Kehoe-Levine Meets Aiyagari: Stationary Equilibrium in the Neoclassical Growth Model with One-Sided Limited Commitment

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Motivation and Objective

- Standard incomplete markets model (SIM):
 - Continuum of households, each solving an income fluctuation problem: idiosyncratic income risk and incomplete markets.
 - Interest rate r determined in stationary equilibrium: Bewley (1986), Imrohoroglu (1989), Huggett (1993), Aiyagari (1994).
- Alternative: explicitly modelled frictions (private information, limited commitment) to rationalize imperfect consumption insurance.

Motivation and Objective

- General equilibrium in limited commitment models: Kehoe and Levine (1993, 2001), Alvarez and Jermann (2000).
 - Continuum of households: Krueger and Perri (2006, 2010).
 - Key question in these models: what happens if households do not honor their promises? Some form of autarky, $U(.; y_t) \ge U^{Aut}(y_t)$. Seems arbitrary.
 - Krueger-Uhlig (2006): Endogenize $U^{Out}(y_t)$ through competition of financial intermediaries.
 - But: exogenous r.
- Goal of this paper: endogenize interest rate in limited commitment economies with endogenously determined outside option.
- Deliver analytically tractable counterpart to Huggett (1993), Aiyagari (1994) with limited commitment and endogenous outside option.

► More on Related Literature

Model: Basics

- Continuous time $t \in [0, \infty)$. Why continuous time?
- Continuum of measure 1 of ex ante identical, potentially infinitely lived individuals.
- Death at constant rate $\gamma > 0$.
- Newborns enter economy at rate γ .
- Thus population size constant at 1.

Model: Preferences and Endowments

• Preferences represented by

$$E\left[\int_0^\infty e^{-\rho\tau} \frac{c(\tau)^{1-\sigma} - 1}{1 - \sigma} d\tau\right]$$

- $\sigma = 1$ corresponds to $u(c) = \log(c)$.
- Expectations taken with respect to idiosyncratic income (y) and survival (γ) risk.
- No aggregate risk.

Model: Endowments

- One unit of time.
- Labor productivity $y(t) \in \{y_l, y_h\}$.
- Transition from y_l to y_h at rate $\nu > 0$.
- Transition from y_h to y_l at rate $\pi > 0$.
- Non-tradeable endowment χ , can only be consumed in autarky. Instanteneous utility from consuming χ given by $\underline{u} = u(\chi) > -\infty$.
- Prevents technical problems for $y_l = 0$ and $\sigma \ge 1$ example.
- Stationary labor productivity distribution: $(\psi_l, \psi_h) = \left(\frac{\pi}{\pi + \nu}, \frac{\nu}{\pi + \nu}\right)$
- Newborns draw productivity from stationary distribution.
- Average labor productivity equal to one: $\frac{\pi}{\pi+\nu}y_l + \frac{\nu}{\pi+\nu}y_h = L = 1$

Model: Technology

• Representative production firm operates aggregate production technology:

$$Y = AF(K, L) = AK^{\theta}L^{1-\theta}$$

- Uses labor L and (if $\theta > 0$) capital K to produce a single output good Y.
- Equilibrium wage in stationary equilibrium $\omega = (1 \theta)AK^{\theta}$.
- Capital depreciates at a constant rate $\delta \geq 0$.
- Endowment economy (as in Huggett, 1993) if $A = 1, \theta = 0, \delta = 0$.
- Production economy (as in Aiyagari, 1994) if $\theta \in (0,1)$.

Financial Market Structure

- Risk-neutral competitive financial intermediaries offer long-term consumption insurance contracts.
- Provide insurance against idiosyncratic labor income risk.
- In every instant intermediary gets $\omega y(t)$, household gets $\omega c(t)$.
- One-sided limited commitment.
 - Intermediaries can commit to the contract into indefinite future.
 - Key friction: agents cannot commit.
 - After observing y(t) agents can leave the contract at any time and sign up with the next intermediary.
- Perfect competition leads to zero profits for intermediaries when signing up an agent. Agent gets lifetime utility $U^{Out}(y)$.

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- Perfect competition leads to zero profits for intermediaries when signing up an agent. Agent gets lifetime utility $U^{Out}(y)$.
 - Similar to Krueger and Uhlig (2006), initially in Harris and Holmstrom (1982), also Phelan (1995).
 - Krueger and Uhlig (2006): this market structure equivalent to complete markets with short-sale constraints $b'(y') \ge 0$. Akin to Alvarez and Jermann (2000).
 - Also equivalent to Gottardi and Kubler (2015)'s market structure with complete insurance contracts but collateral constraints?

Optimal Insurance Contract

Definition

For outside options $\{U^{out}(y)\}\$ with $y\in Y$, a wage ω and interest rate r, an optimal consumption insurance contract $c(\tau;y,U),\ V(y,U)$ solves

$$V(y,U) = \min_{\langle c(\tau) \rangle \ge 0} \mathbf{E} \left[\int_t^\infty e^{-(r+\gamma)(\tau-t)} \left[\omega c(\tau) - \omega y(\tau) \right] d\tau \, \middle| \, y(t) = y \right]$$

subject to

$$\begin{split} \mathbf{E} \left[\int_t^\infty e^{-(\rho + \gamma)(\tau - t)} u(c(\tau)) d\tau \middle| y(t) &= y \right] &\geq & U \\ \mathbf{E} \left[\int_s^\infty e^{-(\rho + \gamma)(\tau - s)} u(c(\tau)) d\tau \middle| y(s) \right] &\geq & U^{out}(y(s)) \text{ for all } s > t \end{split}$$

for all $t \geq 0$, for all $y \in Y$ and all $U \in \left[U^{out}(y), \frac{\bar{u}}{\rho + \gamma}\right]$.

Definition

A Stationary Equilibrium is outside options $\{U^{out}(y)\}_{y\in Y}$, contracts (c,V), a wage ω , interest rate r and stationary consumption density $\phi(c)$ s.t.

- $\mbox{\bf 0}$ Given $\{U^{out}(y)\}_{y\in Y}$ and (ω,r) contract $c(\tau,y,U),V(y,U)$ is optimal.
- **2** Outside options lead to zero profits: for all y we have $V(y, U^{out}(y)) = 0$.
- **3** The interest rate and wage (r, ω) satisfy

$$r = AF_K(K,1) - \delta$$

 $\omega = AF_L(K,1)$

4 The goods and capital market clear

$$\omega \int c\phi(c)dc + \delta K = AF(K,1)$$
$$\frac{\omega \left(\int c\phi(c)dc - 1\right)}{r} = K$$

 $\phi(c)$ is consistent with the dynamics of the optimal consumption contract and with the demographic structure of the economy (birth and death).

Goal: Characterization of Capital Market Equilibrium (as in Aiyagari, 1994)

• Rewrite capital market clearing condition as

$$\left(\frac{K^s}{\omega}\right)(r) := \frac{C(r) - 1}{r} = \left(\frac{K^d}{\omega}\right)(r)$$

where $C(r) = \int c\phi(c)dc$ is aggregate (scaled) consumption demand.

• Capital demand from firms' FOC's:

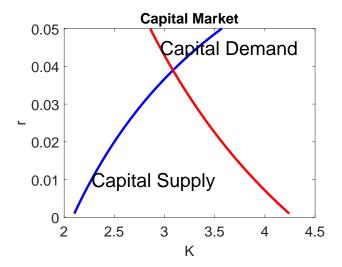
$$\left(\frac{K^d}{\omega}\right)(r) = \frac{\theta}{1-\theta} \frac{1}{r+\delta}$$

• Capital supply by households is given by

$$\left(\frac{K^s}{\omega}\right)(r) := \frac{C(r) - 1}{r}$$

• Now: characterize aggregate consumption demand C(r) and thus capital supply $\left(\frac{K^s}{\omega}\right)(r)$ from risk sharing contracts between households and intermediaries.

Goal: Characterization of Capital Market Equilibrium



Road Map (and Results)

- \bullet Optimal contract and outside options for fixed r
 - $r = \rho$: Full insurance in the limit.
 - $r < \rho$: Partial insurance.
 - $r > \rho$: Super-insurance (not today).
- 2 Stationary consumption distribution for fixed r
 - $r = \rho$: Two point distribution.
 - $r < \rho$: Two mass points, power distribution in between.
- 3 Use resource constraint to determine r
 - Assume $y_l = 0$ and $\sigma = 1$ (plus assumptions on transition probabilities). Then can derive unique equilibrium with $r^* > 0$ and partial insurance in closed form.
 - Conjectures about equilibria (number and properties) in other cases.

Optimal Consumption Contract and Outside Option

- Fix r. Normalize by ω . Consumption is $\omega c(\tau; y, U)$, net contract costs are $v = V/\omega$.
- Basic idea: smooth consumption as much as possible, subject to not violating participation constraints. Off constraints:

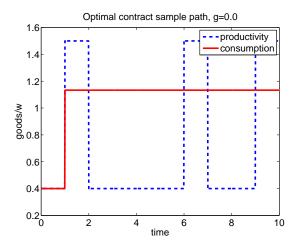
$$\frac{\dot{c}(t)}{c(t)} = -g$$
, where $g = \frac{\rho - r}{\sigma}$

• Define $v_j, j \in \{l, h\}$ as lifetime (net) cost of consumption contract of newborns with $y_j \in \{y_l, y_h\}$. Perfect competition implies

$$v_l = v_h = 0.$$

- Key insight of Krueger and Uhlig (2006) (and many before): payments are front-loaded, consumption is back-loaded.
- They show: if $\theta = 0$ (endowment economy) and $\gamma = 0$ (no newborns), any stationary equilibrium has to be autarkic.
- This paper argues: Birth/death and/or capital might solve the problem.

• $r = \rho$ implies g = 0 and consumption should remain constant whenever participation constraints not binding. Denote by c_l, c_h



Characterization $r = \rho$

Proposition

• The equilibrium consumption insurance contract is given by

$$\begin{array}{rcl} c_l & = & y_l \\ c_h & = & \frac{r+\gamma+\nu}{r+\gamma+\nu+\pi} y_h + \frac{\pi}{r+\gamma+\nu+\pi} y_l < y_h \end{array}$$

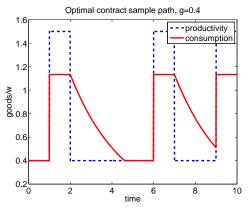
- Contract collects insurance premium $y_h c_h = \frac{\pi(y_h y_l)}{r + \gamma + \nu + \pi}$ from those with $y = y_h$ and uses it to pay consumption insurance $c_h y_l = \frac{(r + \gamma + \nu)(y_h y_l)}{r + \gamma + \nu + \pi}$ to those households that have obtained it.
- The stationary consumption distribution is given by

$$\phi_h = \frac{\gamma \nu + \nu(\pi + \nu)}{(\gamma + \nu)(\pi + \nu)} \in (0, 1)$$

$$\phi_l = \frac{\gamma \pi}{(\gamma + \nu)(\pi + \nu)} \in (0, 1)$$

• Either participation constraint is binding and individual consumes (c_h, c_l) or consumption satisfies complete markets Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = -\frac{\rho - r}{\sigma} = -g < 0$$



- Consumption dynamics: whenever household has $y(t) = y_h$, she consumes c_h . When income switches to y_l consumption drifts down at rate g until it hits c_l .
- Consumption dynamics implies that $c(t) = c_h e^{-gt}$ where t is time elapsed since household last had $y = y_h$.
- Stopping time $T \in (0, \infty)$ such that $c_h e^{-gT} = c_l$.
- Cost v(t) of the contract after "time" t.
- (v_l, v_h) costs of new contracts (to the principal). By perfect competition $v_l = v_h = 0$.

• Hamilton-Jacobi-Bellman equations

$$rv_h = c_h - y_h + \gamma(0 - v_h) + \pi(v(0) - v_h)$$

$$rv_l = c_l - y_l + \gamma(0 - v_l) + \nu(v_h - v_l)$$

$$rv(t) = c(t) - y_l + \gamma(0 - v(t)) + \nu(v_h - v(t)) + \dot{v}(t)$$

where $c(t) = c_h e^{-gt}$.

- Terminal condition $v(T) = v_l = 0$.
- Key unknowns $(c_l, c_h), T$.
- Second equation implies $y_l = y_l$ (since $v_l = v_h = 0$).
- First equation implies

$$\pi v(0) = y_h - c_h$$

• Households with high income pays insurance premium $y_h - c_h$ to compensate financial intermediary for the cost incurred during a low income spell.

• Can solve the differential equation for v(t). Delivers

$$v(t) = \int_{t}^{T} e^{-(r+\gamma+\nu)(\tau-t)} \left(c_{h} e^{-g\tau} - y_{l} \right) d\tau$$

$$= \frac{c_{h} e^{-gt} \left[1 - e^{-(r+\gamma+\nu+g)(T-t)} \right]}{r+\gamma+\nu+g} - \frac{y_{l} \left[1 - e^{-(r+\gamma+\nu)(T-t)} \right]}{r+\gamma+\nu}$$

• Evaluating at t = 0 we obtain

$$v(0) = \frac{c_h \left[1 - e^{-(r+\gamma+\nu+g)T} \right]}{r+\gamma+\nu+g} - \frac{y_l \left[1 - e^{-(r+\gamma+\nu)T} \right]}{r+\gamma+\nu}$$

• Three equations in three unknowns $v(0), c_h, T$ solving previous equation and

$$\pi v(0) = y_h - c_h$$
$$c_h e^{-gT} = y_l$$

Characterization of Optimal Contract for $r < \rho$

Proposition

If $\rho > r$, there exists a unique optimal stopping time $T^* \in (0, \infty]$ solving

$$\frac{y_h}{y_l} = e^{\frac{T}{\sigma}(\rho - r)} + \frac{1 - e^{-\left[r + \gamma + \nu + \frac{\rho - r}{\sigma}\right]T}}{r + \gamma + \nu + \frac{\rho - r}{\sigma}} \pi e^{\frac{T}{\sigma}(\rho - r)} - \frac{1 - e^{-(r + \gamma + \nu)T}}{r + \gamma + \nu} \pi$$

The optimal consumption contract satisfies

$$c_h = y_l e^{\frac{T^*}{\sigma}(\rho - r)}$$

$$c(t) = c_h e^{\frac{-t}{\sigma}(\rho - r)}$$

$$c_l = y_l$$



Characterization of Optimal Contract for $r < \rho$

Proposition

Suppose in addition $y_l = 0$. Then $c_l = 0, T^* = \infty, c(t) = c_h e^{-\frac{t}{\sigma}(\rho - r)}$ and

$$c_h = \frac{r + \gamma + \nu + g}{r + \gamma + \nu + g + \pi} y_h = c_h(r)$$

Proposition

Suppose in addition $y_l = 0, \sigma = 1$. Then $c_l = 0, T^* = \infty$, $c(t) = c_h e^{-t(\rho - r)}$ and

$$c_h = \frac{\rho + \gamma + \nu}{\rho + \gamma + \nu + \pi} y_h$$

Stationary Consumption Distribution

- Can derive stationary consumption distribution from consumption and population (birth and death) dynamics. Have already done so for $r = \rho$. Now consider case $r < \rho$.
- All households with y_h consume c_h . Thus mass point $\phi(c_h) = \frac{\nu}{\nu + \pi}$.
- Consumption of households with currently low income follows diffusion process on (c_l, c_h) with drift $-g = -\frac{\rho r}{\sigma}$. Also experience jumps to c_h (intensity ν) and death (intensity γ).
- Apply Kolmogorov forward equation (augmented by Poisson jumps). On (c_l, ch) stationary consumption distribution is given by

$$\phi(c) = \phi_1 c^{\left(\frac{\gamma+\nu}{g}-1\right)}$$

- (Truncated) Pareto distribution with tail parameter $\frac{\gamma+\nu}{g}-1$. Constant ϕ_1 to be determined.
- Second mass point $\phi(c_l)$ at c_l because of birth and long y_l spells.

Details of Kolmogorov Forward Equation

- Consumption of households with currently low income follows diffusion process on (c_l, c_h) with drift $-g = -\frac{\rho r}{\sigma}$.
- Also experiences jumps to c_h (intensity ν) and death (intensity γ).
- Kolmogorov forward equation

$$0 = -\frac{d\left[-gc\phi(c)\right]}{dc} - (\gamma + \nu)\phi(c)$$

• Since

$$-\frac{d\left[-gc\phi(c)\right]}{dc} = -\left[-g\phi(c) - gc\phi'(c)\right] = g\left[\phi(c) + c\phi'(c)\right]$$

consumption distribution satisfies

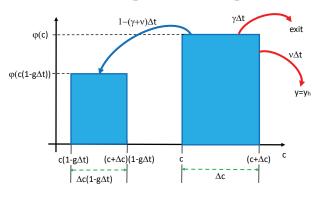
$$g\left[\phi(c) + c\phi'(c)\right] = (\gamma + \nu)\phi(c)$$

• Thus

$$\frac{c\phi'(c)}{\phi(c)} = \frac{\gamma + \nu}{q} - 1$$

• Thus on this interval the stationary consumption distribution is Pareto with tail parameter $\frac{\gamma+\nu}{a}-1$.

Kolmogorov Forward Equation: Graphical View



$$\phi(c(1 - g\Delta t))\Delta c(1 - g\Delta t) = \phi(c)\Delta c(1 - (\gamma + \nu)\Delta t)$$

$$(\phi(c) - cg\Delta t\phi'(c)) (\Delta c - g\Delta c\Delta t) = \phi(c)\Delta c(1 - (\gamma + \nu)\Delta t)$$

$$-g\phi(c) - cg\phi'(c) = -\phi(c)(\gamma + \nu)$$

$$\frac{c\phi'(c)}{\phi(c)} = \frac{\gamma + \nu - g}{g}$$

Stationary Consumption Distribution

Proposition

For any $r < \rho$ the stationary consumption distribution is given by two mass points at $y_l, c_h(r) = y_l e^{gT^*(r)}$ and a Pareto density in between:

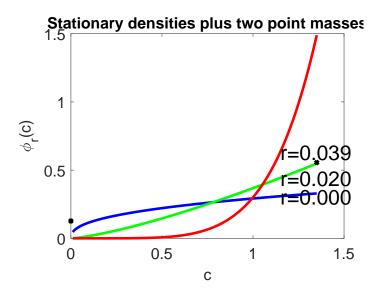
$$\phi_r(c) = \begin{cases} \frac{\pi}{(\nu + \pi)} \left(1 - \frac{\nu}{(\gamma + \nu)} \left[1 - e^{-(\gamma + \nu)T^*(r)} \right] \right) & \text{if} \quad c = y_l \\ \frac{\pi \nu}{g(\nu + \pi)c} \left(\frac{c}{c_h(r)} \right)^{\frac{\gamma + \nu}{g}} & \text{if} \quad c \in (y_l, c_h(r)) \\ \frac{\nu}{(\nu + \pi)} & \text{if} \quad c = c_h(r) \end{cases}$$

Proposition

Suppose in addition $y_l = 0$. Then

$$\phi_r(c) = \begin{cases} \frac{\pi \gamma}{(\nu + \pi)(\gamma + \nu)} & \text{if} \quad c = 0\\ \frac{\pi \nu}{g(\nu + \pi)c} \left(\frac{c}{c_h(r)}\right)^{\frac{\gamma + \nu}{g}} & \text{if} \quad c \in (y_l, c_h(r))\\ \frac{\nu}{(\nu + \pi)} & \text{if} \quad c = c_h(r) \end{cases}$$

Stationary Consumption Distribution for 3 r's



Market Clearing Interest Rate r

• Capital Demand

$$\left(\frac{K^d}{\omega}\right)(r) = \frac{\theta}{1-\theta}\frac{1}{r+\delta}$$

• Capital Supply

$$\left(\frac{K^s}{\omega}\right)(r) = \frac{C(r) - 1}{r}$$

where aggregate consumption demand is given by

$$C(r) = \int c\phi_r(c)dc$$

• Market Clearing

$$\left(\frac{K^s}{\omega}\right)(r) = \left(\frac{K^d}{\omega}\right)(r)$$



Consumption Demand

• Consumption Demand:

$$C(r) = \int c\phi_r(c)dc$$

- General properties of C as a function of r beyond continuity hard to establish.
- In lead numerical example: C is convex.
- Always convex? We have counterexamples.
- But easy (although somewhat magical) to show that C(r = 0) = 1.

Consumption Demand and Capital Supply for $y_l = 0, \sigma = 1.$

• Recall: with $y_l = 0, \sigma = 1$, stopping time $T = \infty$, and upper consumption bound c_h is independent of r.

Proposition

Let $y_l = 0, \sigma = 1$. Then aggregate consumption demand is strictly increasing and strictly convex, and given by

$$C(r) = 1 + \frac{r\pi}{(\gamma + \nu + \rho + \pi)(\gamma + \nu + \rho - r)}$$

Capital demand is determined as

$$\left(\frac{K^s}{\omega}\right)(r) = \frac{C(r) - 1}{r} = \frac{\pi}{(\gamma + \nu + \rho + \pi)(\gamma + \nu + \rho - r)}$$

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Why is C(r) upward sloping? Some Intuition

- C(r): calculated per integration across cross-section.
- Contracts: integration across time. NPV, discounting.
- Higher r: future is discounted more strongly, so intermediary can promise more consumption in the future, at same NPV.
- But wait! Future wage income is also discounted more strongly!
- Resolution: payments are front-loaded, consumption is backloaded: the effect matters more for consumption.

Equilibrium Interest Rate: for $y_l = 0, \sigma = 1$.

Proposition

Assume that $y_l = 0$ and $\sigma = 1$ and $\rho > 0$. Then under assumptions A1 and A2 on parameters there exists exactly one stationary equilibrium, with interest rate $0 < r^* < \rho$, where

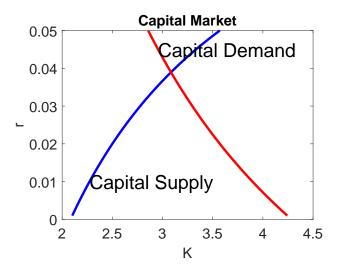
$$r^* = \frac{\theta(\pi + \gamma + \nu + \rho)(\gamma + \nu + \rho) - \pi\delta(1 - \theta)}{\pi + \theta(\gamma + \nu + \rho)}.$$

and partial consumption insurance.

Example:
$$\rho = 0.05, \delta = 0.1, \theta = 0.3, \sigma = 1, A = 1.$$

 $\nu = 0.05, \pi = 0.04, \gamma = 0.02.$ Then $r = 0.039.$

Capital Market Equilibrium



Equilibrium Interest Rate: Assumptions

• Assumption A1 insures $r^* < \rho$

$$\frac{\pi}{(\pi+\gamma+\nu+\rho)(\gamma+\nu)} > \frac{\theta}{(1-\theta)(\rho+\delta)}$$

• Assumption A2 (conditional on A1) insures $r^* > 0$

$$\frac{\pi}{(\pi+\gamma+\nu+\rho)(\gamma+\nu+\rho)}<\frac{\theta}{(1-\theta)\delta}$$

• Note that assumptions A1 and A2 can be combined to:

$$\frac{\delta}{\gamma + \nu + \rho} < \frac{\theta(\pi + \gamma + \nu + \rho)}{(1 - \theta)\pi} < \frac{\delta}{\gamma + \nu + \rho} \times \left(1 + \frac{\rho}{\delta}\right) \times \left(1 + \frac{\rho}{\gamma + \nu}\right)$$

Comparative Statics

Proposition

The equilibrium interest rate $r^* \in (0, \rho)$ is a strictly increasing function of $\rho + \gamma + \nu, \theta$ and a strictly decreasing function of π, δ .

Proposition

The equilibrium capital stock $K^* \in (0, K^{GR})$ is a strictly increasing function of π, θ and a strictly decreasing function of $\rho + \gamma + \nu, \delta$.

How many equilibria are there?

- Benchmark: $\sigma = 1, y_l = 0,$ (A1),(A2): unique $r^* \in (0, \rho)$.
- Suppose $\sigma \neq 1$, $y_l = 0$ and (A1). Equilibrium interest rate r with partial insurance $(r < \rho)$ satisfies quadratic equation:

$$\frac{\theta}{(1-\theta)(r+\delta)} = \frac{\pi}{(\pi+\gamma+\nu+\frac{\rho-r}{\sigma}+r)(\gamma+\nu+\frac{\rho-r}{\sigma})}$$

- Under some conditions (especially $\sigma > 1$), we therefore likely obtain two equilibria.
- $y_l > 0$? Hard analytically: $T^* < \infty$. More than two stationary equilibria?
- For general utility functions, we conjecture that capital supply and capital demand can cross lots of times.

Conclusion

- Long-Term Goal
 - Embed long-term insurance contracts in GE models with idiosyncratic risk.
 - Use them to examine worker-firm or bank-borrower relationships in dynamic GE with idiosyncratic and (eventually) aggregate risk.
- What Have We Done
 - Analyzed Stationary General Equilibrium in limited commitment economy with endogenous outside option.
 - $y_l = 0, \sigma = 1$, assumptions: Unique stationary equilibrium with partial consumption insurance and $r^* \in (0, \rho)$. Comparative statics.
 - Analytical characterization of stationary consumption distribution.
- What is Next?
 - General conditions for existence of multiple stationary equilibria.
 - General conditions for welfare rankings among equilibria.
 - Transitions and stability of stationary equilibria.
 - Role of publicly provided assets (money, government bonds)?

THANK YOU

Related Literature

- Stationary equilibria in standard incomplete markets (SIM) models: Bewley (1986), Huggett (1993), Aiyagari (1994).
- General equilibrium in limited commitment models with exogenous outside option: Kehoe and Levine (1993, 2001), Alvarez and Jermann (2000), also Kocherlakota (1996). With continuum of households: Krueger and Perri (2006, 2010).
- Endogenizing the outside option in limited commitment models: Krueger and Uhlig (2006). But: exogenous interest rate.
- This paper: complete the circle.
- Related equilibrium models with finitely many agents: Hellwig and Lorenzoni (2009), Gottardi and Kubler (2015), Abraham and Laczo (2016).

