

Global Trends in Interest Rates

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June 18, 2018

Abstract

The trend in the world real interest rate for safe and liquid assets fluctuated in a narrow range close to 2 percent for more than a century, but it has dropped significantly over the past three decades. This decline has been common across advanced economies and it has come to dominate the country-specific trends, which have all but vanished since the 1970s. An increase in the convenience yield for safety and liquidity and the reduction in global growth both play important roles in this decline.

JEL CLASSIFICATION: E43, E44, F31, G12

KEY WORDS: Convenience Yield, Interest Rate Parity, VAR, Common Trends

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1 Introduction

Ten years after the most acute phase of the financial crisis, the world economy remains mired in a low interest rate environment. At the time of writing, the nominal yields on ten year government bonds are below 3 percent in the United States, a bit above 1 percent in the UK, around 40 basis points in Germany, and essentially zero in Japan. How unusual is this situation from a historical perspective? What is the role of global factors in depressing interest rates? Does this phenomenon reflect only headwinds still emanating from the global financial crisis, but that will eventually dissipate, or is it connected to secular developments that partly predate the Great Recession?

To address these questions, we study the joint dynamics of short and long term interest rates, inflation, and consumption for seven (now) advanced economies since 1870. We do so through a flexible time-series model—a vector autoregression (VAR) with common trends. This econometric tool allows us to impose long-run restrictions suggested by theory, remaining agnostic on whether these restrictions also hold at other horizons. In particular, we use international arbitrage in the long run to interpret the estimated common trend in real interest rates across countries as the trend in the world real interest rate. The same theoretical framework suggests in turn a decomposition of this trend into some of its potential drivers, such as global consumption growth.

The interest rates in our dataset are either on government securities, or on close substitutes, which are relatively safe and liquid compared to other privately issued assets. Therefore, we allow the convenience yield for safety and liquidity offered by these so-called “safe” assets to play a role in driving the international cross section of returns. To measure this convenience yield, the empirical analysis also includes Moody’s Baa corporate bond yield for the United States, as in Krishnamurthy and Vissing-Jorgensen (2012). Under certain assumptions, which we will discuss, this information is sufficient to capture the long run effect of convenience on the world interest rate. This approach is similar to the one we pursued in Del Negro et al. (2017), though that paper only focused on U.S. data over a much shorter sample.

Four main results emerge from our empirical analysis. First, the estimated trend in the world real interest rate is stable around values a bit below 2 percent through the 1940s. It rises gradually after World War II, to a peak close to 2.5 percent around 1980, but it has been declining ever since, dipping to about 0.5 percent in 2016, the last available year of data. The exact level of this trend is surrounded by much uncertainty, but the drop over the

last few decades is precisely estimated. A decline of this magnitude is unprecedented in our sample. It did not even occur during the Great Depression in the 1930s.

Second, the trend in the world interest rate since the 1960s essentially coincides with that of the U.S. In other words, the US trend *is* the global trend over the past five decades. In fact, this has been increasingly the case for (almost) all other countries in our sample: idiosyncratic trends have been vanishing since the 1970s. This convergence in cross-country interest rates is arguably the result of growing integration in international asset markets.

Third, the trend decline in the world real interest rate over the last few decades is driven to a significant extent by an increase in convenience yields, suggesting an increasing imbalance between the global demand for safety and liquidity and its supply. This contribution is especially concentrated in the period since the mid 1990s, supporting the view that the Asian financial crisis of 1997 and the Russian default as well as the ensuing collapse of LTCM in 1998 were a key turning point in the emergence of the imbalances of the global economy over the past few couple of decades (e.g., Bernanke, 2005; Bernanke et al., 2011; Caballero and Krishnamurthy, 2009; Caballero, 2010; Caballero and Farhi, 2014; Caballero et al., 2015; Gourinchas and Rey, 2016; Hall, 2016; Caballero et al., 2017; Caballero, 2018).

Fourth, a global decline in the growth rate of per-capita consumption is a further notable factor pushing global real rates lower. Its contribution since 1980 is comparable in magnitude to that of the convenience yield, but only about half as important, and less precisely estimated, over the last twenty years.

An important implication of these findings is that the persistent macroeconomic headwinds emanating from the financial crisis, including the effects of the extraordinary policies that were put in place to combat it, are far from the only cause of the low interest rate environment. More long-standing secular forces connected with a decline in economic growth since the early 1980s and the rise of convenience yields since the late 1990s also appear to be crucial culprits, even though these trends might have been exacerbated by the crisis. In fact, Caballero (2018) observes that the causality might run the other way, from the global safe asset imbalances that emerged in the late 1990s, depressing safe returns, to the financial fragility and macroeconomic turbulence that have afflicted the world economy since then. Furthermore, the global nature of the drivers of low interest rates limit the extent to which national policies can address the problem.

This paper is connected to several strands of the literature. The steady decline in real interest rates over the past few decades has been at the center of the academic and

policy debate at least since the early 2000s, when Bernanke (2005) suggested that a global saving glut might be holding interest rates down around the world. Following the Great Recession and slow subsequent recovery, secular stagnation became one of the most popular explanations for this phenomenon, as argued most prominently by Summers (2014).¹ In an attempt to shed light on this debate, a number of empirical papers have investigated the drivers of secular movements in real interest rates. Lunsford and West (2017), for instance, study long time-series for the United States. They find that demographic factors are robustly correlated with real interest rates, while productivity growth is not.²

A concept that has proved useful in the analysis of the causes of low interest rates is the natural real rate of interest, or r^* , which Laubach and Williams (2016) define as “the real short-term interest rate consistent with the economy operating at its full potential once transitory shocks to aggregate supply or demand have abated.” Estimates of r^* therefore focus on the underlying, secular drivers of the movements in interest rates, abstracting from shorter-term influences such as those related to the stance of monetary policy. In Del Negro et al. (2017), we discuss in some detail the theoretical and empirical connections between the concept of r^* proposed by Laubach and Williams (2003), the one obtained in DSGE models with nominal rigidities, and the type of low-frequency movements in real interest rates that are the focus of this paper.³

Compared to Del Negro et al. (2017), where we restrict attention to US data since the 1960s, this paper significantly widens the scope of the analysis by including data from seven advanced economies and dating back to 1870. Aside from generating evidence on the trends in interest rates for a wider set of countries, this paper identifies an explicitly global component in the secular movements of international interest rates, creating a direct connection with several hypothesis on the origin of the low interest rate environment based on worldwide forces, such as Bernanke’s saving glut hypothesis.

Holston et al. (2017) estimate r^* using international data, in their case for the US, Canada, the euro area, and the UK since 1961. Differently from our approach, though, their

¹Eggertsson et al. (2017) provide a quantitative evaluation of the factors leading to secular stagnation in a calibrated overlapping generations model.

²Other work stressing the role of demographics in the movements of interest rates includes Favero et al. (2016), Carvalho et al. (2016), and Gagnon et al. (2016).

³Recent work estimating variants of r^* with data from the United States includes Justiniano and Primiceri (2010), Barsky et al. (2014), Curdia et al. (2015), Kiley (2015), Johannsen and Mertens (2016), Pescatori and Turunen (2015), Christensen and Rudebusch (2016) and Crump et al. (2016). Yellen (2015) and Williams (2018) discuss its monetary policy implications.

estimates do not account explicitly for the global dimension of r^* , treating each country as a closed economy.⁴ The work of Hamilton et al. (2016) and Borio et al. (2017) is also related to ours, as they study the potential drivers of r^* using data from the 1800s for a number of countries. However, neither of these papers explicitly takes into account the comovement in interest rates across countries, which is the focus of our analysis.⁵ Gourinchas and Rey (2018) take a more explicitly global perspective in studying the connection between real returns, consumption, and wealth over long spans of time. They apply a present-value approach to data for the aggregate of the G4 economies, which they treat as a closed world economy.

The simple theoretical framework described in Section 3, from which we deduce restrictions on the long-run behavior of global real rates and their drivers, is based on the vast literature in international economics on the connection between interest rates, inflation and exchange rates across countries. This work has mostly focused on testing interest rate and purchasing power parity conditions, investigating the possible sources of their failures. Engel (2014) surveys the part of this literature that deals with the relationship (or lack thereof) between exchange rates and interest rate differentials—the so-called uncovered interest rate parity (UIP). Unlike us, most of this literature estimates this relationship without distinguishing between fluctuations at different frequencies, even though assumptions on the long-run behavior of some of the variables are often maintained in the analysis. Engel (2016) is a prominent recent example of this approach. He uses a vector auto-correction model (VECM) that embeds long-run restrictions to study possible sources of deviation from UIP.

Burstein and Gopinath (2014) summarize work on pass-through of exchange rate movements into prices. This literature evolved from earlier studies of long-run PPP, the Balassa-Samuelson effect, and their implications for trends in real exchange rates, as surveyed by Froot and Rogoff (1995). A more recent paper in this vein is Chong et al. (2010). Similar to us, they model the joint dynamics of nominal interest rates, inflation, exchange rates and economic growth for a panel of 21 countries spanning data since 1973. However, their results focus on the cointegration between the *level* of the real exchange rate and relative productivity between countries, which they interpret as evidence of a Balassa-Samuelson effect. Their findings are consistent with our maintained assumption that the *growth rate* of

⁴Neri and Gerali (2017) estimate and compare the natural rate of interest in two closed-economy DSGE models for the US and the euro area.

⁵Hamilton et al. (2016) extract trends country by country. Borio et al. (2017) run a panel regression of long term interest rates across countries on a number of possible drivers, but their approach ignores dynamics and hence the distinction between trend and cycle as well as their comovement across countries.

the real exchange rate has no trend, even though its level might be $I(1)$.⁶

Finally, we contribute to the fast growing literature on the role of convenience yields in depressing the returns of safe assets (e.g. Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood and Vayanos, 2014; Greenwood et al., 2015; Nagel, 2016). Several recent papers highlight the role of convenience yields in the international context. Valchev (2017) presents a general equilibrium model linking international convenience yields with monetary and fiscal policy. The resulting endogenous fluctuations in convenience yields can account for the predictability of excess returns of foreign over domestic bonds at various horizons first discovered by Engel (2016).⁷ Jiang et al. (2018) discuss the role of various kinds of preferences by international investors for the safety and liquidity of bonds issued by different countries in driving potential violations of covered interest rate parity. They show that the convenience yield that foreign investors derive from holding US Treasuries accounts for up to 25 percent of the quarterly movements in the dollar exchange rate over the past thirty years.⁸ Du et al. (2017b) measure the difference in the convenience yields between US Treasuries and the bonds of other sovereigns, which they call the US Treasury premium. They find that at medium and long term maturities, this premium rose significantly during the financial crisis, but it has since disappeared. At shorter maturities, on the contrary, the premium remains positive around levels similar to those observed before the financial crisis.

Our results are closely in line with those of this literature in stressing the important role of convenience yields in driving international returns. They are complementary to them because we focus on the contribution of these factors in driving the international comovement of interest rates at low frequencies, and over a much longer span of time. Therefore, we can explicitly address the question of how the global dimension of the demand for safety and liquidity has shaped the secular decline in real rates around the world over the past few decades, which also allows us to make contact with the large literature on r^* discussed above.

⁶Also related is the time-series literature that applies factor analysis to international datasets for inflation (e.g. Ciccarelli and Mojon, 2010) or interest rates (see Nikolaou and Modugno, 2009, and the references therein.)

⁷Itskhoki and Mukhin (2017) similarly stress the role of international asset demand shocks in driving exchange rate disconnect and the observed UIP violations. Those shocks, which they microfound in a general equilibrium model with limits to arbitrage, can also be thought of as reflecting asymmetric preferences for the safety of bonds from different countries.

⁸Du et al. (2017a) extensively document that significant failures of CIP have persisted since the financial crisis. They attribute them to imbalances in the demand for saving and investment across currencies, interacted with the increased cost of financial intermediation.

The rest of the paper proceeds as follows. Section 2 describes the VAR with common trends that we use in the empirical analysis. Section 3 lays out the theory from which we derive long-run restrictions for the joint behavior of real interest rates across countries. This theory also provides us with a well-defined notion of a global interest rate trend. Section 4 presents the empirical results. Section 5 concludes.

2 Econometric Framework

The model we use is essentially the same as that used in Del Negro et al. (2017), down to the specification and parameterization of the priors, with one key difference outlined below. This model is Stock and Watson (1988)’s “VAR with common trends” estimated with Bayesian methods.⁹ It is a state-space model—a multivariate trend-cycle decomposition— whose measurement equation is given by

$$y_t = \Lambda \bar{y}_t + \tilde{y}_t, \quad (1)$$

where y_t is an $n \times 1$ vector of observables, \bar{y}_t is a $q \times 1$ vector of trends, with $q \leq n$, $\Lambda(\lambda)$ is a $n \times q$ matrix of loadings which is restricted and depends on the vector of free parameters λ , and \tilde{y}_t is an $n \times 1$ vector of stationary components. The rank of Λ , which is equal to q , determines the number of common trends, and the number of cointegrating relationships is therefore $n - q$. Both \bar{y}_t and \tilde{y}_t are latent and evolve according to a random walk

$$\bar{y}_t = \bar{y}_{t-1} + e_t \quad (2)$$

and a VAR

$$\Phi(L)\tilde{y}_t = \varepsilon_t, \quad (3)$$

respectively, where $\Phi(L) = I - \sum_{l=1}^p \Phi_l L^l$ and the Φ_l ’s are $n \times n$ matrices, and the $(q + n) \times 1$ vector of shocks (e_t', ε_t') is independently and identically distributed according to

$$\begin{pmatrix} e_t \\ \varepsilon_t \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} 0_q \\ 0_n \end{pmatrix}, \begin{pmatrix} \Sigma_e & 0 \\ 0 & \Sigma_\varepsilon \end{pmatrix} \right), \quad (4)$$

⁹This model is essentially Villani (2009)’s VAR model, except that his deterministic trend is replaced by the stochastic trend (2). Very related approaches have been recently used by Crump et al. (2016), Johannsen and Mertens (2016), and Hasenzagl et al. (2017).

with the Σ 's being conforming positive definite matrices, and where $\mathcal{N}(\cdot, \cdot)$ denotes the multivariate Gaussian distribution.¹⁰ Equations (2) and (3) represent the transition equations in the state space model. The initial conditions \bar{y}_0 and $\tilde{y}_{0:-p+1} = (\tilde{y}'_0, \dots, \tilde{y}'_{-p+1})'$ are distributed according to

$$\bar{y}_0 \sim \mathcal{N}(\underline{y}_0, \underline{V}_0), \quad \tilde{y}_{0:-p+1} \sim \mathcal{N}(0, V(\Phi, \Sigma_\varepsilon)) \quad (5)$$

where $V(\Phi, \Sigma_\varepsilon)$ is the unconditional variance of $\tilde{y}_{0:-p+1}$ implied by (3). Very importantly for this application the procedure straightforwardly accommodates missing observations and can be scaled up to VARs of relatively large dimensions. Section A in the Appendix describes the Gibbs sampler, which takes advantage of Durbin and Koopman (2002)'s "fast simulation smoother."

The priors for the VAR coefficients $\Phi = (\Phi_1, \dots, \Phi_p)'$ and the covariance matrices Σ_ε and Σ_e have standard form, namely

$$p(\varphi | \Sigma_\varepsilon) = \mathcal{N}(\text{vec}(\underline{\Phi}), \Sigma_\varepsilon \otimes \underline{\Omega}), \quad p(\Sigma_\varepsilon) = \mathcal{IW}(\kappa_\varepsilon, (\kappa_\varepsilon + n + 1)\underline{\Sigma}_\varepsilon), \\ p(\Sigma_e) = \mathcal{IW}(\kappa_e, (\kappa_e + q + 1)\underline{\Sigma}_e), \quad (6)$$

where $\varphi = \text{vec}(\underline{\Phi})$, $\mathcal{IW}(\kappa, (\kappa + n + 1)\underline{\Sigma})$ denotes the inverse Wishart distribution with mode $\underline{\Sigma}$ and κ degrees of freedom. The prior for λ is given by $p(\lambda)$, the product of independent Beta, Gamma, or Gaussian distributions for each element of the vector λ .

The one key difference with the model in Del Negro et al. (2017) is that the hyperparameter controlling the tightness of the Minnesota prior in the VAR is treated as an additional unknown parameter. Following the approach of Giannone et al. (2015), we set a (hyper)prior on the degree of shrinkage parameter, a Gamma distribution with mean 0.2 and standard deviation 0.4, and conduct formal inference on it. This prior are is relatively diffuse, and empirical results are confirmed when using completely flat, improper (hyper)prior.

The prior for the covariance Σ_ε of the innovations to the cycles \tilde{y}_t , is a relatively diffuse inverse Wishart distribution with just enough degrees of freedom ($\kappa_\varepsilon = n + 2$) to have a well-defined prior mean, which is set to be a diagonal matrix. The square root of these diagonal elements are set to be equal to 2's — the exception is the inflation cycle, for which the prior mean is set to 4, to reflect the prior belief that the nominal cycles might be more volatile than the other cycles.

¹⁰An important difference with Stock and Watson (1988) is that the shocks affecting the trend and the cycle are assumed to be orthogonal to one another (in Watson (1986)'s parlance, our model features an "independent trend/cycle decomposition").

We use a conservative prior implying limited time variations of the trends. Specifically, we set the prior for Σ_e , the variance-covariance matrix of the innovations to all (common and country specific) trends \tilde{y}_t , to have a mode equal to a diagonal matrix with elements equal to 1/100 for the all real trends — which implies that expected change in the trend over one century has a standard deviation equal to one (percent, since all the variables are measured in percentage points). In different models, we will decompose the trend of the world real interest rate into subcomponents components — such as the convenience yield, stochastic discount factor. We set the prior such that the standard deviation of the innovation of the total is always the same, and at each disaggregation all subcomponents contribute equally. For the inflation trends, we use a value equal to 1/50 — which implies that expected change in the trend over half a century has a standard deviation equal to one. In addition, these priors are quite tight, as we set the degree of freedom $\kappa_e = 100$.

Turning to the initial conditions, for the world trends we use the same priors of Del Negro et al. (2017). The expected values are calibrated to target expected initial values of 0.50 for the real rate, 2 for inflation, 1 for the term spread, 1 for the convenience yield, and 1.5 for consumption growth. The standard deviation for the initial conditions is set to 2 for the world inflation trends and 1 for all the others. The initial conditions for the country specific trends have mean zero and standard deviations equal to half the value of the corresponding world counterparts.

3 Some Theory

This section introduces a simple theoretical framework that guides the specification of the long-run relationships imposed on the time series models estimated in Section 4. This framework is based on standard asset pricing ideas, as captured by a set of international Euler equations augmented to allow for the presence of convenience yield factors. These factors reflect the money-like convenience services offered by assets with special safety and liquidity characteristics—safe assets for short—such as U.S. Treasury bonds. In equilibrium, the willingness of investors to pay for these services gives rise to a wedge between the return on safe assets on the one hand and that on securities with the same pecuniary payoffs, but no such special attributes, on the other. In the international context, the presence of convenience yield differentials between assets denominated in different currencies, and/or originating in different economies, also gives rise to deviations from the usual interest rate

parity conditions, as recently discussed by Valchev (2017), Jiang et al. (2018), and Engel (2014).

3.1 International Arbitrage with Convenience Yields

Consider investors based in two different economies, say the U.S. and the EU for concreteness, trading safe and liquid one-period bonds denominated in dollars (\$) and in euros (€). Call $R_t^{\$}$ the net nominal yield on the former—a three-month U.S. Treasury bill. If M_{t+1}^{US} is the marginal rate of substitution between consumption today and tomorrow for a U.S. investor—the U.S. stochastic discount factor (SDF)—and $P_t^{\$}$ is the dollar price of that consumption, the pricing equation for the bill is

$$E_t \left[M_{t+1}^{US} (1 + CY_{t+1}^{US}) (1 + CY_{t+1}^{\$}) (1 + R_t^{\$}) \frac{P_t^{\$}}{P_{t+1}^{\$}} \right] = 1. \quad (7)$$

The term $(1 + CY_{t+1}^{US})(1 + CY_{t+1}^{\$})$ represents the convenience yield associated with U.S. Treasuries from the perspective of a U.S. investor. We model this convenience yield as having two components. The first one, denoted by $1 + CY_{t+1}^{US}$, stems from the way U.S. investors evaluate the money-like convenience services provided by any asset (hence the *US* superscript). As such, this discount is independent of the currency and the country in which the asset is issued. If U.S. investors have a special motive for holding safe assets in their portfolios, perhaps because of regulatory requirements that uniquely apply to them, the demand generated by these requirements will tend to depress the returns of those securities, regardless of their origin.

The second source of convenience, denoted by $1 + CY_t^{\$}$, stems from asset-specific characteristics, such as the currency of denomination (hence the \$ superscript) and it is independent from who holds the asset. If international investors gain utility from holding U.S. Treasuries, perhaps because they value dollar liquidity or because they put special faith in the U.S. government's ability to repay its obligations, Treasuries will trade at a premium compared to similar assets originating elsewhere.

The equation that captures how U.S. investors price safe and liquid European bonds denominated in euros helps to further clarify the distinction between the two sources of convenience that we have in mind. This equation is

$$E_t \left[M_{t+1}^{US} (1 + CY_{t+1}^{US}) (1 + CY_{t+1}^{\€}) (1 + R_t^{\€}) \frac{S_{t+1}}{S_t} \frac{P_t^{\$}}{P_{t+1}^{\$}} \right] = 1, \quad (8)$$

where R_t^ϵ is the net return on the safe and liquid euro bond and S_{t+1} is the nominal exchange rate that converts that euro return into dollars, so that an increase in S represents a dollar depreciation. We assume that the convenience of European safe assets also has two components. The first one is the same as that found in U.S. Treasuries, since arbitrage implies that the same marginal investor is applying her taste for safety and liquidity ($1 + CY_{t+1}^{US}$) to both assets. The second component ($1 + CY_{t+1}^\epsilon$) instead is specific to the euro bond. It captures the fact that the convenience services offered by this bond to any investor are likely to be different from those of U.S. Treasuries.¹¹

If the joint second moments of the variables that enter the Euler equations have no trend, we can generate useful long-run restrictions from these equations by focusing on a first-order approximation. This approximation results in a modified uncovered interest rate parity (UIP) condition of the form

$$R_t^\$ = R_t^\epsilon + E_t[\Delta s_{t+1}] + cy_t^\epsilon - cy_t^\$, \quad (9)$$

where $s_t \equiv \log S_t$ is the log nominal exchange rate and $cy_t^i \equiv E_t \log(1 + CY_{t+1}^i)$ is the expected net convenience yield for either country or currency i .¹² Although our empirical strategy relies on this linear approximation to hold in the long-run, we make no assumptions on its accuracy at any other frequency. This equation shows that the standard UIP in terms of safe rate of returns does not hold when euro and dollar assets generate different levels of convenience for investors. An increase in the relative convenience of dollar assets depresses their rate of return compared to that of euro assets, even if the dollar is not expected to appreciate. Equivalently, the dollar appreciates on impact if dollar assets become more desirable, even if the safe interest rate differential remains unchanged. On the contrary, if all safe assets were created equal, their interest rates would be lower than those on comparable assets that do not offer the same money-like services, but this effect would be symmetric across countries and currencies, therefore preserving UIP.

We can also write the first order approximation of the two pricing equations for dollar

¹¹This decomposition of the convenience yield into independent investor and asset specific components is fairly flexible, but it does rule out the possibility that EU and U.S. investors might value the money-like services of \$ and € safe assets differently. For such an asymmetric preference to be sustained in equilibrium, though, U.S. and EU investors would need to have other offsetting reasons to hold both assets in their portfolios, such as different exposures to exchange rate risk, as discussed by Jiang et al. (2018). We do not consider this possibility in our subsequent analysis because we are mainly interested in the long-run implications of the theory.

¹²We are also using the approximation $R \approx \log(1 + R)$.

and euro assets as

$$R_t^{\$} - E_t [\pi_{t+1}^{\$}] = m_t^{US} - cy_t^{US} - cy_t^{\$} \quad (10)$$

$$R_t^{\text{€}} - E_t [\pi_{t+1}^{\text{€}}] + E_t [\Delta q_{t+1}] = m_t^{US} - cy_t^{US} - cy_t^{\text{€}}, \quad (11)$$

where $\pi_t^i \equiv \log(P_{t+1}^i/P_t^i)$ is inflation, $m_t^i \equiv -E_t [\log M_{t+1}^i]$ is the negative of the expected growth rate of marginal utility, or the return on bonds that do not provide convenience services, and $q_t \equiv \ln(S_t P_t^{\text{€}}/P_t^{\$})$ is the log real exchange rate. These equations highlight the factor structure in real interest rates implied by international no arbitrage, once those returns are expressed in common consumption units. The fact that the same marginal investor prices both bonds implies that their returns share a common component, given by $m_t^{US} - cy_t^{US}$. In addition, the presence of asset specific convenience yields introduces a wedge between the two interest rates in the form of an idiosyncratic component. This factor structure forms the basis for our empirical analysis of the trends in global interest rates in the next section.

Equations (10) and (11) focus on asset/currency/country specific convenience yields as sources of UIP violations. A vast literature has documented failures of UIP stemming from many other deviations from the very simple assumptions that lead to these equations. The most notable among these assumptions is the effective risk neutrality of investors implied by the log-linear approximation of the Euler equations. If we moved to higher order approximations, heteroskedasticity in the joint distribution of the marginal utility of consumption, the convenience yields, and the exchange rate would result in the addition of several terms to equations (10) and (11), reflecting the presence of time-varying risk. Some of these terms, such as the covariance between the SDF and the exchange rate, would be country-specific and hence give rise to deviations from UIP. When looking at trends in the data, we do not take movements in these second moments into account, since we assume them to be stationary. If they are not, however, their effect will still be captured by the country-specific convenience yields.

So far, we have assumed that U.S. investors are the marginal asset buyers in our economy. What would happen if the EU investors priced them instead? In that case, the Euler equations would become

$$\begin{aligned} R_t^{\$} - E_t [\pi_{t+1}^{\$}] - E_t [\Delta q_{t+1}] &= m_t^{EU} - cy_t^{EU} - cy_t^{\$} \\ R_t^{\text{€}} - E_t [\pi_{t+1}^{\text{€}}] &= m_t^{EU} - cy_t^{EU} - cy_t^{\text{€}}, \end{aligned}$$

which yield the same modified UIP condition as before, but also the further restriction

$$m_t^{US} - cy_t^{US} = m_t^{EU} - cy_t^{EU} + E_t [\Delta q_{t+1}]. \quad (12)$$

Adding the pricing equations for the assets that do not offer any convenience service, we also obtain

$$m_t^{US} = m_t^{EU} + E_t[\Delta q_{t+1}],$$

which together with equation (12) implies $cy_t^{US} = cy_t^{EU}$.

These restrictions have an intuitive interpretation. International arbitrage implies the existence of a unique stochastic discount factor that prices all assets once their returns are expressed in common consumption units and adjusted for the convenience yields that they carry. With risk neutrality, which is implied by the first order approximation to the Euler equations considered above, the global discount factor can be represented interchangeably as the growth rate of the marginal utility of either U.S. or EU investors. Therefore, the answer to the question posed at the beginning of this paragraph is that under our maintained assumptions the “identity” of the investors that price the assets is irrelevant in the long-run.¹³

An important implication of the existence of this global SDF is that only common factors across countries matter for adjusted returns. In the consumption based asset pricing model discussed in Section 4.3, this consideration in turn implies that only global growth can be a priced factor. Allowing for country-specific loadings on this common factor, or for country-specific trends in consumption in the return equations, as it might be tempting to do if approaching the problem from a purely statistical perspective, would imply a violation of arbitrage in the long-run.

3.2 Long-Run Implications

We conclude this section by writing explicitly the factor model for the trends in nominal interest rates across countries implied by equations (10) and (11), together with the restrictions on the EU and U.S. stochastic discount factors that we just derived. Denoting trends with upper bars, those restrictions imply

$$\overline{m}_t^w - \overline{cy}_t^w \equiv \overline{m}_t^{US} - \overline{cy}_t^{US} = \overline{m}_t^{EU} - \overline{cy}_t^{EU}, \quad (13)$$

where \overline{m}_t^w is the trend in the “world” stochastic discount factor and $\overline{cy}^w = \overline{cy}^{US} = \overline{cy}^{EU}$ is the common convenience premium applied by international arbitrageurs to safe assets. Here,

¹³Jiang et al. (2018) discuss some of the frictions that would invalidate this irrelevance result and their implications for UIP.

we imposed the standard assumption that the *growth rate* in the real exchange rate has no trend, or $\overline{\Delta q_t} = 0$. This restriction is implied by, but is weaker than, purchasing power parity ($\bar{q}_t = 1$ in our notation.)¹⁴

Given this definition of the world stochastic discount factor and convenience yield, we can write

$$\bar{R}_{i,t} = \bar{\pi}_{i,t} + \bar{m}_t^w - \bar{c}y_t^w - \bar{c}y_t^i, \quad (14)$$

where $\bar{R}_{i,t}$ is the trend in the nominal interest rate of country i (expressed in terms of that country's currency), $\bar{\pi}_{i,t}$ is the trend in that country's inflation rate, and $\bar{c}y_t^i$ is the trend in the country/currency specific convenience yield. This variable has an i superscript because it represents an idiosyncratic factor in the cross section of real interest rates. Note that, even if segmentation in international asset markets prevented EU investors from engaging in cross-border arbitrage, invalidating (13), equation (14) would maintain its structure as long as U.S. investors can trade the two assets. In that case, the common component in interest rates would reflect the preferences of U.S. investors, as in equations (10) and (11). This is the model for the low frequency co-movement of interest rates that we estimate in the next section.

4 Empirical Results

This section discusses our estimates of the global trends in real interest rates and the models that we use to decompose and interpret some of their drivers. These decompositions rely on the simple economic theory based on international arbitrage described in the previous section. Since we are focusing on trends, we only impose the theoretical restrictions in the long run, making no economic assumptions on short run dynamics. Technically, we take a stance on the structure of the matrix Λ that determines how different trends enter the measurement equations (1), but we leave the VAR matrices in (3) unconstrained. An advantage of this approach is that the restrictions we impose are fairly uncontroversial in the long run, but might easily be violated at other frequencies. But in any case, readers who are skeptical of these restrictions can still interpret our results as a trend-cycle decomposition

¹⁴Most of the literature studying trends in real returns across countries (e.g. Hamilton et al., 2016; Borio et al., 2017) implicitly makes this assumption. Without it, those trends would not be directly comparable, since they would in part reflect secular changes in the value of the consumption bundles in which they are quoted.

obtained from a (mildly restricted) reduced form model, and skip the interpretation of the empirical objects in terms of the economic quantities discussed in Section 3.

The basic building block of our analysis is the long run relationship

$$\bar{R}_{i,t} - \bar{\pi}_{i,t} = \bar{r}_t^w + \bar{r}_t^i, \quad (15)$$

where $\bar{R}_{i,t}$ and $\bar{\pi}_{i,t}$ are trends in nominal short term yields and inflation in country i , \bar{r}_t^w is the trend in the short term world real interest rate, and \bar{r}_t^i is a country-specific trend. This relationship rewrites (14) to highlight that the trend in the world real interest rate reflects trends in the discount factor of the marginal international investor, $\bar{m}_t^w - \bar{c}y_t^w$, including her taste for safety and liquidity, while low frequency movements in the component of the convenience yield specific to country i (i.e. $\bar{c}y_t^i$ for U.S. Treasuries) result in an idiosyncratic trend. We interpret this trend as capturing the different degrees of safety and liquidity of the government paper issued by the various countries in our sample, namely the fact that U.S. treasuries and German bunds are generally considered a safe haven, while Italian government securities may not be considered as such. However, these country-specific factors could be interpreted more generally as capturing any long run deviation from UIP, regardless of its source.

Section 4.1 presents estimates of the different components of the nominal interest rate in (15), \bar{r}_t^w , \bar{r}_t^i , and $\bar{\pi}_{i,t}$, from a baseline model based on data on inflation and nominal yields on short and long term government securities across countries. Since most of these assets are considered both safe and liquid, we interpret \bar{r}_t^w as the global trend for safe and liquid returns. Section 4.2 further decomposes \bar{r}_t^w into the part that we attribute to the global convenience yield for safety and liquidity, $\bar{c}y_t^w$, and a worldwide stochastic discount factor \bar{m}_t^w , using yields on “unsafe” and “illiquid” U.S. securities. Finally, Section 4.3 adds data on consumption growth to split the stochastic discount factor \bar{m}_t^w into a component due to global growth, which we call \bar{g}_t^w , and a residual component unrelated to it.

Our data come from the Jordà-Schularick-Taylor Macrohistory Database, which is described in Jordà et al. (2017).¹⁵ In particular, we use annual data from seven advanced countries (Canada, Germany, France, Italy, Japan, the UK, and the U.S.) on short term and long term interest rates, consumer prices (the database contains an index, which we log-

¹⁵We are very grateful to Oscar Jordà, Moritz Schularick, and Alan Taylor for making their data publicly available.

difference to obtain inflation), and real consumption per capita.¹⁶ We augment this dataset with annual averages of Moody’s Baa corporate bond yield for the U.S., which is available from FRED since 1919. The short term rates are from short term government bills or money market instruments, while the long term rates are all from government bonds. For instance, the long term yield for the U.S. coincides with the 10-year Treasury constant maturity rate from FRED since 1954. Therefore, we consider all these yields as reflecting “safe and liquid” returns, because government paper is generally more liquid than its privately issued equivalents, since it tends to be traded heavily, and safer, since it is backed by taxation. Of course, the degree of safety and liquidity of government securities varies across countries, and we account for that in our analysis.

4.1 A Baseline Model of the World Real Interest Rate

This section presents estimates of the trend in the world real interest rate from a baseline model with data on the nominal yields of short term ($R_{i,t}$) and long term ($R_{i,t}^L$) government (or closely related) securities and inflation ($\pi_{i,t}$) in each of the seven countries in our sample. With three observable variables, we can identify three trends—in inflation, the level of interest rates, and the spread between long and short maturity rates. Moreover, the cross section of countries allows us to identify a common, or “world”, component and a country-specific component in each of these trends. Next, we describe how these six factors enter the trend equations for each of our three variables. These trend equations, which embed the restrictions discussed in the theory section, provide a complete description of the matrix Λ in equation (1) for each of the models that we estimate.

We already discussed the model for the trends in short term real rates in equation (15). Intuitively, we split real returns into a common component and an idiosyncratic component specific to each country. With observations on seven cross-sectional units, we could also estimate country-specific loadings on the common component. However, Section 3 discussed how international arbitrage implies that these loadings must be one because in the long run the free movement of capital across countries will equalize the return on assets with the same characteristics. This common component in the returns induced by arbitrage is therefore the textbook world real interest rate. In addition, our framework includes idiosyncratic factors

¹⁶The time series of interest rates and inflation contain large outliers in the period around the world wars, which are not very informative on the secular trends that we are interested in. Therefore, we treat all observations above 30 percentage points in absolute value as if they were missing data.

that might prevent this equalization to account for the differences in safety and liquidity of the securities issued by different governments, as well as for other potential failures of arbitrage in the long run. As a further check on the no-arbitrage restrictions embedded in equation (15), however, we will also present a model that does allow for country-specific loadings on the world real interest rate. None of those loadings is significantly different from one.

Moving now to the model for the trends in long rates, it is convenient to express it in terms of its implications for the term spread, namely the difference between the trends in long and short rates: $\bar{R}_{i,t}^L - \bar{R}_{i,t}$. Although the vast majority of the finance literature models this spread as stationary, we want our analysis to be robust to the possible failure of this assumption. This approach is dictated both by the longer time series that we are modelling, where low frequency movements in the slope of the yield curve might be more evident, as well as by the special focus of our analysis on secular movements. As we do for the level of rates, we model the trend in the term spread as the sum of a common and a country-specific component:

$$\bar{R}_{i,t}^L - \bar{R}_{i,t} = \bar{t}s_t^w + \bar{t}s_t^i. \quad (16)$$

Both $\bar{t}s_t^w$ and $\bar{t}s_t^i$ are assumed exogenous. In Del Negro et al. (2017), we also considered a specification in which the inflation trend affects the term spread, but found the results little affected by this more general specification. Therefore, we do not consider it here.¹⁷

We are restricting the loadings on the world trend in the term spread to be the same across countries for the same reasons discussed above in relation to the level of rates. International arbitrage implies convergence of interest rates at all maturities, as long as we are comparing identical assets. Therefore, we can interpret $\bar{t}s_t^w$ as the trend in the slope factor in the SDF of the marginal global investor. At the same time, we allow for deviations from this common pricing of term spreads to reflect possible cross-country differences in maturity for the long term bonds in our sample, as well as relative differences in safety and liquidity between long and short term bonds in different countries. These and other potential deviations from perfect arbitrage are captured by the term $\bar{t}s_t^i$.

Similarly, we decompose the trends in inflation in each country ($\bar{\pi}_{i,t}$) into a common and a country-specific component as

$$\bar{\pi}_{i,t} = \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i. \quad (17)$$

¹⁷This model of the trends in short and long term rates implies that the latter are the sum of the former and of the trend in the term spread. Therefore, the trend in the world real interest rate \bar{r}_t^w should be interpreted as having a short term maturity.

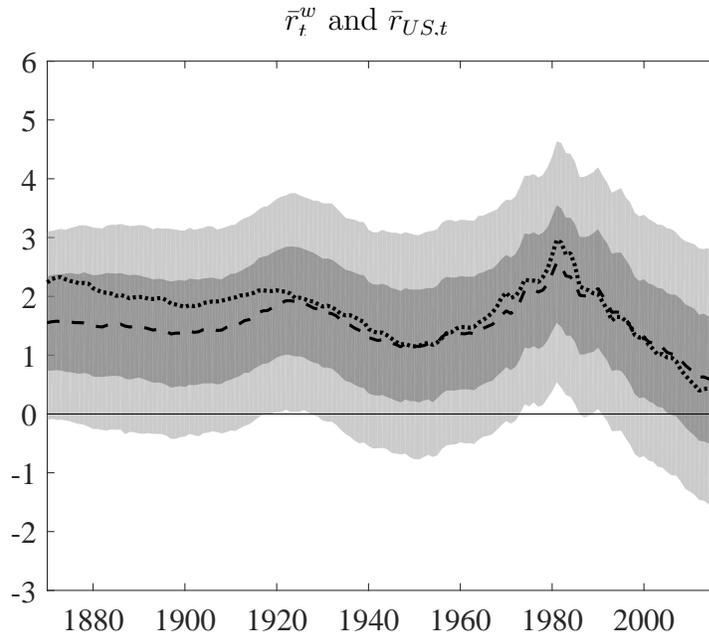
In this case, we do allow different countries to load differently on the global inflation trend $\bar{\pi}_t^w$ through the coefficients λ_i^π , since there is no economic force equivalent to no arbitrage enforcing the convergence of monetary policies and any other potential long-run determinant of inflation across countries. This assumed structure of the inflation trends is the low frequency analog of the global inflation factor model estimated by Ciccarelli and Mojon (2010), for example.

Summarizing, the first model we estimate is:

$$\begin{aligned} R_{i,t} &= \bar{r}_t^w + \bar{r}_t^i + \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i + \tilde{R}_{i,t}, \\ R_{i,t}^L &= \bar{r}_t^w + \bar{r}_t^i + \bar{t}s_t^w + \bar{t}s_t^i + \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i + \tilde{R}_{i,t}^L, \\ \pi_{i,t} &= \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i + \tilde{\pi}_{i,t}, \end{aligned} \quad (18)$$

for $i = 1, \dots, n$. The system is estimated jointly for all countries in the sample.

Figure 1: Trends in Global and U.S. Real Rates: 1870-2016, Baseline Model



Note: The dashed black line shows the posterior median of \bar{r}_t^w and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

4.1.1 Results

Figure 1 plots the posterior median of the trend in the world real interest rate \bar{r}_t^w (dashed line), together with its 68 and 95 percent posterior coverage intervals, as well as the posterior median of the trend in the U.S. real rate $\bar{r}_{US,t}$ (dotted line). This figure delivers the first

two important results of the paper. First, \bar{r}_t^w has fluctuated around 1.5 percent for about a century, but it has been on a steady decline over the past few decades, totaling almost 200 basis points. Second, \bar{r}_t^w and \bar{r}_t^{US} essentially coincide over the last century: the world trend is the U.S. trend.

Table 1: Change in \bar{r}_t^w and Its Components

| | 1980-2016 | 1990-2016 | 1997-2016 |
|-------------------------|----------------------------|----------------------------|----------------------------|
| Baseline Model | | | |
| \bar{r}_t^w | -1.88*** (-3.24, -0.61) | -1.64*** (-2.84, -0.47) | -1.03** (-1.96, -0.09) |
| Convenience Yield Model | | | |
| \bar{r}_t^w | -1.71*** (-2.94, -0.55) | -1.73*** (-2.84, -0.67) | -1.13*** (-2.01, -0.26) |
| $-\overline{cy}_t^w$ | -0.82** (-1.60, -0.05) | -0.97*** (-1.65, -0.24) | -0.66** (-1.25, -0.06) |
| \bar{m}_t^w | -0.90 (-1.89, 0.08) | -0.78* (-1.66, 0.09) | -0.48 (-1.17, 0.22) |
| Consumption Model | | | |
| \bar{r}_t^w | -1.93*** (-3.18, -0.69) | -1.94*** (-3.10, -0.82) | -1.22*** (-2.18, -0.29) |
| $-\overline{cy}_t^w$ | -0.71* (-1.51, 0.11) | -0.92*** (-1.67, -0.19) | -0.65** (-1.25, -0.02) |
| \bar{g}_t^w | -0.74** (-1.50, -0.03) | -0.61* (-1.28, 0.06) | -0.35 (-0.88, 0.19) |
| $\bar{\beta}_t^w$ | -0.47 (-1.21, 0.31) | -0.42 (-1.07, 0.25) | -0.24 (-0.78, 0.30) |

Note: For a variable x , where $x = \{\bar{r}_t^w, \bar{m}_t^w, -\overline{cy}_t^w, \bar{g}_t^w, \bar{\beta}_t^w\}$ depending on the model, the table shows the posterior median of $\Delta x = x_{2016} - x_{t_0}$ for t_0 being equal to 1980 (left column), 1990 (middle column), and 1997 (middle column), and the 90 percent posterior coverage interval for Δx (in parenthesis). The stars next to the posterior median indicate that $Pr\{x \leq 0\}$ is greater or equal to 97.5 % (***) , 95 % (**), or 90 % (*), where $Pr\{\}$ is the posterior probability.

More in detail, our estimates indicate that \bar{r}_t^w fluctuated in a fairly narrow range around 1.5 percent through the post World War II period, reaching a peak close to 2 percent just before the Great Depression and a trough a bit above 1 percent in the early 1950s. From then, it rose steadily through the early 1980s, when it touched 2.5 percent. It has been on a

steady decline since, plunging to about 50 basis points by the end of the sample, way below its previous minimum. The uncertainty on the level of the trend at any point in time is large, so the point estimates should be interpreted with caution. However, the decline over the past few decades is highly significant, as shown in the the top panel of Table 1. This decline has totaled close to 2 percentage points since 1980, with more than 150 basis points of it occurring since 1990, and more than 1 percentage point over the last 20 years of the sample. The 90 percent posterior coverage intervals for the estimated declines over the three periods, which are in parenthesis, all exclude zero. In fact, the posterior probability that the decline is positive is greater than 95% across the board, as indicated by the stars. Table A1 in the Appendix offers even more detail on the posterior distribution of these declines, but the message is clear. The declines are big and they are highly statistically significant.

The dotted line in Figure 1 shows that \bar{r}_t^w and $\bar{r}_{US,t}$, which is the sum of \bar{r}_t^w and the country-specific trend \bar{r}_t^{US} , are very close since the 1920s. This implies that \bar{r}_t^{US} has been small. However, the U.S. overall trend has fallen more than the world interest rate since 1980 and it has been below it since the late 1990s, indicating that the country-specific component \bar{r}_t^{US} has been negative since then, and growing. This evidence suggests that U.S. government bonds have enjoyed a larger convenience yield than those of other sovereigns over the past three decades.

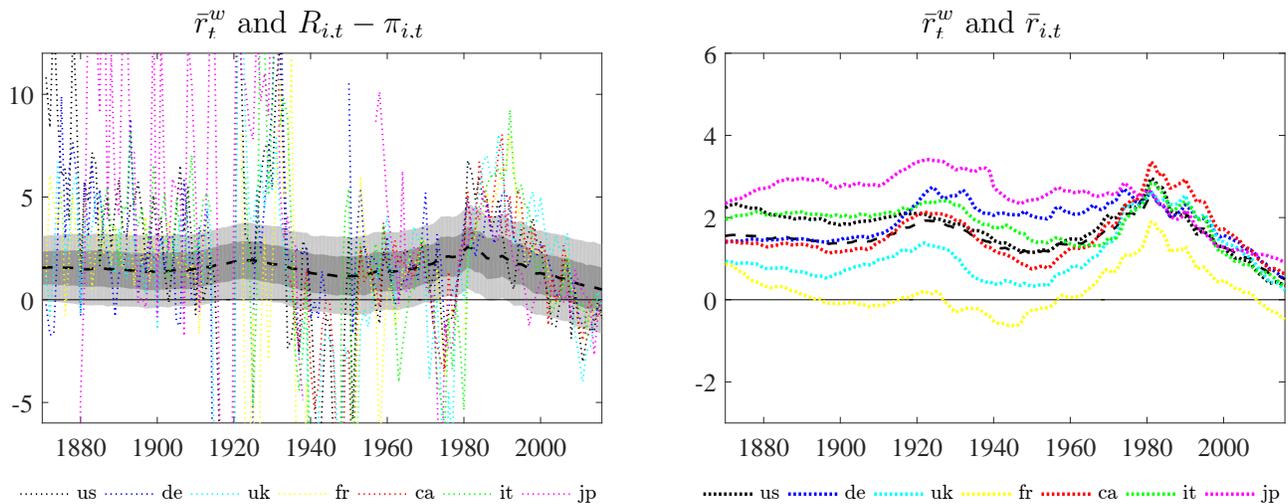
The left panel of Figure 2 presents our estimates of \bar{r}_t^w in the context of some the information that we use to extract this trend, namely the ex-post real short term interest rates ($R_{i,t} - \pi_{i,t}$) for all the countries in the sample. Although these rates fluctuated wildly in the first part of the sample, the movements in \bar{r}_t^w do capture some evident common patterns in the data at least since the 1930s. Real rates fell in the 1930s and 1940s, as well as in the 1970s with high inflation, although the model interprets the latter movement as mostly cyclical. Most notably, real rates have been falling closely together since the 1980s, dragging the world trend down with them. Partly due to our conservative prior on the amount of variation in the trends, the model interprets a good part of this decline as cyclical. Yet, the persistent comovement of real rates over the past four decades is evident to the naked eye. Ultimately, this low frequency comovement is what drives the estimated decline in the trend.¹⁸

The pronounced fluctuations in real rates in the first part of the sample highlighted in

¹⁸Section B.3 in the Appendix shows the results of the baseline model with a looser prior: we use 50 instead of 100 degrees of freedom. Those results are even stronger than those shown in this section in terms of the size of the post-1980 decline in \bar{r}_t^w .

Figure 2 raise the concern that our VAR with constant volatilities might be misspecified. Even if the trends are homoskedastic, as we have argued they are likely to be, their estimates might be affected by ignoring a possible break in the volatilities of the innovations to the stationary VAR (3). To address this concern, we also estimated the baseline model on a restricted sample starting in 1950. As shown in figure A3 in the Appendix, the trend estimates are very similar to those obtained with the longer sample.

Figure 2: Trends and Observables for Short Term Real Rates, Baseline Model

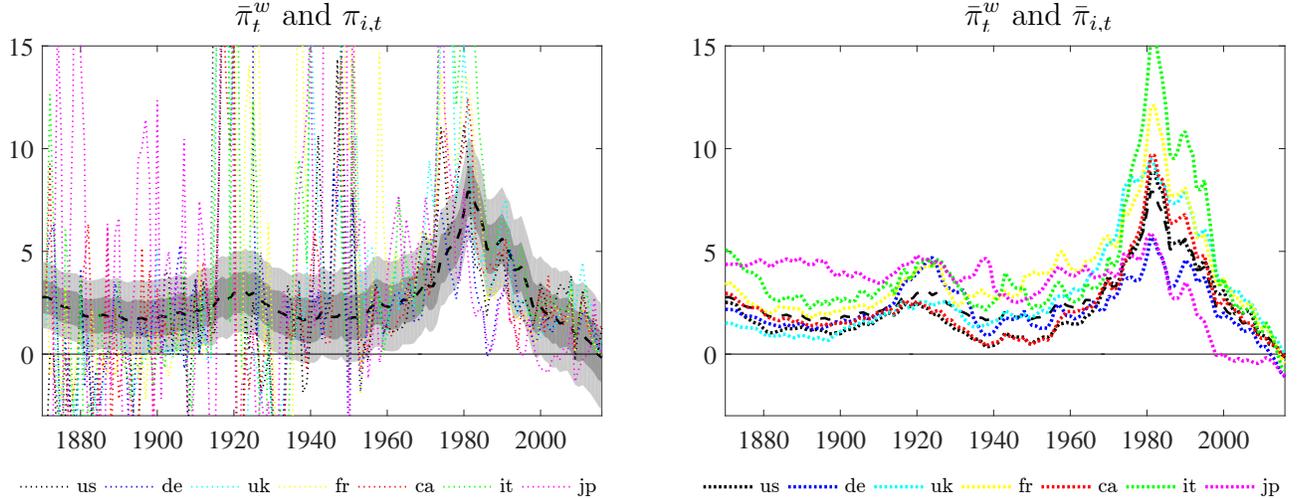


Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines, see legend), together with the trend \bar{r}_t^w (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend \bar{r}_t^w (dashed black line).

The right panel of Figure 2 displays the trend in the world real interest rate (black dashed line) and the overall trend $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country in the sample (dotted lines). Two interesting facts emerge from this figure. First, the country-specific components have shrunk noticeably since the 1960s, bringing the trends much closer together and to the world real interest rate. This long-run convergence in rates of return arguably reflects the increased liberalization of capital movements over the past fifty years, which should move global capital markets closer to the perfect arbitrage ideal. Second, to the extent that we want to interpret \bar{r}_t^i as indicative of a country-specific convenience yield, UK government paper yielded greater convenience than U.S. treasuries for the first century of the sample, but this ranking has been reversed over the last fifty years. One puzzling feature of the figure is the trend for France, which is estimated to be below all others for the entire sample. Figure A2 in the appendix suggests that this is unlikely to be a problem with the model, but it is in fact a feature of the data. The figure compares the estimates of the country-specific trend \bar{r}_t^i

for each country with their closest observed counterpart, the ex-post real rates $R_{i,t} - \pi_{i,t}$ in deviation from the cross-country average $\frac{1}{n} \sum_{i=1}^n (R_{i,t} - \pi_{i,t})$. This comparison suggests that there are no major discrepancies between the estimated idiosyncratic trends and the low frequency movements in the observables, including in the data from France.

Figure 3: Trends and Observables for Inflation, Baseline Model



Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines, see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^{\pi} \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

Our dataset does not include direct observations on real interest rates. Therefore, our estimates of inflation trends are a crucial input in the extraction of the low frequency component of global real returns. In fact, the inflation trends that we compute are of independent interest because they characterize secular movements in inflation across countries and the extent to which these reflect global rather than country-specific forces. In addition, they represent a useful reality check on the ability of our econometric tools to separate trend from cycle.

The left panel of Figure 3 reports one such check by comparing the estimated global trend in inflation $\bar{\pi}_t^w$ with the observed inflation rates of all the countries in the sample. Similar to what we observed for real interest rates, inflation has become an increasingly global phenomenon, at least since World War II. Inflation rates in all countries were low in the 1950s, rose in the 1960s and 1970s, and then fell in the past forty years. The results indicate that the global trend in inflation is at an all-time low.

The right panel of Figure 3 displays the world inflation trend $\bar{\pi}_t^w$ (black dashed line) along with the trend $\bar{\pi}_{i,t} = \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country in the sample (dotted lines). Although the trends, as the raw data, tend to move more closely together in the second half of the sample, this convergence is less pronounced and uniform over time than for real returns, with notable country-specific idiosyncrasies. For instance, Italy is not surprisingly the Country where the inflation trend reaches higher around 1980, followed by France. Germany’s trend, on the contrary, barely touches 5 percent at its peak and it is consistently towards the bottom of the distribution. Japan’s trend, which was often the highest in the first century of data, has been well below all others since the burst of its real estate bubble and the ensuing struggles with deflation and the zero lower bound. With Japan’s exception, all other inflation trends have been extremely close since the 1990s, heading together towards deflationary territory since the global financial crisis. In part, this recent convergence probably reflects the long-run effects on inflation of the adoption of a common monetary policy by Germany, France, and Italy after the introduction of the euro.

The last trend included in our model is that capturing low frequency movements in term spreads. Figure A1 in the Appendix displays the estimated global trend $\bar{t}s_t^w$ together with observations on the term spread $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines) on the left, as well as the estimated trends for each country $\bar{t}s_{i,t}$ on the right. These results demonstrate that the slope of the yield curve does fluctuate at low frequencies and that much of these fluctuations are common across countries, especially over the last few decades.

We close this section by discussing an empirical specification that relaxes the arbitrage condition (15). We do so by letting real returns in each country load on the common factor with a potentially different coefficient, as in

$$\bar{R}_{i,t} - \bar{\pi}_{i,t} = \lambda_i^r \bar{r}_t^w + \bar{r}_t^i. \quad (19)$$

Table A2 in the Appendix reports the estimated loadings under this specification. None of them is significantly different from one.¹⁹ Moreover, Figure A11 shows that the estimates of \bar{r}_t^w and $\bar{r}_{US,t}$ produced by the unconstrained model are very close to those reported above, at least from the 1940s onward. We do not consider these results as a particularly stringent test of perfect international arbitrage in the long run, although they are consistent with it. Their relevant implication for our empirical strategy is that the restricted model, which has the great advantage of being easily interpretable, is not at odds with the data. Therefore,

¹⁹More specifically, all the 90 percent posterior credible intervals include zero. For the UK, the 68 percent credible interval is below one, while it is above for the U.S. (barely) and Japan.

we will maintain the no arbitrage restrictions in the empirical specifications explored in the remaining sections.

4.2 The Role of Convenience Yields

The aim of this section and the next is to identify some of the potential drivers of the trend in the world real interest rate. The baseline model described in the previous section used just enough information to identify this trend and to distinguish it from trends in inflation and the term premium. In what follows, we will bring more information into the estimation that will allow us to decompose the overall trend in the world return on safe assets into some of its fundamental components. The first component that we consider is the convenience yield that distinguishes widely traded government bonds from comparable assets that are less liquid and safe. Going back to equation (14), this distinction is based on the relationship

$$\bar{r}_t^w = \bar{m}_t^w - \bar{c}y_t^w, \quad (20)$$

which splits \bar{r}_t^w into the trend of the stochastic discount factor of the marginal world investors, \bar{m}_t^w , and the low frequency component of her taste for safety and liquidity, the global convenience yield $\bar{c}y_t^w$. Under the maintained no arbitrage assumption, only one SDF and once convenience yield factor, those of the marginal international investor, are relevant to pin down the trend in the world interest rate. Therefore, one extra observable is sufficient to separately identify these two components, given the identification of \bar{r}_t^w that we had already achieved in the baseline model.

The key observable that gives us this identification is the yield on U.S. Baa corporate bonds, as computed by Moody's. Building on the work of Krishnamurthy and Vissing-Jorgensen (2012), we assume that these corporate bonds are both less safe and less liquid than U.S. Treasuries of roughly equivalent maturity. As in Del Negro et al. (2017), we take this assumption to the extreme and postulate that these bonds have no convenience yield whatsoever. Therefore, the long run component of the spread between the Baa yields and those on Treasuries is

$$\bar{R}_{US,t}^{Baa} - \bar{R}_{US,t}^L = \bar{c}y_t^w + \bar{c}y_t^{US}. \quad (21)$$

The assumption that Baa securities have no convenience at all is conservative, in the sense that its failure would result in an *underestimation* of the size of the convenience yield trend, as discussed in Del Negro et al. (2017). In practice, Baa securities are not completely illiquid and there are certainly less safe assets. Therefore, they are likely to earn some fraction of

the convenience yield of Treasuries. If that is the case, the Baa spread will move less than one to one with the convenience yield, thus providing a lower bound on its true size.²⁰

The trend in the U.S. Baa spread provides observations on the *sum* of \overline{cy}_t^w and \overline{cy}_t^{US} , as shown in equation (22). How do we separate the two with no information on the returns of illiquid/unsafe securities in other countries? The answer is that under the assumption that deviations from UIP are due to country-specific convenience yields \overline{cy}_t^i , the estimates of the idiosyncratic component of the trend in the U.S. real interest rate from the baseline model already give us a time series for \overline{cy}_t^{US} . Given this estimate, the U.S. Baa spread is enough to identify \overline{cy}_t^w .²¹

In summary, the second model that we estimate is the same as (18), but now including two common factors (\overline{m}_t^w and \overline{cy}_t^w) as in (20). Those factors are identified by adding $R_{US,t}^{Baa}$ to the list of observables, according to the equation

$$\overline{R}_{US,t}^{Baa} = \overline{m}_t^w + \overline{ts}_t^w + \overline{ts}_t^i + \lambda_i^\pi \overline{\pi}_t^w + \overline{\pi}_t^i + \tilde{R}_{US,t}^{Baa}. \quad (22)$$

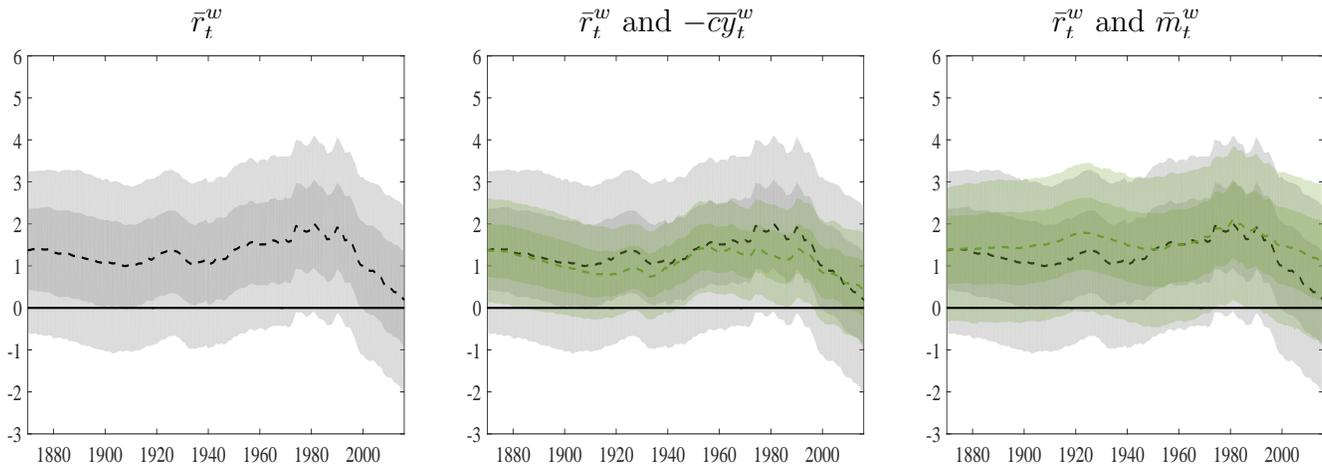
4.2.1 Results

Figure 4 shows the trend in the world real interest rate \overline{r}_t^w from the model that includes Baa yields, as well as its decomposition between the trend in the stochastic discount factor of international investors \overline{m}_t^w and the trend in the global convenience yield \overline{cy}_t^w . The estimates of \overline{r}_t^w are reproduced in all three panels; the levels of $-\overline{cy}_t^w$ in the middle panel and of \overline{m}_t^w in the rightmost one are normalized to coincide with the posterior median of \overline{r}_t^w at the beginning of the sample, so as to provide a visual sense of the contributions of each factor to the secular fluctuations in the world interest rate.

This figure delivers the third result of the paper. Low frequency movements in the global convenience yield are a key driver of the trend in the world real interest rate, and especially

²⁰ Equation (22) ignores trends in the default rate as a potential determinant of the Baa spread. This is because Del Negro et al. (2017) documents that at least in the last forty years default rates for U.S. Baa corporate bonds trended down, based on data from Gilchrist and Zakrajsek (2012). Therefore, accounting for the contribution of this trend to the spread would lead to even larger estimates of the convenience yield.

²¹We did not find long time series for yields on corporate or other similar bonds outside of the U.S. Adding this type of information to our dataset would be valuable, since it would provide a more direct measure of convenience yields in other countries, thus allowing an explicit distinction between this and other potential sources of UIP violation on safe returns. This approach would be tantamount to studying long run UIP violations using returns on unsafe/illiquid assets, rather than government bonds.

Figure 4: \bar{r}_t^w , $\bar{c}y_t^w$, and \bar{m}_t^w 

Note: In all three panels, the dashed black line is the posterior median and the shaded gray areas are the 68 and 95 percent posterior coverage intervals for \bar{r}_t^w . The dashed green line is the posterior median and the shaded green areas are the 68 and 95 percent posterior coverage intervals for $-\bar{c}y_t^w$ (middle panel) and \bar{m}_t^w (right panel).

of its pronounced decline over the past few decades. To a certain extent, this conclusion was already implied by our previous result that the trend in the world real interest rate is very close to that of the safe and liquid return in the U.S. This evidence, together with the findings in Del Negro et al. (2017) that the decline in the U.S. return since the late 1990s is driven in large part by an increase in its convenience yield, already deliver the result qualitatively. Figure 4 and Table 1 formalize and quantify this informal conclusion in the context of our global model.²²

More specifically, the table shows that the trend in the global convenience yield accounts for about half of the secular decline in the world real interest rate since 1980, which in this model totals 171 basis points, and close to 60 percent of the more than 1 percent decline since 1997. These contributions are surrounded by sizable uncertainty, as is the estimate of the overall trend, but they are significantly different from zero at all three horizons. This evidence is therefore consistent with the view expressed by Caballero and Krishnamurthy (2009), Caballero (2010), Caballero et al. (2016), and Bernanke et al. (2011), among others,

²²Figure A12 in the appendix shows that the estimates of $\bar{r}_{US,t}$ from the model with the Baa spread are consistent with those from the baseline model. The fact that they are not identical is not surprising, since the former uses one more piece of information to estimate the U.S. trend. In both models, $\bar{r}_{US,t}$ and \bar{r}_t^w fall closely together since the 1980s, with $\bar{r}_{US,t}$ declining a bit more toward the end of the sample. In addition, Figures A13 and A14 show that the estimated country-specific trends in real rates and in inflation from the model that includes the Baa spread are very similar to those from the baseline model shown in Figures 2 and 3.

that the increased global demand for safe assets since the Asian crisis of 1997 has played a crucial role in driving interest rates lower across the world.

The right panel of Figure 4 shows that the global stochastic discount factor \bar{r}_t^w has also played an important role in the fall of the world real interest rate over the past decades. Table 1 reports that the SDF has declined by about the same amount as the convenience yield since 1980, but by less since 1990 and 1997. Moreover, the changes in the SDF are less precisely estimated than those of the global convenience yield, as clearly illustrated by the wider posterior probability bands in the right panel of Figure 4. As a result, the 90 percent posterior probability intervals for the declines over the three periods that we are considering all include zero.²³ What forces lie behind the estimated decline in the global stochastic discount factor? The next section takes up this question by bringing to the table information on consumption growth, as suggested by asset pricing theory.

4.3 A Model with Consumption

The models that we have discussed so far treat the trend in the stochastic discount factor \bar{m}_t^w as an unobservable variable. In the specification considered in Section 4.2, this was estimated as the common factor among short and long term yields to all securities across the world, regardless of their safety and liquidity characteristics. In this section, we push the decomposition of the world real interest rate one step further. We use data on per capita consumption growth to identify a component of \bar{m}_t^w connected to the global trend in consumption growth. We denote this trend as \bar{g}_t^w . A connection between the stochastic discount factor and some function of consumption growth forms the basis of most macro-finance asset pricing theories, even if the empirical relevance of the resulting relationship between economic growth and rates of return has often been questioned (e.g. Lunsford and West (2017)).

In light of the mixed evidence on the extent to which interest rates and consumption growth actually correlate even at low frequencies, our proposed model of the stochastic discount does not connect the two very tightly, allowing for other factors to shift that relationship. In particular, we assume that

$$\bar{m}_t^w = \bar{g}_t^w + \bar{\beta}_t^w, \quad (23)$$

²³Table A1 in the Appendix provides more details on the posterior distribution of the decline in the world real interest rate and in the factors that drive it.

where \bar{g}_t^w is the trend in consumption growth that is common across countries, while $\bar{\beta}_t^w$ captures influences on the SDF, and hence on interest rates, that are unrelated to consumption, such as demographic changes. We estimate the global consumption trend \bar{g}_t^w from the trends in consumption growth across countries, according to the equation

$$\bar{\Delta}c_{i,t} = \bar{g}_t^w + \bar{\gamma}_t^w + \bar{\gamma}_t^i. \quad (24)$$

Here, we allow for trends in consumption growth that are unrelated to the stochastic discount factor and that therefore do not affect real interest rates. Following the approach that we adopted for inflation and the term premiums, we assume that one of these trends, $\bar{\gamma}_t^w$, is common across countries, while another one, $\bar{\gamma}_t^i$, is idiosyncratic. This specification levers the cross-sectional information contained in our international dataset to generalize a similar model that we estimated in Del Negro et al., 2017 on data for the United States. One possible interpretation of the $\bar{\gamma}_t$ trends is that they reflect the growth rate of consumption of households excluded from international asset markets, as in the limited participation literature (e.g. Vissing-Jorgensen, 2002). From an econometric perspective, we could also identify a country-specific trend in consumption growth, say \bar{g}_t^i , which would enter the expression for the stochastic discount factor. We restrict this term to zero because otherwise the stochastic discount factor would no longer be unique, violating the maintained assumption of perfect international arbitrage discussed in Section 3.

In sum, the third model we estimate includes consumption growth across countries as an observable

$$\Delta c_{i,t} = \bar{g}_t^w + \bar{\gamma}_t^w + \bar{\gamma}_t^i + \widetilde{\Delta}c_{i,t}, \quad (25)$$

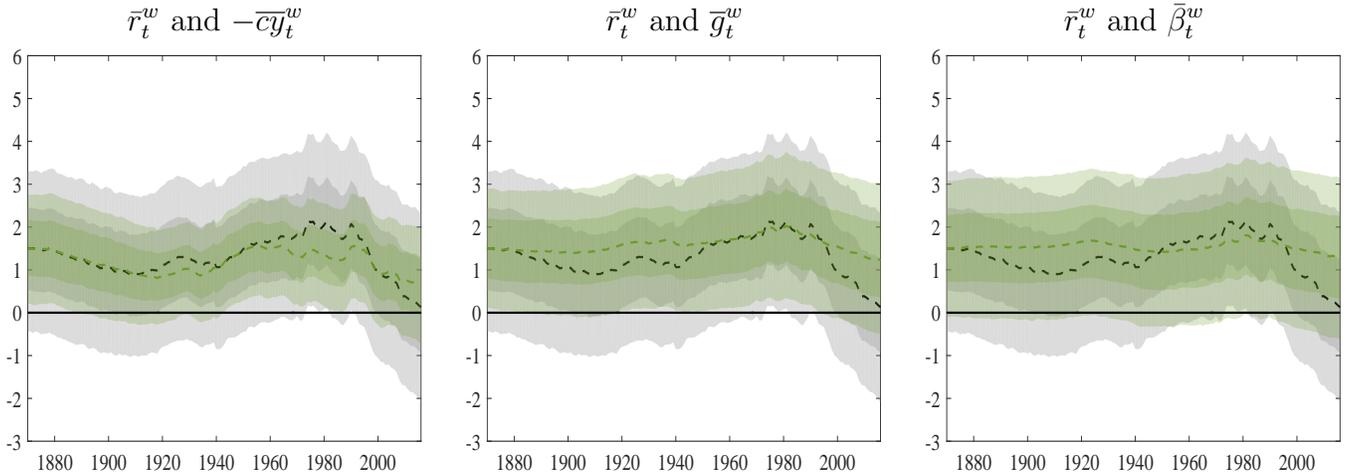
for $i = 1, \dots, n$, in addition to the variables in (18) and (22). Moreover, the safe world real interest rate in this system is decomposed as

$$\bar{r}_t^w = \bar{g}_t^w + \bar{\beta}_t^w - \bar{c}y_t^w. \quad (26)$$

Therefore, this specification includes four global trends: one that is common to all yields and consumption growth rates across countries (\bar{g}_t^w), one that is common to yields only ($\bar{\beta}_t^w$), one that is common only to the yields for liquid assets ($\bar{c}y_t^w$), and finally one that is common only to consumption growth rates ($\bar{\gamma}_t^w$).

4.3.1 Results

Figure 5 shows the trend in the world real interest rate estimated from the model with consumption and its decomposition into global trends in the (negative of the) convenience

Figure 5: \bar{r}_t^w , $\bar{c}y_t^w$, \bar{g}_t^w , and $\bar{\beta}_t^w$ 

Note: In all three panels, the dashed black line is the posterior median and the shaded gray areas are the 68 and 95 percent posterior coverage intervals for \bar{r}_t . The dashed green line is the posterior median and the shaded green areas are the 68 and 95 percent posterior coverage intervals for $\bar{c}y_t$ (middle panel) and \bar{m}_t (right panel).

yield $\bar{c}y_t^w$, the part of consumption growth that prices assets \bar{g}_t^w , and the residual component of the stochastic discount factor unrelated to consumption $\bar{\beta}_t^w$. As in Figure 4, the series are all normalized so that their posterior medians coincide at the beginning of the sample.

The left panel of Figure 5, as well as Table 1, show that the estimated trend in the world real interest rate and the convenience yield trend are very similar to those obtained using the models of Sections 4.1 and 4.2. The middle panel shows that the global decline in consumption growth also plays an important role in bringing down \bar{r}_t^w . The contribution of this factor to the recent trend decline in the world real interest rate is about 75 basis points from 1980, and 60 basis points from 1990. These median estimates are less precise than those of the convenience yield, but the posterior probability that the global consumption growth factor did decline since 1980 is above 95 percent, although it becomes lower after 1990.

The right panel of Figure 5 shows that there is not much left to explain in the secular decline in the world safe return once the convenience yield and consumption growth are accounted for. The residual pricing factor $\bar{\beta}_t^w$ is roughly flat throughout the sample and its mild decline since 1980 is never statistically significant.²⁴

²⁴Figures A17, A18, and A19 reproduce the results shown in Figures 1, 2, and 3 of Section 4.1 for the consumption model.

5 Conclusions

Ten years after the most acute phase of the global financial crisis, interest rates remain at or near historically low levels for many countries. We studied the secular drivers of this low interest rate environment through the lens of a vector autoregression with common trends, using historical data from seven countries dating back to 1870. We found that the trend in the world safe real interest rate, which was roughly stable a bit below 2 percent for more than a hundred years, has dropped significantly over the past three decades. This global trend, which we identified as the common component in the low frequency movements of the real yields on safe and liquid assets (government bonds or close substitutes) in the seven economies in our sample, closely resembles the trends for all advanced economies, including the United States, in the recent periods. We find that country-specific trends have all but vanished since the 1970s.

This secular decline in global real rates is driven primarily by an increase in the premium that international investors are willing to pay to hold safe and liquid assets, as well as by lower economic growth around the world. The latter trend has been putting downward pressure on real rates since around 1980, while the former has emerged in the late 1990s. This timing points to the scarcity of safe assets in the context of a global saving glut as a fundamental secular force behind the low interest rate environment.

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Online Appendix

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A Gibbs Sampler for VARs with Common Trends

Let us use the notation $x_{i:j}$ to denote the sequence $\{x_i, \dots, x_j\}$ for a generic variable x_t . The Gibbs sampler is structured according to the following blocks:

1. $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
 - (a) $\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
 - (b) $\bar{y}_{0:T}, \tilde{y}_{-p+1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
2. $\varphi, \Sigma_\varepsilon, \Sigma_e | \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$
 - (a) $\Sigma_\varepsilon, \Sigma_e | \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$
 - (b) $\varphi | \Sigma_\varepsilon, \Sigma_e, \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$

Details of each step follow:

1. $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

This is given by the product of the marginal posterior distribution of λ (conditional on the other parameters) times the distribution of $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}$ conditional on λ (and the other parameters).

- (a) $\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

The marginal posterior distribution of λ (conditional on the other parameters) is given by

$$p(\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}) \propto L(y_{1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e) p(\lambda),$$

where $L(y_{1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e)$ is the likelihood obtained from the Kalman filter applied to the state space system (1) through (5). $p(\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T})$ does not have a known form so we will use a Metropolis Hastings step.

(b) $\bar{y}_{0:T}, \tilde{y}_{-p+1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

Given λ and the other parameters of the state space model we can use Durbin and Koopman (2002)'s simulation smoother to obtain draws for the latent states $\bar{y}_{0:T}$ and $\tilde{y}_{-p+1:T}$. Note that in addition to $\bar{y}_{1:T}$ and $\tilde{y}_{1:T}$ we also need to draw the initial conditions \bar{y}_0 and $\tilde{y}_{-p+1:0}$ in order to estimate the parameters of (3) and (2) in the next Gibbs sampler step.

Note that missing observations do not present any difficulty in terms of carrying out this step: if the vector y_{t_0} has some missing elements, the corresponding rows of the observation equation (1) are simply deleted for $t = t_0$.

2. $\varphi, \Sigma_\varepsilon, \Sigma_e | \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$

This step is straightforward because for given $\bar{y}_{0:T}$ and $\tilde{y}_{-p+1:T}$ equations (2) and (3) are standard VARs where in case of (2) we actually know the autoregressive matrices. The posterior distribution of Σ_e is given by

$$p(\Sigma_e | \bar{y}_{0:T}) = \mathcal{IW}(\underline{\Sigma}_e + \hat{S}_e, \kappa_e + T)$$

where $\hat{S}_e = \sum_{t=1}^T (\bar{y}_t - \bar{y}_{t-1})(\bar{y}_t - \bar{y}_{t-1})'$. The posterior distribution of φ and Σ_ε is given by

$$p(\Sigma_\varepsilon | \tilde{y}_{0:T}) = \mathcal{IW}(\underline{\Sigma}_\varepsilon + \hat{S}_\varepsilon, \kappa_\varepsilon + T),$$

$$p(\varphi | \Sigma_\varepsilon, \tilde{y}_{0:T}) = \mathcal{N} \left(\text{vec}(\hat{\Phi}), \Sigma_\varepsilon \otimes \left(\sum_{t=1}^T \tilde{x}_t \tilde{x}_t' + \underline{\Omega}^{-1} \right)^{-1} \right),$$

where $\tilde{x}_t = (\tilde{y}'_{t-1}, \dots, \tilde{y}'_{t-p})'$ collects the VAR regressors,

$$\hat{\Phi} = \left(\sum_{t=1}^T \tilde{x}_t \tilde{x}_t' + \underline{\Omega}^{-1} \right)^{-1} \left(\sum_{t=1}^T \tilde{x}_t \tilde{y}_t' + \underline{\Omega}^{-1} \underline{\Phi} \right), \quad \hat{S}_\varepsilon = \sum_{t=1}^T \hat{\varepsilon}_t \hat{\varepsilon}_t' + (\hat{\Phi} - \underline{\Phi})' \underline{\Omega}^{-1} (\hat{\Phi} - \underline{\Phi}),$$

and $\hat{\varepsilon}_t = \tilde{y}_t - \hat{\Phi}' \tilde{x}_t$ are the VAR residuals.

B Additional Tables and Figures

Table A1: Change in \bar{r}_t^w and Its Components – Additional Details

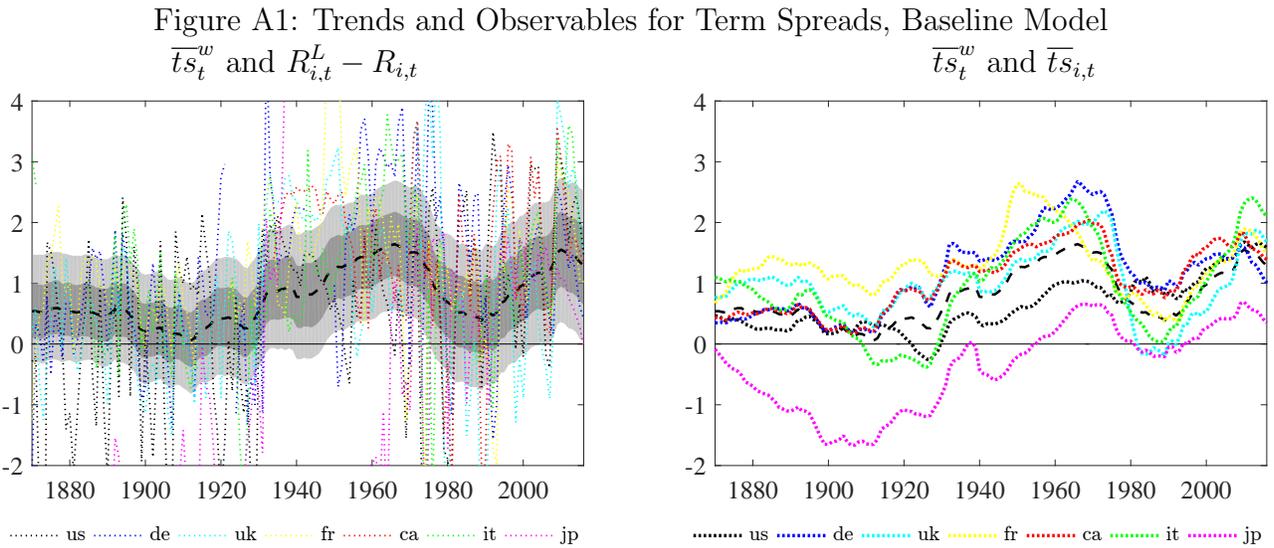
| | 1980-2016 | 1990-2016 | 1997-2016 |
|--------------------------------|----------------|----------------|----------------|
| Baseline Model | | | |
| | −1.88 | −1.64 | −1.03 |
| | [−2.68, −1.11] | [−2.38, −0.93] | [−1.59, −0.45] |
| \bar{r}_t^w | (−3.24, −0.61) | (−2.84, −0.47) | (−1.96, −0.09) |
| | ⟨−3.47, −0.38⟩ | ⟨−3.03, −0.25⟩ | ⟨−2.18, 0.07⟩ |
| | {0.99} | {0.99} | {0.96} |
| Convenience Yield Model | | | |
| | −1.71 | −1.73 | −1.13 |
| | [−2.45, −1.00] | [−2.39, −1.10] | [−1.67, −0.61] |
| \bar{r}_t^w | (−2.94, −0.55) | (−2.84, −0.67) | (−2.01, −0.26) |
| | ⟨−3.16, −0.33⟩ | ⟨−3.08, −0.48⟩ | ⟨−2.19, −0.11⟩ |
| | {0.99} | {1.00} | {0.99} |
| | −0.82 | −0.97 | −0.66 |
| | [−1.29, −0.35] | [−1.38, −0.54] | [−1.02, −0.30] |
| $-\bar{cy}_t^w$ | (−1.60, −0.05) | (−1.65, −0.24) | (−1.25, −0.06) |
| | ⟨−1.74, 0.12⟩ | ⟨−1.79, −0.08⟩ | ⟨−1.36, 0.06⟩ |
| | {0.96} | {0.98} | {0.96} |
| | −0.90 | −0.78 | −0.48 |
| | [−1.48, −0.31] | [−1.32, −0.25] | [−0.91, −0.06] |
| \bar{m}_t^w | (−1.89, 0.08) | (−1.66, 0.09) | (−1.17, 0.22) |
| | ⟨−2.07, 0.28⟩ | ⟨−1.83, 0.26⟩ | ⟨−1.34, 0.35⟩ |
| | {0.94} | {0.93} | {0.87} |

Consumption Model

| | | | |
|---------------------|------------------------------|------------------------------|------------------------------|
| | -1.93 | -1.94 | -1.22 |
| | [-2.66, -1.16] | [-2.63, -1.25] | [-1.78, -0.65] |
| \bar{r}_t^w | (-3.18, -0.69) | (-3.10, -0.82) | (-2.18, -0.29) |
| | \langle -3.44, -0.39 \rangle | \langle -3.35, -0.60 \rangle | \langle -2.38, -0.11 \rangle |
| | {0.99} | {1.00} | {0.98} |
| | -0.71 | -0.92 | -0.65 |
| | [-1.20, -0.22] | [-1.37, -0.47] | [-1.01, -0.26] |
| $-\bar{c}y_t^w$ | (-1.51, 0.11) | (-1.67, -0.19) | (-1.25, -0.02) |
| | \langle -1.68, 0.27 \rangle | \langle -1.80, -0.03 \rangle | \langle -1.37, 0.10 \rangle |
| | {0.92} | {0.98} | {0.96} |
| | -1.22 | -1.02 | -0.58 |
| | [-1.83, -0.60] | [-1.57, -0.48] | [-1.05, -0.15] |
| \bar{m}_t^w | (-2.23, -0.19) | (-1.95, -0.12) | (-1.33, 0.16) |
| | \langle -2.45, 0.03 \rangle | \langle -2.16, 0.10 \rangle | \langle -1.48, 0.31 \rangle |
| | {0.97} | {0.97} | {0.90} |
| | -0.74 | -0.61 | -0.35 |
| | [-1.20, -0.31] | [-1.00, -0.21] | [-0.66, -0.03] |
| \bar{g}_t^w | (-1.50, -0.03) | (-1.28, 0.06) | (-0.88, 0.19) |
| | \langle -1.66, 0.10 \rangle | \langle -1.42, 0.17 \rangle | \langle -0.99, 0.29 \rangle |
| | {0.96} | {0.93} | {0.86} |
| | -0.47 | -0.42 | -0.24 |
| | [-0.91, -0.01] | [-0.82, -0.02] | [-0.56, 0.08] |
| $\bar{\beta}_t^w$ | (-1.21, 0.31) | (-1.07, 0.25) | (-0.78, 0.30) |
| | \langle -1.35, 0.46 \rangle | \langle -1.22, 0.38 \rangle | \langle -0.89, 0.44 \rangle |
| | {0.84} | {0.85} | {0.77} |
| | -1.84 | -1.31 | -0.72 |
| | [-2.53, -1.14] | [-2.00, -0.66] | [-1.31, -0.14] |
| $\bar{\Delta}c_t^w$ | (-2.99, -0.67) | (-2.41, -0.21) | (-1.71, 0.25) |
| | \langle -3.20, -0.43 \rangle | \langle -2.63, 0.03 \rangle | \langle -1.91, 0.46 \rangle |
| | {0.99} | {0.97} | {0.89} |

Note: For a variable x , where $x = \{\bar{r}_t^w, \bar{m}_t^w, -\bar{c}y_t^w, \bar{g}_t^w, \bar{\beta}_t^w, \bar{\Delta}c_t^w\}$ depending on the model, the table shows the posterior median of $\Delta x = x_{2016} - x_{t_0}$ for t_0 being equal to 1980 (left column), 1990 (middle column), and 1997 (middle column), and the 68 ($[\cdot, \cdot]$), 90 ((\cdot, \cdot)), and 95.68 ($\langle \cdot, \cdot \rangle$) percent posterior coverage interval for Δx (in parenthesis). The figures in curly brackets ($\{\cdot\}$) show $Pr\{x \leq 0\}$.

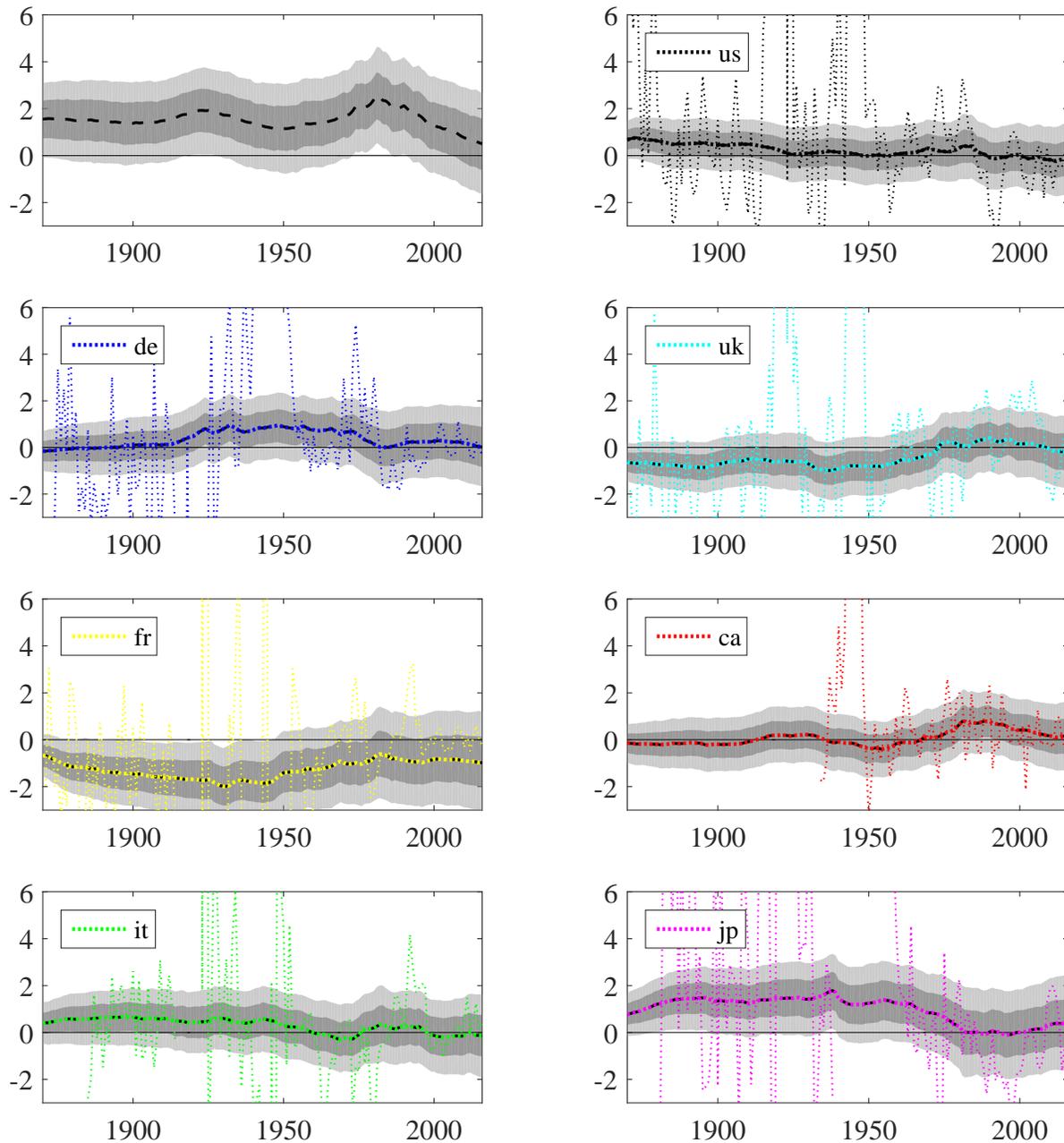
B.1 Baseline Model (Section 4.1) – Additional Results



Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines, see legend), together with the trend \bar{ts}_t^w (the dashed-and-dotted black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{ts}_{i,t} = \bar{ts}_t^w + \bar{ts}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend \bar{ts}_t^w (dashed-and-dotted black line).

Figure A2: Country-specific Trends \bar{r}_t^i and Observables, Baseline Model

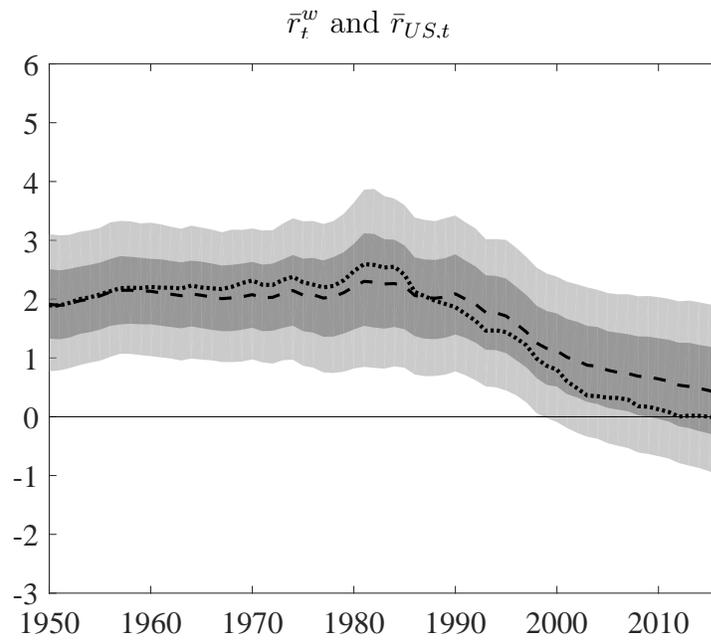
$$\bar{r}_t^i \text{ and } R_{i,t} - \pi_{i,t} - \frac{1}{n} \sum_{i=1}^n (R_{i,t} - \pi_{i,t})$$



Note: The upper-left panel reproduces the estimates of \bar{r}_t^w , so to put in context the magnitude of the fluctuations for the remaining panels, which show the estimates of the country specific effect \bar{r}_t^i for each country together with the ex-post real rates $R_{i,t} - \pi_{i,t}$ in deviation from the average ex-post real rate across countries $\frac{1}{n} \sum_{i=1}^n (R_{i,t} - \pi_{i,t})$. In all panels the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

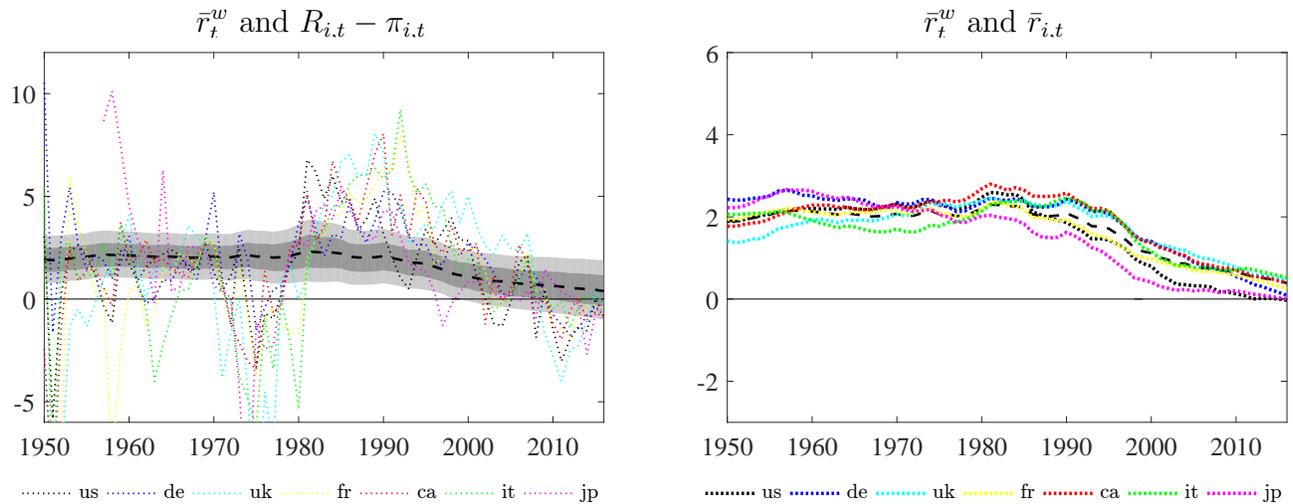
B.2 Baseline Model Estimated from 1950

Figure A3: Trends in Global and U.S. Real Rates: 1870-2016, Baseline Model Estimated from 1950



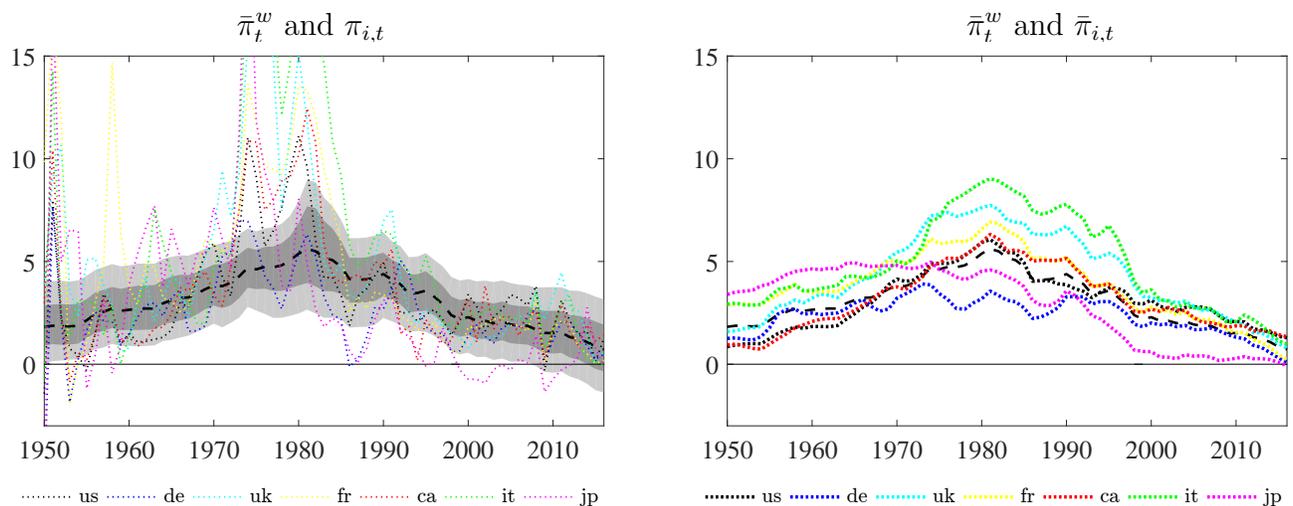
Note: The dashed black line shows the posterior median of \bar{r}_t^w and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

Figure A4: Trends and Observables for Short Term Real Rates, Baseline Model Estimated from 1950



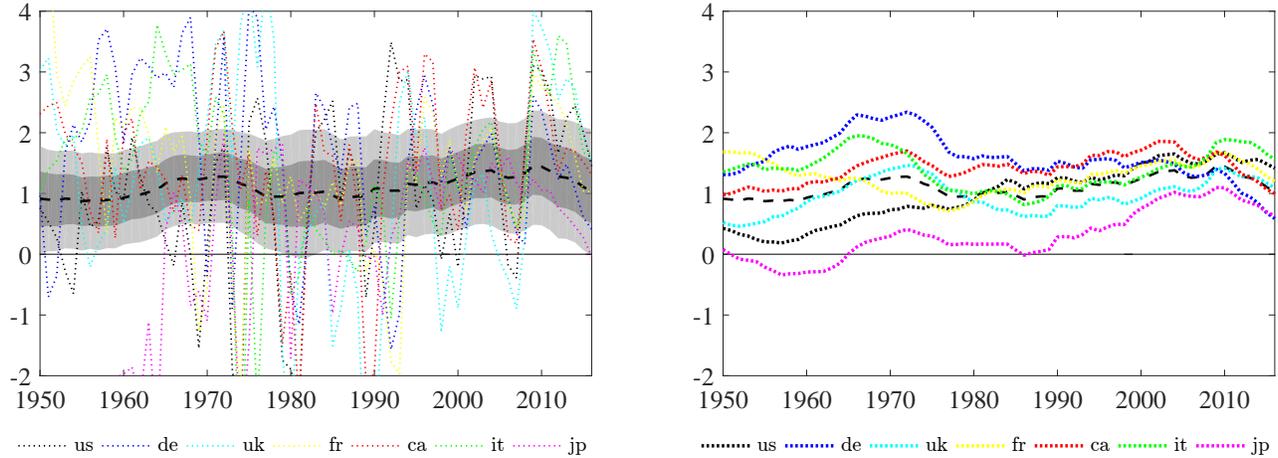
Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines, see legend), together with the trend \bar{r}_t^w (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend \bar{r}_t^w (dashed black line).

Figure A5: Trends and Observables for Inflation, Baseline Model Estimated from 1950



Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines, see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^{\pi} \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

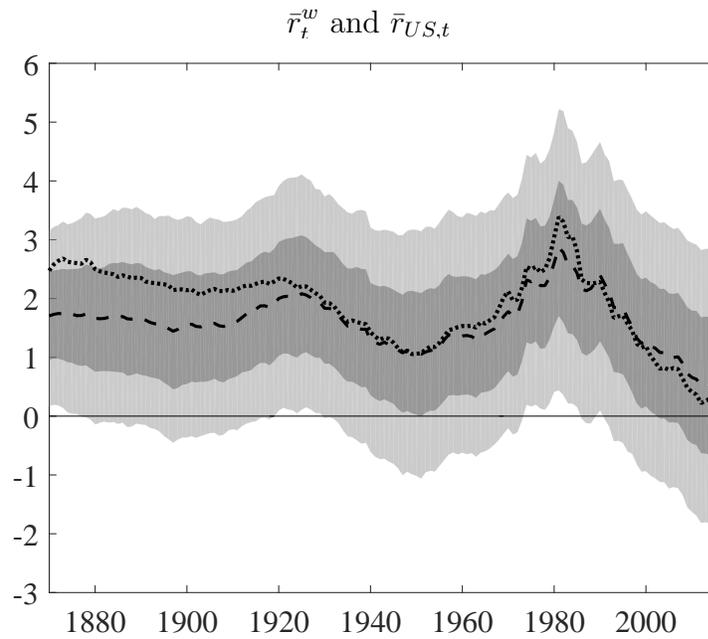
Figure A6: Trends and Observables for Term Spreads, Baseline Model Estimated from 1950
 \bar{ts}_t^w and $R_{i,t}^L - R_{i,t}$ \bar{ts}_t^w and $\bar{ts}_{i,t}$



Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines, see legend), together with the trend \bar{ts}_t^w (the dashed-and-dotted black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{ts}_{i,t} = \bar{ts}_t^w + \bar{ts}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend \bar{ts}_t^w (dashed-and-dotted black line).

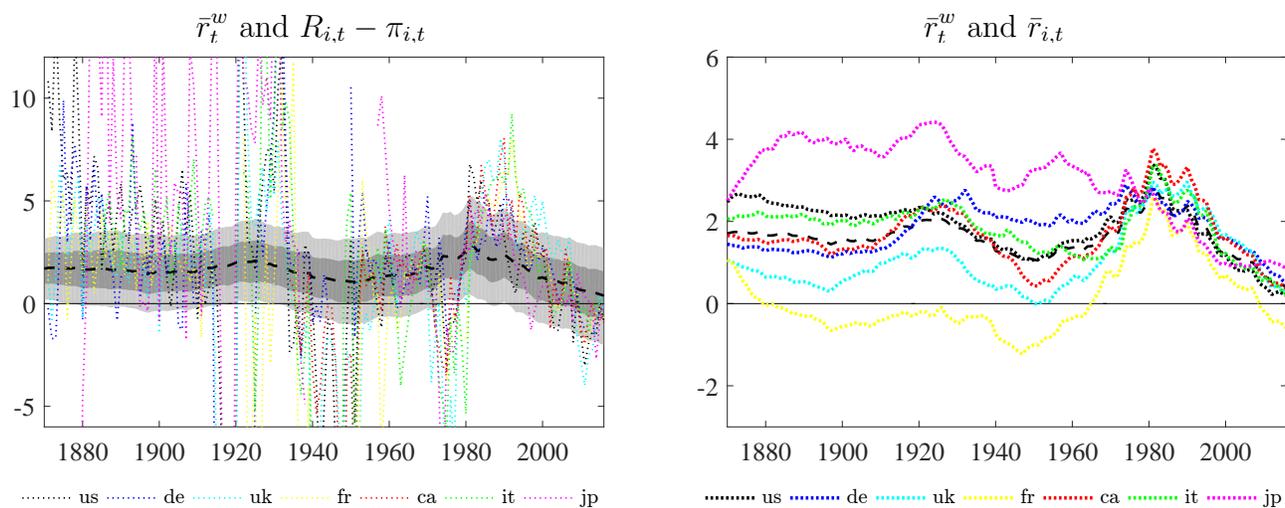
B.3 Baseline Model with 50 Degrees of Freedom

Figure A7: Trends in Global and U.S. Real Rates: 1870-2016, Baseline Model with 50 Degrees of Freedom



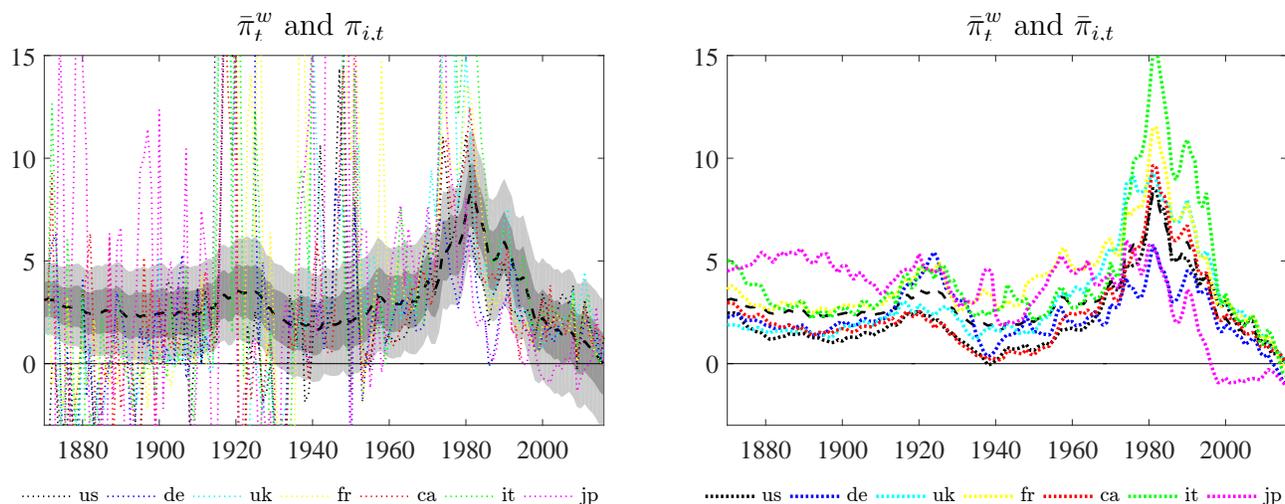
Note: The dashed black line shows the posterior median of \bar{r}_t^w and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

Figure A8: Trends and Observables for Short Term Real Rates, Baseline Model with 50 Degrees of Freedom



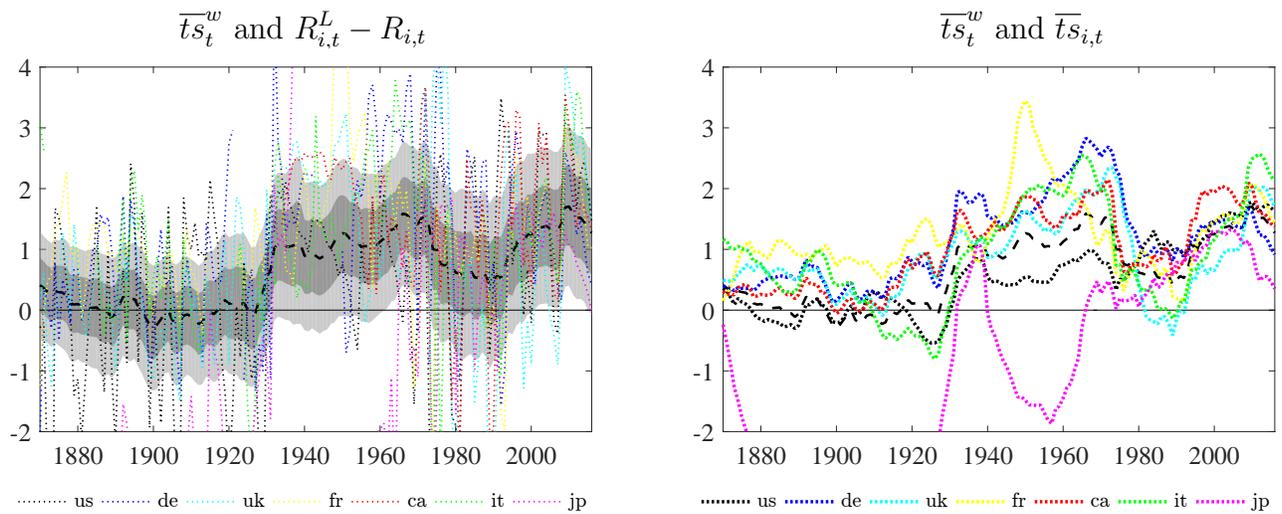
Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines, see legend), together with the trend \bar{r}_t^w (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend \bar{r}_t^w (dashed black line).

Figure A9: Trends and Observables for Inflation, Baseline Model with 50 Degrees of Freedom



Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines, see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^r \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

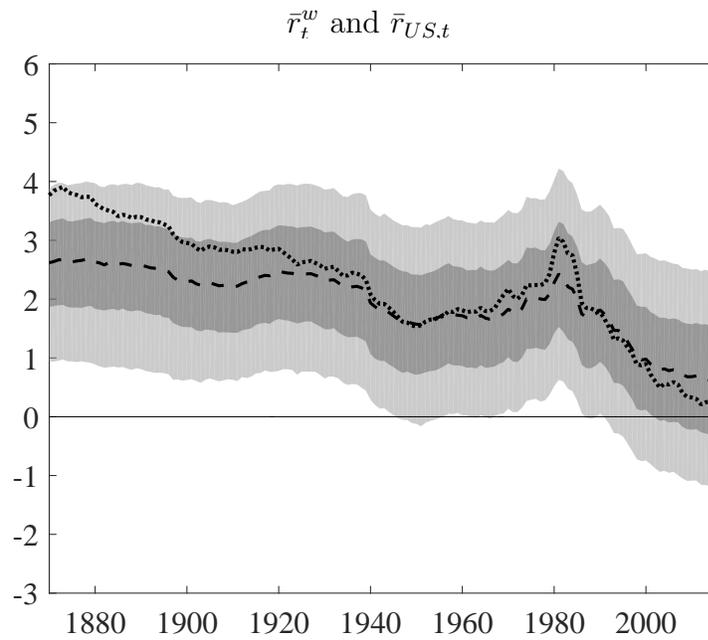
Figure A10: Trends and Observables for Term Spreads, Baseline Model with 50 Degrees of Freedom



Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines, see legend), together with the trend \bar{ts}_t^w (the dashed-and-dotted black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{ts}_{i,t} = \bar{ts}_t^w + \bar{ts}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend \bar{ts}_t^w (dashed-and-dotted black line).

B.4 Baseline Model – Unrestricted Version

Figure A11: Trends in Global and U.S. Real Rates: 1870-2016, Unrestricted Version of the Baseline Model (Section 4.1)



Note: The dashed black line shows the posterior median of \bar{r}_t^w and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

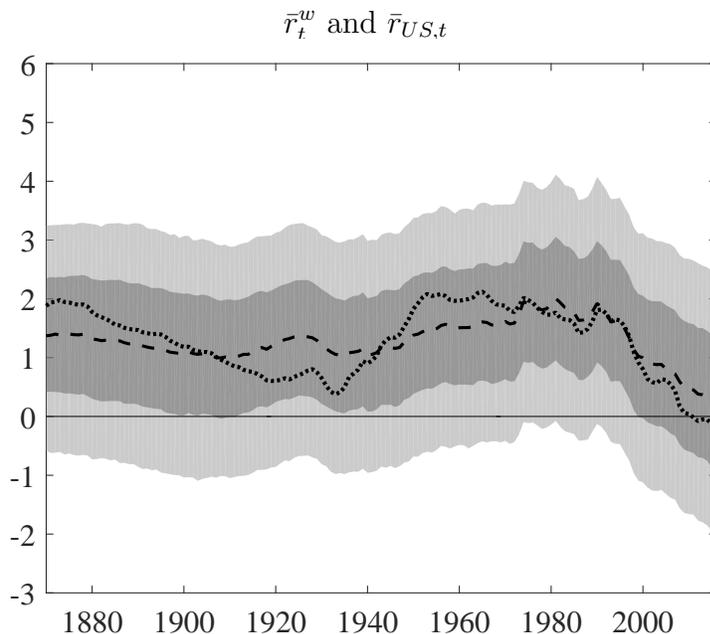
Table A2: Estimates of λ_i^r for the Unrestricted Version of the Baseline Model (Section 4.1)

| | |
|----|--|
| us | 1.23 [1.02, 1.45] $\langle 0.80, 1.68 \rangle$ |
| de | 0.86 [0.68, 1.07] $\langle 0.43, 1.36 \rangle$ |
| uk | 0.72 [0.50, 0.90] $\langle 0.32, 1.18 \rangle$ |
| fr | 0.85 [0.61, 1.08] $\langle 0.38, 1.35 \rangle$ |
| ca | 1.05 [0.84, 1.22] $\langle 0.67, 1.43 \rangle$ |
| it | 1.24 [0.94, 1.50] $\langle 0.68, 1.73 \rangle$ |
| jp | 1.37 [1.15, 1.53] $\langle 0.89, 1.75 \rangle$ |

Note: The table shows the posterior median of λ_i^r and the 68 and 95 percent posterior coverage intervals.

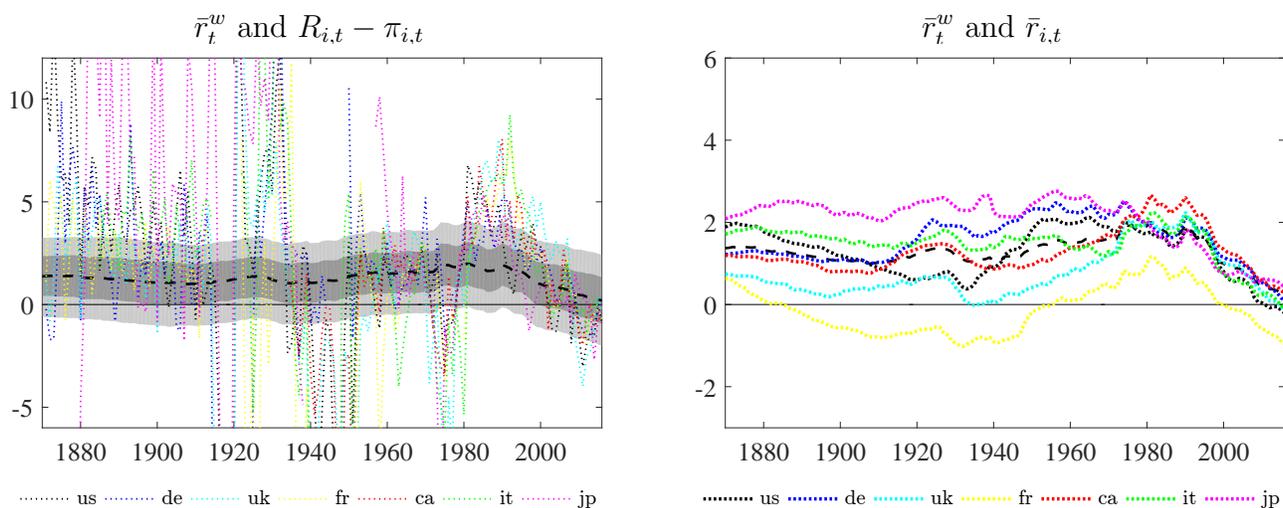
B.5 Convenience Yield Model (Section 4.2) – Additional Results

Figure A12: Trends in Global and U.S. Real Rates: 1870-2016, Convenience Yield Model



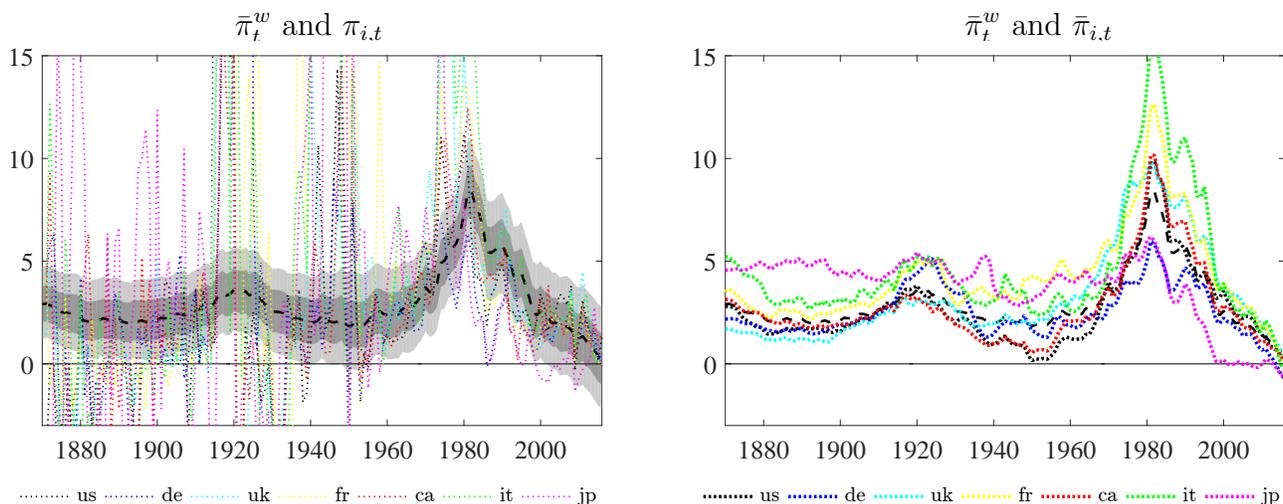
Note: The dashed black line shows the posterior median of \bar{r}_t^w and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

Figure A13: Trends and Observables for Short Term Real Rates, Convenience Yield Model



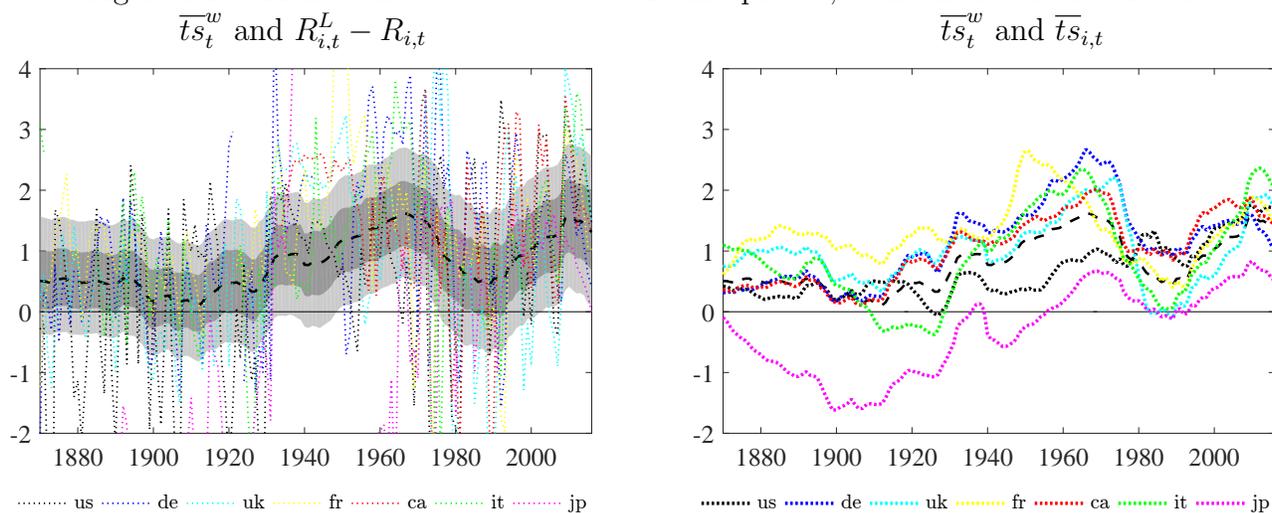
Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines, see legend), together with the trend \bar{r}_t^w (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend \bar{r}_t^w (dashed black line).

Figure A14: Trends and Observables for Inflation, Convenience Yield Model



Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines, see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line) shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^{\pi} \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

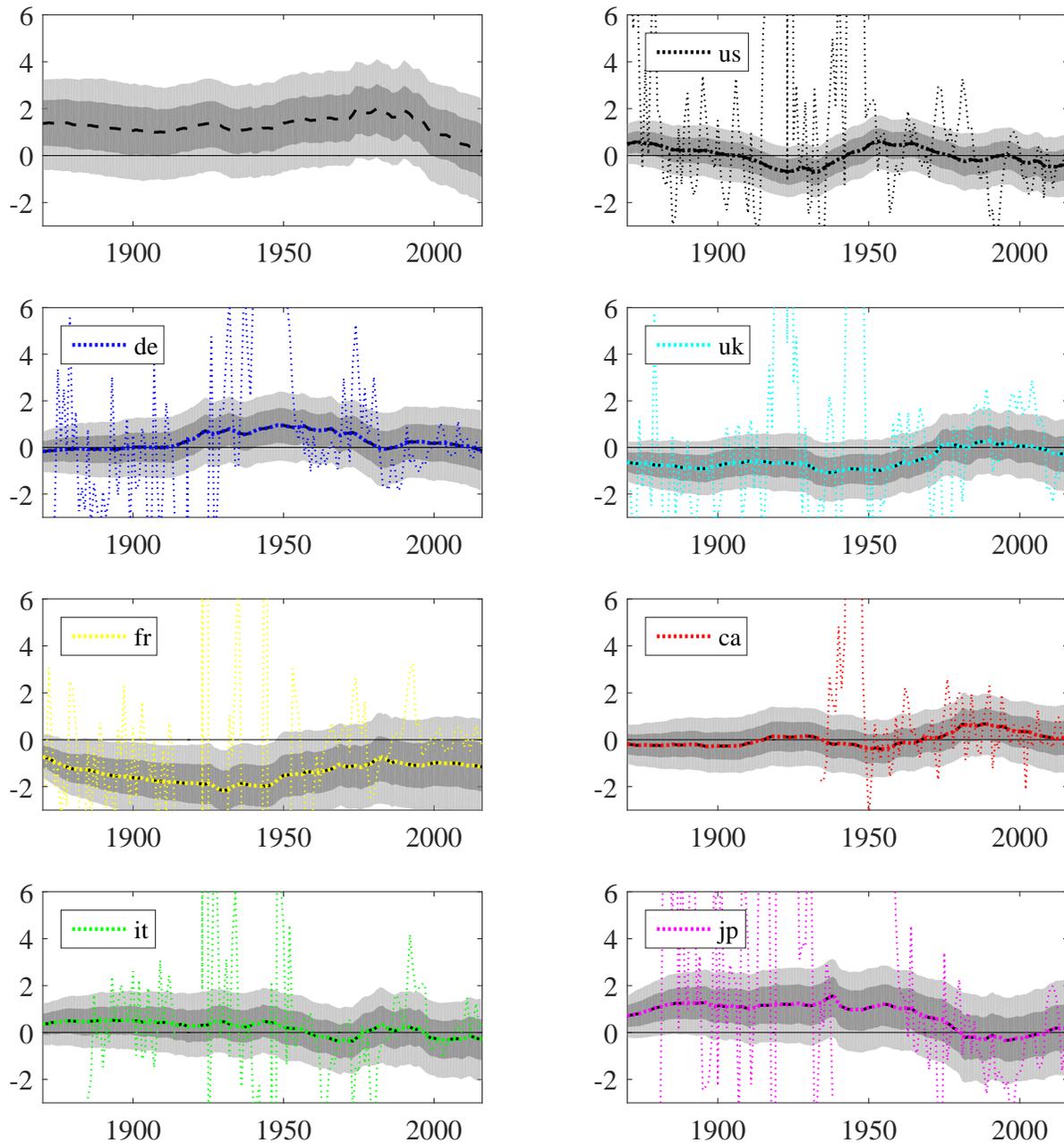
Figure A15: Trends and Observables for Term Spreads, Convenience Yield Model



Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines, see legend), together with the trend \bar{ts}_t^w (the dashed-and-dotted black line) shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{ts}_{i,t} = \bar{ts}_t^w + \bar{ts}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend \bar{ts}_t^w (dashed-and-dotted black line).

Figure A16: Country-specific Trends \bar{r}_t^i and Observables, Convenience Yield Model

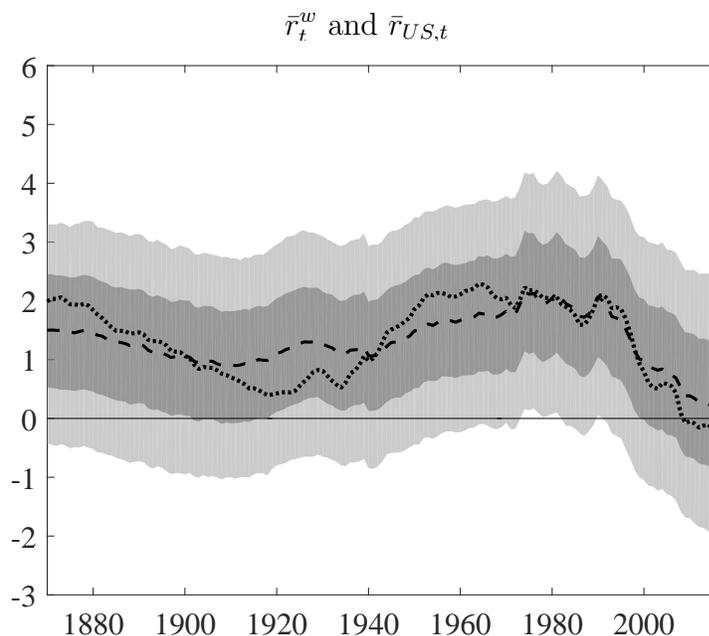
$$\bar{r}_t^i \text{ and } R_{i,t} - \pi_{i,t} - \frac{1}{n} \sum_{i=1}^n (R_{i,t} - \pi_{i,t})$$



Note: The upper-left panel reproduces the estimates of \bar{r}_t^w , so to put in context the magnitude of the fluctuations for the remaining panels, which show the estimates of the country specific effect \bar{r}_t^i for each country together with the ex-post real rates $R_{i,t} - \pi_{i,t}$ in deviation from the average ex-post real rate across countries $\frac{1}{n} \sum_{i=1}^n (R_{i,t} - \pi_{i,t})$. In all panels the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

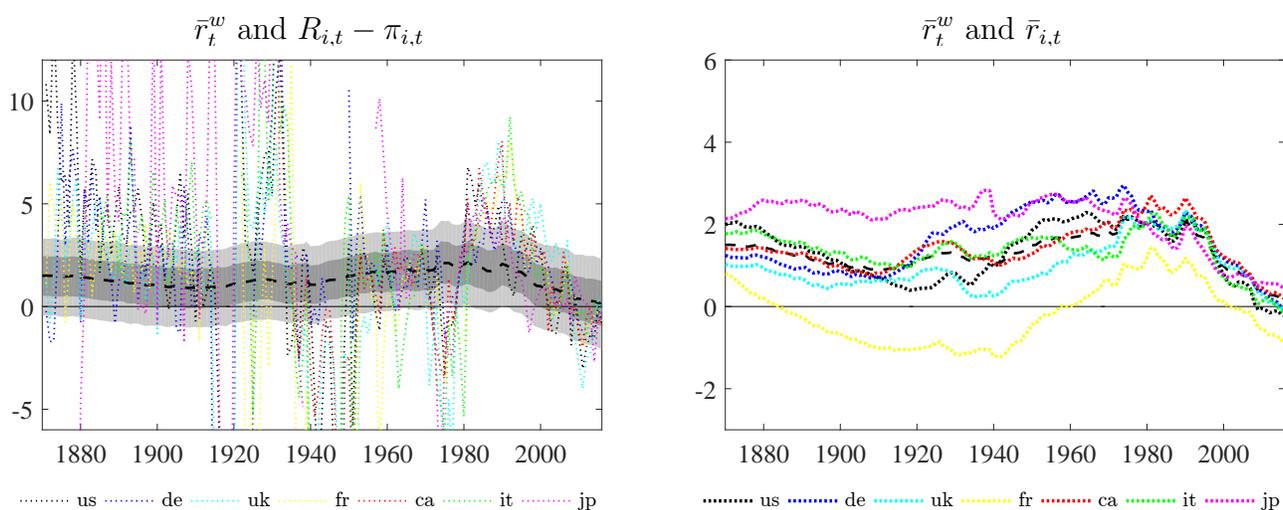
B.6 Consumption Model (Section 4.3) – Additional Results

Figure A17: Trends in Global and U.S. Real Rates: 1870-2016, Consumption Model



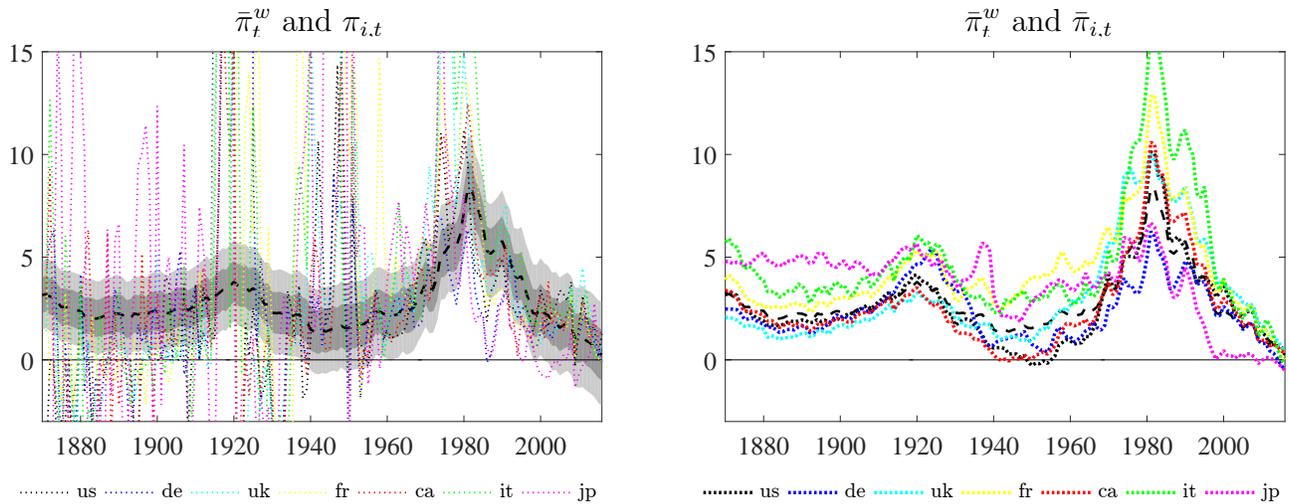
Note: The dashed black line shows the posterior median of \bar{r}_t^w and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

Figure A18: Trends and Observables for Short Term Real Rates, Consumption Model



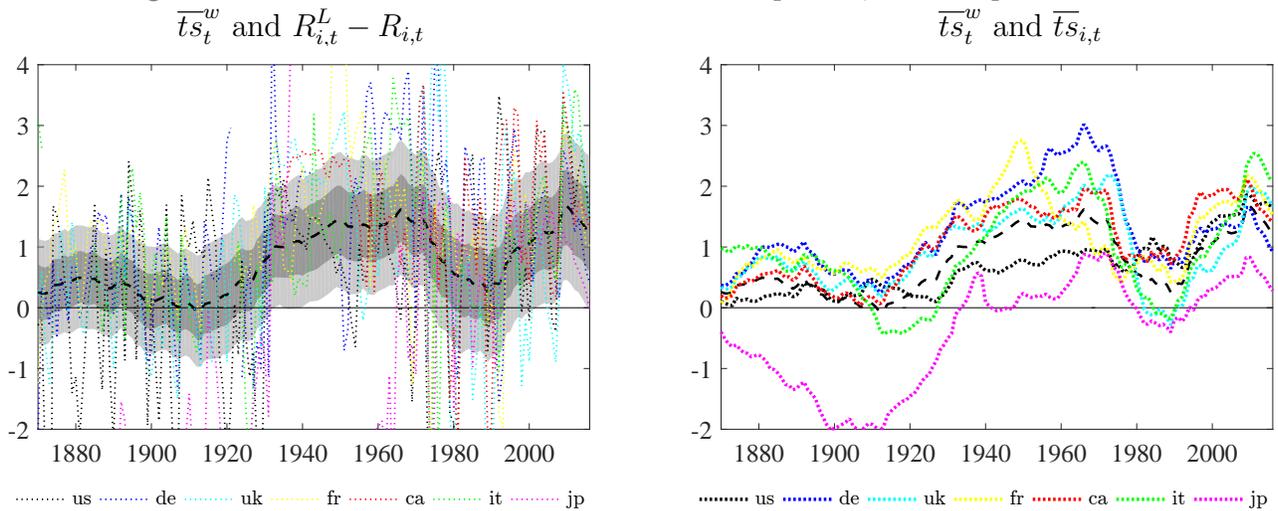
Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines, see legend), together with the trend \bar{r}_t^w (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend \bar{r}_t^w (dashed black line).

Figure A19: Trends and Observables for Inflation, Consumption Model



Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines, see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line) shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t}^w = \lambda_i^{\pi} \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

Figure A20: Trends and Observables for Term Spreads, Consumption Model



Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines, see legend), together with the trend \bar{ts}_t^w (the dashed-and-dotted black line) shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{ts}_{i,t}^w = \bar{ts}_t^w + \bar{ts}_t^i$ for each country i (dotted lines, see legend), together with the posterior median of the trend \bar{ts}_t^w (dashed-and-dotted black line).

Figure A21: Country-specific Trends \bar{r}_t^i and Observables, Consumption Model

$$\bar{r}_t^i \text{ and } R_{i,t} - \pi_{i,t} - \frac{1}{n} \sum_{i=1}^n (R_{i,t} - \pi_{i,t})$$

