

# The interdependence of bank capital and liquidity \*

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## Abstract

We analyze the interdependent effects of bank capital and liquidity on financial stability in a global game model, where banks are exposed to both solvency and liquidity crises with endogenously determined probabilities. We show that capital and liquidity have ambiguous effects on fragility depending on the nature of the crisis and the initial bank balance sheet composition. We then characterize banks' balance sheet decisions and show that these may entail an inefficient liquidation of banks' portfolios and a fire sale externality. Joint capital and liquidity regulation eliminates these inefficiencies only when the cost of bank equity and liquidity is contained and market funding conditions are good.

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# 1 Introduction

The 2007-2009 financial crisis was a milestone for financial regulation, leading to significant reforms to the existing capital regulation and the introduction of a new set of liquidity requirements. In particular, banks have been required to hold higher capital buffers to reduce their exposure to solvency-driven crises and, at the same time, to increase their liquidity holdings to reduce liquidity mismatch and the consequent risk of liquidity-driven crises. The introduction of a new set of liquidity requirements, namely the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR), as complements to the existing and improved capital-based regulation, has led to a debate in the academic and policy arena on the effective need of all these regulatory tools, their interaction, as well as their potential contrasting effects for financial stability.

Bank (il)liquidity and (in)solvency are closely intertwined concepts and often difficult to tell apart when a crisis manifests. On the one hand, liquidity-driven crises can spur solvency issues; on the other hand, fears about bank solvency may precipitate liquidity problems. Furthermore, when a crisis is underway and a bank faces a large outflow of funds, it becomes very difficult to assess the ultimate source of these withdrawals, which, in turn, may limit policymakers' ability to intervene effectively. It is precisely this close link between solvency and liquidity crises that motivates the discussion about the joint effects that capital and liquidity may have on financial stability.

To visualize the issue, consider a simple bank balance as in Table 1. Bank stability depends negatively on both its leverage (i.e.,  $\frac{D}{L+I}$ ) and the proportion of illiquid assets (i.e.,  $\frac{I}{I+L}$ ): A bank with a larger share of short-term funding and a larger proportion of illiquid assets is more exposed to roll over risk than a bank with more equity and more liquid assets. It follows that increasing equity ( $E$ ), while keeping constant the asset side, has a similar effect on stability as increasing the proportion of liquid assets ( $L$ ), while keeping the liability side constant. This means that, in this example, capital and liquidity are substitutes in terms of their effects on stability.

Assets	Liabilities
Liquid assets ( $L$ )	Short-term debt ( $D$ )
Illiquid assets ( $I$ )	Equity ( $E$ )

Table 1: A simplified bank balance sheet

This simple example raises a number of important questions: what are the effects of changes in the level of bank capitalization and portfolio liquidity on bank stability? Are these effects different depending on

the nature of crises— whether they are solvency- or liquidity-driven— and on the bank initial balance sheet composition?

To tackle these questions, we build a two-period model where banks issue short term debt and equity, and invest in a risky portfolio consisting of both liquid and illiquid assets, whose final return depends on the fundamentals of the economy. The portfolio composition shapes the trade-off between intermediate and final date portfolio returns, whereby a higher proportion of liquid assets in the portfolio leads to a higher (safe) return at the interim date, but to a lower (risky) return at the final date. This determines bank available resources and, together with the bank capital structure, affects the likelihood of a bank failure.

In our model, a bank default can be driven by both solvency and liquidity considerations and the probability of each type of crisis is determined endogenously using the global-game methodology. At the interim date, each debt holder receives an imperfect signal regarding bank portfolio return at the final date and, based on this signal, decides whether to roll over or withdraw his debt claim. His decision depends on which of the two actions gives him the highest payoff, which, in turn, depends on the fundamentals of the economy as well as on the expectations about the proportion of debt holders rolling over.

As standard in the global game literature (see e.g., Morris and Shin, 1998, 2003; Goldstein and Pauzner, 2005), the equilibrium outcome is that bank failures take the form of a massive withdrawals of funds by debt holders at the interim date and occur when fundamentals of the economy are below a unique threshold. Within the range of fundamentals where they occur, crises can be classified into either solvency or liquidity crises. The former happen at the lower part of the crisis region where the signal on the fundamentals is so low that not rolling over the debt claim at the interim date is a dominant strategy for debt holders. The latter hinges on the existence of strategic complementarity among debt holders, in that each of them does not roll over out the self-fulfilling belief that others will do the same.

The probability of both a solvency and a liquidity crisis depends on the level of bank capitalization and its portfolio liquidity. Thus, our model delivers a first set of results about the differential effects that a change in the level of bank capitalization and portfolio liquidity has on these probabilities. In particular, we show that an increase in the level of bank capitalization always reduces the likelihood of a solvency crisis, while an increase in the level of liquidity of bank portfolio always increases it.

The effects on the likelihood of a liquidity crisis are more involved, as they depend on whether banks have low, intermediate or high initial levels of bank capitalization and portfolio liquidity. For banks with very little capital and/or very illiquid portfolios, an increase in capital or liquidity worsens the probability of

liquidity crises. As these banks are already facing a high risk of failure at the interim date, higher levels of capitalization or portfolio liquidity raise debt holders' repayment at the interim date, thus increasing further their incentives to run on the bank. By contrast, for banks with intermediate levels of capital or liquidity, higher capitalization or increased portfolio liquidity reduce the occurrence of liquidity crises. As the risk of failing at the interim date is limited for those banks, a capital or liquidity injection increases further their ability to withstand debt holders' withdrawals and, in turn, debt holders' repayment from rolling over the debt claim till the final date. Finally, for banks with high initial levels of capital or liquidity, higher capitalization is beneficial for stability, while more portfolio liquidity is detrimental. The reason is that these banks are very little exposed to strategic complementarities among debt holders and mainly face the risk of solvency crises, for which higher capital is beneficial while liquidity is detrimental.

The comparative statics exercise delivers some initial implications for the design of regulation. First, a one-size-fits-all approach, where all banks are subject to the same requirements, may have undesirable consequences for some banks, especially for those who would need to strengthen their stability the most. Second, capital and liquidity requirements should be designed considering both sides of banks' balance sheets. In this respect, our analysis supports regulatory instruments like the risk-weighted capital ratio, the liquidity coverage ratio and the net stable funding ratio that essentially specify a ratio between banks' assets and liabilities (see, Cecchetti and Kashyap, 2018).

Building on the comparative statics exercise, we then analyze bank's choice of capital structure, portfolio liquidity and debt holders' repayment in the market equilibrium and show that the allocation entails two inefficiencies. First, banks choose intermediate levels of capitalization and portfolio liquidity that expose them to inefficient solvency and liquidity crises. Second, while banks consider market funding conditions at the intermediate date as exogenous to their individual choices, in equilibrium these depend on the aggregate quantity of bank assets on sale and thus on banks' exposure to crises. When market conditions are tight, that is when the assets on sale are excessive relative to the wealth of investors acquiring them, asset early liquidation entails a loss of value due to limited asset redeployment. We refer to this inefficiency as fire sales and show that market tightness depends on banks' initial balance sheet choices through their effect on the amount of portfolios in need of liquidation.

We then turn to analyze whether imposing capital and liquidity requirements allows to restore full efficiency. We show that this depends on the tightness of the market for banks' assets, as well as on how costly capital and liquidity are for individual banks. When market conditions are good and the cost of capital

and liquidity for banks is small, the regulator can enforce the efficient liquidation of banks' portfolios and prevent fire sales with capital or liquidity requirements. By contrast, when the market conditions are tight or raising capital and holding liquidity is costly for banks, it may not be feasible for the regulator to correct both inefficiencies. It follows that the allocation will either feature only efficient liquidation of banks' assets but more severe fire sales or too little liquidation with reduced fire sales.

Our analysis of the impact of capital and liquidity on bank stability is conducted in a framework where the inefficiencies of the unregulated market equilibrium are all associated with the premature liquidation of bank portfolio. In doing this, we disregard other possible sources of inefficiencies that may motivate the use of capital and liquidity regulation, such as, for example, a moral hazard problem on the side of bank managers. Despite this, to the best of our knowledge our framework is the first to allow disentangling the effects of capital and liquidity on solvency and liquidity crises.

A number of recent papers have looked at the role and implications of the newly introduced liquidity regulation, also in connection with capital requirements (see e.g., Walther, 2015; Calomiris, Heider and Hoerova, 2015; and Diamond and Kashyap, 2016). Among those papers, Vives (2014) and König (2015) use global game models to study the implications of capital and liquidity on the probability of banking crises. Both papers build on the bank run model developed by Rochet and Vives (2004) and perform a comparative statics exercise on the run threshold. Vives (2014) finds that both capital and liquidity are beneficial for stability, while, as in our paper, König (2015) shows that the effect of liquidity is more mixed since liquid assets are less profitable than illiquid ones in the long run. Although sharing the global game approach, our framework features a richer structure for debt holders' payoff similar to Goldstein and Pauzner (2005). This introduces additional effects of capital and liquidity on bank stability. Furthermore, we endogenize bank capital structure and portfolio liquidity, as well as the remuneration to debt holders. This allows us to highlight several inefficiencies of the market equilibrium and derive implications for optimal regulation.

In this sense, our paper is related to the recent contribution by Kashyap, Tsomocos and Vardoulakis (2017), who also use global game techniques to pin down bank default probabilities in a framework where banks are subject to moral hazard but there are no fire sales. It follows that, in contrast to our analysis, in Kashyap et al. (2017) both capital and liquidity regulation reduce the probability that a run occurs and improves welfare.

The key aspect of our study is the ability to derive an endogenous probability of (solvency and liquidity) crises and study how it is affected by changes in bank capitalization and portfolio liquidity. To do this, we

rely on the global game techniques as developed in the literature originating with Carlsson and van Damme (1993) (see Morris and Shin, 2003 for a survey on the theory and applications of global games). Our paper is close to two contributions in this literature. First, it shares the idea of rollover game with Eisenbach (2017), although in a framework where banks also raise equity and choose the liquidity-return trade-off of their portfolio. Second, it faces the same technical challenge of characterizing the existence of a unique equilibrium in the absence of global strategic complementarities as in Goldstein and Pauzner (2005).

The paper proceeds as follows. Section 2 presents the baseline model. Section 3 shows the effect that changes in capital and liquidity have on the probability of solvency- and liquidity-driven crises. Section 4 characterized the unregulated equilibrium. Section 5 identifies the inefficiencies of the unregulated equilibrium and analyzes the effectiveness of regulation in addressing them. Section 6 contains concluding remarks. All proofs are contained in the appendix.

## 2 The model

There are three dates ( $t = 0, 1, 2$ ), a continuum  $[0, 1]$  of banks and a continuum  $[0, 1]$  of investors in each bank. Investors are risk-neutral and are endowed with one unit of resources at date 0 each and nothing thereafter.

At date 0, each bank raises a fraction  $k$  of equity at a total cost  $\rho > 1$  per unit and a fraction  $1 - k$  of short term debt at a total per unit (normalized) opportunity cost of 1.<sup>1</sup> In exchange for their funds, debt holders are promised a (gross) interest rate  $r_1$  if they withdraw their investment at date 1 and  $r_2 > r_1$  if they leave their funds until date 2. The (net) interest rates on debt is assumed to be non-negative, so that  $r_1 \geq 1$ . If the bank cannot repay the promised interest rates  $\{r_1, r_2\}$  at either date, the bank fails and debt holders receive a share of the bank's available resources. The deposit market is perfectly competitive so that the bank will always set  $r_1$  and  $r_2$  at the level required for depositors to recover their opportunity cost of funds of 1 and be willing to participate. The assumption that  $\rho > 1$  captures the idea that bank capital is a more expensive form of financing than deposits, as is typically assumed in the literature (see e.g., Hellmann, Murdock and Stiglitz, 2000; Repullo, 2004; Allen, Carletti and Marquez, 2011) and has been recently endogenized on the basis of market segmentation (see e.g., Allen, Carletti and Marquez, 2015) or the existence of costs associated with the issuance of outside equity (see Harris, Opp and Opp, 2017) and

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<sup>1</sup>Banks' reliance on short-term debt has been justified in the literature based on its beneficial effect on asymmetric information problems in credit markets (see e.g., Flannery, 1986 and Diamond, 1991) or its disciplining role for banks' managers and shareholders (see e.g., Calomiris and Kahn, 1991; Diamond and Rajan, 2001 and Eisenbach, 2017).

empirically validated based on a different tax treatment between equity and debt (see e.g., Schepens, 2016).<sup>2</sup>

Banks invest the funds raised by investors in a risky portfolio consisting of both liquid and illiquid assets. The portfolio returns  $\ell\chi$  per unit if liquidated at date 1 and  $R(\theta)(1 - \alpha\ell)$  at date 2, where  $\ell \in [0, 1]$  represents the level of portfolio liquidity the bank chooses at date 0, while the variable  $\theta$  describes the state of the economy and is uniformly distributed over  $[0, 1]$  with  $R'(\theta) > 0$ . Finally,  $\chi \in (0, 1)$  and  $\alpha \in (0, \bar{\alpha})$  with  $\bar{\alpha} < 1$  are constants capturing, respectively, market funding conditions at date 1 (on which we will return below in Section 5) and the cost of liquidity. Our specification entails a liquidity-return trade-off in the choice of the bank portfolio: The more liquid a bank portfolio, the higher its date 1 return, but the lower its expected return at date 2. Yet, we assume  $E_\theta[R(\theta)(1 - \alpha\ell)] > 1$  so that the bank finds it optimal to invest in a portfolio rather than storing.

The state of the economy  $\theta$  is realized at the beginning of date 1, but is not publicly observed until date 2. After  $\theta$  is realized, at date 1 each debt holder receives a private signal  $s_i$  of the form

$$s_i = \theta + \varepsilon_i, \tag{1}$$

where  $\varepsilon_i$  are small error terms that are independently and uniformly distributed over  $[-\varepsilon, +\varepsilon]$ . Based on this signal, debt holders decide whether to withdraw their investment at date 1 or roll it over until date 2.

The timing of the model is as follows. Each bank chooses the terms of the debt contract  $\{r_1, r_2\}$ , the capital structure  $\{k, 1 - k\}$  and the level of portfolio liquidity  $\ell$  at date 0 so as to maximize its expected profit. At date 1, after receiving the private signal about the state of the fundamentals  $\theta$ , each debt holder decides whether to withdraw at date 1 or roll over the debt. At date 2, the bank portfolio return realizes and all claims are paid, if the bank is solvent. The model is solved backwards.

### 3 Market equilibrium: bank fragility

In this section, we analyze debt holders' rollover decision, for given levels of capital and portfolio liquidity. This allows us to pin down the probability of a bank failure at date 1, as well as to distinguish between solvency- and liquidity-driven crises. As we will explain in details below, in equilibrium both types of crisis are triggered by a massive withdrawal of funds by debt holders (i.e., a run). However, a solvency crisis is

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<sup>2</sup>In most jurisdictions, the cost of debt is tax-deductible, while dividends are not. Schepens (2016) shows that a reduction in the tax discrimination between debt and equity financing leads to a significant increase in bank capital ratios.

due to debt holders' expectations of low realizations of  $\theta$ , while a liquidity crisis is triggered by debt holders' fear that other investors will not roll over their debt claim, thus forcing the bank to liquidate its portfolio at date 1. Below we characterize the probability of each type of crisis and its properties.

A solvency crisis occurs in the range of fundamental  $\theta$  in which not rolling over the debt at date 1 is a dominant strategy for every debt holder. This is the case when, upon receiving his signal, a debt holder expects the bank to fail at date 2 and obtain a pro-rata share  $\frac{R(\theta)(1-\alpha\ell)}{(1-k)}$  lower than the return  $r_1$  he would obtain by withdrawing at date 1, even if all other debt holders wait until date 2. We then denote as  $\underline{\theta}(k, \ell, r_1)$  the value of  $\theta$  that solves

$$R(\theta)(1-\alpha\ell) - (1-k)r_1 = 0 \quad (2)$$

so that the interval  $[0, \underline{\theta}(k, \ell, r_1))$  identifies the range of values of  $\theta$  where a banking crisis is driven by solvency considerations.<sup>3</sup> In what follows, we simply refer to  $\underline{\theta}(k, \ell, r_1)$  as the probability of a solvency crisis, which, as shown (2), depends on the level of bank capitalization  $k$  and the liquidity of its portfolio  $\ell$ , as well as on the interest rate  $r_1$ . We have the following result.

**Proposition 1** *The threshold  $\underline{\theta}(k, \ell, r_1)$  is decreasing in  $k$  and increasing both in  $\ell$  and  $r_1$ , i.e.,  $\frac{\partial \underline{\theta}(k, \ell, r_1)}{\partial k} < 0$ ,  $\frac{\partial \underline{\theta}(k, \ell, r_1)}{\partial \ell} > 0$  and  $\frac{\partial \underline{\theta}(k, \ell, r_1)}{\partial r_1} > 0$ .*

The proposition highlights the key role that capital and liquidity play for the emergence of solvency crises. Capital has a beneficial effect because higher capital reduces leverage, thus leaving more resources to repay debt holders. By contrast, liquidity has a detrimental effect on bank solvency. This is due to the negative impact that liquidity has on bank profitability, that is on the date 2 (per unit) portfolio return  $R(\theta)(1-\alpha\ell)$ . The interest rate offered to debt holders at date 1 also has a detrimental effect on the occurrence of solvency crises, as an increase in  $r_1$  makes it more likely for a debt holder to withdraw at date 1.

Besides insolvency, a banking crisis can be also driven by liquidity considerations. Even when the state of the economy  $\theta$  is higher than  $\underline{\theta}(k, \ell, r_1)$ , debt holders may have the incentive not to rollover their debt claims at the interim date as they fear that others would do the same. Their concern is that a large number of withdrawals at date 1 would force a massive liquidation of the bank portfolio, thus depleting the bank's available resources at date 2 and, in turn, their expected payoff.

The signal  $s_i$  plays a key role for debt holders' withdrawal decision and thus for the occurrence of liquidity-

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<sup>3</sup>For the region of solvency crises to exist, it must be the case that there are feasible values of  $\theta$  for which all debt holders receive a signal below  $\underline{\theta}(k, \ell, r_1)$ . Since the noise contained in the signal is at most  $\varepsilon$ , when  $s_i < \underline{\theta}(k, \ell, r_1) - \varepsilon$  all debt holders receive a signal below  $\underline{\theta}(k, \ell, r_1)$ . This holds when  $\theta < \underline{\theta}(k, \ell, r_1) - 2\varepsilon$ .



driven crises. The reason is that the signal provides information on both  $\theta$  and other debt holders' actions. When the signal is high, a debt holder attributes a high posterior probability to the event that the bank portfolio yields a high return and, at the same time, he infers that the other debt holders have also received a high signal. This overall lowers his belief about the likelihood of a bank failure and, as a result, also his own incentive to withdraw at date 1. Conversely, when the signal is low, a debt holder has a high incentive not to roll over the debt, as he attributes a high likelihood to the possibility that the return of the bank portfolio is low and that the other investors withdraw their debt claim at date 1. As a result, in the region for  $\theta \geq \underline{\theta}(k, \ell, r_1)$  a debt holder's decision about whether to roll over its debt claim at date 1 depends on the realization of  $\theta$  as well as on his beliefs regarding the other debt holders' actions. To see this, we specify a debt holder's payoff from withdrawing at date 1 and that from rolling the claim over until date 2.

A debt holder's payoff at date 1 is given by

$$\pi_1 = \begin{cases} r_1 \geq 1 & \text{if } 0 \leq n < \bar{n} \\ \frac{\ell\chi}{(1-k)n} & \text{if } \bar{n} \leq n \leq 1 \end{cases}, \quad (3)$$

where  $\bar{n} = \frac{\ell\chi}{(1-k)r_1}$  corresponds to the solution to  $\ell\chi = (1-k)nr_1$ . At date 1, a debt holder obtains  $r_1 \geq 1$  as long as the value of bank portfolio at date 1  $\ell\chi$  is enough to repay  $r_1$  to all  $(1-k)n$  withdrawing debt holders, i.e., for any  $n < \bar{n}$ . Otherwise, when  $n \geq \bar{n}$ , the bank is forced to liquidate the entire portfolio at date 1 and each debt holder receives a pro-rata share of the bank portfolio equal to  $\frac{\ell\chi}{(1-k)n}$ .

Consider now a debt holder's payoff at date 2. It is given by

$$\pi_2 = \begin{cases} r_2 > r_1 & \text{if } 0 \leq n < \hat{n}(\theta) \\ \frac{R(\theta)(1-\alpha\ell) \left[ 1 - \frac{(1-k)nr_1}{\ell\chi} \right]}{(1-k)(1-n)} & \text{if } \hat{n}(\theta) \leq n \leq \bar{n} \\ 0 & \bar{n} \leq n \leq 1 \end{cases}, \quad (4)$$

where  $\hat{n}(\theta)$  corresponds to the solution to  $R(\theta)(1-\alpha\ell) \left[ 1 - \frac{(1-k)nr_1}{\ell\chi} \right] = (1-k)(1-n)r_2$  and denotes the proportion of investors not rolling over the debt at date 1 pushing the bank at the brick of default at date

2. The threshold  $\hat{n}(\theta)$  is then equal to

$$\hat{n}(\theta) = \frac{R(\theta)(1-\alpha\ell) - (1-k)r_2}{(1-k)\left[\frac{R(\theta)(1-\alpha\ell)r_1}{\ell\chi} - r_2\right]}. \quad (5)$$

At date 2, a debt holder obtains  $r_2$  as long as the bank is solvent, otherwise he obtains a pro-rata share of bank available resources. Whether the bank is solvent or not, it depends on the realization of  $\theta$ , as well as on the proportion  $n$  of debt holders not rolling over at date 1, as they determine both the value of bank portfolio as well as that of the bank liabilities. As the proportion  $n$  of debt holders withdrawing at the interim date 1 increases, a debt holder's incentive to withdraw at date 1 also increases, even if not monotonically. In the range  $[0, \bar{n}]$ , a debt holder's payoff at date 2 is weakly decreasing in  $n$  as long as

$$\ell\chi < (1-k)r_1 \quad (6)$$

The condition (6) guarantees that  $\hat{n}(\theta) < 1$  and is obtained by differentiating  $\frac{R(\theta)(1-\alpha\ell)\left[1 - \frac{(1-k)nr_1}{\ell\chi}\right]}{(1-k)(1-n)}$  with respect to  $n$ . In words, it states that the value of a bank's portfolio  $\ell\chi$  is not enough to repay  $r_1$  if all debt holders were to withdraw at date 1. The condition implies that debt holders' withdrawal decisions are strategic complements and justifies the use of global games techniques to eliminate the associated multiplicity of equilibria and pin down a unique equilibrium (see, e.g., Morris and Shin, 1998, 2003).

As in Goldstein and Pauzner (2005), our model only exhibits the property of one-sided strategic complementarity since in the range  $(\bar{n}, 1]$ , a debt holder's incentive to roll over his debt claim until date 2 increases with  $n$ . This occurs because, when  $n$  is very large (i.e.,  $n > \bar{n}$ ), the more debt holders do not roll over at date 1, the lower a debt holder's payoff from withdrawing at date 1, while the payoff at date 2 is zero. Despite this, as in their framework, there exists a unique threshold equilibrium in which a debt holder withdraws if and only if his signal is below the threshold  $s^*(k, \ell, r_1, r_2)$ . At this signal value, the debt holder is indifferent between withdrawing at date 1 and rolling over his debt claim until date 2. The following result holds.

**Proposition 2** *The model has a unique threshold equilibrium in which debt holders withdraw their debt claim at date 1 if they observe a signal below the threshold  $s^*(k, \ell, r_1, r_2)$  and roll it over above. At the limit, when  $\varepsilon \rightarrow 0$ ,  $s^*(k, \ell, r_1, r_2) \rightarrow \theta^*(k, \ell, r_1, r_2)$  and corresponds to the solution to*

$$\int_0^{\hat{n}(\theta)} r_2 dn + \int_{\hat{n}(\theta)}^{\bar{n}} \frac{R(\theta)(1-\alpha\ell)\left[1 - \frac{(1-k)nr_1}{\ell\chi}\right]}{(1-k)(1-n)} dn - \int_0^{\bar{n}} r_1 dn - \int_{\bar{n}}^1 \frac{\ell\chi}{(1-k)n} dn = 0. \quad (7)$$

Thus, the bank is solvent at date 2 for any  $\theta > \theta^*(k, \ell, r_1, r_2)$ .

The proposition states that in the interval for  $\theta \geq \underline{\theta}(k, \ell, r_1)$ , a debt holder's rollover decision is driven by the fear that others will not roll over, thus reducing a bank's available resources at date 2 and, in turn, his expected repayment. When  $(1 - k)r_1 > \ell\chi$ , a debt holder has the incentive to withdraw when he expects a large proportion of investors to do the same, as otherwise he faces the risk of not receiving anything at date 2. In other words, in the range  $\theta \in [\underline{\theta}(k, \ell, r_1), \theta^*(k, \ell, r_1, r_2))$  a bank fails at date 1 as result of a coordination failure among debt holders spurred by the fear that the bank would not have enough liquidity to repay the debt claims. For any  $\theta > \theta^*(k, \ell, r_1, r_2)$ , all debt holders choose to roll over their debt claims and the bank is solvent. To keep the notation simple, in what follows, we denote the thresholds  $\underline{\theta}(k, \ell, r_1)$  and  $\theta^*(k, \ell, r_1, r_2)$  as  $\underline{\theta}$  and  $\theta^*$ , respectively.

As illustrated in Figure 1, we can then distinguish three regions of fundamental  $\theta$ : For  $\theta \in [0, \underline{\theta})$  a banking crisis occurs and it is solvency-driven; for  $\theta \in [\underline{\theta}, \theta^*)$  a bank failure is liquidity-driven; finally, for  $\theta > \theta^*$  no banking crisis occurs.

Insert Figure 1

Similarly to the threshold for solvency crises  $\underline{\theta}$ , also  $\theta^*$  depends on the level of bank capitalization  $k$  and its portfolio liquidity  $\ell$ , as well as on  $r_1$  and  $r_2$ . While the effects of the interest rates  $r_1$  and  $r_2$  on  $\theta^*$  are straightforward, with the former increasing the crisis threshold and the latter decreasing it, the effects of bank capital and liquidity are more involved. The following lemma illustrates the channels through which changes in the level of bank capitalization and its portfolio liquidity affect the threshold of a liquidity crisis, respectively, for given  $r_1$  and  $r_2$ .

**Lemma 1** *The sign of the effect of capital  $k$  on  $\theta^*$  (i.e.,  $\frac{\partial \theta^*}{\partial k}$ ) is equal to the sign of*

$$\frac{1}{(1-k)^2} \left[ - \int_{\hat{n}(\theta^*)}^{\bar{n}} \frac{R(\theta^*)(1-\alpha\ell)}{(1-n)} dn + \ell\chi \int_{\bar{n}}^1 \frac{1}{n} dn \right], \quad (8)$$

*while that of portfolio liquidity  $\ell$  (i.e.,  $\frac{\partial \theta^*}{\partial \ell}$ ) corresponds to the sign of*

$$\frac{1}{(1-k)\ell} \left[ \int_{\hat{n}(\theta^*)}^{\bar{n}} \alpha\ell \frac{R(\theta^*)}{1-n} dn + \ell\chi \int_{\bar{n}}^1 \frac{1}{n} dn - \int_{\hat{n}(\theta^*)}^{\bar{n}} \frac{R(\theta^*)(1-k)nr_1}{(1-n)\ell\chi} dn \right]. \quad (9)$$

The lemma shows that changes in the level of bank capitalization and portfolio liquidity affect the

likelihood of a liquidity crisis through different channels. The effect of capital on the threshold  $\theta^*$  is twofold. On the one hand, an increase in capital reduces  $\theta^*$  since, by increasing the pro-rata share received by debt holders at date 2 in the range  $[\widehat{n}(\theta), \bar{n}]$ , greater capital increases debt holders' incentives to roll over. This effect, as captured by the first term in (8), leads to a lower threshold  $\theta^*$ . On the other hand, an increase in capital also increases the pro-rata share received by debt holders at date 1 when the bank is facing a run. This effect, which is captured by the second term in (8), increases the threshold  $\theta^*$ . Despite being at odd with common wisdom, this latter effect captures the crowding out associated with strengthening bank capitalization: Increasing capital is a way for the bank to increase the proceeds for debt holders at date 1 at the expenses of equity holders who are wiped out in the case of bank failure.

The effect of liquidity on the threshold  $\theta^*$  is threefold. First, as captured by the first term in (9), an increase in bank portfolio liquidity translates into a lower (per unit) portfolio return at date 2 and, in turn, into a lower pro-rata share if the bank fails at date 2 for the debt holders in the range  $[\widehat{n}(\theta), \bar{n}]$ . This increases debt holders' incentives to withdraw at date 1, thus increasing the threshold  $\theta^*$ . In addition, the threshold  $\theta^*$  increases further with liquidity since, similarly to the case of greater capital, banks with more liquid portfolios can pay a higher pro-rata share to debt holders when the bank defaults at date 1, as captured by the second term in (9). Finally, the third term in (9) represents the beneficial effect that an increase in liquidity has on the pro-rata shares received by debt holders at date 2 in the range  $[\widehat{n}(\theta), \bar{n}]$  and, in turn, on their incentives to roll over. A more liquid portfolio implies that the bank needs to liquidate fewer units of its portfolio at date 1 to meet debt holders' withdrawals, thus leaving more resources for those who roll their debt claim over to date 2. This is the commonly recognized beneficial effect of liquidity and the main rationale behind the newly introduced liquidity regulation.

The overall effect of capital and liquidity on the threshold  $\theta^*$  depends on which of the various effects illustrated above dominates. The following proposition shows that this crucially depends on the initial level of bank capitalization and portfolio liquidity. We have the following result.

**Proposition 3** *The threshold  $\theta^*$  decreases with the level of capital  $k$  for any  $k \in [\widetilde{k}(\ell), 1]$ , and increases otherwise (i.e.,  $\frac{\partial \theta^*}{\partial k} < 0$  if  $k \geq \widetilde{k}(\ell)$  and  $\frac{\partial \theta^*}{\partial k} > 0$  otherwise). The threshold  $\theta^*$  decreases with portfolio liquidity  $\ell$  for any  $k \in (\underline{k}(\ell), \bar{k}(\ell))$ , and increases otherwise (i.e.,  $\frac{\partial \theta^*}{\partial \ell} < 0$  if  $\underline{k}(\ell) < k < \bar{k}(\ell)$  and  $\frac{\partial \theta^*}{\partial \ell} > 0$  otherwise). The boundaries  $\widetilde{k}(\ell)$ ,  $\underline{k}(\ell)$  and  $\bar{k}(\ell)$  are defined in the appendix and lie below the curve  $k^{\max}(\ell)$ , which corresponds to the solution to  $(1 - k)r_1 = \ell\chi$ .*

The proposition, which is also illustrated in Figure 2a and 2b, shows that bank capitalization and portfolio liquidity jointly determine the effect that an increase in capital or liquidity has on bank stability.

Insert Figure 2a and 2b

We start from capital. The proposition shows that a marginal increase in the level of capital increases the threshold  $\theta^*$  for banks positioned in the region below the curve  $\tilde{k}(\ell)$ , while it is beneficial otherwise. The region bounded by the curve  $\tilde{k}(\ell)$  identifies banks that are poorly capitalized and/or hold illiquid portfolios. For those banks, a marginal increase in capital has a detrimental effect on stability because they are very exposed to a failure at date 1, having a large amount of short term debt and/or little liquidity to face withdrawal demands. Thus, debt holders of these banks attach a higher weight to the effect of capital on their date 1 payoff than on that at date 2. The opposite is true for banks falling in the region above the curve  $\tilde{k}(\ell)$ , which are characterized by higher levels of capital and/or portfolio liquidity.

Regarding liquidity, the proposition shows that a marginal increase in liquidity is beneficial only for banks falling in the region between the curves  $\underline{k}(\ell)$  and  $\bar{k}(\ell)$ , that is for banks characterized by an intermediate level of capitalization and/or portfolio liquidity. The reason is that those banks are not as exposed to the same risk of failure at date 1 as banks in the region below  $\underline{k}(\ell)$ , but are still confronted with significant strategic complementarities among debt holders' actions and, in turn, with liquidity problems. Thus, for those banks holding a more liquid portfolio allows to liquidate fewer units at date 1 to repay withdrawing debt holders and so increases the expected payoff for those who decided to roll over.

For banks in the region below  $\underline{k}(\ell)$ , liquidity has a detrimental effect on the threshold  $\theta^*$  for same reason why capital is detrimental in the region below  $\tilde{k}(\ell)$ . By contrast, the negative effect of portfolio liquidity on  $\theta^*$  in the region above  $\bar{k}(\ell)$  hinges on the negative effect that liquidity has on the (per unit) return of bank's portfolio at date 2. Banks in this region are indeed characterized by high levels of capital and/or portfolio liquidity and so are unlikely to fail because of liquidity considerations.

There are a number of interesting implications resulting from Proposition 3. First, increasing capital or liquidity helps the banks that need this the least. An increase in the level of bank capitalization or portfolio liquidity improves stability precisely for banks facing a lower threshold of liquidity crises thanks to their better capitalization and liquidity positions, while it has a destabilizing effect for poorly capitalized banks and those holding very illiquid portfolios.

Second, the result that a marginal increase in liquidity undermines debt holders' incentives to roll over and so increases  $\theta^*$  would also hold in the case of injection of emergency liquidity by a LOLR, which does not affect bank's portfolio returns and so does not give rise to a negative impact on profitability. In other words, the detrimental effect of liquidity for poorly capitalized banks with illiquid portfolio does not depend on the fact that more liquid portfolios tend to be less profitable.

Third, the results in the proposition suggest that the timing of a regulatory/supervisory intervention is key: If banks are asked to recapitalize and/or hold more liquidity when a crisis is already underway (or is likely), this may precipitate rather than contain bank distress.

To sum up, the analysis of the properties of the threshold  $\theta^*$  shows that the same increase in capital or liquidity may have very different effects on stability for weakly, moderately or strongly capitalized banks, as well as for banks with low, moderate or high portfolio liquidity. In other words, it highlights the importance of the interaction between capital and liquidity to assess their effects on stability. As we will show in details below, this is crucial for the bank choice of capital and liquidity as well as for designing and evaluating capital and liquidity regulation.

## 4 Market equilibrium: Bank choice

In this section, we characterize bank date 0 decisions about capital  $k$ , portfolio liquidity  $\ell$  and interest rates on debt  $\{r_1, r_2\}$ . A bank chooses these variables simultaneously to maximize its expected profits as given by

$$\max_{k, \ell, r_1, r_2} \Pi^B = \int_{\theta^*}^1 [R(\theta)(1 - \alpha\ell) - (1 - k)r_2] d\theta - k\rho \quad (10)$$

st

$$IR^D: \int_0^{\theta^*} \frac{\ell\chi}{1 - k} d\theta + \int_{\theta^*}^1 r_2 d\theta \geq 1, \quad (11)$$

$$\Pi^B \geq 0, \quad (12)$$

$$0 \leq k \leq 1, \quad 0 \leq \ell \leq 1. \quad (13)$$

The bank chooses  $k$ ,  $\ell$ ,  $r_1$  and  $r_2$  to maximize its expected profits  $\Pi^B$  subject to a number of constraints. The condition in (11) represents debt holders' participation constraint, which must hold with equality given that banks have all bargaining power. For any  $\theta < \theta^*$ , all debt holders choose not to roll over their debt at

date 1, the bank is forced to liquidate the entire portfolio and each debt holder receives the pro-rata share of bank's available resources  $\frac{\ell\chi}{1-k}$ . For  $\theta \geq \theta^*$ , all debt holders choose to roll over the debt claim until date 2 and they receive the promised repayment  $r_2$ . The condition in (11) states that debt holders' expected payoff from providing funds to the bank must be at least equal to what they can get by storing their funds (i.e., 1). The second constraint (12) is a non-negativity constraint on bank profit, while the last two conditions in (13) are simply physical constraints on the level of capital and portfolio liquidity. The expression for  $\Pi^B$  in (10) reflects bank's limited liability in that the bank only repays debt holders when the return of the project is sufficiently high.

The solution to the bank's maximization problem yields the following result.

**Proposition 4** *The market equilibrium features  $r_1^B = 1$ ,  $r_2^B > 1$  and the pair  $\{k^B, \ell^B\}$  as the solution to*

$$-\left[\frac{\partial\theta^*}{\partial k} + \frac{\partial\theta^*}{\partial r_2} \frac{dr_2}{dk}\right] [R(\theta^*)(1 - \alpha\ell) - \ell\chi] + \rho - 1 = 0, \quad (14)$$

and

$$-\left[\frac{\partial\theta^*}{\partial \ell} + \frac{\partial\theta^*}{\partial r_2} \frac{dr_2}{d\ell}\right] [R(\theta^*)(1 - \alpha\ell) - \ell\chi] + \int_0^{\theta^*} \chi d\theta - \int_{\theta^*}^1 \alpha R(\theta) d\theta = 0. \quad (15)$$

*The equilibrium pair  $\{k^B, \ell^B\}$  identifies a point in the region bounded by the curves  $\underline{k}(\ell)$  and  $\bar{k}(\ell)$  and satisfies  $(1 - k)r_1^B > \ell\chi$  so that liquidity crises occur in equilibrium.*

In choosing its capital structure, a bank trades off the marginal benefit of capital with its marginal cost. The former, as represented by the first term in (14), is the gain in expected profits  $[R(\theta^*)(1 - \alpha\ell) - \ell\chi]$  induced by a lower probability of a liquidity-driven crisis, as measured by  $\frac{\partial\theta^*}{\partial k}$ . The latter, as captured by the last two terms in (14), is the increase in funding cost  $\rho - 1$  associated with an increased reliance on equity financing.

The choice of portfolio liquidity  $\ell$  also trades off marginal benefit and cost. Similarly to the choice of capital, the former is captured by the first term in (15) and represents the gain in expected profits  $[R(\theta^*)(1 - \alpha\ell) - \ell\chi]$  due to the reduced probability of a liquidity-driven crisis, as measured by  $\frac{\partial\theta^*}{\partial \ell}$ . The latter, instead, corresponds to the last two terms in (15) and captures the effect that an increase in liquidity has on bank portfolio return at date 1 and 2.

At the optimum, banks always choose  $k^B$  and  $\ell^B$  in the range where both  $\frac{\partial\theta^*}{\partial k} < 0$  and  $\frac{\partial\theta^*}{\partial \ell} < 0$  as this allows them to reduce exposure to crises and, in turn, financing cost, with an overall positive effect on their

profits. Furthermore, they choose  $k^B$  and  $\ell^B$  so that the inequality  $(1 - k) > \ell\chi$  holds even if this entails liquidity-driven crises. The reason is that when  $k^B$  and  $\ell^B$  are such that  $(1 - k) = \ell\chi$ , the cost of a bank default in terms of reduced expected profits approaches zero.<sup>4</sup> This implies that the marginal benefit of increasing either  $k$  or  $\ell$  in terms of lower crisis probability also approaches zero, while the marginal cost in terms of higher funding costs and reduced portfolio return are still positive. Thus, banks have no incentives to increase  $k^B$  and  $\ell^B$  up to the point where  $(1 - k) = \ell\chi$  holds and find it optimal, instead, to choose lower levels for  $k^B$  and  $\ell^B$ , even though these foster strategic complementarity in debt holders' action and so are consistent with the occurrence of liquidity crises.

## 5 Regulatory intervention

The market equilibrium characterized above entails two inefficiencies. First, banks' choice of  $k$  and  $\ell$  spurs the occurrence of liquidity crises and so entails an output loss. To see this, denote as  $TO$  the total output generated in the market equilibrium. This corresponds to the sum of bank's profit, debt holders' and equity holders' payoffs and is, thus, equal to:

$$\begin{aligned} TO &= \int_{\theta^*}^1 [R(\theta)(1 - \alpha\ell) - (1 - k)r_2] d\theta - k\rho + (1 - k) \int_0^{\theta^*} \frac{\ell\chi}{1 - k} d\theta + (1 - k) \int_{\theta^*}^1 r_2 d\theta + k\rho = \\ &= \int_0^{\theta^*} \ell\chi d\theta + \int_{\theta^*}^1 R(\theta)(1 - \alpha\ell) d\theta. \end{aligned} \quad (16)$$

When the bank is forced to liquidate its portfolio at date 1 in response to a massive withdrawal of funds by debt holders, the resources produced in the economy correspond to the portfolio liquidation value  $\ell\chi$ . Such liquidation is inefficient for any  $\theta > \theta^E$ , where  $\theta^E$  is the solution to

$$R(\theta)(1 - \alpha\ell) = \ell\chi, \quad (17)$$

since, in the absence of liquidation, the bank would generate a higher portfolio return  $R(\theta)(1 - \alpha\ell)$  at date 2. Importantly, from comparing (2), (7) and (17) it holds  $\theta^E < \underline{\theta} < \theta^*$ . This implies that in the market equilibrium liquidity crises are always inefficient, while solvency crises are inefficient for  $\theta^E < \theta < \underline{\theta}$ . Note that only when  $\ell\chi = (1 - k)r_1$ ,  $\theta^E = \underline{\theta}$  and solvency crises are always efficient. The total output loss  $TL$

<sup>4</sup>When  $(1 - k) = \ell\chi$ ,  $\theta^* \rightarrow \underline{\theta}$  and  $r_2^* = 1$ . Thus,  $R(\theta)(1 - \alpha\ell) = (1 - k) = \ell\chi$ , banks make zero profits as all resources are used to repay debt holders.



spurred by inefficient crises at date 1 is then equal to

$$TL = \int_{\theta^E}^{\theta^*} [R(\theta)(1 - \alpha\ell) - \ell\chi] d\theta. \quad (18)$$

From (18), it is easy to see that the higher  $\theta^*$ , the larger the loss  $TL$  in total output.

The second inefficiency of the market equilibrium derives from the fact that individual banks may not internalize the consequences of their portfolio choices on market funding conditions. So far, we have considered that banks choose the liquidity  $\ell$  of their portfolios, thus obtaining a date 1 return  $\ell\chi$ , where  $\chi \leq 1$  is an exogenous parameter independent of banks' choices. We now extend the baseline framework and consider that banks' level of indebtedness and portfolio liquidity affect market conditions as they determine the quantity and type of assets sold in the market by banks at date 1. The idea is that banks access a secondary market where they sell shares of their portfolios to obtain the funds needed to repay withdrawing debt holders at date 1. Bank portfolios are acquired by outside investors endowed with a (finite) amount of wealth  $w > 0$ . As common in the literature (see e.g., Acharya and Yorulmazer, 2008; Acharya, Shin and Yorulmazer, 2010; Eisenbach, 2017), these investors may be less able than banks in managing the portfolios they acquire so that transferring assets outside the banking sector may entail a loss of resources. Such a loss depends positively on the amount of assets on sale relative to investors' overall wealth, as the more assets investors need to manage the lower is their ability to do so.<sup>5</sup>

Assuming that the realization of  $\theta$  is i.i.d. across banks, the quantity of bank portfolios sold in the market, which we denote as  $Q$ , is equal to the fraction of banks failing at date 1 as a consequence of debt holders' withdrawal decisions. Thus, similarly to Eisenbach (2017), we obtain that  $Q$  is a fixed point equal to

$$Q = \Pr(\theta < \theta^*) = \theta^*(k(\chi(Q)), \ell(\chi(Q)), r_2(\chi(Q)), \chi(Q)). \quad (19)$$

The expression in (19) shows that  $Q$  is a function of banks' choices of  $k$ ,  $\ell$  and  $r_2$  as well as of  $\chi$ .

To capture the idea of fire sales due to asset redeployment, we thus assume that the market funding

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<sup>5</sup>This is the case, for example, when banks' assets are used by investors as inputs for production and they have a decreasing return to scale production technology or when assets managing costs increases with the amount of assets to be managed. In both cases, the investors are willing to pay less for banks' portfolios, as the amount of asset on sale increases.

conditions  $\chi$  depend on investors' overall wealth  $w$  and the quantity of bank assets on sale as follows:

$$\begin{cases} \chi = 1 & \text{if } \widehat{\theta}^*(w) > \theta^* \\ \chi(Q) < 1 & \text{if } \widehat{\theta}^*(w) < \theta^* \end{cases}, \quad (20)$$

where  $\widehat{\theta}^*(w)$  represents the maximum amount of assets that outside investors can handle without loss of resources, with  $\frac{\partial \widehat{\theta}^*}{\partial w} > 0$  and  $\frac{\partial \chi(Q)}{\partial Q} < 0$ .

Similarly to the inefficient liquidation of banks' assets, we can specify the output loss associated with fire sales in the case when  $\widehat{\theta}^*(w) < \theta^*$  as given by

$$FS = \int_{\widehat{\theta}^*(w)}^{\theta^*} [\chi(\theta^*) - 1] d\theta. \quad (21)$$

From (21), it is easy to see that the loss associated with fire sales increases with  $\theta^*$ , while it decreases with investors' wealth since  $\frac{\partial \widehat{\theta}^*}{\partial w} > 0$ .

Putting together the two inefficiencies described above, we can distinguish two cases. When  $\theta^E < \theta^* < \widehat{\theta}^*(w)$  the market solution entails no fire sales and is only characterized by the inefficient liquidation of banks' portfolio for  $\theta$  in the interval  $(\theta^E, \theta^*]$ . When, instead,  $\widehat{\theta}^*(w) < \theta^E < \theta^*$ , the market solution entails both the inefficient liquidation of banks' portfolios, since  $\theta^* > \theta^E$ , and fire sales, as  $\widehat{\theta}^*(w) < \theta^*$ . Which of the two cases emerges depends on investors' wealth  $w$ , as well as on the banks' capitalization and portfolio liquidity decisions.

It follows that the analysis of joint capital and liquidity regulation is meaningful only in the case when  $\widehat{\theta}^*(w) < \theta^E < \theta^*$ , since in such a case the market solution entails both the inefficiency related to fire sales and to inefficient portfolio liquidation. Otherwise, that is for  $\theta^E < \theta^* < \widehat{\theta}^*(w)$ , the market solution entails no fire sales and the inefficient portfolio liquidation could be addressed by requiring banks to choose capital and liquidity so that  $1 - k = \ell \chi(\theta^E)$  holds. In fact, for any pair  $\{k, \ell\}$  on this curve, there is no longer strategic complementarity among debt holders' actions so that  $\theta^* \rightarrow \underline{\theta}$  and, as shown above, solvency crises are efficient, i.e.,  $\underline{\theta} = \theta^E$ .

In the next section, we analyze the joint effect of capital and liquidity regulation and study whether a regulator can enforce both the efficient liquidation of banks' assets (i.e.,  $\theta^* \rightarrow \theta^E$ ) and eliminate fire sales (i.e.,  $\chi = 1$ ). The introduction of a regulator modifies the timing as follows. At date 0, the regulator sets the requirement(s) and then banks choose the unregulated variables. At date 1, debt holders receive their

signal  $s_i$  and choose whether or not to roll over their debt claim. The model must be solved backward.

## 5.1 Capital and liquidity regulation

In this section, we analyze the case of a regulator that chooses capital and liquidity requirements to maximize total output as specified in (16), while taking as given debt holders' withdrawal decisions as characterized in Proposition 2, the debt contract  $\{r_1, r_2\}$  chosen by the bank and the non-negative profit constraint of banks. Thus, formally, the regulator chooses the level of bank capital  $k^R$  and portfolio liquidity  $\ell^R$  at date 0 to solve the following problem:

$$\max_{k^R, \ell^R} \int_0^{\theta^*} \ell \chi(Q) d\theta + \int_{\theta^*}^1 R(\theta) (1 - \alpha \ell) d\theta$$

subject to

$$\begin{aligned} r_1^B, r_2^B &= \arg \max \Pi^B, \\ \Pi^B &\geq 0, 0 \leq k^R \leq 1, 0 \leq \ell^R \leq 1. \end{aligned}$$

As discussed in Section 4, the bank promises debt holders a repayment  $r_1^B = 1$  at date 1 and a repayment  $r_2^B \geq 1$  at date 2 that solves the debt holders' participation constraint in (11) with equality. Thus, the banks' profits can be rearranged as

$$\Pi^B = \int_0^{\theta^*} \ell \chi(Q) d\theta + \int_{\theta^*}^1 R(\theta) (1 - \alpha \ell) d\theta - (1 - k) - k\rho, \quad (22)$$

and the regulator's problem can be re-written as follows:

$$\max_{k^R, \ell^R} \int_0^{\theta^*} \ell \chi(Q) d\theta + \int_{\theta^*}^1 R(\theta) (1 - \alpha \ell) d\theta$$

subject to

$$\int_0^{\theta^*} \ell \chi(Q) d\theta + \int_{\theta^*}^1 R(\theta) (1 - \alpha \ell) d\theta \geq (1 - k) + k\rho. \quad (23)$$

Essentially, the regulator's problem boils down to choose  $\ell^R$  and  $k^R$  so to eliminate the two inefficiencies plaguing the market solution whenever this is feasible vis-à-vis the constraint of bank's non-negative profits. Formally, this is the case when  $\theta^E \leq \hat{\theta}^*$  is consistent with  $\Pi^B \geq 0$ . In this case, there are pairs  $\{k, \ell\}$  lying

on the curve  $1 - k = \ell \chi(\theta^E)$  for which the liquidation of banks' assets entails no fire sales and banks accrue non-negative profits. By contrast, eliminating both types of inefficiencies is not feasible when the pairs  $\{k, \ell\}$  corresponding to a threshold  $\theta^E = \widehat{\theta}^*$  would entail negative profits for the banks. It follows that which case emerges depends on the exogenous parameter of the model  $\alpha$ ,  $\rho$  and  $w$  through their effect on the thresholds  $\widehat{\theta}^*$ ,  $\theta^E$  and the financing cost for the bank. Clearly, higher values of  $\alpha$  or  $\rho$  make the bank profit condition in (23) more binding. Furthermore, from (17), it is easy to see that a higher  $\alpha$  is associated, ceteris paribus, with a higher  $\theta^E$ , while a higher  $w$  is associated to a lower  $\widehat{\theta}^*$ . In other words,  $\theta^E \leq \widehat{\theta}^*$  is more likely to emerge in economies in which liquidity and equity are not too costly for banks and where investors hold abundant funds to buy banks' assets. The opposite is true in economies where both capital and liquidity are very costly for banks and there are only limited resources to acquire banks' assets. The effectiveness of regulation in eliminating the inefficiencies of the market solution depends then on whether the economy is in the first or second case. We have the following result.

**Proposition 5** *Capital and liquidity requirements allow to restore full efficiency, i.e.,  $\theta^* \rightarrow \theta^E$  and  $\chi(\theta^E) = 1$  only in economies characterized by low  $\alpha$  and  $\rho$  and high  $w$ . In economies characterized by high  $\alpha$ ,  $\rho$  and low  $w$ , the regulator can either enforce efficient liquidation of banks' assets but having fire sales (i.e.,  $\theta^* \rightarrow \theta^E$  and  $\chi(\theta^E) < 1$ ) or reduce the fire sales  $\chi(\underline{\theta}) < \chi(\theta^E)$ , but induce too little liquidation of banks' assets (i.e.,  $\theta^* \rightarrow \underline{\theta} < \theta^E$ ).*

The proposition shows that the effectiveness of capital and liquidity regulation in correcting the inefficiencies of the market equilibrium depends on the underlying features of the economy: the tightness of the market for banks' assets, as captured by  $w$ , the cost of bank equity  $\rho$  and that of portfolio liquidity  $\alpha$ . By affecting the threshold  $\widehat{\theta}^*$ , market tightness determines the severity of fire sales, that is whether or not fire sales occur in an economy where the liquidation of banks' portfolio is efficient. By contrast, the cost of bank equity and portfolio liquidity affect bank profit and thus determine the feasibility of given capital and liquidity requirements. When market conditions are good and bank equity and liquidity are not too costly, the regulator can eliminate both inefficient portfolio liquidation and fire sales by selecting a pair  $\{k, \ell\}$  such that  $\theta^* \rightarrow \underline{\theta} = \theta^E \leq \widehat{\theta}^*$ . When this is not feasible, as it is the case when market conditions are tight and/or the cost of bank equity and portfolio liquidity is high, the regulator needs to choose between enforcing the efficient liquidation of the banks' asset and reducing the severity of fire sales. This means, in other words, that he must choose between a pair  $\{k, \ell\}$  on the curve  $1 - k = \ell \chi(\theta^E)$ , which would be associated with

$\chi(\theta^E) < 1$ , and a pair  $\{k, \ell\}$  for which  $1 - k < \ell\chi(\underline{\theta})$  holds and  $\chi(\underline{\theta}) < \chi(\theta^E)$ . The regulator will select the option that is associated with the highest level of total output or, in other words, that minimizes the associated output losses. When

$$\int_{\underline{\theta}}^{\theta^E} \ell [\chi(\underline{\theta}) - 1] d\theta > \int_{\underline{\theta}}^{\theta^E} [\ell\chi(\underline{\theta}) - R(\theta)(1 - \alpha\ell)] d\theta, \quad (24)$$

it means that the gains from reducing the fire sales, as captured by the LHS in (24), is larger than the loss associated with the inefficient liquidation of the banks' assets, as given by the RHS in (24). The inequality above can be further rearranged as follows

$$\int_{\underline{\theta}}^{\theta^E} [R(\theta)(1 - \alpha\ell) - \ell] d\theta < 0.$$

Thus, the regulator would choose  $\{k^R, \ell^R\}$  such that  $1 - k < \ell\chi(\underline{\theta})$  holds when  $\int_{\underline{\theta}}^{\theta^E} [R(\theta)(1 - \alpha\ell) - \ell] d\theta < 0$ . The opposite is true if  $\int_{\underline{\theta}}^{\theta^E} [R(\theta)(1 - \alpha\ell) - \ell] d\theta > 0$ . In this case, the regulator would choose  $\{k^R, \ell^R\}$  such that  $1 - k = \ell\chi(\theta^E)$  holds, despite this entails a larger fire sale loss.

## 6 Concluding remarks

In this paper we develop a model where banks are exposed to both solvency and liquidity crises and both banks' and debt holders' decisions are endogenously determined. The paper offers a convenient framework to evaluate the implications of bank capital and liquidity on the likelihood of crises, as it allows to endogenize the probability of crises, distinguish their different type, and account for the different effects that changes in bank capital structure and portfolio liquidity have on each of them.

One of the main implications of the analysis is that, in order to be beneficial for stability, regulation should be designed considering both sides of banks' balance sheet. The same (marginal) increase in capital and liquidity may be beneficial for some banks, while detrimental for others. Real world regulatory tools like risk-weighted capital ratio (RWC), liquidity coverage ratio (LCR) or net stable funding ratio (NSFR) seem to fulfil this criterion, as they specify a ratio between banks' assets and liabilities (see Cecchetti and Kashyap, 2018).

The analysis of the impact of capital and liquidity on bank stability is also the starting point to characterize optimal regulation. In our framework, public intervention in the form of capital and liquidity requirements

is desirable as the market equilibrium is plagued by two inefficiencies. First, banks choose levels of capitalization and portfolio liquidity that are consistent with the occurrence of liquidity crises and, as such, lead to inefficient portfolio liquidation. Second, in choosing their capital structure and portfolio liquidity banks do not internalize the effect that such choices have on market funding conditions that is on the existence and severity of fire sales.

We show that when market funding conditions for the banks are tight, the cost of capital and liquidity for banks are high, the regulator is not able to fully restore efficiency so that the regulator's solution entails either too little liquidation of banks' assets but reduced fire sales or the efficient liquidation of the banks' assets with fire sale.

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## 8 Appendix

**Proof of Proposition 1:** Denote as  $f(\theta, k, \ell, r_1) = 0$  the condition pinning down the threshold  $\underline{\theta}(k, \ell, r_1)$  as given in (2). By using the implicit function theorem, we have that

$$\frac{\partial \underline{\theta}(k, \ell, r_1)}{\partial k} = -\frac{\frac{\partial f(\theta, k, \ell, r_1)}{\partial k}}{\frac{\partial f(\theta, k, \ell, r_1)}{\partial \theta}}, \quad \frac{\partial \underline{\theta}(k, \ell, r_1)}{\partial \ell} = -\frac{\frac{\partial f(\theta, k, \ell, r_1)}{\partial \ell}}{\frac{\partial f(\theta, k, \ell, r_1)}{\partial \theta}}, \quad \text{and} \quad \frac{\partial \underline{\theta}(k, \ell, r_1)}{\partial r_1} = -\frac{\frac{\partial f(\theta, k, \ell, r_1)}{\partial r_1}}{\frac{\partial f(\theta, k, \ell, r_1)}{\partial \theta}}.$$

The denominator  $\frac{\partial f(\theta, k, \ell, r_1)}{\partial \theta} = R'(\underline{\theta})(1 - \alpha\ell) > 0$  as  $R'(\underline{\theta}) > 0$ . Thus, the sign of  $\frac{\partial \underline{\theta}(k, \ell, r_1)}{\partial k}$ ,  $\frac{\partial \underline{\theta}(k, \ell, r_1)}{\partial \ell}$ , and  $\frac{\partial \underline{\theta}(k, \ell, r_1)}{\partial r_1}$  are equal to the opposite sign of the respective numerators. Deriving (2) with respect to  $k$ ,  $\ell$  and  $r_1$ , we obtain

$$\begin{aligned} \frac{\partial f(\theta, k, \ell, r_1)}{\partial k} &= r_1 > 0, & \frac{\partial f(\theta, k, \ell, r_1)}{\partial \ell} &= -R(\underline{\theta})\alpha < 0 \\ \text{and} \quad \frac{\partial f(\theta, k, \ell, r_1)}{\partial r_1} &= -(1 - k) < 0, \end{aligned}$$

which imply  $\frac{\partial \underline{\theta}(k, \ell, r_1)}{\partial k} < 0$ ,  $\frac{\partial \underline{\theta}(k, \ell, r_1)}{\partial \ell} > 0$  and  $\frac{\partial \underline{\theta}(k, \ell, r_1)}{\partial r_1} > 0$ . Thus, the proposition follows.  $\square$

**Proof of Proposition 2:** The proof follows closely that in Goldstein and Pauzner (2005) since the model also exhibits the property of *one-sided* strategic complementarity. We start by characterizing a region where the state of the economy  $\theta$  takes extremely high values and we refer to this region as the upper dominance region. The upper dominance region of  $\theta$  corresponds to the range  $[\bar{\theta}, 1]$  in which fundamentals are so good that all debt holders roll over at date 1. As in Goldstein and Pauzner (2005), we construct this region by assuming that, in this range, the investment is safe and returns  $R(1)(1 - \alpha\ell)$  both at date 1 and 2. Given that  $r_1 < r_2 < R(1)(1 - \alpha\ell)$ , this ensures that repaying  $r_1$  to the  $(1 - k)n$  withdrawing debt holders does not affect bank's ability to repay  $r_2$  to the debt holders rolling over the debt until date 2. Then, when an investor receives a signal such that he believes that the fundamental  $\theta$  are in the upper dominance region, he is certain to receive the promised payment  $r_2$ , irrespective of his beliefs on other debt holders' action and so he does not have any incentive to withdraw early. In what follows, we assume that  $\bar{\theta} \rightarrow 1$ .

The upper dominance region is the mirror image of the range  $[0, \underline{\theta}(k, \ell, r_1))$  characterized in the text in that when both  $\theta < \underline{\theta}(k, \ell, r_1)$  and  $\theta \geq \bar{\theta}$ , debt holders have a dominant strategy and their actions are independent of what others do. Besides these extreme ranges of values of the state of the economy  $\theta$ , a debt holder's rollover decision depends on what he expects the other investors do and so on the signal he receives.

Assume that debt holders behave accordingly to a threshold strategy, that is each debt holder withdraw at date 1 if he receive a signal below  $s^*$  and rolls over otherwise. Then, the fraction of debt holders not rolling over the debt claim  $n$  is equal to the probability of receiving a signal below  $s^*$ . Given that debt holders' signal are independent and uniformly distributed in the range  $[\theta - \varepsilon, \theta + \varepsilon]$ ,  $n(s^*, \theta)$  is equal to

$$n(s^*, \theta) = \begin{cases} 1 & \text{if } \theta \leq s^* - \varepsilon \\ \frac{s^* - \theta + \varepsilon}{2\varepsilon} & \text{if } s^*(k, \ell) - \varepsilon \leq \theta \leq s^*(k, \ell) + \varepsilon \\ 0 & \text{if } \theta \geq s^* + \varepsilon \end{cases} \quad (25)$$

When  $\theta$  is lower than  $s^* - \varepsilon$ , all  $(1 - k)$  debt holders receive a signal below  $s^*$  and they withdraw at date 1. On the contrary, when  $\theta$  is higher than  $s^* + \varepsilon$ , all  $(1 - k)$  debt holders receive a signal above  $s^*$  and, as a result, decide to roll over their debt claim. In the intermediate range of fundamental, when  $s^* - \varepsilon \leq \theta \leq s^* + \varepsilon$ , there is a partial runs, in that only some debt holders withdraw at date 1. The proportion of those not rolling over their debt claim decreases linearly with  $\theta$ , as fewer investors observe a signal below the threshold  $s^*$ .

Denote as  $\Delta(s_i, n(s^*, \theta))$ , a debt holder's expected utility differential between rolling over the debt claim until date 2 and withdrawing it at date 1 when all agents are assumed to behave accordingly to the same threshold strategy  $s^*$ . We have

$$\Delta(s_i, n(s^*, \theta)) = \frac{1}{2\varepsilon} \int_{s_i - \varepsilon}^{s_i + \varepsilon} (\pi_2 - \pi_1) d\theta,$$

with  $\pi_2$  and  $\pi_1$  as given by (4) and (3), respectively. The following lemma states a few properties of the function  $\Delta(s_i, n(s^*, \theta))$ .

**Lemma 2** *i) The function  $\Delta(s_i, n(s^*, \theta))$  is continuous in  $s_i$ ; ii) for any  $a > 0$ ,  $\Delta(s_i + a, n(s^*, \theta) + a)$  is non-decreasing in  $a$ ; iii)  $\Delta(s_i, n(s^*, \theta))$  is strictly increasing in  $a$  if there is a positive probability that  $n < \bar{n}$  and  $\theta < \bar{\theta}$ .*

**Proof of Lemma 2:** The proof follows Goldstein and Pauzner (2005). The function  $\Delta(\cdot)$  is continuous in  $s_i$ , as  $s_i$  only changes the limits of integration in the formula for  $\Delta(s_i, n(s^*, \theta))$ . To show that the function  $\Delta(s_i, n(s^*, \theta))$  is non-decreasing in  $a$ , we need first to show that  $(\pi_2 - \pi_1)$  is non-decreasing in  $\theta$ . As  $\theta$  increases, we have two effects. First, a higher  $\theta$  implies that  $R(\theta)$  is higher, thus increasing the date 2 payoff in the range  $\hat{n}(\theta) < n \leq \bar{n}$ . Second, a change in  $\theta$  affects the threshold  $\hat{n}(\theta)$  as follows.

$$\begin{aligned} \frac{\partial \hat{n}(\theta)}{\partial \theta} &= R'(\theta) (1 - \alpha\ell) \frac{\left[ \frac{R(\theta)(1 - \alpha\ell)r_1}{\ell\chi} - r_2 \right] - [R(\theta)(1 - \alpha\ell) - (1 - k)r_2] \frac{r_1}{\ell\chi}}{(1 - k) \left[ \frac{R(\theta)(1 - \alpha\ell)r_1}{\ell\chi} - r_2 \right]^2} = \\ &= \frac{R'(\theta) (1 - \alpha\ell)}{(1 - k) \left[ \frac{R(\theta)(1 - \alpha\ell)r_1}{\ell\chi} - r_2 \right]^2} r_2 \left[ \frac{r_1}{\ell\chi} (1 - k) - 1 \right] > 0, \end{aligned}$$

since  $R'(\theta) > 0$  and  $(1 - k)r_1 > \ell\chi$ . Thus, since the interval  $[0, \hat{n}(\theta)]$  where the utility differential  $\pi_2 - \pi_1$  is the highest becomes larger, while the range  $(\bar{n}, 1]$  is unaffected by a change in  $\theta$ , the date 2 payoff also increases so that the utility differential  $\pi_2 - \pi_1$  is non-decreasing in  $\theta$ . This also implies that  $\Delta(s_i, n(s^*, \theta))$  is non-decreasing in  $a$ , as when  $a$  increases, debt holders see the same distribution of  $n$  but expects  $\theta$  to be larger. The function  $\Delta(s_i, n(s^*, \theta))$  is strictly increasing in  $a$  since when  $n < \bar{n}$  and  $\theta < \bar{\theta}$ ,  $\pi_2 - \pi_1$  is strictly increasing in  $\theta$ . ■

Since the rest of the proof follows closely that in Goldstein and Pauzner (2005) we omit it here and only specify the condition pinning down the threshold  $s^*$ . A debt holder who receives the signal  $s^*$  is indifferent between rolling over the debt claim until date 2 and withdrawing it at date 1. The threshold  $s^*$  can be

computed as the solution to

$$f(\theta, k, \ell) = \int_0^{\widehat{n}(s^*)} r_2 dn + \int_{\widehat{n}(s^*)}^{\bar{n}} \frac{R(\theta(n))(1-\alpha\ell) \left[1 - \frac{(1-k)nr_1}{\ell}\right]}{(1-k)(1-n)} dn - \int_0^{\bar{n}} r_1 dn - \int_{\bar{n}}^1 \frac{\ell\chi}{(1-k)n} dn = 0, \quad (26)$$

where from (25), we obtain  $\theta(n) = s^* + \varepsilon - 2\varepsilon \frac{n}{1-k}$  and  $\widehat{n}(s^*)$  solves  $R(\theta(n))(1-\alpha\ell) \left[1 - \frac{(1-k)nr_1}{\ell}\right] - (1-k)(1-n)r_2 = 0$ . At the limit, when  $\varepsilon \rightarrow 0$ ,  $\theta(n) \rightarrow s^*$  and we denote it as  $\theta^*$ .

To complete the proof, we need to show that the bank is solvent for any  $\theta > \theta^*$ . To do that we need to exclude the possibility that the bank fails despite debt holders rolling over the debt until date 2. Denote as  $\widehat{\theta}$  the level of fundamental at which, even when all debt holders roll over the debt claim the bank fails at date 2. The threshold  $\widehat{\theta}$  solves

$$R(\theta)(1-\alpha\ell) - (1-k)r_2 = 0.$$

Then, to show that the bank is always solvent for any  $\theta > \theta^*$ , we need to show that the threshold  $\theta^*$  characterized in (26) larger than  $\widehat{\theta}$ . To see this, denote as  $\widetilde{\theta}$  the level of  $\theta$  at which the bank is at the margin between failing and being solvent at date 2. Then,  $\widetilde{\theta}$  is the solution to

$$R(\theta)(1-\alpha\ell) \left[1 - \frac{(1-k)n(s^*, \theta)r_1}{\ell\chi}\right] - (1-k)(1-n(s^*, \theta))r_2 = 0, \quad (27)$$

where  $n(s^*, \widetilde{\theta})$  is given in (25). Rearranging (25) as

$$R(\theta)(1-\alpha\ell) - (1-k)r_2 - n(s^*, \theta) \left[ \frac{R(\theta)(1-\alpha\ell)(1-k)r_1}{\ell\chi} - (1-k)r_2 \right] = 0,$$

it is easy to see that (25) is negative when evaluated at  $\theta = \widehat{\theta}$  when  $(1-k)r_1 > \ell\chi$  holds. Thus, since (25) is increasing in  $\theta$ , it follows that  $\widetilde{\theta} > \widehat{\theta}$ .

The equilibrium in debt holders' withdrawal decision characterized in the proposition corresponds to the pair  $\{s^*, \theta^*\}$  solving (27) and the indifference condition as given by  $\pi_2 - \pi_1 = 0$  after the change of variable using  $\theta(n) = s^* + \varepsilon - 2\varepsilon \frac{n}{1-k}$ . Thus, it is the case that, when  $\varepsilon \rightarrow 0$ ,  $s^* \rightarrow \theta^* = \widetilde{\theta} > \widehat{\theta}$  and the proposition follows.  $\square$

**Proof of Lemma 1:** We compute the effect of capital and liquidity on  $\theta^*$  (i.e.,  $\frac{\partial \theta^*}{\partial k}$  and  $\frac{\partial \theta^*}{\partial \ell}$ ) by using the implicit function theorem as follows:

$$\frac{\partial \theta^*}{\partial k} = - \frac{\frac{\partial f(\theta^*, k, \ell)}{\partial k}}{\frac{\partial f(\theta^*, k, \ell)}{\partial \theta}} \quad \text{and} \quad \frac{\partial \theta^*}{\partial \ell} = - \frac{\frac{\partial f(\theta^*, k, \ell)}{\partial \ell}}{\frac{\partial f(\theta^*, k, \ell)}{\partial \theta}},$$

with  $f(\theta^*, k, \ell)$  is the equation pinning down  $\theta^*$ , as defined in (26). The denominator  $\frac{\partial f(\theta^*, k, \ell)}{\partial \theta}$  is given by

$$\frac{\partial f(\theta^*, k, \ell)}{\partial \theta} = \int_{\hat{n}(\theta)}^{\bar{n}} \frac{R'(\theta^*)(1-\alpha\ell) \left[1 - \frac{(1-k)nr_1}{\ell\chi}\right]}{(1-k)(1-n)} dn > 0$$

since the derivatives of the extremes of the integrals cancel out. Thus, the sign of  $\frac{\partial \theta^*}{\partial k}$  and  $\frac{\partial \theta^*}{\partial \ell}$  are equal to the opposite sign of  $\frac{\partial f(\theta^*, k, \ell)}{\partial k}$  and  $\frac{\partial f(\theta^*, k, \ell)}{\partial \ell}$ , respectively.

We start from  $\frac{\partial f(\theta^*, k, \ell)}{\partial k}$ . Deriving (26) with respect to  $k$  and multiplying it by  $-1$ , we obtain the expression in (8) since the derivatives of the extremes of integrals cancel out.

Similarly, differentiating (26) with respect to  $\ell$ , after a few manipulation and multiplying it by  $-1$ , we obtain the expression in (9), as the derivatives of the extremes of integrals cancel out. Thus, the lemma follows.  $\square$

**Proof of Proposition 3:** The proof proceeds in steps and builds on the results derived in the Lemma 1.

First, denote as  $k^{\max}(\ell)$  the solution to  $(1-k)r_1 = \ell\chi$ . It is easy to see that  $k^{\max}(\ell)$  decreases with  $\ell$ ,  $k^{\max}(0) = 1$  and  $k^{\max}(1) = 1 - \frac{\chi}{r_1}$ . When  $k \rightarrow k^{\max}(\ell)$ , the threshold  $\theta^* \rightarrow \underline{\theta}$ . To see this, we can rearrange the expression in (26) as follows:

$$\begin{aligned} & \int_0^{\tilde{n}(\theta)} \left[ \min \left\{ r_2, \frac{R(\theta)(1-\alpha\ell) \left[1 - \frac{(1-k)nr_1}{\ell\chi}\right]}{(1-k)(1-n)} \right\} - r_1 \right] dn + \\ & + \int_{\tilde{n}(\theta)}^{\bar{n}} \left[ \frac{R(\theta)(1-\alpha\ell) \left[1 - \frac{(1-k)nr_1}{\ell\chi}\right]}{(1-k)(1-n)} - r_1 \right] dn - \int_{\bar{n}}^1 \frac{\ell\chi}{(1-k)n} dn, \end{aligned} \quad (28)$$

with  $\bar{n} = \frac{\ell\chi}{(1-k)r_1}$  and  $\tilde{n}(\theta) = \frac{R(\theta)(1-\alpha\ell) - (1-k)r_1}{(1-k) \left[ \frac{R(\theta)(1-\alpha\ell)}{\ell\chi} - 1 \right] r_1}$  denoting the proportion of debt holders withdrawing at date 1 at which the bank's resources at date 2 are exactly enough to pay  $r_1$  to the debt holders rolling over the debt claim until date 2. When  $k \rightarrow k^{\max}(\ell)$ ,  $\bar{n} \rightarrow \tilde{n}(\theta) \rightarrow 1$  and the expression above simplifies to

$$\int_0^1 \left[ \min \left\{ r_2, \frac{R(\theta)(1-\alpha\ell) \left[1 - \frac{(1-k)nr_1}{\ell\chi}\right]}{(1-k)(1-n)} \right\} - r_1 \right] = 0.$$

Since  $r_1 \geq 1$  and  $r_2 > r_1$ ,  $\theta^*$  solves  $\frac{R(\theta)(1-\alpha\ell)}{(1-k)} - r_1 = 0$ , which is equivalent to the equation pinning down  $\underline{\theta}$ , as given in (2).

Second, we rearrange the expression in (8) as follows:

$$R(\theta^*)(1-\alpha\ell) \text{Log} \left[ \frac{1-\bar{n}}{1-\hat{n}(\theta^*)} \right] - \ell\chi \text{Log}[\bar{n}]. \quad (29)$$

The first term is negative since  $\bar{n} > \hat{n}(\theta^*)$  and so  $\frac{1-\bar{n}}{1-\hat{n}(\theta^*)} < 1$ , while the second one is positive since  $\bar{n} < 1$ . Using  $\bar{n} = \frac{\ell\chi}{(1-k)r_1}$  and  $\hat{n}(\theta^*) = \frac{R(\theta^*)(1-\alpha\ell) - (1-k)r_2}{(1-k) \left[ \frac{R(\theta^*)(1-\alpha\ell)r_1}{\ell\chi} - r_2 \right]}$ , after a few manipulations, the expression above can

be rewritten as follows:

$$\text{Log} \left[ \left( 1 - \frac{\ell \chi r_2}{R(\theta^*)(1-\alpha\ell)r_1} \right)^{\frac{R(\theta^*)(1-\alpha\ell)}{\ell \chi}} \right] - \text{Log} \left[ \left( \frac{\ell \chi}{(1-k)r_1} \right) \right]. \quad (30)$$

Denote as  $\tilde{k}(\ell)$  the solution to  $\text{Log} \left[ \left( 1 - \frac{\ell \chi r_2}{R(\theta^*)(1-\alpha\ell)r_1} \right)^{\frac{R(\theta^*)(1-\alpha\ell)}{\ell \chi}} \right] - \text{Log} \left[ \left( \frac{\ell \chi}{(1-k)r_1} \right) \right] = 0$ . The expression in (30) can be rearranged as

$$\tilde{k}(\ell) = 1 - \frac{\ell \chi}{r_1} (\Lambda)^{-\frac{R(\theta^*)(1-\alpha\ell)}{\ell \chi}}, \quad (31)$$

where  $\Lambda = \left( 1 - \frac{\ell \chi r_2}{R(\theta^*)(1-\alpha\ell)r_1} \right)$ . Since for any pair  $\{k, \ell\}$ ,  $\theta^*$  varies between  $\underline{\theta}$  and  $\bar{\theta} \rightarrow 1$ , it holds that  $\tilde{k}(\ell) < k^{\max}(\ell)$  for any  $\ell \in (0, 1)$ , since  $k^{\max}(\ell) = 1 - \frac{\ell \chi}{r_1}$  and  $(\Lambda)^{-\frac{R(\theta^*)(1-\alpha\ell)}{\ell \chi}} > 1$ . Furthermore, from (31), it follows that  $\tilde{k}(\ell) \rightarrow 1$ , when  $\ell \rightarrow 0$  and that  $\tilde{k}(\ell) = 0$  requires  $\ell > 0$ .

Consider a pair  $\{k, \ell\}$  in the region above  $k^{\max}(\ell)$ . Since in this region,  $\frac{\partial \theta^*}{\partial k} < 0$  and it is zero on the curve  $\tilde{k}(\ell)$ , it must be the case that  $\frac{\partial \theta^*}{\partial k} < 0$  in the region between  $\tilde{k}(\ell)$  and  $k^{\max}(\ell)$ . Consider now a pair  $\{k, \ell\}$  below the curve  $\tilde{k}(\ell)$  and close to the axes origin. For  $k \ll 1$  and  $\ell \rightarrow 0$ , the expression in (30) is positive since the second term approaches to  $-\infty$ , while the is equal to

$$\text{Lim}_{\ell \rightarrow 0} \frac{R(\theta^*)(1-\alpha\ell)}{\ell \chi} \text{Log}[\Lambda] = \text{Lim}_{\ell \rightarrow 0} \frac{\text{Log}[\Lambda]}{\frac{\ell \chi}{R(\theta^*)(1-\alpha\ell)}},$$

and using l' Hopital's rule, after a few manipulations, we obtain

$$\text{Lim}_{\ell \rightarrow 0} \frac{R(\theta^*)(1-\alpha\ell)}{\ell \chi} \text{Log}[\Lambda] = -\frac{\frac{r_2}{r_1} \frac{\chi}{\text{Lim}_{\ell \rightarrow 0}[R(\theta^*)(1-\alpha\ell)]}}{\frac{\chi}{\text{Lim}_{\ell \rightarrow 0}[R(\theta^*)(1-\alpha\ell)]}} = -\frac{r_2}{r_1} < 0,$$

where  $\text{Lim}_{\ell \rightarrow 0} [R(\theta^*)(1-\alpha\ell)]$  is equal to a finite number. This implies that  $\frac{\partial \theta^*}{\partial k} > 0$  for  $k \ll 1$  and  $\ell \rightarrow 0$ . Since the derivative  $\frac{\partial \theta^*}{\partial k}$  is zero on the curve  $\tilde{k}(\ell)$ , by continuity it stays positive below  $\tilde{k}(\ell)$ .

Consider now the effect of liquidity  $\ell$  on  $\theta^*$ . The expression (9) determining the sign of  $\frac{\partial \theta^*}{\partial \ell}$  (i.e.,  $-\frac{\partial f(\theta, k, \ell)}{\partial \ell}$ ) can be rearranged as follows, after adding and subtracting  $\frac{1}{(1-k)\ell} \int_{\hat{n}(\theta^*)}^{\bar{n}} \frac{R(\theta^*)}{1-n} dn$ :

$$\frac{\partial f(\theta^*, k, \ell)}{\partial \ell} = \frac{1}{(1-k)\ell} \left[ \int_{\hat{n}(\theta^*)}^{\bar{n}} \frac{R(\theta^*)(1-\alpha\ell)}{1-n} dn - \ell \chi \int_{\bar{n}}^1 \frac{1}{n} dn - \int_{\hat{n}(\theta^*)}^{\bar{n}} \frac{R(\theta^*)}{(1-n)} \left( 1 - \frac{(1-k)nr_1}{\ell \chi} \right) dn \right].$$

Since, from (8), we have that  $\frac{\partial f(\theta, k, \ell)}{\partial k} = \frac{1}{(1-k)^2} \left[ \int_{\hat{n}(\theta^*)}^{\bar{n}} \frac{R(\theta^*)(1-\alpha\ell)}{1-n} dn - \ell \chi \int_{\bar{n}}^1 \frac{1}{n} dn \right]$ , we can write

$$\begin{aligned} \frac{\partial f(\theta^*, k, \ell)}{\partial \ell} &= \frac{(1-k)}{\ell} \frac{\partial f(\theta, k, \ell)}{\partial k} - \frac{1}{(1-k)\ell} \int_{\hat{n}(\theta^*)}^{\bar{n}} \frac{R(\theta^*)}{(1-n)} \left( 1 - \frac{(1-k)nr_1}{\ell \chi} \right) dn = \\ &= \frac{1}{\ell} \left[ (1-k) \frac{\partial f(\theta, k, \ell)}{\partial k} - \frac{1}{(1-k)} \int_{\hat{n}(\theta^*)}^{\bar{n}} \frac{R(\theta^*)}{(1-n)} \left( 1 - \frac{(1-k)nr_1}{\ell \chi} \right) dn \right]. \end{aligned} \quad (32)$$

From (32), then, it is easy to see that when  $k \leq \tilde{k}(\ell)$   $\frac{\partial f(\theta^*, k, \ell)}{\partial \ell} < 0$ , as  $\frac{\partial f(\theta^*, k, \ell)}{\partial k} \leq 0$ . This implies that  $\frac{\partial \theta^*}{\partial \ell} > 0$ . Furthermore, since for any  $k \geq k^{\max}(\ell)$ , the relevant threshold is  $\underline{\theta}$  and, from (2), it holds  $\frac{\partial \theta}{\partial \ell} > 0$ , we have that an increase in liquidity has a detrimental effect on stability for  $k \leq \tilde{k}(\ell)$  and  $k \geq k^{\max}(\ell)$ .

Consider now the range  $(\tilde{k}(\ell), k^{\max}(\ell))$ . We want to show that in this range there are levels of bank capitalization  $k$  for which increasing liquidity leads to a lower probability of panic-driven runs, i.e.,  $\frac{\partial f(\theta^*, k, \ell)}{\partial \ell} < 0$  for some  $k \in (\tilde{k}(\ell), k^{\max}(\ell))$ . To do this, we need to show that there exist a region of  $k$  and  $\ell$ , where the expression in the bracket in (9) is negative. After a few manipulation, we can rearrange it as follows:

$$R(\theta^*)(1 - \alpha\ell) \text{Log}(\Lambda) \bar{n} - R(\theta^*) \text{Log}(\Lambda) (1 - \bar{n}) + R(\theta^*) \bar{n} - R(\theta^*) \hat{n}(\theta^*) - \ell\chi \text{Log}(\bar{n}) \bar{n},$$

where  $\text{Log}(\Lambda) = -\int_{\hat{n}(\theta^*)}^{\bar{n}} \frac{1}{1-n} dn$  and  $\bar{n} = \frac{\ell\chi}{(1-k)r_1}$ . Adding and subtracting  $R(\theta^*)\alpha\ell \text{Log}(\Lambda)$  to the expression above, it can be further rearranged as follows:

$$\begin{aligned} & -R(\theta^*)(1 - \alpha\ell) \text{Log}(\Lambda) \bar{n} - R(\theta^*) \text{Log}(\Lambda) + R(\theta^*) \bar{n} - R(\theta^*) \hat{n}(\theta^*) + R(\theta^*) \text{Log}(\Lambda) \bar{n} \quad (33) \\ & + [R(\theta^*) \text{Log}(\Lambda) - \ell\chi \text{Log}(\bar{n})] \bar{n}. \end{aligned}$$

The term in the square bracket in (33) is negative for  $k \geq \tilde{k}(\ell)$  because  $R(\theta^*) |\text{Log}(\Lambda)| > R(\theta^*) (1 - \alpha\ell) |\text{Log}(\Lambda)|$  and  $R(\theta^*) (1 - \alpha\ell) |\text{Log}(\Lambda)| = \ell\chi |\text{Log}(\bar{n})|$  at  $k = \tilde{k}(\ell)$ . Denote as  $k^T(\ell)$  the level of capital and liquidity at which the terms in the first four terms sum up to zero. The curve  $k^T(\ell)$  lies below  $k^{\max}(\ell)$  and above  $\tilde{k}(\ell)$ . To see this, we can rearranged the first four terms in (33) as follows:

$$\int_{\hat{n}(\theta^*)}^{\bar{n}} \frac{R(\theta^*)}{1-n} \left[ -(1 - \alpha\ell) \frac{\ell\chi}{(1-k)r_1} + 1 - \frac{n(1-k)r_1}{\ell\chi} \right] dn$$

It is easy to see that the expression in the square bracket is increasing in  $k$ . When  $k = \tilde{k}(\ell)$ , the terms in the bracket sum up to  $-(1 - \alpha\ell) (\Lambda)^{\frac{R(\theta^*)(1-\alpha\ell)}{\ell\chi}} + 1 - n(\Lambda)^{-\frac{R(\theta^*)(1-\alpha\ell)}{\ell\chi}}$ . Evaluating this at  $n = \hat{n}(\theta^*)$ , we obtain  $-(1 - \alpha\ell) (\Lambda)^{\frac{R(\theta^*)(1-\alpha\ell)}{\ell\chi}} + 1 - r_1$ , which is smaller than 0 since  $r_1 \geq 1$ . Then, when  $k = k^T(\ell)$ , it follows that  $\frac{\partial \theta^*}{\partial \ell} < 0$  because  $[R(\theta^*)(1 - \alpha\ell) \text{Log}(\Lambda) - \ell\chi \text{Log}(\bar{n})] < 0$  for any  $k > \tilde{k}(\ell)$ .

Given that  $\tilde{k}(\ell) < k^T(\ell) < k^{\max}(\ell)$  and  $\frac{\partial \theta^*}{\partial \ell} > 0$  for  $k \leq \tilde{k}(\ell)$  and  $k \geq k^{\max}(\ell)$ , by continuity, there must exist two thresholds  $\underline{k}(\ell) \in (\tilde{k}(\ell), k^T(\ell))$  and  $\bar{k}(\ell) \in (k^T(\ell), k^{\max}(\ell))$ , such that  $\frac{\partial \theta^*}{\partial \ell} > 0$  for  $\tilde{k}(\ell) < k < \underline{k}(\ell)$  and  $\frac{\partial \theta^*}{\partial \ell} < 0$  for  $\underline{k}(\ell) < k < \bar{k}(\ell)$ . Thus, the proposition follows.  $\square$

**Proof of Proposition 4:** The choice of  $r_1$  is straightforward as  $r_1$  does not enter in (11) and negatively affects bank profit  $\Pi^B$  since  $\frac{\partial \theta^*}{\partial r_1} > 0$ . Thus, the bank optimally sets  $r_1^B = 1$ . The rest of the proof proceeds in steps. First, we characterize the equilibrium choice of  $k$ ,  $\ell$  and  $r_2$ . Second, we show that in equilibrium banks choose  $k$  and  $\ell$  in such a way that  $(1 - k)r_1 > \ell\chi$  holds in equilibrium so that liquidity crises occur. Finally, we show that the equilibrium  $k$  and  $\ell$  are consistent with  $\frac{\partial \theta^*}{\partial k} < 0$  and  $\frac{\partial \theta^*}{\partial \ell} < 0$ .

Before starting solving the bank's problem, it is important to notice that the interest rate  $r_2$  affects the threshold  $\theta^*$  and it is chosen at date 0 from the debt holder's participation constraint, thus anticipating the

withdrawal threshold  $\theta^*$ . Differentiating the LHS of (11) with respect to  $\theta^*$ , we obtain

$$-\left[r_2 - \frac{\ell\chi}{1-k}\right] + \int_{\theta^*}^1 \frac{dr_2}{d\theta^*} d\theta, \quad (34)$$

where  $\frac{dr_2}{d\theta^*}$  can be computed using the implicit function theorem from (26) and it is then equal to

$$-\frac{\int_{\hat{n}(\theta)}^{\bar{n}} \frac{R'(\theta^*)(1-\alpha\ell)\left[1 - \frac{(1-k)nr_1}{\ell x}\right]}{(1-k)(1-n)} dn}{\int_0^{\hat{n}(\theta)} dn} < 0.$$

This implies that the expression in (34) is negative and so each pair  $\{k, \ell\}$  implements only one  $\theta^*$ .

Now we move on to solve bank's optimal choice. The conditions (14) and (15) in the proposition are obtained by substituting  $r_2$  from (11) into (10) and differentiating it with respect to  $k$  and  $\ell$ .

To prove that the bank's choice is always consistent with  $\frac{\partial\theta^*}{\partial k} < 0$  and  $\frac{\partial\theta^*}{\partial\ell} < 0$ , we show that the effect of a change in  $k$  and  $\ell$  on the threshold  $\theta^*$ , even accounting for the indirect effect of  $k$  and  $\ell$  on  $\theta^*$  via  $r_2$ , is positive when  $\frac{\partial\theta^*}{\partial k} > 0$  and  $\frac{\partial\theta^*}{\partial\ell} > 0$ . Since bank's profits in (10) are strictly decreasing in both  $\theta^*$  and  $r_2$ , it follows that in equilibrium  $k$  and  $\ell$  are chosen such that  $\frac{\partial\theta^*}{\partial k} < 0$  and  $\frac{\partial\theta^*}{\partial\ell} < 0$  hold.

We can compute the total effect of  $k$  on  $\theta^* \frac{d\theta^*}{dk}$  as follows. Implicitly differentiating (11) with respect to  $k$ , we obtain

$$\frac{d\theta^*}{dk} = -\frac{\int_0^{\theta^*} \frac{\ell\chi}{(1-k)^2} d\theta + \int_{\theta^*}^1 \frac{dr_2}{dk} d\theta}{-\left[r_2 - \frac{\ell x}{(1-k)}\right] + \int_{\theta^*}^1 \frac{dr_2}{d\theta^*} d\theta},$$

where  $\frac{\partial r_2}{\partial\theta^*} < 0$  and  $\frac{\partial r_2}{\partial k}$  is obtained by implicitly differentiating (26) and is equal to

$$\frac{dr_2}{dk} = -\frac{\frac{\partial f(\theta^*, k, \ell)}{\partial k}}{\frac{\partial f(\theta^*, k, \ell)}{\partial r_2}}.$$

Given that  $\frac{\partial f(\theta^*, k, \ell)}{\partial r_2} > 0$ , as long as  $\frac{\partial f(\theta^*, k, \ell)}{\partial k} < 0$ ,  $\frac{dr_2}{dk} > 0$  and  $\frac{d\theta^*}{dk} > 0$  since  $-\left[r_2 - \frac{\ell x}{(1-k)}\right] + \int_{\theta^*}^1 \frac{\partial r_2}{\partial\theta^*} d\theta < 0$ . As shown in the proof of Proposition 3,  $\frac{\partial f(\theta^*, k, \ell)}{\partial k} < 0$  when  $k < \bar{k}(\ell)$ . Following the same steps to compute  $\frac{d\theta^*}{d\ell}$ , we have that

$$\frac{d\theta^*}{d\ell} = -\frac{\int_0^{\theta^*} \frac{\ell}{(1-k)} d\theta + \int_{\theta^*}^1 \frac{dr_2}{d\ell} d\theta}{-\left[r_2 - \frac{\ell x}{(1-k)}\right] + \int_{\theta^*}^1 \frac{dr_2}{d\theta^*} d\theta},$$

with

$$\frac{\partial r_2}{\partial\ell} = -\frac{\frac{\partial f(\theta^*, k, \ell)}{\partial\ell}}{\frac{\partial f(\theta^*, k, \ell)}{\partial r_2}}.$$

The derivative  $\frac{dr_2}{d\ell}$  and, in turn,  $\frac{d\theta^*}{d\ell}$  are positive when  $\frac{\partial f(\theta^*, k, \ell)}{\partial\ell} < 0$ . As shown in the proof of Proposition 3, this is the case for any  $k < \underline{k}(\ell)$  and  $k > \bar{k}(\ell)$ . Thus, the bank would only choose a pair  $\{k, \ell\}$  in the region bounded by the curves  $\underline{k}(\ell)$  and  $\bar{k}(\ell)$ .

To complete the proof, we need to show that the equilibrium  $k$  and  $\ell$  satisfy  $(1-k)r_1 > \ell\chi$ . To see this,

we rearrange the first order conditions for  $k$  and  $\ell$ :

$$\begin{aligned} & -\frac{\partial \theta^*}{\partial k} [R(\theta^*) (1 - \alpha \ell) - (1 - k) r_2] + \int_{\theta^*}^1 r_2 d\theta - \rho \\ & + \frac{dr_2}{dk} \left\{ \int_{\theta^*}^1 (1 - k) d\theta - \frac{\partial \theta^*}{\partial r_2} [R(\theta^*) (1 - \alpha \ell) - (1 - k) r_2] \right\} = 0 \end{aligned} \quad (35)$$

and

$$\begin{aligned} & -\frac{\partial \theta^*}{\partial \ell} [R(\theta^*) (1 - \alpha \ell) - (1 - k) r_2] - \int_{\theta^*}^1 \alpha R(\theta) d\theta \\ & + \frac{dr_2}{d\ell} \left\{ \int_{\theta^*}^1 (1 - k) d\theta - \frac{\partial \theta^*}{\partial r_2} [R(\theta^*) (1 - \alpha \ell) - (1 - k) r_2] \right\} = 0 \end{aligned} \quad (36)$$

Assume that a bank sets  $\ell \chi = (1 - k) r_1^* = (1 - k)$ . From (11), it follows immediately that  $r_2^* = 1$ . Then, the expression (35) simplifies to

$$\int_{\theta^*}^1 r_2 d\theta - \rho + \frac{dr_2}{dk} \Big|_{(1-k)=\ell\chi} \int_{\theta^*}^1 (1 - k) d\theta < 0,$$

since  $\rho > 1$  and  $\frac{dr_2}{dk}$  can be computed using the implicit function theorem on (11) and is, then equal to

$$\frac{dr_2}{dk} \Big|_{(1-k)=\ell\chi} = -\frac{\int_0^{\theta^*} \frac{\ell\chi}{(1-k)^2} d\theta}{\int_{\theta^*}^1 d\theta} < 0.$$

Similarly, the expression (36) simplifies to

$$-\int_{\theta^*}^1 \alpha R(\theta) d\theta + \frac{dr_2}{d\ell} \Big|_{(1-k)=\ell\chi} \int_{\theta^*}^1 (1 - k) d\theta < 0,$$

with

$$\frac{dr_2}{d\ell} \Big|_{(1-k)=\ell\chi} = -\frac{\int_0^{\theta^*} \frac{\chi}{(1-k)} d\theta}{\int_{\theta^*}^1 d\theta} < 0.$$

The fact that both (35) and (36) are negative when evaluated at  $(1 - k) = \ell \chi$  implies that the bank will always choose a lower level of  $k$  and  $\ell$ , so that the inequality  $(1 - k) > \ell \chi$  holds in equilibrium. This, in turn, implies that  $r_2^* > 1$  for the (11) to be satisfied. Thus, the proposition follows.  $\square$

**Proof of Proposition 5:** Evaluating the expression for banks' profits as given in (22) when  $1 - k = \ell \chi(\theta^E)$ , we obtain

$$\int_0^{\theta^E} \ell \chi(\theta^E) d\theta + \int_{\theta^E}^1 R(\theta) (1 - \alpha \ell) d\theta - (1 - k) - k\rho, \quad (37)$$

where  $\theta^E$  is given by (17). It easy to see that banks' profits in (37) decreases with  $\rho$ . Similarly, differentiating (37) with respect to  $\alpha$  gives

$$-\frac{\partial \theta^E}{\partial \alpha} [R(\theta^E) (1 - \alpha \ell) - \ell \chi(\theta^E)] + \int_0^{\theta^E} \ell \chi'_\alpha(\theta^E) d\theta - \int_{\theta^E}^1 \ell R(\theta) d\theta < 0,$$



since the first term is zero and  $\chi'_\alpha(\theta^E) = \frac{\partial \chi(\cdot)}{\partial Q} \frac{d\theta^E}{d\alpha} = \frac{\partial \chi(\cdot)}{\partial Q} \frac{\ell}{R'(\theta^E)(1-\alpha\ell)} < 0$  since  $\frac{\partial \chi(\cdot)}{\partial Q} < 0$ . This implies that, for given  $\{k, \ell\}$ , the non-negative profit condition becomes more binding when  $\alpha$  and  $\rho$  increase. Furthermore,  $\theta^E$  increases with  $\alpha$  and  $\hat{\theta}^*$  decreases with  $w$ . Thus, for given  $\rho$  and  $w$ , the larger  $\alpha$ , the higher  $\theta^E$  and the lower  $\Pi^B$ . Similarly, for given  $\alpha$  and  $w$ , the larger  $\rho$ , the lower  $\Pi^B$ . It follows that for large  $\alpha$  and  $\rho$ , there are fewer pairs  $\{k, \ell\}$  consistent with  $\Pi^B \geq 0$  for which  $\theta^E \leq \hat{\theta}^*$  holds. For given  $\alpha$  and  $\rho$ , since  $\frac{\partial \hat{\theta}^*}{\partial w} > 0$ , as  $w$  increases, the set of pairs  $\{k, \ell\}$  consistent with  $\Pi^B \geq 0$  for which  $\theta^E \leq \hat{\theta}^*$  widens. Thus, the proposition follows.  $\square$

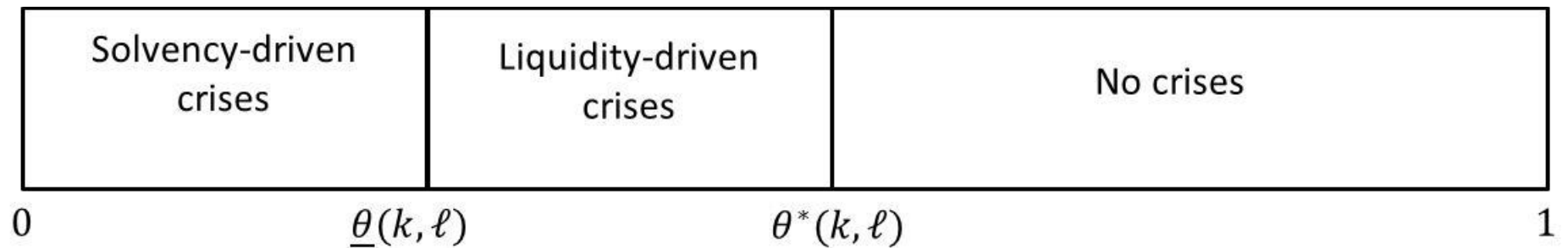


Figure 1: Debt holders' withdrawal decisions and banking crises. The figure illustrates debt holders' withdrawal decisions as a function of the fundamentals of the economy  $\theta$ . In the range for  $\theta < \theta^*(k, \ell)$ , debt holders choose not to roll over their debt claim, thus forcing the bank to default at date 1. In the region in which a bank default occurs, crises can be distinguished into solvency-driven ones for  $0 < \theta < \underline{\theta}(k, \ell)$  and liquidity-driven ones for  $\underline{\theta}(k, \ell) < \theta < \theta^*(k, \ell)$ . The former are driven by debt holders' expectation of a low realization of the fundamentals of the economy  $\theta$ , while the latter are due to debt holders' fear that others will not roll over their debt claim.

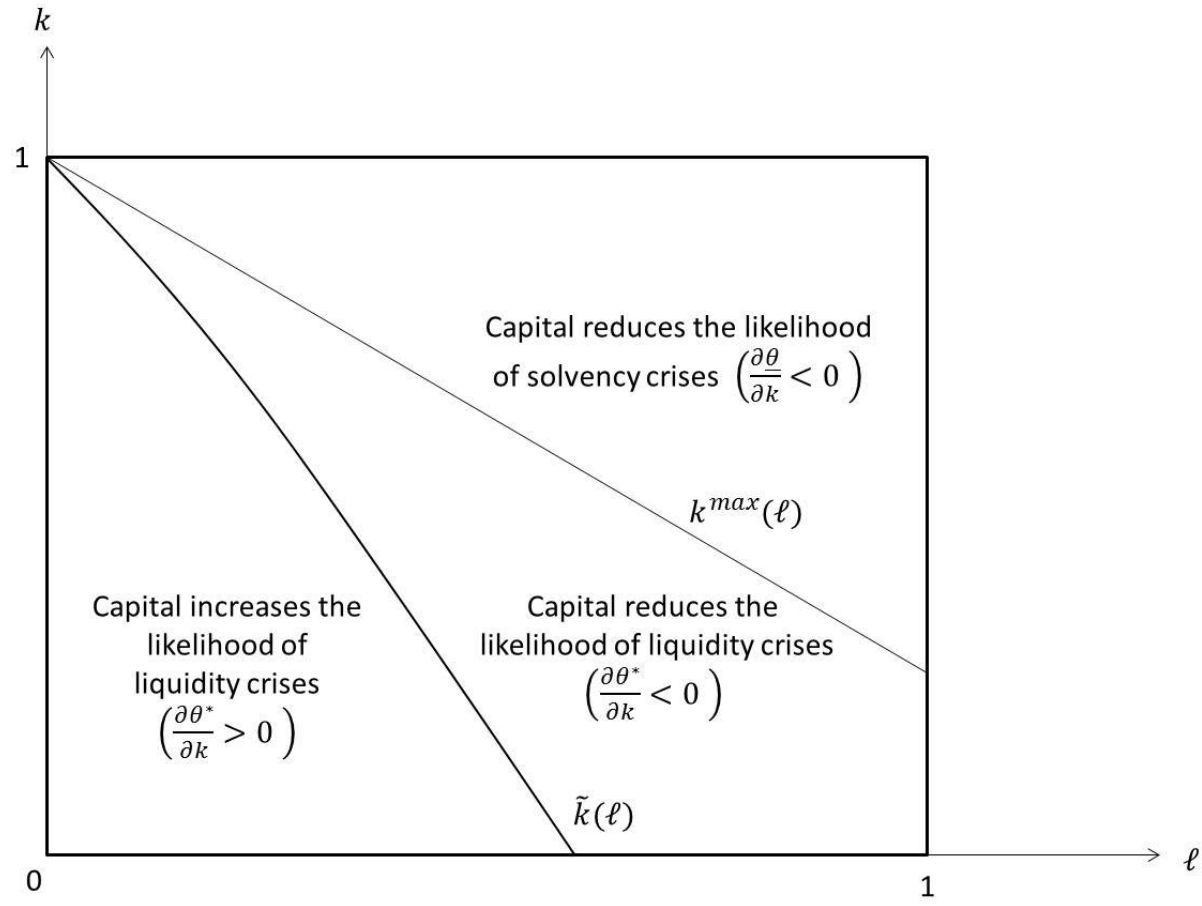


Figure 2a: Effect of capital on financial stability Figure 2b: The figure shows that the effect of capital on financial stability depends on the initial level of bank capitalization  $k$  and portfolio liquidity  $\ell$ . Capital has a detrimental effect on stability (i.e.,  $\frac{\partial \theta^*(k, \ell)}{\partial k} > 0$ ) when the level of bank capitalization or portfolio liquidity are low, as it is the case in the region below  $\tilde{k}(\ell)$ . For high values of bank capitalization and/or portfolio liquidity, as it is the case in the region above the curve  $k > \tilde{k}(\ell)$ , an increase in capital reduces bank failure probability. Thus,  $\frac{\partial \theta^*(k, \ell)}{\partial k} < 0$  in the region between  $\tilde{k}(\ell)$  and  $k^{max}(\ell)$  and  $\frac{\partial \theta(k, \ell)}{\partial k} < 0$  in the region above  $k^{max}(\ell)$ , as in this region only solvency crises occur.

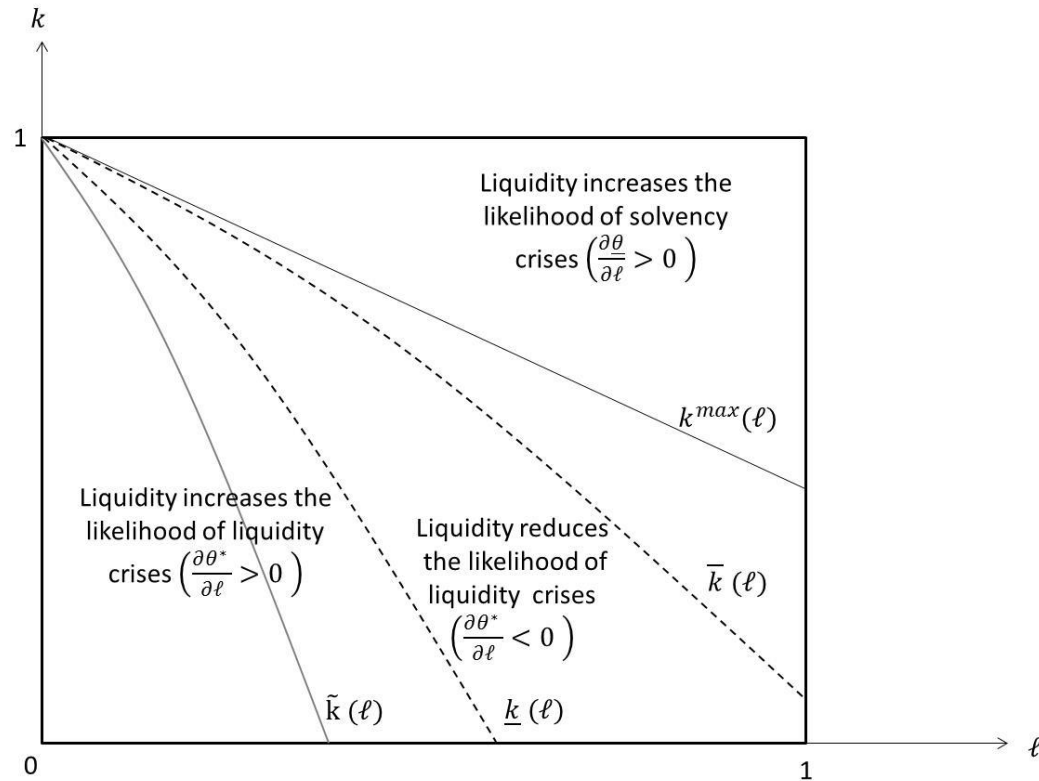


Figure 2b: Effect of liquidity on financial stability. The figure shows that the effect of liquidity on financial stability depends on the initial level of capital  $k$  and on the liquidity of bank portfolio  $\ell$ . Liquidity has a detrimental effect on stability (i.e.,  $\frac{\partial \theta^*(k,\ell)}{\partial \ell} > 0$ ) when the level of bank capitalization and/or portfolio liquidity are either low or high, as it is the case in the regions above the curve  $\bar{k}(\ell)$  and below the curve  $\underline{k}(\ell)$ . For intermediate values of bank capitalization and/or portfolio liquidity, as it is the case in the region between the curves  $\underline{k}(\ell)$  and  $\bar{k}(\ell)$ , higher portfolio liquidity reduces the probability of a liquidity crisis (i.e.,  $\frac{\partial \theta^*(k,\ell)}{\partial \ell} < 0$ ).