Optimal Inflation and the Identification of the Phillips curve*

Michael McLeay †   Silvana Tenreyro‡

27 March 2019

Abstract

Several academics and practitioners have pointed out that inflation follows a seemingly exogenous statistical process, unrelated to the output gap, leading some to argue that the Phillips curve has weakened or disappeared. In this paper we explain why this seemingly exogenous process arises, or, in other words, why it is difficult to empirically identify a Phillips curve, a key building block of the policy framework used by central banks. We show why this result need not imply that the Phillips curve does not hold – on the contrary, our conceptual framework is built under the assumption that the Phillips curve always holds. The reason is simple: if monetary policy is set with the goal of minimising welfare losses (measured as the sum of deviations of inflation from its target and output from its potential), subject to a Phillips curve, a central bank will seek to increase inflation when output is below potential. This targeting rule will impart a negative correlation between inflation and the output gap, blurring the identification of the (positively sloped) Phillips curve. We discuss different strategies to circumvent the identification problem and present evidence of a robust Phillips curve in US data.

*First draft: April 2018. The views expressed in this paper are those of the authors and not necessarily those of the Bank of England. This paper was motivated by a conversation with Ben Broadbent and Jan Vlieghe. We would like to thank Francesco Caselli, Martin Eichenbaum, Andy Haldane, Richard Harrison, Per Krussell, Clare Macallan, Chris Redl, Ricardo Reis, Martin Seneca, Jan Vlieghe and Matt Waldron for helpful comments and suggestions and Oliver Ashtari Tafti for superb research assistance.

†Bank of England (michael.mcleay@bankofengland.co.uk).

‡Bank of England, LSE, CFM and CEPR (s.tenreyro@lse.ac.uk).

1. Introduction

A number of recent papers have pointed out that inflation can be approximated (and forecast) by statistical processes unrelated to the amount of slack in the economy (Atkeson and Ohanian, 2001; Stock and Watson, 2007, 2009; Dotsey, Fujita and Stark, 2017; Cecchetti et al., 2017; Forbes, Kirkham and Theodoridis, 2017). The empirical disconnect between inflation and various measures of slack has been interpreted by some commentators as evidence that the Phillips curve (a positive relation between inflation and the output gap) has weakened or even disappeared (Ball and Mazumder, 2011; IMF, 2013; Hall, 2013; Blanchard, Cerutti and Summers, 2015; Coibion and Gorodnichenko, 2015).[1][2]. On the face of it, a change in the Phillips Curve relationship could have major implications for monetary policy, so the potential causes of any weakening have been an important topic of discussion for policymakers (Draghi, 2017; Carney, 2017*a*; Powell, 2018).

The Phillips curve is one of the building blocks of the standard macroeconomic models used for forecasting and policy advice in central banks. Its empirical elusiveness could challenge the wisdom of these models and the usefulness of their forecasts. Arguably, it even calls in to question part of the rationale for independent, inflation-targeting central banks. Or does it?

In this paper we use a standard conceptual framework to show why:

- the empirical disconnect between inflation and slack is a result to be expected when monetary policy is set optimally; and
- it is also perfectly consistent with an underlying stable and positively sloped Phillips curve.

More specifically, our framework is built under the assumption that the Phillips curve always holds; that is, inflation depends positively on the degree of slack in the economy. We also allow for cost-push shocks that can lead to deviations from the curve, but without altering its slope. Monetary policy is set with the goal of minimising welfare losses (measured as the sum of the quadratic deviations of inflation from its target and of output from its potential), subject to the Phillips curve or aggregate supply relationship. In that setting a central bank will seek to increase inflation when

[1]For a selection of the vast media comment on the issue, see articles in the *Financial Times*, *Wall Street Journal* and *The Economist* and opinion piecies by Alan Blinder, Paul Krugman and Lawrence Summers.

[2]The output gap is defined as the deviation of output from its potential; in the original paper of Phillips (1958), the focus was the negative relationship between wage inflation and unemployment

output is below its potential. This targeting rule imparts a negative correlation between inflation and the output gap, blurring the identification of the (positively sloped) Phillips curve.[3,4]

The paper is extended along five dimensions. First, we study differences in the solutions between discretion – our baseline case in which the monetary authority cannot commit to a future path of inflation and the output gap – and the case of commitment, in which the authority credibly commits to a future plan. We find that the main intuition goes through in both cases. The difference lies in the implied properties of the statistical process for inflation generated by the optimal policy in each case. In the simple framework studied here, the greater degree of inertia under optimal commitment also offers one potential solution to the identification problem.

A second extension introduces shocks to the targeting rule. These shocks can be interpreted as lags in monetary transmission; as shocks to the monetary policy instrument rule; or, in a multi-region setting, as idiosyncratic demand shocks affecting different regions or countries within a monetary union. We show that the relative variance of these shocks vis-a-vis the cost-push shocks is key for the empirical identification of the Phillips curve using standard regression analysis. This result also rationalises the findings of the vast empirical literature that uses identified monetary policy shocks to estimate the transmission of monetary policy. Effectively, well-identified monetary policy shocks should help in retrieving the Phillips curve.

Third, we study a multi-region (multi-country or multi-sector) setting with a common central bank and discuss conditions under which regional (or sectoral) data can help mitigate the bias from the endogeneity of monetary policy. The discussion, however, also underscores some of the limitations faced by regional analysis.

A fourth extension discusses the estimation of a wage-Phillips curve and compares the identification challenges with those faced in the price-Phillips curve.

The final extension departs from the stylised New Keynesian model of Clarida, Gali and Gertler (1999) and studies the aggregate supply constraint in a large-scale DSGE model of the type designed for forecasting and policy analysis in central banks. In such larger models, the concept of a single, structural relationship between inflation and the output gap is no longer well defined: their reduced-

[3]This result follows straightforwardly from the basic New Keynesian model as derived in Clarida, Gali and Gertler (1999), while similar results would obtain in the classic setting of Barro and Gordon (1983).

[4]The original Phillips (1958) curve was estimated over 1861-1913 UK data, a period during which Britain was on the gold standard (King, 2008). The bias we describe here would therefore not have been in effect.

form correlation varies according to which shock hits the economy. Nonetheless, we show that the intuition from the *structural* Phillips curve in the basic model continues to apply to the *reduced-form* Phillips curve in larger-scale DSGE models. In the model of Burgess et al. (2013), designed for policy use at the Bank of England, a positively sloped reduced-form Phillips curve is present when policy is set according to an estimated Taylor rule. But under optimal discretionary policy the slope of the curve changes sign.

We next turn to practical attempts to address the identification issue we raise, focusing on US data. The simultaneity bias arises due to the behaviour of monetary policy in partially accommodating cost-push shocks to the Phillips curve. It is magnified because monetary policy seeks to offset any demand shocks that might otherwise help identify the curve. We discuss three practical solutions that attempt to circumvent these issues by isolating the remaining demand-driven variation in inflation.

First, econometricians can attempt to control for cost-push and other trade-off inducing shocks to aggregate supply, in line with the approach proposed by Gordon (1982). This helps to minimise the remaining cost-push driven variance in the error term, leaving only demand shocks that can correctly identify the Phillips curve. In practice, however, the success of this approach requires successfully controlling for each and every trade-off inducing shock affecting the economy. The ability to do this may be limited in the recent past, where energy price shocks are less dominant than in the 1970s.

Second, if econometricians can find suitable instrumental variables, they can purge their output gap data of any cost-push shocks, leaving only the demand variation needed to consistently estimate the Phillips curve. With highly autocorrelated cost-push shocks (precluding the use of lagged variables as instruments), using measures of monetary policy or other demand shocks may be one set of appropriate external instruments (Barnichon and Mesters, 2019). But if the variance of monetary policy shocks has fallen since the early 1980s, as suggested by Boivin and Giannoni (2006), then these instruments may be too weak to provide a practical solution in the recent data.

We next present evidence on our third solution, using cross-sectional regional variation in unemployment to identify the Phillips curve. Using US metropolitan area price and unemployment data, we estimate a Phillips curve including metropolitan area fixed effects, to control for time-invariant regional heterogeneity in the natural rate of unemployment, as well as time fixed effects to control for variation over time in monetary policy and the aggregate natural rate. Under our preferred specification, a steeper Phillips curve re-emerges, with a slope three times as large as that estimated using aggregate data.

That the empirical Phillips curve may vary with monetary policy was one of the examples given by Lucas (1976), and similar points have been echoed in different forms by other authors since. Similarly, Mankiw, Ball and Romer (1988) showed how increases in average inflation rates, by changing the frequency with which firms reset prices, could change the deep parameters that determine the slope of estimated Phillips curves. Others have modelled a situation where policymakers themselves set policy based on a misspecified or unidentified Phillips curve (Haldane and Quah, 1999; Primiceri, 2006; Sargent, Williams and Zha, 2006). In these papers, mistakes or imperfect information on the part of policymakers can lead to changes in inflation expectations that cause the reduced-form Phillips curve to disappear.

In contrast, we suggest that a disappearing Phillips curve is also a natural consequence of successful monetary policy. The idea that improvements in monetary policy have flattened the slope of the reduced-form Phillips curve is often ascribed to researchers and policymakers at the Federal Reserve.[5] Most articulations of this view have tended to focus on the role of improved monetary policy in anchoring inflation expectations (e.g. Williams, 2006; Bernanke, 2007; Mishkin, 2007; Bernanke, 2010).[6]

Our point is closely related but distinct: even in a purely static setting in which expectations play no role, the structural relationship between slack and inflation can be masked by the conduct of monetary policy. This effect of monetary policy on the Phillips curve has also been highlighted at various times over the years in the literature and by policymakers (e.g. Haldane and Quah, 1999; Roberts, 2006; Mishkin, 2007; Carlstrom, Fuerst and Paustian, 2009; Bullard, 2018). In the context of forecasting, it is an example of the point made by Woodford (1994); if an indicator is a poor predictor of inflation that may just be because monetary policy is already responding to it appropriately.

Despite these contributions, a surprisingly bulky literature has not resisted the temptation to

[5] Gordon (2013) terms it the 'Fed view'.

[6] The effect of monetary policy on inflation expectations also features in some leading explanations of the 'missing disinflation' following the financial crisis, such as Del Negro, Giannoni and Schorfheide (2015).

keep searching, often against all odds, for a Phillips curve in the data. This paper sets out, in a simple framework, the key identification challenge that must be addressed, while also rationalising findings in various strands of the empirical literature and critically evaluating some of the practical solutions.

The paper is organised as follows. Section 2 introduces a simple model of optimal policy embedding the Phillips curve and illustrates the 'exogeneity result' or disconnect between equilibrium inflation and output gap under the assumption that the monetary authority cannot commit to a future path of inflation (discretion). Section 3 illustrates the empirical identification problem. Section 4 presents and discusses extensions of the model and notes some conceptual solutions to the identification problem. Section 5 examines the solutions in practice using national and metropolitan area data for the US. Section 6 contains concluding remarks.

2. Optimal inflation in the basic New Keynesian model

This section uses an optimal monetary policy framework to illustrate why, in equilibrium, one should expect inflation to follow a seemingly exogenous process, unrelated (or even negatively related) to measures of slack.

To explain the intuition as starkly as possible, we use the canonical New Keynesian model, as derived in Clarida, Gali and Gertler (1999), Woodford (2003) and elsewhere. Here we closely follow the textbook exposition from Galí (2008). For now, we dispense with the usual IS equation determining aggregate demand. This equation is necessary only to determine how policy is implemented. In the basic model it does not constrain equilibrium outcomes, so we can equivalently consider the policymaker as directly choosing the output gap as their policy instrument. Our model therefore consists of just two equations: a Phillips curve and a description of optimal monetary policy.

The (log-linearised) New Keynesian Phillips curve is given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \tag{1}$$

where π_t is the deviation of inflation from its target; x_t is the output gap, measured as the difference between output and its potential level[7] and u_t is a cost-push shock that follows an exogenous AR(1) process with persistence ρ ($u_t = \rho u_{t-1} + \epsilon_t$, where ϵ_t are i.i.d. and mean zero). We *assume* that the Phillips curve has a strictly positive slope, denoted by $\kappa > 0$.

The Phillips curve is evidently alive and well in the model: it is the *only* equation making up its non-policy block. By construction, we have a positively sloped Phillips curve. Increases in the output gap clearly increase inflation and falls in the output gap reduce it. Nonetheless, once we augment the model with a description of optimal monetary policy, this relationship will not be apparent in the data. Inflation will instead inherit the properties of the exogenous shock process u_t.

To show this, we assume that the policymaker sets monetary policy optimally under discretion. Period by period, she minimises the following quadratic loss function

$$L_t = \pi_t^2 + \lambda x_t^2$$

subject to the constraint (1) and taking expectations of future inflation as given.[8] The solution to the minimisation problem is the policymaker's optimal targeting rule

$$\pi_t = -\frac{\lambda}{\kappa} x_t \tag{2}$$

When faced with a positive cost-push shock that creates a trade-off between the inflation and output stabilisation objectives, the policymaker balances them, creating a negative output gap to reduce the degree of above-target inflation. The relative weight placed on each objective depends on the policymaker's preference parameter λ.

The Phillips curve (1) and optimal targeting rule (2) together completely determine the path of inflation in the model. We can solve for equilibrium inflation by using (2) to substitute out for x_t in (1), and by iterating forward to obtain

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda(1 - \beta\rho)} u_t \tag{3}$$

[7]In the full model derived in Galí (2008), this is the welfare-relevant gap between output and its efficient level.

[8]Clarida, Gali and Gertler (1999) show how minimising such a loss function is equivalent to maximising the welfare of the representative agent in the model. But it can alternatively be motivated as a simple way to capture the preferences enshrined in the mandates of modern (flexible) inflation targeting central banks: see Carney (2017*b*), for example.

In equilibrium, inflation deviations are at all times perfectly proportional to the exogenous cost-push shock. In other words, with a constant target, equilibrium inflation itself behaves as an exogenous process. In the limit, when the monetary authority does not put any weight on the output gap ($\lambda = 0$), inflation equals the target rate, a point previously made by Haldane and Quah (1999).

This behaviour is entirely consistent with recent empirical work by Cecchetti et al. (2017) and Forbes, Kirkham and Theodoridis (2017) suggesting that inflation data in the US and the UK can be modelled as an exogenous statistical process, unrelated to measures of slack.[9] But crucially, the basic theory is also built under the assumption that monetary policy is at all times constrained by a working Phillips curve. There is no discrepancy between the two results. The Phillips curve may be the correct structural model of the inflation process, but that does not mean that one should observe it in the empirical relationship between (equilibrium levels of) inflation and the output gap.

The reason is simple. The policymaker in the model is able to set policy to achieve any desired level of the output gap. Successful monetary policy should lean against any undesirable deviations in output from potential, which would otherwise cause inflationary or deflationary pressures. Precisely because monetary policy can be used to offset the effect of such output gaps on inflation, their effect should not be visible in the data.

Optimal monetary policy does not seek to eliminate all output volatility: from (2) we can see that in response to cost-push shocks, the policymaker will prefer to tolerate output deviations from potential. But such shocks impart a negative correlation between inflation and output, rather than a positive one. Again, the more successful monetary policy is in managing any trade-offs between inflation and output, the more it will blur the underlying positive Phillips curve correlation.

To summarise, we have shown that with an optimizing monetary policy, equilibrium levels of inflation are unrelated to measures of slack. Instead, inflation inherits the statistical properties of exogenous cost-push shocks. This does not necessarily tell us that the Phillips curve is not present. In the model, the Phillips curve exists and policymakers are completely aware of its existence. But because they know exactly how the curve operates, they are able to perfectly offset its effects on

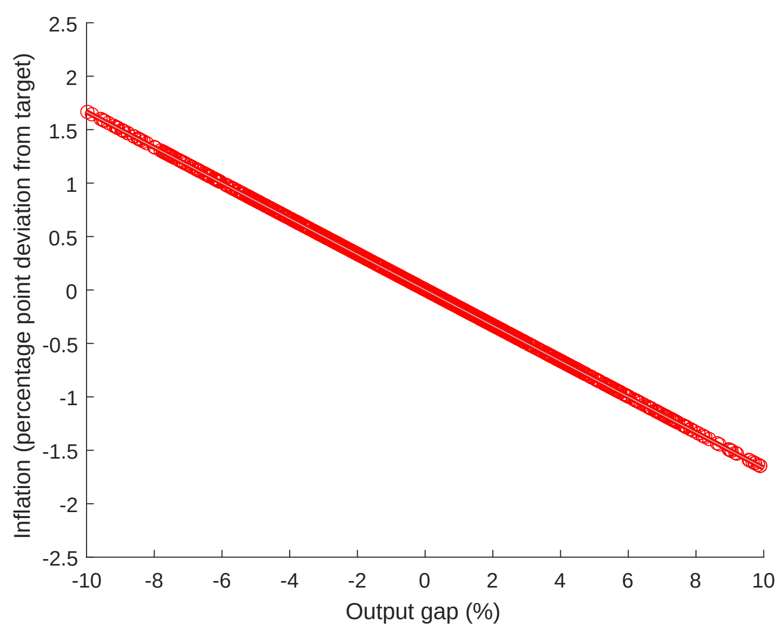
[9] It is also consistent with the observation that in larger DSGE models such as Smets and Wouters (2007), inflation is largely explained by exogenous markup shocks (King and Watson, 2012).

equilibrium inflation.[10]

3. Phillips curve identification

As may already be apparent from the discussion in Section 2, regression analysis will have difficulty in recovering the Phillips curve. Figure 1 shows data simulated from the model described by (1) and (2), with parameters calibrated as in Galí (2008). Specifically, the slope of the Phillips curve is set at $\kappa = 0.1275$, the policymaker's weight on output deviations relative to quarterly inflation is set as $\lambda = 0.0213$, or around one-third relative to annualised inflation. The discount factor is set to $\beta = 0.99$ and the persistence of the cost-push shock to $\rho = 0.5$.

Figure 1: *Inflation/output gap correlation in model-simulated data*

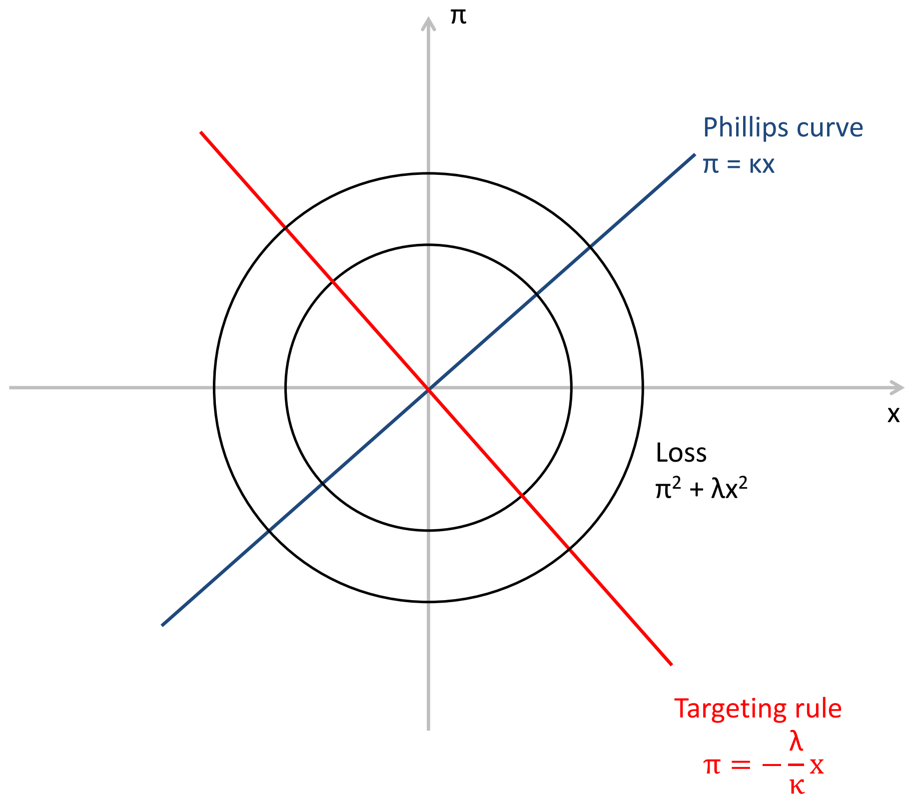


Notes: 1000 periods of data are simulated from the model described by (1) and (2). We draw each ϵ_t from a standard normal distribution.

Of course, there is no Phillips curve visible in the simulated data. As can be seen from the line

[10] Stock and Watson (2009) raise the possibility that, despite its failure to forecast or explain the data, the Phillips curve is still useful for conditional forecasting. They pose the question '...suppose you are told that next quarter the economy would plunge into recession, with the unemployment rate jumping by 2 percentage points. Would you change your inflation forecast?'

Figure 2: *Graphical illustration of optimal monetary policy under discretion*



of best fit, a naive OLS regression of inflation on the output gap,

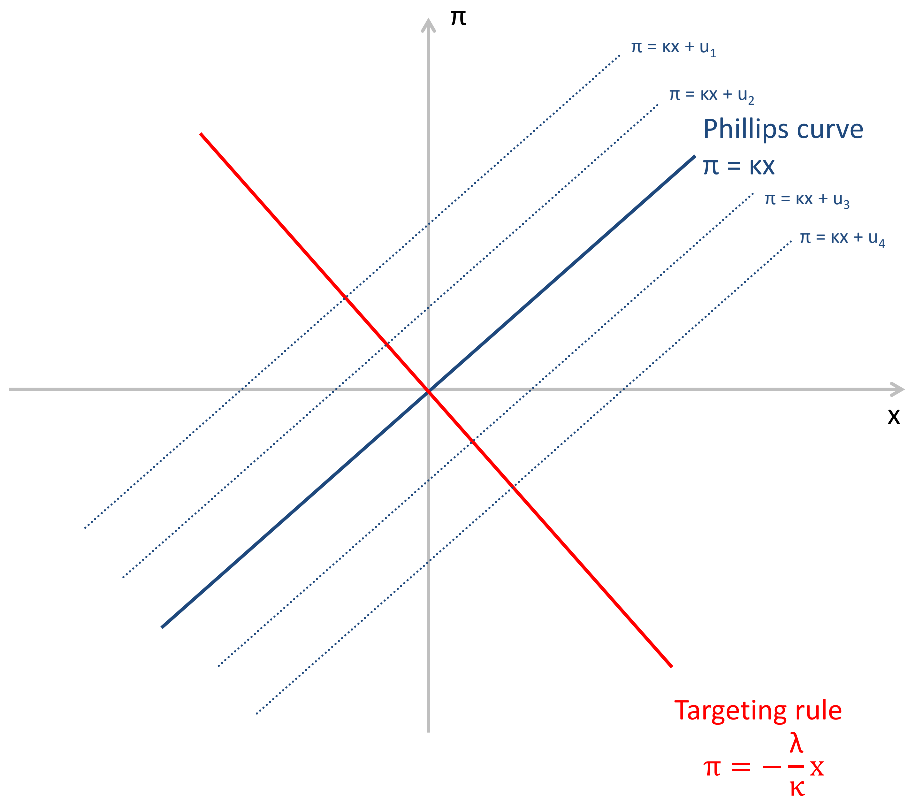
$$\pi_t = \gamma_1 x_t + \varepsilon_t \tag{4}$$

will produce a negative parameter estimate, $\hat{\gamma}_1 = -\frac{1}{6}$, reflecting the targeting rule (2), rather than a consistent estimate of the positive slope of the Phillips curve. Many papers have focused on the difficulty of controlling for inflation expectations in Phillips curve estimation, but the problem here is a more straightfoward one.[11]

The identification problem is a simple case of simultaneity bias. The regressor x_t is correlated with the error term ε_t. The naive econometrician does not observe the Phillips curve in the data. Rather, he or she observes equilibrium inflation and output gap outturns: which are the intersection of the Phillips curve (1) and the targeting rule (2). In fact, the case here is an extreme one: the

[11]See Nason and Smith (2008), Mavroeidis, Plagborg-Møller and Stock (2014) and Krogh (2015) for discussions.

Figure 3: *Graphical illustration of optimal discretionary policy in response to cost-push shocks*



regressor and the error are perfectly negatively correlated.[12]. The issue is completely analagous to the classic case of simultaneity bias: jointly determined supply and demand equations.

To show the identification challenge, we first plot the two model equations in Figure 2.[13] The Phillips curve (1) is in blue, the optimal targeting rule (2) in red, while the black circles index the policymaker's loss function at different levels of loss. The observed inflation-output gap pairs are the equilbrium where the two lines intersect. With no cost-push shocks to the Philips curve, the first-best outcome of at target inflation and no output gap is feasible, so the lines intersect at the origin.

When the upward sloping Phillips curve is subject to cost-push shocks, the equilibrium shifts to different points along the optimal targeting path, shown in Figure 3. But with monetary policy set optimally, there are no shifts along the Phillips curve: at all times the equilibrium remains on the

[12]Using (3) to substitute out for π_t in (2) gives the equilibirum evolution of the output gap $x_t = -\frac{\kappa}{\kappa^2+\lambda(1-\beta\rho)}u_t$. While the regression error term is equal to $\varepsilon_t = u_t + \beta E_t \pi_{t+1} = (1 + \frac{\rho\lambda}{\kappa^2+\lambda(1-\beta\rho)})u_t$

[13]This graphical illustration of optimal discretionary policy is from Seneca (2018): we are grateful to him for making it available to us. A similar graphical exposition also appears in Carlin and Soskice (2005).

negatively sloped optimal targeting rule line. As a result, the simulated data trace out the optimal targeting rule, not the Phillips curve. The estimated coefficient is $\hat{\gamma}_1 = -\frac{\lambda}{\kappa} = -\frac{1}{6}$.

The issue is that the Phillips curve is not identified. Our simple set-up has no exogenous variables shifting monetary policy. Worse, the only shocks are to the equation of interest, so the estimated parameter is almost entirely unrelated to the slope of the Phillips curve.[14] The problem is the same one that arises when trying to identify a supply curve while only observing equilibrium quantities and prices. Without any exogenous demand shifter, there is no way of doing so.

4. Extensions to the basic model and solutions to the estimation challenge

In this section we study a number of extensions to the basic model. For each extension, we discuss whether and how it can help solving the Phillips curve's empirical identification problem. In Subsection 4.1, we discuss the case in which the monetary authority can commit to a path of inflation and output gap. In Subsection 4.2, we allow for shocks to the targeting rule and we discuss how they link to the identified monetary policy shocks in the monetary policy transmission literature. In Subsection 4.3, we study a multi-region setting. In Subsection 4.4 we discuss the mapping into a wage Phillips curve. In Subsection 4.5 we extend our analysis to explore the effect of monetary policy on the Phillips curve in larger DSGE models.

4.1. Commitment

First, we show that our main results are unchanged when the monetary policymaker is able to commit to a future plan for inflation and the output gap. In Sections 2 and 3 we assumed that the policymaker was unable to commit. There are a range of practical issues that may make commitment difficult: monetary policy committees often have changes in membership and future policymakers may not feel bound by prior commitments and perhaps relatedly, successful commitment requires that promises are credible, even when they are time inconsistent. Nonetheless, the optimal commitment policy is able to achieve better outcomes in the face of cost-push shocks than optimal policy under discretion, so it is important to know how this affects our results.

It turns out that the same intuition holds, although the precise details slightly differ. Again

[14] Other than the fact that the slope of the Phillips curve happens to appear in the optimal targeting rule.

following Galí (2008), when the policymaker instead minimises the loss function

$$L = E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2) \tag{5}$$

subject to the sequence of Phillips curves given by (1) for each period. This gives a pair of optimality conditions

$$\pi_0 = -\frac{\lambda}{\kappa} x_0 \tag{6}$$

$$\pi_t = -\frac{\lambda}{\kappa}(x_t - x_{t-1}) \tag{7}$$

These can be combined to give the targeting rule under commitment

$$p_t = -\frac{\lambda}{\kappa} x_t \tag{8}$$

where p_t is the log deviation of the price level from its level in period -1. Substituting $p_t - p_{t-1}$ for π_t in (1) and substituting out x_t using (8) gives a difference equation in p_t. Galí (2008) shows the solution for this in terms of the previous period's price level and the current period cost-push shock. Iterating backwards and then taking the first difference gives equilibrium inflation

$$\pi_t = \frac{\delta}{1 - \delta\beta\rho}(u_t - (1-\delta)\sum_{i=0}^{t-1} \delta^{t-1-i} u_i) \tag{9}$$

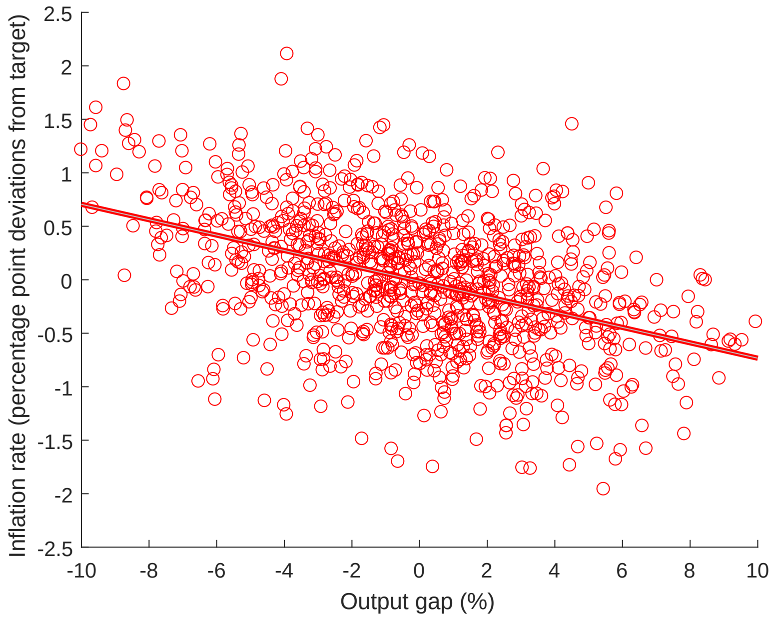
where $\delta \equiv \frac{((\lambda(1+\beta)+\kappa^2)-((\lambda(1+\beta)+\kappa^2)^2-4\beta\lambda^2)^{0.5})}{2\lambda\beta}$. Substituting into (7) and iterating backwards gives the equilibrium output gap

$$x_t = \frac{-\delta\kappa}{\lambda(1-\delta\beta\rho)} \sum_{i=0}^{t} \delta^{t-i} u_i \tag{10}$$

Equilibrium inflation under optimal commitment policy depends solely on the cost-push shock process. The equilibrium path is quite different to that under discretion, however. At any point in time inflation displays history dependence, depending on the entire history of cost-push shocks rather than just the one in the current period.

Simple regressions will again fail to uncover the Phillips curve. The only difference is that under commitment, the optimal targeting rule imposes a negative correlation between the output gap and

Figure 4: *Inflation/output gap correlation in model-simulated data: optimal commitment*



Notes: 1000 periods of data are simulated from the model described by (1) and (7). We draw each ϵ_t from a standard normal distribution.

the *price level*. The relationship between inflation and the output gap in the simulated data shown in Figure 4 is noisier, but shows no sign of the Phillips curve embedded in the model. The OLS estimate of γ in (4) gives the coefficient $\hat{\gamma}_1 = -0.085$.

At least in the simple framework here, the history-dependence of optimal commitment policy also suggests a straightforward solution to the identification problem. From (10), the equilibrium output gap will be correlated with its own lagged values, which can therefore be used as an instrument. Intuitively, the policymaker chooses to create an output gap even after the cost-push shock has disappeared. They commit to do so in order to achieve better inflation outcomes when the shock originally occurs. The policymaker therefore optimally reintroduces the positive Phillips curve relation that is absent under optimal discretion. As a result, in the simple case here, a suitable choice of instrument will be able to recover the true Phillips curve slope.

4.2. Shocks to the targeting rule

The previous sections have illustrated how successful monetary policy might mask the underlying structural Phillips curve in the data. We now show that the opposite is also true in our model: if monetary policy is set far from optimally, the Phillips curve is likely to reappear.

So far we have assumed policymakers can implement monetary policy by directly choosing their desired observable output gap each period. But alas in practice, policymaking is not quite so simple. In empirical studies we observe lags between changing policy and its impact on the output gap and inflation, which means that in practice central banks are inflation *forecast* targeters (Svensson, 1997; Haldane, 1998). Forecast errors will therefore inject noise into the targeting rule. Potential output is unobservable, so the output gap must be estimated (with error). And the effect of the policy instruments actually available (typically the central bank policy rate and forward guidance on its future path; as well as quantitative easing) on the target variables is also unknown. Errors from any of these sources will insert noise into the desired balance between inflation and output gap deviations. These various shocks to the targeting rule correspond closely to the typical interpretations of identified monetary policy shocks in the empirical literature on this topic (Christiano, Eichenbaum and Evans, 1996, 1999; Romer and Romer, 2004*b*; Faust, Swanson and Wright, 2004; Bernanke, Boivin and Eliasz, 2005; Olivei and Tenreyro, 2007; Gertler and Karadi, 2015; Cloyne and Hürtgen, 2016). That literature is able to identify a positively correlated response of inflation and the output gap to monetary policy shocks, in line with the results below.

Returning to optimal policy under discretion, we model implementation errors by including an AR(1) shock process e_t in the targeting rule (2) to give

$$\pi_t = -\frac{\lambda}{\kappa} x_t - e_t \tag{11}$$

where $e_t = \rho_e e_{t-1} + \zeta_t$ and ζ_t is zero-mean and i.i.d. with variance σ_e^2.[15] We can show that equilibrium inflation and the output gap now both have an additional term proportional to e_t. Respectively, they are given by $\pi_t = s_1 \lambda u_t - s_2 \kappa e_t$ and $x_t = -s_1 \kappa u_t - s_2(1-\beta\rho_e)e_t$, where $s_1 \equiv \frac{1}{\lambda(1-\beta\rho)+\kappa^2}$ and $s_2 \equiv \frac{\kappa}{\lambda(1-\beta\rho_e)+\kappa^2}$.

With shocks to the targeting rule, neither equation is identified. The equilibrium values of inflation and the output gap both depend on a combination of both shocks. Consequently, if either

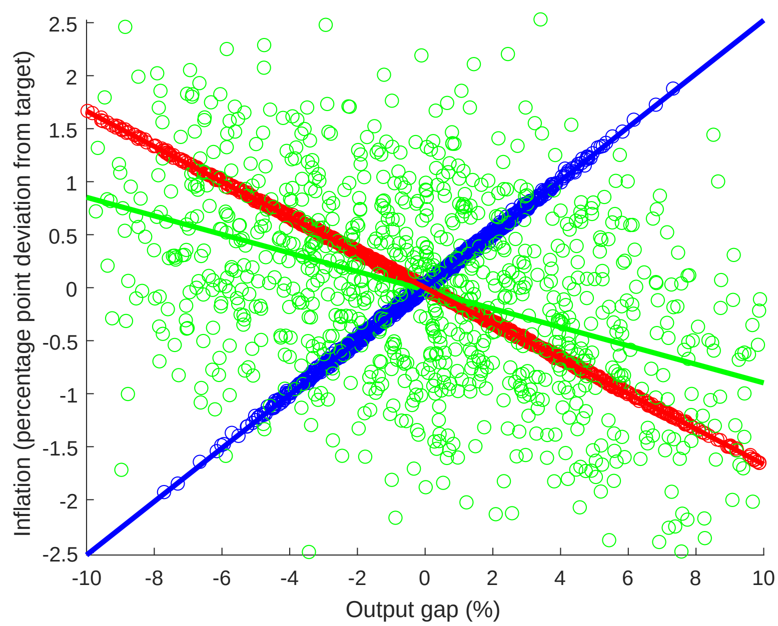
[15] Clarida, Gali and Gertler (1999) and Svensson and Woodford (2004) show in the basic New Keynesian model that when there are policy control lags that mean all variables are predetermined in advance, up to an unforecastable shock, the optimal targeting rule will take exactly this form, where e_t is the forecast error. We subtract it from the right hand side of (11) to match the usual convention that a positive monetary policy shock involves a policy tightening.

equation is estimated by OLS, its regressor will be correlated with the regression error term and the resulting parameter estimate inconsistent. In particular, it follows from substituting the equilibrium values of π_t and x_t into the definition of the OLS estimator in the regression (4) that

$$\text{plim}(\hat{\gamma}) = \frac{\text{plim}(\frac{1}{T}\sum_{t=1}^{T} x_t \pi_t)}{\text{plim}(\frac{1}{T}\sum_{t=1}^{T} x_t^2)} = \frac{\frac{-\lambda}{\kappa}\frac{s_1^2(1-\rho_e^2)}{s_2^2(1-\rho^2)}\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2} + (1-\beta\rho_e)\kappa\frac{\sigma_e^2}{\sigma_u^2+\sigma_e^2}}{\frac{s_1^2(1-\rho_e^2)}{s_2^2(1-\rho^2)}\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2} + (1-\beta\rho_e)^2\frac{\sigma_e^2}{\sigma_u^2+\sigma_e^2}} \tag{12}$$

The size of the simultaneity bias to each equation depends on the relative variances of the shocks.[16]

Figure 5: *Inflation/output gap correlation in model-simulated data: optimal discretion with shocks to the targeting rule*



Notes: 1000 periods of data are simulated from the model described by (1) and (11). The green circles show the case when each ϵ_t and ζ_t is drawn from a standard normal distribution. The blue circles show the case when each ϵ_t is drawn from an N(0,10) distribution and the red circles each ζ_t is instead drawn from an N(0,10) distribution.

Figure 5 plots simulated data for three cases. We set $\rho_e = 0.5$ and set the other parameters as before. First, the red circles show the case where the cost-push shock has a variance 100 times larger than the targeting rule shock. These look almost identical to the case with only a cost-push shock: the circles trace out the targeting rule. Second, the green circles show the case when the shocks have equal variance. The slope is still negative, but flatter. The final case gives the cost-push shock a variance 100 times smaller than the targeting rule shock, and the data trace out a positively sloped

[16] Carlstrom, Fuerst and Paustian (2009) show a similar equation to illustrate the OLS estimate bias in their framework.

line.

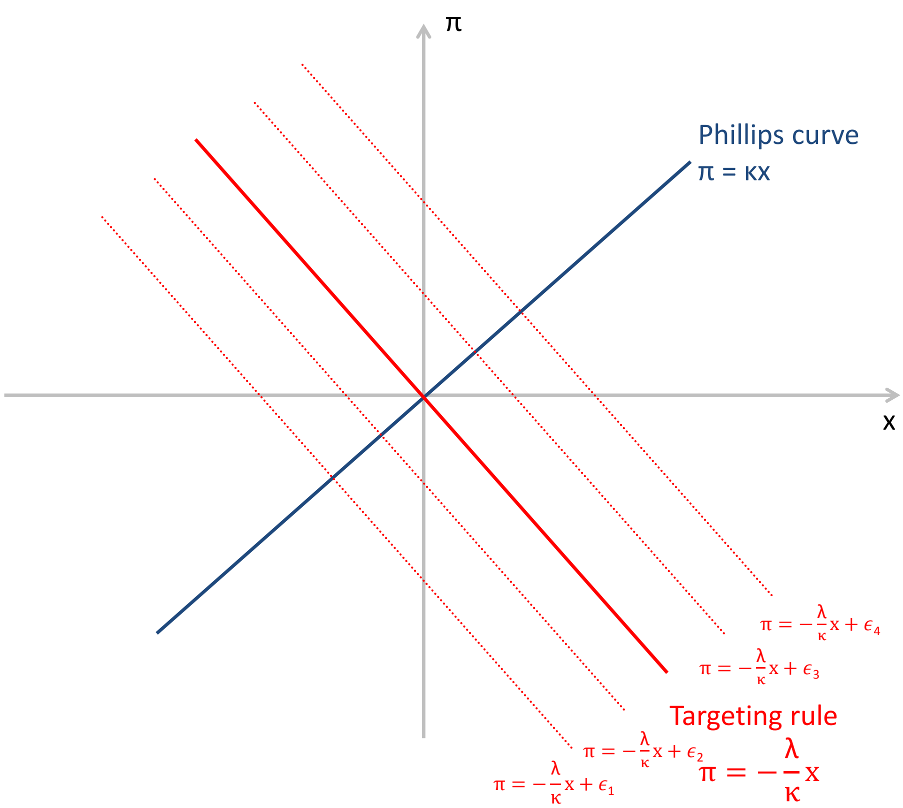
Table 1: *OLS regressions of inflation on the output gap in the simulated data*

	(1)	(2)	(3)	(4)	(5)	(6)
	$\frac{\sigma_u^2}{\sigma_e^2} = 100$	$\frac{\sigma_u^2}{\sigma_e^2} = 100$	$\frac{\sigma_u^2}{\sigma_e^2} = 1$	$\frac{\sigma_u^2}{\sigma_e^2} = 1$	$\frac{\sigma_u^2}{\sigma_e^2} = 0.01$	$\frac{\sigma_u^2}{\sigma_e^2} = 0.01$
LHS variable	π_t	$\pi_t - E_t\pi_{t+1}$	π_t	$\pi_t - E_t\pi_{t+1}$	π_t	$\pi_t - E_t\pi_{t+1}$
x_t	-0.1667	-0.1805	-0.0873	-0.0792	0.2523	0.1275

Notes: Table shows the OLS regression coefficients of OLS for the shock distributions described in the notes to Figure 5. Specifications (2), (4) and (6) (perfectly) control for inflation expectations by subtracting from the dependent variable the true value of $\beta E_t \pi_{t_1}$. The true slope of the Phillips curve is $\kappa = 0.1275$, while the true slope of the optimal targeting rule is $-\frac{\lambda}{\kappa} = -0.1667$.

Looking at the regression coefficients in Table 1, in the first two cases these are both strongly influenced by the endogenous policy response embodied in the optimal targeting rule. It also makes little difference whether or not the econometrician correctly controls for inflation expectations, which also enter the Phillips curve. In the third case however, the regression coefficient turns positive. The estimate is actually upward biased in specification 5, which omits inflation expectations. Once these

Figure 6: *Graphical illustration of optimal discretionary policy in response to targeting-rule shocks*



are controlled for, the bias becomes very small. The regression correctly identifies the slope of the Phillips curve to four decimal places.

The reason the bias disappears is straightforward. When cost-push shocks have a relatively low variance, most of the variation in the simulated data arises from the shocks to the targeting rule. With the Phillips curve stable, these movements in the targeting rule now trace out the Phillips curve, as shown graphically in Figure 6. This suggests that if we can successfully control for the cost-push shocks u_t in (1), then we may be able to limit the bias in estimates of the Phillips curve.

4.3. Regional Phillips curves

Partly to avoid the difficulties associated with identifying the Phillips curve at the national level, a number of authors have estimated Phillips curves at a more disaggregated, regional or sectoral level (Fitzgerald and Nicolini, 2014; Kiley, 2015; Babb and Detmeister, 2017; Leduc and Wilson, 2017; Tuckett, 2018; Vlieghe, 2018; Hooper, Mishkin and Sufi, 2019). In this subsection we show that in an extended version of the basic model, this may also help the econometrician to identify the aggregate Phillips curve.

The key to identification is that at the regional level, the endogenous response of monetary policy to demand shocks is switched off, ameliorating the simultaneity bias in estimating aggregate Phillips curves. This point was made by Fitzgerald and Nicolini (2014) as motivation for their estimation of Phillips curves at a regional level. The same logic can explain why the Phillips curve may be more evident in countries within a monetary union such as the euro area.[17]

We assume that the aggregate Phillips curve (1) continues to hold, but that aggregate inflation and the aggregate output gap also depend on the weighted average of inflation and the output gap in each of n regions

$$\pi_t = \sum_{i=1}^{n} \alpha_i \pi_t^i \tag{13}$$

$$x_t = \sum_{i=1}^{n} \alpha_i x_t^i \tag{14}$$

[17] Nakamura and Steinsson (2014) present evidence that endogenous monetary and tax policies reduce national fiscal multipliers relative to local ones.

where $\sum_{i=1}^{n} \alpha_i = 1$ and regional inflation is determined by a regional Phillips curve analogous to (1)

$$\pi_t^i = \beta E_t \pi_{t+1}^i + \kappa x_t^i + u_t^i \tag{15}$$

with idiosyncratic cost-push shocks $u_t^i = \rho u_{t-1}^i + \epsilon_t^i$ and ϵ_t^i zero-mean and i.i.d over time, but potentially correlated across regions. We must also specify how idiosyncratic demand shocks and aggregate monetary policy affect the regional output gap with an equation analogous to the IS curve in the basic New Keynesian model, given by

$$x_t^i = E_t x_{t+1}^i - \sigma^{-1}(i_t - E_t \pi_{t+1}^i - r_t^i) \tag{16}$$

where the idiosyncratic demand shocks are given by $r_t^i = \rho_r r_{t-1}^i + e_r^i$ and e_r^i are zero-mean and i.i.d over time, but potentially correlated across regions. The equations can be aggregated together to give the usual aggregate IS relation

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_t) \tag{17}$$

We therefore allow inflation and the output gap are determined partly by idiosyncratic shocks to each region, but restrict the monetary policy rate i_t to be the same across all n regions.

We next denote for any regional variable its (log) deviation from the aggregate as $\hat{z}_t^i = z_t^i - \sum_{i=1}^{n} \alpha_i z_t^i$. We can then subtract (1) from (15) to give a Phillips curve in terms of log deviations from aggregate inflation.

$$\hat{\pi}_t^i = \beta E_t \hat{\pi}_{t+1}^i + \kappa \hat{x}_t^i + \hat{u}_t^i \tag{18}$$

Subtracting (17) from (16) gives an equivalent IS curve

$$\hat{x}_t^i = E_t \hat{x}_{t+1}^i + \sigma^{-1}(E_t \hat{\pi}_{t+1}^i + \hat{r}_t^i) \tag{19}$$

Monetary policy is set (under discretion) by minimising the same aggregate period loss function as in Section 2, subject to the aggregate Phillips curve (1).[18] Policy therefore follows the same targeting rule (2) depending solely on aggregate variables.[19]

The crucial difference to the identification problem at the regional level is that while monetary policy perfectly offsets the aggregate demand shocks, $r_t = \sum_{i=1}^{n} \alpha_i r_t^i$, it does not respond at all to the idiosyncratic regional deviations from that average, $\hat{r}_t^i$. The regressor in the Phillips curve equation $\hat{x}_t^i$ is now affected by exogenous demand shocks that do not influence the aggregate Phillips curve. As a result, the endogeneity problem is mitigated.

For each region, we can verify that one solution to the model described by (18) and (19) is

$$\hat{\pi}_t^i = c_1(1-\rho)\hat{u}_t^i + c_2\kappa_i\hat{r}_t^i \tag{20}$$

and

$$\hat{x}_t^i = c_1\rho\sigma^{-1}\hat{u}_t^i + c_2(1-\rho_r\beta)\hat{r}_t^i \tag{21}$$

where $c_1 \equiv \frac{1}{(1-\rho)(1-\rho\beta)-\rho\kappa\sigma^{-1}}$ and $c_2 \equiv \frac{\sigma^{-1}}{(1-\rho_r)(1-\rho_r\beta)-\rho_r\kappa\sigma^{-1}}$.[20] Unlike aggregate inflation, which evolves in line with the exogenous shocks to the Phillips curve, regional inflation also depends on idiosyncratic demand shocks. In the simplest case when the shocks are entirely transitory ($\rho = \rho_r = 0$), the equilibrium output gap deviation will be independent of the idiosyncratic cost-push shocks $\hat{u}_t^i$ and a simple regression of $\hat{\pi}_t^i$ on $\hat{x}_t^i$ will give a consistent estimate of κ.

Away from that special case, inflation expectations present a separate challenge to identifying regional Phillips curves. With $\rho > 0$ or $\rho_r > 0$, there will be omitted variable bias unless the econometrician can control for the effect of regional inflation expectations. While possible in principle, reliable data are likely to be less readily available than at the national level. If cross-sectional variation in inflation expectations is important, there is perhaps likely to be more chance of success when estimating at the country level within a single multi-country monetary authority.

[18] This differs from the monetary policy that would be welfare-optimal in the model, since welfare would also be lowered by dispersion in prices within a region, even if average inflation was zero. Clarida, Gali and Gertler (2001) show in the context of an open economy model that the welfare-optimal policy would minimise a loss function that included the sum across countries of the squared deviations of inflation, rather than the square of the sum of deviations.

[19] Although to ensure determinacy, the policymaker's instrument rule will need to respond to idiosyncratic variables.

[20] While this is one solution, depending on how policy is implemented, there may be a multiplicity of equlibria. It is beyond the scope of this paper to study those, so we assume that the policymaker's instrument rule is able to rule them out. In practice, this will involve responding to deviations of regional inflation or regional output gaps from their equilibrium values, even when those deviations have no impact on aggregate inflation or the aggregate output gap.

Alternatively, if that variation is constant over time, it can be controlled for using region fixed effects. A second difficulty at the regional level is that while the specification will help mitigate the bias from the endogeneity of national monetary policy, insufficient cross-sectional variation in the regional data will lead to imprecise estimates of κ.

4.4. The wage Phillips curve

While identification of the price Phillips curve is complicated by the endogenous response of optimal monetary policy, the focus of the original Phillips study was the correlation between wage inflation and unemployment in the UK. In this subsection we comment on how optimal monetary policy maps into the original wage Phillips curve relationship between wage inflation and unemployment. Intuitively, one might expect the wage Phillips curve to be less vulnerable to identification issues related to the endogeneity of monetary policy, since wage inflation is one step removed from the price-inflation targeting remit of most central banks.

As well as a different dependent variable (wage inflation rather than price inflation), the typical wage Phillips curve attempts to explain inflation using variation in unemployment or the unemployment gap, rather than the output gap. Using unemployment in the equation is unlikely to solve the identification issues arising from the behaviour of monetary policy for at least two reasons.

First, many central banks' remits explicitly specify unemployment or employment as one of their (secondary or dual) target variables. As such, they will optimally set policy to close any gap between unemployment and its natural rate, unless there is a trade-off between that goal and their inflation targets, in which case they will seek to balance the two goals, as was the case with the output gap in Section 2. Monetary policy will therefore blur the structural relationship between inflation and the unemployment gap in a similar way. Second, even for central banks without an explicit mandate to minimise fluctuations in employment, when there is co-movement between the output gap and the unemployment gap, policy will often implicitly seek to stabilise employment.[21]

There are, however, reasons to think that using wage inflation as the dependent variable might lessen some of the identification problems. Nominal wage rigidities can be incorporated into the basic model in an analogous way to price rigidities, as introduced by Erceg, Henderson and Levin

[21] Galí (2011) shows how the basic framework can be easily extended to include unemployment in a way that closely resembles the output gap in the basic model.

(2000). With both wage and price stickiness, some shocks, such as innovations to firms' desired price-markups, will lead to a wedge between the rate of price inflation and the output gap, but not between the rate of wage inflation and the output gap. Since inflation targeting central banks typically target price inflation, policymakers may respond by adjusting the output gap to achieve their desired trade-off with price inflation. But doing so would lead to variation in wage inflation operating via the wage Phillips curve. Put differently, the equilibrium output gap will be a function of the exogenous shocks hitting the economy. But if those shocks only directly affect the price Phillips curve and not the wage Phillips curve, then the output gap will be correlated with the error term in the former but not the latter, which will be consistently estimated.

The wage Phillips curve may not face quite as severe problems, but there remain limits to how easily it can be identified under optimal monetary policy. First, while there may be some shocks that only affect the price Phillips curve, there are likely to be several more that affect both curves (for a given output gap). Wage mark-up shocks will increase both price and wage inflation relative to the prevailing output gap. Erceg, Henderson and Levin (2000) show that shocks to household consumption or leisure preferences, or to total factor productivity, will conversely move price and wage inflation in opposite directions for a given output gap. Since the inflationary impact of these shocks will lead policymakers to attempt to lean against them via the output gap, this will induce a correlation between the output gap and the shocks affecting the wage Phillips curve (for a given output gap). The direction of the bias will differ according to the shock, but the equation will in general not be identified.

Second, even if price inflation shocks are particularly prevalent, many typical examples of such shocks, such as changes in oil prices, have relatively transitory effects on price inflation. Since monetary policy is typically thought to have its peak effect on inflation with some lag, attempting to offset very transitory shocks may not be possible. As a result, policymakers are perhaps less likely to respond to the very shocks that would otherwise have helped econometricians identify the wage Phillips curve. Conversely, when transitory shocks are affecting price inflation, wage inflation can sometimes give a better signal of underlying price pressures, which may lead policymakers to behave at times as if they were targeting wage inflation.[22]

4.5. Larger DSGE models

In addition to nominal wage rigidities, larger macroeconomic models of the type used for policy analysis in central banks usually have a range of other frictions, additional factors of production and a richer dynamic structure.[23] In this subsection we study how the intuition underlying Phillips curve identification in the basic New Keynesian model translates to the aggregate supply relationship in larger models.

An overriding conceptual issue in larger DSGE models is that there typically is no single, stable Phillips curve relationship between inflation and the output gap. In the basic model the output gap is proportional to firms' real marginal costs, but this is a special case that does not generalise to larger models. The reduced-form Phillips curve correlation therefore varies for different shocks.

We illustrate this point in Figure A1 in the Appendix, which shows the inflation-output gap relationship in a large-scale DSGE model *conditional* on each type of shock in the model. We use the COMPASS model, described in Burgess et al. (2013), which was designed for forecasting and policy analysis at the Bank of England. The model is in the tradition of well-known medium-scale DSGE models such as Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007), in which similar findings would emerge, as well as DSGE models used in other central banks. The simulated Phillips curve varies markedly depending on the shock. Conditional on demand-type shocks, such as to government spending or world demand, there is a positive relationship between inflation and the output gap. Conditional on cost-push type shocks to wage or price markups, the correlation turns negative.

Even when we restrict our attention to those shocks we typically think of as demand, there are different reduced-form Phillips curves for different shocks: the investment adjustment cost shock has a slope over twice as steep as a government spending shock, for example. These different reduced-form slopes arise for several reasons. First, the shocks do not all have the same impact

[22] In addition, the welfare optimal policy in models with sticky wages typically involves placing a positive weight on avoiding wage inflation (Erceg, Henderson and Levin, 2000). But we are not aware of any central banks who officially target wage inflation in practice.

[23] See for example Edge, Kiley and Laforte (2010); Burgess et al. (2013); Brubakk and Sveen (2009); Adolfson et al. (2013) for descriptions of models used respectively at the Federal Reserve Board, the Bank of England, Norges Bank and the Riksbank.

on the output gap relative to real marginal costs and inflation. Second, they each have different dynamic effects (some shock processes are estimated to be more persistent than others, for example), which influences the contemporaneous Phillips curve correlations. And related to both points, the simulations incorporate an endogenous monetary policy response via the model's Taylor rule. While the Taylor rule is not sufficient to hide the positive Phillips curve relationships completely, it will be exerting some influence, the scale of which will depend on the specific shock.

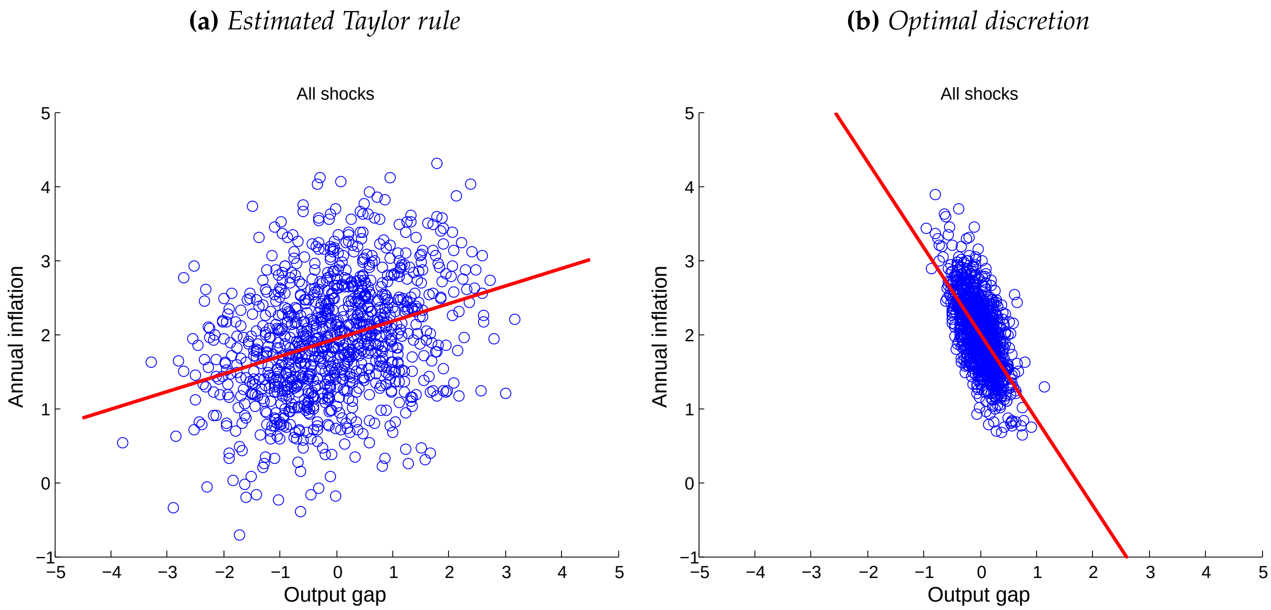
Given these conceptual difficulties, how should we think of the Phillips curve in larger DSGE models? One interpretation, consistent with the Phillips curve's inception as an empirical regularity in the UK data, is that is simply the average reduced-form relationship, conditional on a demand shock having occurred. The slope of such an object would clearly change over time if some types of shock became more or less frequent. It would also be vulnerable to the Lucas critique. But if policymakers judged that such changes were relatively slow-moving, they may still find such an empirical Phillips curve a useful input into their decisions.

Under that interpretation, the logic we have outlined for the basic model continues to complicate estimation of empirical Phillips curves in larger models. Figure 7a shows another DSGE simulation using Burgess et al. (2013), this time for all shocks in the model. Despite the presence of supply shocks and an endogenous monetary policy response, a positively sloped Phillips curve emerges.

Figure 7b runs an otherwise identical simulation with the model's Taylor rule replaced by the optimal monetary policy under discretion. As in the examples from the basic model, the positively sloped Phillips curve disappears and its estimated sign turns negative. This is true irrespective of the shock: Figure A2 in the Appendix shows the correlation under discretion conditional on each shock. In this more complex setting, the reduced-form slope does not represent any single optimal targeting rule. But the same intuition continues to hold: monetary policy will seek to minimise any variation in the output gap that would cause inflation to move in the same direction. Conversely, following a markup (or cost-push) shock, monetary policy will aim to reduce the output gap at times when inflation is above target.

Even in larger models, we would argue one can still interpret the Phillips curve as a structural equation. Although they need not feature a simple structural relationship between inflation and the output gap, larger New Keynesian models will contain some kind of equivalent aggregate supply

Figure 7: *Inflation/output gap correlation in simulated data from a large-scale DSGE model.*



Notes: 1000 periods of data are simulated from the model in Burgess et al. (2013) using the MAPS toolkit described in the same paper. Each period a set of unanticipated shocks are drawn independently from a standard normal distribution. The red lines show the lines of best fit from an OLS regression of the simulated annual inflation data on the (contemporaneous) flexible price output gap. The first panel shows the results using the estimated Taylor rule in the model. The second panel replaces the Taylor rule with the optimal discretionary monetary policy, where the policymaker minimises, period by period, an ad hoc loss function containing the discounted sum of squared deviations of annual inflation from target (with a weight of 1) and the output gap (with a weight of 0.25). The solution is calculated using the algorithm of Dennis (2007).

constraint. Typically this will contain measures of real marginal costs rather than the output gap.[24] It is also likely to have a richer dynamic structure. Given that structure and wider variety of shocks, if one is able to estimate the full structural model and there is enough variation in the data, then it may be possible to recover any structural aggregate supply relationship. But precisely because we do not know the true model of the economy, such an approach may be less robust to misspecification than the empirical Phillips curve described above.

Moreover, as long as the structural aggregate supply relationship can be specified as a relationship between inflation and some measure of slack, then the identification issues we raise in the simple model may still apply. In Burgess et al. (2013), the Phillips curve for consumer price inflation is a function of past and future inflation; the marginal cost of final output production; and a markup shock. Figure A3 in the Appendix shows simulated data from the model under a specification of

[24] In the model simulated above, there is a more stable positive relationship across different shocks between inflation and the relevant measure of real marginal costs than with the output gap.

optimal discretionary policy where the policymaker targets inflation and (instead of the output gap) the marginal cost of final output production. Just as with the effect of demand shocks on the output gap in the basic model, the policymaker is able to perfectly offset the effect of all shocks on the marginal cost. In equilibrium, the only shock that has any effect on the policymaker's chosen target variables is the markup shock, which creates a trade-off between them.

These findings from a larger model designed for practical policy use in central banks suggest another source of variation to identify the structural Phillips curve or aggregate supply relationship. If the measure of slack targeted by the policymaker is different to the one that directly influences inflation, then the policymaker will not seek to offset all variation in the inflation-relevant measure. In the example above, if the policymaker seeks to minimise fluctuations in the output gap this will not always minimise movements in real marginal costs, since the relationship between the two measures of slack will vary according to the mix of shocks. The reasoning is analogous to the discussion of the wage Phillips curve in the previous section. The policymaker's actions will only blur the structural Phillips curve in equilibrium to the extent her policy targets are correlated with the measures of inflation and slack in the aggregate supply relationship.

5. Solutions to the estimation challenge in practice

In this section we examine Phillips curve identification in practice using US data. The previous subsection suggested at least three ways econometricians could recover the structural Phillips curve:

1. Supply shocks: if we can control for these well enough, we should be able to recover the Phillips curve.

2. Instrumental variables: with good instruments for the output gap, uncorrelated with cost-push shocks, then the structural Phillips curve can be recovered.

3. Regional data: monetary policy does not offset regional demand shocks, while time fixed effects can control for aggregate supply shocks.

In summary, the identification challenge arises from the presence of cost-push shocks to the Phillips curve and the partial accommodation of these by monetary policymakers. The size of the simultaneity bias is magnified because monetary policy seeks to offset any demand shocks that, in practice, might otherwise help identify the curve.

Each solution attempts to circumvent these issues by isolating the remaining demand-driven variation in inflation. The first two solutions use aggregate time-series data and the third turns to the regional cross-section. While a large number of papers have estimated Phillips curves without addressing the identification issue we raise here, many others over the years have followed one or more of these approaches, either implicitly or explicitly. Our discussion provides a framework which ties together these different solutions.

The econometric solutions to simultaneity in economics are well known. And econometricians will no doubt continue to come up with other innovative ways to successfully identify Phillips curves.[25] But there are reasons to think that using aggregate data, the task is likely to become ever more difficult. Boivin and Giannoni (2006) first showed that monetary policy shocks had become smaller in the period since the early 1980s, while similar arguments have recently been made by Ramey (2016). Both suggest that in economies such as the US, with established policy frameworks, policy is now largely conducted systematically, which limits the remaining exogenous variation in aggregate demand needed to recover the Phillips curve.

An alternative avenue, therefore, is to turn to cross-sectional data. As in Fitzgerald and Nicolini (2014), we next show that using regional data on inflation and unemployment by metropolitan area, a steeper Phillips curve re-emerges.

5.1. The empirical Phillips curve in the aggregate data

For our empirical exploration, we turn our attention to the US, where Phillips' UK findings were translated by Samuelson and Solow (1960). Our inflation data are the (seasonally adjusted) quarterly annualised log change in *core* CPI inflation. While PCE inflation has been the FOMC's preferred measure since 2000, for most of our sample monetary policy focused on CPI inflation.[26] It also allows us to more readily compare with the US regional price data, which is a CPI measure. Using core inflation rather than headline is a straightforward mechanical way of stripping out a subset of

[25] See Barnichon and Mesters (2019), Galí and Gambetti (2018) and Jorda and Nechio (2018) for some recent examples, discussed further below.

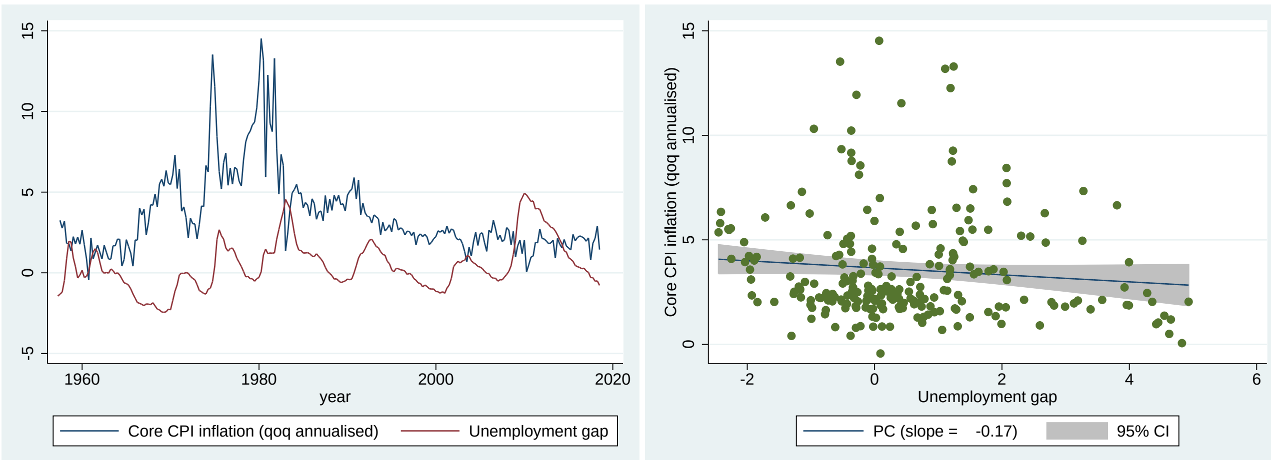
[26] Board of Governors of the Federal Reserve System (2000).

the cost-push shocks affecting headline inflation, in line with our first solution above.

Again for comparability with the regional data, we use the (seasonally adjusted) quarterly unemployment gap as our proxy for slack, measured as the civilian unemployment rate less the CBO estimate of the long-term natural rate of unemployment. Using the unemployment gap, we would therefore expect to see a negative structural relationship with inflation. Figure 8 plots the

Figure 8: *US core CPI inflation and the unemployment gap: 1957 Q1 to 2018 Q2*

(a) *Time series* **(b)** *Scatter plot*



Notes: Figures show plots of quarterly annualised core CPI inflation against the CBO estimate of the unemployment gap. Phillips curve slope and the confidence interval around it is estimated using OLS.

two time series, alongside a simple scatter plot of the data over our sample period of 1957-2018. The reduced-form Phillips curve slope is flat and not significantly different from zero. But as is clear from the time series and has been well-documented elsewhere, the full sample masks a great deal of time variation in the relationship.

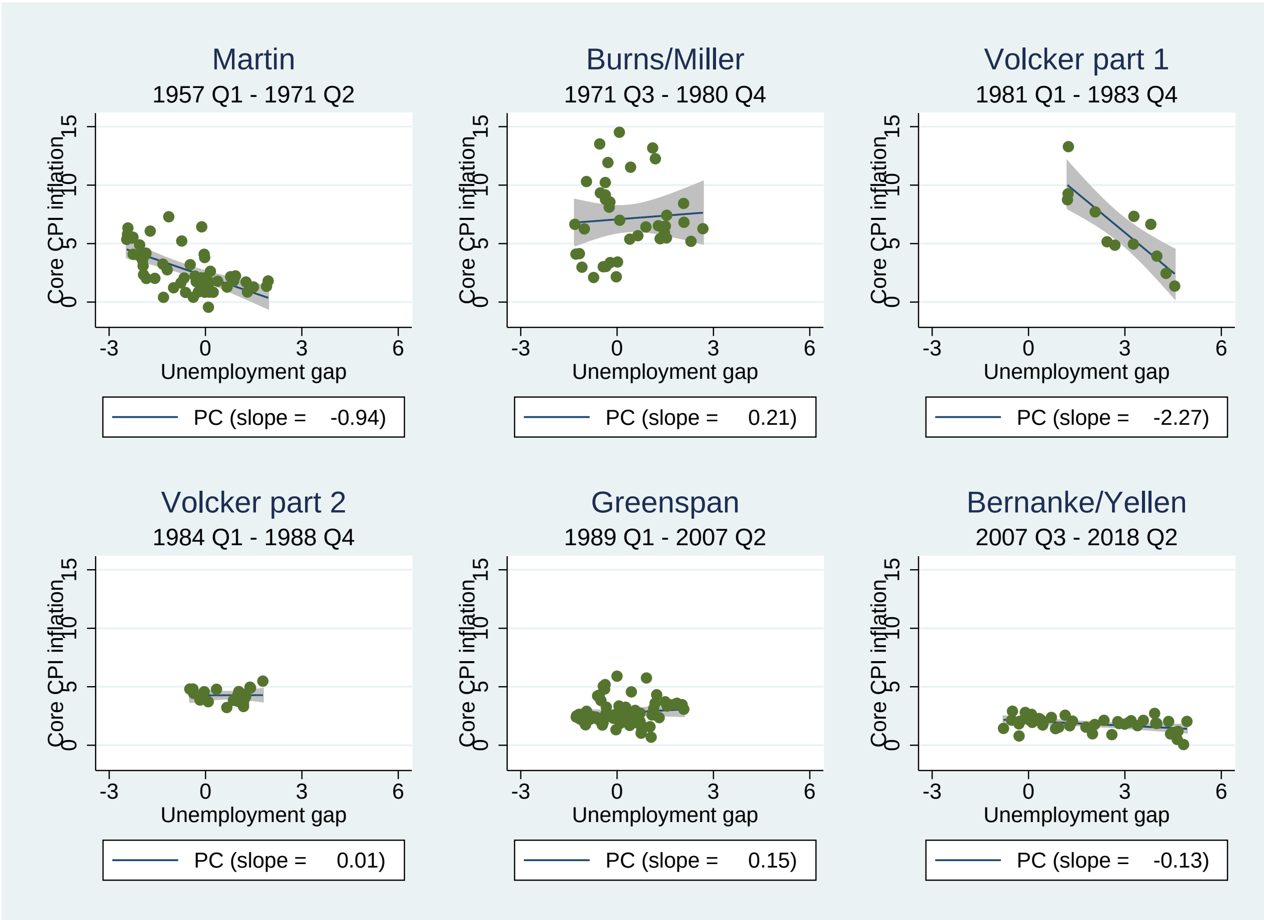
Figure 9 shows how the correlation has varied over time. We split the time periods according to Fed Chair over our sample period.[27] We split Paul Volcker's chairmanship into two periods, given the very different inflation and output dynamics at the start and end of his tenure.[28]

The data can be explained with the traditional narrative of the US Phillips curve over the second half of the 20th century, as discussed in histories by King (2008) and Gordon (2011). In the latter

[27]We also lag the tenure dates by six quarters to reflect the lags between monetary policy actions and their effect on real activity and inflation. Christiano, Eichenbaum and Evans (2005) and Boivin and Giannoni (2006) both find that monetary policy has its peak impact on output after around four quarters, and on quarterly inflation after eight quarters.

[28]We split the sample at the end of 1983 in line with convention in dating the Volcker disinflation (Goodfriend and King, 2005).

Figure 9: *Phillips correlation by Fed Chair*



Notes: Figure shows scatter plots of quarterly annualised core CPI inflation against the CBO estimate of the unemployment gap, split by time period. We lag the tenure dates of each Fed chair by six quarters as a way of reflecting the lags between monetary policy actions and their effect on real activity and inflation. Phillips curve slopes and confidence intervals are estimated using OLS.

years of William McChesney Martin's 23 year term, with the Phillips curve viewed as an exploitable long-run trade off, overly accommodative fiscal and monetary policies led to unemployment falling steadily below today's estimate of its natural rate (Romer and Romer, 2004*a*). Inflation rose at the same time, resulting in a downward sloping Phillips curve visible in the data (driven by rises in x_t in (1)).

During Arthur Burns's tenure in the 1970s, a combination of factors increased both inflation and increased unemployment, leading to a disappearance of any discernible Phillips curve correlation. Those factors were a series of large cost shocks (increases in u_t in (1)) brought about by oil supply disruption[29], and the Federal Reserve's inability, unwillingness[30] or miscalculations[31] in trying to lean against them (falls in e_t in (11)) and their impact on inflation expectations[32] (increases in $E_t \pi_{t+1}$ in (1)).

The beginning of Paul Volcker's tenure saw a re-emergence of a steep negative Phillips curve slope, as tighter monetary policy induced rises in unemployment and a sustained fall in inflation (driven by falls in σ_e^2 or ρ_e in (11), or equivalently a fall in λ and a related fall in $E_t \pi_{t+1}$ in (1): Clarida, Gali and Gertler (2000)).

For the subsequent two decades under Paul Volcker and then Alan Greenspan, the Phillips correlation all but disappeared. The causes of the Great Moderation are often divided into those relating to good policy, good luck (in the form of lower shock variance, particularly of supply shocks), and changes in the structure of the economy (Stock and Watson, 2002).

Despite the Great Moderation coming to an end with the 2008 financial crisis and a large rise in unemployment, the Phillips curve correlation that reappeared under the tenures of Ben Bernanke and Janet Yellen has been at best weak. The lack of a large deflation following the crisis has sparked a burgeoning literature attempting to explain the 'missing disinflation' by appealing to one or more of: a flatter structural Phillips curve slope; better anchored inflation expectations *or* increases in inflation expectations; the inflationary effects of financial frictions; or weaker potential supply growth (see Coibion and Gorodnichenko, 2015, for a discussion).

The reduced-form evidence in Figure 9 has led many commentators to conclude that the Phillips curve has flattened over time. It is also consistent with estimates using more sophisticated techniques. In an influential contribution, Ball and Mazumder (2011) estimate a time-varying Phillips curve using median inflation as a measure of core inflation. They report that the Phillips curve steepened from -0.23 in 1960-72 to -0.69 in 1973-84 and then flattened to -0.14 in 1985-2010. Blanchard, Cerutti and Summers (2015) and Blanchard (2016), extending the non-linear Kalman filter estimates of IMF (2013), find that the Phillips curve slope fell from around -0.7 in the 1970s to around -0.2 from the 1990s onwards.

Over the period since 1990 (spanning the Great Moderation, then the financial crisis and its

[29] Gordon (1977); Blinder (1982).
[30] DeLong (1997).
[31] Orphanides (2002).
[32] Barro and Gordon (1983); Chari, Christiano and Eichenbaum (1998).

Table 2: *OLS Phillips curve regressions using aggregate US data: 1990-2018*

Phillips curve:	(1) Bivariate	(2) New Keynesian	(3) Acceler-ationist	(4) Hybrid $(U_t - U_t^*)$	(5) Hybrid (U_t)	(6) Hybrid $B(L)(U_t - U_t^*)$
Unemployment rate					-0.081**	
					[0.038]	
Unemployment gap	-0.204***	-0.170***	-0.010	-0.078**		0.503*
	[0.074]	[0.048]	[0.042]	[0.037]		[0.272]
First lag						-1.008**
						[0.458]
Second lag						0.291
						[0.437]
Third lag						0.152
						[0.237]
Sum						-0.062*
						[0.037]
Constant	2.583***				-0.054	
	[0.179]				[0.284]	
Inflation expectations		0.943***		0.388***	0.641***	0.384***
		[0.037]		[0.105]	[0.152]	[0.103]
Core CPI inflation						
First lag			0.404***	0.252**	0.223**	0.278***
			[0.091]	[0.103]	[0.096]	[0.097]
Second lag			0.475***	0.343***	0.312***	0.331***
			[0.083]	[0.098]	[0.095]	[0.107]
Third lag			0.092	-0.013	-0.050	-0.029
			[0.089]	[0.083]	[0.091]	[0.079]
Observations	118	118	118	118	118	118
R-squared	0.100	0.950	0.957	0.963	0.745	0.965

Newey-West standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Notes: The first five columns in the Table show the estimated OLS coefficients and standard errors for regressions nested by the hybrid Phillips curve $\pi_t = \alpha + \gamma_1(U_t - U_t^*) + \gamma_2 E_t\pi_{t+1} + \Sigma_{i=1}^{3}\gamma_{2+i}\pi_{t-i} + \varepsilon_t$. Specification (1) constrains $\gamma_2 = 0$, $\gamma_3 = 0$, $\gamma_4 = 0$, and $\gamma_5 = 0$. Specification (2) constrains $\alpha = 0$, $\gamma_3 = 0$, $\gamma_4 = 0$, $\gamma_5 = 0$. Specification (3) constrains $\alpha = 0$ and $\gamma_2 = 0$. Specification (4) constrains $\alpha = 0$ while specification (5) omits U_t^* and uses U_t as the measure of activity. Specification (6) constrains $\alpha = 0$ while also including three lags of $(U_t - U_t^*)$. $B(L)$ represents a third order lag polynomial. Data are quarterly seasonally adjusted measures from 1990 Q1 to 2018 Q2.

aftermath) a flat Phillips curve is common across a range of typical empirical specifications. Table 2 presents simple OLS estimates using data on quarterly annualised core CPI inflation and the unemployment gap/rate, over a sample from 1990-2018. The first column shows a simple bivariate regression of inflation on the CBO measure of the unemployment gap. The second estimates a typical New Keynesian Phillips curve by replacing the constant term with a survey-based measure of forward-looking inflation expectations from the SPF.[33] The third estimates an acelerationist-style Phillips curve (Phelps, 1967; Friedman, 1968) by using (three) lags of inflation as a proxy for inflation expectations. The fourth, fifth and sixth columns nest both models in a hybrid Phillips curve (Galı and Gertler, 1999), which feature both forward-looking expectations and lags of inflation (either as an alternative proxy for inflation expectations or as an additional source of inflation dynamics). The three hybrid curves feature different specifications for unemployment: they use either the unemployment rate; or else the unemployment gap, with or without additional lags.

Across the different specifications, the steepest Phillips curves slopes are only -0.20 (for the bivariate regression) and -0.17 (augmenting with survey-based inflation expectations). These are in line with the flattened Phillips curve slope found by Blanchard, Cerutti and Summers (2015). In all of the specifications featuring lags of inflation (either to proxy for inflation expectations or as an additional source of dynamics), the slope is flatter still and not always significant. The sum of the coefficients on the forward and backward looking inflation terms is close to 1 in each of the estimates (ranging from 0.9-1.1), in line with natural rate theories of unemployment, which predict stable long-run inflation if and only if $U = U^*$.

In all, the results from these "naive" Phillips curves estimates would suggest that the relationship still exists, but that the slope is relatively flat. Since policymakers also pay close attention to similar estimates, then the identification issue we highlight has the potential to provide misleading inferences for monetary policy. A flatter Phillips curve implies a higher "sacrifice ratio" associated with bringing inflation back to target, which could lead policymakers to place greater weight than optimal on avoiding volatility in output and employment relative to inflation (Blanchard, Cerutti and Summers, 2015). At worst, weaker evidence of a clear link between real activity and inflation could be interpreted as a sign that there is no short-run policy trade-off between the two goals, leading policymakers to abandon the natural rate hypothesis (Taylor, 1998; Cogley and Sargent, 2001). Given its importance for policy, we next discuss the different approaches to identifying the Phillips curve using aggregate data.

[33] We use five to ten year ahead inflation expectations, as suggested by Bernanke (2007) and Yellen (2015) as having a stronger empirical fit with the data. See Coibion, Gorodnichenko and Kamdar (2018) for an extensive review of the use of survey expectations in the Philllips curve.

5.2. Identification using aggregate data

In the extensive literature estimating Phillips curves, a number of papers have adopted approaches similar to those we suggest, implicitly or explicitly addressing the identification difficulties we highlight here.[34] Encouragingly, even in the period since the first draft of this paper was circulated, several others have proposed new identification strategies to mitigate simultaneity bias in Phillips curve estimation. In this subsection we discuss the findings from some of those contributions and categorise them according to our conceptual framework.

5.2.1 Controlling for supply shocks

In principle, if econometricians can perfectly control for the effect of any cost-push or other trade-off inducing shocks, then any remaining variation in the output gap and inflation must be due to movements in aggregate demand. As in our estimates above, the many papers that estimate Phillips curves using core inflation are already implicitly controlling for cost-push shocks to some degree, by stripping out their direct effects on the price data.[35] Others include the change in the oil price as a regressor (e.g. Roberts, 1995).

The idea of controlling for supply shocks was even present in the original Phillips (1958) article, which describes periods during which cost-push effects led to deviations from the fitted curve. More recently it has been associated with the "triangle model" of Gordon (1982), originally developed to account for the shift in inflation dynamics in the 1970s.[36] As described in Gordon (2013), the model includes several variables to control for changes in aggregate supply: food and energy price inflation; relative import price inflation; changes in trend labour productivity; and dummies reflecting the start and end of the Nixon price controls in the 1970s.[37]

Despite including these variables to control for supply shocks, Gordon (2013) still finds a flattening in the Phillips curve slope coefficient on the long-term unemployment gap: from -0.50 to

[34] See Mavroeidis, Plagborg-Møller and Stock (2014) for a comprehensive summary.

[35] See Hasenzagl et al. (2019) for evidence on the different channels through which cost-push shocks to energy prices impact inflation.

[36] And subsequently refined in a series of papers, most recently in Gordon (2013).

[37] The model also includes a large number of lags of inflation (up to 6 years) to capture additional dynamic factors affecting inflation.

-0.31 when he extends his sample from 1962-96 to 1962-2013.[38] The smaller absolute coefficient could be due to a flattening in the structural Phillips curve slope, but could also be due to difficulties with the practical implementation of the approach in the recent data. The solution is arguably more suited to helping identify the Phillips curve in a period such as the 1970s, when were large, easily identifiable cost-push shocks *and* a higher variance of monetary policy shocks than more recently.[39]

A related idea is that of Coibion and Gorodnichenko (2015), who argue that the supply shock imparted by higher oil prices also pushed up inflation between 2009 and 2011 by increasing firms' inflation expectations, which they proxy using household expectations.[40] Following Roberts (1995), they use the Michigan Survey of Consumers, and find a stable Phillips curve slope of between -0.2 and -0.3 (using the unemployment gap) in both the 1981-2007 and 1981-2013 periods.

In both cases, the large number of supply variables in Gordon's model point towards a more general practical difficulty, which is that there are many trade-off inducing shocks that need to be controlled for, and which of these are most important may vary over time. As an example, the explanations in the DSGE models of Gilchrist et al. (2017) and Del Negro, Giannoni and Schorfheide (2015) for the lack of disinflation during the financial crisis rely on financial frictions that simultaneously increased inflation and decreased real activity. That suggests one may also need to add a measure of financial frictions as an additional explanatory variable.

In some senses, the many papers that estimate the slope of a Phillips curve as part of a fully specified New Keynesian DSGE model are also adopting a variant of this approach. Schorfheide (2008) shows how full-information maximum likelihood (FIML) estimation of a simple New Keynesian model corrects for the simultaneity bias that markup shocks introduce into the slope of the Phillips curve. But he also reports evidence from the literature on how sensitive such estimates are to model specification, with estimates of the coefficient on the output gap varying from 0 to 4.

[38] Gordon himself emphasises the smaller flattening in the point estimate when using the short-term unemployment rate as the relevant concept of slack, although this measure correlates less closely with estimates of the overall output gap than the total unemployment rate – largely due to the large negative output gap during the financial crisis.

[39] The standard deviation of the Romer and Romer (2004*b*) monetary policy shock series is 2.5 times smaller in the period from 1990 onwards.

[40] See also Hasenzagl et al. (2019).

5.2.2 Instrumental variable estimation

An alternative solution is to use instrumental variable methods. The econometrician must find a valid instrument that correlates with the demand variation in the output gap, and is uncorrelated with the cost-push shock. The fitted value from a first-stage regression will then purge the output gap measure of the endogenous response of monetary policy to the cost-push shock, meaning it can be used to recover the true Phillips curve slope.

Instrumental variable methods, and especially IV-GMM estimation, have been common in much of the literature estimating New Keynesian Phillips curves, including influential papers by Galı and Gertler (1999) and Galı, Gertler and Lopez-Salido (2001). These papers use only lagged variables as instruments, which should be orthogonal to the current period cost-push innovation. But as discussed in Mavroeidis, Plagborg-Møller and Stock (2014) and more recently in Barnichon and Mesters (2019), if the cost-push shocks exhibit autocorrelation, then shocks will still be correlated with the lagged variables and the exclusion restriction will not be satisfied. The instruments used must be of a greater lag length than the lag order of the cost-push shocks, but with highly autocorrelated cost-push shocks, such instruments are likely to have low relevance.

Alternatively, separately identified demand shocks can be used as a set of *external* instruments, as recently proposed by Barnichon and Mesters (2019). To satisfy the exclusion restriction, the candidate instruments should be uncorrelated with the cost-push shocks in (1). Monetary policy shocks, which are not usually thought to affect supply, are a natural candidate.

Essentially, this strategy applies the findings from the large literature on identifying monetary policy shocks to recover the Phillips curve.[41] Given the major focus of that literature has been to try to remove the systematic response of monetary policy to economic developments, it should be able to successfully distill the Phillips curve relationship.

Recent work by Barnichon and Mesters (2019) follows exactly this approach. Using the Romer and Romer (2004*b*) narrative measure of monetary policy shocks as instruments for the output gap, they find a much steeper Phillips curve slope than under OLS.

The approach faces the same challenges as outlined by Ramey (2016) for the monetary policy

[41] For example, Christiano, Eichenbaum and Evans (1996, 1999); Romer and Romer (2004*b*); Uhlig (2005); Bernanke, Boivin and Eliasz (2005); Olivei and Tenreyro (2007); Cloyne and Hürtgen (2016).

shock literature. She argues that in the period since 1990, monetary policy has been set more systematically, and as a result, there is only a limited amount of true exogenous variation in the data, leading to weak instrument issues.

Identification of monetary policy shocks using high-frequency data may offer one solution (Kuttner, 2001; Faust, Swanson and Wright, 2004; Gertler and Karadi, 2015; Nakamura and Steinsson, 2018). The short-time windows over which these shocks are identified help remove any traces of endogenous monetary policy (Nakamura and Steinsson, 2018), which might otherwise be amplified if the shocks were weak instruments for the output gap. Barnichon and Mesters (2019) use the high frequency identified shocks of Gertler and Karadi (2015) for the post 1990 period and find evidence of a flatter Phillips curve slope than in the earlier period.

Other demand shocks, such as fiscal shocks to government spending or taxes, could also be used as external instruments. Relative to monetary policy shocks, one drawback is that some fiscal changes are more likely to affect aggregate supply, and so they may not satisfy the exclusion restriction. The large-scale DSGE model simulations shown in Subsection 4.5 also highlighted that the Phillips curve may vary for different types of demand shock. If so, then the reduced-form curve following a monetary policy shock is arguably the most relevant one for policymakers.

Related to these ideas, a recent paper by Galí and Gambetti (2018) estimates Phillips curves *conditional* on identified demand shocks in a VAR. They find that while endogeneity issues do lead to downward bias in estimates of US wage Phillips curve, there has also been a structural flattening over time.

5.3. Identification using regional data

Given some of the practical difficulties using aggregate data in the presence of systematic monetary policy, an alternative solution is to exploit cross-sectional variation. An interesting recent approach in this vein is Jorda and Nechio (2018), who take advantage of the fact that economies with fixed exchange rates are unable to implement independent monetary policies.

To show the possibility of using regional data to identify the aggregate US Phillips curve, we use a panel of city-level price inflation and unemployment data, as in Fitzgerald and Nicolini (2014). Hooper, Mishkin and Sufi (2019) also make use of US city-level (and state-level data) in their detailed study of the US wage and price Phillips curves. Our city-level dataset, containing price data, is an extended version of the one used by Kiley (2015) and Babb and Detmeister (2017).

5.3.1 Data description

We use data from the 23 US metropolitan areas available from the BLS. Together these areas account for around one-third of the US population (Babb and Detmeister, 2017). There is significant size heterogeneity across the sample - weighted by average labour force, the largest three areas (New York-Newark-New Jersey, Los Angeles-Riverside-Orange County and Chicago-Naperville-Elgin) account for 35% of the total, while the smallest ten areas account for less than 3% each. Our dataset runs from 1990 H1 to 2018 H1, with some gaps for metropolitan areas where the data were not published or discontinued.

The inflation series is the annualised log change in the semiannual CPI excluding food and energy. For the majority of metropolitan areas, data are available at a higher frequency. But for 10 areas, they are only available at a semiannual frequency for some time periods. In order to maximise our cross-sectional sample, we opt to convert the other areas to lower frequency data. For those areas, we take the semiannual average of the monthly or quarterly price level.[42] The city-level CPI data are not seasonally adjusted by the BLS.

For unemployment, we take the BLS's city-level measure of unemployed as a percentage of the share of civilian labour force. As discussed by Babb and Detmeister (2017), there are some differences in geographical coverage of the metropolitan area definitions between the two data series. The BLS publish both seasonally adjusted and unadjusted labour force data at the metro area level - we use the unadjusted series, consistent with the CPI data. We convert the monthly published data to semiannual averages.

We also run specifications using survey-based measures of five to ten year ahead inflation expectations from the University of Michigan Consumer Survey. The Michigan Survey includes data published for four broad geographical regions: the North East, North Central, South, and West. We assign each metropolitan area to its appropriate region (or the region containing most of the metropolitan area's population, for metro areas that span more than one region).

[42] As a robustness check, we have repeated our estimation at a quarterly frequency using the subset of metropolitan areas where the data are available. The results from those estimates point towards a slightly steeper Phillips curve slope.

5.3.2 Regional data results

To motivate our regional empirical specification, first note that we only have data on the unemployment rate at the regional level, rather than the unemployment gap to proxy for the output gap. If the regional Phillips curves are of a form similar to (15), transformed to include the regional unemployment gap $(U_t^i - U^{*i}_t)$:

$$\pi_t^i = \beta E_t \pi_{t+1}^i - \kappa(U_t^i - U^{*i}_t) + u_t^i \tag{22}$$

If, as is likely, the regional equilibrium unemployment rate, U^{*i}_t is positively correlated with the actual unemployment rate, then in a pooled OLS regression such as

$$\pi_{it} = \alpha + \gamma_1 E_t \pi_{it+1} + \gamma_2 U_{it} + \varepsilon_{it}, \tag{23}$$

the omitted variable will bias the estimated coefficient $\hat{\gamma}_2$ towards zero. To partially address this we run specifications including metropolitan area fixed effects (α_i):

$$\pi_{it} = \alpha_i + \gamma_1 E_t \pi_{it+1} + \gamma_2 U_{it} + \varepsilon_{it}, \tag{24}$$

which control for time-invariant regional differences in U^* (as well as time-invariant inflation expectations), although not for time-variation in those regional differences.

Second, as long the regional unemployment rate is correlated with the aggregate unemployment rate, and regional inflation is affected by aggregate cost-push shocks, then the slope estimate will still be biased by the endogenous response of monetary policy to aggregate cost-push shocks. To avoid this, note that our theoretical Phillips curve in terms of regional deviations from the aggregate, (18), can be rearranged to give:

$$\begin{aligned} \pi_t^i &= \pi_t + \beta E_t(\pi_{t+1}^i - \pi_{t+1}) + \kappa(x_t^i - x_t) + \hat{u}_t^i \\ &= \beta E_t \pi_{t+1}^i + \kappa x_t^i + (\pi_t - \beta E_t \pi_{t+1} - \kappa x_t) + \hat{u}_t^i \end{aligned} \tag{25}$$

where x_t^i are uncorrelated with $\hat{u}_t^i$ but are correlated with the aggregate cost-push shock $u_t =$

$\pi_t - \beta E_t \pi_{t+1} - \kappa x_t$. We can therefore remove any monetary-policy induced correlation between the regressor and the error term by also including time fixed effects (δ_t):

$$\pi_{it} = \alpha_i + \gamma_1 E_t \pi_{it+1} + \gamma_2 U_{it} + \delta_t + \varepsilon_{it} \tag{26}$$

which will also control for any time-varying changes in the aggregate equilibrium unemployment rate.

To compare across the different specifications, we estimate each of equations (23), (24) and (26). As additional controls we include seasonal dummies and, given the data are semiannual, just a single lag of inflation. For completeness we also show results including time fixed effects but not including metropolitan area fixed effects. The results are shown in Table 3. All four estimates of the

Table 3: *US Metro area Phillips curve: 1990-2018*

Regression	(1) Pooled OLS	(2) Metro area FE only	(3) Year FE only	(4) Year and Metro area FE
Unemployment rate	-0.136***	-0.148***	-0.271***	-0.367***
	[0.022]	[0.025]	[0.043]	[0.066]
Constant	1.708***	1.911***	5.435***	6.655***
	[0.277]	[0.296]	[0.606]	[0.776]
Inflation expectations	0.278***	0.273***	-0.027	-0.067
	[0.064]	[0.065]	[0.155]	[0.145]
Core CPI inflation				
First lag	0.280***	0.270***	0.110**	0.073
	[0.048]	[0.051]	[0.049]	[0.050]
Observations	1,174	1,174	1,174	1,174
R-squared	0.260	0.298	0.359	0.410
Metro area FE	No	Yes	No	Yes
Year FE	No	No	Yes	Yes
Seasonal dummies	Yes	Yes	Yes	Yes

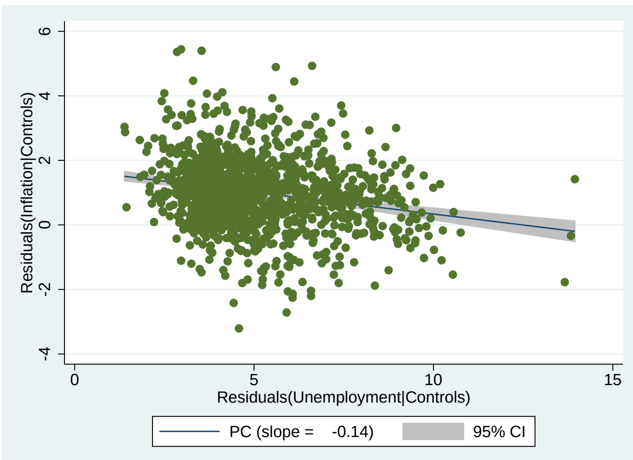
Robust standard errors (clustered by metro area) in brackets
*** p<0.01, ** p<0.05, * p<0.1

Notes: The table shows coefficients and standard errors estimated from four regional Phillips curve specifications. Core CPI inflation is the dependent variable in each case. Specification (1) estimates equation 23 (plus controls) by pooled OLS. Specification (2) estimates equation 24 (plus controls) using group (area) fixed effects. Specification (3) is identical to (1) apart from the inclusion of a set of dummy variables. Specification (4) is identical to (2) apart from the inclusion of a set of year dummy variables. The additional controls are one lag of core CPI inflation and a seasonal dummy variable for each metropolitan area that takes the value of 1 in H2 and 0 in H1. Data are semiannual non-seasonally adjusted measures from 1990 H1 to 2018 H1.

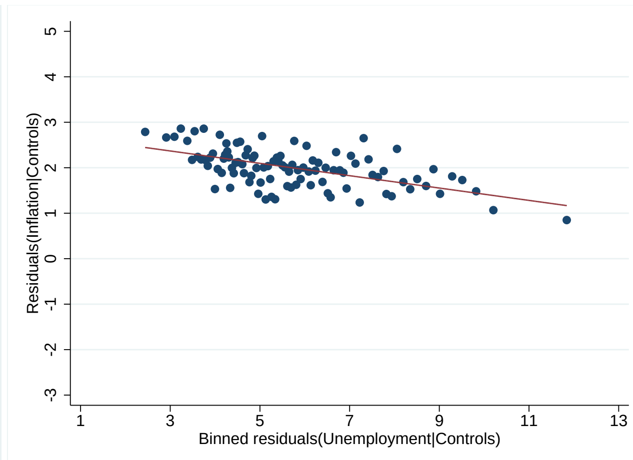
Phillips curve slope are statistically significant and with the correct sign. In the first column, the pooled OLS estimate of -0.14 suggests a flat Phillips curve. It is slightly larger than the estimates with lagged dependent variables using aggregate data in Table 2, but no steeper than the estimates without lagged inflation.

Figure 10: *Pooled OLS: metropolitan area core CPI inflation versus unemployment (both regressed on controls)*

(a) *Raw residuals*



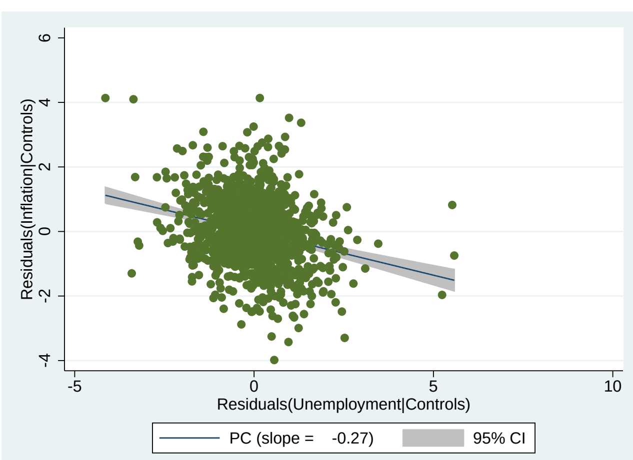
(b) *Residuals grouped into bins*



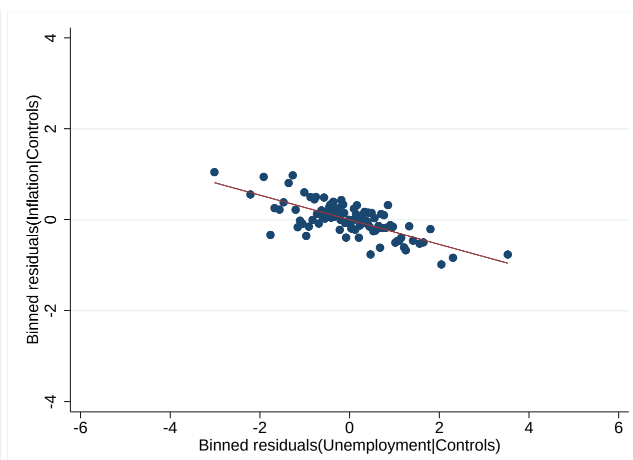
Notes: The figures are a graphical illustration of the Phillips curve slope estimated in specification (1) in table 3. Panel 10a plots the residuals from a regression of core CPI inflation on all regressors other than the unemployment rate, against the residuals from a regression of the unemployment rate on all other regressors. Panel 10b shows averages of the same data, where the unemployment and inflation data are averaged across 100 equal sized bins according to the unemployment rate.

Figure 11: *Year fixed effects only: metropolitan area core CPI inflation versus unemployment (both regressed on controls)*

(a) *Raw residuals*



(b) *Residuals grouped into bins*

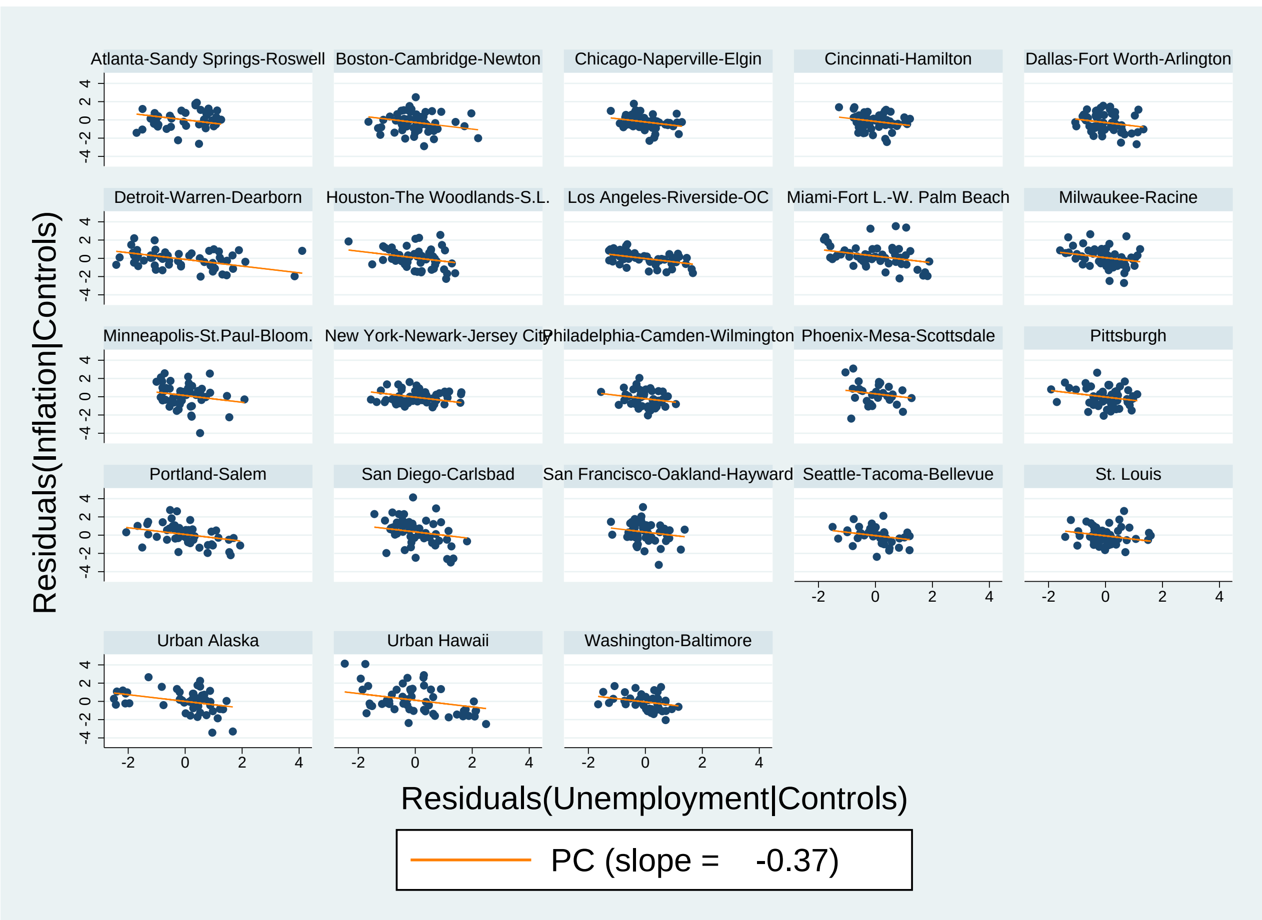


Notes: The figures are a graphical illustration of the Phillips curve slope estimated in specification (3) in table 3. See the notes to Figures 10a and 10b for details.

Figures 10a and 10b illustrate the slope coefficient. In Figure 10a, the scatter plots core inflation against unemployment. Both variables are shown as the residuals following a regression on the other controls in the first column of Table 3, such that the line of best fit shows the estimated Phillips curve slope. Figure 10b shows averages of the same data, where the unemployment and inflation data are averaged across 100 equal sized bins according to the unemployment rate.

In the second column we include area fixed effects and the point estimate of the slope is slightly larger, although not significantly so. In the third column we include year fixed effects but not area fixed effects, purging the data of any aggregate-level variation over time, including changes

Figure 12: *Year and metro area fixed effects: metropolitan area core CPI inflation versus unemployment by metro area (both regressed on controls)*



Notes: The figures are a graphical illustration of the Phillips curve slope estimated in specification (4) in table 3. For each metropolitan area, the figure plots the residuals from a fixed effects regression of core CPI inflation on all regressors other than the unemployment rate, against the residuals from a fixed-effects regression of the unemployment rate on all other regressors.

in monetary policy and in the natural rate of unemployment. The estimated Phillips curve slope steepens to -0.27, as shown in Figures 11a and 11b.

In the fourth column, metro area fixed effects are also included, controlling for any time-invariant unobserved factors such as different average levels of U^* across regions. The resulting Phillips curve is -0.37, 2.7 times larger than the pooled OLS estimate. The effect of including fixed effects is also shown in Figure 12, which plots the estimated Phillips curve by metropolitan area, with different intercept terms for each city.

These results provide evidence of a robust US Phillips curve at the regional level. They are consistent with the idea that because monetary policy endogenously offsets changes in aggregate demand, and leans against cost-push shocks, identification is blurred at the aggregate level.

6. Conclusion

We use standard analytical framework to explain why inflation follows a seemingly exogenous statistical process, or, in other words, why the Phillips curve cannot be easily identified with macroeconomic data. In the framework, a monetary authority minimizes welfare losses, measured as deviations of inflation and output from their targets, subject to a Phillips curve. This leads the authority to follow an optimal targeting rule in which it seeks to increase inflation when the output gap decreases. This imparts a negative relation between inflation and the output gap that blurs the identification of the positively sloped Phillips curve. In equilibrium, inflation inherits the statistical properties of any cost-push shocks affecting the Phillips curves (e.g., energy price shocks, exchange rate changes, and so on).

We show that shocks to the targeting rule are key for the identification of the Phillips curve. These targeting shocks can take the form of monetary policy shocks in a Taylor rule or, in a multi-region setting or a multi-country monetary union, idiosyncratic demand shocks affecting the various regions or countries in different ways. In a univariate regression analysis, if the relative variance of these shocks is sufficiently high, vis-a-vis the remaining variance of the cost-push shocks that cannot be controlled for, the slope of the Phillips curve can be identified. Similarly, identification of monetary policy or other demand shocks allows the positive relationship between inflation and output gap to be distilled.

We have also shown how the simple framework here can jointly rationalise several empirical findings on the Phillips curve. First, it should be weaker in periods when there are large cost shocks – such as the 1970s – *and* when monetary policy is relatively successful in achieving its targets – as in the inflation targeting era. Second, wage Phillips curves should be more evident in the data that price Phillips curves. And third, the Phillips curve relationship should appear stronger in disaggregated panel data than in aggregate data.

To summarise, the paper explains the identification problem posited by the estimation of Phillips curves; rationalises findings in the empirical literature and discusses practical solutions to the identification problem, showing evidence of a steeper Phillips curve in US regional data. In doing so, the paper hopes to address a recent wave of work questioning the existence of a link between inflation and slack, a key building block of the prevalent monetary policy framework.

References

Adolfson, Malin, Stefan Laseen, Lawrence Christiano, Mathias Trabandt, and Karl Walentin. 2013. "Ramses ii-model description." *Sveriges Riksbank Occasional Paper Series*, 12: 1009.

Atkeson, Andrew, and Lee E Ohanian. 2001. "Are Phillips curves useful for forecasting inflation?" *Federal Reserve Bank of Minneapolis. Quarterly Review-Federal Reserve Bank of Minneapolis*, 25(1): 2.

Babb, Nathan R, and Alan K Detmeister. 2017. "Nonlinearities in the Phillips Curve for the United States: Evidence Using Metropolitan Data."

Ball, Laurence, and Sandeep Mazumder. 2011. "Inflation Dynamics and the Great Recession." *Brookings Papers on Economic Activity*, 42(1 (Spring): 337–405.

Barnichon, Regis, and Geert Mesters. 2019. "Identifying modern macro equations with old shocks."

Barro, Robert J, and David B Gordon. 1983. "A positive theory of monetary policy in a natural rate model." *Journal of political economy*, 91(4): 589–610.

Bernanke, Ben. 2007. "Inflation expectations and inflation forecasting." Board of Governors of the Federal Reserve System (US).

Bernanke, Ben S. 2010. "The Economic Outlook and Monetary Policy."

Bernanke, Ben S, Jean Boivin, and Piotr Eliasz. 2005. "Measuring the effects of monetary policy: a factor-augmented vector autoregressive (FAVAR) approach." *The Quarterly journal of economics*, 120(1): 387–422.

Blanchard, Olivier. 2016. "The Phillips Curve: Back to the'60s?" *American Economic Review*, 106(5): 31–34.

Blanchard, Olivier, Eugenio Cerutti, and Lawrence Summers. 2015. "Inflation and activity–Two explorations and their monetary policy implications." National Bureau of Economic Research.

Blinder, Alan S. 1982. "The anatomy of double-digit inflation in the 1970s." In *Inflation: Causes and effects*. 261–282. University of Chicago Press.

Board of Governors of the Federal Reserve System. 2000. "Monetary Policy Report to the Congress."

Boivin, Jean, and Marc P Giannoni. 2006. "Has monetary policy become more effective?" *The Review of Economics and Statistics*, 88(3): 445–462.

Brubakk, Leif, and Tommy Sveen. 2009. "NEMO-a new macro model for forecasting and monetary policy analysis." *Economic Bulletin (Norges Bank)*, 80(1).

Bullard, James. 2018. "The case of the disappearing Phillips Curve." *presentation at the 2018 ECB Forum on Central Banking Macroeconomics of Price-and Wage-Setting.*

Burgess, Stephen, Emilio Fernandez-Corugedo, Charlotta Groth, Richard Harrison, Francesca Monti, Konstantinos Theodoridis, and Matt Waldron. 2013. "The Bank of England's forecasting platform: COMPASS, MAPS, EASE and the suite of models." Bank of England Bank of England working papers 471.

Carlin, Wendy, and David Soskice. 2005. "The 3-equation New Keynesian Model—a graphical exposition." *Contributions in Macroeconomics*, 5(1).

Carlstrom, Charles T, Timothy S Fuerst, and Matthias Paustian. 2009. "Inflation persistence, monetary policy, and the great moderation." *Journal of Money, Credit and Banking*, 41(4): 767–786.

Carney, Mark. 2017*a*. "[De]Globalisation and inflation." *speech delivered as the 2017 IMF Michel Camdessus Central Banking Lecture*.

Carney, Mark. 2017*b*. "Lambda." *speech at the London School of Economics, available online (January 2017)*.

Cecchetti, Stephen G, Michael Feroli, Peter Hooper, Anil K Kashyap, and Kermit L Schoenholtz. 2017. "Deflating Inflation Expectations: the Implications of Inflation's Simple Dynamics."

Chari, Varadarajan V, Lawrence J Christiano, and Martin Eichenbaum. 1998. "Expectation traps and discretion." *Journal of economic theory*, 81(2): 462–492.

Christiano, Lawrence J, Martin Eichenbaum, and Charles Evans. 1996. "The Effects of Monetary Policy Shocks: Evidence from the Flow of Funds." *The Review of Economics and Statistics*, 78(1): 16–34.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 1999. "Monetary policy shocks: What have we learned and to what end?" In . Vol. 1 of *Handbook of Macroeconomics*, 65 – 148. Elsevier.

Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans. 2005. "Nominal rigidities and the dynamic effects of a shock to monetary policy." *Journal of political Economy*, 113(1): 1–45.

Clarida, Richard, Jordi Gali, and Mark Gertler. 1999. "The science of monetary policy: a new Keynesian perspective." *Journal of economic literature*, 37(4): 1661–1707.

Clarida, Richard, Jordi Gali, and Mark Gertler. 2000. "Monetary policy rules and macroeconomic stability: evidence and some theory." *The Quarterly Journal of Economics*, 115(1): 147–180.

Clarida, Richard, Jordi Gali, and Mark Gertler. 2001. "Optimal monetary policy in open versus closed economies: an integrated approach." *American Economic Review*, 91(2): 248–252.

Cloyne, James, and Patrick Hürtgen. 2016. "The Macroeconomic Effects of Monetary Policy: A New Measure for the United Kingdom." *American Economic Journal: Macroeconomics*, 8(4): 75–102.

Cogley, Timothy, and Thomas J Sargent. 2001. "Evolving post-world war II US inflation dynamics." *NBER macroeconomics annual*, 16: 331–373.

Coibion, Olivier, and Yuriy Gorodnichenko. 2015. "Is the Phillips curve alive and well after all? Inflation expectations and the missing disinflation." *American Economic Journal: Macroeconomics*, 7(1): 197–232.

Coibion, Olivier, Yuriy Gorodnichenko, and Rupal Kamdar. 2018. "The formation of expectations, inflation, and the phillips curve." *Journal of Economic Literature*, 56(4): 1447–91.

Del Negro, Marco, Marc P. Giannoni, and Frank Schorfheide. 2015. "Inflation in the Great Recession and New Keynesian Models." *American Economic Journal: Macroeconomics*, 7(1): 168–96.

DeLong, J Bradford. 1997. "America's peacetime inflation: the 1970s." In *Reducing inflation: Motivation and strategy*. 247–280. University of Chicago Press.

Dennis, Richard. 2007. "Optimal policy in rational expectations models: New solution algorithms." *Macroeconomic Dynamics*, 11(1): 31–55.

Dotsey, Michael, Shigeru Fujita, and Tom Stark. 2017. "Do Phillips Curves Conditionally Help to Forecast Inflation?"

Draghi, Mario. 2017. "Accompanying the economic recovery." *speech delivered at the ECB Forum on Central Banking, Sintra, Portugal, June*.

Edge, Rochelle M., Michael T. Kiley, and Jean-Philippe Laforte. 2010. "A comparison of forecast performance between Federal Reserve staff forecasts, simple reduced-form models, and a DSGE model." *Journal of Applied Econometrics*, 25(4): 720–754.

Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin. 2000. "Optimal monetary policy with staggered wage and price contracts." *Journal of Monetary Economics*, 46(2): 281–313.

Faust, Jon, Eric T Swanson, and Jonathan H Wright. 2004. "Identifying VARs based on high frequency futures data." *Journal of Monetary Economics*, 51(6): 1107–1131.

Fitzgerald, Terry J, and Juan Pablo Nicolini. 2014. "Is There a Stable Relationship between Unemployment and Future Inflation? Evidence from US Cities."

Forbes, Kristin, Lewis Kirkham, and Konstantinos Theodoridis. 2017. "A trendy approach to UK inflation dynamics."

Friedman, Milton. 1968. "The Role of Monetary Policy." *The American Economic Review*, 58(1): 1–17.

Galí, J. 2008. *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework.* Princeton University Press.

Galí, Jordi. 2011. "The return of the wage Phillips curve." *Journal of the European Economic Association*, 9(3): 436–461.

Galí, Jordi, and Luca Gambetti. 2018. "Has the US Wage Phillips Curve Flattened? A Semi-Structural Exploration."

Galı, Jordi, and Mark Gertler. 1999. "Inflation dynamics: A structural econometric analysis." *Journal of monetary Economics*, 44(2): 195–222.

Galı, Jordi, Mark Gertler, and J David Lopez-Salido. 2001. "European inflation dynamics." *European economic review*, 45(7): 1237–1270.

Gertler, Mark, and Peter Karadi. 2015. "Monetary policy surprises, credit costs, and economic activity." *American Economic Journal: Macroeconomics*, 7(1): 44–76.

Gilchrist, Simon, Raphael Schoenle, Jae Sim, and Egon Zakrajšek. 2017. "Inflation dynamics during the financial crisis." *American Economic Review*, 107(3): 785–823.

Goodfriend, Marvin, and Robert G King. 2005. "The incredible Volcker disinflation." *Journal of Monetary Economics*, 52(5): 981–1015.

Gordon, Robert J. 1977. "The theory of domestic inflation." *The American Economic Review*, 67(1): 128–134.

Gordon, Robert J. 1982. "Inflation, Flexible Exchange Rates, and the Natural Rate of Unemployment." In *Workers, Jobs, and Inflation*. Brookings Institution.

Gordon, Robert J. 2011. "The history of the Phillips curve: Consensus and bifurcation." *Economica*, 78(309): 10–50.

Gordon, Robert J. 2013. "The Phillips curve is alive and well: Inflation and the NAIRU during the slow recovery." National Bureau of Economic Research.

Haldane, Andrew, and Danny Quah. 1999. "UK Phillips curves and monetary policy." *Journal of Monetary Economics*, 44(2): 259–278.

Haldane, Andrew G. 1998. "On inflation targeting in the United Kingdom." *Scottish Journal of Political Economy*, 45(1): 1–32.

Hall, Robert. 2013. "The Routes Into and Out of the Zero Lower Bound." Vol. 23.

Hasenzagl, Thomas, Filippo Pellegrino, Lucrezia Reichlin, and Giovanni Ricco. 2019. "A Model of the Feds View on Inflation."

Hooper, Peter, Frederic S. Mishkin, and Amir Sufi. 2019. "Prospects for Inflation in a High Pressure Economy: Is the Phillips Curve Dead or is It Just Hibernating?"

IMF. 2013. "The Dog that Didnt Bark: Has Inflation Been Muzzled or Was it Just Sleeping? Chapter 3." *World Economic Outlook (IMF)*.

Jorda, Oscar, and Fernanda Nechio. 2018. "Inflation Globally." Federal Reserve Bank of San Francisco.

Kiley, Michael T. 2015. "An evaluation of the inflationary pressure associated with short-and long-term unemployment." *Economics Letters*, 137: 5–9.

King, Robert G. 2008. "The Phillips curve and US macroeconomic policy: snapshots, 1958-1996." *Economic Quarterly – Federal Reserve Bank of Richmond*, 94(4): 311.

King, Robert G, and Mark W Watson. 2012. "Inflation and Unit Labor Cost." *Journal of Money, credit and Banking*, 44(s2): 111–149.

Krogh, Tord S. 2015. "Macro frictions and theoretical identification of the New Keynesian Phillips curve." *Journal of Macroeconomics*, 43: 191–204.

Kuttner, Kenneth N. 2001. "Monetary policy surprises and interest rates: Evidence from the Fed funds futures market." *Journal of monetary economics*, 47(3): 523–544.

Leduc, Sylvain, and Daniel Wilson. 2017. "Has the Wage Phillips Curve Gone Dormant?" *FRBSF Economic Letter*.

Lucas, Robert E. 1976. "Econometric policy evaluation: A critique." *Carnegie-Rochester Conference Series on Public Policy*, 1: 19 – 46.

Mankiw, N.G., Laurence Ball, and David Romer. 1988. "The New Keynesian Economics and the Output-Inflation Trade-off." *Brookings Papers on Economic Activity*, 1: 1–65.

Mavroeidis, Sophocles, Mikkel Plagborg-Møller, and James H Stock. 2014. "Empirical evidence on inflation expectations in the New Keynesian Phillips Curve." *Journal of Economic Literature*, 52(1): 124–88.

Mishkin, Frederic S. 2007. "Inflation dynamics." *International Finance*, 10(3): 317–334.

Nakamura, Emi, and Jon Steinsson. 2014. "Fiscal Stimulus in a Monetary Union: Evidence from US Regions." *American Economic Review*, 104(3): 753–92.

Nakamura, Emi, and Jón Steinsson. 2018. "High-frequency identification of monetary non-neutrality: The information effect." *The Quarterly Journal of Economics*, 133(3): 1283–1330.

Nason, James M, and Gregor W Smith. 2008. "Identifying the new Keynesian Phillips curve." *Journal of Applied Econometrics*, 23(5): 525–551.

Olivei, Giovanni, and Silvana Tenreyro. 2007. "The Timing of Monetary Policy Shocks." *American Economic Review*, 97(3): 636–663.

Orphanides, Athanasios. 2002. "Monetary-policy rules and the great inflation." *American economic review*, 92(2): 115–120.

Phelps, Edmund S. 1967. "Phillips curves, expectations of inflation and optimal unemployment over time." *Economica*, 254–281.

Phillips, Alban W. 1958. "The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861–1957." *economica*, 25(100): 283–299.

Powell, Jerome H. 2018. "Monetary Policy and Risk Management at a Time of Low Inflation and Low Unemployment." *speech delivered at the "Revolution or Evolution? Reexamining Economic Paradigms" 60th Annual Meeting of the National Association for Business Economics, Boston, Massachusetts, October*.

Primiceri, Giorgio E. 2006. "Why Inflation Rose and Fell: Policy-Makers' Beliefs and U. S. Postwar Stabilization Policy*." *The Quarterly Journal of Economics*, 121(3): 867–901.

Ramey, Valerie A. 2016. "Macroeconomic shocks and their propagation." In *Handbook of Macroeconomics*. Vol. 2, 71–162. Elsevier.

Roberts, John M. 1995. "New Keynesian economics and the Phillips curve." *Journal of Money, Credit and Banking*, 27(4): 975–984.

Roberts, John M. 2006. "Monetary Policy and Inflation Dynamics." *International Journal of Central Banking*.

Romer, Christina D, and David H Romer. 2004*a*. "Choosing the Federal Reserve chair: lessons from history." *Journal of Economic Perspectives*, 18(1): 129–162.

Romer, Christina D, and David H Romer. 2004*b*. "A new measure of monetary shocks: Derivation and implications." *American Economic Review*, 94(4): 1055–1084.

Samuelson, Paul A, and Robert M Solow. 1960. "Analytical aspects of anti-inflation policy." *The American Economic Review*, 50(2): 177–194.

Sargent, Thomas, Noah Williams, and Tao Zha. 2006. "Shocks and Government Beliefs: The Rise and Fall of American Inflation." *American Economic Review*, 96(4): 1193–1224.

Schorfheide, Frank. 2008. "DSGE Model-Based Estimation of the New Keynesian Phillips Curve." *Economic Quarterly-Federal Reserve Bank of Richmond*, 94(4): 397.

Seneca, Martin. 2018. "A graphical illustration of optimal monetary policy in the New Keynesian framework."

Smets, Frank, and Rafael Wouters. 2007. "Shocks and frictions in US business cycles: A Bayesian DSGE approach." *American economic review*, 97(3): 586–606.

Stock, James, and Mark W. Watson. 2007. "Why Has U.S. Inflation Become Harder to Forecast?" *Journal of Money, Credit and Banking*, 39(s1): 3–33.

Stock, James, and Mark W. Watson. 2009. "Phillips Curve Inflation Forecasts." *Understanding Inflation and the Implications for Monetary Policy*, , ed. Jeffrey Fuhrer, Yolanda Kodrzycki, Jane Little and Giovanni Olivei, 99–202. Cambridge:MIT Press.

Stock, James H, and Mark W Watson. 2002. "Has the business cycle changed and why?" *NBER macroeconomics annual*, 17: 159–218.

Svensson, Lars E.O. 1997. "Inflation forecast targeting: Implementing and monitoring inflation targets." *European Economic Review*, 41(6): 1111 – 1146.

Svensson, Lars EO, and Michael Woodford. 2004. "Implementing optimal policy through inflation-forecast targeting." 19–92.

Taylor, John B. 1998. "Monetary policy guidelines for unemployment and inflation stability." *Inflation, unemployment, and monetary policy*. Cambridge:MIT Press.

Tuckett, Alex. 2018. "Regional data and the Phillips Curve." Bank of England Working Paper.

Uhlig, Harald. 2005. "What are the effects of monetary policy on output? Results from an agnostic identification procedure." *Journal of Monetary Economics*, 52(2): 381–419.

Vlieghe, Gertjan. 2018. "From asymmetry to symmetry: changing risks to the economic outlook." *speech*.

Williams, John C. 2006. "Inflation persistence in an era of well-anchored inflation expectations." *FRBSF Economic Letter*, , (oct13).

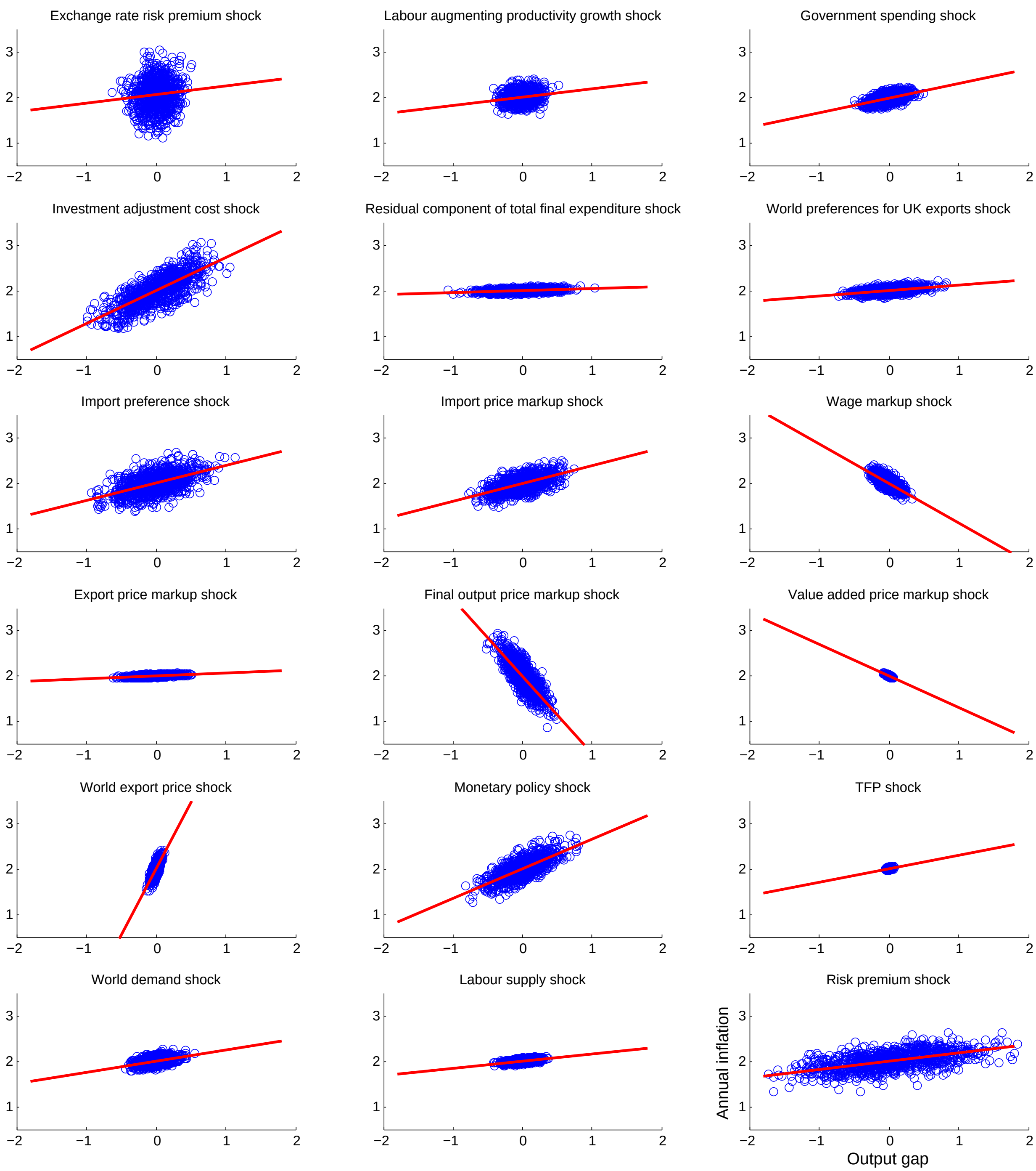
Woodford, Michael. 1994. "Nonstandard indicators for monetary policy: can their usefulness be judged from forecasting regressions?" In *Monetary Policy*. 95–115. The University of Chicago Press.

Woodford, Michael. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.

Yellen, Janet L. 2015. "Inflation dynamics and monetary policy." *speech delivered at the Philip Gamble Memorial Lecture, University of Massachusetts, Amherst, Amherst, Mass., September*, 24.

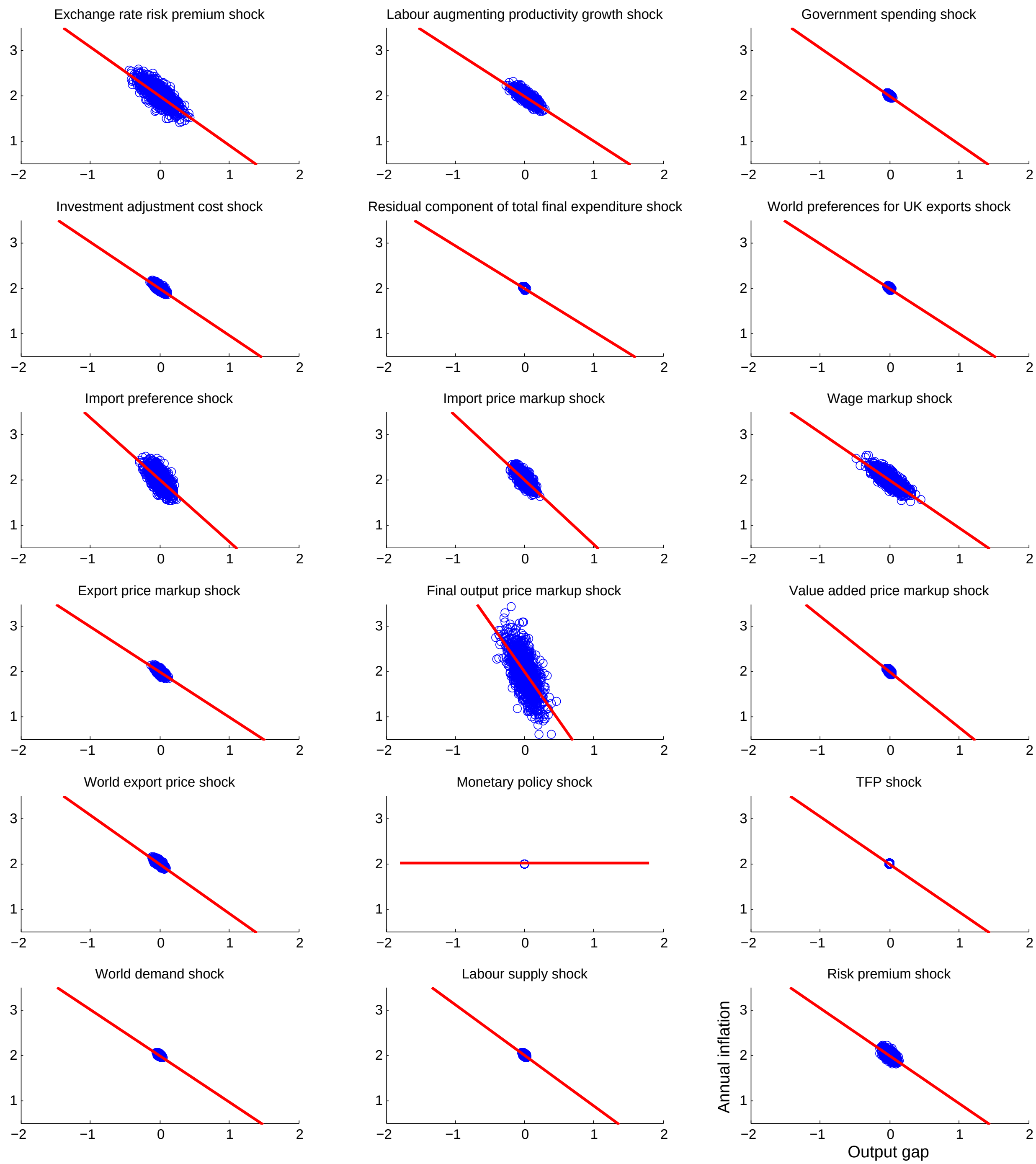
A. Appendix

Figure A1: *Inflation/output gap correlations by shock in a large-scale DSGE model with a Taylor rule*



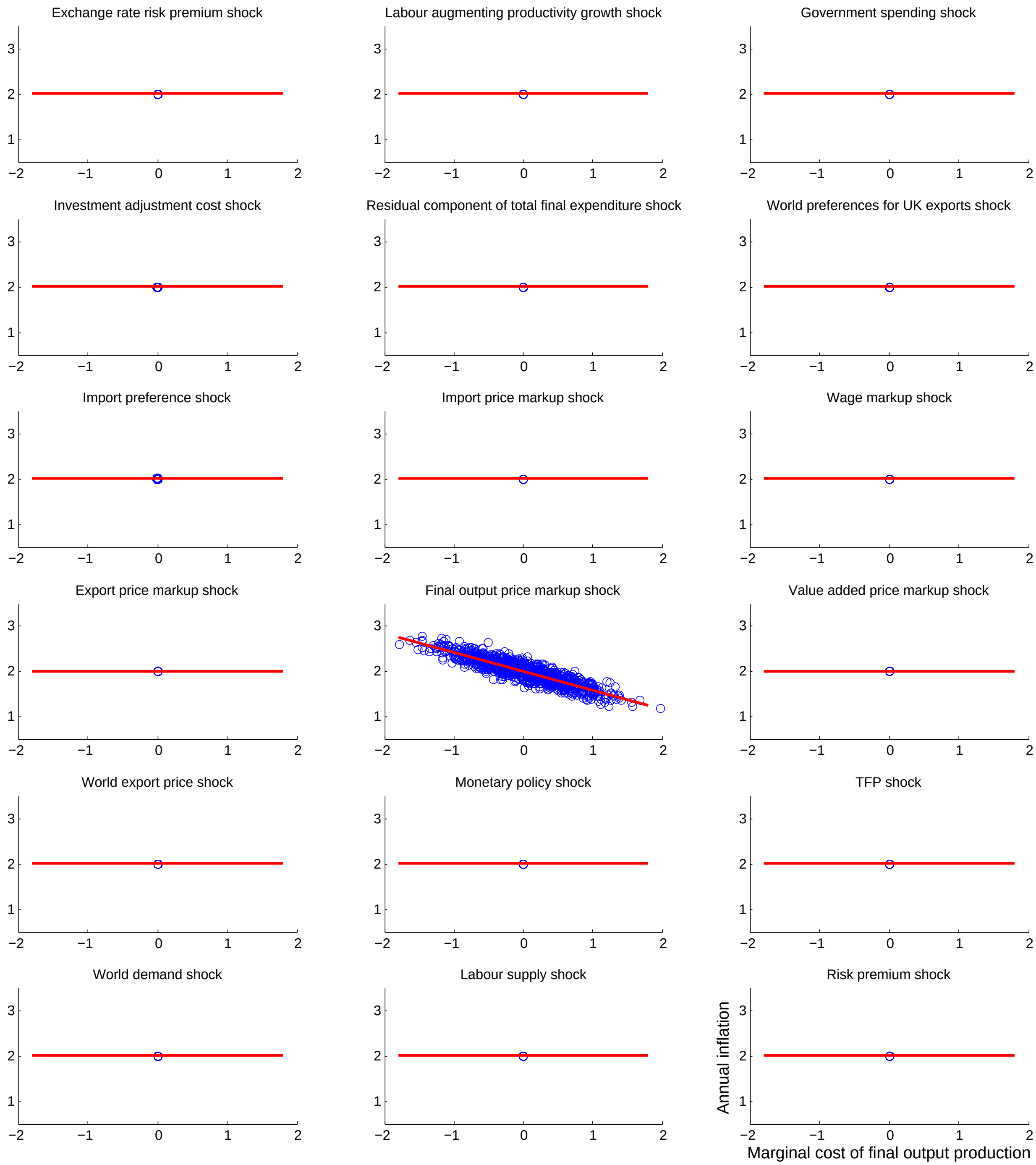
Notes: For each panel, 1000 periods of data are simulated from the model in Burgess et al. (2013) using the MAPS toolkit described in the same paper. For each panel, a realisation for the specified shock is drawn each period from a standard normal distribution. All other shocks are set to zero. The red lines show the lines of best fit from OLS regressions of the simulated annual inflation data on the (contemporaneous) flexible price output gaps. Monetary policy is specified using the estimated Taylor rule in the model.

Figure A2: *Inflation/output gap correlations by shock in a large-scale DSGE model under optimal discretionary policy*



Notes: For each panel, 1000 periods of data are simulated from the model in Burgess et al. (2013) using the MAPS toolkit described in the same paper. For each panel, a realisation for the specified shock is drawn each period from a standard normal distribution. All other shocks are set to zero. The red lines show the lines of best fit from OLS regressions of the simulated annual inflation data on the (contemporaneous) flexible price output gaps. Monetary policy is optimal policy under discretion, , where the policymaker minimises, period by period, an ad hoc loss function containing the discounted sum of squared deviations of annual inflation from target (with a weight of 1) and the output gap (with a weight of 0.25). The solution is calculated using the algorithm of Dennis (2007).

Figure A3: *Inflation/output gap correlations by shock in a large-scale DSGE model under optimal discretionary policy (using real marginal costs as a policy target)*



Notes: For each panel, 1000 periods of data are simulated from the model in Burgess et al. (2013) using the MAPS toolkit described in the same paper. For each panel, a realisation for the specified shock is drawn each period from a standard normal distribution. All other shocks are set to zero. The red lines show the lines of best fit from OLS regressions of the simulated annual inflation data on the (contemporaneous) flexible price output gaps. Monetary policy is optimal policy under discretion, , where the policymaker minimises, period by period, an ad hoc loss function containing the discounted sum of squared deviations of annual inflation from target (with a weight of 1) and the marginal cost of final output production (with a weight of 0.1). The solution is calculated using the algorithm of Dennis (2007).