Network Risk and Key Players: A Structural Analysis of Interbank Liquidity*

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Abstract

We estimate the liquidity multiplier and individual banks' contribution to systemic liquidity risk in an interbank network using a structural model. Banks borrow liquidity from neighbours and update their valuation based on neighbours' actions. When the former (latter) motive dominates, the equilibrium exhibits strategic substitution (complementarity) of liquidity holdings, and a reduced (increased) liquidity multiplier dampening (amplifying) shocks. Empirically, we find substantial and procyclical network-generated risks driven mostly by changes of equilibrium type rather than network topology. We identify the banks that generate most systemic risk and solve the planner's problem, providing guidance to macro-prudential policies.

Keywords: financial networks; liquidity; interbank market; key players; systemic risk.

I Introduction

The 2007-09 financial crisis has stimulated strong interest in understanding financial intermediation and its role in creating liquidity.[1] Intermediaries face their own liquidity management problem, and in particular, banks hold central bank reserves to buffer liquidity shocks (e.g., Bianchi and Bigio (2014)). Their liquidity holding decisions are crucial for payment activities, lending, and asset prices (Piazzesi and Schneider (2017)). Another area that has drawn increasing attention is financial networks. The interbank network, where banks borrow and lend reserves, has been at the heart of studies of systemic risk.[2] These two themes merge in our paper. We study both theoretically and empirically how the interbank network affects banks' liquidity holding decisions, and its implications for systemic risk.

This paper structurally estimates a liquidity-holding game where banks obtain credit from an interbank network and their liquidity management objective incorporates different sources of network externality. Applying our framework to U.K. banks, we find that the dominant type of network externality varies over the business cycle. In the boom before 2008, banks' liquidity holdings exhibit strategic complementarity, and thereby, the network amplifies liquidity shocks. As the financial crisis unfolds, the degree of strategic complementarity declines, and after the introduction of Quantitative Easing (QE) in the U.K., banks' liquidity holdings exhibit strategic substitution, and the interbank network turns from a shock amplifier to a shock buffer. We are not the first to emphasize that shock propagation depends on complementarity versus substitutability (e.g., Jovanovic (1987)). However, this paper is the first, to the best of our knowledge, to estimate such interdependence of players using a network structure and to provide the evidence of time-varying network externalities.

The model in the paper provides a framework to measure the impact of externalities and allow us to compare the decentralised equilibrium with the planner's solution that achieves constrained efficiency. Based on these theoretical findings, the paper offers novel metrics to guide the monitoring of banks and the design of policy intervention during boom and bust cycles. For example, we find during the boom the systemic liquidity risk is too high and the opposite is the case during the crisis, relative to the social optimum, indicating a need for restraining banks' liquidity creation or conducting quantitative easing at the different point of the cycle. We also identify banks that contribute the most to the volatility of aggregate liquidity through network domino effects. Our empirical evidence of time-varying

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[1] In the recent macro-finance literature, intermediaries play the role of marginal investor in asset markets (Brunnermeier and Sannikov (2014); He and Krishnamurthy (2013)), credit supplier (Mark and Kiyotaki (2010); Klimenko, Pfeil, Rochet, and Nicolo (2016)), and money issuer (Brunnermeier and Sannikov (2016); Hart and Zingales (2014); Li (2017); Quadrini (2017)).

[2] Recent theories on interbank network and systemic risk include Freixas, Parigi, and Rochet (2000), Allen, Carletti, and Gale (2008), and Freixas, Martin, and Skeie (2011).

network externalities points out the importance of modelling banks' economic behaviour on network rather than network topology alone. In our setting, we find the variation in network externalities is mostly driven by changes of the type of equilibrium on the network (i.e., strategic complementarity or substitution).

Specifically, we model banks' reserve holding decisions in a linear-quadratic framework (à la Ballester, Calvo-Armengol, and Zenou (2006)), assuming a predetermined but time-varying interbank network where capital and information flow. Bank characteristics and macroeconomic conditions affect banks' decision, but reserve holdings also depend on the network topology, and the nature of network externality captured by structural parameters that are identified by our estimation.

The interbank network generates two counteracting effects for the banks' liquidity management problem. First, interbank relationships allow banks to borrow reserves from their neighbours, so the marginal benefit of holding reserves on their own decreases when neighbours hold more reserves. This free-riding incentive gives rise to strategic substitution (Bhattacharya and Gale (1987)). Consequently, the network acts as a risk buffer for liquidity shocks since neighbouring banks' liquidity holdings are negatively correlated. Second, when banks see neighbouring banks holding more reserves, they positively update their belief on the value of liquidity. Due to such informational spillover, the marginal benefit of holding liquidity increases in neighbours' liquidity, which leads to strategic complementarity. In this case, the network amplifies the liquidity shocks originating from individual banks due to the positive correlation among neighbouring banks' liquidity holding.[3]

In the (unique interior) Nash equilibrium, the overall impact of network on banks' liquidity holdings depends on a parameter ϕ, the network attenuation factor. If strategic substitution dominates, ϕ is negative. If strategic complementarity dominates, ϕ is positive.

Individual banks' equilibrium reserve holdings depend upon the magnitude of liquidity shocks to *all* banks in the network. However, not all shocks are equally important. In particular, the dominant type of network externality, ϕ, and bank i's *indegree* Katz-Bonacich centrality (network topology) determine how a bank weights liquidity shocks to itself and other banks in the network in their optimal liquidity holding decisions. The indegree centrality counts the direct and indirect links from other banks towards bank i, weighting connections by ϕ^k, where k is the number of steps needed to reach bank i.

In other words, the liquidity holding decision of a bank is related to its own shocks, the shocks of its neighbours, of the neighbours of its neighbours, etc., with distant shocks be-

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[3]Another channel through which banks' liquidity holdings may increase in their neighbours' is the leverage stack mechanism in Moore (2012). In Moore (2012), by signalling its credit worthiness through a higher liquidity buffer, banks can borrow more from other banks to finance positive NPV projects. So, when neighbours are ready to lend (i.e., holding more reserves), a bank chooses to hold more liquidity.

coming increasingly less important as they are weighted by ϕ^k.[4] It is important to emphasize that the network topology (e.g. Katz–Bonacich centrality measures) is *not* the only determinant of banks' liquidity holdings. The magnitude of shocks to individual banks and the attenuation factor ϕ are also crucial, and are key structural parameters in our estimation. For example, banks that receive extremely large liquidity shocks, regardless of their network location, may have a large impact on all banks' liquidity holdings in equilibrium.

Based on the equilibrium results, we analyze potential policy interventions to remedy any negative impact of network externalities. Specifically, we characterise the volatility of aggregate liquidity holdings and identify key banks that contributes the most to its volatility – i.e. the *risk key players*. We find that the contribution by each bank to the network risk is related to 1) the network attenuation factor ϕ, 2) the bank specific liquidity risks, and 3) its *outdegree* Katz–Bonacich centrality measure. The outdegree centrality is similarly defined as the indegree centrality but the connections are outbound from bank i to measure the impact of bank i' on its neighbours, neighbours of its neighbours, etc. Moreover, we introduce the concept of network impulse response function (NIRF) that naturally decomposes the volatility of aggregate liquidity into each bank's contribution to it, and we show that the risk key player is precisely the bank with the largest NIRF.

We also solve for the planner's optimum and compare it with the decentralised equilibrium. The discrepancy in the expected level and volatility of liquidity holdings arises from network externalities, and in particular, the fact that banks do not internalise their impact on each other through the outbound linkages.

Using daily data from the Bank of England, we use our model to study the reserve holding decisions of member banks of the sterling large payment system, CHAPS, in the period of 2006 to 2010. Member banks conduct transaction for their own purpose and on behalf of their clients and hundreds of other non-member banks. Their liquidity holdings ensures the functioning of payment system that is crucial for supporting real economic activities. In 2009, on average £272 billions of transactions in the U.K. were settled every day in CHAPS (U.K. nominal GDP every 5.5 days). CHAPS banks regularly have intraday liquidity exposures in excess of £1 billion to counterparties, and they hold reserves to buffer payment imbalances.[5] Variation in payment imbalances is as close as we can get to a pure liquidity shock, because CHAPS transactions are settled in real time and on *gross* terms ("RTGS") to eliminate

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[4]This centrality measure takes into account the number of both direct and indirect connections in a network. For more on the Bonacich centrality measure, see Bonacich (1987) and Jackson (2003). For other economic applications, see Ballester, Calvo-Armengol, and Zenou (2006) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012). For an excellent review of the literature, see Jackson and Zenou (2012).

[5]The U.K. monetary framework leaves reserves management largely at individual banks' discretion (both before and after the Quantitative Easing). In Appendix A.1, we provide background information on the policy framework (reserve regimes) including details on the payment system, and the interbank markets.

counterparty credit risks.[6]

We measure interbank connections using overnight borrowing and lending data. Specifically, a link between two banks is quantified by the fraction of borrowing by one bank from another in the recent past, so the network is directional and its adjacency matrix is weighted (i.e., right stochastic). Note that the links between two banks can be interpreted as (frequentist) probabilities of borrowing-lending transactions. Our results are robust when interbank lending, or the gross amount of borrowing and lending, is used to quantify the interbank relationship. We estimate the impact of this interbank network on banks' reserve holdings.[7]

We exploit the fact that the equilibrium conditions of our model map exactly into the spatial error model (SEM). A bank's liquidity holdings depend on its characteristics, macro variables, its own shock, and through the *entire* network structure, shocks to other banks. Our approach stands in contrast with regression models that project variables of interest on particular network statistics. SEM is a conservative approach, leaving a minimal amount of variation in liquidity holdings to be driven by the interbank network, so we also estimate a spatial Durbin model (SDM), where through the network, banks' liquidity holdings depend on not only shocks to other banks but also other banks' characteristics (e.g., balance-sheet conditions). SDM estimation serves as a model specification test.

We are able to uncover rich, pro-cyclical dynamics of network externality: the network amplified shocks in the pre-crisis period, but as the crisis unfolded, the amplification effect declined, and eventually in the QE period, the network became a shock buffer. Using our estimates of the structural parameters, we quantify this network effect by computing the ratio of aggregate liquidity volatility to the counterfactual volatility when there is no network externality (i.e., the attenuation factor ϕ is zero). We find that this ratio reached 559% in the boom, and declined to 125% during the crisis, and in the QE period, dropped to 89%.

Our finding of time-varying network externality sheds light on the relative importance of different economic forces over a business cycle. Because it is costly to hold liquidity at the expense of forgoing other investments, banks free ride their peers by borrowing reserves in interbank network in response to liquidity shocks (Bhattacharya and Gale (1987)). This common wisdom is only part of the story, because strategic complementarity arises from informational spillovers as previously discussed. Our framework accommodates all these

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[6]CHAPS uses RTGS instead of DNS (deferred net settlement). The DNS model is more liquidity efficient but creates credit risk exposure for recipient banks until the end of a clearing cycle. Such risks do not exist under RTGS since all payments are settled individually and on a gross basis. A detailed description of the RTGS in the U.K. is provided by Dent and Dison (2012) at the Bank of England. In this report, the BoE maintains that RTGS improves financial stability by minimizing credit exposures between banks.

[7]In addition to central bank reserves, banks may pledge government bonds as collateral to borrow from the Bank of England. Therefore, in our definition of reserve holdings, we add banks' holdings of collateral eligible for repo with the Bank of England. Our results are robust if we use actual reserves only.

facets of network externality, so it allows data to reveal the dominant force at different points in time.

In addition, we empirically characterize the shock propagation mechanism, quantify the individual banks' contribution to aggregate liquidity risk, and identify the risk key players. We find that the volatility of aggregate liquidity in the system is largely driven by a small group of banks, and that each bank's contribution varies substantially over time. Moreover, we find that the risk key player is typically *not* the largest net borrower – even net lenders can generate substantial risk in the system. These finding is particularly relevant for monitoring and regulating the banking system, and policy interventions during crisis.

Since in our sample the network topology changes over time, we decompose the time variation of banks' risk contributions into two components: the changes attributed to variation in ϕ, the type of equilibrium on the network, and variation of network topology. We find that the former is clearly the main driver. This suggests that in our context, endogenous network formation plays a limited role in the variation of network effect. It is the type of equilibrium (i.e., strategic complementarity or substitution) that matters.[8]

Finally, we compute the planner's solution based on our estimates of ϕ, the sizes of bank-level structural shocks, and other parameters. We find that during the boom period, the amount of aggregate liquidity held by banks is not too far from the planner's optimum but the network generates too much systemic risk through shock amplification. During the crisis period, the decentralised equilibrium generates smaller aggregate liquidity than the planner's optimum, and the systemic liquidity risk is still too high. After the introduction of QE, banks hold too much liquidity and the volatility of aggregate liquidity is below the social optimum level. These findings may guide policy makers in monitoring banks, designing crisis interventions, and assessing the impact of QE.

This paper contributes to the literature on bank liquidity management. CHAPS banks are at the most fundamental layer of economic transactions. Every transaction ultimately goes through these payment system members. Their decisions have profound influence on the whole economy (Piazzesi and Schneider (2017)). We provide the first evidence on how the liquidity choice of payment system banks depends on interbank network. Our findings can be embedded in the broad discussion of banks' portfolio choice over an economic cycle (e.g., ; Cornett, McNutt, Strahan, and Tehranian (2011)). Importantly, our finding that the

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[8]Our finding is related to the empirical literature that critically examines the systemic consequence of network linkages. While there is an large theoretical literature on network contagion and formation, simulation studies based on reasonably realistic networks show little impact of linkage variation (summarized in Upper (2011)). Using a unique dataset of all Austrian banks, Elsinger, Lehar, and Summer (2006) find that contagion happens rarely and that the necessary funds to prevent contagion are surprisingly small. By applying an Eisenberg and Noe (2001) style model to German banks, Chen, Wang, and Yao (2016) find that the lack of bank capital is the key contributor to bank failure rather than the network contagion.

equilibrium type on the network flips when QE starts contributes to the literature on bank liquidity management and monetary policy (e.g., Bernanke and Blinder (1988); Kashyap and Stein (2000); Bianchi and Bigio (2014); Drechsler, Savov, and Schnabl (2014)).

We contribute to the literature on bank liquidity regulation by providing an empirical framework to attribute systemic risk to individual banks, and by characterizing the wedge between decentralised outcome and the planner's solution. Liquidity regulation has attracted a lot of attention after the financial crisis. Stein (2012) argues that reserves requirement may serve as a tool for financial stability regulation. Diamond and Kashyap (2016) study bank liquidity regulation in the setting of Diamond and Dybvig (1983). Allen and Gale (2017) review earlier theories that may provide foundations (i.e., sources of market failures) for bank liquidity regulations, such as liquidity coverage ratio and net stable funding ratio in Basel III. Our findings of pro-cyclical network externality and banks' time-varying contribution to systemic risk lend support to a macro-prudential perspective on liquidtiy regulation.

Our work also advances the literature on interbank market dynamics and banks' liquidity demand. Fecht, Nyborg, and Rocholl (2010) find that the prices of liquidity depend on counterparties' liquidity levels. Acharya and Merrouche (2010) document evidence of precautionary liquidity demands of U.K. banks during the subprime crisis. Ashcraft, McAndrews, and Skeie (2010) find that in response to heightened payment uncertainty, banks hold excess reserves. There is also a related theoretical literature pioneered by Bhattacharya and Gale (1987). Recent theoretical works in this area highlight the externalities in interbank markets and the associated inefficiencies (e.g. Freixas, Parigi, and Rochet (2000); Allen, Carletti, and Gale (2008); Freixas, Martin, and Skeie (2011); Moore (2012); Castiglionesi, Feriozzi, and Lorenzoni (2017) among others). Our paper differs by modeling banks' liquidity holdings as outcome of a network game, and estimate the time-varying network externality.[9]

Networks have proved to be a useful analytical tool for studying financial contagion and systemic risk from both theoretical and empirical perspectives. Starting from Allen and Gale (2000), recent theories feature increasingly sophisticated networks and shock transmission

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[9]There is a another line of research that focuses on the topology and formation of linkages. Afonso and Lagos (2015) use a search theoretical framework to study the interbank market and banks' trading behaviour. The empirical literature on the topology of interbank networks starts with Furfine (2000, 2003). Other earlier empirical studies of the interbank network topology include Upper and Worms (2004); Boss, Elsinger, Summer, and Thurner (2004); Soramaki, Bech, Arnold, Glass, and Beyeler (2007); Becher, Millard, and Soramaki (2008); Bech and Atalay (2008); and Langfield, Liu, and Ota (2014). Recent works study the impact of the crisis on the structure of these networks, which include (but are not limited to): Gai and Kapadia (2010); Wetherilt, Zimmerman, and Soramaki (2010); Benos, Garratt, and Zimmerman (2010); Ball, Denbee, Manning, and Wetherilt (2011); and Afonso, Kovner, and Schoar (2011). We differ from this literature by studying, rather than network formation, the types of equilibria on a predetermined network. We empirically show that the variation of network externality is driven by the type of equilibrium on the network instead of the changes in network topology.

mechanisms.[10] Recent empirical works also cover a wide range of economic networks.[11] We differ from these papers by taking a linear-quadratic approach of Ballester, Calvo-Armengol, and Zenou (2006) to analyze how economic agents' liquidity holding decisions in a network game generate systemic risk and by structural estimating network externalities. Herskovic, Kelly, Lustig, and Nieuwerburgh (2017) embed a similar spatial autoregressive structure in firms' growth rate to study the comovement of firm volatilities.

The remainder of the paper is organised as follows. In Section II, we present and solve a liquidity holding game in a network, and define key players in terms of aggregate liquidity level and risk. Section III casts the equilibrium of network game into the spatial econometric framework, and outlines the estimation methodology. In Section IV, we describe the data and the features of the network and bank-level and macro control variables. In Section V, we present and discuss the estimation results. Section VI concludes.

II   The Network Model

In this section, we construct a model of banks' liquidity holding decisions that will directly guide our empirical analysis of systemic liquidity risk from interbank network. In this model, a bank's liquidity holding decision depends on its own characteristics as well as all other bank's liquidity holding decisions (and potentially characteristics). The latter dependency is modelled through a network. Structural estimation is presented in the following sections.

The network. There are n banks. The time-t network is predetermined, characterized by an n-square adjacency matrix $\mathbf{G}_t$. If its element $g_{ij\neq i,t} \neq 0$, bank i and j are connected. A link facilitates interbank transactions and information flow. To construct $\mathbf{G}_t$ in the structural estimation, we use interbank borrowing and lending data. This network of overnight credit is likely to be most relevant for the daily variation of banks' liquidity holdings that we focus on. Specifically, $g_{ij\neq i,t}$ will be the fraction of borrowing by bank i from bank j in the month up to day t. The network is therefore weighted and directed, and $\mathbf{G}_t$ is right stochastic.[12]

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[10]This line research includes but is not limited to Leitner (2005); Babus (2009); Babus and Allen (2009) (for a review); Afonso and Shin (2011); Zawadowski (2012); Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012); Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015); Elliott, Golub, and Jackson (2015); Atkeson, Eisfeldt, and Weill (2015); Ozdagli and Weber (2015); Glasserman and Young (2015); Cabrales, Gale, and Gottardi (2015); Cabrales, Gottardi, and Vega-Redondo (2016); Herskovic (forthcoming).

[11]The recent empirical network literature include but is not limited to Diebold and Yilmaz (2009, 2014); Billio, Getmansky, Lo, and Pelizzona (2012); Hautsch, Schaumburg, and Schienle (2012); Aldasoro and Angeloni (2013); Kelly, Lustig, and Nieuwerburgh (2013); Duarte and Eisenbach (2013); Greenwood, Landier, and Thesmar (2015); and Gofman (2017).

[12]We also explore other definitions of the adjacency matrix, where $g_{ij,t}$ is either the sterling amount of borrowing by bank i from bank j, or 1 (0) if there is (no) borrowing or lending between Bank i and j. Note that, in this latter case, the adjacency matrix is unweighted and undirected. Our estimation results are

$g_{ij,t}$ can be interpreted as the frequentist estimate of the probability of bank i's borrowing from bank j. We also consider other measurements of interbank relationships, such as bank i's lending to j and the gross amount of interbank borrowing and lending.

The matrix $\mathbf{G}_t$ keeps track of all direct connections – links of order one – between any pair of banks in the network. Similarly, the matrix $\mathbf{G}_t^k$, for any positive integer k, encodes all links of order k between banks, that is, the paths of length k between any pair of banks in the network. For example, the coefficient in the (i,j)th cell of $\mathbf{G}_t^k$ – i.e. $\left\{\mathbf{G}_t^k\right\}_{ij}$ – gives the exposure of bank i to bank j in k steps. Since $\mathbf{G}_t$ is a right stochastic matrix (with each row summing to 1), it can be interpreted as a Markov chain transition kernel, and $\mathbf{G}_t^k$ as the k-step transition matrix.

Banks and their liquidity preference. We study the amount of liquidity (reserves) banks choose to hold at the beginning of day t. Let $l_{i,t}$ denote the total liquidity held by bank i. We decompose $l_{i,t}$ into two parts: $q_{i,t}$, a benchmark level, and $z_{i,t}$, a network-dependent component. The benchmark level is directly pinned down by bank characteristics ($x_{i,t}^m$, such as leverage, lending and borrowing rates, stock return, CDS spread etc.) and macroeconomic conditions (x_t^p, such as aggregate payment pattern, monetary policy etc.), i.e.,

$$q_{i,t} = \alpha_i + \sum_{m=1}^{M} \beta_m x_{i,t}^m + \sum_{p=1}^{P} \beta_p x_t^p, \tag{1}$$

where α_i is a fixed effect. The interbank network enters into banks' liquidity holdings through $z_{i,t}$. This is a conservative approach to estimate network effects since we limit the impact of the network to $z_{i,t}$. In Appendix A.2.2, we analyze a more general model where banks' liquidity holdings depend on not only their own characteristics but also other banks' characteristics through the network.[13]

Given the predetermined network $\mathbf{G}_t$, bank i chooses $z_{i,t}$ in response to other banks' choices, $\{z_{j,t} : j \neq i\}$.[14] Banks make decisions simultaneously. The vector $\mathbf{z}_t$ records all banks' choices. The network allows banks to borrow and lend reserves, and may also transmits information relevant for liquidity management (more on this later).

Bank i derives utility from an accessible stock of liquidity, which is the sum of its own holdings, $z_{i,t}$, and what can be borrowed from other banks through the interbank network, $\psi \sum_j g_{ij,t} z_{j,t}$. The interbank component is proportional to neighbours' own holdings, $z_{j,t}$,

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robust to these alternative measures.

[13]The optimality conditions of the general model map into a spatial Durbin model, which nests our main model (spatial error model) and serves as a specification test that we report under rolling-window estimation.

[14]Since we model banks' daily liquidity holdings, we take the interbank network as predetermined at the beginning of a day. It is unlikely that interbank relationships change significantly on daily basis.

weighted by the network linkage, $g_{ij,t}$, and a parameter ψ. The benefit of accessible liquidity is $\tilde{\mu}_{i,t}$ per unit. Bank i's objective for choosing $z_{i,t}$ is represented by the following linear quadratic function:

$$u_i(z_{i,t}, \{z_{j,t} : j \neq i\}|\mathbf{G}_t) = \underbrace{\tilde{\mu}_{i,t}}_{\text{Unit Value}} \underbrace{\left(z_{i,t} + \psi \sum_{j \neq i} g_{ij,t} z_{j,t} \right)}_{\text{Accesible Liquidity}} - \frac{1}{2}\gamma \left(z_{i,t} + \psi \sum_{j \neq i} g_{ij,t} z_{j,t} \right)^2, \quad (2)$$

where concavity from the second-order term leads to decreasing marginal utility of liquidity. We decompose the unit value of liquidity, $\tilde{\mu}_{i,t}$, into a bank-specific stochastic component $\hat{\mu}_{i,t}$, and a network component, so

$$\tilde{\mu}_{i,t} := \hat{\mu}_{i,t} + \delta \sum_{j \neq i} g_{ij,t} z_{j,t},$$

where

$$\hat{\mu}_{i,t}/\gamma := \mu_{i,t} = \bar{\mu}_i + \nu_{i,t}, \quad (3)$$

$\bar{\mu}_i$ is the mean of $\mu_{i,t}$, the standalone liquidity value scaled by γ. $\nu_{i,t}$, the ultimate source of uncertainty, is a shock that is independent across banks and over time with variance σ_i^2.

The valuation of liquidity reflects the *net* benefit of liquidity holdings. $\nu_{i,t}$ can be a shock to either the benefits or costs. A liquidity buffer cushions payment imbalances and prepares banks for future investment opportunities, but to hoard liquidity, banks may have to forgo other investments or raise funds at high costs outside the interbank system. $\nu_{i,t}$ is realized before i chooses $z_{i,t}$, and propagated to other banks through the network.

The network-dependent valuation of liquidity can be motivated by an information spillover mechanism. Even though banks may value liquidity differently (due to *private* value), neighbours' liquidity holdings can be informative about the *common* value of reserves. A sufficient statistics from the aggregate trading value is $\delta \sum_j g_{ij,t} z_{j,t}$ where the weights are network weights. This updating rule can be fully rational or in the spirit of opinion formation under bounded rationality (DeGroot (1974); DeMarzo, Vayanos and Zwiebel (2003)).[15] A larger coefficient δ reflects a stronger informational spillover effect. To sum up, we can interpret interbank linkages in our model channelling both fund and information spillover effects.

The bilateral network influences are captured by the following cross derivatives for $i \neq j$:

$$\frac{\partial^2 u_i(\mathbf{z}_t|g_t)}{\partial z_{i,t} \partial z_{j,t}} = (\delta - \gamma\psi) g_{ij,t}.$$

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[15]There is a growing literature on information aggregation in network settings (e.g., Babus and Kondor (2013)).

When the cross derivative is negative, i.e. when $\delta < \gamma\psi$, banks' liquidity holdings exhibit strategic substitution. That is, an individual bank sets aside a smaller amount of liquid assets when its neighbouring banks hold more liquidity, which it can draw upon. This reflects the typical free-riding incentive as in Bhattacharya and Gale (1987).[16] In our model, strategic substitutability arises from the decreasing marginal utility of liquidity, so the degree of strategic substitutability increases when either neighbours' liquidity is more accessible (i.e., higher ψ) or the marginal utility of overall liquidity declines faster (i.e., higher γ).

Strategic complementarity arises when $\delta > \gamma\psi$. Through our interbank network not only flows liquidity (via borrowing and lending) but also information on the common value of reserves. Strategic complementarity arises from such informational spillover. We would expect a higher δ, and stronger strategic complementarity, when the common value of reserves is more prominent than the private value. Even if we restrict the network to be only relevant for fund flows rather than information, strategic complementary may still arise from leverage stack as in Moore (2012). In Appendix A.2.1, we lay out an alternative specification of banks' objective function in such spirit, which leads to the same set of equilibrium conditions.

Equilibrium. We solve banks' optimal reserve holdings in the Nash equilibrium of simultaneous action. Bank i's best response is

$$z_{i,t}^* = \mu_{i,t} + \phi \sum_j g_{ij,t} z_{j \neq i,t}, \tag{4}$$

where $\phi := \delta/\gamma - \psi$, and $\mu_{i,t} = \bar{\mu}_i + \nu_{i,t}$, defined earlier in equation (3). The "network attenuation factor" ϕ is the key parameter that determines the type of equilibrium on network: i.e., strategic substitution if $\phi < 0$ or complementarity if $\phi > 0$. We are agnostic about the the sign of ϕ and we instead estimate it empirically.

Note that the aggregate level of reserves held by banks is fully determined by banks' choices in the model (i.e., the demand side), without being constrained by the reserve supply of the central bank. This assumption of perfectly elastic reserve supply is consistent with the empirical context – under the UK monetary policy framework, the Bank of England accommodates banks' reserve demand to maintain its policy rate (Appendix A.1). Accordingly, we do not model the price of liquidity, but focus on the quantities chosen by banks. In our estimation, interbank rate is controlled as one of the macro variables in $q_{i,t}$.

Proposition 1 *Suppose that $|\phi| < 1$. Then, there is a unique interior solution for the*

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[16]Bhattacharya and Gale (1987) show that banks' liquidity holdings are strategic substitutes, because liquidity holdings come at a cost of forgoing higher return from long-term investments. Banks would like to free-ride their neighbours rather than conducting precautionary savings themselves.

individual equilibrium outcome given by

$$z_{i,t}^* (\phi, g) = \{\mathbf{M}(\phi, \mathbf{G}_t)\}_{i.} \, \mu_t, \tag{5}$$

where $\{\}_{i.}$ is the operator that returns the i-th row of its argument, $\mu_t := [\mu_{1,t}, ..., \mu_{n,t}]'$, $z_{i,t}$ denotes the network-dependent liquidity holdings of bank i, and

$$\mathbf{M}(\phi, \mathbf{G}_t) := \mathbf{I} + \phi \mathbf{G}_t + \phi^2 \mathbf{G}_t^2 + \phi^3 \mathbf{G}_t^3 + ... = \sum_{k=0}^{\infty} \phi^k \mathbf{G}_t^k = (\mathbf{I} - \phi \mathbf{G}_t)^{-1}. \tag{6}$$

where $\mathbf{I}$ is the $n \times n$ identity matrix.

Proof. Since $\gamma > 0$, the first order condition identifies the individual optimal response. Applying Theorem 1, part b, in Calvo-Armengol, Patacchini, and Zenou (2009) to our problem, the necessary equilibrium condition becomes $|\phi \lambda^{max}(\mathbf{G}_t)| < 1$ where the function $\lambda^{max}(\cdot)$ returns the largest eigenvalue. Since $\mathbf{G}_t$ is a stochastic matrix, its largest eigenvalue is 1. Hence, the equilibrium condition requires $|\phi| < 1$, and in this case the infinite sum in equation (6) is finite and equal to the stated result (Debreu and Herstein (1953)). ■

To gain intuition about the above result, note that a Nash equilibrium in pure strategies $\mathbf{z}_t^* \in \mathbb{R}^n$, where $\mathbf{z}_t := [z_{1,t}, ..., z_{n,t}]'$, is such that equation (4) holds for all $i = 1, 2, ..., n$. Hence, if such an equilibrium exists, it solves $(\mathbf{I} - \phi \mathbf{G}_t) \mathbf{z}_t = \mu_t$. If the matrix is invertible, we obtain $\mathbf{z}_t^* = (\mathbf{I} - \phi \mathbf{G}_t)^{-1} \mu_t$. The rest follows by simple algebra. The condition $|\phi| < 1$ states that network externalities must be small enough in order to prevent the feedback triggered by such externalities to escalate without bounds.

The matrix $\mathbf{M}(\phi, \mathbf{G}_t)$ has an important economic interpretation: it aggregates all direct and indirect links among banks using an attenuation factor, ϕ, that penalises (as in Katz (1953)) the contribution of links between distant nodes at the rate ϕ^k, where k is the length of the path between nodes. In the infinite sum in equation (6), the identity matrix captures the (implicit) link of each bank with itself, the second term in the sum captures all the direct links between banks, the third term in the sum captures all the indirect links corresponding to paths of length two, and so on. The elements of $\mathbf{M}(\phi, \mathbf{G}_t)$, given by $m_{ij}(\phi, \mathbf{G}_t) := \sum_{k=0}^{+\infty} \phi^k \{\mathbf{G}_t^k\}_{ij}$, aggregates all paths from j to i, where the kth step is weighted by ϕ^k.

In equilibrium, the matrix $\mathbf{M}(\phi, \mathbf{G}_t)$ contains information about the *centrality* of network players. Multiplying the rows (columns) of $\mathbf{M}(\phi, \mathbf{G}_t)$ by a unit vector of conformable dimensions, we recover the indegree (outdegree) Katz–Bonacich centrality measure.[17] The

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[17] Newman (2004) shows that weighted networks can in many cases be analysed using a simple mapping from a weighted network to an unweighted multigraph. Therefore, the centrality measures developed for unweighted networks apply also to the weighted cases.

indegree centrality measure provides the weighted count of the number of ties directed to each node (i.e., inward paths), while the outdegree centrality measure provides the weighted count of ties that each node directs to the other nodes (i.e., outward paths). That is, the i-th row of $\mathbf{M}(\phi, \mathbf{G}_t)$ captures how bank i loads on the network as whole, while the i-th column of $\mathbf{M}(\phi, \mathbf{G}_t)$ captures how the network as a whole loads on bank i.

To see clearly how the network propagates idiosyncratic shocks, we write equation (5) as,

$$z_t^* = \underbrace{\mathbf{M}(\phi, \mathbf{G}_t)\bar{\mu}}_{\text{level effect}} + \underbrace{\mathbf{M}(\phi, \mathbf{G}_t)\nu_t}_{\text{risk effect}}. \tag{7}$$

The matrix $\mathbf{M}(\phi, \mathbf{G}_t)$ (which includes the network topology and the network attenuation factor ϕ) is *not* enough to determine the systemic importance of a bank. Banks' equilibrium reserve holdings depend on both $\mathbf{M}(\phi, \mathbf{G}_t)$ and μ_t, and μ_t load on bank-specific shocks ν_t. $\mathbf{M}(\phi, \mathbf{G}_t)$ governs the propagation – banks weight its own shock, and through $\mathbf{M}(\phi, \mathbf{G}_t)$, the shocks to neighbouring and centrally located bank more heavily. But, regardless of how shocks are propagated, banks with large liquidity shocks (i.e., large σ_i^2) have a large influence on the other banks' liquidity holdings. This equilibrium equation guides our empirical estimation of the mechanism that propagates idiosyncratic shocks via network $\mathbf{G}_t$. Especially, it gives us a structure to estimate ϕ and a set of σ_i.

The planner's solution. The model captures not only the shock amplification mechanism through the network but also the externalities. Individual banks make their own liquidity provision decisions without internalising the impact on other banks' utilities. We proceed to a formal analysis of planner's problem in this interconnected system to analyse the wedge between the decentralised equilibrium and social optimum. We aim to quantify this wedge using empirical estimates to help regulators with their macro-prudential policy-making decisions. Specially, we model a planner that equally weights the utility of each bank (in equation (2)) choosing the liquidity holdings by solving the following problem:

$$\max_{\{z_{i,t}\}_{i=1}^n} \sum_{i=1}^n u_i(z_{i,t}|\mathbf{G}_t, \{z_{j,t} : j \neq i\}). \tag{8}$$

The first order condition for the liquidity holding of the i-th bank ($z_{i,t}$) yields

$$\underbrace{z_{i,t} = \mu_{i,t} + \phi \sum_{j \neq i} g_{ij,t} z_{j,t}}_{\text{decentralised f.o.c.}} + \psi \sum_{j \neq i} g_{ji,t}\mu_{j,t} + \phi \sum_{j \neq i} g_{ji,t} z_{j,t} - \psi\left(\psi - \frac{2\delta}{\gamma}\right)\sum_{j \neq i}\sum_{m \neq j} g_{ji,t} g_{jm,t} z_{m,t}$$

$$\tag{9}$$

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The first two (indegree) terms are the same as in the decentralised case, while the last three (outdegree) terms reflect that the planner internalizes banks' impact on their neighbours' utilities. The third term arises from the fact that for one unit of liquidity held by bank i, neighbour j can draw $\psi g_{ji,t}$ units. Here, neighbour j values this liquidity at $\mu_{j,t}$, the standalone valuation absent of network impact. The fourth term reflects bank i's impact on the network-dependent part of j's liquidity valuation. When i holds more liquidity, the informational spillover increases j's valuation of liquidity, but because j can borrow more from i, j's marginal value of liquidity holdings changes. The overall impact is summarized by ϕ and weighted by the outbound link $g_{ji,t}$. The fifth term captures the interaction effect of these forces. Rewriting equation (9) in matrix form, we obtain $\mathbf{z}_t = (\mathbf{I} + \psi \mathbf{G}'_t) \mu_t + \mathbf{P}(\phi, \psi, \delta, \mathbf{G}_t) \mathbf{z}_t$ where $\mathbf{P}(\phi, \psi, \delta, \mathbf{G}_t) := \phi (\mathbf{G}_t + \mathbf{G}'_t) - \psi (\psi - 2\delta/\gamma) \mathbf{G}'_t \mathbf{G}_t$. This allows us to formally state the planner's solution.

Proposition 2 *Suppose* $|\lambda^{max}(\mathbf{P}(\phi, \psi, \delta, \mathbf{G}_t))| < 1$. *Then, the planner's optimal solution is uniquely defined and given by*

$$z_i^p(\phi, \psi, \delta, g_t) = \{\mathbf{M}^p(\phi, \psi, \delta, \mathbf{G}_t)\}_{i.} \mu_t, \tag{10}$$

where $\mathbf{M}^p(\phi, \psi, \delta, \mathbf{G}_t) := [\mathbf{I} - \mathbf{P}(\phi, \psi, \delta, \mathbf{G}_t)]^{-1} (\mathbf{I} + \psi \mathbf{G}'_t)$.

Proof. The proof follows the same argument as in the proof of Proposition 1. ∎

Equation (10) directly guides our empirical analysis, especially the comparison between the decentralised outcome and the planner's solution. Rewriting the first order condition, we solve the wedge between the decentralised equilibrium (z^*) and the planner's solution (z^p):

$$\mathbf{z}_t^p - \mathbf{z}_t^* = \mathbf{M}(\phi, \mathbf{G}_t) \left[\psi \mathbf{G}'_t \mu_t + \left(\phi \mathbf{G}'_t - \psi \left(\psi - \frac{2\delta}{\gamma} \right) \mathbf{G}'_t \mathbf{G}_t \right) \mathbf{z}_t^p \right]. \tag{11}$$

The extra terms (in the square brackets) arise from individual banks' failure to internalize the network externalities they generate. The intuition is similar to that of equation (9). The first term arises from a bank's contribution to neighbours' liquidity through the network (valued at μ_t absent of network effects). The second term reflects a bank's impact on neighbours' network-dependent valuation of liquidity. The last term is an interaction effect.[18] The right-hand side of Equation (11) still contains $\mathbf{z}_t^p$, so in Appendix A.2.4, we provide the closed-form solution of the wedge that depends on the parameters and $\mathbf{G}_t$, topology of the network.

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[18] $\phi \mathbf{G}'_t - \psi \left(\psi - \frac{2\delta}{\gamma} \right) \mathbf{G}'_t \mathbf{G}_t$ vanishes only in the unlikely case of $\frac{\phi}{\psi(\psi - \frac{2\delta}{\gamma})}$ being an eigenvalue of $\mathbf{G}_t$.

III Empirical Methodology

In order to estimate the network model presented in Section II, we map the equilibrium conditions of our theoretical model to a spatial error model (SEM). That is, we decompose the total bank liquidity holdings into a function of the observables and a latent term that captures the spatial dependence generated by the network:

$$l_{i,t} = \alpha_t^{week} + \alpha_i^{bank} + \sum_{m=1}^{M} \beta_m^{bank} x_{i,t}^m + \sum_{p=1}^{P} \beta_p^{macro} x_t^p + z_{i,t} \tag{12}$$

$$z_{i,t} = \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{ij,t} z_{j,t} + \nu_{i,t} \sim iid\left(0, \sigma_i^2\right), \ i = 1, ..., n, \quad t = 1, ..., T. \tag{13}$$

We estimate the model using daily data, and include week fixed effects to control for unobserved macro factors. Week fixed effects, bank fixed effects, and bank-level and macro control variables together constitute $q_{i,t}$, the part of liquidity holdings absent of any network effects. On day t, the network is predetermined, and enters into $z_{i,t}$ through the residual equation (13). $g_{ij,t}$ is the fraction of bank i's borrowing from bank j (average over the month up to t). Control variables are lagged by one day for predeterminancy. The estimate of ϕ reveals the type of equilibrium on network, i.e., strategic substitution or complementarity.[19]

Defining $\epsilon_{i,t}$ as the demeaned version of $z_{i,t}$, we have $\sum_{j=1}^{n} g_{ij,t}\epsilon_{j,t}$ as a standard spatial lag term and ϕ being the canonical spatial autoregressive parameter. That is, the model in Equations (12) and (13) is a variation of Anselin (1988) spatial error model (see also Elhorst (2010a, 2010b)). As outlined in the Appendix A.3.1, we estimate the model using a quasi-maximum likelihood approach. To exhibit variation in ϕ and allow for changes in $\{\sigma_i\}_{i=1}^{n}$, we estimate the model in subsamples and rolling windows.

This specification makes clear the nature of network as a shock propagation mechanism: a shock to bank j is transmitted to bank i through $\phi g_{ij,t}$, so if $\phi > 0$ (strategic complementarity), the network amplifies shocks, and if $\phi < 0$ (strategic substitution), the network buffers shocks. The ultimate impact of shocks to all banks is

$$\epsilon_t = (\mathbf{I} - \phi\mathbf{G}_t)^{-1} \nu_t \equiv \mathbf{M}(\phi, \mathbf{G}_t) \nu_t \tag{14}$$

where $\epsilon_t = [\epsilon_{1,t}, ..., \epsilon_{n,t}]'$, and the structural shocks $\nu_t = [\nu_{1,t}, ..., \nu_{n,t}]$. As shown by equation (6), $\mathbf{M}(\phi, \mathbf{G}_t)$ records the routes that propagates ν_t with the direction governed by ϕ.

We can define $(1 - \phi)^{-1}$ as the "average network multiplier". If $\mathbf{G}_t$ is a right stochastic matrix (i.e., $\mathbf{G}_t \mathbf{1} = \mathbf{1}$), which this is the case here since we model $g_{ij,t}$ as the fraction of

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[19]To exhibit potential time variation in ϕ, we perform estimations in subsamples and rolling windows.

borrowing by i from j, then a unit shock to the system equally spread across banks (i.e., $\nu_t = (1/n)\,\mathbf{1}$) would have an ultimate impact on aggregate liquidity equal to $(1-\phi)^{-1}$.[20]

An estimation issue for network models is the well-known reflection problem (Manski (1993)): the neighbouring banks' decisions about their liquidity holdings affect each other, so that we cannot distinguish between whether a given bank's action is the cause or the effect of its neighbouring banks' actions. To address this problem, Bramoullé, Djebbari and Fortin (2009) have shown that the network effect ϕ can be identified if there are two nodes in the network with different average connectivities of their direct connected nodes. This condition is satisfied in our data.[21]

With the estimated parameters at hand, we identify the key contributors of systemic risk ("risk key players"). First, we can decompose the *aggregate* liquidity buffer into a level effect and a risk effect: the total network-dependent liquidity, $Z_t := \sum_i z_{i,t}$, can be written in equilibrium as

$$Z_t^* = \underbrace{\mathbf{1}'\mathbf{M}\left(\phi, \mathbf{G}_t\right)\bar{\mu}}_{\text{level effect}} + \underbrace{\mathbf{1}'\mathbf{M}\left(\phi, \mathbf{G}_t\right)\nu_t}_{\text{risk effect}} \tag{15}$$

where $\bar{\mu} := [\bar{\mu}_1, ..., \bar{\mu}_n]'$, $\nu_t := [\nu_{1,t}, ..., \nu_{n,t}]'$. The first term captures the network level effect, and the second captures the risk effect by aggregating idiosyncratic shocks. Note that even when N is large, idiosyncratic shocks may not vanish in aggregation because of the network effects in $\mathbf{M}\left(\phi, \mathbf{G}_t\right)$ (similar to Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012)).

We measure risk by the conditional volatility of the aggregate liquidity, so we simply work with the demeaned liquidity:

$$Var_t\left(\mathbf{1}'\epsilon_t\right) = \mathbf{1}'\mathbf{M}\left(\phi, \mathbf{G}_t\right)\Sigma_\nu \mathbf{M}\left(\phi, \mathbf{G}_t\right)'\mathbf{1}.$$

Here we have used the fact that $\mathbf{G}_t$ is predetermined with respect to time-t information. The covariance matrix of structural shocks, $\Sigma_v := \mathbb{E}\left[\nu_t\nu_t'\right]$, is a diagonal matrix with the variances, $\{\sigma_i^2\}_{i=1}^n$, on the main diagonal. To identify risk key players, we define the network impulse response function as follows.

Definition 1 (Network Impulse Response Function) *Let $L_t \equiv \sum_{i=1}^N l_{i,t}$ denote the aggregate liquidity in the interbank network. The network impulse response function of aggre-*

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[20]From $\mathbf{1} = (\mathbf{I} - \phi\mathbf{G}_t)^{-1}(\mathbf{I} - \phi\mathbf{G}_t)\,\mathbf{1} = (\mathbf{I} - \phi\mathbf{G}_t)^{-1}\mathbf{1}\,(1-\phi)$, we have $\mathbf{M}\left(\phi, \mathbf{G}_t\right)\mathbf{1} = (1-\phi)^{-1}\mathbf{1}$.

[21]The separate identification of $\bar{\mu}_i$ and α_i^{bank} is more complex, and is discussed in Appendix A.3.1. In particular, when $\mathbf{G}_t$ is a right stochastic matrix, the identification of $\bar{\mu}_i$ and α_i^{bank} requires at least one bank to not borrow from any other bank at some point in the sample (in our data, this happens 13.5% of the time spread over all subsamples and rolling windows we consider). Alternatively, one can normalise one μ_i to zero and identify the remaining ones as deviation from it. But note that the separate identification of these fixed effects does not affect the identification of ϕ that is estimated by maximizing the concentrated likelihood.

gate liquidity, L_t, to a one standard deviation shock to a bank i, is given by

$$NIRF_i\left(\phi, \sigma_i, \mathbf{G}_t\right) \equiv \frac{\partial L_t}{\partial \nu_{i,t}}\sigma_i = \mathbf{1}'\left\{\mathbf{M}\left(\phi, \mathbf{G}_t\right)\right\}_{.i}\sigma_i \qquad (16)$$

where the operator $\left\{\right\}_{.i}$ returns the i-th column of its argument.

The network impulse response is the shock-size weighted *outdegree* centrality of bank i. It measures a bank's contribution to the volatility of aggregate liquidity, and thus, identifies the risk key player by providing a clear ranking of the riskiness of each bank from a systemic perspective. It offers a natural decomposition of volatility, since

$$Var_t\left(\mathbf{1}'\epsilon_t\right) \equiv vec\left(\left\{NIRF_i\left(\phi, \sigma_i, \mathbf{G}_t\right)\right\}_{i=1}^{n}\right)' vec\left(\left\{NIRF_i\left(\phi, \sigma_i, \mathbf{G}_t\right)\right\}_{i=1}^{n}\right). \qquad (17)$$

A bank's risk contribution depends on the size of its own shock σ_i, the network attenuation factor, ϕ, and all the direct and indirect network links. As reminder, for $|\phi| < 1$,

$$\mathbf{1}'\left\{\mathbf{M}\left(\phi, \mathbf{G}_t\right)\right\}_{.i} = \mathbf{1}'\left\{\mathbf{I}+\phi\mathbf{G}_t+\phi^2\mathbf{G}_t^2+...\right\}_{.i} = \mathbf{1}'\left\{\sum_{k=0}^{\infty}\phi^k\mathbf{G}_t^k\right\}_{.i}$$

where the initial element in the series captures direct effects of a unit shock to bank i, the next element is the sum of first-order outbound links, the third element is the sum of second-order outbound links, and so on.

We can isolate the pure network-driven of impulse response, that is, the impact beyond direct effects of bank-level shocks (which we call the "excess NIRF"):

$$NIRF_i^e\left(\phi, \sigma_i, \mathbf{G}_t\right) \equiv NIRF_i\left(\phi, \sigma_i, \mathbf{G}_t\right) - \sigma_i. \qquad (18)$$

The sign of $NIRF_i^e\left(\phi, \sigma_i, \mathbf{G}_t\right)$ depends on the type of equilibrium (strategic substitution or complementarity), i.e., the sign of ϕ. Note that it is straightforward to compute confidence bands for the estimated $NIRF$s using the delta method, since they are functions of $\hat{\phi}$ and $\{\hat{\sigma}_i\}_{i=1}^{n}$ that have canonical asymptotic Gaussian distribution (see the Appendix A.3.2).

As in the decentralised solution, one can decompose the volatility of aggregate liquidity:

$$Var_t\left(Z^p\left(\phi, \psi, \delta, \mathbf{G}_t\right)\right) = \mathbf{1}'\mathbf{M}^p\left(\phi, \psi, \delta, \mathbf{G}_t\right)diag\left(\left\{\sigma_i^2\right\}_{i=1}^{n}\right)\mathbf{M}^p\left(\phi, \psi, \delta, \mathbf{G}_t\right)'\mathbf{1}, \qquad (19)$$

where $\mathbf{M}^p\left(\phi, \psi, \delta, \mathbf{G}_t\right)$ is defined in Proposition 2.

Similarly, we can identify "systemic level key player", whose removal from the system causes the largest reduction of aggregate liquidity level in expectation.[22] A key input is the

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[22]This definition is in the same spirit as the concept of key player in the crime network literature, e.g.,

average liquidity valuation, i.e., $\bar{\mu}$. Since our empirical analysis focuses on shock propagation through the network instead of the sample average of liquidity level, we present the theoretical results on level key player in Appendix A.2.3).

Finally, as a specification test of our model, we consider a more general specification that allows for richer network interactions. That is, we model liquidity holding as a spatial Durbin model (SDM – see, e.g. LeSage and Pace (2009)) where a bank's liquidity depends directly on other banks' liquidity and pairwise control variables

$$l_{i,t} = \alpha_t^{week} + \alpha_i^{bank} + \sum_{m=1}^{M} \beta_m^{bank} x_{i,t}^m + \sum_{p=1}^{P} \gamma_p^{macro} x_t^p +$$

$$+ \rho \sum_{j=1}^{n} g_{i,j,t} l_{j,t} + \sum_{j=1}^{n} g_{i,j,t} x_{i,j,t} \theta + \nu_{i,t} \sim iid \left(0, \sigma_i^2 \right), \tag{20}$$

where $x_{i,j,t}$ denotes match-specific control variables that may include the characteristics of bank j.[23] The above formulation serves as a specification test of our structural model since, setting $x_{i,j,t} := vec(x_{j\neq i,t}^m)'$, and restricting $\theta = -\phi vec(\beta_m^{bank})$, $\gamma_p^{macro} = (1 - \phi) vec(\beta_p^{macro})$, and most importantly $\rho = \phi$, we are back to the benchmark specification. In Section V, we compare the estimates of SDM with those of our benchmark model (SEM)in rolling windows. Given the close mapping from our theoretical model to the spatial error model, the comparison can be viewed as a test of the theoretical model.

IV   Data Description

We study the liquidity holdings of banks that are members of CHAPS, the UK large-value payment system. These eleven banks are at the core of UK banking system, conducting transactions for their own purpose and on behalf of their clients and hundreds of non-member banks (more details in Appendix A.1).[24] Their liquidity holdings serve the critical

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Ballester, Calvo-Armengol, and Zenou (2006), where targeting key players is important for crime reduction.

[23]In Appendix A.2.2 we show that the above formulation is the equilibrium outcome of a network game.

[24]The banks are Halifax Bank of Scotland (owned by Lloyds Banking Group), Barclays, Citibank, Clydesdale (owned by National Australia Bank), Co-operative Bank (owned by The Co-operative Group), Deutsche Bank, HSBC (that acquired Midland Bank in 1999 – one of the historical "big four" sterling clearing banks), Lloyds TSB, Royal Bank of Scotland (including Natwest), Santander (formerly Abbey, Alliance & Leicester and Bradford & Bingley, owned by Banco Santander of Spain), and Standard Chartered. For most of the 20th Century, the phrase "the Big Four" referred to the four largest sterling banks, which acted as clearing houses for bankers' cheques. These were Barclays Bank, Midland Bank (now part of HSBC), Lloyds Bank (now Lloyds TSB Bank and part of Lloyds Banking Group); and National Westminster Bank ("NatWest", now part of The Royal Bank of Scotland Group). Currently, the largest four U.K. banks are Barclays, HSBC, Lloyds Banking Group, and The Royal Bank of Scotland Group, closely followed by Standard Chartered s– and all of these banks in our sample.
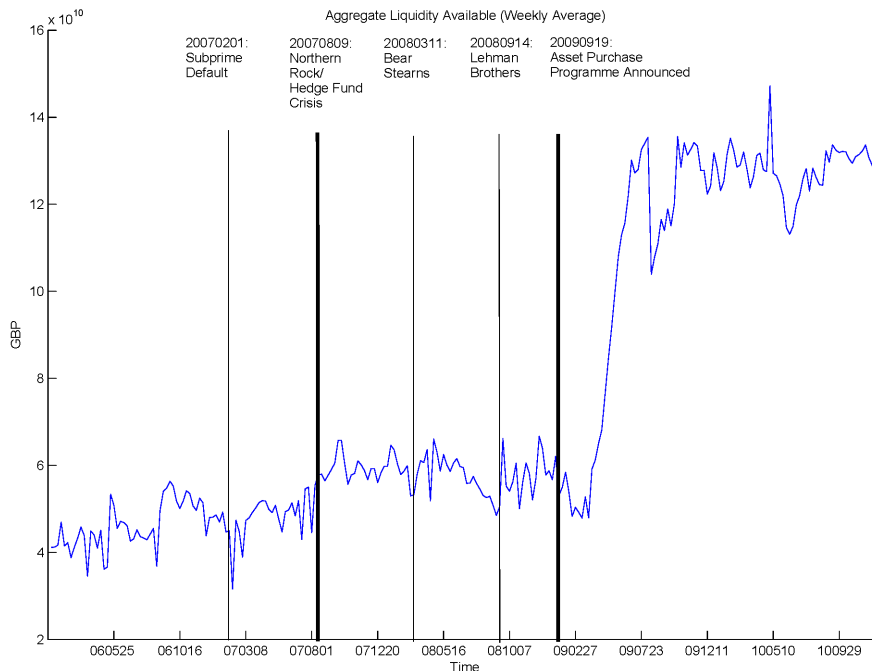
Figure 1: Weekly average of aggregate liquidity available at the beginning of the day.

purpose of buffering intraday flow imbalance, ensuring the functioning of transaction system for the whole economy. Our sample covers the period from January 2006 to September 2010, which allows us to estimate the network effect in the pre-crisis period, during the financial crisis, and later in the era of quantitative easing. We focus on the daily variation of liquidity holdings, so the model is estimated at daily frequency while incorporates variables constructed from higher frequency data.

Liquidity holdings. To measure the dependent variable $l_{i,t}$, that is, the liquidity holdings of each bank, we use central bank reserve holdings (logarithm). We supplement this with the collateral that is repo'ed with the Bank of England in return for intraday liquidity (these repos are unwound at the end of each working day).

The weekly average of aggregate liquidity in the system (the sum of banks' holdings) is reported in Figure 1. The figure shows a substantial upward trend in the period after the U.S. subprime mortgage crisis that saw several market disruptions. This is consistent with the evidence that banks hoard liquidity in crisis (e.g., Acharya and Merrouche (2010)), but this upward trend is dwarfed by the steep run-up in response to the Asset Purchase Programme (aka the Quantitative Easing) that almost tripled the aggregate liquidity. As shown in Figure 10 in the Appendix, this sharp increase in liquidity was associated with a reduction in the velocity, i.e., the ratio of payments to aggregate liquidity. Note that since we include week fixed effects and macro control variables, this trend is unlikely to affect our

estimate of ϕ, and in particular, induce a positive bias in $\hat{\phi}$ due to trend-induced comovement (i.e., spurious strategic complementarity in banks' liquidity holdings).

Interbank network. We construct the interbank network G_t using interbank borrowing data that we extract from overnight interbank payments using the Furfine (2000) algorithm. This algorithm is a common approach in the literature on interbank money market, It identifies pairs of payments between two banks where the outgoing payment are loans and the incoming payment are repayments (equal to the outgoing payment plus an interest rate). It has been tested thoroughly, and accurately tracks the LIBOR rate.[25] Furfine (2000) showed that when applied to Fedwire data, the algorithm accurately identifies the Fed Funds rate.[26]

The loan data are compiled to from an interbank lending and borrowing network. In particular, the element $g_{ij,t}$ of the adjacency matrix $\mathbf{G}_t$ is given by the *average* fraction of bank's i overnight loans from bank j in the previous month ending on day $t-1$.

By construction, $\mathbf{G}_t$ is a square right stochastic matrix. Its largest eigenvalue is therefore equal to one. This implies that the strength of shock propagation on the network depends the second largest eigenvalue of $\mathbf{G}_t$, which is plotted in Figure 2.[27] There was a substantial increase in the crisis period, but what is striking is the large variation of network topology after QE. The variation of G_t is critical for us to empirically identify the network parameters.

Another way to exhibit the variation of G_t is to plot an measure of network cohesiveness, for which we use the Average Clustering Coefficient (ACC – see Watts and Strogatz (1998))

$$ACC_t = \frac{1}{n}\sum_{i=1}^{n} CL_i(\mathbf{G_t}), \quad CL_{i,t} = \frac{\#\{jk \in \mathbf{G}_t \mid k \neq j,\, j \in n_i(\mathbf{G_t}),\, k \in n_i(\mathbf{G_t})\}}{\#\{jk \mid k \neq j,\, j \in n_i(\mathbf{G}_t),\, k \in n_i(\mathbf{G}_t)\}}$$

where $n_i(\mathbf{G}_t)$ is the set of players that have a direct link with i and $\#\{.\}$ is the count operator. The numerator is the number of pairs linked to i that are also linked to each other, while its denominator is simply the number of pairs linked to i. Therefore, ACC measures the

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[25]The data are only available for CHAPS banks. Thus, some loans may be attributed to a settlement bank when in fact the payments are made on behalf of its customers. Moreover, where a loan is made between one customer of a settlement bank and another, this transaction will not be settled through the payment system but rather across the books of the settlement bank (internalisation). Internalised payments are invisible to the BoE, so they are a part of the overnight money market that is not captured here.

[26] As documented in Armantier and Copeland (2012), the Furfine's algorithm can be affected by Type I and, to a lesser extent, Type II, errors. Nevertheless, this is less of a concerns in our application since: first, as documented in Kovner and Skeie (2013), at the overnight frequency that we focus on, interbank exposures measured by the algorithm are highly correlated with the Fed funds borrowing and lending reported in bank quarterly regulatory filings; second, and more importantly, instead of using the daily borrowing and lending data, we smooth these exposures by computing rolling monthly averages, therefore greatly reducing the relevance of false positives and negatives in the identification of interbank relationships. Furthermore, we apply several robustness checks on our measure of interbank linkages (results available upon request).

[27]This is because $\mathbf{G}^k$ can be rewritten in Jordan normal form as $\mathbf{PJ}^k\mathbf{P}^{-1}$ where $\mathbf{J}$ is the (almost) diagonal matrix with eigenvalues (or Jordan blocks in case of repeated eigenvalues) on the main diagonal.
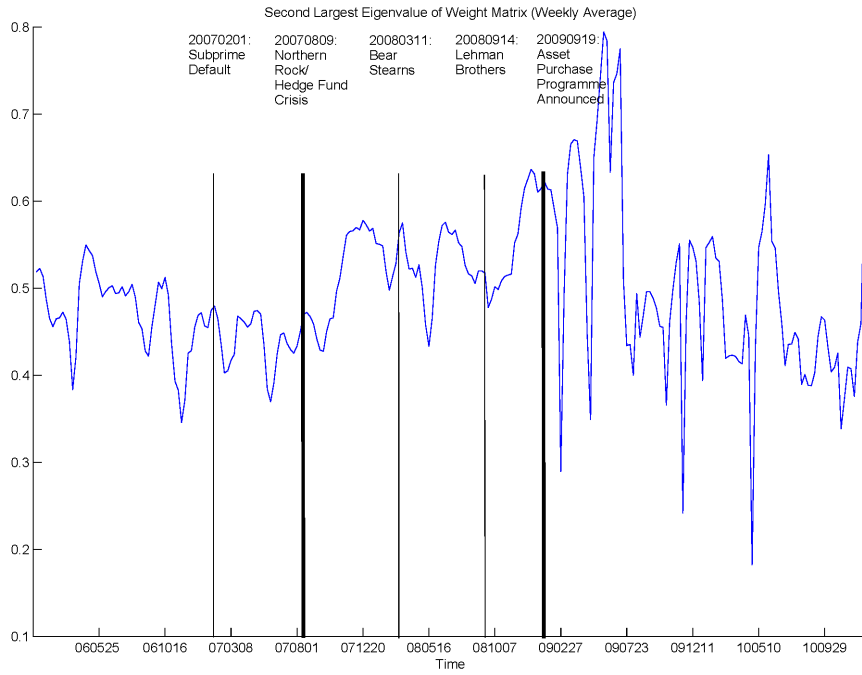
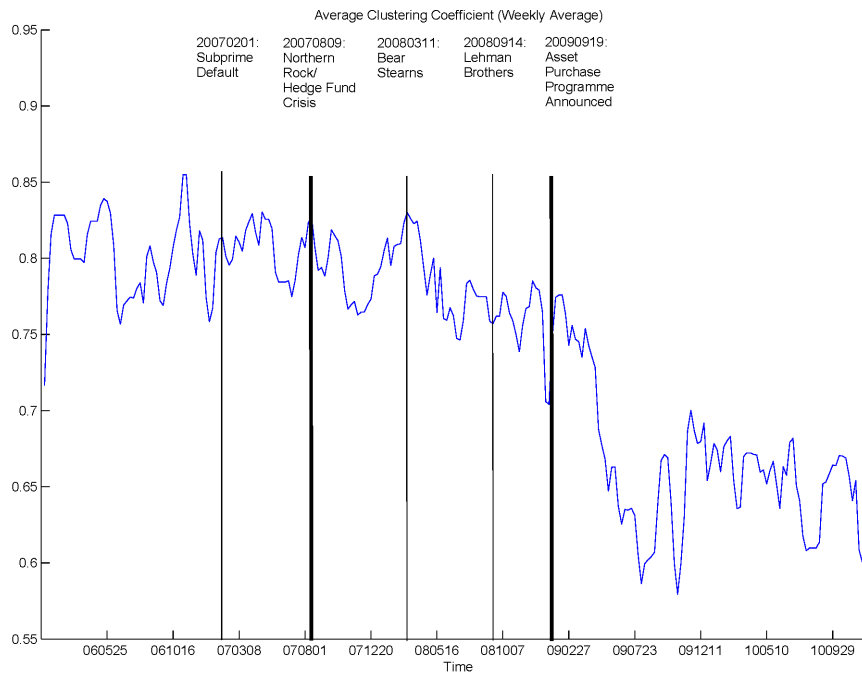Figure 2: Second largest eigenvalue of $\mathbf{G}_t$.

Figure 3: Average clustering coefficient of the interbank network.

average proportion of banks that are connected to i and also connected with each other. By construction it ranges from 0 to 1. A higher value means that the network is more dense.

The time series of ACC is shown in Figure 3. At the beginning of our sample, the

network is highly cohesive since, on average, around 80% of pairs of banks connected to any given bank are also connected to each other. The degree of connectedness seems to have a decreasing trend during 2007–2008, and a substantial and sudden decrease following the Asset Purchase Programme, when ACC dropped by about one-quarter of its pre-crises average. This is related to the reduced interbank borrowing needs during the QE period thanks to the availability of additional reserves from the Bank of England (combined with a move towards increased collateralisation of borrowing and an overall deleveraging, see, e.g. Westwood (2011)). This interpretation is consistent with Figure 9 in the Appendix, the monthly rolling average of daily sterling value of gross borrowing in the network.

Macro control variables. To control for aggregate liquidity condition, we use the LIBOR rate as a proxy for funding cost together with the interbank rate premium (average overnight borrowing rate of the CHAPS banks minus the LIBOR rate).[28] All control variables are lagged by one day so that they are predetermined with respect to time t shocks.

Since banks' decisions to hold liquidity are likely to be influenced by the volatility of their daily payment outflows, we construct a measure of the intraday payments volatility as

$$VolPay_t = \sqrt{\frac{1}{88} \sum_{\tau=1}^{88} \left(P_{t,\tau}^{out}\right)^2} \tag{21}$$

where P^{out} denotes payment outflows and 88 is the number of ten-minute time intervals (the unit of time for payment recording in our sample) within a day. The time series is plotted in Figure 11 in the Appendix. Outflow volatility declined steadily throughout the crisis, suggesting that banks in aggregate smoothed intraday outflow.

We also control for the turnover rate in the payment system. This variable is defined as

$$TOR_t = \frac{\sum_{i=1}^{N} \sum_{\tau=1}^{88} P_{i,t,\tau}^{out}}{\sum_{i=1}^{N} \max \left\{ \max_{\tau \in [1,88]} \left[CNP(\tau; i, t)\right], 0 \right\}}$$

where the cumulative net debit position (CNP) is defined as the difference between payment outflows and inflows (see also Benos, Garratt, and Zimmerman (2010)). The numerator is the total payments in day t, while the denominator is the sum of maximum intraday net debt positions of all banks. The time series is plotted in Figure 12 in the Appendix. The turnover rate increased during the crisis period, and declined after the introduction of QE.

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[28]LIBOR is the average of borrowing rates reported by selected banks, not CHAPS banks. Interbank rate premium can be positively or negatively correlated with banks' liquidity holdings. First, when CHAPS banks face more risks, they may hold more liquidity and face higher borrowing costs. Second, interbank rate premium measures an opportunity cost – CHAPS banks can borrow to lend at LIBOR rate rather than hold reserves. So, when LIBOR is high (interbank rate premium low), banks prefer to hold less liquidity.

Since banks have some discretion on the timing of intraday outflows, they could behave strategically – to preserve liquidity, banks may expedite inflows and delay outflows. Therefore, we control for the right kurtosis (rK_t) of intraday payment time.[29] The time series is plotted in Figure 13 in the Appendix, showing a substantial increase in the QE period.

Beyond these control variables at daily frequency, we add week fixed effects to account for potential missing variables that fluctuate at lower frequencies, such as monetary policy conditions beyond the interbank rates and real economic activities that drive payment flows.

Bank characteristics. Despite the fact that we control for average interest rates, we also control for bank-specific overnight borrowing rate, which is a daily volume-weighted average. As shown by Figure 14 in the Appendix, there was a substantial increase in the cross-sectional dispersion of the overnight borrowing rates during turmoil periods, such as the collapses of Northern Rock and Lehman Brothers. This cross-sectional dispersion persisted during the QE period. Therefore, it is critical to account for the heterogeneity in banks' overnight borrowing rates. As macro variable, all bank-level control variables are lagged by one day.[30]

We also control for other bank-level variables: right kurtosis of intraday payment inflow time $(rK_{i,t}^{in})$ and outflow time $(rK_{i,t}^{out})$; the level of intraday payment outflow $(LevPay_{i,t} \equiv \sum_{\tau=1}^{88} P_{i,t,\tau}^{out})$; the volatility of intraday payment outflow $(VolPay_{i,t}$, constructed as in equation (21) using bank-level flows); the liquidity used $(LU_{i,t}$, as in Benos, Garratt, and Zimmerman (2010) and defined in the Appendix); repo liabilities to total assets ratio; the change of retail deposits to total assets ratio; the total interbank lending and borrowing; an index (cumulative change) of 5-year credit default swap (CDS) spread; daily stock return.

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[29]We define right and left kurtosis (denoted, respectively, by rK_t and lK_t) as the fractions of kurtosis of payment times from, respectively, above and below the average payment time of the day:

$$rK_t = \frac{\sum_{\tau > m_s} (\frac{\tau - m_t}{\sigma_t})^4}{\sum_{\tau=1}^{88} (\frac{\tau - m_t}{\sigma_t})^4}; \quad lK_t = \frac{\sum_{\tau < m_t} (\frac{\tau - m_t}{\sigma_t})^4}{\sum_{\tau=1}^{88} (\frac{\tau - m_t}{\sigma_t})^4};$$

where m_t and σ_t are defined as flow-weighted average payment time and standard deviation, i.e.,

$$m_t = \frac{1}{88} \sum_{\tau=1}^{88} \tau \left(\frac{P_{t,\tau}^{OUT}}{\sum_{t=1}^{T} P_{t,\tau}^{OUT}} \right), \quad \sigma_t^2 = \frac{1}{88-1} \sum_{\tau=1}^{88} \left[\tau \left(\frac{P_{t,\tau}^{OUT}}{\sum_{t=1}^{88} P_{t,\tau}^{OUT}} \right) - m_t \right]^2.$$

[30]For variables available at lower than daily frequency (monthly), we use the latest lagged observation. These variables are: repo liabilities to asset ratio, total assets, and the ratio of retail deposits to total assets.

V Estimation Results

V.1 Subsample estimation

We estimate our model (equations (12) and (13)) in three subsamples of roughly equal size: the period before the Northern Rock and BNP Paribas Fund crisis (9 August 2007), the period after it and before QE (19 January 2009), and the QE period. These three periods are marked by distinct liquidity conditions, and as documented in Section IV, different behaviour of the network and other variables. Period 1 is a relatively tranquil period. Period 2 saw several significant events, such as the subprime mortgage fund crisis (e.g., BNP Paribas fund freezing on 9 August 2007), the run on Northern Rock (the UK's first in 150 years), the Federal Reserve intervention in Bear Stearns and its subsequent sale to JP Morgan Chase, and the bankruptcy of Lehman Brothers. Period 3 began with a regime switch in monetary policy. On 19 January 2009, it was announced that the BoE will purchase up to £50 billion in private assets, which marked the beginning of quantitative easing in the U.K.

The network multiplier. The estimation results for these three subsamples are reported in Panel A of Table 1, where we report only the estimates of the spatial dependency parameter ϕ (first row), the R^2 of the regression (second row), the implied average network multiplier (third row) $1/(1-\phi)$ that was discussed in Section III, as well as the ratio of the volatility of network liquidity to the counterfactual volatility when $\phi = 0$. Omitted from the table are the coefficient estimates of control variables, which are reported in Table A1 of the Appendix.

Recall that $\phi > 0$ (< 0) implies that banks' liquidity holding decisions are strategic complements (substitutes) and that this tends to amplify (reduce) the the impact of bank-level shocks to aggregate liquidity. In the first period, the estimate of ϕ is 0.8137 and highly significant, indicating a substantial network amplification effect: a £1 shock equally spread across banks would result in a $1/\left(1-\hat{\phi}\right) = $ £5.3677 shock to the aggregate liquidity.

In the second period, the coefficient ϕ is substantially lower in magnitude and marginally significant, implying weak strategic complementarity and an average network multiplier of about $1/\left(1 - 0.3031\right) = 1.4349$. This finding suggests that in response to the turbulence in financial markets that have characterised the second period, banks' liquidity management objective increasingly tilted away from responding to informational spillover and towards free riding neighbours, i.e., a shift from strategic complementarity to substitution.

In the third period, $\hat{\phi}$ becomes negative, -0.1794, and statistically significant, implying an average network multiplier of 0.8479. This is particularly interesting since strategic substitution became the dominant force in banks' liquidity holding decisions, as in Bhattacharya and Gale (1987). As a result, the network buffers the impact of shocks from individual banks on the aggregate liquidity. This finding also sheds light on how massive liquidity injection

Table 1: Spatial Error Model Estimation

	Period 1	Period 2	Period 3
Panel A: G_t based on borrowing			
$\hat{\phi}$	0.8137 (21.47)	0.3031 (1.90)	−0.1794 (−4.96)
R^2	66.01%	92.09%	91.53%
$1/\left(1-\hat{\phi}\right)$	5.3677 (4.92)	1.4349 (4.37)	0.8479 (32.61)
$\sqrt{\frac{Var(Z_t\|\hat{\phi})}{Var(Z_t\|\phi=0)}}$	5.59	1.25	0.89
Panel B: G_t based on lending			
$\hat{\phi}$	0.8209 (20.38)	0.2573 (1.23)	−0.3925 (−7.18)
R^2	66.02%	91.63%	91.61%
$1/\left(1-\hat{\phi}\right)$	5.5835 (4.45)	1.3464 (3.54)	0.7181 (25.49)
$\sqrt{\frac{Var(Z_t\|\hat{\phi})}{Var(Z_t\|\phi=0)}}$	5.71	1.45	0.85
Panel C: G_t based on borrowing and lending			
$\hat{\phi}$	0.8204 (19.36)	0.3258 (1.89)	−0.2824 (−6.10)
R^2	63.98%	92.22%	91.70%
$1/\left(1-\hat{\phi}\right)$	5.5679 (4.24)	1.3464 (3.91)	0.7181 (26.74)
$\sqrt{\frac{Var(Z_t\|\hat{\phi})}{Var(Z_t\|\phi=0)}}$	5.94	1.15	0.74

Estimation results for equations (12) and (13). Periods 1, 2 and 3, correspond, respectively, to before the Northern Rock/BNP Paribas Fund Crisis, after it but before the first BoE announcement of Asset Purchase Programme, and the QE period. The t-statistics are reported in parentheses under the estimated coefficients. Standard errors are QMLE-robust ones, and the delta method is used for for the average network multiplier, $1/(1-\hat{\phi})$. In Panel A, the adjacency matrix is computed using the interbank borrowing data, while in Panels B and C we use, respectively, lending and borrowing plus lending (all row-normalized).

by the central bank affects the network effect.[31] Overall, the model fits data fairly well in the three subsamples, with R^2 in the range of $66\% - 92\%$.

The last row of Panel A reports $\sqrt{Var(Z_t|\hat{\phi})/Var(Z_t|\phi=0)}$, i.e., the ratio of the volatility of aggregate liquidity implied by our estimate of ϕ to the counterfactual volatility if there were no network externalities. In the first period, the network multiplier generates a 459% increase in volatility. The excess volatility from network effects dropped to 25% in the crisis period, and turned negative, -11%, in the QE period. Table A2 of the Appendix reports $\sqrt{Var(z_{i,t}|\hat{\phi})/Var(z_{i,t}|\phi=0)}$, the network volatility multiplier for each bank.

For robustness, in Panels B and C, we estimate our network model with two alternative

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[31] Total liquidity injected from the Quantitative Easing program of the BoE was about £435 billion as of December 2017. See https://www.bankofengland.co.uk/monetary-policy/quantitative-easing.

constructions of the adjacency matrix G_t. In Panel B, we use the lending flows, while in Panel C, we use the combined borrowing and lending flows. In both cases, the adjacency matrix is row-normalized (right stochastic). Such an exercise is meaningful because, as we emphasized when constructing the theoretical model, network linkages reflect the interbank relationships, which may transmit information and/or liquidity. Thus, a linkage is not necessarily just about borrowing. If a bank lends to another bank, a relationship formed through this transaction may facilitates future borrowing or information transmission. Overall, the estimates in Panels B and C are very similar to those in Panel A.

Network impulse response and key players. Using the estimates, we compute the network impulse response functions to identify risk key players in each subsample. Results are reported together with banks' net borrowing amount and the network graph.

In the upper panel of Figure 4, we report each bank's excess network impulse response to a unit shock, i.e., $NIRF_i^e\left(\hat{\phi}, 1, \bar{\mathbf{G}}_1\right) = NIRF_i\left(\hat{\phi}, 1, \bar{\mathbf{G}}_1\right) - 1$, defined in equation (18), where $\bar{\mathbf{G}}_1$ denotes the average $\mathbf{G}_t$ in Period 1. It measures Bank i's contribution to systemic risk – the network-induced reaction of aggregate liquidity to a unit shock to Bank i. Note that if either $\phi = 0$ or there is no network linkages ($\bar{\mathbf{G}}_1 = \mathbf{0}$), a unit shock to Bank i is a unit shock to the aggregate liquidity, and thus, the excess response is zero. We also plot one and two standard deviation bands. As a point of reference, we show the *average* excess network multiplier, $(1 - \phi)^{-1} - 1 = 4.3677$ (Panel A of Table 1), i.e., how network as whole amplifies a unit shock equally spread across banks.

A key message from Figure 4 is that a small set of key players are responsible for systemic risk, i.e., the large network multiplier. A shock of £1 to Bank 5, 6 or 9 would generate an excess response of aggregate liquidity equal to £13.9, £8.9, and £13.8 respectively. A shock to Bank 4 would induce an excess response similar to the network average, while the remaining seven banks contribute relatively little to shock amplification.

The comparison between the upper and central panels makes clear that risk key players are not necessarily large net borrowers – large net borrowers and lenders are both likely to be key risk contributors. This is intuitive: a negative shock to a bank that lends to a large part of the network (high outdegree centrality) can be, for the aggregate liquidity buffer, as bad as a negative shock to a bank that borrows from many banks (high indegree centrality).

However, even if we consider both borrowing and lending amounts, it is still not enough to identify key players. For example, the risk contribution of Bank 5 would be underestimated. The reasons behind can be understood by looking at the lower panel of Figure 4, where we present the average network structure in Period 1. The size of ellipses identifying individual banks are (log) proportional to their average gross borrowing, incoming arrows to a node indicate borrowing flows while outgoing arrows indicate lending flows, and the thickness of
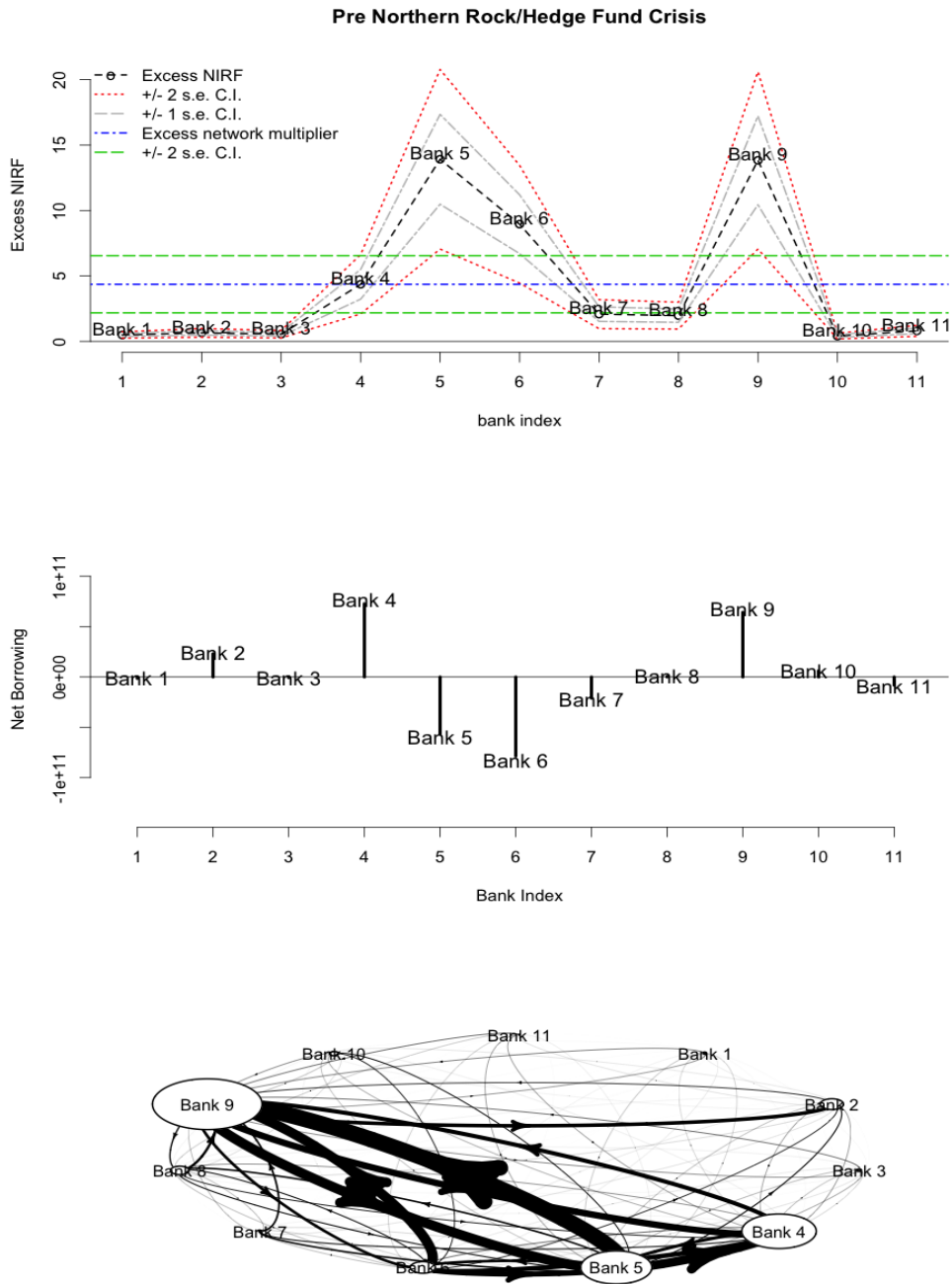
Figure 4: The period before the Northern Rock/Hedge Fund Crisis: Network excess impulse response functions to a unit shock (upper panel); net borrowing (central panel); borrowing and lending flows (lower panel) where the ellipses identifying individual banks are (log) proportional to their average gross borrowing, incoming arrows to a node indicate borrowing flows while outgoing arrows indicate lending flows, and the thickness of arrows is (log) proportional to the sterling value.

arrows is (log) proportional to the sterling value. It shows that key risk contributors tend to be banks with high centrality (e.g., Bank 5), i.e., with thick and many links, especially links to other well-connected banks, but not necessarily the large players by size.

Figure 5 reports excess impulse response functions (upper panel), average net borrowing (central panel), and network flows (lower panel) for Period 2 – the period characterised by a high degree of stress in the financial market. The first thing to notice is that despite the overall increase in borrowing and lending activities in the interbank market (see also Figure 9 in the Appendix), there is a drastic reduction in the average network multiplier reported in the top panel: the network-induced excess reaction to a unit shock is only about 0.43. In a crisis period, banks seem to have radically adjusted their liquidity management objectives, reflected by the estimate of ϕ, and they have done so despite having increased the utilization of interbank network to transfer liquidity. Nevertheless, systemic risk, even though substantially reduced, is still quite high and driven by a few key players. In particular, a unit shock to Bank 5, Bank 9 and Bank 6 trigger an excess reaction of aggregate liquidity equal to 1.77, 1.36 and 0.85 respectively, while a shock to Bank 4 has an average effect, and the remaining banks contribute little.

The results for Period 3 – the one starting at the onset of QE – are reported in Figure 6, and are radically different from the ones of the previous two periods. First, banks' liquidity holdings exhibit strategic substitution ($\hat{\phi} < 0$), and as a result, the network buffers shocks to individual banks, reflected in an average excess multiplier of -0.15 – a unit shock equally spread across banks would result in a shock of $1 - 0.15 = 0.85$ to the aggregate liquidity. But, once again, there is substantial heterogeneity among the banks, in the sense that most banks (1, 3, 7, 8, 10 and 11) contribute little to shock propagation, while a few key players (4, 5, 6, and 9) are responsible for the network buffering effect.

This behaviour arises in a period in which the degree of connectedness of the network was substantially reduced (see Figure 3 and the lower panel of Figure 6), the total borrowing had been substantially reduced (see Figure 9 in the Appendix), and most banks held net borrowing positions close to zero (central panel of Figure 6), but at the same time, the overall liquidity in the system had substantially increased, which is likely due to QE (Figure 1).

What is also interesting to notice is that the same banks that were the riskiest players in the previous two periods (Banks 5, 6 and 9) are now the least risky ones for the system. Thanks to their network centrality, and more importantly, the overall strategic substitution behaviour on the network, these banks become the biggest shock absorbers.

A natural question is whether we can explain the large heterogeneity of individual banks' contribution to system risk using banks' characteristics, and maybe find some proper indicators. Table 2 reports the rank correlations of individual bank characteristics with banks'
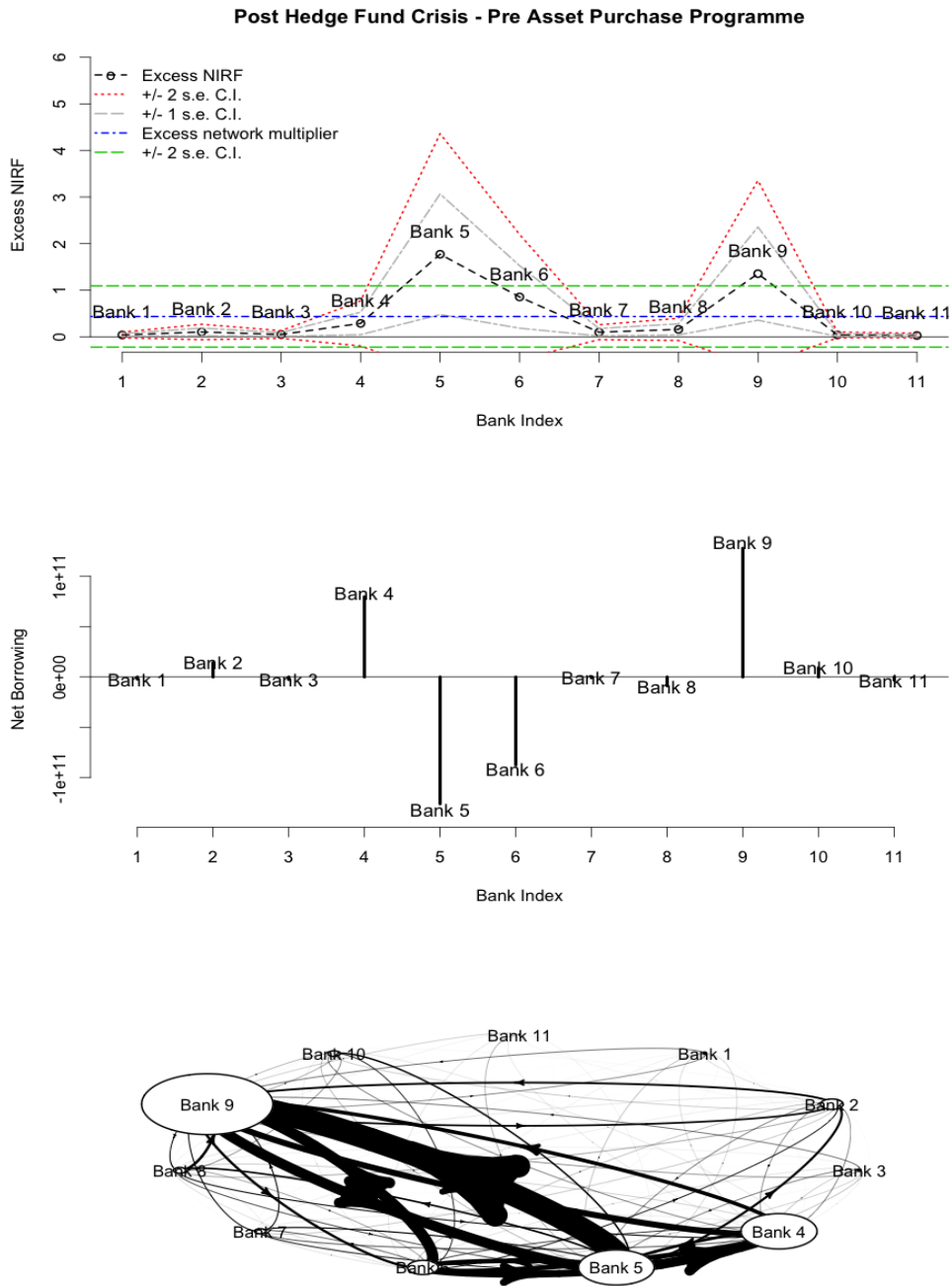
Figure 5: The period after the Norther Rock/Hedge Fund Crisis but before QE: Network excess impulse response functions to a unit shock (upper panel); net borrowing (central panel); borrowing and lending flows (lower panel) where the ellipses identifying individual banks are (log) proportional to their average gross borrowing, incoming arrows to a node indicate borrowing flows while outgoing arrows indicate lending flows, and the thickness of arrows is (log) proportional to the sterling value.

Figure 6: The QE period: Network excess impulse response functions to a unit shock (upper panel); net borrowing (central panel); borrowing and lending flows (lower panel) where the ellipses identifying individual banks are (log) proportional to their average gross borrowing, incoming arrows to a node indicate borrowing flows while outgoing arrows indicate lending flows, and the thickness of arrows is (log) proportional to the sterling value.
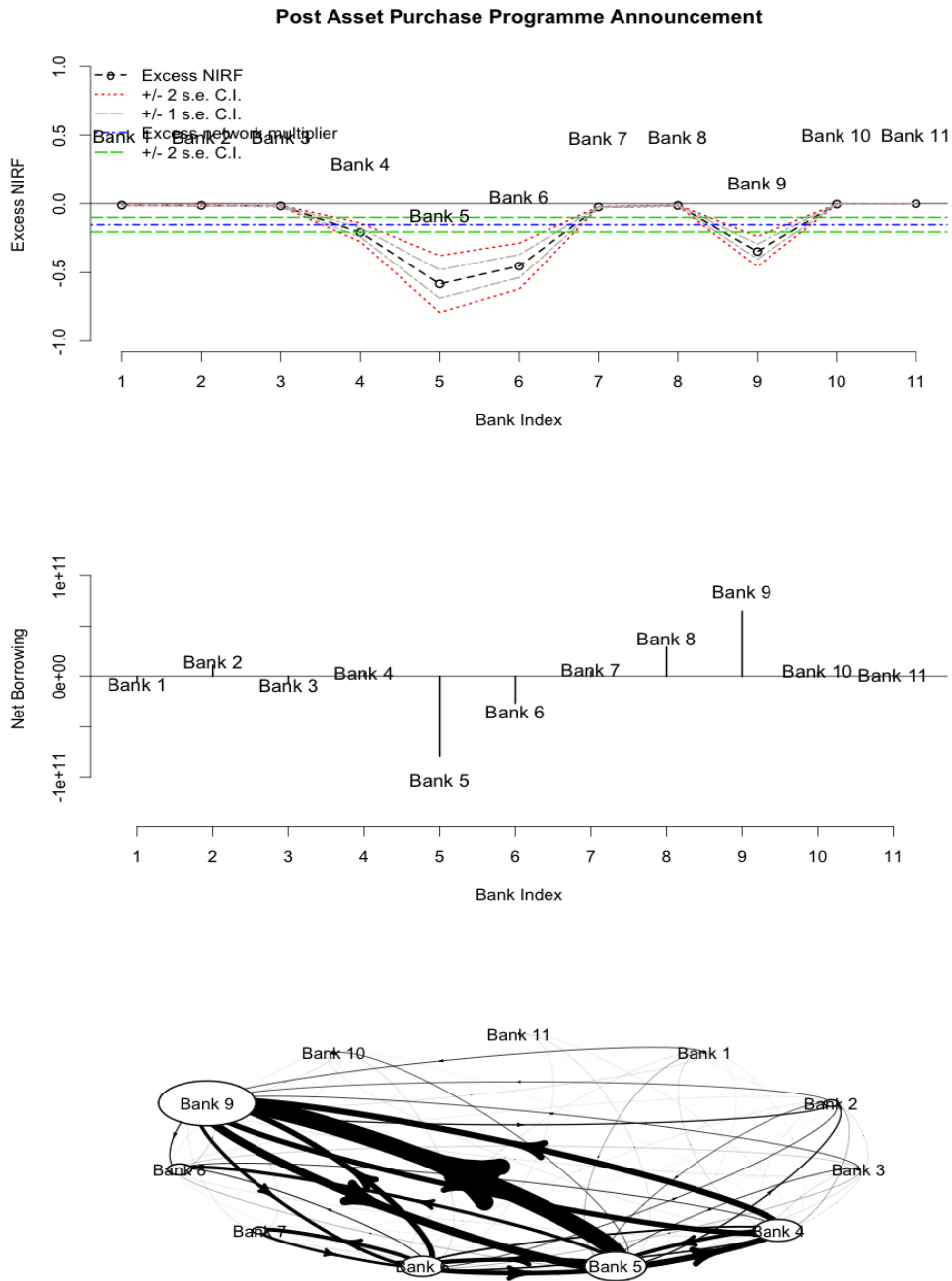
Table 2: Rank Correlation of Bank Characteristics and $NIRF_i^e$

	Period 1	Period 2	Period 3
Interbank Rate	20.91%	37.27%	−64.55%
$\ln LevPay_{i,t-1}$	82.73%***	95.45%***	−85.45%***
$rK_{i,t-1}^{in}$	20.00%	−34.55%	10.91%
$rK_{i,t-1}^{out}$	−45.45%	−89.09%***	73.64%**
$\ln VolPay_{i,t-1}$	48.18%	56.36%*	−54.55%*
$\ln LU_{i,t-1}$	21.82%	35.45%	−23.64%
$\frac{\text{Repo Liability}}{\text{Assets}}$	39.45%	48.18%	−37.27%
Total Assets (log)	12.73%	25.45%	4.55%
$\frac{\Delta\text{Deposits}}{\text{Assets}}$	12.73%	−50%	68.18%**
CDS Spread	38.18%	18.18%	−40.00%
Stock Return	13.64%	−17.27%	−56.36%*
Total Lending and Borrowing (log)	86.36%***	95.45%***	−89.09%***
Total Lending (log)	97.27%***	99.09%***	−89.09%***
Total Borrowing (log)	66.36%**	91.82%***	−76.36%***
Net Borrowing (log)	−17.27%	10.91%	54.55%*

* represents 10% significance, ** 5% significance, and *** 1% significance.

network impulse-response functions in the three subsamples. Only a few characteristics seem to correlate significantly with the magnitude of $NIRF_i^e$. Several observations are in order.

For the total payments channeled by a bank, in periods 1 and 2, the rank correlations for this variable are, respectively, 82.73% and 95.45%, while in period 3 we have -85.45%, suggesting banks that channel a larger amount of payments are likely to be central in the interbank credit network but the implications of its centrality depends on the type of equilibrium, i.e., strategy complementarity ($\phi > 0$) or substitution ($\phi < 0$). In the first two periods, when $\phi > 0$, banks with large payment flows contribute to the volatility of aggregate liquidity, while in the third period, when $\phi < 0$, they dampens the effect of shocks.

The last row of Table 2 shows that *net* borrowing has no significant rank correlation with banks' $NIRF_i^e$, consistent with Figures (4)-(6). Nevertheless, *gross* lending and gross borrowing, and their sum, are all highly correlated with banks' $NIRF_i^e$. That is, banks that borrow and/or lend a lot (in gross terms) tend to be key players in our network. Once again, the sign of correlation depends on the sign of ϕ: large banks, in terms of gross borrowing or lending, can be key risk contributors or absorbers depending on the type of equilibrium on the

Table 3: Central Planner vs. Market Equilibria

	Period 1	Period 2	Period 3
Δ% Volatility of Total Liquidity	−90.8%	−64.8%	30.7%
Δ Level of Total Liquidity (unit: £10bn)	−3.47	15.5	−27.5

The three subsamples are indexed by $j = 1, 2, 3$, and $\bar{\mathbf{G}}_j$ is the average $\mathbf{G}_t$ in subsample j. The table reports: in first row, $100 \times \left[\left(\frac{Var(Z^p(\hat{\phi}_j, \bar{\mathbf{G}}_j))}{Var(Z^*(\hat{\phi}_j, \bar{\mathbf{G}}_j))} \right)^{\frac{1}{2}} - 1 \right]$; in second row, $\mathbf{1}' \left[\mathbf{M}^p \left(\hat{\phi}_j, \bar{\mathbf{G}}_j \right) - \mathbf{M} \left(\hat{\phi}_j, \bar{\mathbf{G}}_j \right) \right] \hat{\bar{\mu}}_j$.

network. How to measure bank size is important. For example, size measured by total assets is only weakly correlated with $NIRF_i^e$. Interestingly, the rank correlations are, in absolute terms, marginally larger for total lending than for total borrowing, suggesting outdegree links are more important for shock propagations. As we have shown in the theoretical model, outbound routes are responsible for the discrepancy between the planner's solution and decentralised equilibrium outcome. Next, we use our estimates to quantify this discrepancy.

V.2   The planner's solution vs. decentralised equilibrium

Using the estimates of structural parameters, we assess the discrepancy between banks' liquidity holdings in the decentralised equilibrium, and the level of liquidity buffer that a benevolent central planner would have wanted the banks to hold. That is, from equations (15), we compute the (expected) difference between the aggregate liquidity of planner's choice and the aggregate decentralised liquidity as $\mathbf{1}' \left[\mathbf{M}^p \left(\phi, \psi, \delta, \mathbf{G} \right) - \mathbf{M} \left(\phi, \mathbf{G} \right) \right] \bar{\mu}$. Similarly, from equations (17) and (19) we compute the difference in the volatility of planner's choice and that of the aggregate decentralised liquidity: $Var(Z^p \left(\phi, \psi, \delta, \mathbf{G} \right))^{\frac{1}{2}} - Var \left(Z^* \left(\phi, \mathbf{G} \right) \right)^{\frac{1}{2}}$.

To compute these quantities, we need ψ and the ratio δ/γ (see Proposition 2). While we cannot estimate ψ directly, we calibrate it to a natural benchmark, that is $\psi = 1$ – banks value the liquidity available from the network (i.e., borrowing from neighbours) in an identical manner as they value its own liquidity holdings. Since we have an estimate of ϕ, the ratio δ/γ can be backed out from the definition of ϕ, i.e., $\delta/\gamma = \phi + \psi$.

Table 3 reports the discrepancies between the central planner's solutions and the market equilibria, based on the estimates of structural parameters in Table 1, and the average value of the adjacency matrix $\mathbf{G}_t$, in the three subsamples.

In Period 1 – when the (average) network multiplier was extremely large – the market equilibrium features excessive risk from the perspective of a central planner: the planner would prefer the volatility of aggregate liquidity to be reduced by almost 91%. Moreover, albeit marginally, the liquidity level in the system is also excessive. Because both the planner and individual banks face the same bank-specific mean valuation of liquidity ($\bar{\mu}$) and shock

variance (σ_i^2), the discrepancy between the planner's solution and decentralised outcome lies in the fact that individual banks do not internalise their impact on each other (i.e., the outdegree linkages), as shown by equation (9).

In Period 2, the market equilibrium produces less volatility than in the Period 1. Nevertheless, the market volatility is still too large (by about 65%) from the central planner's perspective. In comparison with Period 1, the network is less cohesive (see Figure 3), and the network multiplier declines since ϕ is closer to zero. However, this does not mean that the network externalities are eliminated. Quite to the opposite, such externalities through outdegree linkages lead to an expected level of aggregate liquidity buffer that is £15.5 billions lower than what is considered optimal by the central planner.

In the last period, the (average) network multiplier in the market equilibrium is smaller than 1, hence overall the system dampens the volatility of shocks. From the central planner's perspective, not enough volatility is generated (by about 31%) while at the same time the aggregate network liquidity buffer is too high. This implies that banks hold idle reserves, and thus, the transmission of monetary policy (i.e., QE in this context) to the broad economy tends to be less effective than envisioned.

## V.3 Time-varying network effects

The results presented so far indicate a substantial change over time in the role played by the network interactions in determining aggregate liquidity level and risk. In this section, we analyse the drivers of this time variation.

Drivers of variation in network effects. The network impulse response functions depicted in Figures (4)-(6) show substantial time variation in the amplification of shocks across periods. This could be caused by either the time variation in the network topology $\mathbf{G}$ or in the network multiplier ϕ. To examine the relative contribution, we compute the changes in the network impulse response functions across the three periods.

In particular, Panel A of Figure 7 reports the change in NIRFs between Periods 1 and 2 due to the variation of $\mathbf{G}$ ($NIRF_i(\hat{\phi}_1, 1, \bar{\mathbf{G}}_2) - NIRF_i(\hat{\phi}_1, 1, \bar{\mathbf{G}}_1)$, dotted line with triangles), and the change due to the variation of ϕ ($NIRF_i(\hat{\phi}_2, 1, \bar{\mathbf{G}}_1) - NIRF_i(\hat{\phi}_1, 1, \bar{\mathbf{G}}_1)$, dash-dotted line with +). Note that total change is not the sum of ceteris paribus change due to variation in $\mathbf{G}$ and ceteris paribus change due to variation in ϕ, but as a point of reference, the total change is plotted ($NIRF_i(\hat{\phi}_2, 1, \bar{\mathbf{G}}_2) - NIRF_i(\hat{\phi}_1, 1, \bar{\mathbf{G}}_1)$, dashed line with circles).

A striking feature of the graph is that most of the total change comes from the reduction in the network multiplier ϕ for all banks. In fact, ceteris paribus, the NIRF of Bank 5 would have increased from Period 1 to 2 due to the change in $\mathbf{G}$. However, this effect is dwarfed

Panel A: NIRF change decomposition between Period 1 and 2
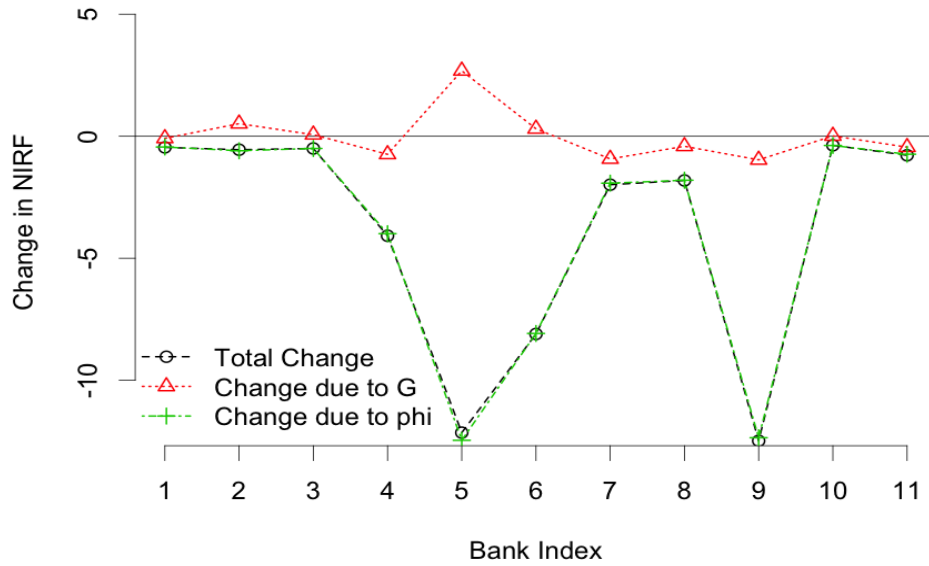
Panel B: NIRF change decomposition between Period 2 and 3
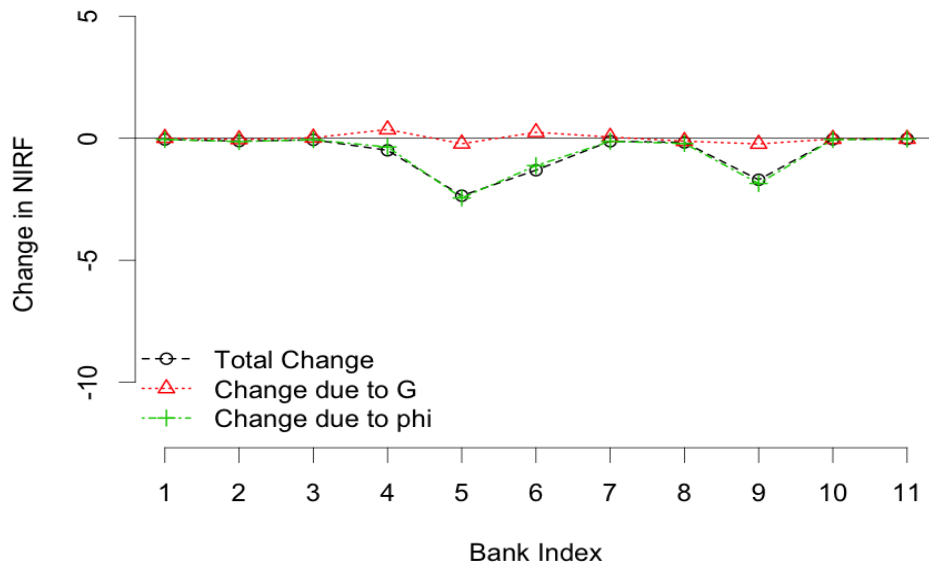
Figure 7: Attribution of the change in NIRFs across periods: the ceteris paribus change due to variation of $\mathbf{G}$ $(NIRF_i(\hat{\phi}_1, 1, \bar{\mathbf{G}}_2) - NIRF_i(\hat{\phi}_1, 1, \bar{\mathbf{G}}_1)$, dotted line with triangles); the ceteris paribus change due to variation of ϕ $(NIRF_i(\hat{\phi}_2, 1, \bar{\mathbf{G}}_1) - NIRF_i(\hat{\phi}_1, 1, \bar{\mathbf{G}}_1)$, dash-dotted line with +); the total change $(NIRF_i(\hat{\phi}_2, 1, \bar{\mathbf{G}}_2) - NIRF_i(\hat{\phi}_1, 1, \bar{\mathbf{G}}_1)$, dashed line with circles).

by the reduction of its NIRF caused by the change in ϕ.

Panel B reports the same decomposition of the change in NIRFs between Periods 2 and 3. Once again the changes are mostly driven by the change in the network multiplier rather than the change in network topology. Overall, Figure 7 shows that the time variation of the network multiplier, i.e., the type of equilibrium on the network (strategic complementarity vs. substitution), has the first order effect on the network amplification mechanism.

Rolling-window estimation and spatial Durbin model. The results in the previous sections indicate the importance of variation of ϕ in determining the network effects. Therefore, to capture this time variation, we now estimate the structural model in equations (12) and (13) using a 6-month rolling window.[32] These rolling estimates of the network coefficient ϕ are reported (blue line), together with 95% confidence bands (red lines), in Figure 8.

The figure also reports the rolling point estimates of the coefficient ϕ implied by the spatial Durbin model (green line) in equation (20) which, as a more general model, serves as a specification test of our benchmark spatial error model. If the two estimated ϕ are close to each other, as shown in Figure 8, this indicates that our theory-driven spatial error specification of the interbank network cannot be rejected for a more general specification.[33] Specifically, we find that less than 95% of the time, the two estimates differ statistically at the 5% confidence level.

At the beginning of the sample, the estimate of ϕ implies an extremely large network multiplier, and thus, the interbank system as a powerful shock amplifier. The estimate has its first sharp reduction around the 18th of May 2006 when the Bank of England introduced the reserve averaging system described in Section A.1. The network multiplier is relatively stable after that, except for a temporary decrease during the 2007 subprime mortgage default, until the Northern Rock bank run when the network multiplier is drastically reduced for several months. After this reduction, the coefficient goes back to roughly the previous average level but exhibits a declining trend that culminates in a slump following the Bear Stearns collapse. From this period onward, and until long after the Lehman Brothers bankruptcy, the coefficient is statistically indistinguishable from zero.

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[32]Recall that when $\mathbf{G}_t$ is a right stochastic matrix, separate identifications of the bank (α^{bank}) and network ($\bar{\mu}$) fixed effects require that there is a subset of banks that does not borrow at least at one point in time in each subsample (see footnote 25 and the Appendix A.3.1). This condition is not satisfied in all the rolling windows. But since the separate identification of these fixed effect does not affect the identification of ϕ, we normalise the bank fixed effects to zero. Moreover, given the very short length of the rolling window, we drop time fixed effects from the specification and heteroskedastic specification of shocks. Estimates with the full sets of fixed effects and heteroscekasticity show a very similar behaviour, but with somewhat larger confidence intervals, due to the increase of number of parameters. We focus on the more parsimonious specification, but results of sull specifications are available upon request.

[33]Likelihood ratio test cannot reject the spatial error model most part of our sample (i.e., except rolling windows ending from May 2009 to May 2010). Results are available upon request.
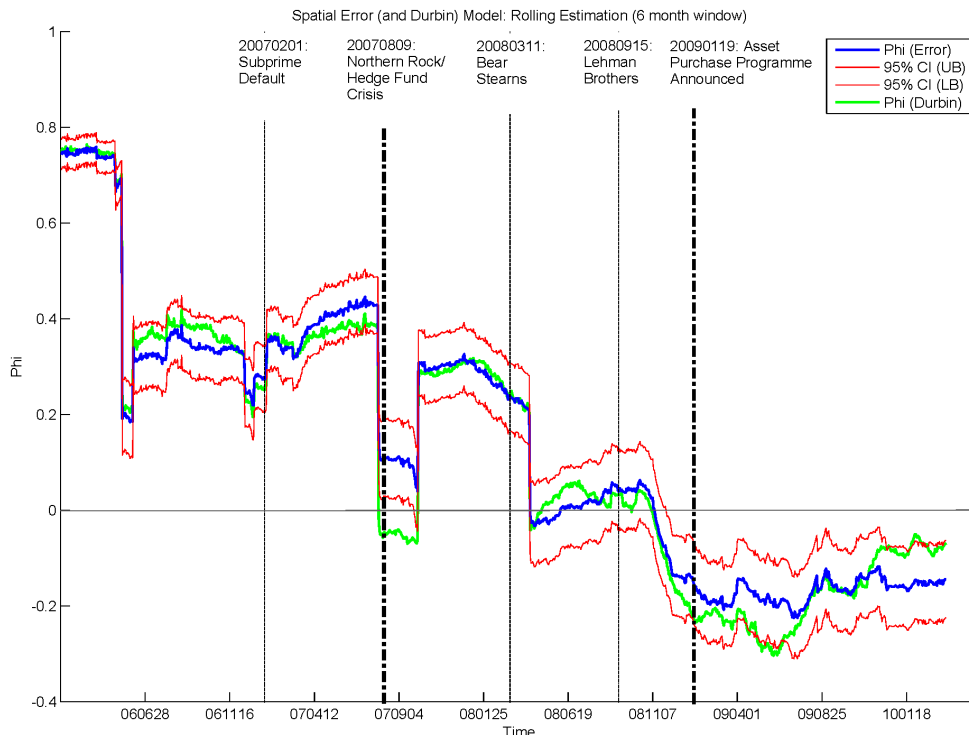
Figure 8: Spatial Error (blue line) and Durbin (green line) rolling estimates of ϕ.

Our estimation suggests that banks' liquidity management objectives change in response to market-wide crisis in a way that reduce the domino effect of shock propagation and amplification on the interbank network. This may happen through the following channels: a) a reduction in the informational spillover effect, i.e., a reduction in δ; b) interbank credit becomes more prominent a source of liquidity, i.e., an increase in ψ; c) the marginal utility from liquidity hoarding decreases faster, i.e., an increase in γ.

Interestingly, the coefficient $\hat{\phi}$ becomes negative right before the announcement of Asset Purchase Programme, and remains negative throughout the QE period. This indicates that during the active liquidity injection by the Bank of England (and also in expectation of it), banks' liquidity holding decisions exhibit strategic substitution in their liquidity holding decision (as in Bhattacharya and Gale (1987)). Note that our results are unlikely to be driven by the direct or mechanical impact of QE on the interbank market of reserves for the following reasons: a) our estimation controls for variation in many prices (e.g., interbank rates), aggregate quantities (e.g., payment patterns, repo, deposits etc.), and time (weekly) fixed effects, so our estimated response functions of individual banks are already conditional on such information; b) as shown in Figure 1, overall, banks hold more liquidity after QE, which favours strategic complementarity (i.e., correlated liquidity holdings) instead of strategic substitution; c) the drop in ϕ actually occurred before the announcement of QE; d) as

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previous emphasized and described in the Appendix A.1, the Bank of England followed an accommodative reserve supply policy throughout our sample period, so the impact of QE is not through alleviating reserve scarcity. Therefore, our finding of a strong QE impact on banks' liquidity management objectives (i.e., our structural parameters) posit a challenge for theoretical research on how monetary policy affects the banking system.

VI   Conclusion

We develop a network model of banks' liquidity holding decisions, and estimate the model to uncover the structural parameters that determine the type of equilibrium on the interbank network, i.e., strategic complementarity or substitution. Using the estimated parameters and the observed network topology, we construct measures of systemic risk and identify banks that contribute the most to the risk of aggregate liquidity buffer in the payment system.

We find that the network effects vary significantly through the sample period of 2006 to 2010. In the pre-crisis period, banks' liquidity holding decisions exhibit strategic complementarity, so shocks are amplified by the network. In contrast, during the crisis, the network multiplier declined significantly, suggesting that banks adjusted their liquidity management objectives in a way that reduce network domino effects. Finally, during the QE period, in response to the large liquidity injection, the equilibrium on interbank network is characterised by strategic substitution, and accordingly, the network became a shock buffer.

To the best of our knowledge, we are the first to provide evidence on the substantial time variation in the nature of equilibrium on a financial network. Moreover, we show that, form a systemic risk perspective, the change in the type of equilibrium is the dominant force (rather than the change in network topology). This could rationalise the empirical puzzle of network changes having little impact on aggregate quantities in the calibration/simulation studies on interbank networks (e.g., Elsinger, Lehar, and Summer (2006)).

From a policy perspective, we are able to identify key risk contributors using our estimates of structural parameters, and show that a subset of players are responsible for systemic risk generated through network connections. Last, but not least, we solve the choice of a benevolent central planner, and quantify the discrepancy between the planner's solution and the decentralised outcome in both expected level and volatility of aggregate liquidity. In particular, we find that during both the pre-crisis and the 2007-2009 crisis periods, the system was characterised by an excessive amount of risk, and during the crisis, too little liquidity buffer held by individual banks relative to the social optimum.

References

ACEMOGLU, D., V. M. CARVALHO, A. OZDAGLAR, AND A. TAHBAZ-SALEHI (2012): "The network origins of aggregate fluctuations," *Econometrica*, 80(5), 1977–2016.

ACEMOGLU, D., A. OZDAGLAR, AND A. TAHBAZ-SALEHI (2015): "Systemic risk and stability in financial networks," *American Economic Review*, 105, 564–608.

ACHARYA, V. V., AND O. MERROUCHE (2010): "Precautionary hoarding of liquidity and inter-bank markets: Evidence from the sub-prime crisis," New York University working paper.

AFONSO, G., A. KOVNER, AND A. SCHOAR (2011): "Stressed, not frozen: The Federal funds market in the financial crisis," *The Journal of Finance*, 66(4), 1109–1139.

AFONSO, G., AND R. LAGOS (2015): "Trade dynamics in the market for Federal funds," *Econometrica*, 83, 263–313.

AFONSO, G., AND H. S. SHIN (2011): "Precautionary demand and liquidity in payment systems," *Journal of Money, Credit and Banking*, 43, 589–619.

ALDASORO, I., AND I. ANGELONI (2013): "Input-output-based measures of systemic influence," Goethe University of Frankfurt working paper.

ALLEN, F., E. CARLETTI, AND D. GALE (2008): "Interbank market liquidity and Central Bank intervention," New York University working paper.

ALLEN, F., AND D. GALE (2000): "Financial contagion," *Journal of Political Economy*, 108(1), 1–33.

——— (2017): "How Should Bank Liquidity be Regulated," Working paper, Imperial College London.

ANSELIN, L. (1988): *Spatial econometrics: methods and models*, Studies in operational regional science. Kluwer Academic Publishers.

ARMANTIER, O., AND A. COPELAND (2012): "Assessing the quality of Furfine-based algorithms," Staff Reports 575, Federal Reserve Bank of New York.

ASHCRAFT, A., J. MCANDREWS, AND D. SKEIE (2010): "Precautionary reserves and the interbank market," Federal Reserve Bank of New York working paper.

ATKESON, A. G., A. L. EISFELDT, AND P.-O. WEILL (2015): "Entry and Exit in OTC Derivatives Markets," *Econometrica*, 83(6), 2231–2292.

BABUS, A. (2009): "The Formation of Financial Networks," Tinbergen Institute Discussion Paper 06-093.

BABUS, A., AND F. ALLEN (2009): "Networks in finance," in *Network-based Strategies and Competencies*, ed. by P. Kleindorfer, and J. Wind. Pearson Education, New Jersey, US.

BABUS, A., AND P. KONDOR (2013): "Trading and Information Diffusion in Over-the-Counter Markets," Working paper, London School of Economics.

BALL, A., E. DENBEE, M. J. MANNING, AND A. WETHERILT (2011): "Intraday Liquidity: Risk and Regulation," *Bank of England Financial Stability Papers*, 11.

BALLESTER, C., A. CALVO-ARMENGOL, AND Y. ZENOU (2006): "Who's who in networks. Wanted: The key player," *Econometrica*, 74, 1403–1417.

BECH, M. L., AND E. ATALAY (2008): "The topology of the federal funds market," Discussion paper.

BECHER, C., S. MILLARD, AND K. SORAMAKI (2008): "The network topology of CHAPS Sterling," Discussion paper.

BENOS, E., R. J. GARRATT, AND P. ZIMMERMAN (2010): "Bank Behavior and Risks in CHAPS Following the Collapse of Lehman Brothers," UCSB working paper.

BERNANKE, B. S., AND A. S. BLINDER (1988): "Credit, Money, and Aggregate Demand," *The American Economic Review*, 78(2), 435–439.

BHATTACHARYA, S., AND D. GALE (1987): "Preference shocks, liquidity, and Central Bank policy," in *New approaches to monetary economics*, ed. by W. Barnett, and K. Singleton. Cambridge University Press, Cambridge, UK.

BIANCHI, J., AND S. BIGIO (2014): "Banks, Liquidity Management and Monetary Policy," Working Paper 20490, National Bureau of Economic Research.

BILLIO, M., M. GETMANSKY, A. W. LO, AND L. PELIZZONA (2012): "Econometric measures of connectedness and systemic risk in the finance and insurance sectors," *Journal of Financial Economics*, 104(3), 535–559.

BONACICH, P. (1987): "Power and centrality: A family of measures," *American Journal of Sociology*, 92, 1170–1182.

BOSS, M., H. ELSINGER, M. SUMMER, AND S. THURNER (2004): "Network topology of the interbank market," *Quantitative Finance*, 4(6), 677–684.

BRAMOULL, Y., H. DJEBBARI, AND B. FORTIN (1993): "Identifications of Endogenous Social Effects: The Refection Problem," *Review of Economic Studies*, 60(1), 531–542.

BRAMOULLÉ, Y., H. DJEBBARI, AND B. FORTIN (2009): "Identification of Peer Effects through Social Networks," *Journal of Econometrics*, 150(1), 41–55.

BRUNNERMEIER, M. K., AND Y. SANNIKOV (2014): "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104(2), 379–421.

——— (2016): "The I Theory of Money," Working paper, Princeton University.

CABRALES, A., D. GALE, AND P. GOTTARDI (2015): "Financial Contagion in Networkss," Discussion paper.

CABRALES, A., P. GOTTARDI, AND F. VEGA-REDONDO (2016): "Risk-sharing and contagion in networks," Working paper, European University Institute.

CALVO-ARMENGOL, A., E. PATACCHINI, AND Y. ZENOU (2009): "Peer effects and social networks in education," *Review of Economic Studies*, 76, 1239–1267.

CASTIGLIONESI, F., F. FERIOZZI, AND G. LORENZONI (2017): "Financial Integration and Liquidity Crises," Working Paper 23359, National Bureau of Economic Research.

CHEN, H., T. WANG, AND D. D. YAO (2016): "Financial Network and Systemic Risk A Dynamic Model," working paper.

CORNETT, M. M., J. J. MCNUTT, P. E. STRAHAN, AND H. TEHRANIAN (2011): "Liquidity risk management and credit supply in the financial crisis," *Journal of Financial Economics*, 101(2), 297 – 312.

DEBREU, G., AND I. HERSTEIN (1953): "Nonnegative Square Matrices," *Econometrica*, 21, 597–607.

DEGROOT, M. (1974): "Reaching a Consensus," *Journal of the American Statistical Association*, 69, 118–121.

DEMARZO, P., D. VAYANOS, AND J. ZWIEBEL (2003): "Persuasion Bias, Social Influence, and Uni-Dimensional Opinions," *Quarterly Journal of Economics*, 118, 909–968.

DENT, A., AND W. DISON (2012): "The Bank of Englands Real-Time Gross Settlement infrastructure," Bank of England Quarterly Bulletin 3, Bank of England.

DIAMOND, D. W., AND P. H. DYBVIG (1983): "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91(3), 401–419.

DIAMOND, D. W., AND A. K. KASHYAP (2016): "Liquidity Requirements, Liquidity Choice and Financial Stability," Working Paper 22053, National Bureau of Economic Research.

DIEBOLD, F., AND K. YILMAZ (2009): "Measuring Financial Asset Return and Volatility Spillovers, with Application to Global Equity Markets," *Economic Journal*, 119(534), 158–171.

Diebold, F. X., and K. Ylmaz (2014): "On the network topology of variance decompositions: Measuring the connectedness of financial firms," *Journal of Econometrics*, 182(1), 119–134.

Drechsler, I., A. Savov, and P. Schnabl (2014): "A Model of Monetary Policy and Risk Premia," Working Paper 20141, National Bureau of Economic Research.

Duarte, F. M., and T. M. Eisenbach (2013): "Fire-sale spillovers and systemic risk," Discussion paper.

Eisenberg, L., and T. H. Noe (2001): "Systemic Risk in Financial Systems," *Journal of Political Economy*, 47, 236–249.

Elhorst, J. P. (2010a): "Applied Spatial Econometrics: Raising the Bar," *Spatial Economic Analysis*, 5(1), 9–28.

———— (2010b): "Spatial Panel Data Models," in *Handbook of Applied Spatial Analysis*, ed. by M. M. Fischer, and A. Getis, pp. 377–407. Springer Berlin Heidelberg.

Elliott, M., B. Golub, and M. O. Jackson (2015): "Financial Networks and Contagion," *American Economic Review*, 104(10), 3115–3153.

Elsinger, H., A. Lehar, and M. Summer (2006): "Risk Assessment for Banking Systems," *Management Science*, 52(9), 1301–1314.

Fecht, F., K. Nyborg, and J. Rocholl (2010): "The price of liquidity: Bank characteristics and market condition," University of Zurich working paper.

Freixas, X., A. Martin, and D. Skeie (2011): "Bank Liquidity, Interbank Markets, and Monetary Policy," *Review Financial Studies*, 44(8), 2656–2692.

Freixas, X., B. Parigi, and J. Rochet (2000): "Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank," *Journal of Money, Credit and Banking*, 32(1), 611–638.

Furfine, C. (2000): "Interbank payments and the daily federal funds rate," *Journal of Monetary Economics*, 46, 535–553.

Furfine, C. H. (2003): "Interbank Exposures: Quantifying the Risk of Contagion," *Journal of Money, Credit and Banking*, 35(1), 111–128.

Gai, P., and S. Kapadia (2010): "Contagion in financial networks," *Proceedings of the Royal Society A: Mathematical, Physical, and Engineering Science*, 466, 2401–2423.

Glasserman, P., and H. Young (2015): "How likely is contagion in financial networks?," *Journal of Banking & Finance*, 50, 383–399.

Gofman, M. (2017): "Efficiency and stability of a financial architecture with too-interconnected-to-fail institution," *Journal of Financial Economics*, forthcoming.

Gorodentsev, A. L. (1994): "Helix theory and nonsymmetrical bilinear forms," in *Algebraic Geometry and its Applications*, pp. 47–59. Vieweg+ Teubner Verlag.

Greenwood, R., A. Landier, and D. Thesmar (2015): "Vulnerable banks," *Journal of Financial Economics*, 115, 471–485.

Hart, O., and L. Zingales (2014): "Banks Are Where The Liquidity Is," Working paper, Harvard University.

Hautsch, N., J. Schaumburg, and M. Schienle (2012): "Financial Network Systemic Risk Contributions," working paper.

Hayashi, F. (2000): *Econometrics*. Princeton University Press: Princeton.

He, Z., and A. Krishnamurthy (2013): "Intermediary Asset Pricing," *American Economic Review*, 103(2), 732–770.

Henderson, H. V., and S. R. Searle (1981): "On Deriving the Inverse of a Sum of Matrices," *SIAM Review*, 23(1), pp. 53–60.

HERSKOVIC, B. (forthcoming): "Networks in Production: Asset Pricing Implications," *Journal of Finance.*

HERSKOVIC, B., B. T. KELLY, H. N. LUSTIG, AND S. V. NIEUWERBURGH (2017): "Firm Volatility in Granular Networks," Chicago Booth Research Paper 12-56, University of Chicago.

JACKSON, M. O. (2003): *Social and Economic Networks.* Princeton University Press, New Jersey.

JACKSON, M. O., AND Y. ZENOU (2012): "Games on Networks," CEPR Discussion Papers 9127, C.E.P.R. Discussion Papers.

JOVANOVIC, B. (1987): "Micro shocks and aggregate risk," *The Quarterly Journal of Economics*, 102(21), 395–409.

KASHYAP, A. K., AND J. C. STEIN (2000): "What Do a Million Observations on Banks Say about the Transmission of Monetary Policy?," *American Economic Review*, 90(3), 407–428.

KATZ, L. (1953): "A new status index derived from sociometric analysis," *Psychometrika*, 18(1), 39–43.

KELLY, B. T., H. N. LUSTIG, AND S. V. NIEUWERBURGH (2013): "Firm Volatility in Granular Networks," Fama-Miller Working Paper; Chicago Booth Research Paper No. 12-56.

KLIMENKO, N., S. PFEIL, J.-C. ROCHET, AND G. D. NICOLO (2016): "Aggregate Bank Capital and Credit Dynamics," Swiss Finance Institute Research Paper Series 16-42.

KOVNER, A., AND D. SKEIE (2013): "Evaluating the quality of fed funds lending estimates produced from Fedwire payments data," Staff Reports 629, Federal Reserve Bank of New York.

LANGFIELD, S., Z. LIU, AND T. OTA (2014): "Mapping the UK interbank system," *Journal of Banking & Finances*, 45, 288–303.

LEE, L.-F., AND J. YU (2010): "A Spatial Dynamic Panel Data Model With Both Time And Individual Fixed Effects," *Econometric Theory*, 26(02), 564–597.

LEITNER, Y. (2005): "Financial networks: Contagion, commitment, and private sector bailouts," *The Journal of Finance*, 60, 2925–2953.

LESAGE, J., AND R. PACE (2009): *Introduction to spatial econometrics*, Statistics, textbooks and monographs. CRC Press.

LI, Y. (2017): "Procyclical Finance: The Money View," Working paper, Columbia University Graduate School of Business.

MARK, G., AND N. KIYOTAKI (2010): "Chapter 11 - Financial Intermediation and Credit Policy in Business Cycle Analysis," in *Handbook of Monetary Economics*, ed. by B. M. Friedman, and M. Woodford, vol. 3 of *Handbook of Monetary Economics*, pp. 547 – 599. Elsevier.

MOORE, J. (2012): "Leverage stacks and the financial system," Edinburgh University and London School of Economics working paper.

NEWMAN, M. E. J. (2004): "Analysis of weighted networks," *Physical Review*, 70, 1–9.

OZDAGLI, A. K., AND M. WEBER (2015): "Monetary Policy Through Production Networks: Evidence from the Stock Market," working paper.

PIAZZESI, M., AND M. SCHNEIDER (2017): "Payments, Credit and Asset Prices," Working paper, Stanford University.

QUADRINI, V. (2017): "Bank liabilities channel," *Journal of Monetary Economics*, 89, 25 – 44, Carnegie-Rochester-NYU Conference Series on the Macroeconomics of Liquidity in Capital Markets and the Corporate Sector.

SORAMAKI, K., M. L. BECH, J. ARNOLD, R. J. GLASS, AND W. E. BEYELER (2007): "The topology of interbank payment flows," *Physica A*, 379(1), 317–333.

STEIN, J. C. (2012): "Monetary Policy as Financial-Stability Regulation," *Quarterly Journal of Economics*, 127(1), 57–95, forthcoming in the Quarterly Journal of Economics Draft Date: Revised May 2011.

UPPER, C. (2011): "Simulation methods to assess the danger of contagion in interbank markets," *Journal of Financial Stability*, 7(3), 111–125.

UPPER, C., AND A. WORMS (2004): "Estimating bilateral exposures in the German interbank market: Is there a danger of contagion?," *European Economic Review*, 48(4), 827 – 849.

WATTS, D. J., AND S. H. STROGATZ (1998): "Collective dynamics of 'small-world' networks," *Nature*, 393, 440–442.

WESTWOOD, B. (2011): "The Money Market Liaison Group Sterling Money Market Survey," *Bank of England Quarterly Bulletin*, 51(3), 247–250.

WETHERILT, A., P. ZIMMERMAN, AND K. SORAMAKI (2010): "The sterling unsecured loan market during 2006-08: insights from network theory," Bank of England working papers 398, Bank of England.

ZAWADOWSKI, A. (2012): "Entangled financial systems," Boston University working paper.

A   Appendix

A.1   Reserves schemes, payment systems, and interbank liquidity

Banks in the UK choose the amount of central bank reserves that they hold to support a range of short term liquidity needs. Reserves are the ultimate settlement asset for interbank payments. Whenever payments are made between the accounts of customers at different commercial banks, they are ultimately settled by transferring central bank money (reserves) between the reserves accounts of those banks. Reserve balances are used to buffer against intraday payment imbalances (i.e., cumulative outflows larger than inflows). Additionally, central bank reserves are the most liquid asset that banks can draw upon in the presence of unexpected outflows of funds. Since 2006, the starting year of our sample, banks choose their reserve holdings on a discretionary basis, i.e., reserve holdings are not mandatory. However, their reserve holding decisions depend on the policy framework in which they operate.

A.1.1   Monetary policy framework

Since the 1998 Banking Act, the Bank of England (BoE) has had independent responsibility for setting interest rates to ensure that inflation, as measured by the Consumer Price Index (CPI), meets the inflation target of 2%. Each month the Monetary Policy Committee (MPC) meets to decide the appropriate level of the Bank rate (the policy interest rate) to meet the inflation target in the medium term. The Sterling Monetary Framework changed over time. During our sample period, the Bank of England had three distinct monetary frameworks: prior to 18 May 2006, the Bank of England operated an unremunerated reserve scheme; this was then replaced by a reserves average scheme; since March 2009 and the initiation of Quantitative Easing, the reserves average scheme has been suspended.

Pre-2006 Reform: Prior to the 2006 reforms, the Sterling Monetary Framework (SMF) was based upon a voluntary unremunerated reserves. There were no reserve requirements and no reserve averaging over a maintenance period. The only requirement was that banks were obliged to maintain a minimum zero balance at the end of each day. In practice, due to uncertainties from end of day cash positions, banks opted for small positive reserve balances.

Reserve Averaging: In May 2006, the Bank of England undertook a major reform of the Sterling Monetary Framework. The new scheme was voluntary remunerated reserves with a period-average maintenance requirement. Each maintenance period – the period between two meetings of the Monetary Policy Committee – banks were required to decide upon a reserves target. This voluntary choice of reserves target is a feature unique to the UK system. Over the course of each maintenance period, the banks would manage their balance sheets so that, on average, their reserve balances hit the target. Where banks were unable to hit the

target, standing borrowing and deposit facilities were available. Within a range of $\pm 1\%$ of the target, reserves are remunerated at the Bank Rate.[34] Holding an average level of reserves outside the target range attracts a penalty charge.[35] But an SMF participant can ensure it hits its target by making use of the Bank's Operational Standing Facilities (OSFs). These bilateral facilities allow SMF participants to borrow overnight from the Bank (against high-quality collateral) at a rate above Bank Rate or to deposit reserves overnight with the Bank at a rate below Bank Rate. The possibility of arbitrage between interbank market rates and reserves remunerated at Bank Rate is the main mechanism through which market rates are kept in line with Bank Rate. In both schemes before Quantitative Easing (QE), the BoE would ensure sufficient reserves supply for banks to meet their reserves target. Banks then use the interbank market to reallocate reserves from banks in surplus to banks in deficit.

Post Quantitative Easing: Quantitative Easing in UK started in March 2009 when the MPC decided that in order to meet the inflation target in the medium term, it would need to supplement the use of interest rate (which had hit the practical lower bound of 0.5%) with the purchase of assets using central bank reserves. This consisted of the BoE's boosting the money supply by creating central bank reserves and using them to purchase assets, predominantly UK gilts. Furthermore, the BoE suspended the average reserve targeting regime, and now remunerates all reserves at the Bank rate.

A.1.2 Payment and settlement systems

Banks use central bank reserves to, inter alia, meet their demand for intraday liquidity in the payment and settlement systems. Reserves act as a buffer to cover regular timing mismatches between incoming and outgoing payments, for example, due to exceptionally large payments, operational difficulties, or stresses that impact upon a counterparty's ability, or willingness to send payments. There are two major payment systems in the UK: CHAPS and CREST.[36] These two systems play a vital role in the UK financial system. On average, in 2011, £700 billion of transactions was settled every day within the two systems. This equates to the UK 2011 nominal GDP being settled every two days.

CHAPS is the UK's large-value payment system. It is used for real time settlement of payments between its member banks. These banks settle payments on behalf of hundreds of other banks through correspondent relationships. Typical payments are business-to-business payments, home purchases, and interbank transfers. Payments relating to unsecured inter-

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[34]At various points during the crisis, this $\pm 1\%$ range was increased to give banks more flexibility to manage their liquidity.

[35]Settlement banks also pay a penalty if their reserves account is overdrawn at the end of any day.

[36]There are also four retail payment systems (Bacs, the Faster Payments Service (FPS), Cheque and Credit Clearing (CCC) and LINK) that are operated through the BoE.

bank money markets are settled in CHAPS. CHAPS opens for settlement at 8 am and closes at 4:20 pm. Payments made on behalf of customers cannot be made after 4 pm. The system has throughput guidelines which require members to submit 50% of their payments by noon and 75% by 14:30. This helps ensure that payments are settled throughout the day and do not bunch towards the end of the day.

In 2011, CHAPS settled an average of 135,550 payments each day valuing £254bn. CHAPS is a real-time gross settlement (RTGS) system. This means that payments are settled finally and irrevocably in real time. To fund these payments, banks have to access liquidity intraday. If a bank has, at any point during the day, cumulatively sent more payments than it has received, then it needs liquidity to cover this difference. This comes either from central bank reserves or intraday borrowing from the BoE. Furthermore, when a bank sends funds to another bank in the system, it exposes itself to liquidity risk. That is, the risk that the bank may not get those fund inflows back during the day, and so will run down their own liquidity holdings or borrow from the BoE. Therefore, it is important to choose an appropriate level of liquidity buffer. Besides maintaining a liquidity buffer, banks manage liquidity by borrowing from and lending to each other in the unsecured overnight markets. According to Bank of England estimates, payments relating to overnight market activity (advances and repayments) account for about 20% of CHAPS values (Wetherilt, Zimmerman, and Soramaki (2010)).

CREST is a securities settlement system. Its Delivery-vs-Payment (DVP) mechanism ensures simultaneous transfer of funds and securities. When a liquidity need is identified, the CREST system's intraday liquidity mechanism with the BoE works automatically through "Self Collateralising Repos" (SCRs): if a CREST settlement bank would otherwise have insufficient funds to settle a transaction, a secured intraday loan is generated using as eligible collateral either the purchased security (if eligible) or other securities.

A.1.3 The sterling unsecured overnight interbank market

Interbank markets are the markets where banks and other financial institutions borrow and lend assets, typically with maturities of less than one year. At the shortest maturity, overnight, banks borrow and lend central bank reserves. Monetary policy aims at influencing the rate at which these markets transact, so as to control inflation in the wider economy. There is limited information available about the size and the structure of the sterling money markets. The Bank of England estimates suggest that the overnight unsecured market is approximately £20–30 billion per day during our sample period. Wetherilt, Zimmerman, and Soramaki (2010) describe the network of the sterling unsecured overnight money market. They find that the network has a small core of highly connected participants, surrounded by

a wider periphery of banks loosely connected with each other, but with connections to the core. It is believed that prior to the recent financial crisis, the unsecured market was much larger than the secured one. We identify interbank borrowing and lending transactions in CHAPS settlement data.

A.2 Model discussion and additional theoretical results

A.2.1 An alternative bank objective function

Even if we restrict the network to be only relevant for fund flows rather than information, strategic complementary may still arise from leverage stack as in Moore (2012). In the Appendix, we lay out an alternative specification of banks' objective function in the spirit of (2012).

Moore (2012) models a chain of borrowing/lending relationships that starts from the bank who borrows from households and ends at the bank with investment project. Interbank loans can be pledged to upstream lenders as collateral, so δ is lower if the collateral haircut is higher. Under this alternative formulation, we may posit bank i's objective function as follows:

$$
u_i(\mathbf{z}_t|g_t) = \hat{\mu}_{i,t} \underbrace{\left(z_{i,t} + \psi \sum_{j \neq i} g_{ij,t} z_{j,t} \right)}_{\text{Accesible Liquidity}} - \frac{1}{2}\gamma \left(z_{i,t} + \psi \sum_{j \neq i} g_{ij,t} z_{j,t} \right)^2 + \underbrace{z_{i,t}\delta \sum_{j \neq i} g_{ij,t} z_{j,t}}_{\text{``Collateralised'' Liquidity}} \quad , \ (22)
$$

The "collateralised" liquidity term, $z_{i,t}\delta \sum_j g_{ij,t} z_{j,t}$ has two parts: the available reserves that could be borrowed from neighbours, $\sum_j g_{ij,t} z_{j,t}$, and the multiplication factor $z_{i,t}\delta$, which can be thought of as collateral posted by bank i with parameter δ reflecting a haircut. Since in the empirical context banks borrow and lend reserves on an unsecured basis, we may also interpret the multiplication factor as "information collateral," i.e., by holding liquidity $z_{i,t}$, bank i signals its creditworthiness to neighbouring banks in the interbank network. Note that whether banks' objective is from equation (2) or (22) does not change the equilibrium outcome (i.e., their first-order conditions, or best response functions, stay the same).[37]

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[37]The only difference between these two objective functions is that equation (2) has an additional second-order term $\left(\delta \sum_{j \neq i} g_{ij,t} z_{j,t} \right) \left(\psi \sum_{j \neq i} g_{ij,t} z_{j,t} \right)$, but it only contains *other* banks' choice of liquidity holdings, not bank i's, so this additional term does not affect bank i's first-order condition. In the planner's problem that we discuss later, since the planner internalizes any spillover effect, the solutions differ slightly depending on whether we take equation (2) or equation (22) as banks' objective function.

A.2.2 A more general model

In this section, we present an alternative model where the network effect on banks' liquidity holding decisions is not modelled as a residual. Specifically, we let the total liquidity holding by bank i, i.e., $l_{i,t}$, to be accessible to the network. Hence, the valuation of liquidity for bank i in network g_t becomes:

$$\underbrace{\tilde{\mu}_{i,t}}_{\text{Per Unit Value}} \underbrace{\left(l_{i,t} + \psi \sum_{j \neq i} g_{ij,t} l_{j,t} \right)}_{\text{Accesible Liquidity}} \tag{23}$$

and the per unit value $\tilde{\mu}_{i,t}$ is specified as

$$\tilde{\mu}_{i,t} := \hat{\mu}_{i,t} + \delta \sum_{j} g_{ij,t} l_{j,t} + \sum_{m=1}^{M} \tilde{\beta}_m x_{i,t}^m + \sum_{j} g_{ij,t} x_{i,j,t} \tilde{\theta} + \sum_{p=1}^{P} \tilde{\gamma}_p x_t^p \tag{24}$$

where $x_{i,j,t}$ denotes match specific control variables and the characteristics of other banks, and $\tilde{\theta}$ is a vector of suitable dimension. That is, in addition to the aggregate information embedded in the neighbouring banks' holdings, also macro variables and the neighbouring banks' characteristics affect the per unit valuation of the liquidity.

In this setup, bank i's utility from holding liquidity is specified as:

$$
\begin{aligned}
u_i(\mathbf{l}_t | g_t) &= \left(\hat{\mu}_{i,t} + \delta \sum_{j} g_{ij,t} l_{j,t} + \sum_{m=1}^{M} \tilde{\beta}_m x_{i,t}^m + \sum_{j} g_{ij,t} x_{i,j,t} \tilde{\theta} + \sum_{p=1}^{P} \tilde{\gamma}_p x_t^p \right) \left(l_{i,t} + \psi \sum_{j \neq i} g_{ij,t} l_{j,t} \right) \\
&\quad - \frac{1}{2} \gamma \left(l_{i,t} + \psi \sum_{j \neq i} g_{ij,t} l_{j,t} \right)^2.
\end{aligned}
\tag{25}
$$

The optimal response function for each bank is then:

$$
\begin{aligned}
l_{i,t}^* &= \frac{\hat{\mu}_{i,t} + \sum_{m=1}^{M} \tilde{\beta}_m x_{i,t}^m + \sum_{j} g_{ij,t} x_{i,j,t} \tilde{\theta} + \sum_{p=1}^{P} \tilde{\gamma}_p x_t^p}{\gamma} + \left(\frac{\delta}{\gamma} - \psi \right) \sum_{j \neq i} g_{ij,t} l_{j,t} \\
&= \mu_{i,t} + \sum_{m=1}^{M} \beta_m x_{i,t}^m + \sum_{j} g_{ij,t} x_{i,j,t} \theta + \sum_{p=1}^{P} \gamma_p x_t^p + \phi \sum_{j} g_{ij,t} l_{j \neq i,t}
\end{aligned}
\tag{26}
$$

where $\phi := \delta/\gamma - \psi$, $\mu_{i,t} := \hat{\mu}_{i,t}/\gamma := \bar{\mu}_i + \nu_{i,t}$, $\beta_m = \tilde{\beta}_m/\gamma$, $\gamma_p = \tilde{\gamma}_p/\gamma$, and $\theta = \tilde{\theta}/\gamma$. Note that the empirical counterpart of the above best response is the spatial Durbin model in equation (20).

Let us denote $\mu_{i,t} + \sum_{m=1}^{M} \beta_m x_{i,j,t}^m + \sum_{j} g_{ij,t} x_{i,t} \theta + \sum_{p=1}^{P} \gamma_p x_t^p$ by $\breve{\mu}_t$. The following result is immediate following similar steps of the proof in the main text.

Proposition 3 *Suppose that* $|\phi| < 1$. *Then, there is a unique interior solution for the individual equilibrium outcome given by*

$$l_{i,t}^*(\phi, g) = \{\mathbf{M}(\phi, \mathbf{G}_t)\}_{i.} \, \breve{\mu}_t, \tag{27}$$

where $\{\}_{i.}$ *is the operator that returns the i-th row of its argument,* $\breve{\mu}_t := [\breve{\mu}_{1,t}, ..., \breve{\mu}_{n,t}]'$, $l_{i,t}$ *denotes the total liquidity holding by bank i.*

The above result implies that, even in this more general model, the definitions of conditional volatility of liquidity, risk key player, and level key player[38], as well as their dependency on the network topology and equilibrium parameter ϕ, stay unchanged.

A.2.3 Level key player

Similar to risk key player, we can identify "systemic level key player", whose removal from the system causes the largest aggregate liquidity reduction in expectation.[39]

Definition 2 (Level key player) *The level key player* τ_t^* *is the player that, when removed, causes the maximum expected reduction in the overall level of total liquidity. We use* $\mathbf{G}_{\backslash \tau, t}$ *to denote the new adjacency matrix obtained by setting to zero all of* $\mathbf{G}_t$*'s τ-th row and column coefficients. The resulting network is* $g_{\backslash \tau, t}$. *The level key player* τ_t^* *is found by solving*

$$\tau_t^* = \arg\max_{\tau=1,...,n} \; \mathbb{E}\left[\sum_i z_i^*(\phi, g_t) - \sum_{i \neq \tau} z^*(\phi, g_{\backslash \tau, t}) \,\middle|\, g_t, \tau\right] \tag{28}$$

where $\mathbb{E}$ *is the expectation operator.*

We define level key player under the assumption that the removal of banks does not trigger immediately the formation of new links. Hence, we capture the short-run effects of a bank's sudden failure. Since we do not observe bank failure in our sample, we cannot provide a precise time frame for link formation after removal. Yet, our definition can be operational from a policy perspective, especially during a crisis when banks shun each other and link formation becomes less likely. Using Proposition 1, we have the following corollary.

---

[38]In this case, l^* should replace z^* in equation (28).

[39]This definition is in the same spirit as the concept of key player in the crime network literature, e.g., Ballester, Calvo-Armengol, and Zenou (2006), where targeting key players is important for crime reduction. Here, it is useful to consider the ripple effect on the aggregate liquidity when a bank fails and exits from the system. Injecting liquidity to out key level players might be necessary to avoid major disruptions to the whole system.

Corollary 1 *A player τ_t^* is the level key player that solves (28) if and only if*

$$\tau_t^* = \underset{\tau=1,\ldots,n}{\arg\max} \quad \underbrace{\{\mathbf{M}(\phi, \mathbf{G}_t)\}_{\tau.}\bar{\mu}}_{\text{Indegree effect}} + \underbrace{\mathbf{1}'\{\mathbf{M}(\phi, \mathbf{G}_t)\}_{.\tau}\bar{\mu}_\tau}_{\text{Outdegree effect}} - \underbrace{m_{\tau\tau}(\phi, \mathbf{G}_t)\bar{\mu}_\tau}_{\text{Double count correction}} \quad , \tag{29}$$

where $m_{\tau\tau}(\phi, \mathbf{G}_t)$ is the τ-th element of the diagonal of $\mathbf{M}(\phi, \mathbf{G}_t)$.

When bank τ is removed, its liquidity disappears from the system. This is the first component, the indegree effect, which depends on neighbours' $\bar{\mu}$ through $\{\mathbf{M}(\phi, \mathbf{G}_t)\}_{\tau.}$, the routes from neighbours to τ. The second component reflects bank τ's impact on other banks, so its own $\bar{\mu}_\tau$ is multiplied by the sum of routes from τ to neighbours, i.e., $\mathbf{1}'\{\mathbf{M}(\phi, \mathbf{G}_t)\}_{.\tau}$. This outdegree effect captures network externality. The level key player metric is particularly relevant for a central planner who decides on which bank to help to sustain the aggregate liquidity buffer. Such a decision depends on a bank's own contribution to aggregate liquidity and the spillover effects through network linkages. As in the risk key player metric, focusing on network alone is not enough. Both the attenuation factor ϕ and bank-specific characteristics, now captured by $\bar{\mu}$, are important inputs in computing key players.

A.2.4 Planner's solution vs. decentralised outcome: a close-form solution

The following corollary offers a closed-form characterization of the wedge between the planner's solution and the decentralised outcome.

Corollary 2 *Let $\mathbf{H}_t := \phi\mathbf{G}_t' - \psi(\psi - 2\delta/\gamma)\mathbf{G}_t'\mathbf{G}_t$. Then, the aggregate network liquidity in the planner's solution can be expressed as*

$$Z_t^p = Z_t^* + \mathbf{1}'\left[\psi\mathbf{M}_t\mathbf{G}_t' + \mathbf{M}_t\mathbf{H}_t\mathbf{M}_t(\mathbf{I} - \mathbf{H}_t\mathbf{M}_t)^{-1}(\mathbf{I} + \psi\mathbf{G}_t')\right]\mu_t \tag{30}$$

where Z_t^* denotes the aggregate bilateral liquidity in the decentralised equilibrium in equation (15) and $\mathbf{M}_t := \mathbf{M}(\phi, \mathbf{G}_t)$. Moreover, if $\mathbf{H}_t$ is invertible, we have

$$Z_t^p = Z_t^* + \mathbf{1}'\left[\psi\mathbf{M}_t\mathbf{G}_t' + \mathbf{M}_t(\mathbf{H}_t^{-1} - \mathbf{M}_t)^{-1}\mathbf{M}_t(\mathbf{I} + \psi\mathbf{G}_t')\right]\mu_t. \tag{31}$$

Proof. If $\mathbf{H}_t$ is invertible, observing that

$$\mathbf{M}^p(\phi, \psi, \delta, \mathbf{G}_t) \equiv \left[\mathbf{M}(\phi, \mathbf{G}_t)^{-1} - \phi\mathbf{G}_t' + \psi(\psi - 2\delta/\gamma)\mathbf{G}_t'\mathbf{G}_t\right]^{-1}(\mathbf{I} + \psi\mathbf{G}_t')$$

and using the Woodbury matrix identity (see, e.g. Henderson and Searle (1981)) gives

$$\mathbf{M}^p(\phi, \psi, \delta, \mathbf{G}_t) = \mathbf{M}_t + \mathbf{M}_t(\mathbf{H}_t^{-1} - \mathbf{M}_t)^{-1}\mathbf{M}_t,$$

hence the result is immediate. If $\mathbf{H}_t$ is not invertible, using equation (26) in Henderson and Searle (1981), we obtain

$$\mathbf{M}^p\left(\phi, \psi, \delta, \mathbf{G}_t\right) = \mathbf{M}_t + \mathbf{M}_t\mathbf{H}_t\mathbf{M}_t(\mathbf{I} - \mathbf{H}_t\mathbf{M}_t)^{-1}$$

and the result follows. ∎

The above implies that the discrepancy between the planner solution and the decentralised solution can be positive or negative depending on the parameters and topology of the network (the eigenvalues of the canonical operator of $\mathbf{G}_t$).[40]

A.3 Details of the empirical methodology

A.3.1 Quasi-maximum likelihood formulation and identification

Writing the variables and coefficients of the spatial error model in equations (12) and (13) in matrix form as[41]

$$B := [\alpha_1^{time}, ..., \alpha_t^{time}, ..., \alpha_T^{time}, \alpha_1^{bank}, ..., \alpha_i^{bank}, ..., \alpha_N^{bank},$$
$$\beta_1^{bank}, .., \beta_m^{bank}, ..., \beta_M^{bank}, \beta_1^{time}, ..., \beta_p^{time}, ..., \beta_P^{time}]',$$

$$L := [l_{1,1}, ..., l_{N,1}, ..., l_{i,t}, ..., l_{1,T}, ..., l_{N,T}], \quad \mathbf{z} := [z_{1,1}, ..., z_{N,1}, ..., z_{i,t}, ..., z_{1,T}, ..., z_{N,T}]'$$

$$\nu := [\nu_{1,1}, ..., \nu_{N,1}, ..., \nu_{i,t}, ..., \nu_{1,T}, ..., \nu_{N,T}]', \quad \underline{\mu} := \mathbf{1}_T \otimes [\bar{\mu}_1, ..., \bar{\mu}_N]',$$

$$\mathbf{G} := diag\left(\mathbf{G}_t\right)_{t=1}^T = \begin{bmatrix} \mathbf{G}_1 & \mathbf{0} & ... & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2 & ... & ... \\ ... & ... & ... & \mathbf{0} \\ \mathbf{0} & ... & \mathbf{0} & \mathbf{G}_T \end{bmatrix}, \quad \mathbf{X} := \left[\mathbf{D}, \mathbf{F}, \mathbf{X}^{bank}, \mathbf{X}^{time}\right],$$

where $\mathbf{D} := I_T \otimes \mathbf{1}_N$, $\mathbf{F} := \mathbf{1}_T \otimes I_N$, and

$$\mathbf{X}^{time} = \begin{bmatrix} x_1^1 & ... & x_1^p & ... & x_1^P \\ ... & ... & ... & ... & ... \\ x_t^1 & ... & x_t^p & ... & x_t^P \\ ... & ... & ... & ... & ... \\ x_T^1 & ... & x_T^p & ... & x_T^P \end{bmatrix} \otimes \mathbf{1}_N, \quad \mathbf{X}^{bank} = \begin{bmatrix} x_{1,1}^1 & ... & x_{1,1}^m & ... & x_{1,1}^M \\ ... & ... & ... & ... & ... \\ x_{N,1}^1 & ... & x_{N,1}^m & ... & x_{N,1}^M \\ ... & ... & ... & ... & ... \\ x_{N,T}^1 & ... & x_{N,T}^m & ... & x_{N,T}^M \end{bmatrix},$$

---

[40]See, e.g., Gorodentsev (1994) for the definition of canonical operator. The proof of this result is very involved, hence we present it in an appendix available upon request.

[41]This is similar to the spatial formulation in Lee and Yu (2010).

we can then rewrite the empirical model as

$$L = \mathbf{X}B + \mathbf{z}, \quad \mathbf{z} = \underline{\mu} + \phi \mathbf{G} \mathbf{z} + \nu, \quad \nu_{i,t} \sim iid\left(0, \sigma_i^2\right).$$

This, in turn, implies that

$$\nu\left(B, \underline{\mu}, \phi\right) = \left(I_{N \times T} - \phi \mathbf{G}\right)\left(L - \mathbf{X}B\right) - \underline{\mu}. \tag{32}$$

Finally, using the Gaussian distribution to model the exogenous error terms ν yields the log likelihood

$$\ln \mathcal{L}\left(B, \phi, \underline{\mu}, \left\{\sigma_i^2\right\}_{i=1}^N\right) \equiv -\frac{TN}{2}\ln\left(2\pi\right) - \frac{T}{2}\sum_{i=1}^N \ln \sigma_i^2 - \sum_{i=1}^N \frac{1}{2\sigma_i^2}\sum_{t=1}^T \nu_{i,t}\left(B, \underline{\mu}, \phi\right)^2, \tag{33}$$

and the above can be estimated using standard optimization methods.

In the above formulation, the identification of ϕ is ensured by the usual conditions on $\mathbf{G}$ (see, e.g. Bramoullé, Djebbari, and Fortin (2009)). Instead, the separate identification of the bank fixed effects, $\alpha^{bank} := \left[\alpha_1^{bank}, ..., \alpha_N^{bank}\right]'$, and the network-bank fixed effects, $\bar{\mu} := \left[\bar{\mu}_1, ..., \bar{\mu}_N\right]'$, deserve some further remarks. Isolating the role of these fixed effects, equation (32) can be rewritten as

$$\nu\left(B, \underline{\mu}, \phi\right) = \left(I_{N \times T} - \phi \mathbf{G}\right)\left(L - \tilde{\mathbf{X}}\tilde{B} - \mathbf{F}\alpha^{bank}\right) - \underline{\mu}$$
$$= \left(I_{N \times T} - \phi \mathbf{G}\right)\left(L - \tilde{\mathbf{X}}\tilde{B}\right) - \mathbf{1}_T \otimes \left(\bar{\mu} + \alpha^{bank}\right) + \phi \mathbf{G} \mathbf{F} \alpha^{bank}$$

where $\tilde{\mathbf{X}} := \left[\mathbf{D}, \mathbf{X}^{bank}, \mathbf{X}^{time}\right]$ and $\tilde{B}$ is simply the vector B without the α^{bank} elements. Several observations are in order. First, the above implies that if $\phi = 0$, then $\bar{\mu}$ and α^{bank} cannot be separately identified (nevertheless the parameters $\tilde{B}$ are still identified). Second, if there is no time variation in the network structure, i.e. if $\mathbf{G}_t = \bar{\mathbf{G}} \ \forall t$, $\bar{\mu}$ and α^{bank} cannot be separately identified even if $\phi \neq 0$. Third, if a bank never lends to any other bank in the sample, its fixed effects $\bar{\mu}_i$ and α_i^{bank} cannot be separately identified. Fourth, if $\mathbf{G}_t$ is a right stochastic matrix, separate identification of $\bar{\mu}$ and α^{bank} can be achieved only up to a parameter normalization, since for any scalar κ and vector $\bar{\kappa} := \mathbf{1}_N \otimes \kappa$, we have

$$\nu\left(B, \underline{\mu}, \phi\right) = \left(I_{N \times T} - \phi \mathbf{G}\right)\left(L - \tilde{\mathbf{X}}\tilde{B}\right) - \mathbf{1}_T \otimes \left(\bar{\mu} + \alpha^{bank} + \phi \bar{\kappa}\right) + \phi \mathbf{G} \mathbf{F}\left(\alpha^{bank} + \bar{\kappa}\right)$$

The above also makes clear that a handy normalisation is to set one of the network-bank fixed effect (say the i-th one) to zero since it would imply the restriction $\left\{\alpha^{bank} + \phi \bar{\kappa}\right\}_i = \left\{\alpha^{bank} + \bar{\kappa}\right\}_i$ that, for any $\phi \neq 0$ and 1, can only be satisfied with $\kappa = 0$. Under this

normalisation, the remaining estimated bank-network fixed effects are then in deviation from the normalised one. Fifth, note that the lack of separate identification for $\bar{\mu}$ and α^{bank} is due to the fact that when $\mathbf{G}_t$ is a right stochastic matrix, and if all banks borrow from at least one bank at each point in time (i.e. $\mathbf{G}_t$ has no rows of zeros), then $\mathbf{G}_t \mathbf{1}_N = \mathbf{1}_N$ and $\mathbf{G}\mathbf{1}_{N \times T} = \mathbf{1}_{N \times T}$. Fortunately, in our dataset, the condition $\mathbf{G}_t \mathbf{1}_N = \mathbf{1}_N$ does not hold every day in the sample because there are periods in which certain banks do not borrow (in this case, the corresponding rows of $\mathbf{G}_t$ contain all zeros and sum to zero, instead of one). In our sample, except for bank 7 and bank 11, all the other banks borrow every period from at least one of their counterparties. There are fourteen days when bank 7 does not borrow at all, and 145 days in which bank 11 does not borrow at all. Moreover, the no borrowing days of bank 7 and bank 11 do not overlap, so we have a total of 159 days in which either the sum of the 7th row of $\mathbf{G}_t$ or the sum of the 11th row of $\mathbf{G}_t$ is equal to zero, not one (13.5% of the days).

A.3.2   Confidence bands for the network impulse response functions

The ϕ estimator outlined in the previous section has an asymptotic Gaussian distribution with variance s_ϕ^2 (that can be readily estimated from the QMLE covariance matrix based, as usual, on the Hessian and gradient of the log likelihood in equation (33)). That is, $\sqrt{T}\left(\hat{\phi} - \phi_0\right) \xrightarrow{d} N\left(0, s_\phi^2\right)$, where ϕ_0 denotes the true value of ϕ. Writing

$$a_1\left(\phi\right) := \frac{\partial \mathbf{1}'\left\{(\mathbf{I} - \phi\mathbf{G})^{-1}\right\}_{.i}}{\partial \phi}, \quad a_2\left(\phi\right) = \frac{\partial \mathbf{1}'\left\{(\mathbf{I} - \phi\mathbf{G})^{-1}\phi\mathbf{G}\right\}_{.i}}{\partial \phi}$$

we have from Lemma 2.5 of Hayashi (2000) that

$$\sqrt{T}\left[NIRF_i\left(\hat{\phi}, 1\right) - NIRF_i\left(\phi_0, 1\right)\right] \xrightarrow{d} N\left(0, a_1\left(\phi_0\right)^2 s_\phi^2\right),$$

$$\sqrt{T}\left[NIRF_i^e\left(\hat{\phi}, 1\right) - NIRF_i^e\left(\phi_0, 1\right)\right] \xrightarrow{d} N\left(0, a_2\left(\phi_0\right)^2 s_\phi^2\right).$$

Therefore, since $a_j\left(\hat{\phi}\right) \xrightarrow{p} a_j\left(\phi_0\right)$, $j = 1, 2$, by the continuous mapping theorem, and by Slutsky's theorem, $a_j\left(\hat{\phi}\right)\hat{s}_\phi^2 \xrightarrow{p} a_j\left(\phi_0\right)^2 s_\phi^2$, where $\hat{s}_\phi^2$ is a consistent variance estimator, we can construct confidence bands for the network impulse response functions using the sample estimates of ϕ and s_ϕ^2.

A.3.3   Details on variable construction

Macro control variables

- rK_{t-1}: lagged right kurtosis of the intraday time of aggregate payment outflow:

$$rK_t = \frac{\sum_{\tau > m_t} \left(\frac{\tau - m_t}{\sigma_t}\right)^4}{\sum_{\tau=1}^{88} \left(\frac{\tau - m_t}{\sigma_t}\right)^4}$$

where

$$m_t = \frac{1}{88} \sum_{\tau=1}^{88} \tau \left(\frac{P_{t,\tau}^{OUT}}{\sum_{\tau=1}^{88} P_{t,\tau}^{OUT}}\right), \quad \sigma_t^2 = \frac{1}{88-1} \sum_{\tau=1}^{88} (\tau - m_t)^2 \left(\frac{P_{t,\tau}^{OUT}}{\sum_{\tau=1}^{88} P_{t,\tau}^{OUT}}\right)$$

and $P_{t,\tau}^{OUT}$ is the aggregate payment outflow at time interval τ. Note that transactions are recorded for 88 10-minute time intervals within each day (from 5:00 to 19:30). The variable m_t is the average of payment time weighted by the payment outflow.

- $\ln VolPay_{t-1}$: intraday volatility of aggregate liquidity available (lagged by one day, in logarithms). "Liquidity available" is defined below at bank level.

- TOR_{t-1}: lagged turnover rate in the payment system. To define the turnover rate, we need first to define the Cumulative Net (Debit) Position (CNP):

$$CNP(T, i, s) = \sum_{t=1}^{T} (P_{i,s,t}^{OUT} - P_{i,s,t}^{IN}),$$

where $P_{i,s,t}^{OUT}$ is bank i's total payment outflow at time t in day s. $P_{i,s,t}^{IN}$ is the payment inflow. The turnover rate (in day s) is defined as

$$TOR_s = \frac{\sum_{i=1}^{N} \sum_{t=1}^{88} P_{i,s,t}^{OUT}}{\sum_{i=1}^{N} \max\{\max_T[CNP(T; i, s)], 0\}}$$

The numerator is the total payment made in the system at day s. The denominator sums the maximum cumulative net debt position of each bank at day s. Note that in the denominator, if the cumulative net position of a certain bank is always below zero (that is, this bank's cumulative inflow alway exceeds its cumulative outflow), this bank actually absorbs liquidity from the system. If there are banks absorbing liquidity from the system, there must be banks injecting liquidity into the system. When we calculate the turnover rate (the ratio between the total amount circulating and the base), we should only consider one of the two. That's why we take the first (outside) maximum operator. The reason for the inside operator goes as follows: any increase in the cumulative net debit position (wherever positive) incurs an injection of liquidity into the system, so the maximum of the cumulative net position is the total injection

from the outside to the payment system. And, the sum over different banks gives the total injection through all the membership banks. A higher turnover rate means a more frequent reuse of liquidity injected from outside into the payment system.

- *LIBOR*: daily LIBOR rate, lagged by one day.

- *Interbank Rate Premium:* average interbank rate minus LIBOR, lagged by one day.

Bank-level control variables

- Intraday liquidity available (LA) is the amount of liquidity to meet payment requirements and is measured as the sum of reserves ($SDAB$, Start of Day Account Balance) plus the value of intraday repo available with the BoE (PC, Posting of Collateral). As time passes, the liquidity available in CHAPS is calculated by subtracting the money moved to CREST from the liquidity available in the previous time interval. In this way, we can trace for bank i the liquidity available at any time t in day s:

$$LA(t, i, s) = SDAB_{i,s} + PC_{i,s} - \sum_{\tau=1}^{t} CREST_{i,s,\tau}$$

- Liquidity holdings ($l_{i,t}$, i.e., the dependent variable): the logarithm of reserve balances plus collateral posted to the BoE at the beginning at the day.

- *Interbank Rate*: interbank borrowing rate, lagged by one day.

- $\ln LevPay_{i,t-1}$: total intraday payment level (lagged by one day, in logarithms).

- $rK_{i,t-1}^{in}$: right kurtosis of incoming payment time, lagged by one day.

- $rK_{i,t-1}^{out}$: right kurtosis of outgoing payment time, lagged by one day.

- $\ln VolPay_{i,t-1}$: log intraday volatility of liquidity available, lagged by one day.

- $\ln LU_{i,t-1}$: liquidity used is defined as follows

$$LU(i, s) = \max\{\max_{T}[CNP(T; i, s)], 0\}.$$

The first (inside) maximum operator picks the highest level of cumulative net debit position (CNP) within a day. About the second (outside) maximum operator, if the highest level is negative, the bank i is always a liquidity contributor on day s, so LU is equal to zero. When used in estimation, it is lagged by one day and in logarithms.

- $\frac{repo\ Liability}{Assets}$: repo liability to total asset ratio (lagged, monthly).

- *Total Assets (log)*: total asset (lagged, monthly, in logarithms).

- $\frac{\Delta Deposits}{Assets}$: the change of retail deposits to total assets ratio $\times$ 100 (lagged, monthly).

- *Total Lending and Borrowing (log)*: total lending and borrowing in the interbank market (lagged by one day, in logarithms).

- *CDS Spread*: an index (log cumulative change) of 5-year senior unsecured CDS spread.

- *Stock Return*: gross stock return including dividends (lagged by one day).

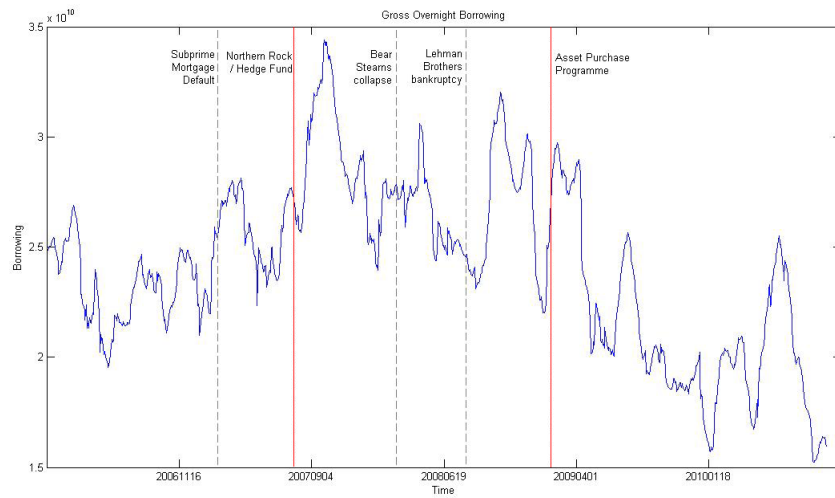## A.4   Additional figures and tables

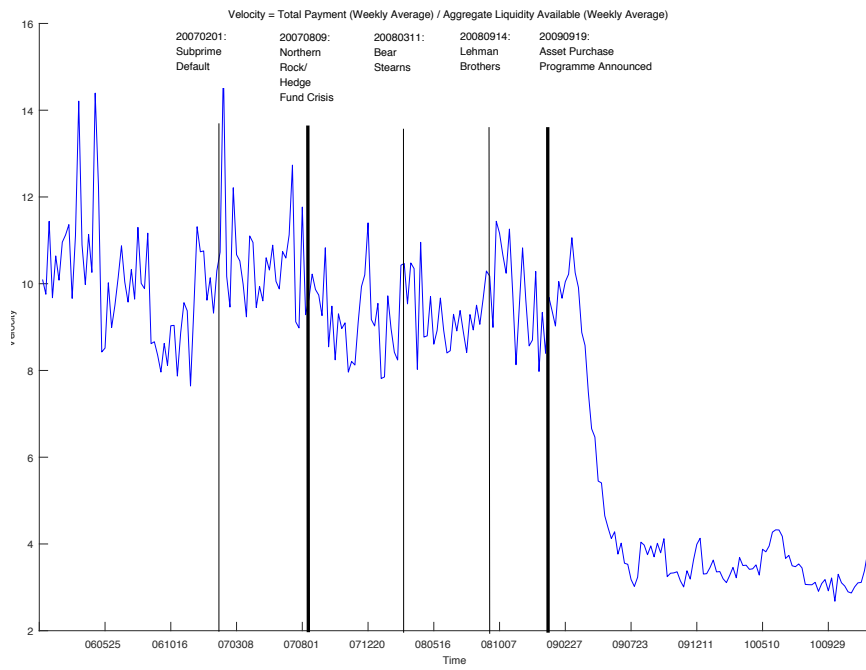Figure 9: Daily gross interbank borrowing (rolling monthly average).

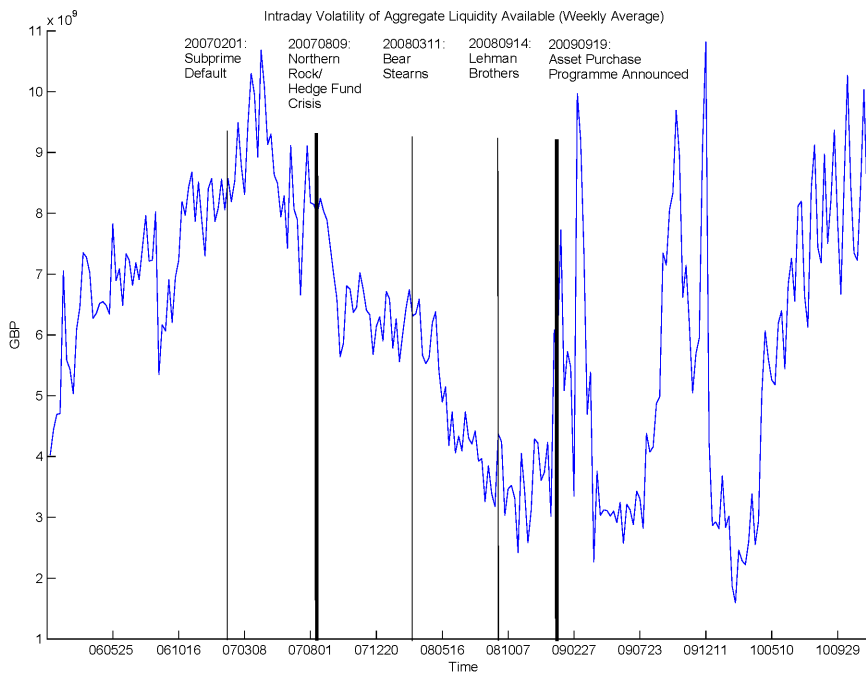Figure 10: Velocity of liquidity in the payment system.

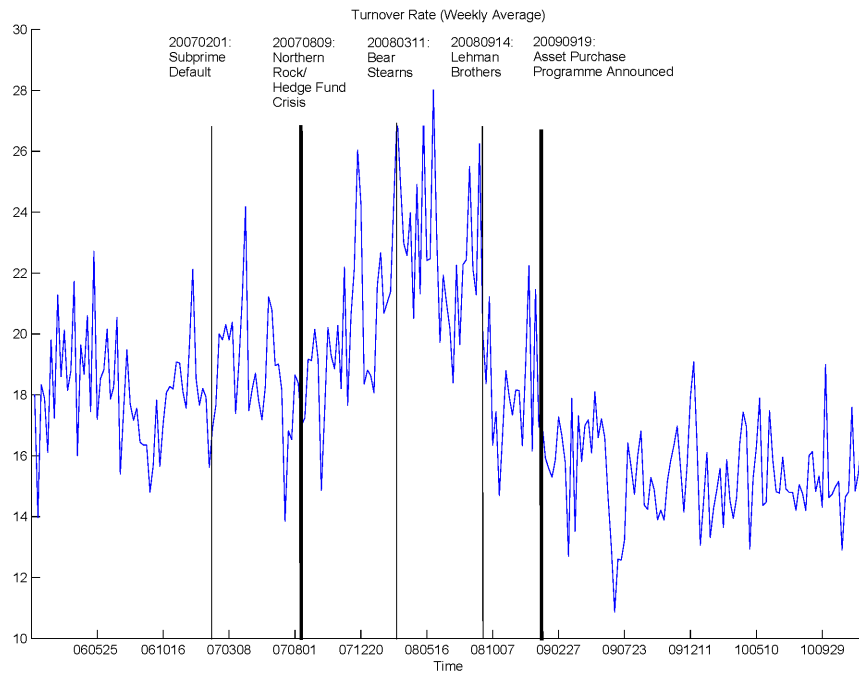Figure 11: Intraday volatility of aggregate outflows.

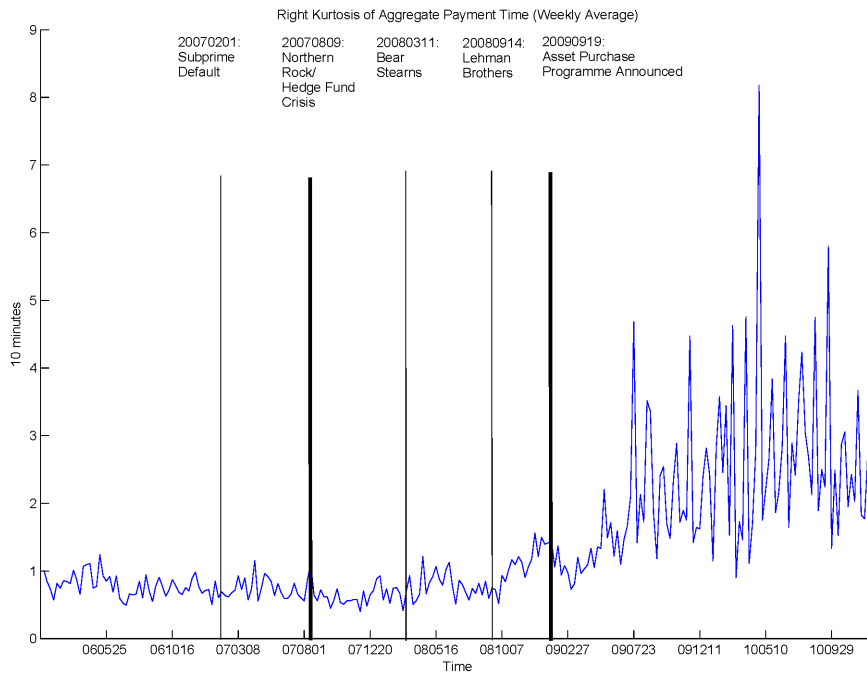Figure 12: Turnover rate in the payment system.

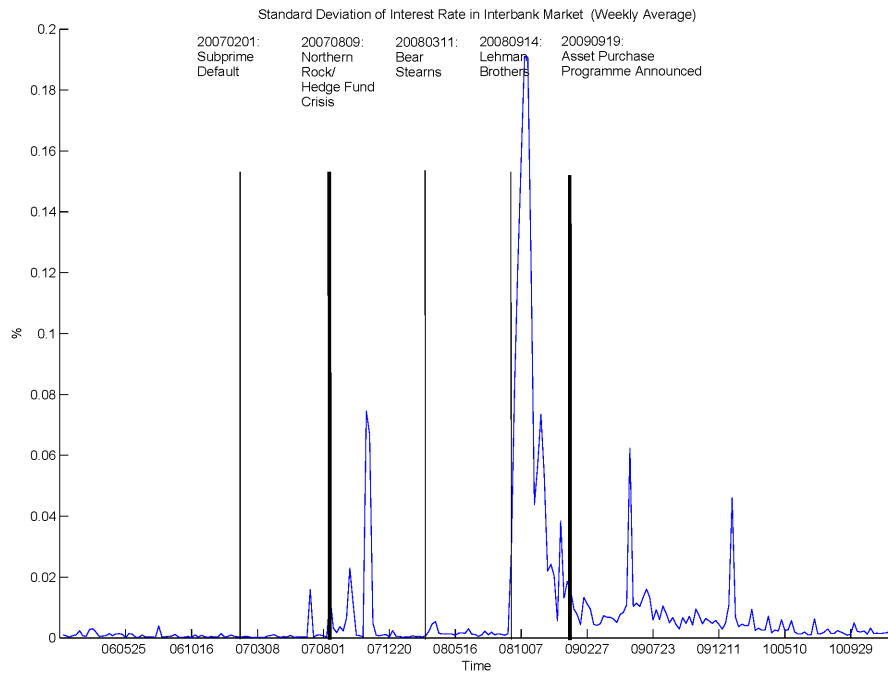Figure 13: Right kurtosis of aggregate payment times.

Figure 14: Cross-sectional dispersion of interbank rates.

Table A1: Spatial Error Model Estimation Results

	Period 1	Period 2	Period 3
$\hat{\phi}$	0.8137^* (21.47)	0.3031^* (1.90)	-0.1794^* (-4.96)
$1/\left(1-\hat{\phi}\right)$	5.3677^* (4.92)	1.4349^* (4.37)	0.8479^* (32.61)
Macro Control Variables			
rK_{t-1}	0.1845 (1.30)	0.0084 (0.55)	-0.0032^* (-3.88)
$\ln VolPay_{t-1}$	-0.4451 (-1.00)	0.0308 (1.17)	0.0291 (1.72)
TOR_{t-1}	0.0166 (1.80)	0.0007 (0.69)	0.0018 (1.75)
LIBOR	0.2378 (0.27)	0.0928 (1.28)	0.5800^* (2.52)
Interbank Rate Premium	3.8845 (1.61)	-0.0405 (-0.33)	0.6973^* (3.00)
Bank Characteristics/Mircro Control Variables			
Interbank Rate	-0.2081 (-0.98)	-0.0473 (-1.03)	-0.0880 (-1.92)
$\ln LevPay_{i,t-1}$	-0.0235 (-0.62)	0.0802^* (3.29)	0.0808^* (5.09)
$rK_{i,t-1}^{in}$	0.0010 (0.14)	-0.0086 (-0.63)	0.0045 (1.03)
$rK_{i,t-1}^{out}$	0.0090 (0.92)	0.0320^* (3.62)	-0.0061 (-1.32)
$\ln VolPay_{i,t-1}$	0.0129^* (4.59)	0.0039 (1.92)	0.0196^* (5.96)
$\ln LU_{i,t-1}$	-0.0038^* (-2.86)	-0.0039^* (-3.41)	-0.0027^* (-3.79)
$\frac{\text{Repo Liability}}{\text{Assets}}$	-5.5625^* (-3.61)	0.0282 (0.43)	-0.3057 (-1.45)
Total Assets (log)	1.2590^* (5.39)	0.6328^* (10.31)	1.0170^* (18.92)
$\frac{\Delta\text{Deposits}}{\text{Assets}}$	-0.0014 (-0.20)	0.0149^* (5.15)	0.0481^* (11.76)
Total Lending and Borrowing (log)	-0.1882^* (-5.57)	0.0612^* (2.95)	-0.0025 (-1.27)
CDS Spread	0.0051 (0.13)	-0.1212^* (-6.61)	-0.0383^* (-4.00)
Stock Return	-0.5667 (-0.88)	0.1927 (1.49)	0.2574 (1.88)
R^2	66.01%	92.09%	91.53%

Estimation results for equations (12) and (13). Periods 1, 2 and 3, correspond, respectively, to before the Northern Rock/BNP Paribas Fund Crisis, after it but before the first BoE announcement of Asset Purchase Programme, and the QE period. The t-statistics are reported in parentheses under the estimated coefficients, and * denotes statistically significant estimates at a 10% or higher confidence level. Standard errors are QMLE-robust ones, and for the average network multiplier, $1/(1-\hat{\phi})$, the delta method is employed.

Table A2: ratio of network to idiosyncratic volatility.

	Period 1	Period 2	Period 3
Bank 1	2.54	1.05	0.98
Bank 2	2.10	1.04	1.02
Bank 3	1.83	1.06	0.87
Bank 4	2.62	1.06	1.08
Bank 5	2.41	1.10	1.02
Bank 6	1.65	1.09	1.03
Bank 7	1.47	0.97	1.13
Bank 8	1.69	1.09	1.03
Bank 9	2.12	1.09	1.03
Bank 10	1.62	0.99	1.09
Bank 11	2.04	1.14	1.31
Mean	2.01	1.06	1.05

The table reports $\sqrt{\frac{Var(\hat{z}_{i,t})}{Var(\hat{v}_{i,t})}}$ for each bank and each period considered.