Private Information Distributions in Securities Markets

Kerry Back, Kevin Crotty, Tao Li

Rice University & City University of Hong Kong

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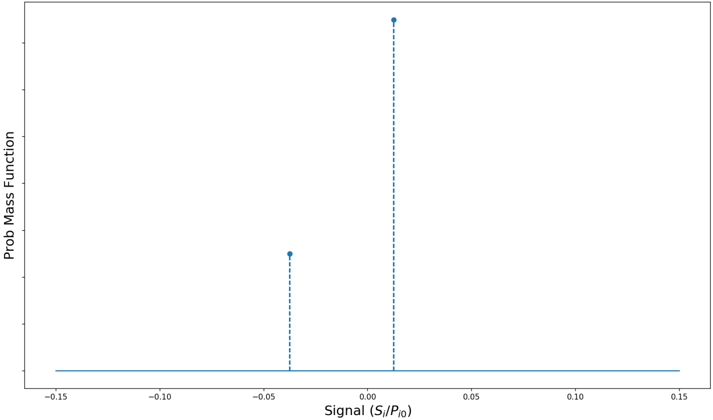
- How can we use theory and the price and/or order flow data to measure private information?
- In Back, Crotty, Li (2018), we develop and estimate a structural model of informed trading to address this question.
- Unexplored question: **From what distribution is private information drawn?**

Back, Crotty, Li (2018)

- We develop a structural model of informed trading and an estimation procedure to identify information asymmetry.
    - Continuous-time Kyle model with uncertain information event and magnitude
    - Propose ML estimation that allows use of intraday observations
    - Estimation utilizes the joint distribution of returns and order flows
- For computational reasons, BCL (2018) makes simplifying assumption that signal is binary in order to facilitate estimation.

Some possible signal distributions



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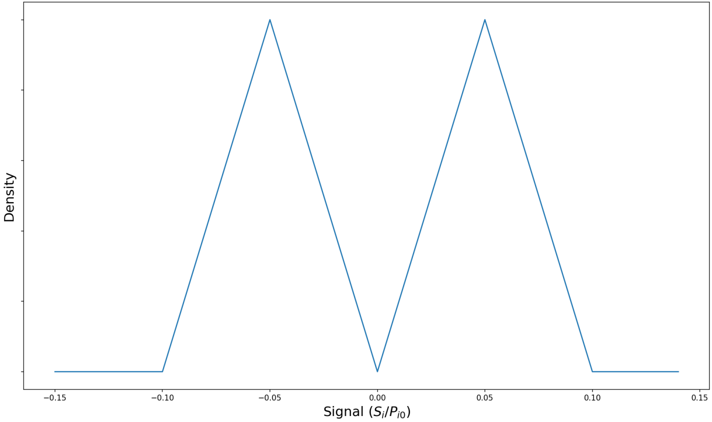

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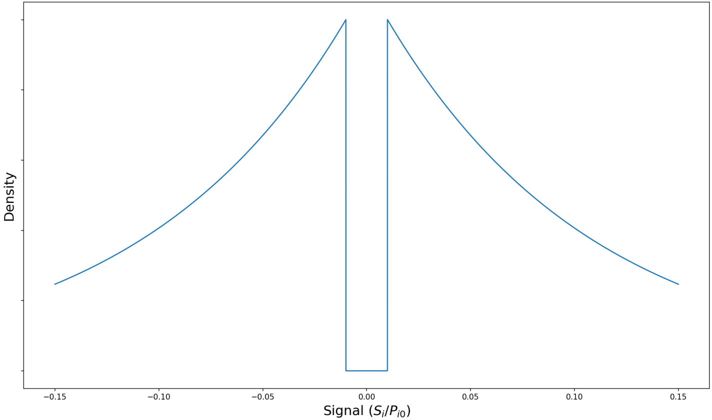

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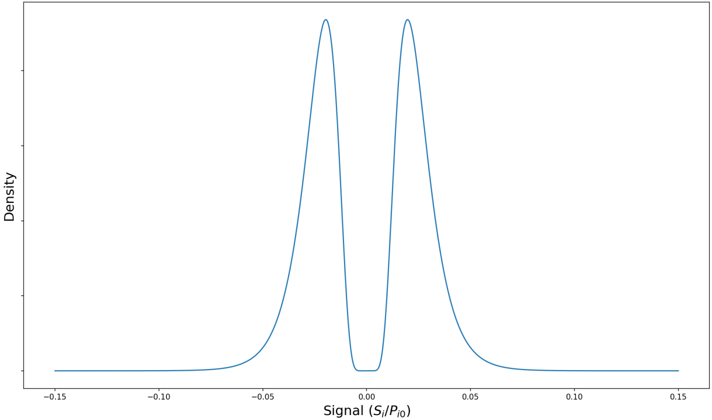

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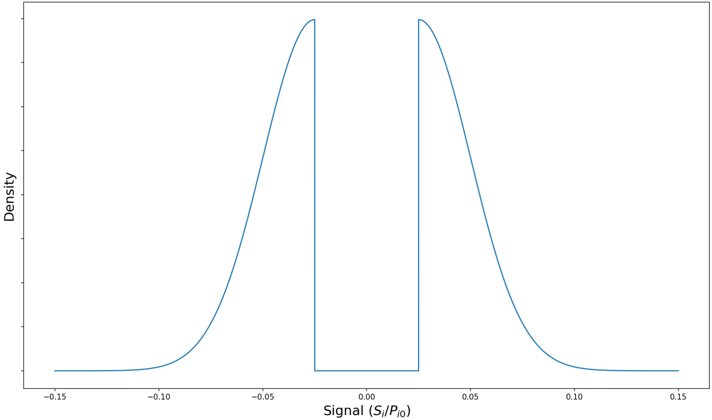

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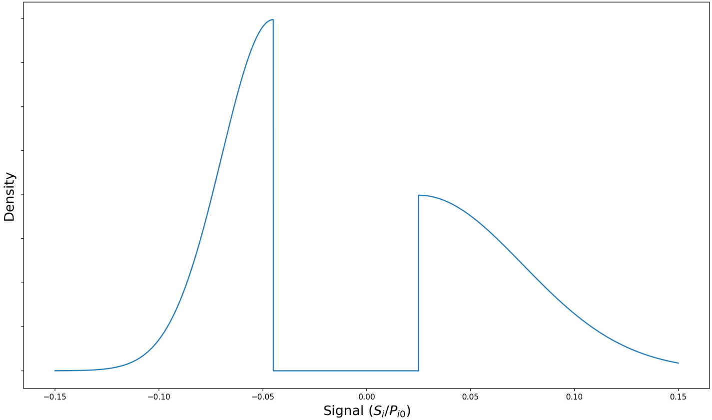

Research questions

- What types of distributions are private signals drawn from?
- Are these distributions systematically linked to firm characteristics?
- What explains time-series variation in private signals?
- Are private signal distributions related to trading frictions (e.g., short-sale costs)?
- What are the asset-pricing implications of private information distributions?

Computational issues

- Estimating the model for non-binary signals involves optimizing a likelihood function containing numerical integration (in the pricing function).

	Computing Time in Python	
Signal Type	Single Firm-Year	25-yr panel of 2500 firms
Triangular	15 hrs	1,070 yrs
Exponential	7 hrs	500 yrs
Normal	4 hrs	285 yrs
Lognormal	22 hrs	1,570 yrs

Model

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- Liquidity trades: Brownian motion Z with standard deviation σ
- Risk-neutral competitive market makers
- Information event at beginning with probability α
    - If there is an information event, a single trader sees a zero-mean signal S
    - If there is no information event, the trader is still present in the market as a value trader.
- Public information = martingale V

Definition of Equilibrium

- Let $\xi =$ indicator of information event (1 if yes, 0 if no).
- Let $Y = X + Z$ where $X =$ strategic trader's inventory.
- Market makers observe cumulative order imbalances Y and public information V.

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- Strategic trades must be optimal. The strategic trader chooses a rate of trade to maximize expected profits.

Order Imbalances in Equilibrium – Brownian Bridge

- Let F denote the distribution function of the normally distributed variable Z_1.
- Let G denote the continuous distribution function of the signal S.
- Set $y_L = F^{-1}(\alpha G(0))$ and $y_H = F^{-1}(1 - \alpha + \alpha G(0))$, so

$$\underbrace{\alpha\,\text{prob}(S \leq 0)}_{\text{Uncond. Prob. of Bad News}} = \text{prob}(Z_1 \leq y_L)\,,$$

and

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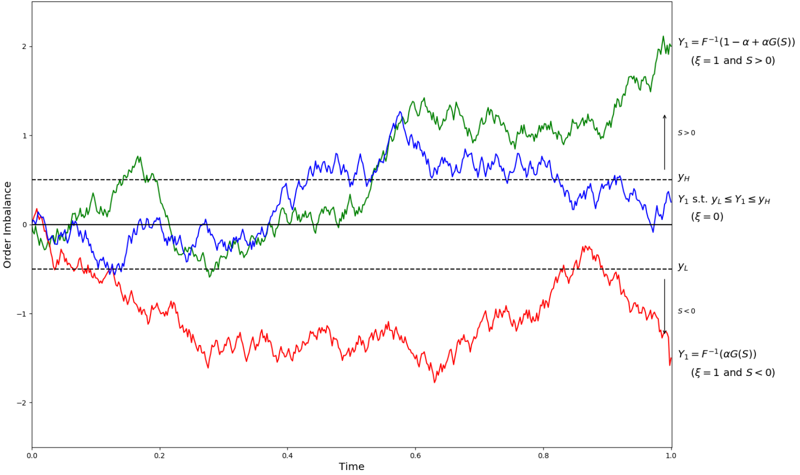
- In equilibrium, final cumulative order flows (Y_1) satisfy:
  - $Y_1 = F^{-1}(\alpha G(S)) < y_L$ when there is a low signal ($\xi S < 0$),
  - $Y_1 = F^{-1}(1 - \alpha + \alpha G(S)) > y_H$ when there is a high signal ($\xi S > 0$),
  - $y_L \leq Y_1 \leq y_H$ when there is no info event ($\xi = 0$).

Order Imbalance Paths in Equilibrium



Equilibrium Trades

- Equilibrium rate of trade depends on t, Y_t, and whether $\xi S < 0$, $\xi S = 0$, or $\xi S > 0$:

$$\mathrm{E}[Z_1 - Z_t \mid Z_t = Y_t, \xi S] =$$

$$\begin{cases} F^{-1}(\alpha G(s)) - Y_t & \text{if } \xi S < 0\,, \\ \mathrm{E}[Z_1 \mid Z_t = Y_t, y_L \le Z_1 \le y_H] - Y_t & \text{if } \xi = 0\,, \\ F^{-1}(1 - \alpha + \alpha G(s)) - Y_t & \text{if } \xi S > 0\,. \end{cases} \tag{1}$$

divided by $1 - t$.

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divided by $1 - t$.

- Market makers perceive order flows Y as a Brownian motion with zero drift and std deviation σ.

Equilibrium Prices (and a computational challenge)

- Given the history of Y through time t, the equilibrium price is

$$p(t, Y_t) = \int_{-\infty}^{y_L} \overbrace{G^{-1}\left(\frac{F(z)}{\alpha}\right)}^{\text{Signal Value}} \overbrace{f(z \mid t, Y_t)}^{\text{Density of } Z_1 \text{ cond. on } Z_t = Y_t} \, \mathrm{d}z + \int_{y_H}^{\infty} G^{-1}\left(\frac{F(z) - 1 + \alpha}{\alpha}\right) f(z \mid t, Y_t) \, \mathrm{d}z \,.$$

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- BCL (2018) makes a simplifying assumption about the signal distribution - signal is either high or low: $L < 0 < H$.
- Simplified pricing function:

$$p(t, Y_t) = L \times \overbrace{\mathrm{N}\left(\frac{y_L - Y_t}{\sigma\sqrt{1-t}}\right)}^{\Pr(\text{low info} \mid t, Y_t)} + H \times \overbrace{\mathrm{N}\left(\frac{Y_t - y_H}{\sigma\sqrt{1-t}}\right)}^{\Pr(\text{high info} \mid t, Y_t)} .$$

Estimation

- Use the joint distribution of intraday prices and order imbalances.
- Timing assumptions:
  - trading period corresponds to a day.
  - parameters are stable across year.
- The gross return through time t is

$$\frac{P_{it}}{P_{i0}} = \frac{V_{it}}{V_{i0}} + p(t, Y_{it})\,.$$

- V_{it} is geometric Brownian motion with volatility δ.

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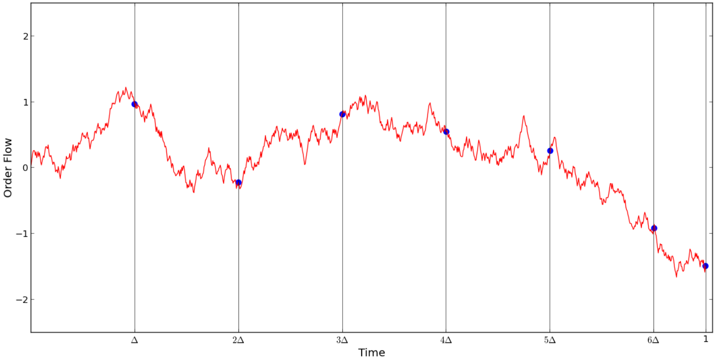
- Some signal parametrization: BCL (2018) assumes a binary zero-mean signal with magnitude parameter κ such that

$$(H_i - L_i)/P_{i0} = 2\kappa$$

Timing of Observations



- Observe $k+1$ daily prices (P_{ij}) and order-flows (Y_{ij}) at $t_1, \ldots, t_{k+1}$
  - $t_{k+1} = 1$ being the close
  - Evenly spaced intraday observations: $t_j = j\Delta$ for $\Delta > 0$ and $j \leq k$.

Day-i likelihood

- The log-likelihood of observing day-i sample is:

$$\mathcal{L}_i = \log\left(f\left(\frac{P_{i1}}{P_{i0}}, \ldots, \frac{P_{i,k+1}}{P_{i0}} | Y_i\right) f(Y_i)\right)$$

- On each day i, the vector $Y_i = (Y_{i,t_1}, \ldots, Y_{i,t_{k+1}})'$ is normally distributed with mean 0 and covariance matrix $\sigma^2 \Delta \Sigma$.

$$\Sigma = \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ 1 & 2 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 2 & \cdots & k & k \\ 1 & 2 & \cdots & k & 1/\Delta \end{pmatrix}.$$

Day-i likelihood

- The density function of $(P_{i1}/P_{i0}, \ldots, P_{i,k+1}/P_{i0})$ conditional on Y_i is

$$f(U_{i1}, \ldots U_{i,k+1})\mathrm{e}^{-\sum_{j=1}^{k+1} U_{ij}},$$

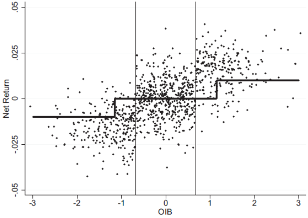
where f denotes the multivariate normal density function with mean vector $-(\delta^2\Delta/2)\Gamma$ and covariance matrix $\delta^2\Delta\Sigma$ and

$$\Gamma = \begin{pmatrix} 1 \\ 2 \\ \vdots \\ k \\ 1/\Delta \end{pmatrix}$$

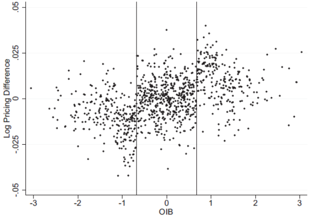
and

$$U_{ij} = \log\left(\frac{P_{ij}}{P_{i0}} - p(t_j, Y_{ij})\right) \tag{2}$$

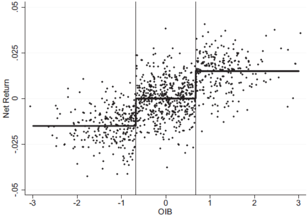
Intuition



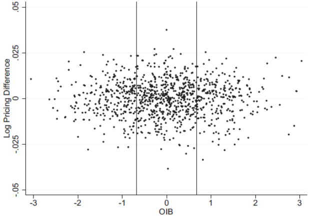
(a) $\hat{\alpha} = 0.25, \hat{\kappa} = 0.01, \hat{p}_L = 0.5$



(b) $\hat{\alpha} = 0.25, \hat{\kappa} = 0.01, \hat{p}_L = 0.5$

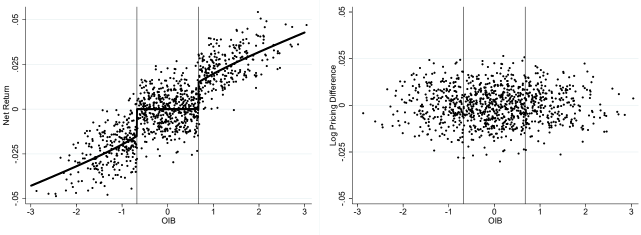


(c) $\hat{\alpha} = 0.50, \hat{\kappa} = 0.015, \hat{p}_L = 0.5$



(d) $\hat{\alpha} = 0.50, \hat{\kappa} = 0.015, \hat{p}_L = 0.5$

More general distributions



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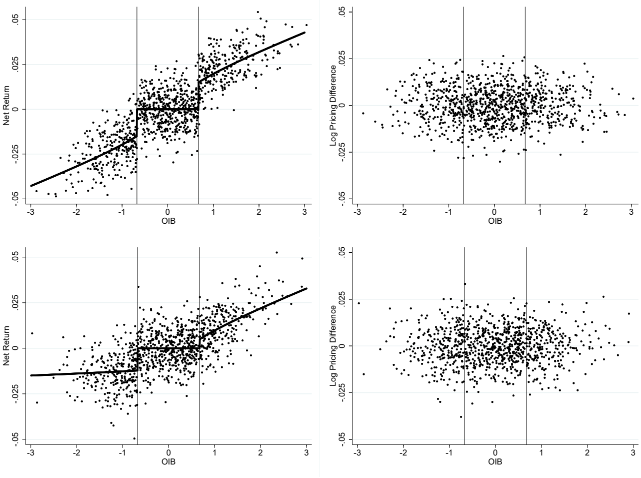

- Estimating private information distributions from prices and trading data could provide insights into the information environment in financial markets.

Thanks for your feedback!