The True Cost of Social Security *

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August 24, 2018

Abstract

Implicit government obligations represent the lion's share of government liabilities in the U.S. and many other countries. Yet these liabilities are rarely measured, let alone properly adjusted for their risk. This paper shows, by example, how modern asset pricing can be used to value implicit fiscal debts taking into account their risk properties. The example is the U.S. Social Security System's net liability to working-age Americans. Marking this debt to market makes a big difference. Its market value is 86 percent higher than the Social Security trustees' valuation method suggests.

*We thank Deborah Lucas and Steven Zeldes for very helpful comments and MIT and Boston University for research support. This paper is an updated version of NBER WP 14427 by Alex Blocker, Laurence Kotlikoff, and Stephen Ross, who passed away in 2017. The WP was written in 2008 but never published. Kotlikoff writes, "Steve Ross will forever be a giant in economics and finance. And for those who knew Steve, he'll be our ever brilliant North Star, guiding our path and sparkling with joy."

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1 Introduction

In most developed countries and in many developing ones, commitments to make transfer payments and collect receipts represent the lion's share of government obligations and resources. Often referred to as implicit liabilities and assets, they typically are either ignored in assessing fiscal sustainability or valued on a piecemeal basis using ad hoc techniques. The justification for this practice generally offered is twofold. First, implicit fiscal commitments do not represent legal liabilities. Second, implicit commitments are difficult to value given their uncertain and extended nature.

This rationale may assuage accountants, but it offers little comfort to economists or, indeed, to anyone concerned with economic policy. The immense gulf between countries' true indebtedness and what's being measured means countries are largely driving blind with respect to their fiscal affairs. Generational accounting, developed by Auerbach, Gokhale, and Kotlikoff (1991), attempts to remedy this situation. Its framework is the government's intertemporal budget constraint, and it treats all government commitments on a consistent basis regardless of their legal status.

These advantages notwithstanding, a major shortcoming of generational accounting as well as related measurements[1] is the failure to adjust future government flows properly for risk. Generational accountants usually value the government's future payments and receipts by adding a risk premium to the risk-free discount rate. But their choice of risk premiums has no clear theoretical or empirical basis.

This paper presents a method for properly valuing implicit government debt. It treats government benefit obligations and tax claims as non-traded financial assets and applies what are now standard asset-pricing techniques to their valuation. In particular, we use Arbitrage Pricing Theory (APT) from Ross (1976b) and Ross (1976a) and its associated risk-neutral, derivative-pricing and process-free pricing theories (see Cox and Ross (1976) and Ross (1978)). Our method treats future government payments and receipts as securities whose returns comprise two components – a market component, which is spanned by traded securities, and an idiosyncratic component, which can be fully diversified.

We apply our pricing method (henceforth referenced as APT) to value Social Security's net retirement benefit liability to working-age Americans (those aged 26 to 60).[2] Our valuation determines how much the U.S. government would have to pay private parties or foreign governments to retire this liability. Marking this implicit debt to market makes a big difference – an 86 percent difference to be precise. The 2015 (our benchmark year) liability equals 48.0 trillion when marked to market, but $25.8 trillion when valued using Social Security Administration (SSA) methodology.

Finding such a large discrepancy could be expected. The Social Security Trustees' unfunded liability calculation, reported in their annual Trustees Report, makes no adjustment for uncertain future economy-wide average wage growth, which is so determinative of workers' future benefits and taxes.[3] The Trustees also make no attempt to mark their risk-free benefit obligations to market notwithstanding the availability of risk-free securities to do such pricing. These obligations include retirement benefits being made to current retirees as well as retirement benefits that will be made to current workers once their initial benefit level is

[1]See, for example, the 75-year and infinite horizon liability calculations reported in the annual OASDI Trustees Reports.

[2]Net retirement liability refers to Social Security's obligation to pay OAI retirement benefits net of OAI taxes.

[3]The trustees do examine the sensitivity of their liability measures to alternative economic and demographic assumptions. But this is no substitute for proper risk adjustment.

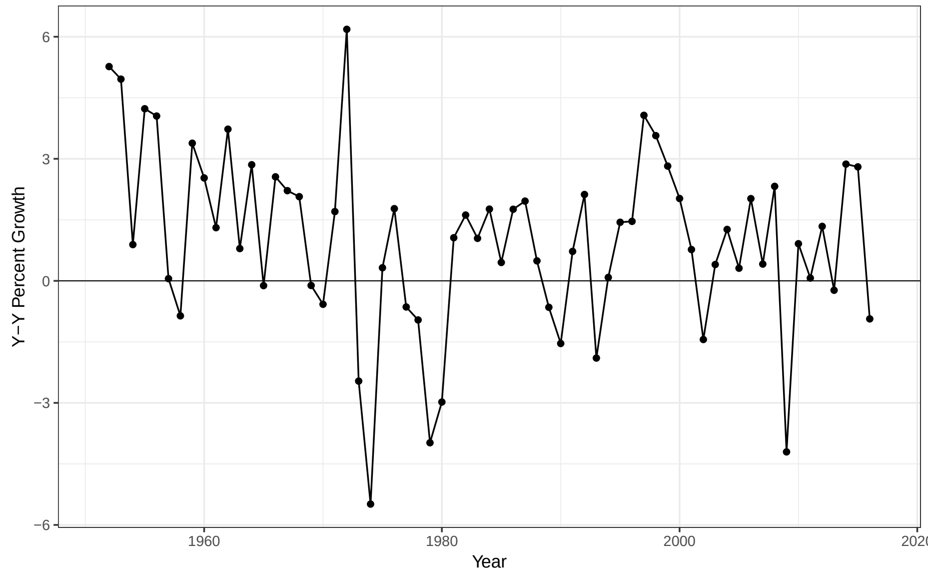
determined.

Social Security's failure to adjust formally for uncertainty in average real-wage growth is surprising given that a) this growth rate has been highly variable and b) Social Security's benefit obligations and tax receipts represent, in large part, wage-growth financial derivatives.

Figure 1 documents annual swings in average real wage growth rates between 1951 and 2015. Over this period, the average real wage has grown by as much as 6.2 percent in a single year and declined by as much as 5.5 percent.

Figure 1

Growth Rates of the Average U.S. Real Wage, 1951-2016



Source: `https://www.ssa.gov/oact/cola/AWI.html`

The variability of real wage growth suggests there could be risk here to price, but it tells us nothing about the degree to which implicit claims to growth in the real wage are valued in the market. For it is the covariance of real wage growth and market returns, together with the mean real-wage growth rate that determines the current price of an implicit wage-growth security.

As we show, a one-year, \$1 investment in the wage-growth security is worth \$0.999 according to APT. Social Security's valuation of such a claim is quite similar – \$0.985.[4] But the APT and SSA valuations of multi-year wage growth securities (and the associated valuations of out-year benefits and taxes) diverge to an increasing degree the longer the duration of the relevant wage-growth security. In the case of a 35-year wage growth security, the APT and SSA valuations are \$1.03 and \$0.598, respectively.

Since the benefits to be paid to current workers postdate the taxes to be collected from them, the duration-dependent divergence of APT and SSA wage-growth valuations and the dependence of benefits and taxes on wage growth suggest that Social Security is systematically understating its net liability to current workers. But the trustees make a second valuation mistake that more than offsets the first.

This second mistake involves the valuation of the benefits beyond those received in the first year of eligibility. These benefits are paid out as inflation-indexed annuities and should

[4]Social Security assumes real wages will grow, on average, by 1.2 percent and discounts, as indicated, at a 2.7 percent rate.

be actuarially priced using the prevailing Treasury Inflation Protected Securities (TIPS) term structure.[5] In 2018 the average annual real yields on TIPS was 0.75 percent, 0.76 percent, 0.78 percent, 0.85, and 0.92 percent for 5, 7, 10, 20, and 30 year maturities, respectively. Each of these yields is considerably lower than the 2.7 percent real yield used by SSA in its 2018 unfunded liability calculation. Using the TIPS term structure, the value of a $1 dollar single-life real annuity for a 62 year-old woman in 2017 is $22.02. This is 24.8 percent higher than SSA's $17.64 valuation.[6]

Thus, Social Security's trustees appear, in part, to be undervaluing benefits relative to taxes and, in part, overvaluing benefits relative to taxes, with the latter mistake outweighing the former.[7]

Our paper proceeds in section 2 with a literature review. Section 3 observes, following Baxter (2001) and Baxter and King (2001), that a household's future Social Security benefits and, by extension, its future Social Security taxes, can be viewed as financial assets, albeit ones with special market and idiosyncratic return properties. We clarify these return properties and show how to value Social Security net retirement benefits using risk-neutral pricing and arbitrage-pricing theory.

Section 4 presents our wage-growth valuation regressions and compares APT valuation of a $1 wage-growth security with SSA valuation. Section 5 describes our use of the Panel Study of Income Dynamics (PSID) data to estimate a random effects model of individuals' annual relative earnings – their annual earnings relative to Social Security's measure of economy-wide, average annual earnings. We use this model to determine the predictable, idiosyncratic component of future relative earnings and, thus, of future benefit claims and tax obligations. When it comes to taxes, we treat 8.5 percentage points of the 10.6 percentage point combined employer and employee OASI (Old Age Survivor Insurance) payroll tax rate as the tax used to finance OAI retirement benefits.[8] We then combine market-pricing and idiosyncratic-pricing elements to calculate the average value of benefit claims and tax obligations by age, sex, and education. Finally, we apply age-, sex-, and education-specific population weights to determine, using both APT and SSA methodologies, the aggregate values of future Social Security net benefits payable to working-age Americans.

Since our main focus is on market-pricing differences in valuing Social Security, we incorporate the same idiosyncratic component in both our SSA and APT valuations. Thus, the aforementioned 86 percent difference in aggregate APT and SSA net liability measures is purely attributable to differences in market pricing, specifically how APT and SSA value wage growth and inflation-indexed annuities.

Section 6 illustrates section 5's analysis by comparing APT and SSA 2015 benefit, tax, and net retirement-liability valuations for selected demographic groups. We then present the aggregate valuations under the APT and SSA methodologies, decomposing the 86 percent net liability difference into benefit and tax components. Section 7 responds to potential criticisms

[5] See `www.federalreserve.gov/release/h15.data.htm`

[6] Inflation-indexed annuities are sold on the market. Indeed, Principal Financial Group's price for this annuity is $24.36, which is 28 percent higher than our valuation (and 56 percent higher than Social Security's). Using these market prices for real annuities, while tempting, would, we think be inappropriate given that adverse selection surely explains much of the 28 percent differential and doesn't come into play in valuing annuities provided to all members of particular cohorts.

[7] The former mistake involves failing to account for risk with respect to initial benefit awards and tax payments, whereas the latter mistake involves failure to account for safety with respect to benefit payments once they commence.

[8] This is 10.6 percent times .797, which is the 2015 ratio of retiree benefits to total OASI benefits as reported in the 2016 OASDI Trustees Report.

of our approach. And section 8 concludes by pointing out that the valuation methods used here can be applied to other government implicit claims and obligations.

2 Literature Review

There are two approaches to pricing risky government promises, whether positive or negative. One illustrated by Hasanhodzic and Kotlikoff (2018a) is to specify and simulate a full CGE model and then use consumption-asset pricing to value government promises. Their findings suggest using prevailing safe bond rates to discount short as well as long-term promises even when those promises can be far larger or far smaller than expected.

Unfortunately, simulating highly detailed, large scaled CGE models runs afoul of the Curse of Dimensionality. Although recent computational breakthroughs by Marcet (1988); Marcet and Marshall (1994); Judd, Maliar, and Maliar (2009, 2011); Brumm and Scheidegger (2017) are permitting the inclusion of up to 300 state variables (Hasanhodzic and Kotlikoff (2018a) use 80), these may still fall far short of what's needed to handle highly detailed specifications. Moreover, there is no guarantee that a necessarily parsimonious CGE model is capturing all the key salient factors underlying the valuation of unpriced promises.

The alternative to structurally pricing government promises is reduced-form, empirical pricing based on Arbitrage Pricing Theory (APT) (Ross (1976b, 1976a) and its associated risk-neutral, derivative-pricing and process-free pricing theories.[9] Lucas and Zeldes (2006) is an early paper that applies modern asset-pricing theory and APT techniques to value pension promises in a realistic setting. Their focus is on private-sector defined benefit pensions. But their approach extends automatically to government-provided pensions.

In their APT approach, Lucas and Zeldes (2006) posit a diffusion process for wages and stock values. This process produces a significant long-term correlation between earnings growth and stock returns a là Goetzmann (2005). Geanakoplos and Zeldes (2010, 2011) also value Social Security promises using a diffusion process, pointing to Benzoni et al. (2007) to support their assumption of a low short-run, but high long-run correlation between wage growth rates and stock returns. Geanakoplos and Zeldes (2011) provide an interesting Lucas tree-type model involving the early revelation of news about future productivity shocks. This information acquisition process does indeed produce small short-run but high long-run correlation between wage growth and stock returns.

Yet, as Hasanhodzic and Kotlikoff (2018b) show, OLG models with significant ergodic properties can evince correlations between current economic activity and economic outcomes in the distant past. This can arise in models in which agents do not learn ahead of time about future productivity shocks, i.e., they do not learn from past (current) outcomes about current (future) shocks.

Our study takes a different tack from those of Geanakoplos and Zeldes. We relate the growth rate of wage rates to either to current only or to one-period-only lagged asset returns. We find very similar pricing of wage growth0rate securities regardless of the choice of contemporaneous or lagged regressors. And, unlike Geanakoplos and Zeldes (2011), we find a major understatement of Social Security's unfunded retirement benefit liability.

The different approaches generate quite different assessments of Social Security's unfunded

[9] See Cox and Ross (1976), Cox, Ingersoll, and Ross (1977), Ross (1978), and Cox, Ross, and Rubinstein (1979). We say "reduced form" because the operationalization of this pricing method requires positing and estimating how government promises co-vary with either marketed assets or a subset of economic factors meant to capture undiversifiable economic risk.

liabilities. We find a very significant understatement of these liabilities. Social Security's mistake, in our view, is not its failure to adjust for risk, but its failure to adjust for safety. Social Security's actuaries discount benefits, once they've been received and become sure liabilities, at a rate far above the market rate on long-term TIPS (Treasury Inflation Protected Securities). Although Geanakoplos and Zeldes (2010, 2011) report that Social Security's liabilities are significantly overstated, they adopt Social Security's overly high safe discount rate. Hence, their measure of Social Security's liabilities appears biased upward.

3 Valuing Social Security Retirement Benefits and Taxes

Let b_i stand for the full retirement benefit or Primary Insurance Amount (PIA) available to worker i[10]. This benefit is a concave function of the worker's Average Indexed Monthly Earnings (AIME). The AIME is, in turn, calculated by first accumulating worker i's past covered earnings in each year starting from the year the worker was age 16 and continuing to the year the worker reaches age 60. The accumulation factor is based on the economy-wide growth in average total (uncovered as well as covered) monthly earnings. Next the 35 largest values of these indexed earnings plus worker i's nominal earnings received after age 60 are averaged to form the AIME.

The rate of accumulation is determined by the real growth in economy-wide averaged earnings. For simplicity, we assume that workers' 35 years of highest earnings occur between ages 26 through 60 and that the worker retires at age 60.

The PIA is inflation-adjusted to ensure the same real PIA is used regardless of when the worker elects to start collecting benefits. Workers can begin collecting benefits as early as age 62 and as late as age 70. Deviations in workers' initial collection ages from their ages of full retirement trigger actuarial reductions or increases in the retirement benefit. We assume that all current workers begin collecting their benefits at age 62.[11]

In (1), $w_{i,j}$ denotes the covered (up to the Social Security earnings ceiling) wage earned by worker i in year j, τ_i references worker i's year of birth, g_k stands for the growth rate of average real earnings in year k, and 420 refers to 35 years times 12 months.

$$b_i = f_{\tau_i+60}\left(\frac{\sum_{j=\tau_i+25}^{\tau_i+60} w_{i,j} \prod_{k=j}^{\tau_i+60}(1+g_k)}{420}\right), \tag{1}$$

where f_{τ_i+60} captures the PIA benefit formula and its argument is worker i's AIME.

Let $\bar{w}_j$ stand for the level of economy-wide, real average earnings in year j. Define the ratio of worker i's covered earnings in year j to $\bar{w}_j$ by $z_{i,j}$, i.e.,

$$w_{i,j} \equiv z_{i,j}\bar{w}_j. \tag{2}$$

[10]The PIA formula is described in detail at `https://www.ssa.gov/oact/cola/piaformula.html`

[11]Assuming that Social Security's actuarial adjustment is based on the same real interest rate as used in its unfunded liability valuation, SSA valuation of its net liability to current workers should be independent of when workers collect their benefits. On the other hand, the APT valuation of this net liability will be larger the later workers collect because Social Security will provide larger benefit increases in return for delaying benefit collection that the market indicates is actuarially fair. Thus, in assuming that current workers begin collecting retirement benefits at age 62, we are biasing down our estimate of Social Security's understatement of its net liability to current workers.

Substituting (2) into (1) yields

$$\begin{aligned} b_i &= f_{\tau_i+60}\left(\frac{\sum_{j=\tau_i+25}^{\tau_i+60} z_{i,j}\bar{w}_j \prod_{k=j}^{\tau_i+60}(1+g_k)}{420}\right) \\ &= f_{\tau_i+60}(\bar{z}_i\bar{w}_{\tau_i+60}), \end{aligned} \tag{3}$$

where $\bar{z}_i$ is one twelfth the average annual value of $z_{i,j}$.

Social Security indexes not just a worker's past earnings to economy-wide average covered earnings; it also indexes the brackets in its year-t benefit function $f_t()$. Thus, other things equal, if $\bar{w}_{\tau_i+60}$ is twice as large, the value of b_i will be twice as large; i.e.,

$$f_{\tau_i+60}(\bar{z}_i\bar{w}_{\tau_i+60}) = f(\bar{z}_i\bar{w}_{\tau_i+60}, \bar{w}_{\tau_i+60}), \tag{4}$$

where $f(,)$ is homogeneous of degree one in the second argument. Using this property, we can write

$$b_i = h(\bar{z}_i)\bar{w}_{\tau_i+60}, \tag{5}$$

where

$$h(\bar{z}_i) = f(\bar{z}_i, 1). \tag{6}$$

In what follows, we assume that $h(\bar{z}_i)$ and $\bar{w}_{\tau_i+60}$ are independently distributed.

Note that for someone born in year τ, the current (year t) average covered wage, $\bar{w}_t$, is related to the average covered wage in year $\tau+60$ according to

$$\bar{w}_{\tau+60} = \bar{w}_t \prod_{k=t}^{\tau+60}(1+g_k). \tag{7}$$

Equations (5) and (7) imply

$$b_i = h(\bar{z}_i)\bar{w}_t \prod_{k=t}^{\tau_i+60}(1+g_k). \tag{8}$$

According to (8), worker i's full retirement benefit, b_i, is equivalent to what would be earned by investing the amount $h(\bar{z}_i)\bar{w}_t$ at time t and holding it until time τ_i+60 in a wage-growth security, i.e., a security that compounds at the rate of growth of average real wages. Of course, as with any time-t risky investment, the ultimate value of $h(\bar{z}_i)$ at time τ_i+60 is unknown. Yet enough is known at time t to determine the value of b_i.

Note that the cross-sectional average of $\bar{z}_i$ is 1. We assume that the terms $\bar{z}_i - 1$ have no value much like the deviations of individual insurance claims from the industry average have no value. What matters for value, then, is simply the expected value of $h(\bar{z}_i)$, $E[h(\bar{z}_i)]$.

Given that $h(\bar{z}_i)$ is independent of $\bar{w}_{\tau_i+60}$ and that the valuation operator is linear, we can write the value of the benefit claim as

$$V_t(b_i) = V\left(\bar{w}_t h(\bar{z}_i)\left[\prod_{k=t}^{\tau_i+60}(1+g_k)\right]\right), \tag{9}$$

and further as

$$V_t(b_i) = \bar{w}_t E(h(\bar{z}_i)) V\left(\prod_{k=t}^{\tau_i+60}(1+g_k)\right), \tag{10}$$

where $V()$ stands for the valuation function.

A straightforward argument, laid out in the appendix and motivated by financial pricing theory, lets us determine the current value of $\prod_{k=1}^{\tau_i+60}(1+g_k)$, which we refer to as a \$1 wage-growth security with maturity τ_i+60–t. To form the valuation, all we need find is a portfolio of traded securities whose payoff mimics the final real wage up to some idiosyncratic terms. These terms are assumed not to matter for valuation because they are uncorrelated with the returns to marketed securities.

Assume that the annual growth rate, g_t, of the real wage has the following structure where $f_{i,t}$ denotes the time-t value of marketed asset i, and ϵ_t is an unpriced, idiosyncratic shock:

$$g_t = \alpha + \sum_i \beta_i \frac{\Delta f_{i,t}}{f_{i,t-1}} + \epsilon_t. \tag{11}$$

The appendix demonstrates that the final real-wage payment can be replicated by a portfolio of the (real) bond and the assets, $f_{i,t}$, that is rebalanced every year. The cost of doing so is the value of the terminal real wage, which the appendix shows is

$$V\left(\prod_{k=t}^{\tau_i+60}(1+g_k)\right) = \left(\frac{1+\alpha+r\sum_i \beta_i}{1+r}\right)^{\tau_i+60-t}, \tag{12}$$

where r denotes the real rate of interest, which we assume is constant. If there is a term structure of real rates available from, say, inflation-protected bonds, then the formula becomes:

$$V\left(\prod_{k=t}^{\tau_i+60}(1+g_k)\right) = \prod_{t}^{\tau_i+60}\left(\frac{1+\alpha+r_s\sum_i \beta_i}{1+r_s}\right). \tag{13}$$

Combining (10) and (13) gives:

$$V_t(b_i) = \bar{w}_t E_t h(\bar{z}_i) \prod_{t}^{\tau_i+60}\left(\frac{1+\alpha+r_s\sum_i \beta_s}{1+r_s}\right). \tag{14}$$

These formulas assume that only contemporaneous asset returns price the wage growth security. As also shown in the Appendix, the formulas are more complex if lagged as well as contemporaneous asset returns predict current wage growth. The complexity involves the need to adjust for the fact that lagged returns predict current and future wage growth. While the case of multiple lags is quite complex, the formulas follow quite naturally in the case of a single lag:

$$V(b_i) = \bar{w}_t E_t h(\bar{z}_i)\left(\frac{1+\alpha+\sum_j \beta_j \frac{\Delta f_{j,t-1}}{f_{j,t-1}}}{1+r_{t-1}}\right)\prod_{s=t}^{T-1}\left(\frac{1+\alpha+r_s\sum_j \beta_j}{1+r_s}\right), \tag{15}$$

where the differences between equations (14) and (15) reflect the impact of lagged asset returns on expected future wage growth. To be more precise, the β_js are the loadings on one-year lagged returns in the following modification of (10).

$$g_t = \alpha + \sum_j \beta_j \frac{\Delta f_{j,t-1}}{f_{j,t-2}} + \epsilon_t. \tag{16}$$

3.1 Incorporating Survival to Age 62 and Annuity Valuation

The above treats Social Security benefits as a one-year payoff of a wage-growth security derivative, with the payoff occurring in the year the worker reaches age 60. This is inappropriate for four reasons. First, under our assumption that workers take early retirement benefits, benefits commence at 62. Second, the worker may not survive from her current age to age 62. Third, receiving benefits at age 62, rather than full retirement age, triggers an actuarial reduction, the size of which depends on the worker's year of birth. Fourth, the benefit starting at age 62 is not a one-year payment, but continues each year in the future conditional on the worker's survival.

Equation (17) modifies (15) to arrive at $V_t(B_i)$ – the time-t APT value of worker i's lifetime benefits, B_i. In the formula, we multiply $V_t(b_i)$ by a) c – a two year real discount factor that discounts, at the market's safe real rate, for the fact that benefits don't begin at age 60, but rather at age 62, b) $q_{i,t,\tau_i,62}$ – the time-t probability that worker i, who was born at time τ_i, survives to age 62, c) μ_{τ_i} – the early-retirement benefit reduction factor for workers born in year τ_i who begin benefit receipt at age 62, and d) δ_{i,τ_i} – the actuarially discounted present value of a \$1 real annuity beginning at age 62 payable to worker i who is born in year τ_i, where the discounting is at the market's safe real term structure and goes back to age 62.

$$V_t(B_i) = cq_{i,t,\tau_i,62}\mu_{\tau_i}\delta_{i,\tau_i}\bar{w}_t E_t(h(\bar{z}_i)) \left(\frac{1+\alpha+\sum_j \beta_j \frac{\Delta f_{j,t-1}}{f_{j,t-1}}}{1+r_{t-1}} \right) \prod_t^{T-1} \left(\frac{1+\alpha+r_t \sum_j \beta_j}{1+r_t} \right). \tag{17}$$

3.2 SSA's Benefit Valuation Formula

The corresponding SSA valuation, $\hat{V}_t(B_i)$, is given by (

$$\hat{V}_t(B_i) = \hat{c}q_{i,t,\tau_i,62}\mu_{\tau_i}\hat{\delta}_{i,\tau_i}\bar{w}_t E_t(h(\bar{z}_i)) \left(\frac{1+\bar{g}}{1+\bar{r}} \right)^{\tau_i+60-t}, \tag{18}$$

where $\bar{r}$ and $\bar{g}$ reference, respectively, Social Security's assumed 2.7 percent real discount rate and 1.2 percent real-wage growth rate. Clearly the final terms in equations (17) and (18) differ, which reflects differences in APT and SSA valuations of real wage growth. But the two-year discount factor, $\hat{c}$, and the actuarial value of the annuity, $\hat{\delta}_{i,\tau_i}$, also differ from their equation (17) counterparts because they too incorporate SSA's assumed 2.7 percent discount rate rather than the prevailing TIPS term structure.

3.3 Measuring the Idiosyncratic Component of Benefit Valuation

The value of $\bar{w}_t$ for our base year, t=2015, is reported by the Social Security administration, so the remaining question is how to determine the value of the idiosyncratic component, $E_t h(\bar{z}_i)$. Our method is to use our aforementioned random- effects model of relative earnings to simulate the average value of $h(\bar{z}_i)$ by individual age in 2015, sex, and education group. The education groups are less than high school, high school, and college or more.

Our random effects model, which we estimate separately for each of the six education and sex groups, is given by

$$\begin{aligned} &z_{it} = \phi_i + \theta_0 + \theta_1 b_1 + \theta_2 a_{it} + \theta_3 a_{it} b_i + \theta_4 a_{it}^2 + \theta_5 a_{it}^2 b_i + \theta_6 a_{it}^3 + \theta_7 a_{it}^3 b_i + \epsilon_{it}, \\ &\theta_i \sim N(0, \sigma_\alpha^2), \epsilon_{it} \sim N(0, \sigma_\epsilon^2), \end{aligned} \tag{19}$$

where ϕ_i is the random effect, and a_{it} and b_t reference, respectively, worker i's age in year t and her year of birth (i.e., $t - a_{it}$). The term ϵ_{it} is a transitory error.

To determine the average value of $h(\bar{z}_i)$ for agents in 2015 of a given sex and education group who were born in year b_i (who were a given age in 2005), we draw 100,000 values of α_i ; i.e., we consider 100,000 agents with specific random effects. For each agent we draw 35 values of ϵ_{it} – starting for the year the agent was age 26 and continuing through the year the agent will be age 60. For each year, m, of these 35 years, we evaluate the right-hand-side of (19) using the values of the agent's α_i and b_i as well as the value of ϵ_{it} drawn for that year. In this evaluation, a_{im} is set to $m - b_i$. Next we form the average over the 35 simulated values of z_{it} to form the value the agent's $\bar{z}_i$. This value of $\bar{z}_i$ is then run through Social Security's benefit formula to calculate the value of $h(\bar{z}_i)$.[12] The average of the $h(\bar{z}_i)$ values across all 100 agents provides our sex-, education-, and cohort-specific estimates of $E_t h(\bar{z}_i)$.

3.4 Calculating the Aggregate Value of Benefits

The total value of benefits for Americans age 26 through 60 is calculated by forming

$$\sum_{i=1}^{N} \omega_i V_t(B_i), \tag{20}$$

where ω_i is the CPS population weight for sex-, education-, and cohort-population cell i.

3.5 Valuing Taxes

Taxes, $T_{i,l}$, paid by worker i in year l equal the tax rate in year l, ν_l, multiplied by worker i's covered wages in year l, $z_{i,l}\bar{w}_l$.

$$T_{i,l} = \nu_l z_{i,l} \bar{w}_l. \tag{21}$$

Following the above lines of argument, we can write the time-t value of taxes paid in year l as

$$V_t(T_{i,l}) = q\nu_l E_t(z_{i,l})\bar{w}_t \left(\frac{1 + \alpha + \sum_j \beta_j \frac{\Delta f_{j,t-1}}{f_{j,t-1}}}{1 + r_{t-1}} \right) \prod_t^l \left(\frac{1 + \alpha + r_t \sum_j \beta_j}{1 + r_t} \right). \tag{22}$$

To determine the value of $E_t z_{it}$ in (22), we again resort to averaging draws generated from our random-effects model within each sex-, education-, and cohort-specific cell. Letting T_i stand for the remaining lifetime OAI taxes of worker i and $V_t(T_i)$ stand for the market value of these taxes,

$$V_t(T_i) = \sum_t^{\tau_i + 60} V_t(T_{i,l}). \tag{23}$$

[12] This function includes the early retirement reduction factor since we are computing reduced benefits assumed to be taken at age 62.

3.6 Calculating the Aggregate Value of Taxes

The total value of taxes for Americans age 26 through 60 is calculated by forming

$$\sum_{i=1}^{S} \omega_i V_t(T_i), \tag{24}$$

where, again, ω_i is the CPS weight for sex-, education-, and cohort-cell i.

4 Valuing the Wage-Growth Security

The first step in valuing the real wage-growth security is estimating the parameters of (11) on data covering 1952 through 2015. We consider two sets of APT-factor regressions for estimating our intercept α and the factor loadings (the β coefficients). The first set of regressions, reported in table 1, incorporate contemporaneous and lagged nominal equity returns of indexes of large and small cap stocks as well as of short- and long-term government nominal bond returns.[13] The second set, reported in table 2, substitute contemporaneous and lagged Fama-French factors for the asset-index returns.

When lagged regressors are included, the adjusted R^2s are quite high, ranging from .217 to .314 across the two tables. Omitting lagged regressors lowers these values dramatically. The table 1 regressions with lagged returns provide better fits than the corresponding table 2 regressions based on Fama-French factors.

For each regression, we used the estimated parameters to calculate the implied present value of \$1 invested in the wage growth security for 1 and 35 year horizons based on the appendix formula (a13). These valuations are provided in the tables. Consider the results in table 1, which include lagged regressors and provide the best fits to the data. For a 1-year, \$1 wage-growth security, our valuations range from 99.8 cents to 1.01 cents. SSA's valuation is 1.012 divided by 1.027, or 98.5 cents, based on SSA's assumed 1.2 percent average real-wage growth rate and 2.7 percent real discount rate. In the case of the 35-year, \$1 wage-growth security, our table 1 lagged regression valuations range from 95.2 cents to 133 cents. The corresponding SSA valuation is $(1.012/1.027)^{35}$ or 59.8 cents.

Clearly the discrepancy between the SSA and APT valuations grows the farther out is the wage-growth security's duration. This is clear from the third model in table 1. Its 1-year wage-growth security valuation is almost identical to Social Security's. But its 35-year valuation of 133.1 cents is 123 percent higher than SSA's 59.8 cent figure. The reason the APT and SSA valuations diverge more for longer duration wage-growth securities is due to the fact that contemporaneous and past real market returns affect future expected real wage growth differently through time in the APT valuation. This is clear from considering how the security's duration enters into equations (17) and (18).

We base our APT valuations on the regression appearing in the last column in table 1 (Restricted 2). This model values the \$1, one-year wage-growth security at 99.9 cents and the \$1, 35-year wage-growth security at 103.0 cents. Our selection of this model was guided by the Bayesian information criterion (BIC, also known as Schwartz's Criterion) and Akaike's information criterion (AIC) as well as Akaike's criterion corrected for small-sample bias (AICc).[14]

[13] Regressing a real growth rate against nominal returns may seem surprising, but the short-term nominal bond return is highly correlated with, and thus controls for, the inflation rate.

[14] See Andrews and Monahan (1992), Hurvich and Tsai (1989), and Schwarz (1978). These criteria represent ways to

5 Modeling Relative Earnings

Table 4 presents parameter estimates by demographic group for our random-effects model. The dependent variable is the natural logarithm of relative earnings (z). The table's estimated coefficients were used to simulate lifetime paths for relative covered earnings as described above. Our data come from the Panel Survey of Income Dynamics (PSID) for the years 1968 through 2015. We include observations reporting educational attainment and positive labor income.[15]

Figures 2 and 3 shows the average age-relative earnings profiles for different cohorts holding sex and education constant as predicted by table 4's results. The profiles tell some interesting stories. First, all the profiles peak between ages 35 and 50 with the exception of females with high school educations; their profiles increase monotonically. Second, the female (male) profiles are significantly higher (lower) for younger cohorts, indicating that successive cohorts have experienced a smaller gender gap in earnings. Third, females with college or more education experience relative limited declines in their relative earnings after age forty. Each of the other groups experience very sharp declines.

6 Benefit and Tax Valuations

Tables 5 through 13 show the results of our calculations of benefit and tax values for individuals aged 26, 40, and 55 in 2005. The APT and SSA benefit values are calculated based on (17) and (18), respectively. The APT tax value is calculated based on (23), and the SSA tax values are calculated using wage-growth security values analogous to those in (18). Table 14 presents aggregate values of tax and benefits based on (20) and (24).

The values in tables 5 through 13 make sense. The market values of benefit obligations are larger for those with more education, but so are the market values of tax obligations. The net liabilities are smaller for those with more education in the case of 26 year olds, but larger for the better educated in the case of 40 and 55 year olds. This simply reflects the fact that older workers have all their benefits coming, but only a portion of their lifetime taxes left to pay.

The differences in SSA and APT valuations of net liabilities to particular groups can be quite sizeable. Take 40 year-old females with a high school education. SSA's methodology places Social Security's average liability to these women at \$166,055, whereas the APT valuation is 2.1 times higher at \$347,481.

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trade-off model complexity and goodness-of-fit in model selection. All three criteria are increasing in the number of parameters in the model and decreasing in the maximized log-likelihood. In the case of normal errors, the latter can be recast as increasing in the sum of squared residuals. Hence, minimization of these criteria is a logical guide for model selection. Furthermore, the use of model selection criteria avoids the problems of multiple testing and non-nested model comparison that are commonly seen with other approaches. The criteria have similar forms (see Schwarz (1978) and Hurvich and Tsai (1989)), although each behaves somewhat differently. AIC and AICc are based on an information- theoretic derivation and are asymptotically equivalent to likelihood-based selection. BIC is based on Bayesian as well as minimum-description-length arguments and is not asymptotically equivalent to selection based purely upon maximized likelihood. This is because BIC penalizes the addition of parameters more heavily than AIC (the coefficient on the number of parameters is 2 for AIC vs. $log(n)$ for BIC). Hence, BIC tends to select more parsimonious models than AIC.

[15] Selecting observations in this manner excluded over half of potential observations depending on the year in question. This selection results in an unbalanced panel and raises the issue of coefficient bias, although it's not clear how such bias, were it to exist, would cut with respect to our comparison of APT and SSA valuations.

A final point concerns the absolute size of the benefit obligations to older workers. As table 11 shows, whether one uses APT or SSA valuations, the amounts are sizeable when set against the relatively small values of financial wealth held by typical older workers.

Table 14 reports our main finding, namely that Social Security's valuation method appears to understate the market value of the net liability to working-age Americans by approximately \$22 trillion or 86 percent. The main source of this undervaluation involves the valuation of benefits. SSA-based valuation leads to a \$35.3 trillion figure, whereas APT valuation puts the figure at \$59.5 trillion. This is a \$24.1 trillion differential. In contrast, the APT value of taxes owed by working-age Americans is only \$1.9 trillion larger than the SSA value.

Were we to value benefits by simply marking annuities to market (using TIPS rates), but retaining SSA's wage-growth security valuation, we'd arrive at an SSA aggregate benefit valuation of \$45.7 trillion. Since doing so would leave the SSA valuation of taxes unchanged, it would increase the SSA valuation of net liabilities to \$36.1 trillion. Hence, proper annuity valuation would, by itself, eliminate approximately 47 percent of the difference in APT and SSA valuations. Social Security Trustees could, therefore, significantly improve their net liability measure simply by using TIPs returns to value the system's promised annuities.

7 Critiquing the Approach

There are at least five objections to the approach taken here. The first is that, given their size, any actual attempt to market Social Security's net liabilities would dwarf the financial markets. Our response is that valuation is a marginal exercise; we routinely establish values for total stocks of financial and real assets as well as financial liabilities based on the going price in the market. Take, for example, the Federal Reserve's Flow of Funds valuation of owner-occupied homes. All of these homes are all carried at marginal market price even though the immediate sale of all U.S. homes could greatly alter values. Like most homes, Social Security liabilities are currently being held, rather than actively traded. Moreover, although Social Security's net liabilities are large relative to U.S. net worth, they are a small component of total world net worth.

The second objection involves what we take to be the idiosyncratic component of real wage growth. Does the market value this component, which accounts for about half of wage growth variability? Arguably not. If this risk were significant to investors it would, presumably, be marketed and priced by the major financial securities we've included in our analysis. The opposite view must maintain that financial markets are profoundly incomplete and fail to span aggregate risks of major importance to investors. Were the opposite view correct, our results would be incomplete and potentially biased. But the direction of such bias cannot be determined a priori.

This second objection is reminiscent of the old debate in international trade about whether there are more factors of production or goods being produced; it is fundamentally unresolvable. Our position, though, should be clear. We have a practical problem, and we offer a consistent and robust practical solution that closely aligns with decades of research in modern asset pricing. The alternative of relying on economic projections of real wages many years in the future is similar to relying on analysts' forecasts of a company's earnings and the using the resulting discounted cash flow to determine the value of the company's stock rather than simply using its trading price. Such practice bets against the market and represents a highly questionable foundation on which to base generational and fiscal policy.

The third objection is our failure to take account of potential future policy changes; i.e., changes to the $h()$ function. Here we plead guilty; but determining which policy changes are likely to arise and the impact on different parties of such changes is not our objective. Our objective is valuing Social Security's net claims taking current policy as given and determining whether that policy is sustainable. Were we instead to incorporate future policy changes, our valuation exercise would be trivial; we'd necessarily find the government to be intertemporally balanced. The reason is that along any path the economy travels government spending will necessarily be financed by the private sector. From this perspective, the government can never be intertemporally insolvent. That said, many of these paths, all of which entail ex-post satisfaction of the government's intertemporal budget constraint, will entail terrible economic and fiscal conditions, including policy changes described as explicit or implicit defaults.

A fourth objection is that the sum of workers' valuations of their net Social Security's benefit claims may differ dramatically from the valuation we measure. As demonstrated in Liu, Rettenmaier, and Saving (2007), individual valuations are based on wealth- equivalent changes in expected utility and take into account workers' idiosyncratic risks. Admittedly, how much today's workers would be willing to pay to keep Social Security is an interesting question. But that amount is potentially quite different from what the market would be willing to pay.[16]

Finally, there is the question of relevance. Does it matter if the market thinks a country is financially troubled, while its government proclaims its solvency? The answer is surely yes. Over the years, scores of countries have experienced abrupt runs on their currencies and financial instruments because the market made a decision that their policies were unsustainable. Argentina's 2002 fiscal/financial meltdown, following a decade of excellent economic growth, is a good example. This crisis came as a shock to its leaders who had forecasts aplenty for how the government was going to reverse its prolonged fiscal slide and pay its bills.

8 Conclusion

No one would suggest that the prices of explicit financial securities are independent of their risk properties. Such a proposition would deny fact, let alone theory. But the same financial laws that determine the prices of marketed securities govern the pricing of non- marketed assets and liabilities; they cannot be priced by treating their variable returns as sure things and discounting at safe rates.[17] Nor can safe government payments and receipts be valued using discount factors that differ from the discounts associated with safe marketed securities. This, however, has been standard U.S. practice since our government began considering its implicit debts.

Were marking to market implicit government liabilities and assets of minor import, the

[16] The consumption of grapefruit provides a useful analogy. What workers are willing to pay to have access to grapefruit, if the alternative is never eating another grapefruit, is the sum of their consumer surplus from grapefruit. This is not the same as the cost of buying, at market prices, the grapefruit the workers intend to consume.

[17] Fortunately, government officials aren't asked to value the stock market. Were they to do so, they'd badly misprice the market. Indeed, were Social Security's valuation method applied to the S&P, its price- earnings ratio would equal 34.5 – more than twice the ratio observed at our writing. To see this, note that Social Security uses an assumed safe 2.7 percent discount rate for its liability valuations. Let e stand for the expected earnings on the S&P per dollar invested. Then Social Security would value the S&P by setting P, the price per dollar invested, equal to the $e/.027$; i.e., the value of a perpetual safe stream e discounted at 2.9 percent. Since $P = e/.027$, $P/e = 1/.027 = 37.0$.

government's ad-hoc valuation methods would be of little concern. But the example considered here – the valuation of Social Security's net retirement liability to working- age Americans – suggests the opposite. Proper asset pricing delivers a measure of this net liability that exceeds SSA's valuation by 86 percent.

Of course, Social Security's net retirement liability to working-age Americans is only part of its overall implicit debt. And Social Security is only one part of a much broader set of future U.S. government receipts and payments, whose market values need to be assessed. The ultimate goal, in this regard, is valuing all components of the government's intertemporal budget to determine whether its overall current policy is sustainable, i.e., whether the government's entire fiscal enterprise breaks even as a matter of present valuation. Answering this broader question is a much bigger task, but one that can surely be approached using techniques similar to those considered here.

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Appendix A Calculations

Suppose that the real wage follows the growth process:

$$\frac{dw}{w} = \alpha dt + \sum_i \beta_i \frac{df_i}{f_i} + \sigma_\epsilon d\epsilon, \tag{A.1}$$

where f is the vector of priced assets, e.g., the S&P 500, and ϵ is the process for the residual and unpriced component. The symbol df/f denotes the total returns (dividends included) on the priced assets.

Assuming w and f_i all follow a geometric Brownian motion (allowing non-zero covariance between asset prices) and converting this into logs we have:

$$\begin{aligned} d(\ln w) &= \frac{dw}{w} - \frac{1}{2}\sigma_w^2 dt \\ d(\ln f_i) &= \frac{df_i}{f_i} - \frac{1}{2}\sigma_i^2 dt, \end{aligned} \tag{A.2}$$

where σ^2 denotes the appropriately subscripted instantaneous variance.[18]

For reference, letting Ω denote the instantaneous variance covariance matrix for the returns on the priced assets we have:

$$\sigma_w^2 = \beta'\Omega\beta + \sigma_\epsilon^2. \tag{A.3}$$

Now we integrate to obtain the stochastic wage at time T:

$$w_T = w_0 \exp\left(\alpha T - \frac{1}{2}\sigma_w^2 T + \frac{1}{2}\left(\sum_i \beta_i \sigma_i^2\right) T + \sigma_\epsilon \int_0^T d\epsilon\right) \prod_i \left(\frac{f_{iT}}{f_{i0}}\right)^{\beta_i}. \tag{A.4}$$

In other words, the terminal real wage is an exponential in some time terms multiplied by power functions of the total accumulated values of the priced assets.

To obtain the current value of the wage at time T, we take the expected discounted value under the martingale measure, i.e., we take the expectation of the discounted value assuming that all of the priced assets have an expected growth rate equal to the risk free rate.

$$\begin{aligned} V &= E^*(e^{-rT} w_T) \\ &= w_0 \exp\left(-rT + \alpha T - \frac{1}{2}\sigma_w^2 T + \left(\frac{1}{2}\sum_i \beta_i \sigma_i^2\right) T + \frac{1}{2}\sigma_\epsilon^2 T + \left(\sum_i r\beta_i\right) T - \left(\frac{1}{2}\sum_i \beta_i \sigma_i^2\right) T + \frac{1}{2}(\beta'\Omega\beta) T\right) \\ &= w_0 \exp\left(-rT + \alpha T + r\left(\sum_i \beta_i\right) T\right). \end{aligned} \tag{A.5}$$

In the valuation all of the variance terms have dropped out and only the betas and the alpha remain. The easiest way to understand this is to build it up. Suppose first that there were no betas or they were all zero. Then the wage will grow at the rate α to T and be discounted back at the rate r. Now suppose there is only one marketed asset and its beta is 1. Then the wage is just the same as something that has the terminal value, $w_0 e^{\alpha T}$, invested in the asset. In that case the value is just the invested value, $w_0 e^{\alpha T}$, which is precisely what the formula gives. If beta isn't one, then the formula just corrects for the difference.

[18] Note, the second term in the *log* derivatives arises because the process instantaneously has infinite movement and its variance is of order dt. Since *log* is concave, we have the second order negative correction.

A.1 A Discrete Time Approach

While the above analysis is somewhat formal, the result indicates that a simpler intuitive approach applies. Furthermore, while the above may appear to depend on distributional assumptions, an elementary discrete time analysis will verify that it is in fact independent of any such assumptions.

In discrete time, we have

$$\frac{w_{t+1}}{w_t} = 1 + g_t = 1 + \alpha + \sum_i \beta_i f_{i,t} + \epsilon_t. \tag{A.6}$$

To value the future payment of w_{t+1} we can ask how much we would have to invest in marketed assets to replicate it. Consider investing A_t in the risky asset and B_t in the riskless asset. The return on that portfolio will be

$$\sum_i A_{it}(1 + f_{i,t}) + B_t(1 + r) = 1 + g_t = 1 + \alpha + \sum_i \beta_i f_{i,t} + \epsilon_t. \tag{A.7}$$

This will replicate the priced portion for any realization of the returns, $f_{i,t}$, if

$$\begin{aligned} A_{i,t} &= \beta_i, \\ B_t &= \frac{1 + \alpha - \sum_i \beta_i}{1 + r}, \end{aligned} \tag{A.8}$$

for a total expenditure of

$$V_1 = \sum_i A_{i,t} + B_t = \frac{1 + \alpha + r \sum_i \beta_i}{1 + r}. \tag{A.9}$$

The above expression tells us that a claim to one dollar invested in the wage-growth security for one year has an immediate and, therefore, sure value of V_1. At the end of one year the expected value of this claim is just $V_1(1 + r)$. Having this amount for sure in a year also has this same expected value. The value after one period of investing one dollar for two years in the wage-growth security is $V_1^2(1 + r)$. Discounted to the present, the value is just V_1^2. In general, the value, V^T, of a dollar invested in the wage-growth security for T years is V_1^T ; hence we can write

$$V^T = w_0 \left(\frac{1 + \alpha + r \sum_i \beta_i}{1 + r} \right)^T \to w_0 e^{T\left(\alpha + r(\sum_i \beta_i - 1)\right)}, \tag{A.10}$$

which is the continuous time formula.

Some care should be exercised in interpreting formulas a9 and a10. They would appear to imply that a riskier wage, i.e., one with a higher beta would actually be more valuable than a less risky one. This apparently anomalous result comes about because the return on the asset in a6 has not been demeaned. As a consequence, the intercept, α, is actually the intercept from a demeaned regression, γ, less β times the expected return on the asset,

$$a = \gamma - \beta E_f = \gamma - \beta(r + \pi),$$

where π is the risk premium on the asset. Substituting this result into a9 gives

$$V_1 = \sum_i A_{i,t} + B_t = \frac{1 + \alpha + r \sum_i \beta_i}{1 + r} = \frac{1 + \gamma - r \sum_i \beta_i \pi_i}{1 + r},$$

which clearly reflects the decline in value with increasing risk.

A.2 An Extension to Lagged Variable

Suppose the regression is in discrete terms (e.g., yearly), and we have only lagged variables:

$$\frac{\Delta w_t}{w_t} = \alpha + \sum_j \beta_j \frac{\Delta f_{j,t-1}}{f_{j,t-1}} + \epsilon_t. \tag{A.11}$$

A similar analysis produces the amended formula:

$$V(w_t) = w_0 \left(\frac{1 + \alpha + \sum_j \beta_j \frac{\Delta f_{j,-1}}{f_{j,-1}}}{1+r} \right) \left(\frac{1 + \alpha + r \sum_j \beta_j}{1+r} \right)^{T-1}, \tag{A.12}$$

which is the same as when the returns are contemporaneous but with the addition of the multiplying term containing the past year's returns. Without concerning ourselves with the issues of a stochastic interest rate, using the term structure of interest rates this formula becomes:

$$V(w_T) = w_0 \left(\frac{1 + \alpha + \sum_j \beta_j \frac{\Delta f_{j,-1}}{f_{j,-1}}}{1+r_{-1}} \right) \prod_{t=0}^{T-1} \left(\frac{1 + \alpha + r_t \sum_j \beta_j}{1+r_t} \right). \tag{A.13}$$

Extending this analysis to multiple lags is more difficult and explicitly involves the covariance structure of the returns. As an alternative we could change to a different formulation in terms of the unit root process:

$$w_T = w_0 \left((1+\alpha)^T + \sum_j \sum_{s=0}^{k} \beta_j f_{j,T-s} + \epsilon_T \right). \tag{A.14}$$

While this is more difficult to estimate that the usual sort of regression, it allows for a simple valuation equation:

$$V(w_T) = \left(\frac{1+\alpha}{1+r} \right)^T + \sum_j f_{j,0} \sum_{s=0}^{k} \frac{\beta_j}{(1+r)^s}. \tag{A.15}$$

We won't make use of this formulation since our estimations involve a single lag.

Appendix B Tables and Figures

Table 1

Regression of Average Wage Growth on Nominal Returns

	Unrestricted	Contemp. only	Lagged only	Restricted 1	Restricted 2
	Dependent variable: Real Average Wage Growth				
	(1)	(2)	(3)	(4)	(5)
Constant	0.008	0.019***	0.008	0.009*	0.010**
	(0.005)	(0.007)	(0.006)	(0.005)	(0.005)
STGovtBond	−0.309	−0.139		−0.372***	
	(0.243)	(0.314)		(0.122)	
LTGovtBond	−0.245	−0.076			
	(0.333)	(0.296)			
SmallCapStock	0.004	−0.017			
	(0.014)	(0.015)			
LargeCapStock	0.019	0.036			
	(0.032)	(0.028)			
L.STGovtBond	0.210		−0.165	0.147	
	(0.141)		(0.207)	(0.153)	
L.LTGovtBond	0.140		0.024	0.039	−0.145
	(0.211)		(0.202)	(0.209)	(0.101)
L.SmallCapStock	0.005		−0.006		
	(0.013)		(0.013)		
L.LargeCapStock	0.062***		0.068***	0.066***	0.060***
	(0.020)		(0.019)	(0.011)	(0.010)
1 yr. Valuation		1.008	0.998		0.999
35 yr. Valuation		1.331	0.952		1.03
AIC	-320.8	-307	-317.8	-324.4	-320.7
AICc	-317.4	-306	-316.8	-323.3	-320.3
BIC	-299.3	-294.1	-305	-311.5	-312.1
R^2	0.401	0.122	0.287	0.358	0.275
Adjusted R^2	0.312	0.062	0.238	0.314	0.251
Residual Std. Error	0.017 (df = 54)	0.021 (df = 59)	0.018 (df = 58)	0.017 (df = 58)	0.018 (df = 60)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 2

Regression of Average Wage Growth on Fama-French Factors

	Dependent variable:				
	Real Average Wage Growth				
	Unrestricted	Contemp. only	Lagged only	Restricted 1	Restricted 2
	(1)	(2)	(3)	(4)	(5)
Constant	0.004	0.009^{**}	0.006^{**}	0.006^{**}	0.005^{**}
	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)
MktRF	0.020	0.022			
	(0.024)	(0.023)			
HML	0.007	0.003			
	(0.027)	(0.023)			
SMB	−0.005	-0.038^{*}			
	(0.019)	(0.023)			
L.MktRF	0.064^{***}		0.063^{***}	0.064^{***}	0.059^{***}
	(0.010)		(0.011)	(0.010)	(0.010)
L.HML	−0.008		−0.009		
	(0.019)		(0.018)		
L.SMB	−0.026		−0.028	−0.028	
	(0.031)		(0.031)	(0.029)	
1 yr. Valuation		1	1.012	1.011	1.008
35 yr. Valuation		1.017	0.94	0.926	0.922
AIC	-318.9	-306.7	-322.5	-324.2	-324.1
AICc	-316.9	-306	-321.8	-323.8	-323.9
BIC	-301.5	-295.8	-311.6	-315.5	-317.5
R^2	0.290	0.061	0.264	0.261	0.236
Adjusted R^2	0.217	0.015	0.228	0.237	0.224
Residual Std. Error	0.019 (df = 58)	0.022 (df = 61)	0.019 (df = 61)	0.019 (df = 62)	0.019 (df = 63)
Note:					*p<0.1; **p<0.05; ***p<0.01

Table 3

Social Security Valuation Elements

Valuation	Annuity Factor (δ) for F, b. 1955	Early Retirement Factor (μ)	Discount for waiting to 62 (c)	2015 Average Wage ($\bar{w}$)
APT	22.02	0.733	0.983	\$48,099
SSA	17.64	0.733	0.948	\$48,099

Table 4

Coefficient Estimates from Random Effects Model of Relative Earnings

	Dependent variable:					
	log(z)					
	Male,<HS	Male,HS	Male,≥Col	Female,<HS	Female,HS	Female,≥Col
Constant	94.001***	100.027***	32.299***	49.654***	122.076***	62.210***
	(13.085)	(11.005)	(8.401)	(17.801)	(14.055)	(10.024)
AGE	−3.886***	−8.869***	−1.519***	−2.767**	−9.908***	−7.229***
	(0.847)	(0.740)	(0.557)	(1.101)	(0.927)	(0.657)
YAGE	0.002***	0.005***	0.001***	0.001***	0.005***	0.004***
	(4.3×10^{-4})	(3.8×10^{-4})	(2.8×10^{-4})	(5.6×10^{-4})	(4.7×10^{-4})	(3.3×10^{-4})
AGE2	0.071***	0.269***	0.041***	6.6×10^{-4}	0.194***	0.173***
	(0.017)	(0.015)	(0.011)	(0.021)	(0.019)	(0.013)
YAGE2	-4.0×10^{-5}***	-1.4×10^{-4}***	-2.7×10^{-5}***	-6.1×10^{-7}	-1.0×10^{-4}***	-9.3×10^{-5}***
	(8.8×10^{-6})	(7.8×10^{-6})	(5.7×10^{-6})	(1.1×10^{-5})	(9.6×10^{-6})	(6.4×10^{-6})
AGE3	-4.6×10^{-4}***	−0.002***	-4.6×10^{-4}***	4.1×10^{-4}***	−0.001***	−0.001***
	(1.2×10^{-4})	(1.1×10^{-4})	(7.7×10^{-5})	(1.5×10^{-4})	(1.3×10^{-4})	(8.2×10^{-5})
YAGE3	2.6×10^{-7}***	1.3×10^{-6}***	2.7×10^{-7}***	-2.1×10^{-7}***	7.0×10^{-7}***	7.1×10^{-7}***
	(6.2×10^{-8})	(5.5×10^{-8})	(4.0×10^{-8})	(7.7×10^{-8})	(6.7×10^{-8})	(4.2×10^{-8})
YOB	−0.051***	−0.056***	−0.021***	−0.028***	−0.065***	−0.036***
	(0.007)	(0.006)	(0.004)	(0.009)	(0.007)	(0.005)
Std. Dev. of Random Effects	0.85	0.73	0.67	1.04	0.88	0.77
Std. Dev. of Residuals	4.04	4.61	5.01	4.52	5.3	5.83
Individuals	3100	4067	4666	2499	3899	5251
Observations	21,768	35,980	47,918	15,269	33,788	46,877

Note: *p<0.1; **p<0.05; ***p<0.01

Table 5

Social Security Benefit Obligations to 26 Year-Olds

Sex	Valuation	<HS	HS	Col
Male	APT	226,483	278,290	486,923
Male	SSA	103,342	126,981	222,178
Female	APT	320,663	591,113	613,816
Female	SSA	144,162	265,750	275,957

Table 6

Social Security's Tax Claims on 26 Year-Olds

Sex	Valuation	<HS	HS	Col
Male	APT	61,105	88,485	213,677
Male	SSA	48,005	71,422	161,967
Female	APT	77,983	211,030	210,108
Female	SSA	58,794	154,537	159,357

Table 7

Social Security's Net Liability to 26 Year-Olds

Sex	Valuation	<HS	HS	Col
Male	APT	165,378	189,805	273,247
Male	SSA	55,336	55,559	60,211
Female	APT	242,680	380,083	403,708
Female	SSA	85,369	111,213	116,600

Table 8

Social Security Benefit Obligations to 40 Year-Olds

Sex	Valuation	<HS	HS	Col
Male	APT	240,720	293,898	478,645
Male	SSA	137,448	167,812	273,301
Female	APT	270,305	450,018	499,327
Female	SSA	151,940	252,957	280,674

Table 9

Social Security's Tax Claims on 40 Year-Olds

Sex	Valuation	<HS	HS	Col
Male	APT	38,709	48,460	139,640
Male	SSA	33,937	42,437	120,576
Female	APT	41,210	102,537	103,829
Female	SSA	35,637	86,902	89,082

Table 10

Social Security's Net Liability to 40 Year-Olds

Sex	Valuation	<HS	HS	Col
Male	APT	202,010	245,438	339,004
Male	SSA	103,511	125,376	152,724
Female	APT	229,095	347,481	395,498
Female	SSA	116,303	166,055	191,593

Table 11

Social Security Benefit Obligations to 55 Year-Olds

Sex	Valuation	<HS	HS	Col
Male	APT	264,392	322,601	483,241
Male	SSA	192,121	234,418	351,148
Female	APT	230,962	347,802	413,263
Female	SSA	165,074	248,583	295,369

Table 12

Social Security's Tax Claims on 55 Year-Olds

Sex	Valuation	<HS	HS	Col
Male	APT	6,953	10,354	27,962
Male	SSA	6,666	9,913	26,778
Female	APT	6,259	18,073	18,743
Female	SSA	5,998	17,280	17,930

Table 13

Social Security's Net Liability to 55 Year-Olds

Sex	Valuation	<HS	HS	Col
Male	APT	257,439	312,247	455,280
Male	SSA	185,454	224,505	324,370
Female	APT	224,703	329,729	394,520
Female	SSA	159,076	231,303	277,439

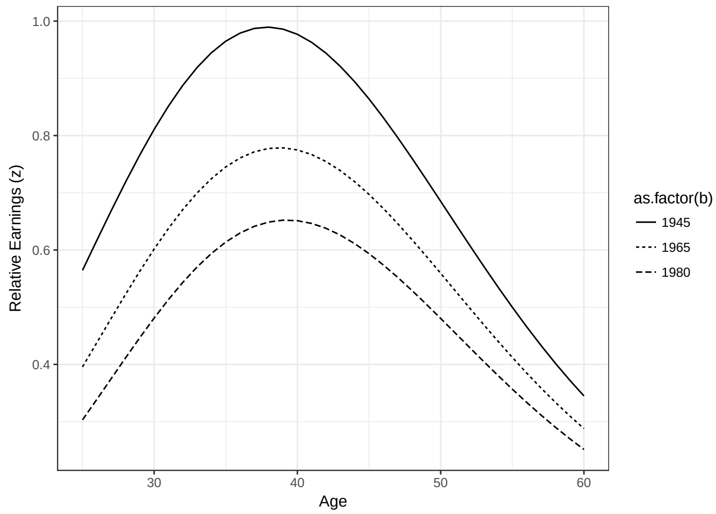
Table 14

SSA's Aggregate Net Liability to Working-Age Americans

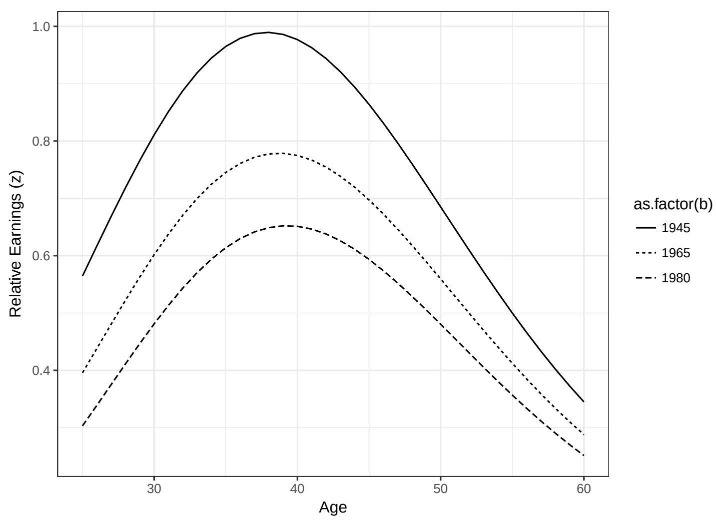
	APT	SSA
Aggregate Benefits Owed by SSA	59.450 Trillion	35.335 Trillion
Aggregate Tax Obligations Owed to SSA	11.480 Trillion	9.558 Trillion
SSA's Aggregate Net Liability	47.970 Trillion	25.777 Trillion

Figure 2

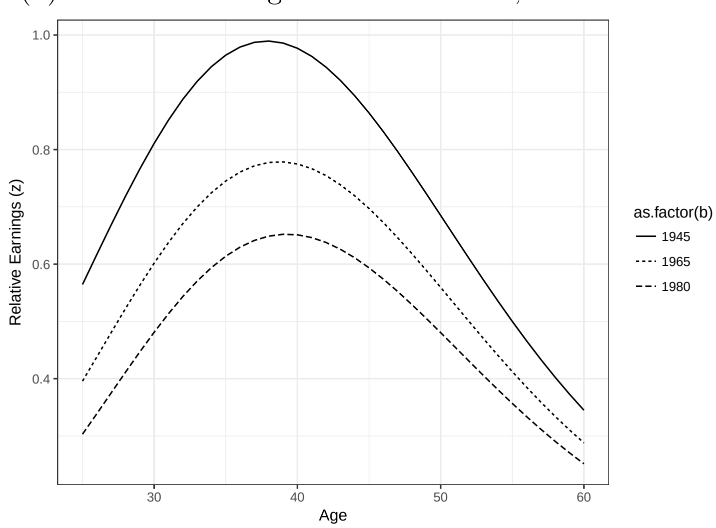
Earnings Profiles for Males



(a) Relative Earnings Profile for Males, Less than HS

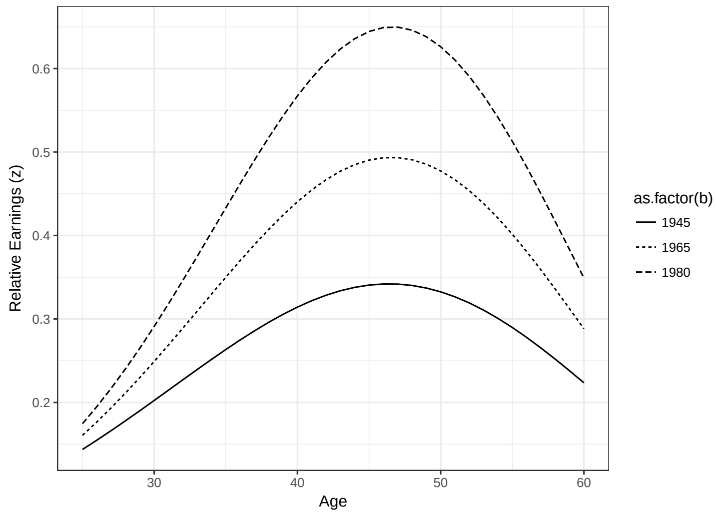


(b) Relative Earnings Profile for Males, HS Graduates

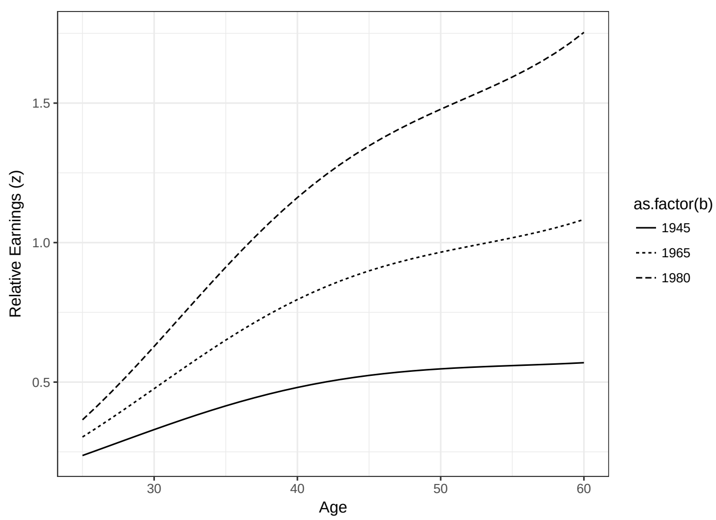


(c) Relative Earnings Profile for Males, College or more

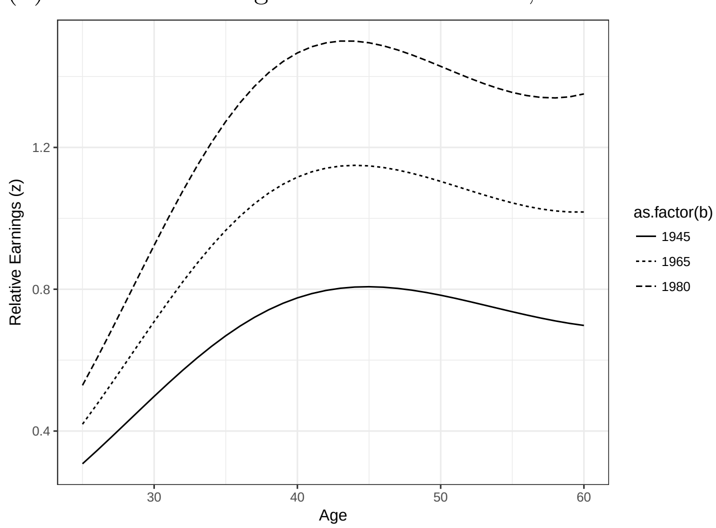
Figure 3
Earnings Profiles for Females



(a) Relative Earnings Profile for Females, Less than HS



(b) Relative Earnings Profile for Females, HS Graduates



(c) Relative Earnings Profile for Females, College or more