \in \mathcal{K}$. In addition, $p(s',j)$ depends on the endogenous wage paid in s', $w(j')$, and the prices that j' itself pays for inputs purchased from each source s'', $p(s'',j')$, times an exogenous trade cost $\tau(s'',j') \geqslant 1$ governing how many units melt when shipped from s'' to j'.

Production is specialized in that s' puts in specific shares of domestic value-added and inputs from each s'' into its exports to each market j. That is, of every dollar sold from s' to j a share $\beta(s',j)$ is domestic value-added embedded directly by s' while the expenditure share on s'' inputs is endogenous and given by

$$a(s''|s',j) = \frac{\alpha(s''|s',j)\left(p(s'',j')\,\tau(s'',j')\right)^{1-\sigma(k'')}}{\sum_{t'' \in \mathcal{J} \times k''} \alpha(t''|s',j)\left(p(t'',j')\,\tau(t'',j')\right)^{1-\sigma(k'')}} \times \gamma(k''|s',j). \tag{14}$$

These input expenditure shares are disciplined by the parameters $\alpha(s''|s',j)$ and I interpret this heterogeneity as a simple way of (exogenously) capturing the interdependencies across different stages of the value chain.[19] Note, however, that input specificity is *eroded* as goods flow down the value chain. That is, every country j' has access to specific inputs from each source s'', available at unit cost $p(s'',j')\,\tau(s'',j')$, but can use them to produce new goods for any downstream market j. Further, note this microfoundation is slightly more restrictive than the specialized inputs described in (9) and (10) since the input shares in both intermediate inputs and final goods are common and only vary across destinations: $a_X(s''|s',s) = a_F(s''|s',j) = a(s''|s',j)$. While generalizing is straightforward, I focus on this case for parsimony.

3.1.2 Consumers

As is standard, I assume consumers aggregate goods across industries using an upper tier Cobb-Douglas aggregator with $\zeta(k'|j)$ denoting the expenditure share on industry k' final goods by consumers in country j. Further, within each industry consumers aggregate varieties across source countries into a CES composite with the same elasticity of substitution $\sigma(k') \geqslant 1$ as above and with the free parameters $\varphi(s'|j)$ disciplining the share of final goods from s' purchased by consumers in each j. The price index is then

$$P(j) = \prod_{k'\in\mathcal{K}} \left(\sum_{s'\in\mathcal{J}\times k'} \varphi(s'|j)\,\left(p(s',j)\,\tau(s',j)\right)^{1-\sigma(k')} \right)^{\frac{\zeta(k'|j)}{1-\sigma(k')}}, \tag{15}$$

and the expenditure share on final goods from each source country-industry s' equals

$$\pi_F(s'|j) = \frac{\varphi(s'|j)\,(p(s',j)\,\tau(s',j))^{1-\sigma(k')}}{\sum_{t'\in\mathcal{J}\times k'} \varphi(t'|j)\,(p(t',j)\,\tau(t',j))^{1-\sigma(k')}} \times \zeta(k'|j). \tag{16}$$

3.1.3 Mapping the Model to Input-Output Data

Mapping the model to the data requires building the model's analogs of the input-output table elements. From the consumer's side, final good purchases in j from source s' equal a share of aggregate income

$$F(s',j) = \pi_F(s'|j) \times w(j)\,L(j).$$

The intermediate input side is constructed by noting that a share of the dollar exports to a given market is used to pay for the inputs embedded in them. Thus, aggregate intermediate input sales from s'' to s' must equal the total value of inputs used by s' to produce exports sold to all destinations

$$X(s'',s') = \sum_{j\in\mathcal{J}} a(s''|s',j) \left(\sum_{s\in j\times\mathcal{K}} X(s',s) + F(s',j) \right). \tag{17}$$

Given the input shares and final good flows, these $\mathcal{S}\times\mathcal{S}$ equations implicitly define the $\mathcal{S}\times\mathcal{S}$ input flows.[20]

There are multiple parameterizations of this model that can perfectly fit the input-output data. Specifi-

[19] These parameters can be further unpacked and made endogenous in richer models, for example Antràs and de Gortari (2017).

[20] Alternatively, input flows can be computed directly with linear algebra through $\mathbf{X} = \mathbf{a}\,[\mathbb{I} - \mathbf{a}]^{-1}\,\mathbf{F}$. This approach is reminiscent of the Leontief inverse matrix but requires a matrix of size $\mathcal{S}^2\times\mathcal{S}^2$ instead of size $\mathcal{S}\times\mathcal{S}$.

cally, conditional on any vector of iceberg trade costs $\tau(s', j) \geqslant 1$ and elasticities of substitution $\sigma(k) \geqslant 1$, the parameters $\varphi(s'|j)$ adjust to match final good flows, the input mix parameters $\alpha(s''|s', j)$ adjust to match intermediate input flows, and the Cobb-Douglas shares $\beta(s', j)$, $\gamma(k''|s', j)$, and $\zeta(k'|j)$ adjust to match GDP and gross output. Since the microstructure permits destination-specific input expenditure shares, it can produce many different GVC networks that aggregate up to the same input-output data.

In particular, the roundabout model corresponds to the knife-edge case of no specialization in which exports to all markets use the same input mix.[21] With these restrictions, (17) delivers the property of roundabout models that input shares are proportional to bilateral trade shares. That is, when value-added and input expenditure shares are common across markets, then there is a single parameterization that fits the data and delivers the exact same GVC network in equilibrium as the one given by input-output analysis

$$\begin{gathered} \text{if } \beta\left(s', j\right) = \beta\left(s'\right), \gamma\left(k''\left|s', j\right.\right) = \gamma\left(k''\left|s'\right.\right), \text{ and } \alpha\left(s''\left|s', j\right.\right) = \alpha\left(s''\left|s'\right.\right), \forall j \in \mathcal{J}, \\ \Rightarrow a\left(s''\left|s', j\right.\right) = a\left(s''\left|s'\right.\right) = \frac{X\left(s'', s'\right)}{GO\left(s'\right)}. \end{gathered} \tag{18}$$

Hence, while roundabout models may fit the data perfectly, this *cannot* be interpreted as evidence *for* the roundabout approach since many other specialized inputs models also fit it perfectly. Moreover, input-output data contains no information identifying which GVC networks are most accurate.

3.2 The Gains from Trade

Building on the insights of ACR, the welfare change following any shock to trade barriers depends on a set of domestic expenditure shares. I derive this formula using the exact hat-algebra approach in four steps. Specifically, let a hat variable denote the ratio of a given variable x across two equilibria, i.e. $\hat{x} = x_1/x_0$, and let $\hat{\tau}(s', j)$ denote the (exogenous) change in trade costs of goods shipped from s' to j. As is standard, to make notation cleaner I assume that domestic trade costs do not change, i.e. $\hat{\tau}(s', j) = 1\ \forall s' \in j \times \mathcal{K}$.

First, I derive the change in expenditure shares. From (14), the change in input expenditures from source s'' used by s' for goods sold to j as a share of overall expenditure on industry k'' inputs equals

$$\frac{\hat{a}(s''|s', j)}{\gamma(k''|s', j)} = \frac{\left(\hat{p}(s'', j')\,\hat{\tau}(s'', j')\right)^{1-\sigma(k'')}}{\sum_{t'' \in \mathcal{J} \times k''} a(t''|s', j) \times \left(\hat{p}(t'', j')\,\hat{\tau}(t'', j')\right)^{1-\sigma(k'')}}. \tag{19}$$

Analogously, from (16), the change in the share of final good expenditures from source s' by consumers in j relative to overall expenditure on industry k' final goods equals

$$\frac{\hat{\pi}_F(s'|j)}{\zeta(k'|j)} = \frac{\left(\hat{p}(s', j)\,\hat{\tau}(s', j)\right)^{1-\sigma(k')}}{\sum_{t' \in \mathcal{J} \times k'} \pi_F(t'|j) \times \left(\hat{p}(t', j)\,\hat{\tau}(t', j)\right)^{1-\sigma(k')}}. \tag{20}$$

[21] To be clear, I am using the term roundabout when referring to production processes in which all output uses the same input mix *and* in which the model is implemented literally in that the industries in the theory are mapped one-to-one to the industries in the data (for example, as in Costinot and Rodríguez-Clare 2014, Caliendo and Parro 2015, and Caliendo et al. 2017). More generally, this specialized inputs model can also be interpreted as a more disaggregate multi-industry roundabout model in which country j has $\mathcal{K} \times \mathcal{J}$ industries in which the goods produced by industry k for country j are only sold to country j. The mapping to the data is not one-to-one, however, since the theory has $\mathcal{K} \times \mathcal{J}$ industries per country whereas the data has $\mathcal{K}$.

Both expenditure changes (19) and (20) depend on the exogenous Cobb-Douglas and elasticity parameters, the exogenous change in trade costs, the initial GVC network, and the endogenous change in unit prices.

Second, to derive price changes in terms of domestic expenditures, substitute (19) into (13) to obtain

$$\hat{p}\left(s',j\right) = \hat{w}\left(j\right)^{\beta\left(s',j\right)} \prod_{k''\in\mathcal{K}} \left(\hat{a}\left(s''\left|s',j\right.\right)^{-\frac{1}{1-\sigma\left(k''\right)}} \times \hat{p}\left(s'',j'\right)\hat{\tau}\left(s'',j'\right)\right)^{\gamma\left(k''|s',j\right)}, \tag{21}$$

where s'' can be a source located in any country, that is $s'' \in \mathcal{J} \times k''$. Then take (21) defined in terms of domestic industries of j, i.e. $s' = \{j, k'\}$ and $s'' = \{j, k''\}$, and substitute (21) repeatedly into itself. In the limit, domestic unit prices depend exclusively on changes in domestic expenditure shares

$$\hat{p}\left(s',j\right) = \prod_{s''\in j\times\mathcal{K}} \left(\hat{w}\left(j\right)^{\beta\left(s'',j\right)} \times \prod_{s'''\in j\times\mathcal{K}} \hat{a}\left(s'''\left|s'',j\right.\right)^{-\frac{\gamma\left(k'''|s'',j\right)}{1-\sigma\left(k'''\right)}}\right)^{\delta\left(k''|s',j\right)}, \tag{22}$$

with s', s'', and s''' domestic industries of j. The change in domestic prices thus depends on the change in domestic wages and in expenditures on domestic inputs used in the production of domestically-sold goods. Further, (22) captures the domestic expenditure change across all stages of the supply chain through

$$\delta\left(k''\left|s',j\right.\right) = 1_{[k''=k']} + \gamma\left(k''\left|s',j\right.\right) + \sum_{s'''\in j\times\mathcal{K}} \gamma\left(k''\left|s''',j\right.\right)\gamma\left(k'''\left|s',j\right.\right) + \ldots.$$

That is, $\delta\left(k''|s',j\right)$ captures the aggregate (gross) use of k'' inputs used in all upstream production stages of a purely domestic supply chain for inputs that are eventually embedded in goods sold by s' domestically.[22]

Third, the change in the price index of country j can be written in terms of the change in final expenditure shares from some source s' by substituting in (20) into equation (15)

$$\hat{P}\left(j\right) = \prod_{k'\in\mathcal{K}} \left(\hat{\pi}_F\left(s'|j\right)^{-\frac{1}{1-\sigma\left(k'\right)}} \times \hat{p}\left(s',j\right)\hat{\tau}\left(s',j\right)\right)^{\zeta\left(k'|j\right)}. \tag{23}$$

Finally, substituting the price changes in (22) into the price index change in (23), defined domestically with $s' \in j\times\mathcal{K}$, delivers the welfare change $\hat{W}\left(j\right) = \hat{w}\left(j\right)/\hat{P}\left(j\right)$ in terms of domestic expenditure changes

$$\hat{W}\left(j\right) = \prod_{s'\in j\times\mathcal{K}} \left(\hat{\pi}_F\left(s'|j\right)^{\frac{1}{1-\sigma\left(k'\right)}} \times \prod_{s''\in j\times\mathcal{K}} \prod_{s'''\in j\times\mathcal{K}} \hat{a}\left(s'''\left|s'',j\right.\right)^{\frac{\gamma\left(k'''|s'',j\right)\delta\left(k''|s',j\right)}{1-\sigma\left(k'''\right)}}\right)^{\zeta\left(k'|j\right)}. \tag{24}$$

This formula incorporates various elements found previously such as the GVC elements from Antràs and de Gortari (2017), the domestic expenditure shares from ACR, and the multi-industry input-output

[22]Note that $\delta\left(k''|s',j\right)$ contains value-added counted multiple times. Since the focus is on domestic shares, writing the Cobb-Douglas shares $\gamma\left(k''|s',j\right)$ for country j as a $\mathcal{K}\times\mathcal{K}$ matrix $\boldsymbol{\gamma}$ delivers the corresponding $\delta\left(k''|s',j\right)$ shares as $\boldsymbol{\delta} = \left[\mathbb{I}-\boldsymbol{\gamma}\right]^{-1}$.

linkages of Caliendo and Parro (2015). Specifically, first, in a single-industry world this formula becomes

$$\hat{W}(j) = \left[\hat{\pi}_F(j|j) \times \hat{a}(j|j,j)^{\frac{1-\beta(j,j)}{\beta(j,j)}}\right]^{\frac{1}{1-\sigma}}, \tag{25}$$

and the change in welfare depends on the change in the share of final goods purchased domestically and the change in the share of domestic inputs used in the production of domestically-sold inputs. Each term captures the relative importance of domestic goods in the production of a purely domestic supply chain.[23] This formula is similar to that derived by Antràs and de Gortari (2017) in a multi-stage Ricardian model where welfare depends on the expenditure share on goods produced through purely domestic supply chains.

Second, ACR's benchmark analysis without intermediate inputs is nested here by imposing $\beta(j,j) = 1$

$$\hat{W}(j) = \hat{\pi}_F(j|j)^{\frac{1}{1-\sigma}}.$$

Further, ACR's generalization to intermediate inputs under the roundabout solution — corresponding to the assumptions in (18) — while imposing symmetry, i.e. $\hat{\pi}_F(j|j) = \hat{a}(j|j)$, is also nested and given by

$$\hat{W}(j) = \hat{\pi}_F(j|j)^{\frac{1}{\beta(j)(1-\sigma)}}. \tag{26}$$

Hence, ACR's insight that the gains from trade depend on some form of domestic expenditure shares is also true in the world of specialized inputs. Third, and finally, imposing the roundabout assumptions in (18) directly on (24) delivers the formula of Caliendo and Parro (2015). In sum, (24) extends the roundabout multi-industry ACR formula with input-output linkages to specialized inputs.

3.3 The Import Demand System is Not CES

Before delving further, it is helpful to pause and analyze why specialized inputs imply that aggregate expenditure shares are insufficient for tracing the implications of changes in trade barriers. In a nutshell, this occurs because GVCs play a role in propagating trade shocks and specialized inputs determine the structure of these trade linkages. In words, if both Ford and Volkswagen assemble vehicles in Mexico but have different supply chains, then changes in Mexican trade costs with different export partners have asymmetric effects on input suppliers depending on the structure of Ford and Volkswagen's supply chains.

Formally, this can be stated in terms of ACR's restriction concerning how third country trade shocks pass through into relative imports; I discuss only the intuition, the proof is in appendix section B. In a single-industry world, the partial elasticity of imports in j' from source $j'' \neq j'$ relative to domestic purchases (i.e. from j') with respect to changes in trade costs with a third country $i'' \neq j'$ depends on (i) the direct effect on relative imports present when $j'' = i''$, (ii) a substitution effect from i'' inputs into both

[23] The exponents capture the gross domestic output used in the production of a dollar of final goods: The power 1 on $\hat{\pi}_F(j|j)$ is the dollar of final goods while the power on $\hat{a}(j|j,j)$ equals the use of intermediate inputs across all stages of the supply chain

$$(1-\beta(j,j)) \times 1 + (1-\beta(j,j)) \times (1-\beta(j,j)) \times 1 + \cdots = (1-\beta(j,j))/\beta(j,j).$$

j'' and j' inputs, and (iii) a supply chain effect into j'' and j' inputs derived from the change in downstream production. Crucially, the latter two effects depend on the differential importance of each export market j for inputs from j'' relative to j' and on how trade costs with i'' affect exports to each j.

The latter two effects thus illustrate how changes in third-country trade barriers affect imports asymmetrically depending on the depth of supply chain integration. These channels are in line with the empirical evidence suggesting that specialized inputs play a crucial role in propagating trade shocks. For example, Barrot and Sauvagnat (2016), Carvalho et al. (2016), and Boehm et al. (2018) show that supply chain disruptions due to natural disasters are propagated by input specificity through trade networks. Increases in suppliers' marginal costs mostly affect tightly-linked firms, rather than entire industries symmetrically as in roundabout models. The knife-edge roundabout model, however, is the one case in which the effect is symmetric since all exports get built with the same inputs. In other words, in roundabout models all export markets j are equally important for inputs from j'' and j' and so the two latter effects disappear. This is the very special case in which model satisfies the ACR condition "the import demand system is CES".

Finally, note that the gravity equation's empirical success is not evidence for the roundabout model. Appendix section B.1 shows that gravity regressions fare well across simulations of the specialized inputs model even though structural gravity does not hold: While third country trade costs shift bilateral trade flows asymmetrically, on aggregate the bilateral terms dominate. In practice, this misspecification leads to attenuated trade elasticity estimates and is similar to introducing classical measurement error — thus suggesting a downward bias in gravity-based elasticities when deep supply chain linkages are pervasive.

3.4 Bounding Counterfactuals

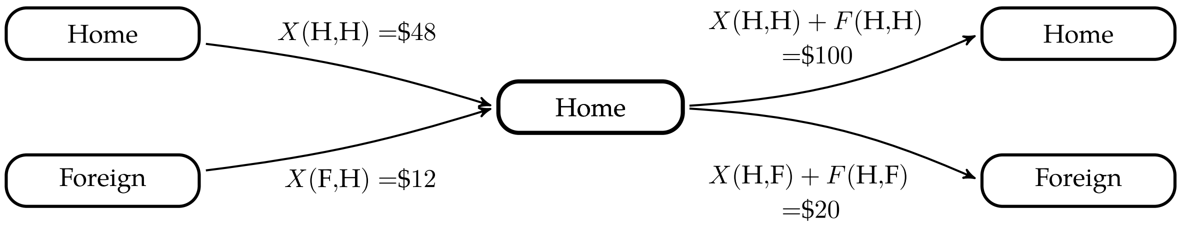
3.4.1 Autarky Gains from Trade - Single Industry Bounds

I begin by showcasing the bounds approach to counterfactuals in a simplified setting. For now, I ignore the data's industrial dimension and assume there is a single industry per country, i.e. $\mathcal{S} = \mathcal{J}$, and compute the bounds on the gains relative to autarky. In this case, the change in expenditure shares equals the observed equilibrium's expenditure shares, i.e. $\hat{\pi}_F(j'|j') = \pi_F(j'|j')/1$ and $\hat{a}(j'|j',j') = a(j'|j',j')/(1-\beta(j',j'))$. Since $\pi_F(j'|j')$ is observed in the data and $\beta(j',j') = 1 - \sum_{j''\in\mathcal{J}} a(j''|j',j')$, the only endogenous variables are the input shares $a(j''|j',j)$.

The autarky bounds for country j' in any model delivering a welfare formula as in (25) are given by

$$
\begin{aligned}
\min/\max_{\{a(j''|j',j)\}_{j''\in\mathcal{J},j\in\mathcal{J}}} \quad & \frac{\sum_{j''\in\mathcal{J}} a(j''|j',j')}{1-\sum_{j''\in\mathcal{J}} a(j''|j',j')} \times \ln \frac{a(j'|j',j')}{\sum_{j''\in\mathcal{J}} a(j''|j',j')}, \\
\text{subject to} \quad & X(j'',j') = \sum_{j\in\mathcal{J}} a(j''|j',j)\left(X(j',j)+F(j',j)\right), \ \forall j'', \\
& \sum_{j''\in\mathcal{J}} a(j''|j',j) \leqslant 1, \ \forall j, \\
& a(j''|j',j) \geqslant 0, \ \forall j'', j.
\end{aligned}
\tag{27}
$$

The objective function is a concave transformation of (25), while the constraints restrict the search to GVCs



	Roundabout	Lower Bound	Upper Bound
$\alpha(H \mid H, H)$: Share of H inputs in sales to H	40%	48%	38%
$\alpha(H \mid H, F)$: Share of H inputs in sales to F	40%	0%	50%
$\mathcal{X}(H \mid H, H)$: Dollar value of H inputs in sales to H	$40	$48	$38
$\mathcal{X}(H \mid H, F)$: Dollar value of H inputs in sales to F	$8	$0	$10
$\hat{W}(H)$: Autarky gains from trade in H	7.6%	3.7%	8.7%

Figure 4: GVC Networks in a Simple Home vs Foreign Example: For simplicity, let Home's value-added share be common across destinations and given by $\beta(H) = \$60/\$120 = 50\%$, while its domestic final good share is $\pi_F(H \mid H) = \$52/\$60 = 87\%$. The gains are relative to autarky and computed using (25) with $1 - \sigma = -5$. Since home is a relatively closed economy, the upper bound is mechanically close to the roundabout estimates. That is, the latter assign a lot of domestic inputs into all output and so many domestic inputs can be shifted out of exports into domestically sold goods ($8) but few domestic inputs can be shifted into exports from domestically sold goods ($2).

that replicate the input-output data.[24] This optimization is relatively easy to solve since the objective function is well-behaved and the constraints are linear. In the special case with constant value-added shares, i.e. $\beta(j', j) = \beta(j') \ \forall j \in \mathcal{J}$, this becomes a simple linear program bounding $\alpha(j' \mid j', j')$ directly.

Crucially, computing these bounds requires only zooming in on all import-export linkages *within* country j'. That is, while the world economy depends on $\mathcal{J} \times \mathcal{J} \times \mathcal{J}$ input shares, (27) solves only for $\mathcal{J} \times \mathcal{J}$ endogenous variables. This occurs because this model features little specialization in that country j' buys specific inputs from j'', but can then use them to produce exports to any market. Hence, the specialized inputs linkages through country j' extend at most from its immediate import suppliers to its direct export markets. In other words, the observed bilateral trade flows to and from country j' curtail its domestic network and computing the bounds requires only searching for extremal domestic GVC linkages.[25] Figure 4 illustrates this in a simple two-country network with constant value-added shares.

Figure 5 plots the gains from trade relative to autarky in the roundabout model (ACR) and in specialized inputs models with both common and destination-specific value-added shares (note the log scale) using the 2014 WIOD. Since the latter class of models nest the former the bounds are wider and any value within the bounds is feasible since the optimization constraints are linear and any convex combination of the lower and upper bounds is a possible initial trade equilibrium. Now, while the optimization does not depend

[24]Computing autarky bounds incorporates trade imbalances automatically. First, the imbalances observed in the benchmark equilibrium are fed in through the input-output data. Second, the autarky equilibrium assumes, by construction, no imbalances.

[25]This breaks down with a higher degree of specialization. For example, bounds on GVCs built with third-order Markov chains, i.e. $\alpha(j''' \mid j'', j', j)$, would be wider, but computing them comes at a substantial cost in dimensionality (see footnote 13).

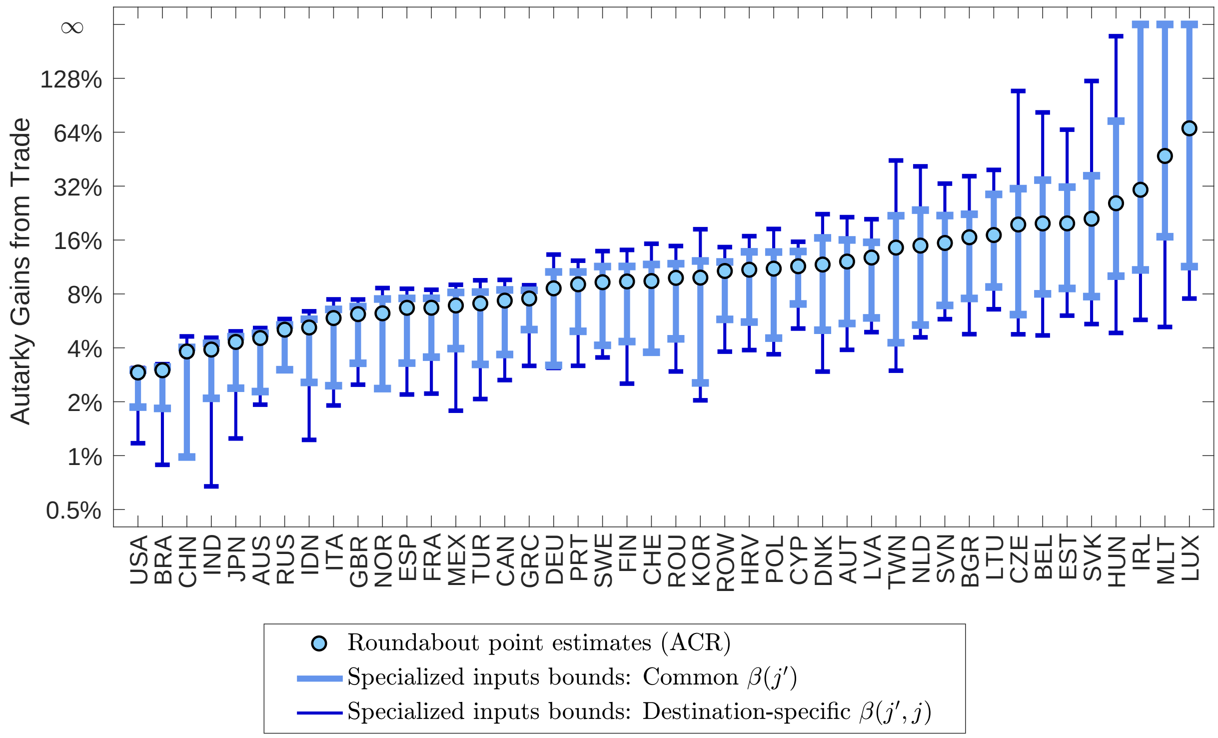


Figure 5: Single-Industry Autarky Welfare Gains from Trade: Both roundabout estimates and specialized inputs bounds based on (25); the latter computed with (27). All counterfactuals use roundabout trade elasticity $1 - \sigma = -5$. Note the log scale. Data is from 2014 WIOD (at country level).

on the trade elasticity, the latter is necessary for transforming the solutions into bounds. However, while using specialized inputs models to measure elasticities is a fascinating research topic, it is beyond this paper's scope. Thus, I simply set a roundabout trade elasticity of $1 - \sigma = -5$, in line with mainstream estimates (Anderson and van Wincoop 2003, Costinot and Rodríguez-Clare 2014, Head and Mayer 2014); the reader can transform any of these numbers x to an elasticity $1 - \sigma$ through $(1 + x)^{(1-6)/(1-\sigma)} - 1$.

The bounds on the gains from trade are wide and increasing in trade openness. For example, the U.S. ACR gains, a relatively closed economy with only 10% of its total inputs purchased abroad, are low at 2.9% while the range with destination-specific value-added shares lies between 1.2-3.1% indicating the gains might actually be 60% lower or 10% higher. The range is relatively small, however, with a ratio between the upper and lower bounds of 2.6. In contrast, very open economies are consistent with a wide range of domestic GVC networks since one can find both trade equilibria in which goods sold domestically use either mostly domestic inputs or almost no domestic inputs. For example, Taiwan imports about 40% of its total inputs and has a bounds ratio of $45\%/3\% = 15$. Full results are reported in appendix section A.3.[26]

[26] These bounds feature a mechanical correlation where distance between the ACR gains and the upper bound increases with trade openness. This occurs because trade equilibria where domestically-sold goods use arbitrarily few domestic goods (a high upper bound) can only be found in countries that trade a lot (figure 4 provides further intuition). In practice, extremely open economies like the small European markets on the right of figure 5 feature upper bounds that are quite literally off the charts.

3.4.2 Autarky Gains from Trade - Multiple Industry Bounds

Computing the autarky bounds with multi-industry data is analogous but more complex numerically. In particular, incorporating destination-specific (Cobb-Douglas) value-added and industry shares is challenging since the welfare gains in (24) are highly nonlinear in these terms. First, because the direct and indirect linkages captured by $\delta(k''|s',j)$ are a function of these shares (see footnote 22). Second, because the Cobb-Douglas shares capture cross-industry linkages and so the optimization has to be done globally across all of a country's GVC network and cannot be done in isolation within each country-industry. Hence, to make things simple, I avoid these issues and focus on the special case where industry-level expenditures are fixed and given by the data — that is, $\beta(s',j) = \beta(s') = GDP(s')/GO(s')$ and $\gamma(k''|s',j) = \gamma(k''|s') = \sum_{s''\in\mathcal{J}\times k''} X(s'',s')/GO(s')\ \forall j \in \mathcal{J}$.

The autarky bounds for country j' in any multi-industry model delivering the welfare formula (24) are found by solving for the extremal GVC networks within every pair of industries separately. Specifically, the extremal domestic shares in country j' for the industry pair k'' and k' are found through

$$\begin{aligned}
&\min/\max_{\{a(t''|s',j)\}_{t''\in\mathcal{J}\times k'',j\in\mathcal{J}}} && a\left(s''\left|s',j'\right.\right), \\
&\text{subject to} && X\left(t'',s'\right) = \sum_{j\in\mathcal{J}} a\left(t''\left|s',j\right.\right)\left(\sum_{s\in j\times\mathcal{K}} X\left(s',s\right) + F\left(s',j\right)\right), \forall t'' \in \mathcal{J}\times k'', \\
& && \sum_{t''\in\mathcal{J}\times k''} a\left(t''\left|s',j\right.\right) = \gamma\left(k''\left|s'\right.\right), \forall j \in \mathcal{J}, \\
& && a\left(t''\left|s',j\right.\right) \geqslant 0, \forall t'' \in \mathcal{J}\times k'', j \in \mathcal{J},
\end{aligned} \tag{28}$$

where $s'' = \{j',k''\}$ and $s' = \{j',k'\}$ are domestic country-industries. Optimization problem (28) is a linear program with $\mathcal{J}\times\mathcal{J}$ endogenous variables and easy to solve numerically. The bounds are then constructed by solving this problem for the $\mathcal{K}\times\mathcal{K}$ industry pairs and inserting the solutions into the terms $\hat{a}(s''|s',j') = a(s''|s',j')/\gamma(k''|s')$ and $\hat{\pi}_F(s'|j') = \pi_F(s'|j')/\zeta(k'|j')$ in (24).

It is well known that multi-industry models deliver larger gains from trade (Costinot and Rodríguez-Clare 2014) and figure 6 shows the same is true for the bounds. This is not by construction, rather, the multi-industry bounds are larger and overlap little with the single-industry bounds because heterogeneity in openness across industries leads to disproportionate effects on the gains from trade. To understand this better, imagine a world in which the input mix used across all industries were common. As shown by Ossa (2015), the multi-industry gains in this very special world are still higher as long as there is heterogeneity in the elasticities of substitution across industries. However, with common elasticities, the gains are the same in both the multi- and single-industry data. Yet, figure 6 is built with common elasticities and delivers higher multi-industry gains. In other words, even relatively closed countries might have some very open industries consistent with many GVC networks — thus increasing the gains from trade even in the presence of zero heterogeneity in the trade elasticities.[27] Full results are reported in appendix section A.3.

[27]Mixing both approaches and incorporating heterogeneity in elasticities may produce even wider bounds.

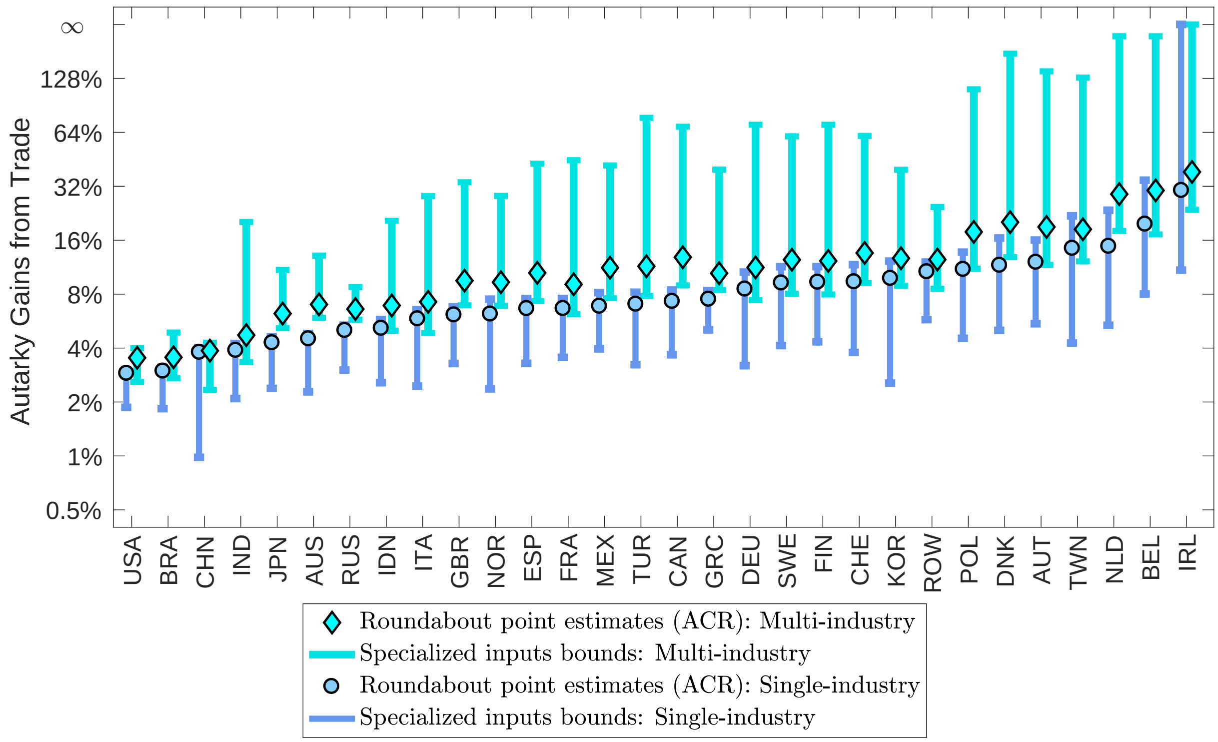


Figure 6: Multi-Industry Autarky Welfare Gains from Trade: Both roundabout estimates and specialized inputs bounds based on (24); the latter computed with (28). All counterfactuals use roundabout trade elasticities $1-\sigma(k) = -5\ \forall k \in \mathcal{K}$. Note the log scale. Data is from 2014 WIOD and aggregated to $\mathcal{J} = 30$ largest economies with $\mathcal{K} = 25$ industries each (see appendix section A.2).

3.4.3 Arbitrary Changes in Trade Costs

Ultimately, perhaps the most relevant exercise is computing bounds on counterfactuals based on real world and policy-motivated events. Relative to the autarky case, however, computing bounds following arbitrary changes to trade barriers requires much heavier numerical work for three reasons. First, the counterfactual gains in general depend on the full GVC network whereas in autarky they depend only on the domestic GVC network. Hence, while the multi-industry autarky bounds relied on solving $\mathcal{K} \times \mathcal{K}$ separate linear optimization problems of size $\mathcal{J} \times \mathcal{J}$ each, computing bounds in general requires solving a single global problem for all input-shares $a(s''|s', j)$; that is, with $\mathcal{J} \times \mathcal{K} \times \mathcal{J} \times \mathcal{K} \times \mathcal{J}$ endogenous variables. Second, the objective function is now given by the full nonlinear formula in (24). Third, the constraints are also nonlinear since they include fixed points for the counterfactual changes in unit prices and wages.

While describing the procedure for constructing exact bounds is straightforward and described in appendix section C, its implementation is left open to the future because its dimensionality and nonlinearity prevent it being solved with current computing power. Having said this, I have also developed a procedure for constructing approximate bounds that is much less computationally intensive and applied it to studying a NAFTA trade war in which trade barriers between Mexico and the U.S. increase by 50%. While instructive, it will become clear in section 5 that the approximate bounds are not very informative about the true bounds and so the exercise is relegated to appendix for the interested reader. Instead, section 5 will show how to construct alternative and arguably better informed point estimates for these counterfactuals.

Overall, the bounds show that quantitative counterfactual predictions based on input-output data may vary substantially depending on how the data is used to construct GVCs.[28] Further, since all GVC networks perfectly fit the same data, the latter can shed no further light on which specific estimates are most reasonable — an important point since the literature often takes the roundabout point estimates at face value. I now take a detour through a parallel literature using input-output data to measure the fragmentation of production and show that similar insights apply there. The paper's last section revisits both literatures and shows that additional information can be used to obtain better informed point estimates.

4 GVCs and Measures of Globalization

A second strand of the GVC literature is concerned with developing measures that better capture the extent of fragmentation of production across borders and stages of the supply chain than those based on traditional gross trade flow statistics. The most influential measures are those based on value-added trade (Hummels et al. 2001, Johnson and Noguera 2012, Koopman et al. 2014, Wang et al. 2013), which capture where value is created rather than where value is shipped from, and those based on upstreamness (Fally 2012, Antràs et al. 2012, Antràs and Chor 2013), which capture a country's average position along the value chain. Without being exhaustive, the literature has also developed measures to capture the factor content of trade (Trefler and Zhu 2010), value-added exchange rates (Bems and Johnson 2017), international inflation spillovers (Auer et al. 2017), and business cycle synchronization (di Giovanni and Levchenko 2010, Johnson 2014b, Duval et al. 2016, di Giovanni et al. 2017). I refer to these generically as *measures of globalization.*

This section shows that measures of globalization vary substantially across GVC networks built from the same input-output dataset. For clarity, I focus on value-added trade decompositions but the same ideas hold generally. I proceed in three steps. First, I show that any measure of globalization can be defined using the general theory of GVCs from section 2. This contrasts with the conventional approach which defines these measures directly with the roundabout solution. The more general definition proves useful since this permits the comparison of different theories of production in terms of their implications on these measures. Second, I show how to construct bounds when imposing the specialized inputs solution. Third, and finally, I use the WIOD to construct the bounds on the share of U.S. value-added in imported Mexican final goods and the U.S.-China value-added deficit and find they are very wide. This suggests that conventional roundabout value-added estimates may be highly mismeasured.

A key difference between this section on measures of globalization and the previous section on counterfactuals is that here I no longer need to take a stand on a specific microfoundation in order to do empirical analysis. That is, while I still need to assume that the theory of production is given by either the roundabout or specialized inputs solution, I need only assume that the theory of production delivers these GVCs

[28] Note that the autarky bounds do not depend explicitly on the Armington microfoundation and only on the sufficient statistics formula (24). That said, more realistic models incorporating elements such as tariff income, fixed costs, and monopoly power may not be consistent with ACR-type formulas. My goal has been to illustrate in the simplest way how quantitative results differ depending on how input-output datasets are interpreted. While outside the scope of this paper, I conjecture that richer microfoundations — which may or may not deliver ACR-type formulas — deliver similar qualitative results in the sense that multiple GVC networks both replicate the input-output data and deliver different quantitative counterfactual estimates.

in equilibrium but can disregard the specific microfoundation that delivers such equilibrium. In this sense, this section is much more flexible than the previous one in that it requires much fewer assumptions.

4.1 Decomposing Value-Added Trade

Decomposing final good consumption into where value-added is produced is useful for understanding how final consumption in some country, say the U.S., is linked to the production of another, say China, through final good exports of a third country, say Mexico. Further, this decomposition is useful for constructing value-added trade imbalances such as the difference between the aggregate flow of, say, Chinese value-added consumed in the U.S. arriving through final good exports of any country and the total U.S. value-added that is eventually consumed in China. More specifically, in the most general form, the value-added from s'' that arrives through final good exports of s' and is consumed in country j is defined as

$$\begin{aligned} VA\left(s'' \left| s', j\right.\right) = 1_{[s''=s']} &\left[\mathcal{G}^1\left(s', j\right) - \sum_{s^2 \in \mathcal{S}} \mathcal{G}^2\left(s^2, s', j\right)\right] \\ + \sum_{N=3}^{\infty} \sum_{s^{N-2} \in \mathcal{S}} \cdots \sum_{s^2 \in \mathcal{S}} &\left[\mathcal{G}^{N-1}\left(s'', s^{N-2}, \ldots, s^2, s', j\right) - \sum_{s^N \in \mathcal{S}} \mathcal{G}^N\left(s^N, s'', s^{N-2}, \ldots, s^2, s', j\right)\right]. \end{aligned} \tag{29}$$

The first term imputes value-added created directly at the assembly stage, appearing only if $s'' = s'$, while the remaining terms impute value-added created by s'' at all further upstream stages of production and which eventually arrives, through any possible sequence, to s' to be shipped to consumers in j.

Value-added trade can be rewritten in terms of a model's equilibrium GVC network once one takes a stand on the equilibrium theory of production solving the GVC challenge in (1). To exemplify this, I begin by showing how this decomposition simplifies once one assumes the specialized input solution in (9) and (10), and then show how it relates to the other solutions. To make the exposition clearer, I derive the decomposition separately for the value-added created at each stage N. First, the value-added by s'' at the most downstream stage, $N = 1$, into final good sales of s' to j equals

$$VA^1\left(s'' \left| s', j\right.\right) = 1_{[s''=s']} \beta_F\left(s', j\right) F\left(s', j\right), \tag{30}$$

where the superindex on VA is meant to index the stage at which this value-added is produced. Clearly, since the most downstream stage is that of final production, s'' adds value-added to final good sales of s' if and only if $s'' = s'$, and the decomposition is given by the share of value-added $\beta_F\left(s', j\right)$ in each dollar of output times the sales of final goods. Second, the value-added generated at the $N = 2$ upstream stage is given by

$$VA^2\left(s'' \left| s', j\right.\right) = \beta_X\left(s'', s'\right) \alpha_F\left(s'' \left| s', j\right.\right) F\left(s', j\right), \tag{31}$$

and equals the intermediate input value-added share times the level of inputs from s'' used in final good sales from s' to j. Third, and finally, the value-added created at any further upstream stage $N \geqslant 3$ equals

$$VA^N\left(s''\left|s',j\right.\right) = \sum_{s^{N-1}\in\mathcal{S}} \dots \sum_{s^2\in\mathcal{S}} \beta_X\left(s'',s^{N-1}\right)\left[\prod_{n=3}^{N} a_X\left(s^n\left|s^{n-1},s^{n-2}\right.\right)\right] a_F\left(s^2\left|s',j\right.\right) F\left(s',j\right), \quad (32)$$

with $s^N = s''$ and $s^1 = s'$. Hence, the total value-added of s'' embedded in final good sales of s' to j is given by the sum of value-added by s'' created at all stages of production

$$VA\left(s''\left|s',j\right.\right) = \sum_{N=1}^{\infty} VA^N\left(s''\left|s',j\right.\right).$$

While writing the decomposition in terms of summations across stages of production is useful for illustrating the intuition behind it, in practice it is tedious to implement numerically. This can be avoided by writing the definitions compactly with linear algebra. To see this, first, organize final good flows $F\left(s',j\right)$ into a vector $\mathbf{F}$ of size $1\times\mathcal{S}\mathcal{J}$. Second, organize the input shares $a_X\left(s''\left|s',s\right.\right)$ into a matrix $\mathbf{a}_X$ stacked as

$$\mathbf{a}_X = \begin{pmatrix} a_X(1|1,1) & a_X(1|1,2) & \dots & a_X(1|1,\mathcal{S}) & a_X(1|2,1) & \dots & a_X(1|\mathcal{S},\mathcal{S}) \\ a_X(2|1,1) & a_X(2|1,2) & \dots & a_X(2|1,\mathcal{S}) & a_X(2|2,1) & \dots & a_X(2|\mathcal{S},\mathcal{S}) \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ a_X(\mathcal{S}|1,1) & a_X(\mathcal{S}|1,2) & \dots & a_X(\mathcal{S}|1,\mathcal{S}) & a_X(\mathcal{S}|2,1) & \dots & a_X(\mathcal{S}|\mathcal{S},\mathcal{S}) \end{pmatrix},$$

of size $\mathcal{S}\times\mathcal{S}^2$, and let $\mathbf{a}_F$ be an analogous matrix of elements $a_F\left(s''\left|s',j\right.\right)$ but of size $\mathcal{S}\times\mathcal{S}\mathcal{J}$. Third, let $\boldsymbol{\beta}_X$ and $\boldsymbol{\beta}_F$ be vectors of elements $\beta_X\left(s',s\right)$ and $\beta_F\left(s',j\right)$ and of size $1\times\mathcal{S}^2$ and $1\times\mathcal{S}\mathcal{J}$. Finally, denote the Kronecker product with $\otimes$ and the Khatri-Rao, or column-wise Kronecker, product with $*$ to define the following auxiliary matrices. Let $\tilde{\mathbf{F}} = \mathbf{F} * \left(\mathbb{I}_{\mathcal{S}\mathcal{J}\times\mathcal{S}\mathcal{J}}\right)$ be of size $\mathcal{S}\mathcal{J}\times\mathcal{S}\mathcal{J}$, $\tilde{\mathbf{a}}_X = \mathbf{a}_X * \left(\mathbb{I}_{\mathcal{S}\times\mathcal{S}}\otimes\mathbf{1}_{1\times\mathcal{S}}\right)$ of size $\mathcal{S}^2\times\mathcal{S}^2$, $\tilde{\mathbf{a}}_F = \mathbf{a}_F * \left(\mathbb{I}_{\mathcal{S}\times\mathcal{S}}\otimes\mathbf{1}_{1\times\mathcal{J}}\right)$ of size $\mathcal{S}^2\times\mathcal{S}\mathcal{J}$, $\tilde{\boldsymbol{\beta}}_X = \boldsymbol{\beta}_X * \left(\mathbb{I}_{\mathcal{S}\times\mathcal{S}}\otimes\mathbf{1}_{1\times\mathcal{S}}\right)$ of size $\mathcal{S}\times\mathcal{S}^2$, and $\tilde{\boldsymbol{\beta}}_F = \boldsymbol{\beta}_F * \left(\mathbb{I}_{\mathcal{S}\times\mathcal{S}}\otimes\mathbf{1}_{1\times\mathcal{J}}\right)$ of size $\mathcal{S}\times\mathcal{S}\mathcal{J}$. The matrix with elements $VA\left(s''\left|s',j\right.\right)$ of size $\mathcal{S}\times\mathcal{S}\mathcal{J}$, stacked as $\mathbf{a}_X$, is given by

$$\mathbf{VA} = \tilde{\boldsymbol{\beta}}_F\tilde{\mathbf{F}} + \tilde{\boldsymbol{\beta}}_X\left[\mathbb{I} - \tilde{\mathbf{a}}_X\right]^{-1}\tilde{\mathbf{a}}_F\tilde{\mathbf{F}}. \quad (33)$$

The relation between matrix and full notation is that the term $\tilde{\boldsymbol{\beta}}_F\tilde{\mathbf{F}}$ summarizes the value-added created at the most downstream stage and is the matrix representation of (30). Analogously, $\tilde{\boldsymbol{\beta}}_X\left(\tilde{\mathbf{a}}_X\right)^{N-2}\tilde{\mathbf{a}}_F\tilde{\mathbf{F}}$ is the matrix representation of $VA^N\left(s''\left|s',j\right.\right)$ for $N \geqslant 2$ so that the second term in (33), given by $\sum_{N=2}^{\infty}\tilde{\boldsymbol{\beta}}_X\left(\tilde{\mathbf{a}}_X\right)^{N-2}\tilde{\mathbf{a}}_F\tilde{\mathbf{F}} = \tilde{\boldsymbol{\beta}}_X\left[\mathbb{I}-\tilde{\mathbf{a}}_X\right]^{-1}\tilde{\mathbf{a}}_F\tilde{\mathbf{F}}$, is the matrix representation of (31) and (32).

Since the roundabout solution is a special case of specialized inputs, its value-added decomposition is nested in (33). Indeed, imposing the GVC mapping in (5) on (29) delivers the value-added decomposition

$$\mathbf{VA} = \boldsymbol{\beta}\left[\mathbb{I} - \mathbf{a}\right]^{-1}\tilde{\mathbf{F}}, \quad (34)$$

where now $\tilde{\mathbf{F}} = \mathbf{F} * \left(\mathbb{I}_{\mathcal{S}\times\mathcal{S}}\otimes\mathbf{1}_{1\times\mathcal{J}}\right)$ is a matrix of size $\mathcal{S}\times\mathcal{S}\mathcal{J}$, $\boldsymbol{\beta}$ is a diagonal matrix of elements $\beta\left(s\right)$ of size $\mathcal{S}\times\mathcal{S}$, and $\mathbf{a}$ is the matrix of technical coefficients $a\left(s'\left|s\right.\right)$ of size $\mathcal{S}\times\mathcal{S}$. This is the standard formula used in the GVC literature and mirrors those in Johnson and Noguera (2012) and Koopman et al. (2014).

The key difference between the specialized inputs and roundabout decomposition of value-added trade is that the former depends on an inverse matrix $[\mathbb{I} - \tilde{\mathbf{a}}_X]^{-1}$ of size $\mathcal{S}^2 \times \mathcal{S}^2$ while the latter depends on the Leontief inverse matrix $[\mathbb{I} - \mathbf{a}]^{-1}$ of size $\mathcal{S} \times \mathcal{S}$. The former is larger since it summarizes the larger set of information contained in the specialized inputs technical coefficients in which input shares vary depending on the use and destination of output.[29] Finally, decomposing value-added by source in the world of 'only trade in final goods' is trivial since, by construction, all value-added is created at the assembly stage. That is, imposing the GVC mapping in (4) implies that the value-added decomposition in (29) becomes

$$VA\left(s'' \left| s', j\right.\right) = 1_{[s''=s']} F\left(s', j\right).$$

This discussion illustrates the value of section 2's general theory of GVCs. While value-added trade has been conventionally defined directly in terms of roundabout GVCs as in (34), defining this measure generally in (29) is useful for deriving this decomposition in other equilibrium theories of production.[30]

4.2 Bounding Value-Added Trade

Conditional on an input-output dataset and an equilibrium theory of production, measures of globalization can be bounded. In particular, the specialized inputs bounds on the value-added from country-industry t'' embedded in the final goods shipped from country-industry t' to consumers in country i are given by

$$\begin{aligned}
\min/\max_{\{a_X(s''|s',s),\, a_F(s''|s',j)\}} \quad & \sum_{N=1}^{\infty} VA^N\left(t'' \left| t', i\right.\right), \\
\text{subject to} \quad & X\left(s'', s'\right) = \sum_{s \in \mathcal{S}} a_X\left(s'' \left| s', s\right.\right) X\left(s', s\right) + \sum_{j \in \mathcal{J}} a_F\left(s'' \left| s', j\right.\right) F\left(s', j\right), \ \forall s'', s', \\
& \sum_{s'' \in \mathcal{S}} a_X\left(s'' \left| s', s\right.\right) \leqslant 1, \ \forall s', s, \\
& \sum_{s'' \in \mathcal{S}} a_F\left(s'' \left| s', j\right.\right) \leqslant 1, \ \forall s', j, \\
& a_X\left(s'' \left| s', s\right.\right), a_F\left(s'' \left| s', j\right.\right) \geqslant 0, \ \forall s'', s', s, j.
\end{aligned} \tag{35}$$

The endogenous variables are the destination-specific input shares for the production of both inputs and final goods. Similarly to the autarky bounds optimization (27) and (28), the constraints are all linear and ensure that the constructed GVC network replicate the observed input-output data. In contrast, however, this optimization searches globally over the full GVC network. That is, whereas the autarky optimization searches separately within each industry s', here the objective function depends on how value flows across all stages and sequences of production and thus needs to account for the world economy as a whole.

[29] The invertibility of these matrices can be shown with the arguments of Hawkins and Simon (1949). In the words of Solow (1952), the necessary condition is that no group of industries be "self-exhausting".

[30] Further, defining concepts cleanly at this general level should also prove useful for resolving outstanding debates in the literature based on specific equilibrium theories of production. For example, the ongoing debate about how to define certain (roundabout) value-added measures between Koopman et al. (2014), Los et al. (2016), Johnson (2017), and Koopman et al. (2018).

In practice, this problem cannot be solved exactly with current computing power since the optimization is highly nonlinear and highly dimensional. Specifically, since $VA^N(t''|t',i)$ is a polynomial of order N in the endogenous variables, the objective function is an infinite sum of polynomials of every order. Further, with $\mathcal{J}$ countries and $\mathcal{K}$ industries, $\mathcal{JK} \times \mathcal{JK} \times (\mathcal{JK} + \mathcal{J})$ endogenous variables need to be solved for.

A straightforward alternative is to, instead, truncate the objective function and implement an approximate bounds approach. Define the $\overline{N}$-th order bounds as the solutions to the optimization problem

$$\min/\max \sum_{N=1}^{\overline{N}} VA^N\left(t''\left|t',i\right.\right), \tag{36}$$

subject to the same constraints in (35). Though a solution to (36) does not deliver the bounds on (35) when $\overline{N} < \infty$, the bounds are close when $\overline{N}$ is big. This occurs because value created in very upstream stages of production represents a small share of final good output and so the terms corresponding to higher-order polynomials beyond $\overline{N}$ are relatively unimportant when $\overline{N}$ is large enough.[31] In the limit $\overline{N} \to \infty$, the approximate bounds converge to the true bounds.

The approximate bounds approach is more easily implemented numerically since the first-order bounds, when $\overline{N} = 1$, are characterized by a linear program while the second-order bounds, when $\overline{N} = 2$, are defined by a quadratic program. Both can be feasibly solved in high dimensions. The problem can be further simplified by focusing on the heterogeneity in input shares while keeping constant value-added shares. That is, when imposing that $\beta_X(s',s) = \beta_F(s',j) = GDP(s')/GO(s')$, the second-order bounds are solved by a linear program while the third-order bounds, with $\overline{N} = 3$, are solved by a quadratic program.[32] In practice, I will show that even the second-order bounds are quite informative about the true bounds.

4.3 Value-Added Trade in the World Input-Output Database

4.3.1 U.S. Value-Added Returned Home Through Imported Mexican Final Goods

One of the most important features of trade in the NAFTA region is that supply chains have become deeply integrated. This integration is often proxied with measures such as the amount of U.S. value-added that returns home through final good imports from its NAFTA partners and, in particular, these statistics received widespread attention during the recent NAFTA renegotiation (see footnote 7). These measures matter because, first, they say something about how each NAFTA country is exploiting its comparative advantage by specializing on specific segments of the supply chain instead of on specializing on different goods and, second, because it informs how changes in trade barriers ripple across country borders.[33]

But how much U.S. value actually returns home through, say, Mexican imports? Figure 7 shows that current estimates might be off by a wide margin. Specifically, figure 7 provides estimates for the U.S. con-

[31]Formally, the gross value traded at upstream stage N decays at least at rate $(1 - \min\{\beta_X(s',s)\})^{N-2}(1 - \min\{\beta_F(s',j)\})$.

[32]In this case the linear inequality constraints in (35) are replaced by $\sum_{s''\in\mathcal{S}} a_X(s''|s',s) = \sum_{s''\in\mathcal{S}} a_F(s''|s',j) = 1 - \beta(s')$.

[33]Policymakers typically interpret a high share of U.S. content in, say, Mexican imports as higher supply chain integration and so higher costs of disruption. The exact quantitative effects, of course, depend on elasticities of substitution and the costs of relocating supply chains across countries. However, Blanchard et al. (2017) showed this basic intuition holds formally. Specifically, they show that countries like the U.S. should set lower tariffs on imports containing a high share of their own domestic content.

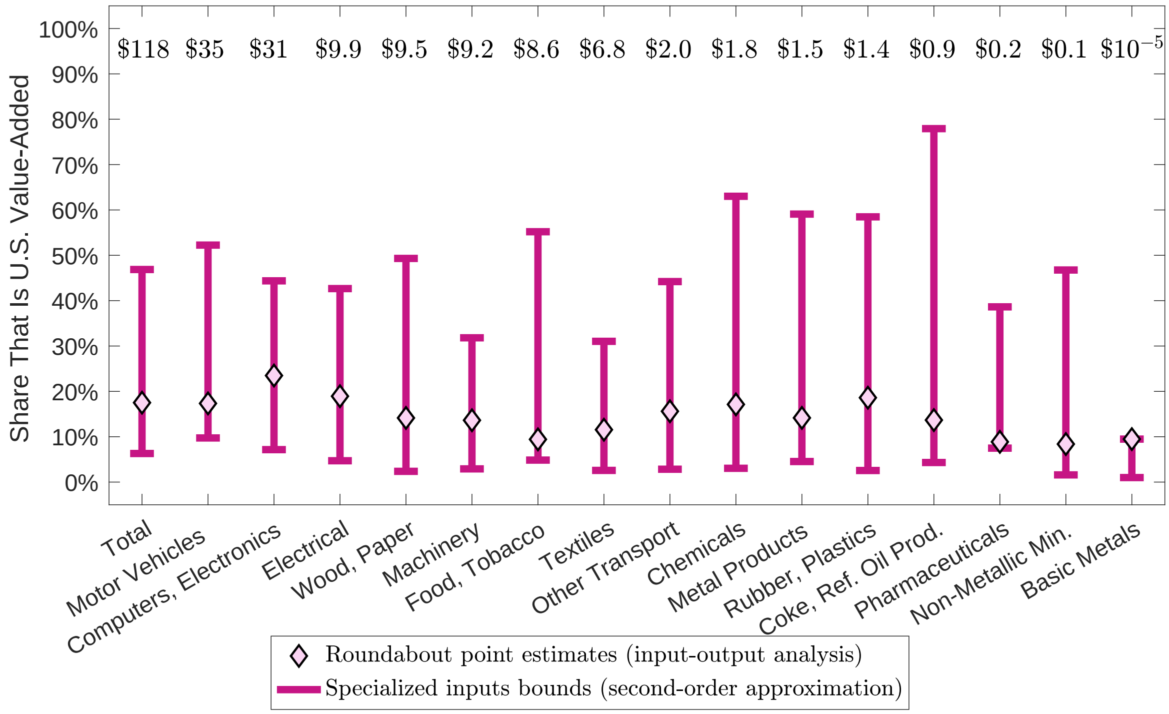


Figure 7: Share of U.S. Value-Added in U.S. Imported Mexican Final Goods: Roundabout point estimates are based on the input-output analysis decomposition in (34). Specialized inputs bounds correspond to second-order bounds on the decomposition in (33) computed with (36) when $\overline{N} = 2$ and with common value-added shares. Numbers at top are gross Mexican final good imports in each manufacturing industry (in billion dollars). Data is from the 2014 WIOD.

tent in imported Mexican manufacturing final goods in 2014. The roundabout point estimates correspond to the conventional estimates used in both academia and policy in which, for example, about 17% of the $118 billion of imported Mexican manufactures corresponds to U.S. value-added created at upstream stages of production. Figure 7 also provides the second-order bounds when using the specialized inputs decomposition in (33) together with the optimization problem in (36). This shows that the true share may be as low as 6% or as high as 47%. Intuitively, the shares vary because, conditional on the level of U.S. imported Mexican goods, the upper bound corresponds to GVCs in which Mexico uses a lot of U.S. inputs to produce these goods while the lower bound corresponds to GVCs in which Mexico uses very few U.S. inputs.

Of course, the true bounds are wider and thus the true estimates may not even be contained within these second-order bounds. However, the latter are so wide that I hope this is sufficient to convince the reader that, in practice, the share of U.S. value in imported Mexican goods may be highly mismeasured.

4.3.2 U.S.-China Trade Imbalances

Another widely cited value-added trade measure is the value-added trade balance between the U.S. and China. In particular, Johnson and Noguera (2012) showed the trade deficit looks less extreme if it is computed as the difference between the U.S. value consumed in China and the Chinese value consumed in the U.S. instead of the difference in gross exports between the two countries. The reasoning is that the former

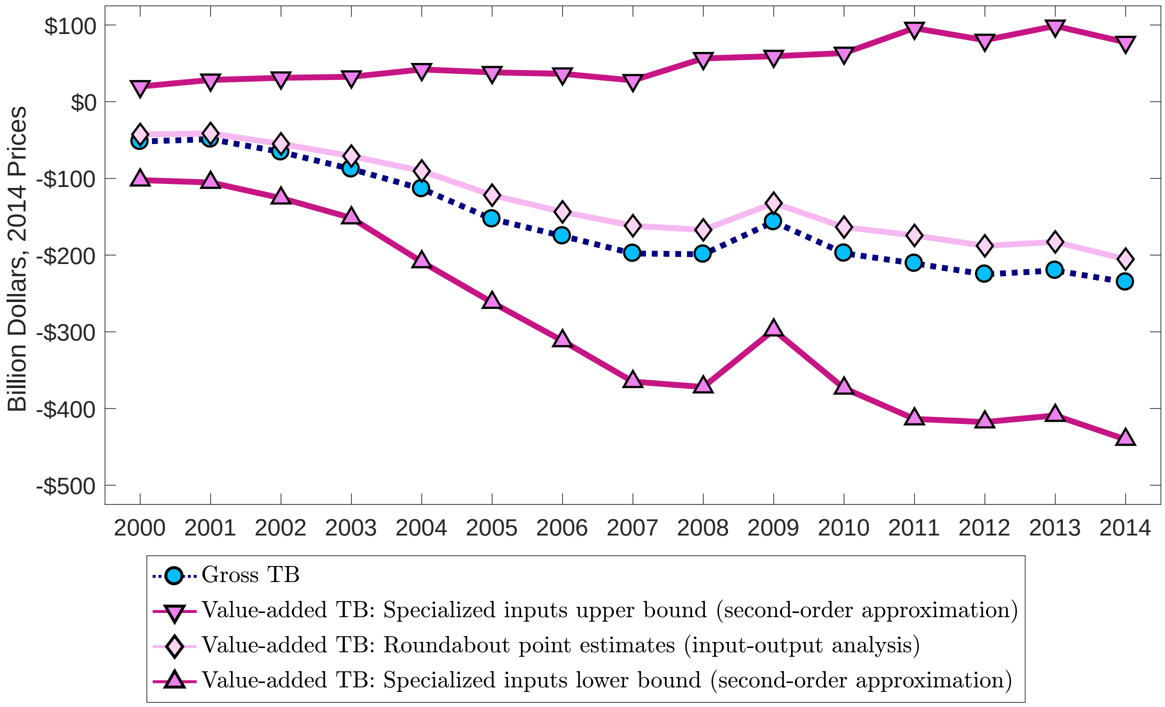


Figure 8: U.S.-China Trade Imbalances: The series with circles corresponds to the gross trade balance. The other three series correspond to the value-added trade balance. Roundabout point estimates (diamonds) are based on the input-output analysis decomposition in (34). Specialized inputs bounds (triangles) correspond to second-order bounds on the decomposition in (33) computed with (36) when $\overline{N} = 2$ and with common value-added shares. Data is from the 2000-2014 WIOD.

provides a better measure of how both economies are relatively linked, and one that does so in terms of the final consumer's perspective, since gross exports from, say China, in principle may say little about China's importance to those exports (i.e. how much value-added China contributes).

But is it really true that the U.S.-China trade deficit is smaller when computed in value-added terms? Figure 8 plots the U.S.-China trade balance both in gross and value-added terms between 2000-2014. The difference between the gross trade balance (circles) and the value-added trade balance based on the roundabout solution (diamonds) replicate the finding in Johnson and Noguera (2012) that the gross deficit in 2004 overstates the value-added deficit by about 25%. Furthermore, the evolution of both series across time looks exactly like the third figure in Johnson (2014a). However, specialized inputs tell a potentially different story.[34] The second-order bounds show these findings may actually be more pronounced in that the value-added balance at the upper bound is a surplus. Alternatively, these findings may actually be reversed in that the value-added deficit at the lower bound is larger than the gross deficit. Intuitively, a

[34] Since the value-added trade balance is based on the decomposition of value-added trade $VA(s''|s', j)$, the bounds are found by replacing the objective function in (35) with

$$\sum_{N=1}^{\infty} \sum_{t'' \in USA \times \mathcal{K}} \sum_{t' \in \mathcal{J} \times \mathcal{K}} VA^{N}(t''|t', CHINA) - \sum_{N=1}^{\infty} \sum_{t'' \in CHINA \times \mathcal{K}} \sum_{t' \in \mathcal{J} \times \mathcal{K}} VA^{N}(t''|t', USA).$$

value-added surplus means that in reality China is exporting back to the U.S. a lot more U.S. value than is currently accounted for. Meanwhile, a larger value-added deficit means that in reality China is exporting back so little U.S. value that this imbalance is much larger than when studied in gross terms.

As before, figure 8 does not necessarily imply that the value-added deficit is not smaller than the gross deficit. But it does illustrate how facts that are part of conventional wisdom may be pure artifice of how the roundabout solution constructs GVCs. The key point being that more knowledge about the GVC networks underlying input-output datasets is required in order to make either statement convincingly. Indeed, the bounds in figure 8 deliver a stark message: The difference between the conventional estimates of value-added and gross trade balances is dwarfed by the the potential mismeasurement in the former.

5 GVCs and Measurement: Bringing in New Sources of Information

This last section is devoted to a third strand of the GVC literature concerned with measuring GVC flows. The motivation is that both academics and policymakers often need an informed best guess of the effects of a policy change or of the value of a specific measure of globalization. But, since input-output datasets are consistent with wide ranges of values, delivering point estimates requires making additional assumptions. Instead of focusing on the roundabout solution, I propose an alternative approach in which information beyond that contained in input-output data is exploited in order to obtain better informed best guesses.

I proceed in three steps. First, I show how to incorporate the additional information in the objective function of an optimization problem that delivers GVC point estimates that are both consistent with input-output data but more accurate than the roundabout GVCs. Second, I implement this approach using Mexican customs data and revisit the counterfactuals and value-added decompositions of sections 3 and 4. Third, I discuss an alternative approach in which the additional information is used to narrow the bounds on the latter exercises by imposing additional constraints on the bounds optimization problems.

5.1 Disciplining GVCs with New Sources of Information

I propose a new GVC measurement framework that constructs a specific GVC network through an optimization problem that exploits both the input-output data and other sources of information as follows

$$\begin{aligned} \min \quad & h\left(\left\{\alpha_X\left(s''\left|s',s\right.\right)\right\}_{s''\in\mathcal{S},s\in\mathcal{S}},\left\{\beta_X\left(s',s\right)\right\}_{s\in\mathcal{S}},\left\{\alpha_F\left(s''\left|s',j\right.\right)\right\}_{s''\in\mathcal{S},j\in\mathcal{J}},\left\{\beta_F\left(s',j\right)\right\}_{j\in\mathcal{J}}\right), \\ \text{subject to} \quad & X\left(s'',s'\right)=\sum_{s\in\mathcal{S}}\alpha_X\left(s''\left|s',s\right.\right)X\left(s',s\right)+\sum_{j\in\mathcal{J}}\alpha_F\left(s''\left|s',j\right.\right)F\left(s',j\right),\ \forall s'', \\ & \sum_{s''\in\mathcal{S}}\alpha_X\left(s''\left|s',s\right.\right)+\beta_X\left(s',s\right)=1,\ \forall s, \\ & \sum_{s''\in\mathcal{S}}\alpha_F\left(s''\left|s',j\right.\right)+\beta_F\left(s',j\right)=1,\ \forall j, \\ & \alpha_X\left(s''\left|s',s\right.\right),\alpha_F\left(s''\left|s',j\right.\right),\beta_X\left(s',s\right),\beta_F\left(s',j\right)\geqslant 0,\ \forall s'',s,j. \end{aligned} \tag{37}$$

The objective function $h(\cdot)$ depends on the endogenous input and value-added expenditure shares and (potentially) some exogenous parameters. For example, a simple and tractable objective function is given by targeting exogenous values for each share and minimizing the weighted sum of squared deviations

$$h(\cdot) = \sum_{s'' \in \mathcal{S}} \sum_{s \in \mathcal{S}} \omega_X^0(s''|s', s)\left[a_X(s''|s', s) - a_X^0(s''|s', s)\right]^2 + \sum_{s \in \mathcal{S}} \omega_X^0(s', s)\left[\beta_X(s', s) - \beta_X^0(s', s)\right]^2$$
$$+ \sum_{s'' \in \mathcal{S}} \sum_{j \in \mathcal{J}} \omega_F^0(s''|s', j)\left[a_F(s''|s', j) - a_F^0(s''|s', j)\right]^2 + \sum_{j \in \mathcal{J}} \omega_F^0(s', j)\left[\beta_F(s', j) - \beta_F^0(s', j)\right]^2.$$

In this case, $a_X^0(s''|s', s)$, $a_F^0(s''|s', j)$, $\beta_X^0(s', s)$, and $\beta_F^0(s', j)$ are targets for the endogenous variables and $\omega_X^0(s''|s', s)$, $\omega_F^0(s''|s', j)$, $\omega_X^0(s', s)$, and $\omega_F^0(s', j)$ correspond to the weights on each target. While other objective functions can be used, I will focus throughout on this quadratic form since it is the simplest nonlinear function that can be solved in high dimensions (i.e., it has linear first-order conditions).[35]

Both the targets and the weights in $h(\cdot)$ are chosen by the researcher and this is where the additional information is used to discipline the GVC network. That is, the constraints on (37) restrict the optimization problem to only search across GVC networks consistent with the input-output data and it is in this sense that the optimization exhausts the information contained in the latter. The additional information is then used to pin down a specific GVC network, among all networks consistent with the input-output data, through the objective function by ensuring that the constructed GVC network is the one closest to the researcher's targets in the sense of minimizing the weighted sum of squared deviations.

This approach is useful when a researcher has some information about the GVCs underlying input-output data that is insufficient for fully measuring the flows directly. For example, a researcher with access to the universe of firm-to-firm transaction data for a whole country, say Belgium, could build segments of the GVC flows crossing through Belgium directly with the microdata — for example, the shares $a_F(s''|s', j)$ for $s' \in \text{BEL} \times \mathcal{K}$. Since the microdata should be, in principle, consistent with the aggregate input-output data then this GVC network will perfectly aggregate up to the latter (i.e. the input-output data for Belgium is redundant). However, in practice, many firm-level datasets are informative about GVC flows but too limited for measuring the latter directly. For example, most countries construct customs-level datasets but these cannot be used to measure GVC flows directly since they lack the universe of domestic transactions. To see why, note that the information in figure 1 corresponds to expenditure shares in terms of overall foreign input expenditure shares. That is, the 74% in figure 1 corresponds to

$$\frac{\sum_{k'' \in \mathcal{K}} a_F(\{\text{USA}, k''\}|\{\text{MEX}, \text{cars}\}, \text{USA})}{\sum_{j'' \in \mathcal{J} \setminus \text{MEX}} \sum_{k'' \in \mathcal{K}} a_F(\{j'', k''\}|\{\text{MEX}, \text{cars}\}, \text{USA})} = 74\%.$$

The denominator cannot be measured in customs data because it requires knowing what share of overall output value is spent on foreign inputs — which requires knowing the expenditure on domestic input purchases and domestic value-added.[36] The same is true for the other input and value-added shares.

[35]This approach follows a long tradition of exploiting linearity to solve for high-scale optimization problems in economics and developed by such giants as Kantorovich (1939), Koopmans and Beckmann (1957), Dorfman et al. (1958), and Dantzig (1963).

[36]Note that while figures 1 and 2 provide foreign expenditure shares from specific countries, the data describes import shipments at the country-industry level and so the shares can be constructed at this level (i.e. from a specific $s'' \in \mathcal{J} \times \mathcal{K}$).

Hence, customs data cannot be used to measure a specific segment of the GVC network directly. Instead, the measurement framework (37) provides a workaround by letting a researcher take a stand on how to map the customs data into the input shares through the targets. However, in general, the researcher's targets will not aggregate up to the input-output data since they are based on additional information plus some auxiliary assumptions that are not fully accurate. Thus, the optimization problem is useful because it reallocates flows in order to deliver a GVC network that aggregates up to the input-output data while also respecting the researcher's targets. In sum, while both the roundabout GVCs and the GVCs built through (37) ultimately depend on some assumptions, the latter are built by replacing some assumptions with additional information and thus should be closer to the true GVCs underlying input-output data.

In terms of numerical implementation, careful inspection of (37) reveals the problem is defined for a specific country-industry s'. That is, (37) delivers input shares from all sources s'' in the production of intermediates sold to all country-industries s and countries j. This problem is larger than the multi-industry counterfactual bounds in (28) since the latter has more structure because the upper tier Cobb-Douglas production function requires only searching for the input shares from suppliers within industry k''. However, it is a smaller problem than the multi-industry value-added bounds in (35) which requires searching over the full GVC network and thus for input shares across all country-industries s' simultaneously. While the measurement problem (37) is also large and nonlinear, exact solutions can be computed (see appendix section D.1 for implementation details). Overall, measuring a full GVC network when minimizing the weighted sum of squared deviations requires choosing a set of weights and targets and solving $\mathcal{S}$ optimization problems, one for each $s' \in \mathcal{S}$, of size $(\mathcal{S}+1) \times (\mathcal{S}+\mathcal{J})$ each.[37]

5.2 Improving Measurement with Mexican Customs Shipment-Level Data

I now construct GVCs with (37) while disciplining the targets using Mexican customs data. In order to map this data to the targets I take the stand that Mexico only does processing trade — i.e. that imported inputs are only used to produce exports and that exports only use imported inputs.[38] While this assumption is restrictive and not fully accurate in reality, it provides an useful and, in my view, reasonable starting point for showing how this measurement framework can be used. More generally, (37) provides a common ground for beginning a conversation on the best practices for GVC measurement: For example, this application can be studied with other auxiliary assumptions while keeping the same numerical procedure.

Assuming processing trade implies that the foreign input distribution becomes the overall input expenditure distribution. Specifically, index firms by f and let $\mathcal{X}_X^f(s''|s', s)$ be the dollar value of inputs from s'' used by a firm f producing in Mexican industry s' selling to s. Analogously, let $\mathcal{X}_F^f(s''|s', j)$ be the flow but for inputs used to produce final good exports. Both these objects can be constructed using the customs data as described in section 2.4.1 and can be interpreted in dollar values when assuming that

[37] The problem cannot be made smaller without further structure because the input shares across suppliers and destinations of s' are interlinked through the constraints. The problem can be generalized, though, by choosing an objective function featuring complementarities across country-industries $s' \in \mathcal{S}$ and thus solving for the full GVC network in a single optimization problem.

[38] Mexico and China are the largest countries where processing trade is widely prevalent. For example, De La Cruz et al. (2011) show that in 2003 about 96.6% of transportation equipment exports were processing trade and of this 74% is foreign and 26% is domestic value-added. More generally, that exports use a higher share of imported inputs than domestically-sold goods appears to be a fairly common feature across countries (see Kee and Tang 2016 for evidence on China and Tintelnot et al. 2017 on Belgium).

exporting firms do only processing trade. I define the targets for the Mexican industry-level input shares $s' \in \text{MEX} \times \mathcal{K}$ as

$$\begin{aligned} a_X^0\left(s''\left|s', s\right.\right) &= \frac{\sum_f \mathcal{X}_X^f\left(s''|s', s\right)}{\sum_{t'' \in \mathcal{S}} \sum_f \mathcal{X}_X^f\left(t''|s', s\right)} \times \left(1 - \beta_X^0\left(s', s\right)\right), \\ a_F^0\left(s''\left|s', j\right.\right) &= \frac{\sum_f \mathcal{X}_F^f\left(s''|s', j\right)}{\sum_{t'' \in \mathcal{S}} \sum_f \mathcal{X}_F^f\left(t''|s', j\right)} \times \left(1 - \beta_F^0\left(s', j\right)\right), \end{aligned} \tag{38}$$

where, for now, I target the roundabout value-added shares given by input-output data: $\beta_X^0\left(s', s\right) = \beta_F^0\left(s', j\right) = \text{GDP}\left(s'\right)/\text{GO}\left(s'\right)$. From the processing trade assumption, the targets in (38) equal zero when inputs are domestic, i.e. $s'' \in \text{MEX} \times \mathcal{K}$, or the destination is domestic, i.e. $s \in \text{MEX} \times \mathcal{K}$ or $j = \text{MEX}$. However, since processing trade does not hold fully in reality, these targets deliver a GVC network that does not aggregate up to the observed input-output data. The optimization problem (37) resolves this issue by reallocating GVC flows to ensure that they fit both the input-output data while respecting (as much as possible) the targeted shares. Finally, since computing counterfactuals and measures of globalization requires the full GVC network, I proceed conservatively and maintain the roundabout shares in all other countries. That is, when $s' \in (\mathcal{J} \backslash \text{MEX}) \times \mathcal{K}$ I set[39]

$$a_X\left(s''\left|s', s\right.\right) = a_F\left(s''\left|s', j\right.\right) = \frac{X\left(s'', s'\right)}{\text{GO}\left(s'\right)}. \tag{39}$$

Table 1 presents the share of U.S. value in Mexican manufacturing imports and the costs of a NAFTA trade war according to the roundabout solution (column I), the approximate bounds on specialized inputs (II and III), and the GVCs obtained when using Mexican customs data (IV and V). The difference between the latter two columns is that IV targets common value-added shares across all output, while V targets 50% more value-added in Mexican domestic sales and 50% less in exports than at the overall industry level — in line with De La Cruz et al. 2011 who show exports use domestic value-added less intensively.[40]

The main takeaway is that Mexican-American supply chains are more integrated and disrupting them is more costly than suggested by conventional estimates. For example, in motor vehicles — the largest imported manufacturing industry — the U.S. imports back around 38% of its own domestic value whereas the roundabout estimates predict a smaller share of only 17%. For overall manufacturing, the U.S. share is about 30% and also substantially higher than the roundabout estimate of 17%. These differences are in line with the input shares observed in figure 2: While the customs data show that Mexico uses a high share of American inputs to produce exports to the U.S., these estimates trace value across all stages of production and confirm that a large part of these exports is American value-added. In contrast, the roundabout approach waters down the U.S. content in exports to the U.S. since it assumes a common share of U.S. content in exports to all countries. With specialized inputs, however, since the U.S. content in exports to the U.S.

[39] To fix ideas, note that solving (37) with the shares in (39) as targets in all countries delivers the roundabout GVCs.

[40] Formally, in column V for $s' \in \text{MEX} \times \mathcal{K}$ if $s \in \text{MEX} \times \mathcal{K}$ or $j = \text{MEX}$ then $\beta_X^0\left(s', s\right) = \beta_F^0\left(s', j\right) = 1.5 \times \beta\left(s'\right)$ and if $s \in (\mathcal{J} \backslash \text{MEX}) \times \mathcal{K}$ or $j \in \mathcal{J} \backslash \text{MEX}$ then $\beta_X^0\left(s', s\right) = \beta_F^0\left(s', j\right) = 0.5 \times \beta\left(s'\right)$. In both columns IV and V, the weights are given by $\omega_X^0\left(s''|s', s\right) = a_X^0\left(s''|s', s\right) X\left(s', s\right)$, $\omega_F^0\left(s''|s', j\right) = a_F^0\left(s''|s', j\right) F\left(s', j\right)$, $\omega_X^0\left(s', s\right) = \beta_X^0\left(s', s\right) X\left(s', s\right)$, and $\omega_F^0\left(s', j\right) = a_F^0\left(s', j\right) F\left(s', j\right)$. These weights are designed to put more weight on the targets corresponding to bilateral trade flows which are more important and to the most important input suppliers therein.

	Roundabout	Specialized Inputs Common V.A.			Specialized Inputs Low Export V.A.
		Approx. Bounds Lower	Upper	Mex. Customs + Proc. Trade	Mex. Customs + Proc. Trade
U.S. Value-Added in Mexican Imports (%)	I	II	III	IV	V
Total Manufactures	17	6	47	27	30
Basic Metals	9	1	9	5	5
Chemicals	17	3	63	33	37
Coke, Refined Oil Prod.	14	4	78	43	25
Computers, Electronics	23	7	44	13	16
Electrical	19	5	43	25	28
Food, Tobacco	9	5	55	37	39
Machinery	14	3	32	20	24
Metal Products	14	5	59	27	32
Motor Vehicles	17	10	52	37	39
Non-Metallic Minerals	8	2	47	24	17
Other Transport	16	3	44	24	30
Pharmaceuticals	9	7	39	15	24
Rubber, Plastics	19	3	58	34	41
Textiles	12	3	31	24	26
Wood, Paper	14	2	49	33	38
Welfare Cost of Unilateral U.S-Mexico Trade War (%)					
Mexico	2.43	2.15	3.02	2.16	1.74
U.S.	0.19	0.14	0.22	0.37	0.46
If Mexico Retaliates (%)					
Mexico	3.68	3.25	4.66	3.24	3.31
U.S.	0.26	0.19	0.29	0.50	0.62

Table 1: U.S. Value-Added in Mexican Manufacturing Imports and the Welfare Costs of a NAFTA Trade War: Column I is computed with the roundabout GVCs while II and III correspond to the approximate specialized inputs bounds depicted in figures 7 and 11. Columns IV and V correspond to GVCs obtained through (37) when disciplining the targets with Mexican customs data and a processing trade assumption; IV targets common value-added shares and V targets 50% more direct Mexican value-added in domestic sales and 50% less in exports. Data is from the 2014 WIOD.

increased then the U.S. content in exports to other countries is now lower.

In terms of counterfactuals, table 1 uses section 3's model to study the welfare cost of a NAFTA trade war in which the U.S. unilaterally increases trade barriers on Mexican imports by 50% and of a war in which Mexico retaliates and also increases trade barriers on U.S. imports by 50%. Columns IV and V show that incorporating the deep integration between the U.S and Mexico doubles the U.S. welfare cost of a NAFTA trade war relative to the roundabout (ACR) estimates and decreases the cost for Mexico. These opposing effects are due to the asymmetric implications of Mexican customs data for each country: By increasing the share of U.S. inputs in U.S.-bound Mexican exports, U.S. consumers are more exposed to a NAFTA trade war while Mexican consumers are less. However, note that I targeted the roundabout input shares within the U.S. since I do not have U.S. customs data. This suggests that incorporating the latter — which would likely reveal a high share of Mexican inputs in Mexican-bound exports — might increase the welfare cost for Mexico. Finally, note that the counterfactual estimates in columns IV and V are substantially outside of the approximate bounds in II and III computed in appendix section C.3. This provides evidence that the procedure for computing approximate bounds on counterfactuals following arbitrary changes to trade barriers discussed in section 3.4.3 produces ranges that are not very informative about the true bounds.[41]

Finally, while I have focused on incorporating Mexican customs data, this new GVC measurement framework is flexible enough to let researchers incorporate whatever additional information is both relevant and available in each context (all code is online here). In particular, table 1's analysis can readily be improved by further including other countries' customs data. Moreover, researcher's can incorporate more abstract forms of information as long as one takes a stand on how to map such information into the optimization targets. For example, since NAFTA supply chains are shaped by rules of origin (Conconi et al. 2018), a researcher without access to customs data could still obtain a GVC network with highly integrated NAFTA supply chains by mapping the (abstract) rules-of-origin criteria into the optimization targets.

5.3 Disciplining the Bounds with New Sources of Information

The measurement framework (37) provides an alternative to the roundabout point estimates by constructing GVCs that are disciplined by both input-output data and additional information. However, there is another way of doing this through a procedure that is closer to the bounds approach of sections 3 and 4.

Additional information can be exploited in order to narrow the bounds by using it to impose additional constraints on the optimization problems used to construct bounds on counterfactuals and measures of globalization in (28) and (35). This approach is conceptually different from the measurement framework in (37) for three reasons. First, instead of delivering an alternative to the roundabout point estimates it provides narrower ranges than the bounds based solely on input-output data. Second, this approach exploits the additional information through the constraints instead of the objective function and this has important implications on the feasibility of the problem. Third, this approach delivers bounds that are specific to a given statistic of interest such as a counterfactual or value-added trade decomposition while the measurement in (37) minimizes a feature of the GVCs themselves and thus provides GVC estimates that can then be used to construct any statistic of interest.

While constructing narrower bounds is an interesting alternative approach to GVC measurement, I believe it is less useful in practice for three reasons. First, policy questions often require having a precise best guess to specific questions so that constructing ranges of values is unappealing in certain contexts. Second, this approach is hard to implement numerically in practice since, as in the measurement framework in (37), the additional information needs to be mapped into additional constraints using some auxiliary assumptions and this can lead to the optimization problem having no feasible solutions. In contrast, incorporating

[41] The counterfactual estimates reported in columns IV and V are built with a slightly more general model than that of section 3. Specifically, while that model assumes $\alpha_X(s''|s',s) = \alpha_F(s''|s',j) = \alpha(s''|s',j)$, columns IV and V allow $\alpha_X(s''|s',s)$ and $\alpha_F(s''|s',j)$ to be different. Thus, the approximate bounds in II and III might also be off because they correspond to a more restricted model. Crucially, though, the relevant roundabout estimates for this more general model are still those in column I.

the additional information through the objective function does not affect the problem's feasibility in any way. Third, and most importantly, current computing power is insufficient for constructing exact bounds on both counterfactuals following arbitrary changes in trade barriers and measures of globalization even without the additional information. Hence, this approach cannot be implemented for many interesting questions and so I believe that the measurement framework in (37) is a more practical way of incorporating additional information into GVC measurement. Still, for the interested reader, appendix section D.2 illustrates how additional information can be used to narrow the bounds in a stylized example.

6 Conclusion

In sum, this paper's message is twofold. First, that conventional GVC flow estimates are potentially mismeasured and that this trickles down into the answers to both quantitative counterfactuals and the measures of globalization used to quantify the fragmentation of production across countries. Second, that by incorporating more information and improving GVC measurement, researchers can then go back and answer these questions more precisely. While this point is, perhaps, obvious, it has mostly been ignored.

In particular, Mexican customs data confirms the anecdotal evidence that Mexican-American supply chains are highly integrated and measuring GVCs while incorporating this information increases the U.S. content of imported Mexican manufactures and the U.S. welfare cost of a NAFTA trade war. These facts are in line with the intuition on how GVC linkages magnify the effects of economic shocks (Yi 2003) and dampen a country's incentive to manipulate its terms-of-trade: Import tariffs are more costly when imports have more domestic content because they ripple back and hurt domestic suppliers (Blanchard et al. 2017).

The avenue for future research is extraordinarily rich. First, section 5's measurement framework is quite versatile and can be used to conduct many new exercises. For example, it can be used with the same Mexican customs data but when defining the targets differently, it can incorporate more microdata such as other countries' customs data or any other information that a researcher may find relevant, and it can be used with other objective functions. Second, this new measurement framework may be a useful steppingstone towards future alternative and more effective frameworks that also incorporate new information. In this sense, this paper is only a first step in a research line on the best methods and practices for conducting GVC measurement. By abandoning the roundabout approach, the possibilities become endless.

Finally, there remains the issue of constructing multi-country input-output datasets themselves accurately. While this paper's theoretical and numerical procedures are derived with an accurate dataset in mind, the empirical results are all based on the WIOD. The latter is not free of measurement assumptions and so, in principle, its input-output flows might be mismeasured. Going forward, new dataset releases are getting more accurate so that this issue becomes less important and, further, I believe this paper's insights might be used in future research to better measure both input-output data and GVCs simultaneously.[42]

[42]For example, in the spirit of Batten (1982), Golan et al. (1994), Canning and Wang (2005), and Wang et al. (2010).

References

Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012): "The Network Origins of Aggregate Fluctuations," *Econometrica*, 80, 1977–2016.

Allen, T., C. Arkolakis, and Y. Takahashi (2017): "Universal Gravity," *NBER Working Paper 20787.*

Anderson, J. E. (1979): "A Theoretical Foundation for the Gravity Equation," *The American Economic Review*, 69, 106–116.

Anderson, J. E. and E. van Wincoop (2003): "Gravity with Gravitas: A Solution to the Border Puzzie," *The American Economic Review*, 93, 170–192.

Antràs, P. and D. Chor (2013): "Organizing the Global Value Chain," *Econometrica*, 81, 2127–2204.

Antràs, P., D. Chor, T. Fally, and R. Hillberry (2012): "Measuring the Upstreamness of Production and Trade Flows," *The American Economic Review*, 102, 412–416.

Antràs, P. and A. de Gortari (2017): "On the Geography of Global Value Chains," *NBER Working Paper 23456.*

Antràs, P., T. C. Fort, and F. Tintelnot (2017): "The Margins of Global Sourcing: Theory and Evidence from U.S. Firms," *American Economic Review*, 107, 2514–2564.

Antràs, P. and R. W. Staiger (2012): "Offshoring and the Role of Trade Agreements," *American Economic Review*, 102, 3140–3183.

Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012): "New Trade Models, Same Old Gains?" *American Economic Review*, 102, 94–130.

Auer, R. A., A. A. Levchenko, and P. Saur (2017): "International Inflation Spillovers Through Input Linkages," *NBER Working Paper 23246.*

Balistreri, E. J., R. H. Hillberry, and T. F. Rutherford (2011): "Structural estimation and solution of international trade models with heterogeneous firms," *Journal of International Economics*, 83, 95–108.

Baqaee, D. R. (2014): "Cascading Failures in Production Networks," 1–60.

Baqaee, D. R. and E. Farhi (2017): "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem," *NBER Working Paper 23145.*

Barrot, J.-N. and J. Sauvagnat (2016): "Input Specificity and the Propagation of Idiosyncratic Shocks in Production Networks," *The Quarterly Journal of Economics*, 1543–1592.

Bartelme, D. and Y. Gorodnichenko (2015): "Linkages and Economic Development," *NBER Working Paper 21251.*

Bastos, P. and J. Silva (2010): "The quality of a firm's exports: Where you export to matters," *Journal of International Economics*, 82, 99–111.

Bastos, P., J. Silva, and E. Verhoogen (2018): "Export Destinations and Input Prices," *American Economic Review*, 108, 353–392.

Basu, S. (1995): "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare," *The American Economic Review*, 85, 512–531.

Batten, D. F. (1982): "The Interregional Linkages between National and Regional Input-Output Models," *International Regional Science Review*, 7, 53–67.

Bems, R. (2014): "Intermediate inputs, external rebalancing and relative price adjustment," *Journal of International Economics*, 94, 248–262.

Bems, R. and R. C. Johnson (2017): "Demand for Value Added and Value-Added Exchange Rates," *American Economic Journal: Macroeconomics.*

Bigio, S. and J. La'O (2016): "Financial Frictions in Production Networks," .

Blanchard, E. J., C. P. Bown, and R. C. Johnson (2017): "Global Value Chains and Trade Policy," .

Blaum, J., C. Lelarge, and M. Peters (2017a): "Firm Size and the Intensive Margin of Import Demand," .

——— (2017b): "The Gains From Input Trade With Heterogeneous Importers," *American Economic Journal: Macroeconomics*, Forthcomin.

Boehm, C. E., A. Flaaen, and N. Pandalai-Nayar (2018): "Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the 2011 Tohoku Earthquake," *The Review of Economics and Statistics*, Forthcomin.

Brambilla, I., D. Lederman, and G. Porto (2012): "Exports, Export Destinations, and Skills," *The American Economic Review*, 102, 3406–3438.

Brambilla, I. and G. G. Porto (2016): "High-income export destinations, quality and wages," *Journal of International Economics*, 98, 21–35.

Caliendo, L. and F. Parro (2015): "Estimates of the Trade and Welfare Effects of NAFTA," *Review of Economic Studies*, 82, 1–44.

Caliendo, L., F. Parro, and A. Tsyvinski (2017): "Distortions and the Structure of the World Economy," *NBER Working Paper 23332.*

Canning, P. and Z. Wang (2005): "A Flexible Mathematical Programming Model To Estimate Interregional Input-Output Accounts," *Journal of Regional Science*, 45, 539–563.

Carvalho, V. and X. Gabaix (2013): "The Great Diversification and its Undoing," *The American Economic Review*, 103, 1697–1727.

Carvalho, V. M. (2014): "From Micro to Macro via Production Networks," *Journal of Economic Perspectives*, 28, 23–48.

Carvalho, V. M., M. Nirei, Y. U. Saito, and A. Tahbaz-Salehi (2016): "Supply Chain Disruptions: Evidence from the Great East Japan Earthquake," .

Conconi, P., M. García-Santana, L. Puccio, and R. Venturini (2018): "From Final Goods to Inputs : the Protectionist Effect of Rules of Origin," *American Economic Review*, Forthcomin.

Costinot, A. and A. Rodríguez-Clare (2014): "Trade Theory with Numbers: Quantifying the Consequences of Globalization," in *Handbook of International Economics*, Elsevier B.V., vol. 4, 197–261.

Costinot, A., J. Vogel, and S. Wang (2012): "An Elementary Theory of Global Supply Chains," *The Review of Economic Studies*, 80, 109–144.

Cuñat, A. and R. Zymek (2017): "International Value-Added Linkages in Development Accounting," *Working paper.*

Dantzig, G. B. (1963): *Linear Programming and Extensions*, Princeton University Press.

de Gortari, A. (2017): "Disentangling Global Value Chains," .

De La Cruz, J., R. B. Koopman, Z. Wang, and S.-J. Wei (2011): "Estimating Foreign Value-Added in Mexico's Manufacturing Exports," *U.S. International Trade Commission Working Paper.*

Dean, J. M., K. C. Fung, and Z. Wang (2011): "Measuring Vertical Specialization: The Case of China," *Review of International Economics*, 19, 609–625.

di Giovanni, J. and A. A. Levchenko (2010): "Putting the Parts Together: Trade, Vertical Linkages, and Business Cycle Comovement," *American Economic Journal: Macroeconomics*, 2, 95–124.

——— (2013): "Firm entry, trade, and welfare in Zipf's world," *Journal of International Economics*, 89, 283–296.

di Giovanni, J., A. A. Levchenko, and I. Mejean (2014): "Firms, Destinations, and Aggregate Fluctuations," *Econometrica*, 82, 1303–1340.

——— (2017): "The Micro Origins of International Business Cycle Comovement," *NBER Working Paper 21885.*

Domar, E. D. (1961): "On the Measurement of Technological Change," *The Economic Journal*, 71, 709–729.

Dorfman, R., P. A. Samuelson, and R. M. Solow (1958): *Linear Programming and Economic Analysis*, McGraw-Hill.

Dornbusch, R., S. Fischer, and P. A. Samuelson (1977): "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *The American Economic Review*, 67, 823–839.

Duval, R., N. Li, R. Saraf, and D. Seneviratne (2016): "Value-added trade and business cycle synchronization," *Journal of International Economics*, 99, 251–262.

Eaton, J. and S. Kortum (2002): "Technology, Geography, and Trade," *Econometrica*, 70, 1741–1779.

Fally, T. (2012): "Production Staging: Measurement and Facts," .

Fally, T. and R. Hillberry (2016): "A Coasian Model of International Production Chains," *NBER Working Paper 21520*, 1–53.

Fieler, A. C., M. Eslava, and D. Y. Xu (2017): "Trade, Quality Upgrading, and Input Linkages: Theory and Evidence from Colombia," *American Economic Review*, Forthcomin.

Golan, A., G. Judge, and S. Robinson (1994): "Recovering Information from Incomplete or Partial Multi-sectoral Economic Data," *The Review of Economics and Statistics*, 76, 541–549.

Gopinath, G. and B. Neiman (2014): "Trade Adjustment and Productivity in Large Crises," *American Economic Review*, 104, 793–831.

Hanson, G. H., R. J. Mataloni, and M. J. Slaughter (2005): "Vertical Production Networks in Multinational Firms," *The Review of Economics and Statistics*, 87, 664–678.

Hatanaka, M. (1952): "Note on Consolidation Within a Leontief System," *Econometrica*, 20, 301–303.

Hawkins, D. and H. A. Simon (1949): "Note: Some Conditions of Macroeconomic Stability," *Econometrica*, 17, 245–248.

Head, K. and T. Mayer (2014): "Gravity Equations: Workhorse, Toolkit, and Cookbook," in *Handbook of International Economics*, Elsevier B.V., vol. 4, 131–195.

Hulten, C. (1978): "Growth Accounting with Intermediate Inputs," *The Review of Economic Studies*, 45, 511–518.

Hummels, D., J. Ishii, and K.-M. Yi (2001): "The nature and growth of vertical specialisation in world trade," *Journal of International Economics*, 54, 75–96.

Johnson, R. C. (2014a): "Five Facts about Value-Added Exports and Implications for Macroeconomics and Trade Research," *Journal of Economic Perspectives*, 28, 119–142.

——— (2014b): "Trade in Intermediates and Business Cycle Comovement," *American Economics Journal: Macroeconomics*, 6, 39–83.

——— (2017): "Measuring Global Value Chains," *NBER Working Paper 24027*.

Johnson, R. C. and A. Moxnes (2016): "Technology, Trade Costs, and the Pattern of Trade with Multistage Production," .

Johnson, R. C. and G. Noguera (2012): "Accounting for Intermediates: Production Sharing and Trade in Value Added," *Journal of International Economics*, 86, 224–236.

Jones, C. I. (2011): "Intermediate Goods and Weak Links in the Theory of Economic Development," *American Economic Journal: Macroeconomics*, 3, 1–28.

——— (2013): "Misallocation, Input-Output Economics, and Economic Growth," in *Advances in Economics and Econometrics*, ed. by D. Acemoglu, M. Arellano, and E. Dekel, Cambridge University Press, tenth worl ed.

Kantorovich, L. V. (1939): *Mathematicheskie Metody Organizatsii i Planirovania Proizvodstva*, Leningrad State University Publishers.

Kee, H. L. and H. Tang (2016): "Domestic Value Added in Exports: Theory and Firm Evidence from China," *American Economic Review*, 106, 1402–1436.

Kikkawa, A. K., G. Magerman, and E. Dhyne (2017): "Imperfect Competition and the Transmission of Shocks: The Network Matters," .

Koopman, R., Z. Wang, and S.-J. Wei (2012): "Estimating domestic content in exports when processing trade is pervasive," *Journal of Development Economics*, 99, 178–189.

——— (2014): "Tracing Value-Added and Double Counting in Gross Exports," *American Economic Review*, 104, 459–494.

Koopman, R., Z. Wang, S.-J. Wei, and K. Zhu (2018): "A Comparison of Two Approaches to Gross Trade Decomposition," .

Koopmans, T. C. and M. Beckmann (1957): "Assignment Problems and the Location of Economic Activities," *Econometrica*, 25, 53–76.

Krugman, P. and A. J. Venables (1995): "Globalization and the Inequality of Nations," *The Quarterly Journal of Economics*, 110, 857–880.

Leontief, W. (1941): *The Structure of American Economy 1919-1929: An Empirical Application of Equilibrium Analysis*, Oxford University Press.

Linden, G., K. Kraemer, and J. Dedrick (2011): "Innovation and Job Creation in a Global Economy: The Case of Apple's iPod," *Journal of International Commerce and Economics*, 3, 223–239.

Long, J. B. and C. I. Plosser (1983): "Real Business Cycles," *Journal of Political Economy*, 91, 39–69.

Los, B., M. P. Timmer, and G. J. de Vries (2016): "Tracing Value-Added and Double Counting in Gross Exports: Comment," *The American Economic Review*, 106, 1958–1966.

Ludema, R. D., A. M. Mayda, M. Yu, and Z. Yu (2018): "The political economy of protection in GVCs: Evidence from Chinese micro data," .

Manova, K. and Z. Yu (2016): "How firms export: Processing vs. ordinary trade with financial frictions," *Journal of International Economics*, 100, 120–137.

Manova, K. and Z. Zhang (2012): "Export Prices and Destinations," *The Quarterly Journal of Economics*, 127, 379–436.

McManus, M. (1956): "On Hatanaka's Note on Consolidation," *Econometrica*, 24, 482–487.

Nunn, N. (2007): "Relationship-Specificity, Incomplete Contracts, and the Pattern of Trade," *The Quarterly Journal of Economics*, 122, 569–600.

Oberfield, E. (2018): "A Theory of Input-Output Architecture," *Econometrica*, 86, 559–589.

Ossa, R. (2015): "Why trade matters after all," *Journal of International Economics*, 97, 266–277.

Popper, K. R. (1959): *The Logic of Scientific Discovery*, London: Hutchinson.

Rauch, J. E. (1999): "Networks versus markets in international trade," *Journal of International Economics*, 48, 7–35.

Samuelson, P. A. (1951): "Abstract of a Theorem Concerning Substituteability in Open Leontief Models," in *Activity Analysis of Production and Allocation*, ed. by T. C. Koopmans, John Wiley & Sons, Inc., 142–146.

——— (1952): "Spatial Price Equilibrium and Linear Programming," *The American Economic Review*, 42, 283–303.

Schott, P. K. (2004): "Across-Product versus Within-Product Specialization in International Trade," *The Quarterly Journal of Economics*, 119, 647–678.

Solow, R. M. (1952): "On the Structure of Linear Models," *Econometrica*, 20, 29–46.

Timmer, M. P., E. Dietzenbacher, B. Los, R. Stehrer, and G. J. de Vries (2015): "An Illustrated User Guide to the World Input-Output Database: The Case of Global Automotive Production," *Review of International Economics*, 23, 575–605.

Tintelnot, F., A. K. Kikkawa, M. Mogstad, and E. Dhyne (2017): "Trade and Domestic Production Networks," .

Trefler, D. and S. C. Zhu (2010): "The structure of factor content predictions," *Journal of International Economics*, 82, 195–207.

Wang, Z., M. Gehlhar, and S. Yao (2010): "A globally consistent framework for reliability-based trade statistics reconciliation in the presence of an entrepôt," *China Economic Review*, 21, 161–189.

Wang, Z., W. Shang-Jin, and K. Zhu (2013): "Quantifying International Production Sharing at the Bilateral and Sector Levels," *NBER Working Paper 19677*.

Yi, K.-M. (2003): "Can Vertical Specialization Explain the Growth of World Trade?" *Journal of Political Economy*, 111, 52–102.

——— (2010): "Can Multistage Production Explain the Home Bias in Trade?" *The American Economic Review*, 100, 364–393.

A Data and Additional Results

A.1 Evidence for Specialized Inputs from Domestic Input-Output Tables

Figure 9 summarizes the industry aggregation bias across all U.S. manufacturing as proxied by the coefficient of variation—standard deviation relative to mean—of input shares from each source within each 3-digit code.[43] In the absence of aggregation bias, there is no heterogeneity in input shares at the 6-digit level and the coefficient of variation is zero. Alternatively, when the aggregation is done across industries with substantial heterogeneity the coefficient of variation is large. Each column in figure 9 corresponds to a given 3-digit manufacturing industry, with each circle corresponding to the coefficient of variation of input shares from some 6-digit input supplier across the 6-digit subindustries of the 3-digit industry; the size of each circle is proportional to the importance of each input supplier. Figure 9 reveals one key takeaway: There is substantial variation in input shares within each 3-digit industry. For example, the five biggest circles for computers and electronics are those from the sources in figure 3. The largest circle corresponds to other electronic components (the most important supplier) and, as figure 3's right panel shows, since there is relatively little variation in input shares the coefficient of variation is 0.8. In contrast, the high variation in computer storage devices visible in figure 3 yields a coefficient of variation of 2.7.

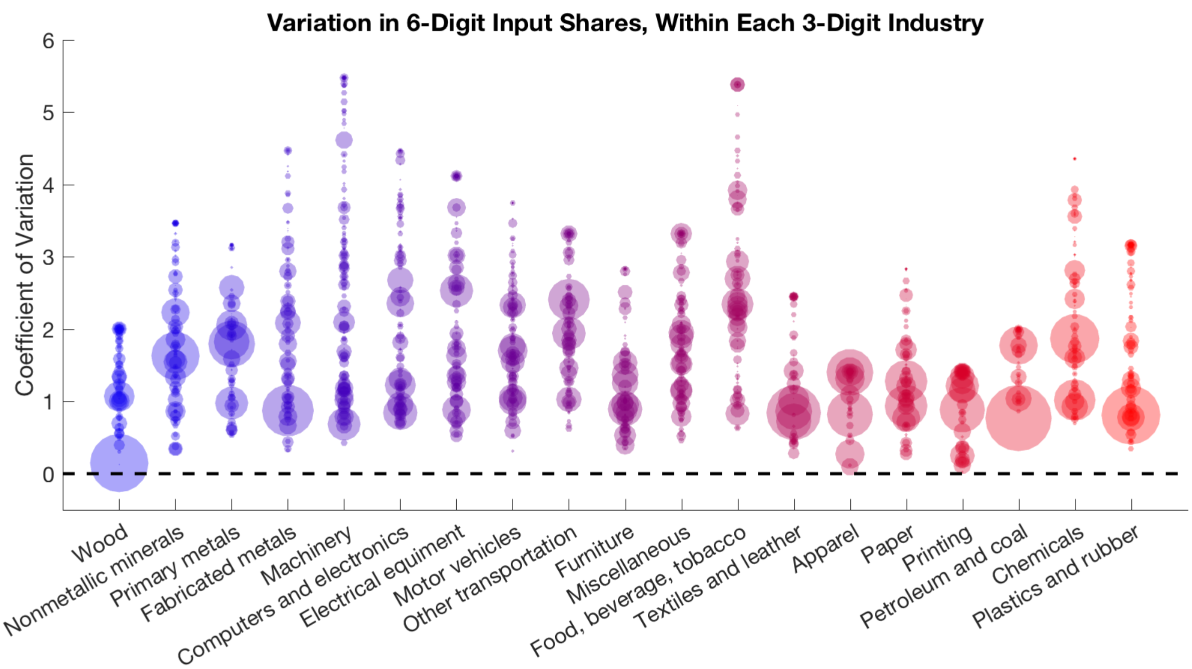


Figure 9: Variation in Domestic Industry Input Shares in U.S. Manufacturing Sales Across Domestic Industries: Each circle corresponds to the coefficient of variation — standard deviation relative to mean — of the input shares from a specific 6-digit input supplier across all 6-digit subindustries within each 3-digit industry on the x-axis. Circle size is proportional to the share of aggregate input purchases by the 3-digit industry from each source. In contrast to this chart, assuming the roundabout solution at the 3-digit industry-level implies zero variation across all 6-digit subindustries. Data is from 2007 U.S. input-output tables from the Bureau of Economic Analysis.

[43] Specifically, for each 3-digit industry $k^{3dig} \in \mathcal{K}^{3dig}$ I compute the coefficient of variation of the input shares $\alpha(t|k)$ from a given source $t \in \mathcal{K}^{6dig}$ across all the 6-digit subindustries k bundled in k^{3dig}. For example, for the 3-digit industry computers and electronics, figure 9 plots one circle for the coefficient of variation of α (printed circuit assembly $|k$) across all 6-digit subindustries indexed by k, and another circle for α (computer storage devices $|k$). Analogously, across all 6-digit suppliers $t \in \mathcal{K}^{6dig}$.

A.2 WIOD Aggregation

Throughout the paper I aggregate the WIOD's 56 industries slightly in order to eliminate some very small industries and reduce the size of the numerical optimization problems. The following table presents the aggregation across industries.

Industries	Raw WIOD		Aggregated WIOD	
	#	% of GDP	#	% of GDP
Crop, animal production, hunting, related service activities	1	3.1	1	7.2
Forestry and logging	2	0.2	1	7.2
Fishing and aquaculture	3	0.3	1	7.2
Mining and quarrying	4	3.7	1	7.2
Manufacture of food products, beverages and tobacco	5	4.3	2	4.3
Manufacture of textiles, wearing apparel and leather	6	1.7	3	1.7
Manufacture of wood and of products of wood and cork	7	0.6	4	2.3
Manufacture of paper and paper products	8	0.6	4	2.3
Printing and reproduction of recorded media	9	0.3	4	2.3
Manufacture of coke and refined petroleum products	10	2.4	5	2.4
Manufacture of chemicals and chemical products	11	2.6	6	2.6
Manufacture of basic pharmaceutical products	12	0.8	7	0.8
Manufacture of rubber and plastic products	13	1.1	8	1.1
Manufacture of other non-metallic mineral products	14	1.2	9	1.2
Manufacture of basic metals	15	2.8	10	2.8
Manufacture of fabricated metal products	16	1.5	11	1.5
Manufacture of computer, electronic and optical products	17	2.5	12	2.5
Manufacture of electrical equipment	18	1.5	13	1.5
Manufacture of machinery and equipment n.e.c.	19	2.2	14	2.4
Manufacture of motor vehicles, trailers and semi-trailers	20	2.8	15	2.8
Manufacture of other transport equipment	21	0.9	16	0.9
Manufacture of furniture; other manufacturing	22	0.8	4	2.3
Repair and installation of machinery and equipment	23	0.2	14	2.4
Electricity, gas, steam and air conditioning supply	24	3.3	17	3.9
Water collection, treatment and supply	25	0.2	17	3.9
Sewerage; waste collection, treatment and disposal	26	0.3	17	3.9
Construction	27	7.5	18	7.5
Wholesale and retail trade and repair of motor vehicles	28	0.9	19	8.9
Wholesale trade, except of motor vehicles and motorcycles	29	4.9	19	8.9
Retail trade, except of motor vehicles and motorcycles	30	3.1	19	8.9
Land transport and transport via pipelines	31	2.6	20	4.5
Water transport	32	0.4	20	4.5
Air transport	33	0.5	20	4.5
Warehousing and support activities for transportation	34	1.0	20	4.5
Postal and courier activities	35	0.2	21	7.1

Industries	Raw WIOD		Aggregated WIOD	
	#	% of GDP	#	% of GDP
Accommodation and food service activities	36	2.4	21	7.1
Publishing activities	37	0.4	22	8.1
Motion picture, video, television, sound recording, music	38	0.4	22	8.1
Telecommunications	39	1.5	22	8.1
Computer programming, consultancy, information service	40	1.3	22	8.1
Financial service, except insurance and pension funding	41	2.9	23	4.7
Insurance, reinsurance and pension funding	42	1.3	23	4.7
Activities auxiliary to financial services and insurance	43	0.5	23	4.7
Real estate activities	44	5.4	24	5.4
Legal, accounting; head offices; management consultancy	45	2.2	22	8.1
Architectural and engineering; technical testing and analysis	46	0.7	22	8.1
Scientific research and development	47	0.5	22	8.1
Advertising and market research	48	0.3	22	8.1
Other professional, scientific and technical; veterinary	49	0.7	22	8.1
Administrative and support service	50	2.3	21	7.1
Public administration, defense, compulsory social security	51	5.4	25	11.8
Education	52	2.3	25	11.8
Human health and social work activities	53	4.0	25	11.8
Other service activities	54	2.1	21	7.1
Activities of households as employers	55	0.1	21	7.1
Activities of extraterritorial organizations and bodies	56	0.0	21	7.1

Table 2: WIOD Industrial Classification: The shares refer to percent of world GDP.

A.3 Autarky Gains from Trade

Country	% of World GDP	Aggregate Domestic Share	Single-Industry ACR	Single-Industry Common Bounds		Single-Industry Dest.-Spec. Bounds		Multi-Industry ACR	Multi-Industry Common Bounds	
AUS	1.8	87.9	4.5	2.3	4.8	1.9	5.2	7.0	5.9	13.1
AUT	0.5	68.6	12.0	5.5	16.0	3.9	21.5	19.0	11.7	140.1
BEL	0.7	57.6	19.6	8.0	34.6	4.7	82.5	30.3	17.2	227.6
BGR	0.1	67.3	16.4	7.5	22.3	4.8	36.4	26.5	19.7	223.0
BRA	3.0	88.2	3.0	1.8	3.1	0.9	3.2	3.6	2.7	4.9
CAN	2.3	78.3	7.3	3.7	8.4	2.6	9.6	12.8	8.9	68.8
CHE	0.9	77.0	9.3	3.8	11.7	3.8	15.3	13.6	9.2	61.1
CHN	13.7	93.6	3.8	1.0	4.0	1.0	4.6	3.9	2.3	4.3
CYP	0.0	68.5	11.3	7.0	13.8	5.1	15.6	19.9	18.2	51.6
CZE	0.3	64.4	19.3	6.1	31.0	4.8	108.8	31.4	21.1	261.2
DEU	4.9	76.0	8.5	3.2	10.6	3.1	13.3	11.3	7.4	70.5
DNK	0.4	65.1	11.6	5.0	16.4	2.9	22.3	20.2	12.9	175.7
ESP	1.7	80.3	6.6	3.3	7.6	2.2	8.6	10.5	7.3	42.8
EST	0.0	60.3	19.6	8.6	31.6	6.1	66.2	55.9	37.9	287.9
FIN	0.3	74.1	9.3	4.3	11.4	2.5	14.1	12.3	8.0	70.5
FRA	3.5	79.0	6.6	3.5	7.6	2.2	8.4	9.1	6.2	44.7
GBR	3.7	82.1	6.1	3.3	6.8	2.5	7.4	9.5	6.9	33.7
GRC	0.3	75.1	7.5	5.1	8.4	3.2	8.9	10.5	8.4	39.7
HRV	0.1	69.7	10.8	5.6	13.7	3.9	16.8	24.9	20.5	121.6
HUN	0.2	48.6	25.4	10.1	73.9	4.8	817.3	39.9	22.7	474.5
IDN	1.2	81.8	5.1	2.6	5.8	1.2	6.4	6.9	5.0	20.5
IND	2.8	83.9	3.9	2.1	4.2	0.7	4.6	4.7	3.3	20.2
IRL	0.3	39.7	30.2	10.9	∞	5.7	∞	38.4	23.7	∞
ITA	2.7	83.4	5.8	2.5	6.6	1.9	7.5	7.2	4.8	28.2
JPN	6.0	85.3	4.3	2.4	4.6	1.2	4.9	6.2	5.2	10.9
KOR	1.8	78.2	9.8	2.5	12.2	2.0	18.4	12.7	8.9	39.6
LTU	0.1	55.8	16.9	8.7	28.8	6.6	39.4	26.3	19.8	176.2
LUX	0.1	41.1	66.7	11.4	∞	7.5	∞	99.5	65.4	∞
LVA	0.0	74.6	12.6	5.9	15.5	4.9	20.9	32.3	26.0	140.3
MEX	1.7	71.6	6.8	4.0	8.2	1.8	9.0	11.2	7.6	41.8
MLT	0.0	41.2	46.7	16.7	∞	5.2	∞	59.9	42.9	∞
NLD	1.1	63.2	14.7	5.4	23.5	4.6	41.3	28.9	18.0	217.3
NOR	0.6	78.0	6.2	2.4	7.5	2.4	8.6	9.3	6.9	28.3
POL	0.7	74.6	11.0	4.5	13.7	3.7	18.4	17.8	11.1	111.2
PRT	0.3	74.3	9.0	4.9	10.6	3.2	12.3	14.3	9.9	104.0
ROU	0.3	75.4	9.7	4.5	11.8	3.0	14.8	14.0	11.6	51.8
RUS	2.3	90.6	5.0	3.0	5.4	3.0	5.8	6.6	5.8	8.7

Country	% of World GDP	Aggregate Domestic Share	Single-Industry					Multi-Industry		
			ACR	Common Bounds		Dest.-Spec. Bounds		ACR	Common Bounds	
SVK	0.1	59.5	20.8	7.7	36.6	5.4	123.7	40.9	28.4	318.5
SVN	0.1	65.1	15.3	6.9	21.9	5.8	33.1	24.4	16.3	185.1
SWE	0.7	74.5	9.2	4.1	11.4	3.5	13.9	12.4	8.0	60.8
TUR	1.0	78.7	7.0	3.2	8.2	2.1	9.5	11.4	7.8	77.1
TWN	0.7	66.5	14.4	4.3	21.9	3.0	44.5	18.4	12.2	129.4
USA	23.1	89.7	2.9	1.9	3.0	1.2	3.1	3.5	2.6	4.0
ROW	14.2	79.5	10.6	5.8	12.1	3.8	14.6	12.5	8.6	24.5
Mean	2.3	72.0	12.8	5.2	14.9	3.4	42.5	20.3	14.0	102.8
Weighted	10.6	83.8	6.3	3.0	7.3	2.1	10.6	8.4	5.8	30.0

Table 3: Welfare Gains from Trade Relative to Autarky: The aggregate domestic share refers to the aggregate share of inputs purchased domestically and is a good proxy for trade openness. Common bounds refers to common value-added shares across destinations in the single-industry case and common value-added and industry-level expenditure shares across destinations in the multi-industry case. The weighted means use world GDP shares as weights. Data is from 2014 WIOD.

B The Import Demand System is Not CES

For the sake of clarity, I restrict attention to a single industry world (the multi-industry extension is immediate). To begin, define the dollar value of inputs from source j'' used by country j' to produce exports for market j as

$$\mathcal{X}\left(j''\,|j',j\right) = a\left(j''\,|j',j\right)\left(X\left(j',j\right) + F\left(j',j\right)\right),$$

and note that $X\left(j'',j'\right) = \sum_{j\in\mathcal{J}} \mathcal{X}\left(j''\,|j',j\right)$. Take $j'' \neq j'$ and $i'' \neq j'$. Start with the identity $X\left(j'',j'\right) = \sum_{j\in\mathcal{J}} \mathcal{X}\left(j''\,|j',j\right)$ and differentiate with respect to trade costs with a third country to obtain

$$\frac{\partial \ln X\left(j'',j'\right)}{\partial \ln \tau\left(i'',j'\right)} = \sum_{j\in\mathcal{J}} \frac{\mathcal{X}\left(j''\,|j',j\right)}{X\left(j'',j'\right)} \frac{\partial \ln \mathcal{X}\left(j''\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)}.$$

From the definition of $\mathcal{X}\left(j''\,|j',j\right)$, differentiate and obtain

$$\begin{aligned}\frac{\partial \ln \mathcal{X}\left(j''\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)} &= \frac{\partial \ln a\left(j''\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)} + \frac{\partial \ln\left(X\left(j',j\right) + F\left(j',j\right)\right)}{\partial \ln \tau\left(i'',j'\right)},\\ &= \frac{\partial \ln \mathcal{X}\left(j'\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)} + \frac{\partial \ln a\left(j''\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)} - \frac{\partial \ln a\left(j'\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)}.\end{aligned}$$

From the definition of input expenditures in (14), differentiate and obtain

$$\frac{\partial \ln a\left(j''\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)} = (1-\sigma)\left(1_{[j''=i'']} - a\left(i''\,|j',j\right)\right).$$

Substituting these two equations into the ratio of bilateral imports yields

$$\begin{aligned}&\frac{\partial \ln X\left(j'',j'\right)/X\left(j',j'\right)}{\partial \ln \tau\left(i'',j'\right)}\\ &\quad= \sum_{j\in\mathcal{J}}\left(\frac{\mathcal{X}\left(j''\,|j',j\right)}{X\left(j'',j'\right)} \frac{\partial \ln \mathcal{X}\left(j''\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)} - \frac{\mathcal{X}\left(j'\,|j',j\right)}{X\left(j',j'\right)} \frac{\partial \ln \mathcal{X}\left(j'\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)}\right),\\ &\quad= \sum_{j\in\mathcal{J}}\left(\frac{\mathcal{X}\left(j''\,|j',j\right)}{X\left(j'',j'\right)}\left(\frac{\partial \ln \mathcal{X}\left(j'\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)} + (1-\sigma)\,1_{[j''=i'']}\right) - \frac{\mathcal{X}\left(j'\,|j',j\right)}{X\left(j',j'\right)} \frac{\partial \ln \mathcal{X}\left(j'\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)}\right).\end{aligned}$$

Hence, the partial elasticity of imports in j' from source $j'' \neq j'$ relative to domestic purchases with respect to changes in trade costs with a third country $i'' \neq j'$ equals

$$\frac{\partial \ln X\left(j'',j'\right)/X\left(j',j'\right)}{\partial \ln \tau\left(i'',j'\right)} = (1-\sigma)\,1_{[j''=i'']} + \sum_{j\in\mathcal{J}}\left(\frac{\mathcal{X}\left(j''\,|j',j\right)}{X\left(j'',j'\right)} - \frac{\mathcal{X}\left(j'\,|j',j\right)}{X\left(j',j'\right)}\right)\frac{\partial \ln \mathcal{X}\left(j'\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)}. \tag{40}$$

The first term captures the direct effect on relative imports present when $j'' = i''$; this is the only effect in roundabout models. More generally, however, GVC linkages play a role. The partial elasticity $\partial \ln \mathcal{X}\left(j'\,|j',j\right)/\partial \ln \tau\left(i'',j'\right)$ captures the change in domestic input purchases due to both a substitution from domestic inputs towards more imports from i'' and a supply chain effect derived from the change in downstream production as proxied by the change in exports to each j. That is

$$\frac{\partial \ln \mathcal{X}\left(j'\,|j',j\right)}{\partial \ln \tau\left(i'',j'\right)} = -(1-\sigma)\,a\left(i''\,|j',j\right) + \frac{\partial \ln\left(X\left(j',j\right) + F\left(j',j\right)\right)}{\partial \ln \tau\left(i'',j'\right)}.$$

Further, the term in parenthesis in (40) amplifies/dampens the effect on relative imports depending on the differential importance of each export market j for inputs from j'' relative to j'.

In words, if Mexican exports to Germany use mostly Japanese inputs, then a reduction in Mexico-Germany shipping costs reduces both imports from Japan and domestic input sales following the substitution towards more German inputs. However, imports from Japan fall relatively more since exports to Germany use Japanese inputs intensively. On net, the supply chain effect exerts an opposing force and increases Japanese imports relatively more than domestic sales following the rise in exports to Germany.

The supply chain effect thus illustrates how changes in third-country trade barriers affect imports asymmetrically depending on the depth of supply chain integration. The knife-edge roundabout model, however, is the one case in which the effect is symmetric since all exports get built with the same inputs. In other words, the roundabout model satisfies the ACR condition "the import demand system is CES"

$$\text{the conditions in (18)} \quad \Rightarrow \frac{\partial \ln X\left(j'', j'\right)/X\left(j', j'\right)}{\partial \ln \tau\left(i'', j'\right)} = \left(1-\sigma\right) 1_{[j''=i'']}.$$

B.1 Gravity Regressions in Specialized Inputs Models

I illustrate the effects of running gravity regressions on data generated by specialized inputs models through simulations. Assume that there are $\mathcal{J} = 25$ countries and a single industry per country. In each simulation I sample parameters from random distributions. Specifically, I take draws $\beta\left(j', j\right) \sim \text{Uniform}(0,1)$, $\alpha\left(j'' | j', j\right) \sim \text{Lognormal}(0,1)$, $\varphi\left(j' | j\right) \sim \text{Lognormal}\left(0,1\right)$. I normalize the latter two shares so that $\sum_{j''\in\mathcal{J}} \alpha\left(j''|j', j\right) = 1$ and $\sum_{j'\in\mathcal{J}} \varphi\left(j'|j\right) = 1$. To obtain symmetric trade costs I take $\rho\left(j', j\right) \sim \text{Uniform}\left(0, 1/2\right)$ and define $\tau\left(j', j\right) = 1 + \rho\left(j', j\right)\rho\left(j, j'\right)$ for $j' \neq j$ and $\tau\left(j, j\right) = 1$. For the roundabout model I run similar simulations while imposing common input shares $\alpha\left(j''|j'\right)$.

The only missing parameter is the elasticity of substitution that I set to $\sigma = 6$ so that the roundabout trade elasticity is $1 - \sigma = -5$. In each simulation I construct the input-output table and run the following regression

$$\ln X\left(j', j\right) = \delta_0 + \delta_{exp}\left(j'\right) + \delta_{imp}\left(j\right) + \theta \ln \tau\left(j', j\right),$$

where δ_0 is the intercept, and $\delta_{exp}\left(j'\right)$ and $\delta_{imp}\left(j\right)$ are exporter and importer fixed effects. The coefficient θ equals the trade elasticity in the roundabout model if input shares are driven entirely by trade costs, i.e. if $\alpha\left(j'|j\right) = 1/\mathcal{J}$ for all pairs. More generally, the coefficient θ will differ from the trade elasticity since these parameters do vary.

Figure 10 presents the range of estimates for θ across 10,000 simulations of the roundabout and specialized inputs models. As discussed, θ differs from the trade elasticity because of the exogenous input share parameters but note that on average it exactly equals the trade elasticity. In contrast, in specialized inputs models structural gravity does not hold and thus the recovered value for the trade elasticity θ does not match its structural interpretation. That is, on average, $\theta \neq 1 - \sigma$. While the average estimate in the roundabout model hits precisely the structural trade elasticity value of $1-\sigma = -5$, the average estimate in specialized inputs is lower at -4.47 reflecting the fact that trade costs with third countries affect bilateral trade flows through supply chain linkages. This attenuation can be understood as introducing classical measurement error by consequence of model misspecification.

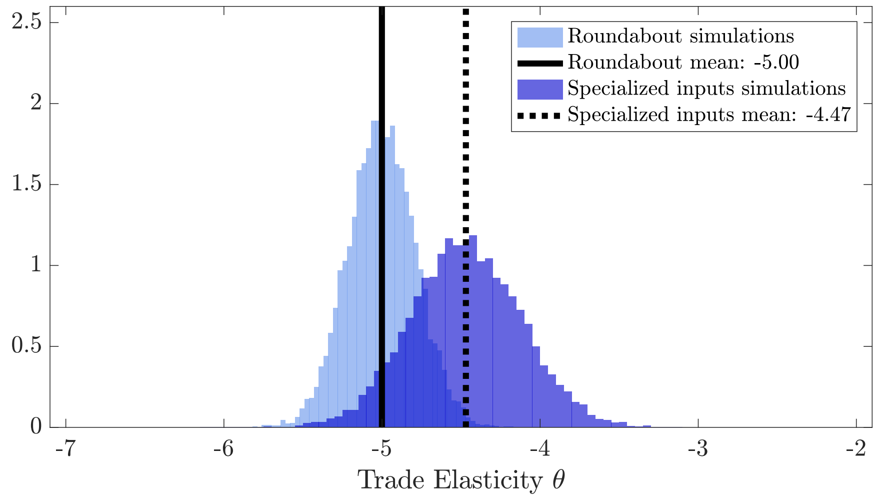


Figure 10: Gravity Regressions: The histograms correspond to trade elasticity estimates across 10,000 simulations of roundabout and specialized inputs models. All simulations use $1 - \sigma = -5$.

C Bounds on Welfare: Arbitrary Changes in Trade Costs

C.1 Solving for an Equilibrium, Given a GVC Network

Suppose, for now, that we have an observed GVC network described by the input shares $a(s''|s',j)$ and $\pi_F(s'|j)$. Remember that the input-output flows $X(s',s)$ and $F(s',j)$ are known, that the elasticities $\sigma(k)$ are given, and that we seek to solve for the change in all endogenous variables following an arbitrary change to trade barriers $\hat{\tau}(s',j)$. This can be done in six steps. First, the Cobb-Douglas expenditure shares can be recovered from the GVC network as follows

$$\beta(s',j) = 1 - \sum_{s'' \in \mathcal{J} \times \mathcal{K}} a(s''|s',j),$$
$$\gamma(k''|s',j) = \sum_{s'' \in \mathcal{J} \times k''} a(s''|s',j).$$
$$\zeta(k'|j) = \sum_{s' \in \mathcal{J} \times k'} \pi_F(s'|j).$$

Clearly, the Cobb-Douglas shares add up to one

$$\beta(s',j) + \sum_{k'' \in \mathcal{K}} \gamma(k''|s',j) = 1,$$
$$\sum_{k' \in \mathcal{K}} \zeta(k'|j) = 1.$$

Second, conditional on a change in wages $\hat{w}(j)$, the change in unit prices $\hat{p}(s',j)$ is found through the following fixed point

$$\hat{p}(s',j) = \prod_{s'' \in j \times \mathcal{K}} \left(\hat{w}(j)^{\beta(s'',j)} \times \prod_{s''' \in j \times \mathcal{K}} \hat{a}(s'''|s'',j)^{-\frac{\gamma(k'''|s'',j)}{1-\sigma(k''')}} \right)^{\delta(k''|s',j)},$$
$$\hat{a}(s''|s',j) = \frac{(\hat{p}(s'',j')\,\hat{\tau}(s'',j'))^{1-\sigma(k'')}}{\sum_{t'' \in \mathcal{J} \times k''} a(t''|s',j) \times (\hat{p}(t'',s')\,\hat{\tau}(t'',j'))^{1-\sigma(k'')}} \times \gamma(k''|s',j).$$

Third, this delivers the change in final good shares and final good flows

$$\hat{\pi}_F(s'|j) = \frac{(\hat{p}(s',j)\,\hat{\tau}(s',j))^{1-\sigma(k')}}{\sum_{t' \in \mathcal{J} \times k'} \pi_F(t'|j) \times (\hat{p}(t',j)\,\hat{\tau}(t',j))^{1-\sigma(k')}} \times \zeta(k'|j),$$
$$\hat{F}(s',j) = \frac{\hat{\pi}_F(s'|j)\,\pi_F(s'|j) \times \hat{w}(j)\,GDP(j)}{F(s',j)}.$$

Fourth, the change in bilateral intermediate input flows can be found with the fixed point

$$\hat{X}(s'',s') = \frac{1}{X(s'',s')} \sum_{j \in \mathcal{J}} \hat{a}(s''|s',j)\,a(s''|s',j) \left(\sum_{s \in j \times \mathcal{K}} \hat{X}(s',s)\,X(s',s) + \hat{F}(s',j)\,F(s',j) \right).$$

Fifth, the change in unit wages can be updated through

$$\hat{w}(j) = \frac{1}{\text{GDP}(j)} \sum_{s' \in j \times \mathcal{K}} \left(\sum_{s \in \mathcal{J} \times \mathcal{K}} \hat{X}(s', s) X(s', s) + \sum_{j \in \mathcal{J}} \hat{F}(s', j) F(s', j) - \sum_{s'' \in \mathcal{J} \times \mathcal{K}} \hat{X}(s'', s') X(s'', s') \right),$$

Sixth, and finally, repeat steps two to five using the new guess for the change in unit wages until a fixed point is found. This delivers a new equilibrium in which the endogenous variables for any country can be found as the product of the benchmark variable times the hat variable. The change in welfare can be found by substituting the change in the shares of the GVC network into the welfare formula (24).

C.2 Solving for the Exact GVC Bounds

The previous subsection showed how to compute the change in welfare following an arbitrary change to trade barriers for a given benchmark GVC network. Computing the bounds requires, in addition, searching across all GVC networks consistent with a given input-output dataset and finding the ones that minimize or maximize these gains. In other words, the bounds are found by solving

$$\begin{aligned}
&\text{min/max} && \text{the welfare formula in } (24), \\
&\text{subject to} && X(s'', s') = \sum_{j \in \mathcal{J}} a(s'' | s', j) \left(\sum_{s \in j \times \mathcal{K}} X(s', s) + F(s', j) \right), \forall s'', s', \\
& && \sum_{s'' \in \mathcal{J} \times k''} a(s'' | s', j) = \gamma(k'', | s'), \forall k'', s', j \\
& && a(s'' | s', j) \geqslant 0, \forall s'', s', j, \\
& && \text{the fixed point for unit prices and wages holds.}
\end{aligned}$$

The optimization problem solves jointly for a GVC network that fits the input-output data in the benchmark equilibrium and the counterfactual equilibrium following the exogenous change to trade barriers. The solution is given by the combination of benchmark and counterfactual equilibria that minimize or maximize the gains from trade for some country j. Solving this problem is very hard numerically because the objective function is highly nonlinear, the constraints are highly nonlinear, and the problem is very large because it depends on solving for the full GVC network across all country-industries in the world.

C.3 Solving for Approximate GVC Bounds

While finding the extremal GVC networks — i.e. the exact bounds — is hard numerically, approximate bounds can be constructing by noting that computing general counterfactuals for a given GVC network is straightforward. In other words, computing counterfactuals across a given set of GVC networks produces a range of values for any exercise that may, in principle, be informative about the true exact bounds. For example, this can be done with the previously constructed GVC networks underlying the autarky bounds but when shocking the model with a different set of changes in trade costs. Further, since the mapping to input-output data depends on the linear restrictions in equation (17), additional GVC networks can easily be constructed by taking convex combinations of the extremal autarky GVC networks.

The results in figure 11 are found in this way. First, I take the GVC networks underlying the autarky bounds and roundabout point estimates in figure 6 and then construct a new GVC network by taking convex combinations of the three benchmark networks within each $s' \in \mathcal{S}$. This delivers a full GVC network characterized by $a(s''|s', j)$ and $\pi_F(s'|j)$. I then assume that either trade costs from Mexico

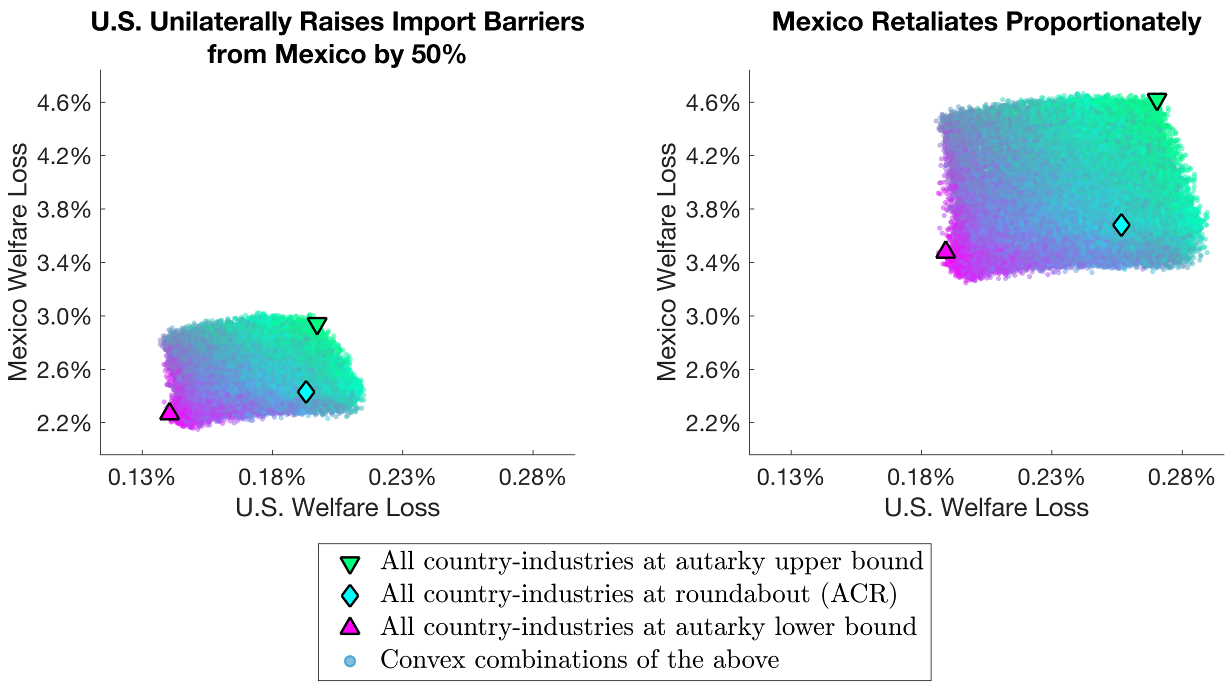


Figure 11: Approximate Bounds of a NAFTA Trade War: The left panel plots the U.S. and Mexico welfare loss following a 50% increase in trade barriers on U.S. imported Mexican goods across 10,000 GVC networks that replicate the 2014 WIOD. Each dot is a GVC network constructed as a random convex combination, within each country-industry, of the GVCs corresponding to the autarky lower bound, roundabout solution, and upper bound underlying figure 6. The right panel plots the welfare losses when Mexico retaliates and trade barriers on Mexican imported U.S. goods also increase by 50%. Note the true bounds on these counterfactuals are wider than these sets.

to the U.S. increase by 50% or that both trade costs from Mexico to the U.S. and from the U.S. to Mexico increase by 50% and compute the new equilibrium in changes using the above approach. I then compute the welfare loss for Mexico and the U.S using the welfare formula (24).

Figure 11 shows the results across 10,000 convex combinations of these three benchmark networks. The sets depicted in figure 11 show substantial, but not huge, deviations to the predictions of a roundabout model.[44] In this particular example, the roundabout model tends to underpredict the costs of NAFTA trade war for Mexico and overpredict for the U.S. relative to the overall set. Though, of course, the true GVC bounds might look quite different and, further, the true GVC network may be quite different from the average network. Indeed, the point estimates in columns IV and V of table 1 are pretty far from these approximate bounds. This indicates that while this exercise is instructive, the approximate bounds are in practice quite far away from the true bounds (at least for this specific exercise).

[44]The curious reader may wonder why these sets are not convex combinations of the three benchmark estimates. There are two reasons. First, the convex combinations are within industry pairs so, for example, there are some GVC networks in which the U.S. is close to the lower bound input shares in some industries but close to the upper bound in other industries. Second, the autarky gains are monotonic in the domestic input shares and so there is a one-on-one relation between convex combinations (at the country level) and the gains from trade. This is not true in general, though, since the counterfactual equilibrium depends nonlinearly (and possibly non-monotonically) on the observed equilibrium through the fixed point in unit prices and wages.

D GVCs and Measurement

D.1 Disciplining GVCs with New Sources of Information: Numerical Implementation

I write the optimization problem in (37), when minimizing the weighted sum of squared deviations, in terms of linear algebra. I proceed in five steps. First, since the optimization is done separately within each country-industry $s' \in \mathcal{S}$, fix s'. Second, use the input-output data to define the vectors $\mathbf{X}_1 = [X(s', s)]$, $\mathbf{X}_2 = [X(s'', s')]$, and $\mathbf{F}_1 = [F(s', j)]$ of size $1 \times \mathcal{JK}$, $\mathcal{JK} \times 1$, and $1 \times \mathcal{J}$. Third, define the endogenous variables as vectors

$$\boldsymbol{\alpha}_X = \begin{bmatrix} \alpha_X(1|s',1) \\ \vdots \\ \alpha_X(1|s',\mathcal{S}) \\ \alpha_X(2|s',1) \\ \vdots \\ \alpha_X(\mathcal{S}|s',\mathcal{S}) \end{bmatrix}, \quad \boldsymbol{\beta}_X = \begin{bmatrix} \beta_X(s',1) \\ \vdots \\ \beta_X(s',\mathcal{S}) \end{bmatrix}, \quad \boldsymbol{\alpha}_F = \begin{bmatrix} \alpha_F(1|s',1) \\ \vdots \\ \alpha_F(1|s',\mathcal{J}) \\ \alpha_F(2|s',1) \\ \vdots \\ \alpha_F(\mathcal{S}|s',\mathcal{J}) \end{bmatrix}, \quad \boldsymbol{\beta}_F = \begin{bmatrix} \beta_F(s',1) \\ \vdots \\ \beta_F(s',\mathcal{J}) \end{bmatrix},$$

of sizes $\mathcal{JKJK} \times 1$, $\mathcal{JK} \times 1$, $\mathcal{JKJ} \times 1$, and $\mathcal{J} \times 1$. Fourth, stack the targets and weights into analogous vectors and call them $\boldsymbol{\alpha}_X^0$, $\boldsymbol{\beta}_X^0$, $\boldsymbol{\alpha}_F^0$, $\boldsymbol{\beta}_F^0$, $\boldsymbol{\omega}_{\alpha_X}^0$, $\boldsymbol{\omega}_{\beta_X}^0$, $\boldsymbol{\omega}_{\alpha_F}^0$, and $\boldsymbol{\omega}_{\beta_F}^0$. The optimization problem in (37) can be written as a quadratic program as follows

$$\min \quad \begin{pmatrix} \boldsymbol{\alpha}_X - \boldsymbol{\alpha}_X^0 \\ \boldsymbol{\beta}_X - \boldsymbol{\beta}_X^0 \\ \boldsymbol{\alpha}_F - \boldsymbol{\alpha}_F^0 \\ \boldsymbol{\beta}_F - \boldsymbol{\beta}_F^0 \end{pmatrix}^T \operatorname{diag}\begin{Bmatrix} \boldsymbol{\omega}_{\alpha_X}^0 \\ \boldsymbol{\omega}_{\beta_X}^0 \\ \boldsymbol{\omega}_{\alpha_F}^0 \\ \boldsymbol{\omega}_{\beta_F}^0 \end{Bmatrix} \begin{pmatrix} \boldsymbol{\alpha}_X - \boldsymbol{\alpha}_X^0 \\ \boldsymbol{\beta}_X - \boldsymbol{\beta}_X^0 \\ \boldsymbol{\alpha}_F - \boldsymbol{\alpha}_F^0 \\ \boldsymbol{\beta}_F - \boldsymbol{\beta}_F^0 \end{pmatrix},$$

$$\text{subject to} \quad \begin{pmatrix} \mathbb{I}_{\mathcal{JK}\times\mathcal{JK}} \otimes \mathbf{X}_1 & \mathbf{0}_{\mathcal{JK}\times\mathcal{JK}} & \mathbb{I}_{\mathcal{JK}\times\mathcal{JK}} \otimes \mathbf{F}_1 & \mathbf{0}_{\mathcal{JK}\times\mathcal{J}} \\ \mathbf{1}_{1\times\mathcal{JK}} \otimes \mathbb{I}_{\mathcal{JK}\times\mathcal{JK}} & \mathbb{I}_{\mathcal{JK}\times\mathcal{JK}} & \mathbf{0}_{\mathcal{JK}\times\mathcal{JKJ}} & \mathbf{0}_{\mathcal{JK}\times\mathcal{J}} \\ \mathbf{0}_{\mathcal{J}\times\mathcal{JKJK}} & \mathbf{0}_{\mathcal{J}\times\mathcal{JK}} & \mathbf{1}_{1\times\mathcal{JK}} \otimes \mathbb{I}_{\mathcal{J}\times\mathcal{J}} & \mathbb{I}_{\mathcal{J}\times\mathcal{J}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha}_X \\ \boldsymbol{\beta}_X \\ \boldsymbol{\alpha}_F \\ \boldsymbol{\beta}_F \end{pmatrix} = \begin{pmatrix} \mathbf{X}_2 \\ \mathbf{1}_{\mathcal{JK}\times 1} \\ \mathbf{1}_{\mathcal{J}\times 1} \end{pmatrix},$$

$$\begin{pmatrix} \boldsymbol{\alpha}_X \\ \boldsymbol{\beta}_X \\ \boldsymbol{\alpha}_F \\ \boldsymbol{\beta}_F \end{pmatrix} \geqslant 0,$$

where $\operatorname{diag}\{\cdot\}$ is a diagonal matrix, $\otimes$ is the Kronecker product, $\mathbb{I}$ is the identity matrix, and $\mathbf{0}$ and $\mathbf{1}$ are matrices of zeros and ones. Often, numerical quadratic programming solvers define the objective function as $\frac{1}{2}x^T Q x + c^T x$, in which case the above objective function can be rewritten in these terms as

$$x = \begin{pmatrix} \boldsymbol{\alpha}_X \\ \boldsymbol{\beta}_X \\ \boldsymbol{\alpha}_F \\ \boldsymbol{\beta}_F \end{pmatrix}, \qquad Q = \operatorname{diag}\begin{Bmatrix} \boldsymbol{\omega}_{\alpha_X}^0 \\ \boldsymbol{\omega}_{\beta_X}^0 \\ \boldsymbol{\omega}_{\alpha_F}^0 \\ \boldsymbol{\omega}_{\beta_F}^0 \end{Bmatrix}, \qquad c = -\operatorname{diag}\begin{Bmatrix} \boldsymbol{\omega}_{\alpha_X}^0 \\ \boldsymbol{\omega}_{\beta_X}^0 \\ \boldsymbol{\omega}_{\alpha_F}^0 \\ \boldsymbol{\omega}_{\beta_F}^0 \end{Bmatrix} \begin{pmatrix} \boldsymbol{\alpha}_X^0 \\ \boldsymbol{\beta}_X^0 \\ \boldsymbol{\alpha}_F^0 \\ \boldsymbol{\beta}_F^0 \end{pmatrix}.$$

D.2 Disciplining the Bounds with New Sources of Information

Given the practical and numerical challenges described in section 5, I implement this exercise only illustratively and hope that future research might use these tools to conduct more serious analysis. First, I restrict attention to narrowing the single-industry autarky gains from trade bounds when incorporating new sources of information. Analogous procedures can be implemented for the bounds with multi-industry data, the bounds on general counterfactuals, and the bounds on measures of globalization. Second, since the main text already shows how to use Mexican customs data to construct alternative point estimates, here I take other forms of new sources of information from the literature in order to narrow the bounds. Analogous procedures can be implemented for narrowing the bounds with the Mexican customs data.

Take China, Germany, and the United States — the largest economies of the American, Asian, and European continents. Figure 12 shows how figure 5's ranges become smaller when imposing additional restrictions into the optimization problem in (27). For example, suppose that after careful analysis of firm-level data, it becomes clear that exports to a given market use a high share of imports from that same market. Column B shows that the upper bounds tend to fall, relative to the benchmark in **A**, when imposing a *home-bias* share of at least 20% of inputs from each j'' as a share of total inputs in the exports to j'' itself since now fewer intermediate input imports can be put into goods sold on the domestic market.[45] Alternatively, suppose firms offshore production to countries that produce particularly suitable inputs. Column C shows that imposing a share of at least 50% of domestic inputs in all exports increases the lower bound since now a certain amount of domestic inputs must be exported.

Measuring value-added shares more accurately also delivers sharper bounds. For example, De La Cruz et al. (2011) and Koopman et al. (2012) show that when processing trade is pervasive, exports often contain little domestic value-added. Column E presents the ranges when imposing twice as much domestic value-added in domestically sold goods than in exported goods, while column F further restricts the range by imposing a high share of inputs from the same market to which goods are exported to. The upper bound falls dramatically because both restrictions require that each dollar of exports contain a high share of imported inputs. Alternatively, Kee and Tang (2016) documented an upward trend in the share of domestic value in Chinese exports over the last decade as China began exploiting its comparative advantage in high domestic content industries. Column H shows that the lower bounds increase when imposing a high domestic value-added share and also a high domestic input share in exports since disrupting trade becomes more costly once domestic consumption becomes more tightly linked with international trade.

While incorporating new sources of information to obtain alternative point estimates as discussed in section 5 or to narrow the bounds as described here is complementary, I view the former approach as more practical. The reason is that the optimization problem (37) includes the new information through the objective function and is thus always well defined. Narrowing the bounds, however, can be problematic because it includes the new information through additional constraints. The latter can sometimes lead to there not existing any solutions to the optimization problem (or in other words, the bounds are not even consistent with a single point estimate). Further, since current computing power is too limited for computing exact bounds on counterfactuals following arbitrary changes to trade barriers and on measures of globalization this implies that narrowing the bounds cannot be done either.

[45] This could be driven by multinational firms offshoring a production stage within otherwise domestic supply chains (Hanson et al. 2005), or by compatibility in quality (Bastos et al. 2018), or rules-of-origin (Conconi et al. 2018).

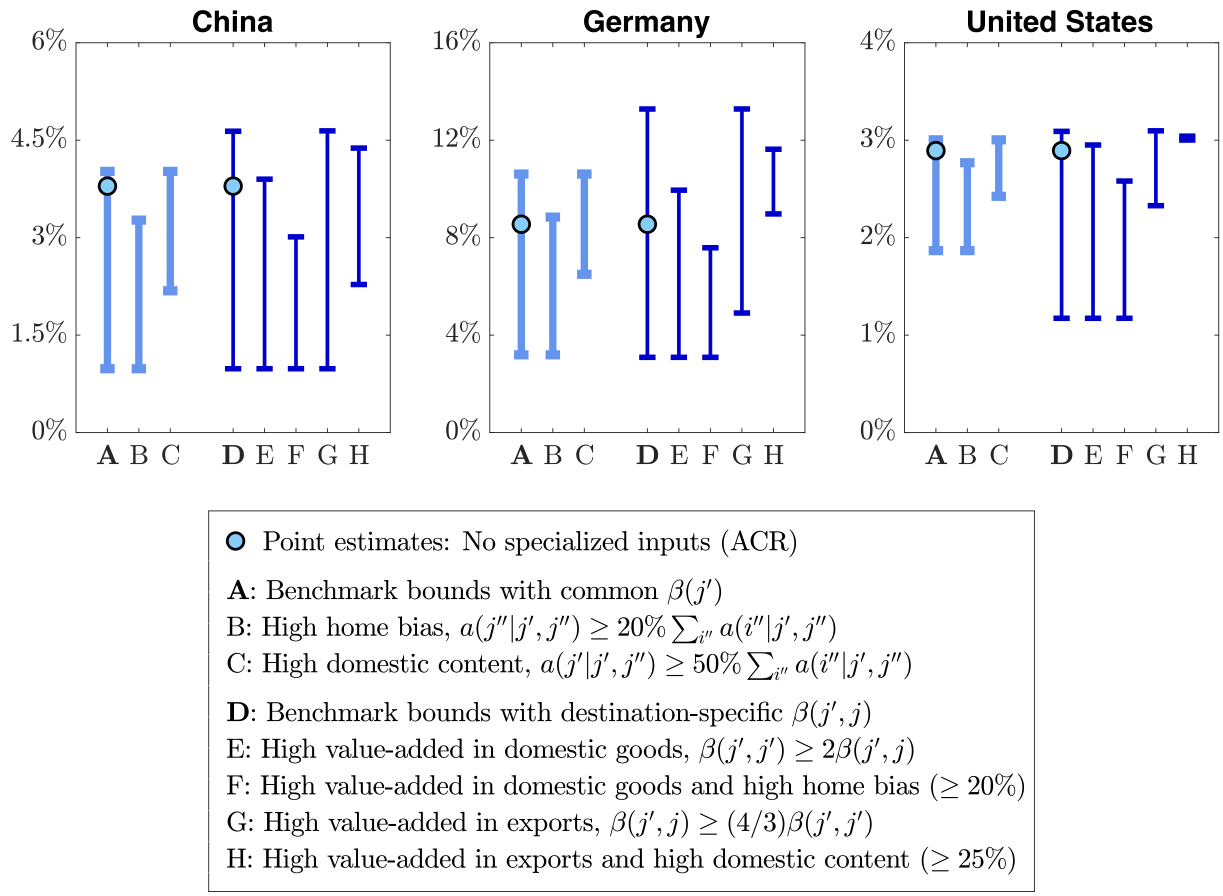


Figure 12: Narrowing the Bounds on the Welfare Gains from Trade Relative to Autarky: The benchmark bounds in **A** and **D** correspond to the ranges in figure 5. The bounds in B and C impose common value-added shares and include additional restrictions. The bounds in E, F, G, and H incorporate destination-specific value-added shares and include additional restrictions. In every exercise, I omit the restrictions on the domestic shares $a(j'|j',j')$ in order to not restrict the ranges directly. Further, in order to get feasible solutions, I only apply the restrictions to the top trading partners.