Private Information Distributions in Securities Markets

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In Back, Crotty, Li (2018), we develop and estimate a structural model of informed trading to address this question.

Unexplored question: From what distribution is private information drawn?
We develop a structural model of informed trading and an estimation procedure to identify information asymmetry.

- Continuous-time Kyle model with uncertain information event and magnitude
- Propose ML estimation that allows use of intraday observations
- Estimation utilizes the joint distribution of returns and order flows

For computational reasons, BCL (2018) makes simplifying assumption that signal is binary in order to facilitate estimation.
Some possible signal distributions
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Research questions

- What types of distributions are private signals drawn from?
- Are these distributions systematically linked to firm characteristics?
- What explains time-series variation in private signals?
- Are private signal distributions related to trading frictions (e.g., short-sale costs)?
- What are the asset-pricing implications of private information distributions?
Computational issues

- Estimating the model for non-binary signals involves optimizing a likelihood function containing numerical integration (in the pricing function).

<table>
<thead>
<tr>
<th>Signal Type</th>
<th>Single Firm-Year</th>
<th>25-yr panel of 2500 firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>15 hrs</td>
<td>1,070 yrs</td>
</tr>
<tr>
<td>Exponential</td>
<td>7 hrs</td>
<td>500 yrs</td>
</tr>
<tr>
<td>Normal</td>
<td>4 hrs</td>
<td>285 yrs</td>
</tr>
<tr>
<td>Lognormal</td>
<td>22 hrs</td>
<td>1,570 yrs</td>
</tr>
</tbody>
</table>
Model

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- Liquidity trades: Brownian motion $Z$ with standard deviation $\sigma$
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- Continuous-time Kyle model with asset traded on time interval $[0, 1]$
- Liquidity trades: Brownian motion $Z$ with standard deviation $\sigma$
- Risk-neutral competitive market makers
- Information event at beginning with probability $\alpha$
  - If there is an information event, a single trader sees a zero-mean signal $S$
  - If there is no information event, the trader is still present in the market as a value trader.
- Public information $= \text{martingale } V$
Definition of Equilibrium

- Let $\xi =$ indicator of information event (1 if yes, 0 if no).
- Let $Y = X + Z$ where $X =$ strategic trader’s inventory.
- Market makers observe cumulative order imbalances $Y$ and public information $V$. 
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- Price must equal the expected value of the asset conditional on the market makers’ information and given the trading strategy of the strategic trader:

$$P_t = V_t + \mathbb{E} \left[ \xi S \mid \mathcal{F}_t^V, Y \right]$$
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- Strategic trades must be optimal. The strategic trader chooses a rate of trade to maximize expected profits.
Order Imbalances in Equilibrium – Brownian Bridge

Let $F$ denote the distribution function of the normally distributed variable $Z_1$.

Let $G$ denote the continuous distribution function of the signal $S$.

Set $y_L = F^{-1}(\alpha G(0))$ and $y_H = F^{-1}(1 - \alpha + \alpha G(0))$, so

$$\alpha \text{ prob}(S \leq 0) = \text{prob}(Z_1 \leq y_L),$$

Uncond. Prob. of Bad News

and

$$\alpha \text{ prob}(S > 0) = \text{prob}(Z_1 > y_H).$$

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In equilibrium, final cumulative order flows ($Y_1$) satisfy:

- $Y_1 = F^{-1}(\alpha G(S)) < y_L$ when there is a low signal ($\xi S < 0$),
- $Y_1 = F^{-1}(1 - \alpha + \alpha G(S)) > y_H$ when there is a high signal ($\xi S > 0$),
- $y_L \leq Y_1 \leq y_H$ when there is no info event ($\xi = 0$).
Order Imbalance Paths in Equilibrium

\[ Y_L = F^{-1}(1 - \alpha + \alpha G(S)) \]
(\(\xi = 1\) and \(S > 0\))

\[ Y_H \]

\[ Y_L \leq Y_1 \leq Y_H \]
(\(\xi = 0\))

\[ Y_L = F^{-1}(\alpha G(S)) \]
(\(\xi = 1\) and \(S < 0\))
Equilibrium Trades

- Equilibrium rate of trade depends on $t$, $Y_t$, and whether $\xi S < 0$, $\xi S = 0$, or $\xi S > 0$:

$$E[Z_1 - Z_t \mid Z_t = Y_t, \xi S] =$$

$$\begin{cases} 
F^{-1}(\alpha G(s)) - Y_t & \text{if } \xi S < 0, \\
E[Z_1 \mid Z_t = Y_t, y_L \leq Z_1 \leq y_H] - Y_t & \text{if } \xi = 0, \\
F^{-1}(1 - \alpha + \alpha G(s)) - Y_t & \text{if } \xi S > 0.
\end{cases}$$

(1)

divided by $1 - t$. 
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- Market makers perceive order flows $Y$ as a Brownian motion with zero drift and std deviation $\sigma$. 
Equilibrium Prices (and a computational challenge)

Given the history of $Y$ through time $t$, the equilibrium price is

$$p(t, Y_t) = \int_{-\infty}^{Y_L} G^{-1}\left(\frac{F(z)}{\alpha}\right) f(z \mid t, Y_t) \, dz$$

$$+ \int_{Y_H}^{\infty} G^{-1}\left(\frac{F(z) - 1 + \alpha}{\alpha}\right) f(z \mid t, Y_t) \, dz.$$
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$$

- BCL (2018) makes a simplifying assumption about the signal distribution - signal is either high or low: $L < 0 < H$.

- Simplified pricing function:

$$
p(t, Y_t) = L \times N \left( \frac{y_L - Y_t}{\sigma \sqrt{1 - t}} \right) + H \times N \left( \frac{Y_t - y_H}{\sigma \sqrt{1 - t}} \right).
$$
Estimation

- Use the joint distribution of intraday prices and order imbalances.
- Timing assumptions:
  - trading period corresponds to a day.
  - parameters are stable across year.
- The gross return through time $t$ is
  \[
  \frac{P_{it}}{P_{i0}} = \frac{V_{it}}{V_{i0}} + p(t, Y_{it}).
  \]
- $V_{it}$ is geometric Brownian motion with volatility $\delta$. 


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Some signal parametrization: BCL (2018) assumes a binary zero-mean signal with magnitude parameter $\kappa$ such that

$$\frac{(H_i - L_i)}{P_{i0}} = 2\kappa.$$
Observing $k+1$ daily prices ($P_{ij}$) and order-flows ($Y_{ij}$) at $t_1, \ldots, t_{k+1}$

- $t_{k+1} = 1$ being the close
- Evenly spaced intraday observations: $t_j = j\Delta$ for $\Delta > 0$ and $j \leq k$. 
The log-likelihood of observing day-\(i\) sample is:

\[
\mathcal{L}_i = \log \left( f \left( \frac{P_{i1}}{P_{i0}}, \ldots, \frac{P_{i,k+1}}{P_{i0}} \mid Y_i \right) \right) f(Y_i)
\]

On each day \(i\), the vector \(Y_i = (Y_{i,t_1}, \ldots, Y_{i,t_{k+1}})'\) is normally distributed with mean 0 and covariance matrix \(\sigma^2 \Delta \Sigma\).

\[
\Sigma = \begin{pmatrix}
1 & 1 & \cdots & 1 & 1 \\
1 & 2 & \cdots & 2 & 2 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
1 & 2 & \cdots & k & k \\
1 & 2 & \cdots & k & 1/\Delta
\end{pmatrix}.
\]
The density function of \((P_{i1}/P_{i0}, \ldots, P_{i,k+1}/P_{i0})\) conditional on \(Y_i\) is

\[
f(U_{i1}, \ldots U_{i,k+1})e^{-\sum_{j=1}^{k+1} U_{ij}},
\]

where \(f\) denotes the multivariate normal density function with mean vector \(-((\delta^2 \Delta/2)\Gamma)\) and covariance matrix \(\delta^2 \Delta \Sigma\) and

\[
\Gamma = \begin{pmatrix} 1 \\ 2 \\ \vdots \\ k \\ 1/\Delta \end{pmatrix}
\]

and

\[
U_{ij} = \log \left( \frac{P_{ij}}{P_{i0}} - p(t_j, Y_{ij}) \right) \quad (2)
\]
Intuition

(a) $\hat{\alpha} = 0.25, \hat{\kappa} = 0.01, \hat{p}_L = 0.5$

(b) $\hat{\alpha} = 0.25, \hat{\kappa} = 0.01, \hat{p}_L = 0.5$

(c) $\hat{\alpha} = 0.50, \hat{\kappa} = 0.015, \hat{p}_L = 0.5$

(d) $\hat{\alpha} = 0.50, \hat{\kappa} = 0.015, \hat{p}_L = 0.5$
More general distributions
More general distributions
• Estimating private information distributions from prices and trading data could provide insights into the information environment in financial markets.

Thanks for your feedback!