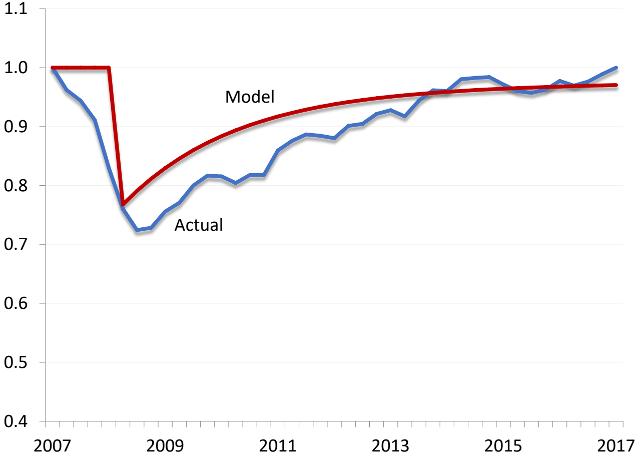
Investment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Model
Actual
2007
2009
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2013
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The idea of this paper

Real business cycle-style model

extended to comprehend unemployment

with confidence as the single driving force of aggregate fluctuations

Fairly successful in accounting for fluctuations of the past decade, flowing from the collapse of confidence during the financial crisis and the return to normal more recently

Scope

Explore the macroeconomics of fluctuations in confidence

not a contribution to knowledge of the sources of the collapse of confidence

Basics

Collapse of investment, including consumer durables

Collapse of job-creation, as in Hall *AER* (2017)

Small percentage decline in consumption of nondurables and services, not yet captured by the model (Barro-King)

Ways to model a drop in confidence

1. The value that investors perceive from future payoffs declines—myopia with higher utility discount rate
2. Changing beliefs about future adverse tail events

A simple model suggests that a higher utility discount works and a worsening belief about adverse events does not work

The economy lasts for two periods

An investor has a consumption endowment of 1 unit in the first period

In the second, the endowment is random: 1 unit in state N (normal) and $1 - q$ units in state B (bad)

State N arises with probability $1 - \pi$ and the bad state B arises with probability π

Equilibrium

State prices p_N and p_B, denominated in terms of first-period output, for the receipt of one unit of value in the future state

Utility is

$$u(1) + \frac{1}{1+\rho}[(1-\pi)u(1) + \pi u(1-q)]$$

First-order conditions determine the state prices

$$\frac{1}{1+\rho}(1-\pi) = p_N$$

$$\frac{1}{1+\rho}\pi\mu = p_B$$

$\mu > 1$ is the ratio of marginal utility in state B, $u'(1-q)$, to marginal utility in period 1, $u'(1)$

The state prices measure the payoff to a hypothetical investment

The payoff, measured as of period 1, to a risky claim that delivers one unit of output in period 2 under state N and $1-q$ units of output under state B, is

$$\frac{1}{1+\rho}[1-\pi+\pi\mu\cdot(1-q)]$$

With a constant coefficient of relative risk aversion of γ, $\mu=(1-q)^{-\gamma}$, so $\mu\,(1-q)=(1-q)^{-(\gamma-1)}$

Under the reasonable assumption that $\gamma>1$, $\mu\cdot(1-q)$ is an increasing function of q and the payoff to the investment is correspondingly an increasing function of q

Because $q>0$, $(1-q)^{-(\gamma-1)}>1$ and the payoff is an increasing function of the disaster probability π

Under the assumption that $\gamma > 1$, the incentive to invest will decline if

- the utility discount rate ρ rises
- the disaster shortfall of consumption, q, falls
- the disaster probability π falls

Risk

The model's approach to risk is to consider the discounts applied to risky payoffs

Rather than an explicit stochastic discounter, each stochastic payoff has its own discount rate reflecting its covariance with the latent stochastic discounter

In financial equilibrium, the discount rate for a risky investment equals the expected return to that investment

The financial discount is a feature of general equilibrium and is not generally the same as the utility discount

Example

In the two-period, two-future-state example, the expected return ratio for an investment that pays off one unit of value in the second period in state 1 and $1-q$ units in state 2 is

$$R = \frac{1-\pi+\pi(1-q)}{p_N + p_B(1-q)}$$

Under the earlier assumption marginal utility is one in state N and $(1-q)^{-\gamma}$ in state B, the expected return ratio is

$$R = (1+\rho)\,\frac{1-\pi+\pi(1-q)}{1-\pi+\pi(1-q)^{-(\gamma-1)}}$$

The financial discount rate

$$r = R - 1$$

so, if $q = 0$, the financial rate would be the utility discount rate, ρ

The financial discount rate differs from the utility discount rate in the presence of risk, when $q > 0$ and $\pi > 0$

Three financial determinants can be driving forces

1. the utility discount rate, ρ, which raises r
2. the disaster probability π, which lowers r, and
3. the disaster consumption shortfall q, which also lowers r

The model in this paper considers ρ as the exclusive driving force

The model portrays the utility discount factor as fluctuating smoothly in continuous time

Its change is governed by the instantaneous discount rate ρ—the discount applied to time-t utility as of time zero is

$$\beta(t) = \exp\left(-\int_0^t \rho(\tau)d\tau\right)$$

Technology and capital

The only variation in employment, n, arises from variations in unemployment

Production is Cobb-Douglas with capital elasticity α

Capital depreciates at rate δ

The law of motion of the capital stock, k is

$$\dot{k} = k^{\alpha} n^{1-\alpha} - \delta k - c$$

Investment

The supply of installed capital relates to Tobin's q, with cost κ, as

$$\dot{k} = \frac{k}{\kappa}(q-1)$$

The demand for installed capital satisfies

$$\dot{q} = q(r+\delta) - \alpha\left(\frac{n}{k}\right)^{1-\alpha}$$

Consumption

Infinitely-lived investor-households spread consumption over time according to an Euler equation with an elasticity of intertemporal substitution σ and a time-varying utility discount rate ρ:

$$\dot{c} = \sigma(r - \rho)c$$

Here r is the economy's time-varying financial discount

Labor market

Employers place a value J on an employee

J is the present value of the future stream of value the worker will contribute, the difference between the worker's marginal product and the worker's wage; the difference is normalized at one:

$$\dot{J} = (r + s)J - 1$$

In the DMP model, J determines the flow rate of vacancy-filling, which determines the tightness of the market, which in turn determines the job-finding rate, which determines the unemployment rate, and the employment rate is the complement of the unemployment rate

Summarize this chain by a linear relation between J and n:

$$n = \bar{n} + \phi J$$

Driving force

The single driving force in the model is ρ, the utility discount

It begins at ρ_0 and returns at rate ω to its normal level, ρ^*:

$$\dot{\rho} = -\omega(\rho - \rho^*)$$

Uncontroversial parameters

- $\alpha = 0.4$
- $\delta = 0.1$,
- $s = 0.18$,
- $\rho^* = 0.05$
- $\bar{n}$ chosen to equate the stationary value of employment, n, to its long-run average value of 0.945

Parameters less well pinned down by data and research

The adjustment cost parameter, κ: A standard value at an annual rate from research based on the first-order condition for optimal investment is 2, but the results in this paper point toward a considerably higher value, 8; use $\kappa = 2$ as a variant

The intertemporal elasticity of substitution, σ; the results here point toward a low value of 0.2; use 0.5 as a variant

The sensitivity ϕ of the employment rate n to the job value J; results here point toward a value of 0.05; this value overcomes the Shimer puzzle; use $\phi = 0$ as a variant, interpreted as a growth model without employment volatility

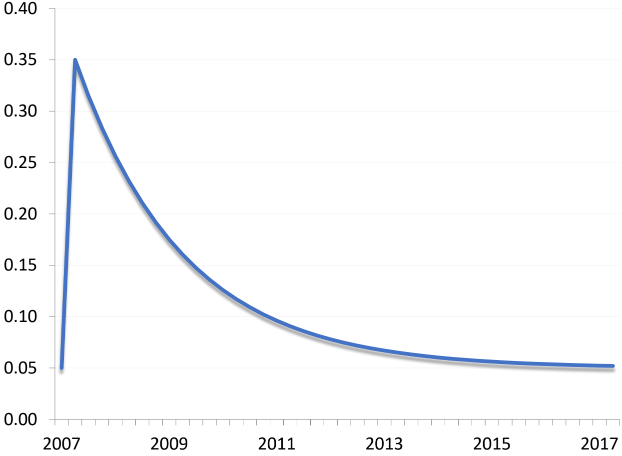
The model contains two distinct discount rates

One is ρ, the rate households apply to future utility—the discount applied to time-t utility as of time zero is

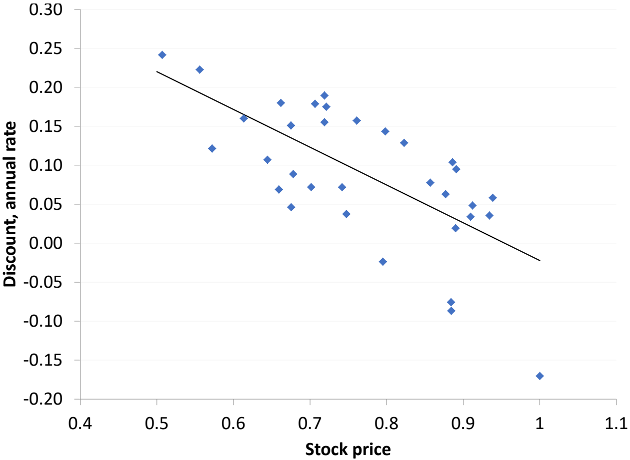
$$\exp\left(-\int_0^t \rho(\tau)d\tau\right)$$

The other is the financial discount rate, the asset-specific rate r that discounts future expected payoffs back to the present; it is also the expected return to the asset

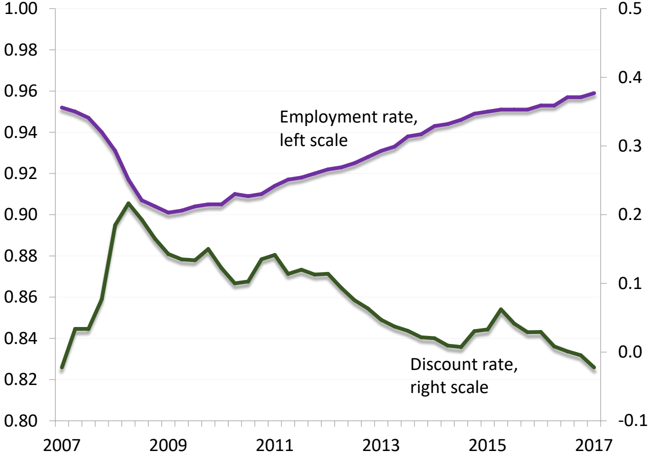
The Utility Discount Rate, Driving Force of the Model



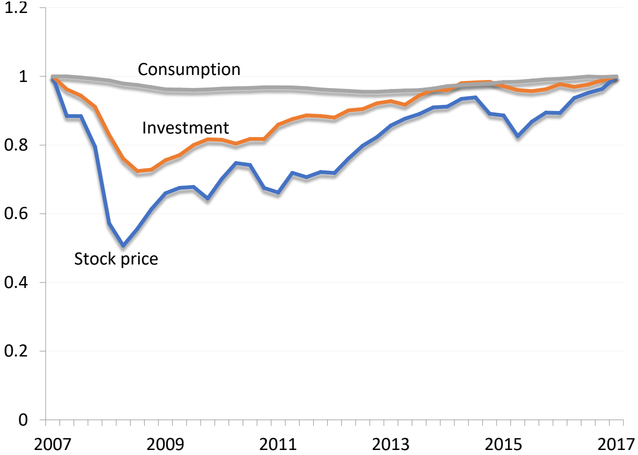
Campbell-Shiller Analysis of Expected Returns in the Stock Market



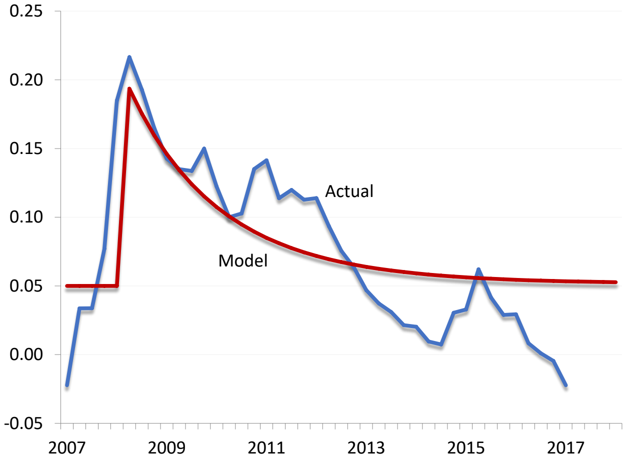
Data for the Employment Rate and the Discount Rate for the Stock Market, 2007 through 2017



Normalized and Detrended Data for Consumption, Investment, and Stock Price, 2007 through 2017



Financial Discount Rate

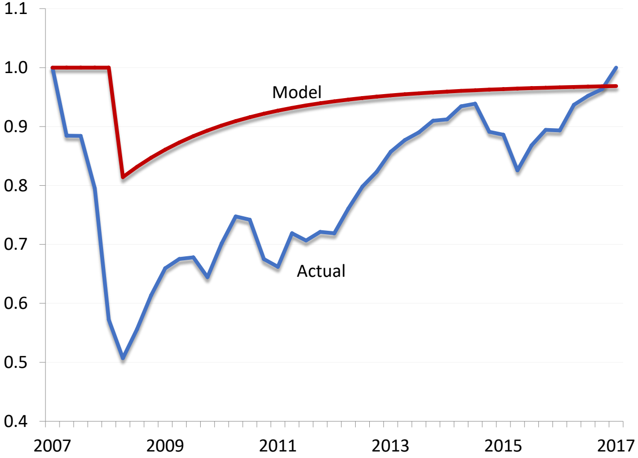


The stock market

The value of the stock market in the model is the product, $q\,k$, of the market price of installed capital and the quantity of capital

plus the value of the established relationships with workers, $n\,J$

Stock Price



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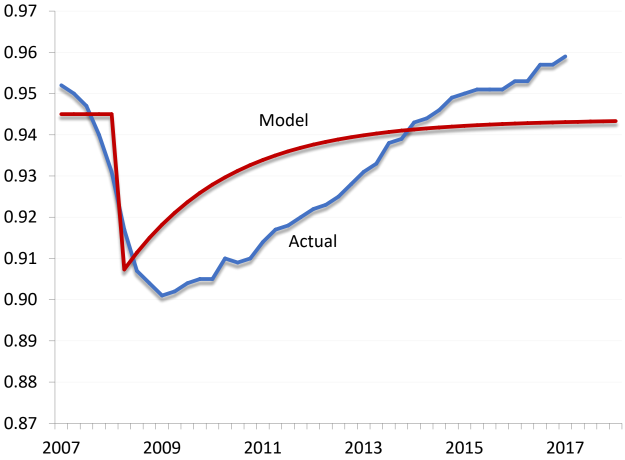
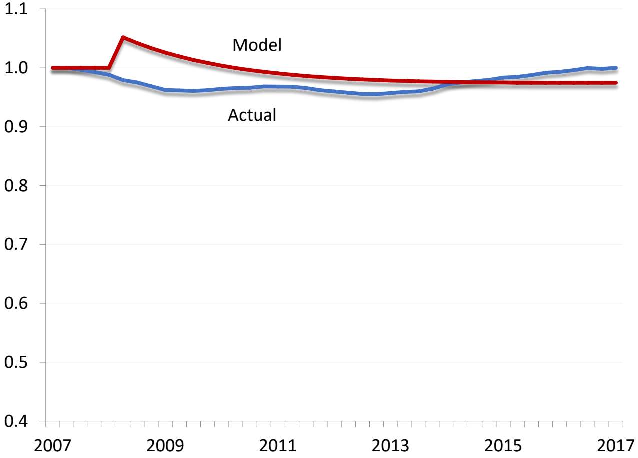
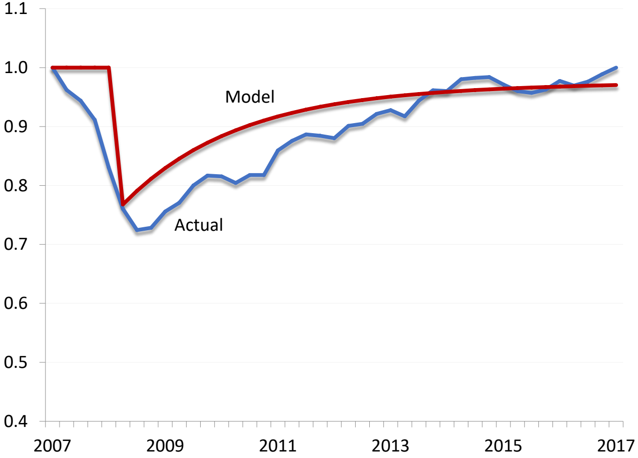
Employment Rate



Consumption



Investment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Results for Variants of Parameter Values

	Actual	*Base model*	*Lower adjustment cost*	*Growth model*	*Higher intertempral substitution*
			$\kappa = 2$	$\varphi = 0$	$\sigma = 0.5$
Stock price	0.50	0.82	0.90	0.85	0.74
Investment	0.73	0.77	0.74	0.81	0.67
Consumption	1.00	1.05	1.08	1.07	1.07
Employment rate	0.910	0.907	0.923	0.945	0.892
Discount rate	0.20	0.19	0.13	0.17	0.27