Trading Up and the Skill Premium

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Abstract

We study the role that increases in the quality of the goods consumed (“trading up”) might have played in the rise of the skill premium that occurred in the last four decades. Our empirical work shows that high-quality goods are more intensive in skilled labor than low-quality goods and that household spending on high-quality goods rises with income. We propose a model consistent with these facts and argue that it accounts for the observed rise in the skill premium with more plausible rates of skill-biased technical change than those required by the canonical model.

Keywords: Recessions, quality choice, business cycles.

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1 Introduction

It is well known that the skill premium increased in the last four decades despite a large expansion in the supply of skilled workers. These observations have been interpreted by Katz and Murphy (1992), Autor, Katz and Krueger (1998) and a large subsequent literature as evidence for the presence of skill-biased technical change (SBTC). One major drawback of this interpretation is that it requires large rates of SBTC to rationalize the observed rise in the skill premium. For example, the annual rate of SBTC required to account for the rise in the skill premium estimated by Acemoglu and Autor (2011) for the 1970-2008 period is 5.5 percent per year.

In this paper, we study the role that increases in the quality of the goods consumed ("trading up") can play in explaining the rise in the skill premium. In our empirical work, we document two facts. First, high-quality goods are more intensive in skilled labor than low-quality goods. Second, household spending on high-quality goods rises with income.

We propose a simple model consistent with these facts. In our model, both SBTC and Hicks-neutral technical change (HNTC) increase real income, thus expanding the demand for quality. Since quality is intensive in high-skill workers, the demand for these workers rises, leading to an endogenous rise in the skill premium and in income inequality.

Using Fernald’s (2014) estimates of the rate of HNTC, we compute the rate of SBTC consistent with the rate of change in the quality of goods consumed estimated by Bils and Klenow (2001).\textsuperscript{1} We find that the model accounts for the observed rise in the skill premium with an annual rate of SBTC of roughly one percent per year, which is arguably more plausible than that implied by the canonical model.

Acemoglu and Autor (2011) argue that the distinction between skilled and unskilled

\textsuperscript{1}These estimates are obtained by regressing the Bureau of Labor Statistics’ inflation measure on the growth rate of expenditures instrumented with the slope of the quality Engel curve.
workers may be less relevant to study forms of automation such as artificial intelligence which may replace routine tasks performed by high-skill workers. We study the relation between quality and the intensity of routine and non-routine work. We find that low-quality goods are intensive in routine work and that high-quality goods are intensive in non-routine, abstract work.

Our work is related to four strands of literature. The first strand, is the large body of work on SBTC surveyed by Acemoglu and Autor (2011). The second strand, is the literature on the role of capital deepening in explaining the evolution of the skill premium (e.g. Krusell, Ohanian, Ríos-Rull, and Violante (2000), Polgreen and Silos (2008), and Burstein and Vogel (2017)). We show in the Appendix that incorporating the trading-up phenomenon in these models reduces the resulting estimates of the elasticity of substitution between unskilled labor and capital. The third strand, is work on skill-biased structural change (e.g. Acemoglu and Guerrieri (2008), Buera and Kaboski (2012), Buera, Kaboski and Rogerson (2015), Boppart (2015), and Alon (2018)). This work emphasizes how rises in income shift demand towards sectors that are more intensive in skilled work. In contrast, we emphasize that, as income rises, the demand for quality increases, raising the demand for skilled labor within a given sector. One important difference between these two mechanisms is that the process of upgrading quality within a sector is presumably unbounded while sectoral reallocation is likely to be bounded. The fourth strand, is work on the importance of quality choice in growth models (e.g. Grossman and Helpman (1991a,b), Stokey (1991)), trade models (e.g. Verhoogen (2008) and Fieler, Eslava and Xu (2017)) and macro models (e.g. Jaimovich, Rebelo, and Wong (2019)).

This paper is organized as follows. Section 2 contains our empirical work. In Section 3 we consider a simple model and discuss its implications for the rate of SBTC required to explain the rise in the skill premium. Section 4 concludes.
2 Quality and Skill Intensity

In this section, we measure the intensity of skilled labor in establishments that are in the same sector but produce products of different quality. To accomplish this goal, we first construct measures of both the quality of goods produced and skill intensity by establishment. We then study the relation between quality and skill intensity and the relation between quality and income.

There are three approaches used in the literature to measure quality. The first approach, which we adopt in this paper, is to use relative prices as proxies for quality. The idea underlying this approach is that if consumers are willing to pay more for an item, they perceive it to be of higher quality.\(^2\) The second approach, is to infer quality from the materials and labor costs used in production (e.g. Veerhoogen (2008)). The third approach, is to structurally estimate quality using data on prices and quantities combined with functional form assumptions about household utility (e.g. Veerhoogen (2008) and Hottman, Redding and Weinstein (2016)). We focus on relative prices as measures of quality because it allows us to use a broader sample of goods and firms in the OES data set.\(^3\)

There is strong evidence that relative prices are positively correlated with the quality measures produced by the other two approaches. For example, Veerhoogen (2008) shows that higher quality items, which have higher costs of production, also have higher prices. Hottman, Redding and Weinstein (2016) and Khandelwal (2010) find that quality is strongly positively correlated with relative prices within product groups.

\(^2\)See Jaimovich, Rebelo and Wong (2019) for evidence that supports this assumption.

\(^3\)Structural estimation generally requires price shifters to instrument for price changes. A commonly used price shifter for an item sold in a particular county is the average price of the item in other counties. Using this shifter restricts the analysis to items sold in many counties which results in a substantial reduction in sample size.
2.1 Measuring Skill Intensity

Our measure of the intensity of skilled labor is based on two data sets. The first, is the Microdata of Occupational Employment Statistics (OES) collected by the Bureau of Labor Statistics (BLS). It covers 1.1 million establishments, representing 62 percent of total employment and spanning all sectors of the North American Industry Classification System at a 6-digit level. Unfortunately, the OES does not contain information about education attainment by worker. For this reason, we calculate the distribution of employees across twelve wage bins for each occupation and establishment from the OES and relate this wage distribution to the wages of skilled workers estimated using the U.S. Department of Commerce’s Current Population Survey (CPS). We use the information regarding wages, education and industry in the CPS as follows. For every industry in the CPS, we compute the average wage of college graduates. We classify a worker in the OES as skilled if her wage exceeds the average wage of college graduates for her industry. We then compute for each establishment in the OES data the fraction of employment and of the wage bill accounted for by skilled workers.

We proceed similarly to construct two other measures of the share of skilled workers. For our second measure, we classify workers as skilled if their wage exceeds the average wage of workers with “some college or more” for the worker’s industry. For our third measure, we classify workers as skilled if their wage exceeds the average wage for all workers in the respective industry.

Table 1 displays our results for different sectors. Consider for example the sector of manufacturing. Using the first measure of skill, we find that the fraction of manufacturing workers who are skilled is 13.9 percent and that these workers earn 43.1 percent of the wage bill. Using the second, broader classification of skill, we find that the fraction of manufacturing workers share of the wage bill earned by these workers are 20.9 and 49.4 percent, respectively. Using the third and broadest classification of skill, we find that the fraction of manufacturing workers share of the wage bill earned
by these workers are 29.8 and 59.6 percent, respectively.

**Table 1: Establishments’ Share of Skilled Workers**

<table>
<thead>
<tr>
<th>Sample</th>
<th>#Est.</th>
<th>Skilled 1</th>
<th>Skilled 2</th>
<th>Skilled 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#Est.</td>
<td>Emp</td>
<td>Wage</td>
<td>Emp</td>
</tr>
<tr>
<td>All Sectors</td>
<td>1,131,170</td>
<td>16.7</td>
<td>36.9</td>
<td>23.7</td>
</tr>
<tr>
<td>NAICS Sector:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Management</td>
<td>13,997</td>
<td>50.3</td>
<td>53.6</td>
<td>63.5</td>
</tr>
<tr>
<td>Educational</td>
<td>39,385</td>
<td>33.6</td>
<td>25.4</td>
<td>38.0</td>
</tr>
<tr>
<td>Information</td>
<td>33,176</td>
<td>29.3</td>
<td>45.4</td>
<td>34.8</td>
</tr>
<tr>
<td>Utilities</td>
<td>6,217</td>
<td>29.8</td>
<td>30.3</td>
<td>35.9</td>
</tr>
<tr>
<td>Professional</td>
<td>106,407</td>
<td>28.9</td>
<td>29.1</td>
<td>34.3</td>
</tr>
<tr>
<td>Transportation</td>
<td>43,934</td>
<td>28.3</td>
<td>25.7</td>
<td>40.5</td>
</tr>
<tr>
<td>Construction</td>
<td>82,188</td>
<td>23.8</td>
<td>44.2</td>
<td>29.4</td>
</tr>
<tr>
<td>Finance</td>
<td>56,599</td>
<td>23.6</td>
<td>53.8</td>
<td>30.1</td>
</tr>
<tr>
<td>Wholesale</td>
<td>86,176</td>
<td>19.8</td>
<td>31.3</td>
<td>26.8</td>
</tr>
<tr>
<td>Health Care</td>
<td>124,463</td>
<td>16.4</td>
<td>55.1</td>
<td>27.1</td>
</tr>
<tr>
<td>Other Services</td>
<td>73,062</td>
<td>15.4</td>
<td>10.4</td>
<td>24.4</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>107,826</td>
<td>13.9</td>
<td>43.1</td>
<td>20.9</td>
</tr>
<tr>
<td>Entertainment</td>
<td>26,549</td>
<td>12.0</td>
<td>38.9</td>
<td>20.0</td>
</tr>
<tr>
<td>Real Estate Rental</td>
<td>37,750</td>
<td>10.3</td>
<td>49.9</td>
<td>16.1</td>
</tr>
<tr>
<td>Retail</td>
<td>121,065</td>
<td>9.6</td>
<td>42.7</td>
<td>17.8</td>
</tr>
<tr>
<td>Administrative</td>
<td>77,873</td>
<td>8.8</td>
<td>74.6</td>
<td>17.2</td>
</tr>
<tr>
<td>Accommodation</td>
<td>50,700</td>
<td>3.2</td>
<td>31.7</td>
<td>10.4</td>
</tr>
</tbody>
</table>
2.2 Quality Measures

We use prices of goods as a proxy for their quality. Our price measures come from two sources. The first is data from Yelp!, a website where consumers post review information about different goods and services. The second, is Nielsen Homescan data.

2.2.1 Yelp!-based Quality Measures

For each store and location pair, Yelp! asks users to classify the price of the goods and services they purchased into one of four categories: $ (low), $$ (middle), $$$ (high), and $$$$ (very high). Because there are few observations in the very-high category, we merge the last two categories into a single high-price category.

We match Yelp! establishments to the OES establishments in three steps. First, we match the contact phone numbers of Yelp! establishments to the contact phone number of the OES establishments. Whenever this match is not possible, we match the Yelp! to the OES establishments based on name, industry (NAICS 3-digit), and zip code. When multiple OES establishments are matched to one (or multiple) Yelp! establishment(s), we average the skill measures of all the OES establishments that are matched to each Yelp! establishment, and assign that average skill measure to the Yelp! establishment. Third, because zip codes are not available for every establishment in the OES database, we conduct a matching procedure similar to that used in our second step based on name, industry (NAICS 3-digit) and county. With these three steps, we obtain the share of skilled labor for 9,908 Yelp! establishments. These data covers the retail, accommodation, entertainment, and information services sectors.

4A major challenge is that the phone number in the OES database can be either the phone number of the establishment or the phone number of the contact person for the establishment, such as the person’s mobile phone number. We obtain industry codes of Yelp establishments by matching the Yelp establishments to the ReferenceUSA database, which covers the near universe of establishments in the U.S. Establishments from the two data sets with the same industry, zip code, and similar names based on bigram are also matched. This fuzzy name matching increases the match rate of the two data sets.

5For instance, two Starbucks coffee shops may locate in the same zip code. In this case, our approach assumes that the two coffee shops share the same skill measure.
2.2.2 Nielsen-based Quality Measures

In order to extend our analysis to the manufacturing sector, we use the Nielsen Home-scan data. This data set contains prices paid and quantities of groceries purchased at a barcode (UPC) level for 113 thousand households over the period 2004-10. Nielsen organizes bar codes into 613 product modules according to where they would likely be stocked in a store.

To construct a measure of quality for each manufacturing firm, we proceed as follows. First, we link each item $k$ (defined at a UPC level) in Nielsen with the manufacturing firm $f$ that produced the UPC using information obtained from GS1 US. We focus on the 2006 data set to match the sample period of the OES data.

Second, we compute the sales-weighted average price across all transactions made during the month $t$, $p_{kft}$, for each item $k$ produced by firm $f$. Similarly, for each product module $m(k)$ that item $k$ belongs to, we calculate $p_{m(k)t}$, the sales-weighted average price within the product module. For each item $k$, we then calculate the price $p_{kt}$ in month $t$ relative to the average price in the product module, $p_{m(k)t}$:

$$R_{kft} = \frac{p_{kft}}{p_{m(k)t}}.$$ 

By dividing prices by the average price in the product module, we can compare the relative prices of items across different categories of goods.

For single-product firms, we use the average relative price for item produced by the firm in 2006 as the measure quality. For multi-product firms, we compute the firm $f$’s relative price as a weighted average of the relative price of different products, weighted by sales in 2006 ($w_{kf}$):

$$R_{f,2006} = \sum_{k \in \Omega} w_{k,f,2006} R_{k,f,2006},$$

where $\Omega$ denotes the set of all products in the Nielsen data.

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6We thank David Argente, Munseob Lee and Sara Moreira for sharing their code to link UPCs to firms with us. These links between items and firms are also used in Hottman, Redding and Weinstein (2016) and Argente, Lee and Moreira (2019).
Third, we link each manufacturing firm $f$ in the Nielsen-GS1 database to the OES firms. To do so, we perform a fuzzy merge of the first component of the firm names from GS1-Nielsen firm with the first component of the OES legal or trade names. We take a conservative approach in classifying a successful fuzzy merge. A merge between Nielsen-GS1 and the OES is defined as successful if one of the following three situations occur: (i) the similarity score is above 95 percent, (ii) the similarity score is above 90 percent and two names share the same first two words; or (iii) the similarity score is above 85 percent and one name contains the other name. This approach yields about 1,600 firms, which include over 29,000 OES establishments.

### 2.3 Quality and Skill Intensity

Table 2 documents our first fact using the Yelp! data set: the share of high-skill workers employed increases with the quality of the goods produced by the firm. Consider, for example, the results we obtain using our measure of skilled workers based on the average wage of college graduates in the industry. The fraction of high-skill workers is 3.54, 6.38 and 9.49 in our low-, middle- and high-quality tier, respectively. Our estimates of skill intensity naturally vary with the breadth of the definition of skill. But, as Table 2 shows, our three alternative definitions of high skill generate similar estimates of the differences in skill intensity of high versus low quality goods.

Table 2: Quality and share of high-skill workers in retail

<table>
<thead>
<tr>
<th>Sample</th>
<th>#Est.</th>
<th>Skilled 1</th>
<th>Skilled 2</th>
<th>Skilled 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Emp</td>
<td>Wage</td>
<td>Emp</td>
</tr>
<tr>
<td>Yelp Sample</td>
<td>9,908</td>
<td>6.01</td>
<td>16.9</td>
<td>13.94</td>
</tr>
<tr>
<td>By Quality:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>2,316</td>
<td>3.54</td>
<td>11.15</td>
<td>9.60</td>
</tr>
<tr>
<td>Middle</td>
<td>6,089</td>
<td>6.38</td>
<td>17.28</td>
<td>14.94</td>
</tr>
<tr>
<td>High</td>
<td>1,503</td>
<td>9.49</td>
<td>23.72</td>
<td>19.40</td>
</tr>
</tbody>
</table>
Table 3 provides results analogous to those in Table 2 obtained using Nielsen’s Homescan data. Here too, the share of high-skill workers employed is an increasing function of the quality of the goods produced by the firm. Consider, for example, the results we obtain using our measure of skilled workers based on the average wage of college graduates in the industry. The fraction of high-skill workers is 1.2 to 1.5 times higher in the high-quality tier when compared with the low-quality tier. The difference between the skill intensity of high- and low-quality products is lower than in the Yelp! data. This property is likely to reflect smaller differences in the quality of groceries, which are the most important category of goods in Nielsen Homescan, than in other categories such as durables.

Table 3: Quality and share of high-skill workers

<table>
<thead>
<tr>
<th>Sample</th>
<th>#Firms</th>
<th>Skilled 1</th>
<th>Skilled 2</th>
<th>Skilled 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Emp</td>
<td>Wage</td>
<td>Emp</td>
</tr>
<tr>
<td>Nielsen Sample</td>
<td>1,097</td>
<td>12.64</td>
<td>30.76</td>
<td>22.04</td>
</tr>
<tr>
<td>By Quality:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>384</td>
<td>10.46</td>
<td>25.89</td>
<td>20.47</td>
</tr>
<tr>
<td>Middle</td>
<td>339</td>
<td>11.63</td>
<td>29.30</td>
<td>21.14</td>
</tr>
<tr>
<td>High</td>
<td>374</td>
<td>15.79</td>
<td>37.08</td>
<td>24.48</td>
</tr>
</tbody>
</table>

2.4 Quality and Routine Work

Acemoglu and Autor (2011) argue that the distinction between skilled and unskilled workers may be less relevant to study new forms of automation. These forms of automation, such as artificial intelligence, might replace routine tasks that are performed by high-skill workers (e.g. radiologists).
In Table 4, we study the relation between quality and the intensity of routine and non-routine work. Two patterns emerge from our data. First, the share of routine workers is lower in the production of high-quality goods when compared to that low quality goods. So quality is intensive in non-routine work. Second, the part of non-routine work that rises with quality is abstract work, not manual work.

<table>
<thead>
<tr>
<th>Sample</th>
<th>#Firms</th>
<th>Routine Emp</th>
<th>Routine Wage</th>
<th>Non-Routine Manual Emp</th>
<th>Non-Routine Manual Wage</th>
<th>Non-Routine Abstract Emp</th>
<th>Non-Routine Abstract Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>384</td>
<td>76.66</td>
<td>62.78</td>
<td>5.24</td>
<td>3.36</td>
<td>18.10</td>
<td>33.87</td>
</tr>
<tr>
<td>Middle</td>
<td>339</td>
<td>80.62</td>
<td>62.77</td>
<td>2.35</td>
<td>1.57</td>
<td>17.03</td>
<td>35.66</td>
</tr>
<tr>
<td>High</td>
<td>374</td>
<td>69.16</td>
<td>51.44</td>
<td>7.60</td>
<td>3.95</td>
<td>23.24</td>
<td>44.60</td>
</tr>
</tbody>
</table>

3 Quality and Income

Our second empirical fact is that the quality of the goods and services consumed by households rises with income. We document this fact using data from the Consumer Expenditure Survey (CEX) and the Nielsen Homescan Data. Our findings corroborate previous results in the literature. Bils and Klenow (2001) show that for a wide range of durable goods in the CEX households with higher total expenditures consume higher-quality goods. Similarly, Faber and Fally (2018), Jaravel (2018), and Argente and Munseob (2016) show that higher-income households consume higher-quality goods. There is also a large trade literature that shows that as countries get richer, they increase the quality of what they consume (see, e.g. Verhoogen (2008) and Fieler, Eslava and Xu (2017)).
3.1 CEX

Our first data set, are durable expenditures from the CEX. Our data covers 73,000 households over the period 1980-2013. Durables are defined as categories whose items have a life that exceeds 2 years. These categories include home furnishing (e.g. carpeting, curtains, mattresses, and sofas), appliances (e.g. dryers, microwaves, stoves, and radios), electronics, and vehicles. The advantage of using durables expenditures is that, given that households are unlikely to buy more than one item at a time, we can, as in Bils and Klenow (2001), use expenditures as a measure of the price paid for each item.

We estimate the quality Engel curve as follows. As in Bils and Klenow (2001), we express the unit price of an item paid by household $h$ at time $t$ as $x_{ht} = z_{ht}q_{ht}$, where $z_{ht}$ is the quality-adjusted price and $q_{ht}$ is the quality of the item. We estimate $\theta$, the elasticity of quality with respect to income, using the following specification:

$$\ln(q_{ht}) = \beta_0 + \theta \ln(y_{ht}) + \epsilon_{ht}$$

where $y$ denotes the income of household $h$ in period $t$, and $\epsilon_{ht}$ denotes the residual.

We can rewrite this specification as

$$\ln(x_{ht}) = \beta_0 + \theta \ln(y_{ht}) + \ln(z_{ht}) + \epsilon_{ht}.$$ 

The logarithm of quality-adjusted price, $\ln(z_{ht})$, is an unobservable variable that reflects differences in prices across time and across households that are not related to the choice of quality. It can, for instance, be due to differences in shopping intensity across households which affects the prices paid for the same item. It may also be due to differences in the discounts available in different locations. As in Bils and Klenow (2001), we include demographic controls to account for these unobservable factors that may affect prices paid. These controls include the age of the households, family size, household fixed effects, and time fixed effects.

Table 5 reports our regression results using 5 income quintile dummies, so that $\theta$ is a vector. This table shows that high-income households pay on average 80 percent more
than low-income households for items of a given category. In our view, these differences are too large to be explained by price discrimination or different search intensities. For example, Aguiar and Hurst (2006) estimate that doubling the shopping frequency lowers the price paid for a given good by only 7 to 10 percent.

### Table 5: Prices and income: CEX

<table>
<thead>
<tr>
<th>Consumer Expenditure Survey Durables</th>
<th>log(Price, Category)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I)</td>
</tr>
<tr>
<td>Relative to income quintile 1:</td>
<td></td>
</tr>
<tr>
<td>Income quintile 2</td>
<td>0.205***</td>
</tr>
<tr>
<td>Income quintile 3</td>
<td>0.368***</td>
</tr>
<tr>
<td>Income quintile 4</td>
<td>0.533***</td>
</tr>
<tr>
<td>Income quintile 5 (top)</td>
<td>0.834***</td>
</tr>
<tr>
<td>Time fixed effects</td>
<td>Yes</td>
</tr>
<tr>
<td>Category fixed effects</td>
<td>Yes</td>
</tr>
<tr>
<td>Demographic controls</td>
<td>Yes</td>
</tr>
</tbody>
</table>

### 3.2 Nielsen Homescan

We supplement the empirical evidence on the relation between income and quality for durable goods with a second data set, the Nielsen Homescan data, that focuses on non-durable goods such as grocery products.

We compute an average price across households, \( \bar{P}_{imt} \), for every item \( i \) in product module \( m \) and time \( t \). By using this average price we ensure that differences in overall prices paid by households reflect differences in choice of item-store, rather than shopping intensity (i.e. using coupons and taking advantage of promotions). For each household,
we compute the price of module \( h \) at time \( t \) as:

\[
\log (P_{hmt}) = \sum_i w_{iht} \log (\bar{P}_{imt}).
\]

We then estimate the following regression:

\[
\log (P_{hmt}) = \beta_0 + \sum_k \beta_k 1(y_{ht} \in k) + \gamma X_{ht} + \lambda_t + \lambda_m + \varepsilon_{hmt}, \tag{1}
\]

where \( 1(y_{ht} \in k) \) is a dummy variable equal to one if the household income is in quintile \( k \), \( X_{ht} \) denotes demographic controls (age group, employment status, size of family, and ethnicity), \( \lambda_t \) denotes time fixed effects, \( \lambda_m \) denotes product-module fixed effects, and \( \varepsilon_{hmt} \) is the error term.

Table 6: Prices and income: Nielsen data

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative to income quintile 1:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income quintile 2</td>
<td>0.0399***</td>
<td>0.0398***</td>
</tr>
<tr>
<td>Income quintile 3</td>
<td>0.0911***</td>
<td>0.0908***</td>
</tr>
<tr>
<td>Income quintile 4</td>
<td>0.151***</td>
<td>0.150***</td>
</tr>
<tr>
<td>Income quintile 5 (top)</td>
<td>0.227***</td>
<td>0.224***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Product module fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Demographic controls</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>

Table 6 reports our estimates. The first and second columns report results without and with demographic controls, respectively. The results are similar. Households in the top quintile choose items that are 22.7 percent more expensive than households in the
bottom quintile. This difference is economically large given that most of the products included in Nielsen’s Homescan Data are groceries. This is an expenditure category in which price differentials are relatively small when compared with to categories such as durable goods.

4 A Simple Model

In this section, we consider a simple model that is consistent with our two empirical facts. First, firms that produce higher quality goods employ a higher share of high-skill workers. In other words, quality is skill intensive. Second, the quality of the goods a household consumes rises with income. In other words, quality is a normal attribute. We then consider the implications of our model for the measurement of SBTC.

Before we delve into the details of our model, we review the key features of the canonical model used to explains the rise in the skill premium. In this model, output, $Y$, is produced according to the following production function:

$$Y = A \left[ \alpha (SH)^\rho + (1 - \alpha)L^\rho \right]^{1/\rho}, \quad (2)$$

where $S$ denotes the level of SBTC, $A$ denotes the level of HNTC, $H$ is the supply of skilled work, and $L$ is the supply of unskilled work.

Output is produced by firms that are competitive in product and factor markets. The optimization conditions for these firms imply that the skill premium, defined as the ratio of the wage rate of skilled, $W_H$, and unskilled, $W_L$, workers, is given by:

$$\frac{W_H}{W_L} = \frac{\alpha}{1 - \alpha} S_l^\rho \left( \frac{H}{L} \right)^{\rho-1}. \quad (3)$$

This expression implies that changes in $A$ have no impact on the skill premium. Computing logarithmic growth rates of the two sides of equation (3), we obtain:

$$\Delta \log \left( \frac{W_H}{W_L} \right) = \rho \Delta \log (S) + (\rho - 1) \Delta \log \left( \frac{H}{L} \right). \quad (4)$$

14
According to the estimates in Acemoglu and Autor (2010), the relative supply of effective labor of skilled workers $H/L$ increased by about 110 percent and the skill premium increased 22 percent between 1970 and 2002. As in Acemoglu and Autor (2010), we assume that $\rho$ equals 0.41.\footnote{Estimates for $\rho$ generally range from 0.16 (Card and Lemieux (2001) to 0.5 (Angrist (1995)).} Equation (4) implies that $\Delta \log(S) = 210.8$ percent, which corresponds to an average annual rate of SBTC of 5.5 percent.

4.1 Homogeneous households

We make some simplifying assumptions so that our model is as similar as possible to the canonical model used in the SBTC literature. These assumptions are: (i) no capital in production; (ii) a single production sector; and (iii) households consume a single unit of the consumption good, so changes in expenditures translate fully into changes in the quality of the goods consumed. These assumptions allow us to derive results without taking a stand on the form of the utility function.

Household We consider a representative household composed by the same fraction of skilled and unskilled workers present in the population. The household pools its resources and buys a single unit of a consumption good of quality $q$ at a price $P(q)$. The household budget constraint is given by

$$P(q) = W_H H + W_L L,$$

where $H$ and $L$ denote high-skill and low-skill workers respectively. We treat the supply of high- and low-skill workers as exogenous and assume that workers are identical within each skill group. Household utility is given by

$$U = V(q),$$

where $V'(q) > 0$, $V''(q) \leq 0$. 

\footnote{Estimates for $\rho$ generally range from 0.16 (Card and Lemieux (2001) to 0.5 (Angrist (1995)).}
Production Final goods are produced by competitive firms using skilled and unskilled labor according to the production function

\[ Y = A \left[ \alpha (SH)^\rho + q^{-\gamma \rho} (1 - \alpha) (L)^\rho \right]^{\frac{1}{\rho}}, \]  

(5)

where \( \rho > 0 \), \( \gamma \geq 0 \), and \( q \) denotes the quality of the good produced. When \( \gamma = 0 \) this production function is identical to the one used in the canonical model (equation (2)).

The equilibrium price of a good of quality \( q \) is given by:

\[ P_q = \frac{1}{A} \left[ \alpha^{\frac{1}{1-\rho}} (S)^{\frac{\rho}{1-\rho}} W_H^{\frac{\rho}{1-\rho}} + (1 - \alpha)^{\frac{1}{1-\rho}} (q)^{\frac{\rho}{1-\rho}} W_L^{\frac{\rho}{1-\rho}} \right]^{\frac{\rho-1}{\rho}}. \]

The production function (5) with \( \gamma > 0 \) together with the perfect-competition assumption implies two key properties. First, \( P_q' > 0 \), i.e. the price of a final good is increasing in its quality. Second, quality is intensive in high-skill labor, i.e., the labor share of high-skill labor, \( W_H H / (W_H H + W_L L) \) is increasing in \( q \).

Skill Premium The firms’ optimization conditions imply that the skill premium is given by:

\[ \frac{W_H}{W_L} = \alpha q^\gamma (S)^\rho \left( \frac{H}{L} \right)^{\rho-1}. \]  

(6)

Computing growth rates, we see that the change in the skill premium is the one obtained in the canonical model (equation (4)) plus the effect of trading up on the skill premium, which is given by the term \( \gamma \rho \Delta \log (q) \) in the expression below:

\[ \Delta \log \left( \frac{W_H}{W_L} \right) = \rho \Delta \log (S) + (\rho - 1) \Delta \log \left( \frac{H}{L} \right) + \gamma \rho \Delta \log (q). \]  

(7)

Equations (6-7) show that quality plays a role that is similar to that of SBTC. Other things equal, a rise in quality increases the demand for skilled labor raising the skill premium.
**SBTC and HNTC** Combining the household budget constraint and the firms’ first-order condition we obtain the following equation:

$$A \times S = \frac{\left(\frac{W_H}{W_L}\right)^{\frac{1}{\rho}} \left(\frac{H}{L}\right)^{\frac{1-\rho}{\rho}}}{\alpha^\rho L \left[\frac{W_H H}{W_L L} + 1\right]^{\frac{1}{\rho}}}.$$  

Equation (8) implies that, holding constant $H$ and $L$, a change in $A$ has the same impact on the skill premium as a change in $S$. This equivalence holds because in our model a change in $A$ triggers a change in $q$:

$$q = \left[A(1-\alpha)^{1/\rho}\left(\frac{W_H}{W_L} H + L\right)\left(\frac{W_H H}{W_L L} + 1\right)^{(1-\rho)/\rho}\right]^{1/\gamma},$$  

and this change in $q$ has an effect on the skill premium that is similar to that of a rise in $S$ (see equation (6)).

### 4.1.1 Quantitative results

The estimates in Acemoglu and Autor (2010) suggest that between 1970 and 2008 the skill premium increased by 25 percentage points, while the effective ratio of high-skill labor to low-skill labor increased by 110 percentage points. Below, we use our model to characterize the combinations of SBTC and HNTC that are consistent with these observed changes in $W_H/W_L$, $H$ and $L$.

Using equation (8) to compute logarithmic growth rates, we obtain

$$\Delta A + \Delta S = \frac{1}{\rho} (\Delta W_H - \Delta W_L) + \frac{1-\rho}{\rho} (\Delta H - \Delta L) - \Delta L + \frac{1}{\rho} \Delta \left(1 + \frac{W_H H}{W_L L}\right).$$  

Following Acemoglu and Autor (2010), we set $\rho = 0.41$. Given this value of $\rho$, we can compute the right-hand side of equation (10), which is equal to 1.92 percent on an annual basis. The left-hand side of the equation gives us the combinations of $\Delta A$ and $\Delta S$ that match the right-hand side.
Table 7 reports results for both our model and the canonical model for different combinations of HNTC and SBTC. For each combination of SBTC and HNTC there is a value of $\gamma$, given by equation (8), such that our model is consistent with Bils-Klenow estimates for $\Delta q$ (3.8 percent per year). The value of $\gamma$ is lower the higher is the level of SBTC. The reason for this property is as follows. Quality is intensive in skilled labor and SBTC lowers the cost of skilled labor. The parameter $\gamma$ controls the response of prices to a rise in quality. The higher the level of SBTC the lower is the value of $\gamma$ required to be consistent with the Bils-Klenow estimates for $\Delta q$.

<table>
<thead>
<tr>
<th>$\Delta A$</th>
<th>$\Delta S$</th>
<th>$\gamma$</th>
<th>Cumulative $\Delta (W_H/W_L)$ (percent)</th>
<th>Trading-up model</th>
<th>Canonical model</th>
<th>Trading-up model with $\Delta S = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.92</td>
<td>−46.0</td>
<td>−65.0</td>
<td>−46.0</td>
<td>−46.0</td>
</tr>
<tr>
<td>0.00</td>
<td>1.92</td>
<td>0.92</td>
<td>25.0</td>
<td>−34.0</td>
<td>−46.0</td>
<td>−46.0</td>
</tr>
<tr>
<td>0.87</td>
<td>1.05</td>
<td>1.15</td>
<td>25.0</td>
<td>−48.0</td>
<td>−25.0</td>
<td>−22.0</td>
</tr>
<tr>
<td>0.98</td>
<td>0.94</td>
<td>1.18</td>
<td>25.0</td>
<td>−50.0</td>
<td>−25.0</td>
<td>−22.0</td>
</tr>
<tr>
<td>1.92</td>
<td>0.00</td>
<td>1.42</td>
<td>25.0</td>
<td>−65.0</td>
<td>25.0</td>
<td></td>
</tr>
</tbody>
</table>

Notes: All numbers in percentages. $\Delta A$ and $\Delta S$ are reported on an annual basis. $\Delta (W_H/W_L)$ for $\Delta S = 0$ is the value for the entire sample.

Comparing columns 4 and 5 of Table 7, we see that our model generates sizable increases in the skill premium and in income inequality when compared with the canonical model. Line 1 corresponds to the case where there is no HNTC or SBTC. In the canonical model, the skill premium falls by 65 percent. In our model, this fall is only 46 percent. The reason for this result is that in our sample more workers became skilled over time, creating a rise in income that induces households to trade up, expanding the demand for high-skill labor and increasing the skill premium.
Line 2 pertains to the case where there is no HNTC and the rate of SBTC is such that our model matches the observed rise in the skill premium. This rate of SBTC results in a 34 percent fall in the skill premium in the canonical model. Comparing these two rates shows the degree of amplification generated by our model.

In line 3, we choose the rate of HNTC to match Fernald’s (2014) estimates for TFP growth for the period 1970 to 2008 without controlling for capital utilization (0.87 percent per year). The annual growth rate of SBTC consistent with the rise in the skill premium is equal to 1.05 percent per year, a much more plausible number that the 5.5 percent obtained in the absence of the trading-up channel. In the canonical model this configuration of HNTC and SBTC results in a 48 percent fall in the skill premium. Even though in our benchmark calibration the rate of SBTC is only 1.05 percent per year, the presence of SBTC is essential to produce a rise in the skill premium. Column 6 shows that in the absence of SBTC the skill premium falls by 25 percent because the rise in the demand for skilled workers is not strong enough to overcome the increase in the relative supply of skilled workers. In this calibration HNTC accounts for 30 percent of the rise in the skill premium while in the canonical model it accounts for zero percent of this rise,\(^8\)

In line 4, we choose the rate of HNTC to match Fernald’s (2014) estimates for TFP growth for the period 1970 to 2008 controlling for capital utilization (0.98 percent per year). This higher rate of HNTC translates into a lower rate of SBTC required to match the skill premium (0.94 percent per year).

Line 5 corresponds to the case where our model matches the rise in the skill premium with HNTC and no SBTC. The required rate of HNTC is 1.92 percent. With this configuration of parameters the canonical model generates a fall of 65 percent in the skill premium. Recall that in this model the change in the skill premium is not affected

\(^{8}\)In the absence of HNTC and SBTC, the skill premium falls by -46 percent (line 1 of column 6). HNTC and SBTC together raise the change in the skill premium from -46 to +25 percent. With HNTC and no SBTC, the skill premium falls by 25 percent. So the fraction of the rise in the skill premium accounted for by HNTC is \([-25 - (-46)]/(25 - (-46)) = 30\) percent.
by the rate of HNTC.

4.2 Heterogenous Households

In the model described above we make several simplifying assumptions to derive analytical results about the impact of trading up on the skill premium. In particular, there is only one quality level produced in equilibrium. So the model cannot, by construction, match the cross-sectional relation between quality and the share of high-skill workers used in production.

In this subsection, we extend the model by considering an economy where there is more than one consumption good produced in equilibrium. This version of our model has two types of households. The first type has only low-skill workers and the second type only high-skill workers. These households receive different income levels and, as a result, they consume goods of different quality.

Production Household type \( j \in \{L, H\} \) consumes goods of quality \( q_j \) which are produced according to

\[
Y_{q_j} = A \left[ \alpha (SH_j)^\rho + q_j^{-\gamma} (1 - \alpha) L_j^\rho \right]^{1/\rho}.
\]

The price of a good of quality \( q_j \) is

\[
P_{q_j} = \frac{1}{A} \left[ \alpha^{1/(1-\rho) S^\rho/(1-\rho) W_H^\rho/(1-\rho)} + q_j^{\gamma - 1} (1 - \alpha)^{1/(1-\rho)} W_L^\rho/(1-\rho) \right]^{(\rho-1)/\rho}.
\]

We assume perfect labor mobility which implies that the skill premium is identical in both the high- and low-quality production sector. The first-order conditions for competitive output producers imply that the skill premium in each sector \( j \) is given by:

\[
\frac{W_H}{W_L} = \frac{\alpha}{1 - \alpha} (\bar{q}_j^n \times S)^\rho \left( \frac{H_j}{L_j} \right)^{\rho - 1}.
\]

For future reference, we note that using the expression for the skill premium implies that total expenditure in each of the goods produced is given by

\[
P_{q,j} Y_{q,j} = W_L L_{q,j} \left[ 1 + \frac{W_H}{W_L} \left( \frac{H_{q,j}}{L_{q,j}} \right) \right].
\]
Households There is a measure $\Gamma(H)$ of skilled workers and $\Gamma(L) = 1 - \Gamma(H)$ of unskilled workers. The supply of skilled work measured in efficiency units is: $H = H_{q,L} + H_{q,H}$. Similarly, the supply of unskilled workers in efficiency units is: $L = L_{q,L} + L_{q,H}$.

The maximization problem of high-skill households is:

$$\max U = V(q_H),$$
subject to

$$P(q_H) = W_H H.$$

Similarly, the maximization problem of low-skill households is:

$$\max U = V(q_L),$$
subject to

$$P(q_L) = W_L L.$$

Equilibrium Using the budget constraints of low- and high-skill workers, we obtain:

$$L_{q,j} \left[ 1 + \frac{W_H}{W_L} \left( \frac{H_{q,L}}{L_{q,L}} \right) \right] = L,$$

$$L_{q,H} \left[ 1 + \frac{W_H}{W_L} \left( \frac{H_{q,H}}{L_{q,H}} \right) \right] = \frac{W_H}{W_L} H.$$

As in the previous section we look for combinations of SBTC and HNTC that are consistent with the observed changes in the skill premium and aggregate changes in $L$ and $H$.

To solve this two-sector model we proceed as follows. We guess $H_{q,L}/L_{q,L}$. The budget constraint of low-skill workers implies a value for $L_{q,L}$. Using this value, we can compute $H_{q,L}$ and $H_{q,H}$. We can then compute the ratio of the two qualities produced:

$$\frac{q_H \left( \frac{W_H}{W_L} \frac{H_{q,H}}{L_{q,H}} + 1 \right)^{\frac{1}{\rho}} \Gamma_H}{q_L \left( \frac{W_H}{W_L} \frac{H_{q,L}}{L_{q,L}} + 1 \right)^{\frac{1}{\rho}} \Gamma_L} = \frac{W_H}{W_L} \frac{H}{L}.$$
Given the observed skill premium, the overall measures of high- and low-skill workers, and the equilibrium sectoral, these identify the ratio of qualities, \( q_H/q_L \). With this ratio we then verify that the skill premium is identical in both sectors.

Before commenting on the implications for the measurement of the SBTC, we note that we can compute the share of high-skill workers in labor income \((W_H H / (W_L H + W_H L))\) in the production of the high- and low-quality good. Importantly, the model has no free parameters that allows us to target these ratios, so it is of interest to see whether the model comes close to the observed numbers in the data. The ratio of these shares is 2.4 in the beginning of the sample and 2.5 in the end of the sample. We also compare the share of high-skill labor \((H / (H + L))\) used in the production of the high- and the low-quality good. The ratio of these shares is 2.8 in the beginning of the sample and 3.6 in the end of the sample. These values are somewhat higher than those reported in Table 2 for the ratio of highest quality ($$$) and the lowest quality ($).

Once we find the equilibrium sectoral allocation we use the fact that each consumer purchases one unit of the good and thus, \( Y_{q,j} = \Gamma(j) \) from which it follows that for each sector the following equation holds

\[
\Delta A - \gamma \Delta q_H = \frac{\rho - 1}{\rho} \Delta \left( 1 + \frac{W_H H H}{W_L L_H} \right) + \Delta \Gamma_H - \Delta \left( \frac{W_H H}{W_L L} \right).
\]

The right hand side is a given number (given the data and the equilibrium sectoral allocation). Hence given different values of \( \Delta A \) we can recover \( \Delta q \) for each of the two sectors. Then from the skill premium equation,

\[
\frac{W_H}{W_L} = \frac{\alpha}{1 - \alpha} (q_j \times S)^\rho \left( \frac{H_j}{L_j} \right)^{\rho - 1}.
\]

we can recover the consistent value of \( \Delta S \).

We re-do the analysis for the same configurations of \( \gamma, \Delta A \) and \( \Delta S \) used in Table 7. The results are almost identical, showing that our results are robust to introducing the form of income heterogeneity embodied into this model.
5 Conclusions

In this paper, we show empirically that as income rises, households trade up to higher quality goods and that these goods are intensive in skilled labor. As a result, the demand for high-skill labor rises, increasing the skill premium. This trading-up phenomenon amplifies the effect of skill-biased technical change by creating an endogenous rise in the demand for skilled workers.

In the canonical model, technical change has to be skill biased to produce a rise in the skill premium. In our model, any factor that raises income increases the skill premium. This idea has important implications for the future evolution of the skill premium. Growth in income that is not accompanied by an increase in the supply of skilled workers is likely to increase the skill premium even in the absence of skill-biased technological change.
References


6 Appendix: Capital Deepening and the Skill Premium

Krusell, Ohanian, Ríos-Rull, and Violante (2000), which we refer to as KORV, argue that capital deepening associated with investment-specific technical progress explains much of the rise in the skill premium. To re-examine their analysis, we adopt their functional form for the production function. Output with quality $q$ is produced according to the following nested CES function

$$Y_q = K_S^\gamma \left[ \alpha (\lambda K_E + (1 - \lambda) H)^{\frac{\sigma}{\sigma + 1}} + q^{-\gamma \rho/(1 - \alpha)} (L)^{\rho} \right]\frac{1 - \gamma}{\rho},$$

(11)

where $K_S$ is the stock of structures and $K_E$ is the stock of equipment.

To retain the one-sector character of the model, we assume that investing $I_{qt}$ units of output with quality $q_t$ yields $P(q_t)$ units of installed capital. The capital accumulation equation takes the form

$$K_{St+1} = P_{qt}I_{qSt} + (1 - \delta)K_{St},$$
$$K_{Et+1} = P_{qt}I_{qEt} + (1 - \delta)K_{Et},$$

where $I_{qSt}$ and $I_{qEt}$ denote the investment in structures and equipment, respectively.

The market clearing condition for output is:

$$Y_{qt} = 1 + I_{qSt} + I_{qEt}/z_t,$$

where $1/z_t$ is the relative price of equipment. Recall that the representative household buys one unit of a good with quality $q_t$.

We use the wage of low-skill workers as the numeraire,

$$W_L = 1,$$

but for clarity we retain the symbol $W_L$ in the derivations below.
Output producers are competitive in goods and factors markets. Profit maximization implies the following first-order conditions

\[ P_q \gamma K_S^{\gamma-1} \left[ \alpha (\lambda K_E^\sigma + (1 - \lambda) H^\sigma) \frac{q}{\sigma} + q^{-\gamma\rho}(1 - \alpha) (L)^{\rho} \right]^{\frac{1-\gamma}{\rho}} = R_S, \]

\[ P_q(1 - \gamma)K_S^\gamma \left[ \alpha (\lambda K_E^\sigma + (1 - \lambda) H^\sigma) \frac{q}{\sigma} + q^{-\gamma\rho}(1 - \alpha) (L)^{\rho} \right]^{\frac{1-\gamma-\rho}{\rho}} \times (12) \]

\[ \alpha (\lambda K_E^\sigma + (1 - \lambda) H^\sigma) \frac{q}{\sigma} \frac{\lambda K_E^\sigma}{K_E} = R_E, \]

\[ P_q(1 - \gamma)K_S^\gamma \left[ \alpha (\lambda K_E^\sigma + (1 - \lambda) H^\sigma) \frac{q}{\sigma} + q^{-\gamma\rho}(1 - \alpha) (L)^{\rho} \right]^{\frac{1-\gamma-\rho}{\rho}} \times (13) \]

\[ \alpha (1 - \lambda) \left( \lambda K_E^\sigma + (1 - \lambda) H^\sigma \right) \frac{1}{\sigma} H^\sigma \frac{H^\sigma}{H} = W_H, \]

\[ P_q(1 - \gamma)K_S^\gamma \left[ \alpha (\lambda K_E^\sigma + (1 - \lambda) H^\sigma) \frac{q}{\sigma} + q^{-\gamma\rho}(1 - \alpha) (L)^{\rho} \right]^{\frac{1-\gamma-\rho}{\rho}} \times (14) \]

\[ q^{-\gamma\rho}(1 - \alpha) (L)^{\rho-1} = W_L. \]

where \( R_S \) and \( R_E \) are the rental rates on structures and equipment, respectively.

These first-order conditions imply that the price of a good with quality \( q \) is

\[ P_q = \frac{R_S^\gamma \left[ \alpha^{\frac{1}{1-\rho}} \left( \lambda^{1-\sigma} R_E^\sigma + (1 - \lambda)^{1-\sigma} W_H^\sigma \right) \frac{q}{\sigma} (\frac{q}{\sigma})^{\frac{1}{\sigma-\rho-1}} + (1 - \alpha)^{\frac{1}{1-\rho}} q^{\frac{\gamma\rho}{\rho-1}} (W_L)^{\frac{1}{\rho-1}} \right]^{\frac{1}{1-\gamma}}} {\gamma^{\gamma(1-\gamma)^{1-\gamma}}}. \]  

Equations (13) and (14) imply that the skill premium is given by

\[ \frac{\alpha (1 - \lambda) \left( \lambda K_E^\sigma + (1 - \lambda) H^\sigma \right) \frac{q}{\sigma} \frac{H^\sigma}{H}} { (1 - \alpha) q^{-\gamma\rho} L^{\rho-1}} = \frac{W_H}{W_L}. \]  

Combining equations (12) and (13), we obtain

\[ \frac{(1 - \lambda)}{\lambda} \left( \frac{K_E}{H} \right)^{-\sigma} R_E K_E = W_H H. \]
Combining equations (16) and (17), the skill premium can be written as

\[
\frac{\alpha(1 - \lambda)}{(1 - \alpha)} \times \frac{1 + R_E K_E / (W_H H)}{q^{\gamma \rho}} \left( \frac{H}{L} \right)^{\rho - 1} = \frac{W_H}{W_L}.
\] (18)

If we abstract from the impact of quality by assuming that \( q \) is constant and equal to one, we obtain the same expression for the skill premium equation used in KORV.

### 6.1 Some Analytics

Recall that \( \sigma \) is the parameter that governs the elasticity of substitution between capital and high-skill workers. To see the effects of the presence quality on the point estimates for \( \sigma \), it is useful to log-linearize equation (18),

\[
(\rho - \sigma) \frac{1}{1 + W_H H / (R_E K_E)} \left( \hat{K}_E - \hat{H} \right) + (\rho - 1) \left( \hat{H} - \hat{L} \right) + \gamma \rho \hat{q} = \hat{W}_H - \hat{W}_L.
\] (19)

where \( \hat{x} \) denotes the logarithmic growth rate of \( x \). Solving equation (19) for \( \sigma \), we obtain

\[
\sigma = \frac{1 + W_H H / (R_E K_E)}{\hat{K}_E - \hat{H}} \times \left[ \frac{\rho (\hat{K}_E - \hat{H})}{1 + W_H H / (R_E K_E)} + (\rho - 1)(\hat{H} - \hat{L}) - \hat{W}_H + \hat{W}_L + \gamma \rho \hat{q} \right].
\] (20)

Capital-skill complementarity requires \( \sigma \leq \rho \). How are these estimates affected by trading up? The answer to this question depends on the value of \( \gamma \rho \hat{q} [1 + W_H H / (R_E K_E)] / (\hat{K}_E - \hat{H}) \). In the KORV data \( \hat{K}_E - \hat{H} > 0 \). Since \( \rho > 0 \), an increase in \( \hat{q} \) implies that the right-hand side is overall a higher number. Since \( \sigma < 0 \), then the degree of capital skill complementarity required to match the same empirical facts is reduced.

Trading up also affects the point estimates of \( \rho \), the parameter that governs the degree of substitutability between unskilled workers and the composite good of equipment
capital and skilled workers. To see this effect, note that the value of \( \rho \), as a function of a given value of \( \sigma \) can be expressed as

\[
\rho = \frac{\sigma (\bar{K}_E - \bar{H})/(1 + \frac{W_H H}{R_E K_E}) + (\bar{W}_H + \bar{H} - \bar{W}_L - \bar{L})}{1 + \frac{\gamma q (\bar{K}_E - \bar{H})/(1 + \frac{W_H H}{R_E K_E}) + (\bar{H} - \bar{L})}{(\bar{K}_E - \bar{H})/(1 + \frac{W_H H}{R_E K_E}) + (\bar{H} - \bar{L})}}. \tag{22}
\]

The change in the level of quality affects only the denominator reducing the value of \( \rho \). As a result, the degree of substitutability of unskilled labor and the composite good of equipment and skilled worker \((1/(1 - \rho))\) falls.

In this analysis we were holding constant the value of \( \sigma \) when analyzing the effects of quality on the measurement of \( \rho \) and vice versa when analyzing the effect of quality for the measurement of \( \sigma \). Naturally, both of these estimates can change as a results of quality. We thus proceed by jointly estimating these two parameters.

### 6.2 Estimation

In this section, we estimate the production function using the approach proposed by Polgreen and Silos (2008).\(^9\) In this approach, the posterior distribution is obtained by combining a prior distribution for the vector of parameters with a measurement-error-based likelihood function for the data.

The estimation is based on three conditions. The first is the equation for the labor share in income

\[
\frac{W_L L}{P_q Y_q} + \frac{W_H H}{P_q Y_q} = (1 - \gamma) Y_q^{-\rho/(1 - \gamma)} \left[ \alpha (1 - \lambda) (\lambda K_E^\sigma + (1 - \lambda) H^\sigma)^{\frac{\sigma - \bar{\sigma}}{\bar{\sigma}}} H^\sigma + q^{-\gamma \rho} (1 - \alpha) L^\rho \right].
\]

The second condition is the equation for the ratio of labor income of skilled and unskilled agents,

\(^9\)We are extremely grateful to Pedro Silos for kindly sharing with us his code and for various consultations.
\[
\frac{W_H H}{W_L L} = \frac{\alpha(1 - \lambda) \left( \lambda K_E^\sigma + (1 - \lambda) H_q^\sigma \right)^{\frac{\rho - \sigma}{\sigma}} H^\sigma}{q^{-\gamma \rho (1 - \alpha)} L_q^\rho}.
\]

The third condition equates the rate of return to investing in structures and equipment.

\[
\gamma K^\gamma_S^{-1} \left[ \alpha \left( \lambda K_E^\sigma + (1 - \lambda) H^\sigma \right) \gamma + q^{-\gamma \rho (1 - \alpha)} (L)^{\rho} \right] \left[ \frac{1 - \gamma}{\sigma} + (1 - \delta) \right]
= (1 - \gamma) K^\gamma_S \left[ \alpha \left( \lambda K_E^\sigma + (1 - \lambda) H^\sigma \right) \gamma + q^{-\gamma \rho (1 - \alpha)} (L)^{\rho} \right] \left[ \frac{1 - \gamma - \rho}{\rho} \right] \times 
\alpha \left( \lambda K_E^\sigma + (1 - \lambda) H^\sigma \right)^{\frac{\rho - \sigma}{\sigma}} \lambda K^\sigma_E^{-1} + (1 - \delta) z_{t-1}/z_t.
\]

**Estimating \( \rho \) and \( \sigma \)** We begin by replicating the analysis of Polgreen and Silos (2008) and estimate \( \rho \) and \( \sigma \) in a model without quality choice. The resulting estimates are \( \rho = 0.4470 \), \( \sigma = -0.3871 \).

We now estimate \( \rho \) and \( \sigma \) in a model with quality choice. Since we do not have a time series for \( \Delta q \), we consider a constant trend in quality at the annual growth rate estimated by Bils and Klenow (2001). We obtain the following estimates: \( \rho = 0.2485 \) and \( \sigma = -0.3730 \). As suggested by our analytical results, incorporating quality choice into the model implies a fall in the degree of substitutability of unskilled labor and the composite good of equipment and skilled worker \( 1/(1 - \rho) \). This elasticity of substitution between unskilled labor and capital falls from \( 1/(1 - 0.4470) = 1.81 \) to \( 1/(1 - 0.2485) = 1.33 \).