

# Dynamic Financial Constraints: Which Frictions Matter for Corporate Policies? \*

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## Abstract

We build, solve, and estimate a range of dynamic models of corporate investment and financing. We focus on limited enforcement, moral hazard, and tradeoff models. All models share a common technology, but differ in the friction generating financial constraints. Using panel data on Compustat firms for the period 1980-2015 and a more recent dataset on private firms from Orbis, we determine which features of the observed data allow to distinguish among the models, and we assess which model performs best at rationalizing observed corporate investment and financing policies across various samples. Our tests, based on empirical policy function benchmarks, favor trade-off models for larger Compustat firms, limited commitment models for smaller firms, and moral hazard models for private firms. Our estimates point to significant financing constraints due to agency frictions.

Keywords: Financial frictions, moral hazard, limited enforcement, tradeoff, structural estimation, empirical policy function estimation, corporate policies

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# 1 Introduction

Corporate finance revolves around the study of financial frictions. Indeed, as pointed out forcefully by Modigliani and Miller, in the absence of frictions restricting firms' access to external financing, corporations financing decisions are irrelevant to their valuations and real policies. But what are the sources of financial frictions? While there is little disagreement about the relevance of financing constraints, their nature is much more debated. Is firms' access to external finance restricted because tax advantages make it attractive to firms to issue loans that they might not be able to pay back after adverse shocks, as in a trade-off model? Is it because firms cannot commit to honor their obligations, as in a limited commitment model? Or is it because firms might misreport their effective performance to lenders and divert cash flows, as moral hazard models have suggested? And, are these frictions equally important across firms, if at all?

In this paper, we propose to take a step towards providing guidance regarding the sources of financial frictions by structurally estimating a host of dynamic financing models as proposed in the literature, across a variety of different data samples, and assessing their relative fit. In every estimation, we ask: Which of the proposed models provides the best description of the actual behavior of a given set of companies, if any? Do our data allow us to discriminate between the relevance of these models for a particular set of firms?

Our approach relies on recent advances in computation and estimation. On the computational side, the linear programming approach to dynamic programming, introduced by Trick and Zin (1993) and recently extended in the context of dynamic corporate finance models in Nikolov, Schmid, and Steri (2016), enables us to efficiently solve a large number of dynamic models of firm financing, such as trade-off, limited commitment and dynamic moral hazard models, that are computationally challenging because of the high-dimensionality of the set of choice variables. Regarding estimation, we adopt a novel approach to structural estimation, empirical policy function estimation, introduced by Bazdresch, Kahn, and Whited (2016), that identifies novel empirical benchmarks and allows to succinctly trace out firms' dynamic behavior<sup>1</sup>. Based on these benchmarks, we develop tests that allows us to empirically discriminate among sets of models.

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<sup>1</sup>Gala and Gomes (2016) offer a related approach.

More specifically, we start by laying out in a unified environment a triplet of models of dynamic firm financing that have received attention in both empirical and theoretical literature. The first is a standard trade-off model, similar to Hennessy and Whited (2007), in which tax advantages of debt encourage firms to issue defaultable debt. The relevance of tax considerations for the determination of firms' capital structures has long been highlighted in the literature. More recently, in contrast, the theoretical literature has emphasized the role of financial contracting in determining firm policies and dynamics.<sup>2</sup> Financial contracts arise to mitigate incentive conflicts between firms' insiders and outsiders and affect corporations' financial structures, investment policies and valuations. We account for these developments by considering, first, a model in which an optimal lending contract between lenders and shareholders determine firm financing when the latter cannot commit to honor their obligations, as in Albuquerque and Hopenhayn (2004) or more recently in Rampini and Viswanathan (2010) and Rampini and Viswanathan (2013), and second, a model in which firms' access to external financing is curtailed by moral hazard in the presence of asymmetric information about cash flows, so that shareholders can divert cash flows from lenders, similar to Clementi and Hopenhayn (2006), Quadrini (2004) or Verani (2016). In contrast to tax-based tradeoff models, the latter dynamic agency models emphasize the state-contingent nature of financial instruments as important features of optimal financial contracts.

Our strategy is to evaluate all of these models by estimating their policy functions on a variety of samples and assess their relative fit. To that end, we consider full samples and subsamples drawn from both the standard Compustat universe, as well as a more recent dataset on US private firms from Orbis. Our data thus allow us to evaluate model fit across small and large firms, public and private firms, profitable and unprofitable firms, among others. Following Bazdresch, Kahn, and Whited (2016), we estimate empirical policy functions on each of these samples, so that our estimator picks model parameters that minimizes the distance between policy functions recovered from simulations and the empirical benchmarks. Our implementation is based on investment, leverage and payout policy functions. Policy function estimation is an attractive approach to model comparison in this context, as the procedure has excellent power to detect misspecification, as emphasized in Bazdresch, Kahn, and Whited (2016).

Our estimation results favor trade-off models as best rationalizations of the behavior of large

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<sup>2</sup>See e.g. the surveys of Harris and Raviv (1991) and Zingales (2000).

public firms. When it comes to smaller public firms, limited commitment models provide an adequate descriptions of corporate behavior. The relative fit of trade-off and limited commitment models across samples captures the relative importance of state-contingent financing instruments for corporate policies. In this sense, our results highlight how such financing options are reflected in large firms' behavior. Indeed, our results are thus consistent with the recent literature documenting that smaller firms rely more heavily on secured bank debt in the form of credit lines, exhibiting a state-contingent flavor, as in Nikolov, Schmid, and Steri (2016) . Larger firms more readily have access to corporate bonds, whose defaultable nature is more accurately captured by means of trade-off models.

Intriguingly, our estimation procedure strongly favors moral hazard models over all others in the case of private firms. In our dataset, private firms tend to be smaller than public firms, they tend to invest less, and be significantly more levered. Our results suggests that cash flow diversion is highly relevant for this class of firms. Realistically, cash flow diversion is likely to be interpreted not narrowly as outright stealing of profits, but more broadly perhaps as conflicts of interest about the proper use of funds in firms that are less transparent lacking the scrutiny of the public spotlight. One way to interpret our results then is that leverage arises as an effective device to discipline such conflicts.

Our structural estimation procedure also yields point estimates of the relevant parameters across models, and thus also allow us to gauge the magnitudes of the financial frictions and agency costs necessary to rationalize observed firm behavior, both in the full sample as well as across subsamples. Reassuringly, we find that the estimates of the technological parameters are remarkably similar and consistent across model specifications, so that differences in fit can be attributed almost exclusively to their financing behavior, allowing us to identify the relevant sources of financial frictions. The parameter estimates of particular concern, therefore, relate to the unobserved agency conflicts that drive financial structure and investment, such as the degree of cash flow diversion in moral hazard models, and the amount of capital firms can abscond with in limited commitment models. Regarding cash flow diversion, our results indicate that in order to rationalize observed corporate policies, firm owners need to be able to divert about 10 or 13 cents on the dollar of profits, for public and private firms, respectively. On the other hand, consistent with earlier results in Nikolov, Schmid, and Steri

(2016), firms can collateralize about 74 or 88 percent of their capital stock in limited commitment models, for public and private firms, respectively.

**Literature Review** Our paper belongs to a small, but growing literature which tries to estimate, or quantitatively evaluate, the empirical implications of the literature on dynamic agency conflicts across a variety of economic environments. In our attempt to distinguish and discriminate across different models of dynamic financial constraints, we are inspired by Karaivanov and Townsend (2014) and Karaivanov and Wright (2011), who use numerical techniques and estimation methods to distinguish among different sources of financing constraints that inhibit risk sharing among households in Thai villages, and international capital flows, respectively. Our work is extending this agenda in the context of the literature of financial contracting in dynamic corporate finance. While our work revolves around endogenously complete optimal contracts, Matvos (2013) estimates the benefits of completing debt contracts by means of covenants.

A number of recent papers have used structural estimation to evaluate models based on a single source of agency conflicts for firm dynamics and financing. For example, Li, Whited, and Wu (2016) investigate a dynamic limited commitment model to gauge the relative importance of tax advantages and agency conflicts for firm financing. Relatedly, Ai, Kiku, and Li (2016) estimate a dynamic moral hazard model in general equilibrium to assess the severity of the agency conflicts arising from effort provision for the real economy. We differ from these papers by our focus on model comparison, and estimation technique. Verani (2016) presents and estimates a model combining moral hazard and limited commitment and estimates it on macroeconomic data from Colombia, while we focus on firm-level panel data.

Regarding structural estimation and quantitative evaluation in dynamic corporate finance, our work builds on a growing and maturing literature that estimates and gauges the magnitude of financial frictions, as in Gomes (2001), Cooley and Quadrini (2001), Hennessy and Whited (2005, 2007), Moyen (2004, 2007), Hackbarth and Mauer (2012), Taylor (2010), Taylor (2013), Korteweg (2010), Glover (2015), Morellec, Nikolov, and Schurhoff (2012), and Morellec, Nikolov, and Schurhoff (2016).

In our implementation of a limited commitment model, we follow the work of Albuquerque and Hopenhayn (2004), and especially Rampini and Viswanathan (2010, 2013), Ai and Li (2016), Zhang

(2016), and Sun and Zhang (2016). Perhaps unsurprisingly, our approach is closest to our recent companion paper, Nikolov, Schmid, and Steri (2016), which emphasizes the implementation of a limited commitment model with state-contingent debt with a mixture of real world securities such as straight loans and credit lines.

Our implementation of a discrete time moral hazard model follows the work of Clementi and Hopenhayn (2006) and especially, Quadrini (2004), who quantitatively examines a dynamic moral hazard model when shocks can be persistent. Doepke and Townsend (2006) develop computational techniques to deal with the challenging case when persistent shocks are privately observed only. In contrast, we assume that persistent shocks are publicly observable. For the case with privately observed persistent shocks, Fu and Krishna (2016) develop an implementation of the corresponding cash flows by means of real world securities.

## 2 A Triplet of Models

In this section, we present the models that we take to the data and attempt to empirically evaluate and discriminate. The models themselves are fairly standard and have been widely used in the literature to address a variety of questions in corporate finance, growth and development, among others. To facilitate comparison, we present them in a unified setup that emphasizes similarities, and readily allows to identify differences.

The models are i) a standard trade-off model where tax advantages encourages financing with one-period debt which is limited by an endogenous collateral constraint, ii) a limited commitment model which retains the tax advantages of debt financing, but allows for a more flexible implementation of debt structure by means of state-contingent debt repayment schedules, which may be thought of straight debt and credit lines or derivatives, as well as iii) a model where external financing from lenders is limited by asymmetric information, in that lenders do not observe shock realizations and financial contracts thus need to be structured such that borrowing firms are induced to revealing the truth in the presence of moral hazard.

We keep the specification of technology identical across models, so that differences in observed policies stem exclusively from different financial frictions. We outline the technology first, and then

describe in detail the different sources of frictions. Perhaps slightly deviating from the standard formulations of the models familiar from the literature, we cast them as firm rather than equity value maximization problems, to facilitate comparison. We establish the equivalence of these formulations in the appendix.

## 2.1 Technology and Investment

We consider the problem of value-maximizing firms in a perfectly competitive environment. Time is discrete. We assume that all agents are risk-neutral, so that the one period interest rate  $r$  is constant.

After-tax operating profits for firm  $i$  in period  $t$  depend upon the capital stock  $k_{it}$  and shocks  $z_{it}$  and  $\eta_{it}$ , respectively, and are given by

$$\pi(k_{it}, z_{it}, \eta_{it}) = (1 - \tau)((z_{it} + \eta_{it})k_{it}^\alpha - f), \quad (1)$$

where  $0 < \tau < 1$  denotes the corporate tax rate,  $0 < \alpha < 1$  is the capital share in production, and  $f > 0$  is a fixed cost incurred in the production process. Note that a capital share less than unity captures decreasing returns to scale. The variable  $z_{it}$  reflects shocks to demand, input prices, or productivity and follows a stochastic process with bounded support  $Z = [\underline{z}, \bar{z}]$ , with  $-\infty < \underline{z} < \bar{z} < \infty$ , and described by a transition function  $Q_z(z_{it}, z_{it+1})$ <sup>3</sup>. Finally,  $\eta_{it}$  is an iid disturbance, which takes values  $\eta$  with probability  $p$  and  $-\eta$  with probability  $1 - p$ . The shock  $\eta_{it}$  only plays a major role in the context of dynamic moral hazard, in which it allows us to introduce asymmetric information in a tractable manner.

At the beginning of each period the firm is allowed to scale its operations by choosing its next period capital stock  $k_{it+1}$ . This is accomplished through investment  $i_{it}$ , which is defined by the standard capital accumulation rule

$$k_{it+1} = k_{it}(1 - \delta) + i_{it}, \quad (2)$$

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<sup>3</sup>In our empirical work, we parameterize  $z_{it}$  so as to provide a discrete approximation to a continuous AR(1) process with persistence  $\rho_z$  and conditional volatility  $\sigma_z$ .

where  $0 < \delta < 1$  is the depreciation rate of capital. Given our modeling of corporate taxation, we account for a depreciation tax allowance in the form of  $\tau\delta k_{it}$ .

Investment is subject to capital adjustment costs. As in Bolton, Chen, and Wang (2011), for example, we follow the neoclassical literature (Hayashi, 1982) and consider convex adjustment costs for simplicity. We parameterize capital adjustment costs with the functional form

$$\Psi(k_{it+1}, k_{it}) \equiv \frac{1}{2}\psi \left(\frac{i_{it}}{k_{it}}\right)^2 k_{it}, \quad (3)$$

where the parameter  $\psi$  governs the severity of the adjustment cost.

## 2.2 Trade-off

Our first model is a standard trade-off model in which firms aim at exploiting the tax advantage of debt financing available in the US tax code, similar to e.g. Hennessy and Whited (2007). In this setup,  $\eta_{it}$  is public information.

**Financing** At time  $t$ , firms have to the option to issue one-period bonds  $b_{it+1}$ , that are due at the beginning of the next period, with interest. Limited liability implies that there are states in which firms will be unable to fully repay their debt obligations at time  $t + 1$ . This is because internal funds after a sequence of bad shocks are so low that they are not sufficient to cover repayments. In such states, shareholders default on their commitments, creditors take over and recover a fraction of firms cash flows and assets net of bankruptcy costs. In anticipation of such states, creditors adjust the yields on debt so as to break even in expectation. In other words, they will charge a default premium  $\Delta_{it+1}$  above the risk-free rate to be compensated for potential losses in default, so that the effective interest rate on bonds amounts to  $r + \Delta_{it+1}$ . We determine  $\Delta_{it+1}$  endogenously below.

In line with the US tax code, we assume that interest payments are tax deductible, so that the effective repayment due in period  $t + 1$  amounts to  $(1 + (1 - \tau)(r + \Delta_{it+1}))b_{it+1}$  only. The amount  $\tau(r + \Delta_{it+1})b_{it+1}$  therefore represents a tax shield.



We assume that firms have to repay debt commitments at the beginning of the period, after realization of the shocks, and before issuing new debt and taking investment decisions. At that point, the firm is solvent if and only if

$$(1 - \tau)\pi(z_{it}, k_{it}, \eta_{it}) + (1 - \delta)k_{it} + \tau\delta k_{it} - (1 + (r + \Delta_{it})(1 - \tau)) b_{it} \geq 0 \quad (4)$$

. We assume that the lenders liquidate the firm if the pair of shocks  $(z_{it}, \eta_{it})$  and the policy  $(k_{it}, \Delta_{it})$  violate this solvency constraint. We can thus define the default set as  $D_{it} \equiv \{(z_{it}, \eta_{it}, k_{it}, \Delta_{it}) \in \bar{Z} \times \bar{N} \times \mathcal{R}^+ \times \mathcal{R}^+ : (4) \text{ does not hold}\}$ , where  $\bar{Z}$  and  $\bar{N}$  denote the support for the shocks  $z$  and  $\eta$  respectively. To save on notation, we denote the indicator function for default as  $\mathcal{I}_{D,it}$ . Creditors will anticipate default states, and determine the default premium accordingly. Given risk neutrality, creditors break even in expectation if

$$\int_{\bar{D}} (1 + r + \Delta_{it}) d(z_{it}, \eta_{it}, k_{it}, \Delta_{it}) + \int_D \overbrace{\frac{\xi(1 - \delta)k_{it}}{b_{it-1}}}^{\text{recovery rate}} d(z_{it}, \eta_{it}, k_{it}, \Delta_{it}) = 1 + r$$

where  $\xi$  denote deadweight costs incurred in bankruptcy.

With the default premium at hand, we can determine firms' payouts. Debt and internal resources can be used to fund investment expenditures, or distributions  $d_{it}$  to shareholders. Given limited liability, seasoned equity offerings are effectively precluded. While this may initially appear restrictive, in the data equity issuances are often employee-initiated issues<sup>4</sup>. Employee-initiated issues are not part of our model and characterize the exercise of stock options. With this caveat in mind, then, we have that

$$d_{it} \equiv (1 - \tau)\pi(z_{it}, k_{it}, \eta_{it}) - k_{it+1} + (1 - \delta)k_{it} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} - (1 + (r + \Delta_{it})(1 - \tau)) b_{it} + b_{it+1} \geq 0.$$

**Firm problem** Investment and financing policies are set to maximize firm value. Capital accumulation and financing needs reflect the persistent profitability shocks  $z_{it}$ , while debt policies additionally exploit the tax advantage, and constraints. More formally, firm value  $W(k_{it}, b_{it}, z_{it})$

<sup>4</sup>McKoen (2015) documents the empirical relevance of employee-initiated equity issuances.

satisfies the following Bellman equation:

$$W(k_{it}, b_{it}, z_{it}, \eta_{it}) \equiv \frac{1}{1+r} \max_{k_{it+1}, b_{it+1}} -k_{it+1} + (1-\delta)k_{it} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} \\ + \tau(r + \Delta_{it})b_{it}\mathcal{I}_{1-D,it} - \xi((1-\delta)k_{it} + \tau\delta k_{it})\mathcal{I}_{D,it} + E_t[(1-\tau)\pi(k_{it}, z_{it}, \eta_{it}) + W(k_{it+1}, b_{it+1}, z_{it+1}, \eta_{it+1})]$$

subject to

$$(1-\tau)\pi(z_{it}, k_{it}, \eta_{it}) - k_{it+1} + (1-\delta)k_{it} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} - (1 + (r + \Delta_{it})(1-\tau))b_{it} + b_{it+1} \geq 0,$$

$$\int_{\overline{D}} (1+r+\Delta_{it}) d(z_{it}, \eta_{it}, k_t, \Delta_{it}) + \int_D \frac{\xi(1-\delta)k_{it}}{b_{it-1}} d(z_{it}, \eta_{it}, k_{it}, \Delta_{it}) = 1+r.$$

### 2.3 Limited Commitment

We now relax the perhaps slightly stark assumption that firms only source of external financing is one-period debt. This assumption immediately precludes any instruments with a more state-contingent flavor such as credit lines, derivatives or even external equity. We relax this in the context of a limited commitment model, in which, formally, we allow the payoffs of the securities available to outside investors to be contingent on the realization of the profitability shock  $z_{it}$ . While we do not take a stand on the precise implementation of this instrument by means of real world securities for the sake of this paper, we refer to our companion paper Nikolov, Schmid, and Steri (2016), or the recent work by Rampini and Viswanathan (2013) or Li, Whited, and Wu (2016) for examples and estimation evidence. In this context,  $\eta_{it}$  is public information.

**Financing** Formally, in every period firms can sell a portfolio of securities whose payoffs  $b_{it+1}(z_{it+1}, \eta_{it+1})$  to investors are contingent on the realization of next periods profitability shocks  $z_{it+1}$  and  $\eta_{it+1}$ . Selling such a portfolio at time  $t$  thus raises an amount  $b_{it+1} \equiv E_t[b_{it+1}(z_{it+1}, \eta_{it+1})]$ . For the sake of our analysis here, we think of these state contingent payments as repayments to a lender, which need to be fully collateralized. That is, we require that

$$(1+r(1-\tau))b_{it+1}(z_{it+1}, \eta_{it+1}) \leq \theta(1-\delta)k_{it+1}, \quad \forall z_{it+1}, \eta_{it+1}.$$

Similar to the case of straight debt considered in the trade-off model above, these state-contingent debt instruments can be used to fund investment expenditures and distributions to shareholders,  $d_{it}$ , jointly with internal resources. We retain the assumption of limited liability on the shareholders' side, which requires that

$$d_{it} \equiv (1 - \tau)\pi(z_{it}, k_{it}, \eta_{it}) - k_{it+1} + (1 - \delta)k_{it} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} \\ - (1 + r(1 - \tau))b_{it}(z_{it}, \eta_{it}) + E_t[b_{it+1}(z_{it+1}, \eta_{it+1})] \geq 0$$

**Firm problem** The firm's problem is then to choose investment and state-contingent financing plans so as to maximize firm value, subject to constraints. Formally, firm value  $W(k_{it}, b_{it}, z_{it}, \eta_{it})$  satisfies the following Bellman equation:

$$W(k_{it}, b_{it}, z_{it}, \eta_{it}) \equiv \frac{1}{1+r} \max_{k_{it+1}, b_{it+1}(z_{it+1}, \eta_{it+1})} - k_{it+1} + (1 - \delta)k_{it} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} + \tau r b_{it}(z_{it}, \eta_{it}) \\ + E_t[(1 - \tau)\pi(k_{it}, z_{it}, \eta_{it}) + W(k_{it+1}, b_{it+1}(z_{it+1}, \eta_{it+1}), z_{it+1}, \eta_{it+1})]$$

subject to

$$(1 - \tau)\pi(z_{it}, k_{it}, \eta_{it}) - k_{it+1} + (1 - \delta)k_{it} - \Psi(k_{it+1}, k_{it}) + \tau\delta k_{it} \\ - (1 + r(1 - \tau))b_{it}(z_{it}, \eta_{it}) + E_t[b_{it+1}(z_{it+1}, \eta_{it+1})] \geq 0,$$

$$(1 + r(1 - \tau))b_{it+1}(z_{it+1}, \eta_{it+1}) \leq \theta(1 - \delta)k_{it+1}, \forall z_{it+1}, \eta_{it+1}.$$

## 2.4 Moral Hazard

We now embed a dynamic moral hazard problem into our setup by assuming that lenders cannot observe the realization of all shocks and therefore has to rely on shareholders' report. Naturally, all else equal, shareholders have an incentive to underreport realized shocks as it allows them to pocket a larger share of realized cash flows by repaying less debt. An incentive compatible contract therefore designs a repayment schedule such that shareholders are always better off truthfully reporting. This

can be achieved by implementing state-contingent repayments satisfying a number of constraints as detailed below.

Given that shocks are unobservable to lenders, some care must be taken with respect to the timing of decisions. Further, as previously, we realistically want to allow for serially correlated shocks  $z_{it}$ . To keep the analysis transparent, we assume, as before, that  $z_{it}$  follows a Markov Chain, which is publicly observable<sup>5</sup>, especially for the lender. However, conditional on a particular realization of  $z_t$ , realized cash flows are also impacted by the iid disturbance  $\eta_{it}$ . To give rise to a meaningful dynamic moral hazard problem, critically, we assume that  $\eta_{it}$  is observable by shareholders, but *unobservable by* lenders.

In this context, a lending contract amounts to a sharing rule that splits a firm's resources between payments to the lender,  $p_{it}$ , and payments to the shareholder, that is, dividends,  $d_{it}$  in a fully state-contingent manner. An optimal contract between shareholders and lenders maximizes the firm value  $W_{it}$ , subject to incentive constraints, promise keeping, as well as limited liability constraints. In this context, the incentive constraints amount to requiring that under the contract shareholders are always better off sticking to the contract and revealing the true realization of  $\eta_{it}$ , rather than underreporting realized cash flows and diverting cash flows. At this stage, we capture the cash flows that can be diverted by misreporting  $\hat{\eta}_{it}$  rather than the true  $\eta_{it}$  by the general 'diversion function'  $\mathcal{D}(k_{it}, z_{it}, \eta_{it}, \hat{\eta}_{it})$ . We will discuss economically motivated functional forms below.

In this setting with dynamic moral hazard, it is convenient to use the equity value of the firm,  $V_{it}$ , as a state variable. Clearly, then, the value of debt can be recovered, from  $b_{it} = W_{it} - V_{it}$ . Similarly, the contract picks state-contingent dividend payments  $d_{it}$  as controls, so that the payments to the lender  $p_{it}$  are recovered from the resource constraint, as detailed below.

Regarding timing, let us denote the firm value at the end of period  $t - 1$  (that is, after the realization of all the  $t - 1$  and *before* the  $t$  shocks) as  $W(k_{it-1}, V_{it-1}, z_{it-1})$ . The state variable  $z_{it-1}$  is informative about the conditional distribution of  $z_{it}$  in period  $t$ , which affects the expected returns to capital. For tractability, we assume that shareholders decide at the end of period  $t$  about their investment expenditures at the beginning of period  $t$ , which entails adjustment costs as before and which leaves them with a depreciated capital stock after production. Finally, we accommodate tax

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<sup>5</sup>See Doepke and Townsend, etc. for analyses of privately observable persistent shocks

deductability of interest on debt,  $\tau r b_{it} = \tau r(W_{it} - V_{it})$ , which translates into an adjusted discount rate for the firm ( $1/(1 + (1 - \tau)r)$  rather than  $1(1 + r)$ ) and a penalty for foregone tax deductions on debt for the amount of  $\tau r V_{it}$ .

More formally, the firm value function satisfies

$$W(k_{it-1}, V_{it-1}, z_{it-1}) = \max_{k_{it}, V_{z_{it}, \eta_{it}}, d_{z_{it}, \eta_{it}}} \frac{1}{1 + (1 - \tau)r} [-k_{it} - \Psi(k_{it}, k_{it-1}) + (1 - \delta)k_{it} + \tau \delta k_{it} - r\tau V_{it-1} + E_{t-1}[(1 - \tau)\pi(k_{it}, z_{it}, \eta_{it}) + W(k_{it}, V_{z_{it}, \eta_{it}}, z_{it})]]$$

subject to

$$V_{it-1} = \frac{1}{1 + r} E_{t-1}[d_{z_{it}, \eta_{it}} + V_{z_{it}, \eta_{it}}], \quad (5)$$

$$d_{z_{it}, \eta_{it}} + V_{z_{it}, \eta_{it}} \geq d_{z_{it}, \hat{\eta}_{it}} + V_{z_{it}, \hat{\eta}_{it}} + \mathcal{D}(k_{it}, z_{it}, \eta_{it}, \hat{\eta}_{it}), \quad \forall z_{it}, \forall \hat{\eta}_{it}, \quad (6)$$

$$d_{z_{it}, \eta_{it}} \geq 0, \quad \forall z_{it}, \forall \eta_{it}, \quad (7)$$

$$V_{z_{it}, \eta_{it}} \geq 0, \quad \forall z_{it}, \forall \eta_{it}. \quad (8)$$

Here,  $V_{it-1}$  is the equity value at the end of period  $t - 1$ , and the promise keeping constraint states that the (state-contingent) dividend payments to shareholders and equity value at the end of period  $t$  have to add up to  $V_{t-1}$  in expectation. The incentive constraints state that shareholders are always better reporting the true realization of the iid shock  $\eta_{it}$  and receiving a dividend  $d_{z_{it}, \eta_{it}}$  and continuation value  $V_{z_{it}, \eta_{it}}$ , rather than misreporting  $\hat{\eta}_{it}$  and pocketing the diverted cash flow, captured by the diversion function  $\mathcal{D}(k_{it}, z_{it}, \eta_{it}, \hat{\eta}_{it})$ , as well as the dividends and continuation values under the misreported cash flows,  $d_{z_{it}, \hat{\eta}_{it}} + V_{z_{it}, \hat{\eta}_{it}}$ . The diversion function, in its most straightforward specification, is just  $\lambda(\pi(k_{it}, z_{it}, \eta_{it}) - \pi(k_{it}, z_{it}, \hat{\eta}_{it}))$ , where  $1 - \lambda$  captures potential losses in cash flow diversion. Finally, in every state, dividend payments and equity values have to be non-negative, reflecting shareholders' limited liability.

Given our timing assumption, all the cash flows accrue intra-period, so that payments  $p_{it}$  to the lender are simply the mirror image of contractually designed dividend payments to shareholders. In other words, we must have that these respective payments exhaust available resources, so that,

state by state,

$$p_{it} = -k_{it} - \Psi(k_{it}, k_{it-1}) + (1 - \delta)k_{it} + \tau\delta k_{it} + \tau r(W_{it} - V_{it}) + \pi(k_{it}, z_{it}, \eta_{it}) - d_{it}.$$

This formulation emphasizes the trade-off between payments to shareholders and lenders, which must respect promise keeping and incentive constraints laid out above.

### 3 Model Computation and Estimation

In this section, we describe the two key steps that enable us to bring the models we laid out in Section 2 to the data. Section describes the numerical solution method we use for model computation. Section presents the structural estimation procedure and the statistical tests we use for model comparison.

#### 3.1 Solution Method: Linear Programming

The triplet of models we laid out in Section 2 has no closed-form solution. Thus, we solve the dynamic programs all of them numerically. In addition, the numerical solution of the limited commitment and moral hazard models is computationally challenging. The presence of state-contingent policies introduces a large number of control variables that makes the curse of dimensionality excessively severe for standard iterative computational methods.<sup>6</sup>

We overcome this difficulty by adopting the linear programming (LP) representation of dynamic programming problems with infinite horizon (Ross (1983)), building on Trick and Zin (1993), and Trick and Zin (1997). We exploit and extend linear programming methods to efficiently solve for the value and policy functions. Linear programming methods, while common in operations research, have been introduced into economics and finance in Trick and Zin (1993, 1997). We follow Nikolov, Schmid, and Steri (2017) to extend the LP approach to setups common in dynamic

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<sup>6</sup>Because of the presence of several occasionally non-binding collateral constraints, the models cannot be solved numerically by interior point methods. In principle, all models can be solved on a discrete grid by standard iterative methods as value and policy function iteration. However, as discussed above, the application of these methods to the contracting models in Section 2 is computationally problematic.

corporate finance. More specifically, we exploit a separation oracle, an auxiliary mixed integer programming problem, to deal with large state spaces and find efficient implementations of Trick and Zin’s constraint generation algorithm.

To start with, any finite dynamic programming problem with infinite horizon can be equivalently formulated as a linear programming problem (LP). The LP representation associates every feasible decision at each grid point on the state space with a constraint. Specifically, the three models can be formulated as LP problems as follows:

$$\min_{W_{k,u,z}} \sum_{k=1}^{nk} \sum_{u=1}^{nu} \sum_{z=1}^{nz} W_{k,u,z} \quad (9)$$

*s.t.*

$$W_{k,u,z} \geq R_{k,u,z,a} + \sum_{z'=1}^{nz} \beta Q(z'|z) W_{k'(a),u'(a),z'} \quad \forall k, u, z, a, \quad (10)$$

where  $u$  denotes the promised utility variable, namely  $b_{it}$  for the tradeoff and the limited commitment model and  $v_{it}$  for the moral hazard model;  $nk$ ,  $nu$ , and  $nz$  are the number of grid points on the grids for  $k_{i,t}$ ,  $u_{i,t}$ , and  $z_{i,t}$  respectively;  $W_{k,u,z}$  is the value function on the grid point indexed by  $k$ ,  $u$  and  $z$ ,  $a$  is an index for a *feasible* action on the grid for both capital, promised utility, and payouts, and  $R_{k,u,z,a}$  denotes the return function corresponding to the action  $a$  starting from the state indexed by  $k$ ,  $u$  and  $z$ ;  $\beta$  is the appropriate discount rate;  $Q(z'|z)$  is the transition matrix of the Markov chain driving profitability shocks;  $k'(a)$  and  $u'(a)$  denote the future values for the state variables given the current firm’s decisions. For a formal proof, we refer to Ross (1983).

The solution of the LP above would require to store an extremely large matrix, because state-contingent decisions render the number of constraints in the problem enormous. Precisely, the set of feasible actions  $a$  is a highly dimensional object for both the limited commitment model (due to state-contingent debt repayments) and the moral hazard model (due to state-contingent dividends and promised equity values). Computational requirements would therefore be excessive. Thus, we implement constraint generation, a standard operation research technique to attack problems with a large number of constraints. First, we solve a relaxed problem with the same objective function. Second, we use the current solution to identify the constraints it violates. Third, we add one of

the violated constraints, namely the most violated one, to the relaxed problem. We iterate the procedure until all constraints are satisfied.

To practically implement the constraint generation procedure, we need to deal with another computational issue. The selection of the most violated constraint involves searching over an extremely large vector of grid points for all the state-contingent control variables. The computational burden would still be excessive for the two contracting models we solve. To do so, we implement a separation oracle, an auxiliary mixed-integer programming problem to identifies the most violated constraint<sup>7</sup>. Appendix A outlines the constraint-generation algorithm and the separation oracle for the three models.

We implement the codes with Matlab<sup>®</sup> and CPLEX<sup>®</sup> as a solver for the linear and mixed-integer programming problems. Our workstation has a CPU with 24 cores and 124GB of RAM. The models are solved with five grid points for the idiosyncratic shock, 21 grid points for capital, 17 grid points for current promised utility. All control variables are choses on a continuous grid up to CPLEX numerical precision, which is 1e-6.

### 3.2 Estimation Method: Empirical Policy Functions

The dynamic corporate finance literature relies typically on two estimation methods: the Simulated Method of Moments or SMM, (Hennessy and Whited, 2005, 2007, Taylor, 2010) and the Simulated Maximum Likelihood or SML (Morellec, Nikolov, and Schuerhoff, 2012, 2016). We depart from the literature and rely on an alternative method that is most readily identified as Indirect Inference or II in the terminology of Gourieroux, Monfort, and Renault, 1993.

Unlike, SMM or SML, II relies on an auxiliary model for estimating the structural parameters. The auxiliary model is an approximation of the true data generating process. Typically, this approach is suitable in cases where the likelihood is not available in closed form and is computationally infeasible. In our case, we choose II as it constitutes a natural framework for comparing models. In particular, for the auxiliary model, we choose the model policy functions. The competing models

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<sup>7</sup>Separation oracles are standard tools in operation research, as described in Schrijver (1998) and Vielma and Nemhauser (2011).



that we consider do not share the same parameters, but their policy functions are common. We can thus compare in this framework both nested and non-nested models. Below we further motivate our choice of estimation method and describe its implementation.

### 3.2.1 Empirical Policy Functions

A policy function<sup>8</sup> is an association between an optimal choice of the firm, for example investment or financing, and its currently observable state. In accordance with this definition, we write the policy function as

$$\mathbf{w} = P(\mathbf{x}) \tag{11}$$

where  $\mathbf{x}$  is the vector of state variables and  $\mathbf{w}$  is the vector of policy variables of the model. For example, in our models  $k_{it}$ ,  $b_{it}$ , and  $z_{it}$  are the state variables and  $k_{it+1}$ ,  $b_{it+1}$ , and  $d_{it+1}$  are the control variables, so we have  $\mathbf{x}_{it} = \{k_{it}, b_{it}, z_{it}\}$  and  $\mathbf{w}_{it} = \{k_{it+1}, b_{it+1}, d_{it+1}\}$ .

One challenge when working with policy functions is that some state or control variables are unobservable. We tackle this challenge by working with observable transformations of these variables. For example, the state variable  $z$  is unobservable. In this case, we use  $zk^\alpha/k$ , firm profitability, that is observable.

We now characterize the empirical counterpart of the policy function  $\mathbf{w} = P(\mathbf{x})$ . One way to do so is linear approximation. This approach however will fail to capture the non linearities embedded in our models. We select then a semiparametric approach.

We consider the following specification for each control variable

$$\mathbf{w}_{it}^n = P^n(\mathbf{x}_{it}) + \mathbf{u}_{it}^n \tag{12}$$

where  $n$  is the  $n^{th}$  element of the policy vector  $\mathbf{w}_{it}^n$ ,  $u_{it}^n$  is the specification error with  $E[\mathbf{u}_{it}^n | \mathbf{x}_{it}] = 0$ . We estimate the function  $P(\mathbf{x}_{it})$

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<sup>8</sup>The exposition follows Bazdresch, Kahn, and Whited (2016).

We use a series approximation functions  $p_j(\mathbf{x}_{it})$  where  $j = 1, \dots, J$  to estimate the policy function  $P(\mathbf{x}_{it})$ . In particular, as  $J \rightarrow \infty$ , the expected mean square difference between the  $P(\mathbf{x}_{it})$  and a linear combination of  $p_j(\mathbf{x}_{it})$  approaches zero, that is

$$\lim_{J \rightarrow \infty} E \left( \sum_{j=1}^J h_j p_j(\mathbf{x}_{it}) - P(\mathbf{x}_{it}) \right)^2. \quad (13)$$

We find that a power series with linear, quadratic, and all cross-products performs well. We have experimented with several alternative series functions. We observe that our results are immune to the particular choice of the series functions.

### 3.2.2 Structural estimation: Indirect inference

We use Indirect Inference to structurally estimate our set of models. The Indirect Inference method relies on an auxiliary model. While the auxiliary model is an approximation of the true data generating process, it captures the most important features of the data. Empirical policy functions constitute a natural candidate for an auxiliary model as they characterize the solution of the model. Below we detail the estimation procedure.

We define the vector of observed data  $\mathbf{v}_{it} \equiv (\mathbf{w}_{it}, \mathbf{x}_{it})$ , where  $i = 1, \dots, N$  indexes firms and  $t = 1, \dots, T$  indexes time. Similarly, we define the vector of simulated data  $\mathbf{v}_{it}^s$ , where  $s = 1, \dots, S$  is the number of times we simulate the model. The simulated data vector,  $\mathbf{v}_{it}^s(\beta)$ , depends on the vector of structural parameters  $\beta$ . We define the estimating equation as

$$g(\mathbf{v}_{it}, \beta) = \frac{1}{nT} \sum_{i=1}^n \sum_{t=1}^T \left[ \mathbf{h}(\mathbf{v}_{it}) - \frac{1}{S} \sum_{s=1}^S \mathbf{h}(\mathbf{v}_{it}^s(\beta)) \right] \quad (14)$$

where  $\mathbf{h}(\cdot)$  is the parameter vector from (13) defining the empirical policy functions. The dimension of  $\mathbf{h}$  is larger than the dimension of the vector of structural parameters  $\beta$ . The Indirect Inference estimator for  $\beta$  is given by

$$\hat{\beta} = \arg \min_{\beta} g(\mathbf{v}_{it}, \beta)' \widehat{W}_{nT} g(\mathbf{v}_{it}, \beta)$$

where  $\widehat{W}_{nT}$  is a positive definite weighting matrix that converges in probability to a deterministic positive definite matrix  $W$ .

### 3.2.3 Testing and Model Selection

We now build a set of tests that help us evaluate the relative performance of the competing models.<sup>9</sup> We consider two separate estimations for which we use the same set of data moments,  $\mathbf{h}(\mathbf{v}_{it})$ , but that use different economic models, so that the simulated data differ across estimations. Let the parameter vector from the first estimation be  $\beta_1$ , and let the parameter vector from the second estimation be  $\beta_2$ . We want to test the null hypothesis that  $E(g(\mathbf{v}_{it}, \beta_1) - g(\mathbf{v}_{it}, \beta_2)) = 0$ . Equation (14) implies that this test is equivalent to the test of the null that

$$H(\beta_1, \beta_2) \equiv \mathbf{h}(\mathbf{v}_{it}^s(\beta_2)) - \mathbf{h}(\mathbf{v}_{it}^s(\beta_1)) = 0. \quad (15)$$

Because the expression  $H(\beta_1, \beta_2)$  in (15) is a function of  $(\beta_1, \beta_2)$ , we can use a standard Wald test. The hurdle is calculating the asymptotic variance of  $H(\beta_1, \beta_2)$  because it contains two parameter vectors that are estimated separately.

We use the influence function technique from Erickson and Whited (2002) To calculate the asymptotic variance of  $H(\beta_1, \beta_2)$ , we note that it equals, by definition, the asymptotic variance of the influence function of  $H(\beta_1, \beta_2)$ . To calculate this influence function we use the delta method as follows.

Let  $G(\mathbf{v}_i, \beta)$  be the Jacobian of  $g(\mathbf{v}_i, \beta)$ . The influence function for observation  $i$  for  $\beta$  is given by

$$\psi_\beta = (G(\mathbf{v}_i, \beta)'WG(\mathbf{v}_i, \beta))^{-1}G(\mathbf{v}_i, \beta)'Wg(\mathbf{v}_i, \beta).$$

Therefore, the delta method gives the influence function for (15) as

$$\psi_{H(\mathbf{v}_i, \beta_1, \beta_2)} = G(\mathbf{v}_i, \beta_1)(G(\mathbf{v}_i, \beta_1)'WG(\mathbf{v}_i, \beta_1))^{-1}G(\mathbf{v}_i, \beta_1)'Wg(\mathbf{v}_i, \beta_1) \quad (16)$$

$$- G(\mathbf{v}_i, \beta_2)(G(\mathbf{v}_i, \beta_2)'WG(\mathbf{v}_i, \beta_2))^{-1}G(\mathbf{v}_i, \beta_2)'Wg(\mathbf{v}_i, \beta_2). \quad (17)$$

To calculate the variance of  $H(\beta_1, \beta_2)$ , we simply need to calculate the covariance of  $\psi_{H(\mathbf{v}_i, \beta_1, \beta_2)}$ . To simplify this calculation, we note that under the null, the influence functions for  $g(\mathbf{v}_i, \beta_1)$  and

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<sup>9</sup>This Section is from Nikolov and Whited (2014) with minor notational modifications.

$g(\mathbf{v}_i, \beta_2)$  both equal  $\psi_{h(\mathbf{v}_i)}$ , which is the influence function for the data moment vector  $h(\mathbf{v}_i)$ . Making this substitution we have

$$\psi_{H(\mathbf{v}_i, \beta_1, \beta_2)} = (G(\mathbf{v}_i, \beta_1)(G(\mathbf{v}_i, \beta_1)'WG(\mathbf{v}_i, \beta_1))^{-1}G(\mathbf{v}_i, \beta_1)'W \quad (18)$$

$$- G(\mathbf{v}_i, \beta_2)(G(\mathbf{v}_i, \beta_2)'WG(\mathbf{v}_i, \beta_2))^{-1}G(\mathbf{v}_i, \beta_2)'W) \psi_{h(\mathbf{v}_i)}. \quad (19)$$

The variance of  $H(\beta_1, \beta_2)$  can then be obtained by covarying the influence function  $\psi_{H(\mathbf{v}_i, \beta_1, \beta_2)}$  with itself,

$$\text{avar}(H(\mathbf{v}_i, \beta_1, \beta_2)) = \frac{1}{n} \left(1 + \frac{1}{K}\right) E [\psi_{H(\mathbf{v}_i, \beta_1, \beta_2)} \psi_{H(\mathbf{v}_i, \beta_1, \beta_2)}'],$$

where the term  $(1 + \frac{1}{K})$  accounts for simulation error. Finally, the Wald test for the null that any particular element of  $H(\mathbf{v}_i, \beta_1, \beta_2)$  is zero can be constructed as the ratio of that element to the corresponding element of  $\text{avar}(H(\mathbf{v}_i, \beta_1, \beta_2))$ . This statistic is distributed as a  $\chi^2$  with one degree of freedom.

A Wald statistic for the test that  $\beta_1 - \beta_2 = 0$  can be derived analogously. As above, the delta method gives us

$$\text{avar}(\beta_1 - \beta_2) = \frac{1}{n} \left(1 + \frac{1}{K}\right) E [(\psi_{b_1} - \psi_{b_2})(\psi_{b_1} - \psi_{b_2})'].$$

The construction of the Wald tests proceeds exactly as above.

## 4 Empirical Results

We start by describing our datasets, and then discuss identification, estimation results, as well as model comparisons.

### 4.1 Data and Sample Splits

Our main sample is drawn from the Compustat database for the 1965 to 2015 period. We exclude firms that do not have a stock exchange code (EXCHG) equal to 10 or 11. We remove firms that operate in the financial sector (four-digit SIC code between 4900 and 4999) or in regulated

sectors (four-digit SIC code between 6000 and 6999 or between 9000 and 9999). We also drop firms with less than two consecutive year of data to be able to compute growth rates when required. Our resulting sample includes 59,143 firm/year observations.

We then compute the following variables. Investment is  $CAPX/AT$ ; net book leverage is  $(DLTT+DLC-CHE)/AT$ ; profitability is  $OIBDP/AT$ ; dividends are  $(DVT+PSSTKC-PSTKRV)/AT$ ; log size is the natural logarithm of  $PPENT$ ; market-to-book is  $(DLTT+DLC+PRCCF \times CHSO)/AT$ . Following, for example, Hennessy and Whited (2005) and Hennessy and Whited (2007), for the estimation procedure we remove firm fixed effect of each state and control variable. To reconcile the average levels of the control variables in the data and in the model, we add the sample mean into each variable after removing fixed effects.

To quantify the importance of the different types of frictions behind each model, we estimate the three models on different subsamples of firms. These subsamples are formed by splitting the sample in two on each on the (rescaled) state variables, namely (log) size, profitability, and leverage, and market-to-book<sup>10</sup>. Firms in the top and bottom 20th percentiles are respectively assigned to the subsamples with high and low values for the sorting variable.

In addition, we also consider a subsample of private firms. We ascertain data on US private firms from the Orbis database in the period from 2003 to 2012, as described in Kalemli-Ozcan, Sorensen, Villegas-Sanchez, Volosovych, and Yesiltas (2015). We remove observations with missing, zero, or negative values for total assets. Due to limited data availability, data on dividends are not available. In addition, market-to-book ratios cannot be computed for private firms in that there are no market share prices for them. We proxy investment as the growth rate of total assets, leverage as the ratio of the difference between total assets and shareholder funds and total assets, log size as the natural logarithm of total assets, and profitability as the ratio between profit and losses before taxes and total assets. We drop firm-year observations with missing values for investment, leverage, and profitability. We are left with an unbalanced panel with 277,074 firm/year observations.<sup>11</sup>

Table 1 reports the summary statistics of the aforementioned variables in the model. We winsorize all variables at the one percent level. In line with several existing studies, as a fraction of

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<sup>10</sup>We consider the sample split on book-to-market equity because the market value of equity appears as a state variable in the moral hazard model.

<sup>11</sup>Not surprisingly, private firms are numerous in comparison to public firms in a given time period.

total assets, the average profitability of our Compustat sample is around 15 %, investment around 7.5 %, net book leverage around 16 %, payouts around 2.2 %. The average log size is slightly above 5, while the market-to-book equity is around 1.3. Compared to this benchmark, some patterns are worth noting. In particular, the table suggests that large firms are more mature than small ones. This is reflected in a lower profitable, a higher leverage in line with the observation of Gomes and Schmid (2010), they pay more dividends and have a lower market-to-book ratio. More profitable firms, possibly because of persistence in investment opportunities, are typically smaller, invest more, are less levered, pay more dividends, and have higher market-to-book ratios. Highly levered firms invest more, are larger, less profitable and have lower market-to-book ratios than low leverage firms. Finally, in comparison to public firms, private firms are more profitable, significantly smaller, invest more and are far more levered, with a remarkable debt-to-asset ratio around 60 %. While this figure appears extremely high, it is in line with Huynh, Paligorova, and Petrunia (2012), who discuss that it is mainly driven by higher short-term leverage than public firms. In addition, private firms display a high heterogeneity across firms, as a standard deviation around thirty percent suggests.

[Insert Table 1 Here]

## 4.2 Identification and Model Comparison

For each model, global identification of the parameter vector requires a one-to-one mapping between the vector of model deep parameters  $\beta$  and a subset of the parameters of the auxiliary model with the same dimension. Local identification requires the gradient  $\frac{\partial h(v_{it}^s(\beta))}{\partial \beta}$  of the auxiliary model with respect to the deep parameters to have full rank. This condition has an intuitive interpretation. Identification requires that every estimated deep parameter in  $\beta$  has a differential impact on the firms' investment, financing, and payout policies as characterized by the set of auxiliary parameters  $h(v_{it}^s(\beta))$ .

[Insert Figure 1 Here]

Because the choice of  $\beta$ , and in particular of the financing parameters  $\xi$ ,  $\theta$ , and  $\lambda$  affects the optimal policy functions, the aptness of the three models to rationalize observed corporate policies depends on how parameters can be chosen to minimize the distance between the policies implied by the model and their real-data counterparts, which are depicted in Figure 1 for the full sample of Compustat firms and are described by  $h(v_{it})$ . This is the goal of the estimation procedure formally described in Section 3. For each model, the estimation procedure selects parameters to minimize the distance function in Equation 14. In this context, the tests of Section 3.2.3 provide a statistical procedure to test differences in the ability of quantitatively describe observed corporate policies across the three models.

[Insert Figure 2 Here]

Importantly, different financial frictions have a different importance for different types of firms, that in turn make different corporate decisions. Intuitively, the financial constraints that restrict the most the access to external financing for large mature firms are likely to be different than those that restrict small young firms. As a consequence, the observed investment, financing, and payout policies of different subsets of firms can significantly differ, and the model that better describes such policies is likely not to be the same for all firms in the economy. For example, Figure 2 depicts real-data policy for our sample of private firms from the Orbis dataset. Specifically, in comparison to public firms, for private firm the leverage-size relationship is clearly positive and steep (Panel B of Figure 1 versus Panel B of Figure 2), investment is positively related to leverage (Panel D of Figure 1 versus Panel C of Figure 2), leverage exhibits less hysteresis (Panel E of Figure 1 versus Panel D of Figure 2), and leverage is more sensitive to profitability (Panel H of Figure 1 versus Panel F of Figure 2), which resembles the prediction of the moral hazard model. For this reason, in the following analyses we consider sample splits according to the candidate determinants of policies in the models, namely the transformed state-variables (small versus large firms, high versus low leverage firms, profitable versus unprofitable firms), and we contrast public to private firms.

[Insert Table 2 Here]

Finally, although the technological parameters are common across all models, their impact on corporate policies can also differ because of the different financial frictions firms face. Table 2 presents a heatmap in which, for all three models, the sensitivity of some moments, namely average profitability, investment, leverage and payouts to the technological parameters is represented using a “traffic light” color scale. The numbers in the figure report the elements of the Jacobian matrix that capture the sensitivity of these moments to the parameters. While the reported moments only provide a crude description of the policy functions considered in the estimation, Table 2 shows that while some patterns are qualitatively similar across models, others can differ quantitatively and even qualitatively because of how shocks are propagated and amplified endogenously in the presence of different financial constraints. For example, the relationship between profitability and the technological parameters, which is mainly driven by the exogenous productivity shocks, is largely similar across models and in line with the neoclassical models analyzed in Strebulaev, Whited, et al. (2012). Instead, other sensitivities can differ even qualitatively, consistent with the interaction between investment and financing that translates into different policy functions.

### 4.3 Model Comparison through Empirical Policy Functions

Figures 3, 4, and 5 flesh out the economic intuition behind the formal statistical comparison of models through the indirect inference procedure described in the previous section.

[Insert Figure 3 Here]

To illustrate the effect of different dynamic financial constraints on corporate policies for large public firms, Figure 3 shows how the predicted empirical policy functions for the tradeoff (solid lines), limited enforcement (dashed lines), and moral hazard (dotted lines) models compare to their data counterparts under the estimated parameters.

Panels A to C refer, respectively, to firm’s investment, financing and payout policies policy as a function of the first transformed state variable, namely (standardized, log) size. In these panels, the other transformed state variables, namely leverage and profitability, are set to the average value in the simulated sample each model generates in correspondence of its deep parameters. Similarly,



panels D to F depict firms' policies as a function of (standardized) leverage, and panels G to I report (standardized) profitability on the horizontal axis.

Two remarks about the figure are in order. First, not all policy features help discriminating among models. For example, Panel E shows that all models predict hysteresis in leverage because of the estimated persistence in profitable investment opportunities and in the model state variables, as observed in the data. In addition, average levels for the optimal investment, leverage, and payout policies generally do not suffice to discriminate among models. All panels show that in correspondence of the value of zero on the horizontal axis, the intercept for all models is roughly zero. Because of the standardized scales, this means that all models are fairly successful in quantitatively match average observed investment, leverage, and payouts.

Second, although some policy features are qualitatively different among models, some of them differ only from a quantitative standpoints. Panel D shows that, under the estimated parameters, the tradeoff model predict a roughly flat investment-size relationship, the limited enforcement an upward-sloping one, and the moral hazard a downward-sloping one. On the contrary, Panel A shows that while all models agree on large and mature firms investing less, the limited commitment model estimates a steeper relationship between investment and size.

Panels A, D and G compare the investment policy among models. Panel A shows that, because of decreasing returns to scale, the investment-size relationship is negative for all models. This relationship is however steeper, for the limited enforcement, because firms' borrowing capacity to finance investment is directly linked to firms' assets in the collateral constraints. Accordingly, Panel D shows that leverage positively affects investment for the limited enforcement model, while the relationship is roughly flat for both the tradeoff model and the data. This data feature gives traction to the tradeoff model, consistent with the intuition that large public firms are less constrained in financing investments, while tax shields represent an important motive to raise debt. Finally, as in Clementi and Hopenhayn (2006), in the moral hazard model more equity (i.e. less leverage) gives more "skin in the game" to the entrepreneurs and credibly reduces their incentive to divert resources. Thus, equity helps firms relax financing constraints, seize profitable investment opportunities, and increase its value. Panel G instead show that, qualitatively, the estimated positive persistence in profitability shocks leads to more investment when profitability is high for all models. Intuitively,

the moral hazard model overestimates the slope on this relationship for large public firms, because profitability directly affects firms' financing constraints through the diversion function.

Panels B, E, and H pertain to firms' financing policies. The curves in Panel B show that all models estimate that large firms have more leverage. The tradeoff model predicts a higher slope in that large firms issue debt to earn tax shields rule, and their higher recovery rates give them better access to debt markets. As anticipated before, Panel E shows that the pronounced persistence in leverage ratios is not a very helpful data feature to distinguish among models. Panel H shows that the moral hazard model predicts a counterfactual steep and positive leverage-profitability relationship. This relationship is instead flatter for both the tradeoff and the limited enforcement model. Possibly contrasting economic forces influence the leverage-profitability relationship in the models. When current profitability is high, equity is relatively less expensive compared to the marginal value of investment. On the contrary, high profitability reduces the need for external financing because of persistence in internally generated profits. In addition, in the moral hazard model, high current profits increase the diversion value, but also increase the continuation value that can be credibly promised to entrepreneur. The positive steep relationship predicted by the moral hazard model suggests that profitability helps promising less equity to aid external financing to profitable investments and create value.

Finally, Panels C, F, and I refer to the payout policy. In all models, large firms pay out more dividends in the presence of a tradeoff between allocating resources to investment versus payouts (Panel C). Panel F depicts a positive payout-leverage relationship for both the tradeoff and limited enforcement models, which predict that large, mature, and levered, firms pay out more dividends. Instead, as for the investment policy in Panel D, the moral hazard model predicts that less investment-constrained firms here those with high equity, leading to a negative relationship. Panel I depicts a positive dividend-profitability relationship for the tradeoff model, and a U-shaped one for both the limited enforcement and the moral hazard model. In all models, this relationship is influenced by contrasting effect. Firms tend to pay out more in good times because they have more resources, but investments needs are also higher in those times. As in the data, the relationship is monotone and steep for the tradeoff model. In the model, in good times firms are less financially constrained because of their low probability of default and can allocate more resources to dividend payments.

All in all, as the statistical evidence in the next section establishes, the tradeoff model appears to provide the best description of corporate policies for large public firms. In the model, tax shields motives play a first-order effect to determine firms' financing and, as a consequence, investment and payout policies. On the contrary, firms are on average less financially constrained. The structure of the model implies relatively flat investment-size and investment-leverage relationships, as well as positive payout-size and payout-profitability relationship. These relationship resemble those observed in the data, and drive our results.

[Insert Figure 4 Here]

Figure 4 refers to small public firms. While, in the data, many features of observed policies are similar to those for large public firms, there are noticeable differences. In particular, investment is more sensitive to size (Panel A), while investment, leverage, and payouts are less sensitive to profitability (Panels G, H, I). Small firms are more constrained and have, on average, larger growth opportunities. The limited enforcement model appears to do a better job to generate policies that resemble the observed ones, while the tradeoff model loses explanatory power in comparison to the case of Figure 3.

[Insert Figure 5 Here]

Finally, Figure 5 refers to the case of private firms. Panel F shows that the moral hazard model does a good job rationalizing how profitability affects leverage in the data. In addition, for private firms with relatively high leverage and profitability, the moral hazard model delivers investment policies similar to the observed ones.

## 4.4 Estimation Results

Table 3 offers a first glimpse of our estimation results, both regarding parameter estimates and sample moments implied by our model specifications. To begin with, and to establish a benchmark, we focus on the entire compustat sample.

[Insert Table 3 Here]

Panel A shows that parameters related to firms' technology are estimated remarkably consistently across models. This is reassuring as it suggests that the source of variation and identification across models stems from the particular financial frictions embedded in each model. In particular, the estimates of the degree of scale,  $\alpha$ , and the persistence of shocks,  $\rho_z$ , are within the range of values obtained previously in the literature, thus providing additional validation for our approach.

Perhaps more importantly, the models also deliver parameter estimates that allows us to gauge the magnitudes of the respective financial frictions that are required to replicate the relevant statistics in the data. Regarding collateralizability, our point estimates suggest that firms can collateralize around fifty percent of their tangible capital, in line with the companion estimates in Nikolov, Schmid, and Steri (2017). Regarding moral hazard, a good empirical fit requires that insiders can divert about fifteen cents on the dollar.

Panel B reports the moments implied by our estimation procedure. Not that in our indirect inference approach, these moments are not directly targeted, in contrast to the SMM estimators more commonly used in the literature. Nevertheless, our empirical policy function benchmark estimation provides reasonable fits in terms of sample moments. Interestingly, the moral hazard model tends to overpredict both average investment as well as leverage on the compustat sample, while both tradeoff and limited commitment models are remarkably close to their empirical counterparts in that respect. That turns out to be a virtue in the sense, that it suggests a way to account for the comparably higher investment rates and leverage ratios commonly found in private firms, as documented earlier. Intuitively, this suggests that dynamic moral hazard may provide a better representation of the financial frictions shaping private firms' policies. We establish this point more formally and statistically in the next section.

#### 4.4.1 Subsample Estimation

We next present parameter estimates obtained from policy function estimation of the three models across various subsamples and sample splits. This not only provides further validation of our approach, but also provides the basis and gives intuition regarding the mechanisms driving our model fit results.

[Insert Table 4 Here]

We start, in Table 4, by considering estimation results obtained by splitting the sample along the size dimension. In particular, we re-estimate all our models both on the top, and the bottom tercile of Compustat firms sorted by size. We start by noting that the estimates of the main technological parameters  $\alpha$ ,  $f$ , and  $\delta$  are remarkably consistent across models, suggesting that differences in corporate behavior are mainly driven by differences in financial frictions across models. Interestingly, the differences in size across samples is primarily absorbed in the fixed costs, with estimates of  $f$  significantly higher for smaller firms. Similarly, smaller firms exhibit more volatile cash flows as measured by  $\sigma_z$ , as well as  $\eta$ .

Regarding financial frictions, we find that, consistent with intuition, our estimates indicate that within the context of a trade-off model, larger firms have higher recovery rates than small ones, as measured by  $\xi$ . This is in line with the observation that larger firms have more leverage, facilitated by their access to the corporate bond market. That result is mirrored in the estimates of collateralizability ( $\theta$ ) within the context of the limited commitment model we specify, resulting in a lower  $\theta$  for smaller firms. Finally, our estimates suggest that cash flow diversion, as measured by  $\lambda$  within the context of our moral hazard specification is quantitatively less of a concern in smaller Compustat firms. While this may initially be surprising, it reflects the observation that such firms tend to be less profitable, so that a friction based on cash flow diversion naturally has less bite in such a context.

Finally, we can assess the empirical adequacy of our model specifications across samples by the goodness of fit, measured as the minimized criterion for the empirical policy function estimation. As shown in the table, our estimations favor trade-off models for larger firms, and limited commitment models for smaller firms. This suggests that tax savings considerations are critical for understanding the behavior of larger, more profitable firms, while smaller firms' investment is often restricted by collateral requirements that come with bank debt in the form of credit lines dominating their debt structure.

[Insert Table 5 Here]

Table 5 reports results from estimating our models both on the top, and the bottom tercile of Compustat firms sorted by leverage. The trade-off model fits the high leverage firms quite well, as measured by the minimized objective function. On the other hand, the moral hazard model has trouble fitting corporate behavior in that sample. In the latter case, the estimation points to a very low level of potential cash flow diversion, the estimates for the recovery rates in the trade-off model are more natural across samples. Indeed, a high recovery rate for larger firms makes debt relatively attractive, so that such firms access debt markets more readily. While the goodness of fit is only modest across these samples, the point estimates for the limited commitment model are intuitive, in that firms in the high leverage sample exhibit a higher degree of collateralizability, as measured by  $\theta$ .

[Insert Table 6 Here]

We turn to profitability sorts in Table 6. Notably, from the point of view of our models, across the board, low profitability is driven by very high fixed operating costs. Indeed, these are quite consistently estimated to be an order of magnitude higher than those for high profitability firms, and similarly in comparison to all other subsamples. Given uniformly higher operating costs, the estimated financial parameters exhibit less variation across the samples, albeit in a natural manner. Highly profitable firms tend to have slightly higher recovery rates in the trade-off specification, reflecting higher incentives for tax shielding effects of leverage, which is also mirrored in the higher collateral parameter in the context of the limited commitment model. Higher profitability also comes with extended scope for cash flow diversion, which is reflected in a slightly higher estimated  $\lambda$  in the context of the moral hazard specification.

[Insert Table 6 Here]

Finally, in Table 7, we turn to a direct comparison of estimation results based on public and private firms, respectively. It is apparent from the table that across all models, public firms and private firms are succinctly different. Indeed, there is typically a lot less variation in parameter estimates across models than across firm types. For example, fixed costs come out an order of magnitude higher across models for private firms, reflecting their lower average profitability. The overall

higher investment rates of public firms comes with lower curvature and higher depreciation rates. The higher debt of private firms is reflected in a higher estimated recovery rate and pledgeability. Interestingly, the estimated cash flow diversion parameter for private firms is an order of magnitude higher than the estimate for public firms, suggesting a relevant role for debt as a disciplining device in private firms. Regarding overall fit, public firms appear to be best described overall by trade-off models, in which case moral hazard problems seem to play a limited role only. Perhaps this is a reflection of the more highly developed corporate governance practices of firms in the public spotlight. For private firms, in turn, moral hazard models provide the most accurate overall description. This points to the relevance of cash flow diversion in such firms. Realistically, cash flow diversion is likely to be interpreted not narrowly as outright stealing of profits, but more broadly perhaps as conflicts of interest about the proper use of funds in firms that are less transparent. A potential narrative in this context arises in the case of entrepreneurial firms where conflicts between entrepreneurs and founders and outside financiers appear most prevalent and relevant.

The last table provides an overview and summary of estimation results across models and samples. Our tests, based on empirical policy function benchmarks, favor trade-off models for larger Compustat firms, limited commitment models for smaller firms, and moral hazard models for private firms. Our estimates point to significant financing constraints due to agency frictions.

## 5 Conclusion

We develop, solve, and estimate a range of dynamic models of corporate investment and financing, with the objective of empirically identifying the quantitatively most prevalent sources of financial frictions. Our approach encompasses tax and default based models of firms' financial structure, as well as dynamic financial contracting models featuring limited commitment and dynamic moral hazard in the presence of asymmetric information. Critically, our estimation procedure based on empirical policy function benchmarks readily lends itself to developing tests that allow to empirically discriminate between proposal models across various samples. Specifically, we evaluate and compare the fit of our proposal models both on the standard Compustat universe and a dataset on private firms coming from Orbis, as well as various subsample splits. Our tests, based on empirical

policy function benchmarks, favor trade-off models for larger Compustat firms, limited commitment models for smaller Compustat firms, and moral hazard models for private firms.

In addition, our estimation procedure allows to gauge the magnitude of various financial frictions proposed in the dynamic contracting literature. The parameter estimates of particular concern relate to the unobserved agency conflicts that drive financial structure and investment, such as the degree of cash flow diversion in moral hazard models, and the amount of capital firms can abscond with in limited commitment models. Regarding cash flow diversion, our results indicate that in order to rationalize observed corporate policies, firm owners need to be able to divert about 15 cents on the dollar of profits. On the other hand, consistent with earlier results in Nikolov, Schmid, and Steri (2016), firms can collateralize about 50 percent of their capital stock in limited commitment models.



## References

- Ai, Hengjie, Dana Kiku, and Rui Li, 2016, Quantifying the impact of moral hazard: Evidence from a structural estimation, Working Paper, University of Minnesota.
- Ai, Hengjie, and Rui Li, 2016, Investment and CEO compensation under limited commitment, *Journal of Financial Economics*, forthcoming.
- Albuquerque, Rui, and Hugo Hopenhayn, 2004, Optimal lending contracts and firm dynamics, *The Review of Economic Studies* pp. 285–315.
- Bazdresch, Santiago, Jay Kahn, and Toni Whited, 2016, Estimating and testing dynamic corporate finance models, Working Paper, University of Rochester.
- Bolton, Patrick, Hui Chen, and Neng Wang, 2011, A unified theory of Tobin’s  $q$ , corporate investment, financing, and risk management, *The Journal of Finance* 66, 1545–1578.
- Clementi, Gian Luca, and Hugo Hopenhayn, 2006, A theory of financing constraints and firm dynamics, *Quarterly Journal of Economics* 121, 229–265.
- Cooley, Thomas, and Vincenzo Quadrini, 2001, Financial markets and firm dynamics, *American Economic Review* 91, 1286–1310.
- Doepke, Matthias, and Robert Townsend, 2006, Dynamic mechanism design with hidden income and hidden actions, *Journal of Economic Theory* 126, 235–285.
- Fu, Shiming, and Vijay Krishna, 2016, Dynamic financial contracting with persistent private information, Working Paper, University of Rochester.
- Gala, Vito, and Joao F. Gomes, 2016, Investment without  $q$ , Working Paper, University of Pennsylvania.
- Glover, Brent, 2015, The expected cost of default, *Journal of Financial Economics* pp. 284–299.
- Gomes, Joao F., 2001, Financing investment, *The American Economic Review* 91, 1263–1285.

- Hackbarth, Dirk, and David Mauer, 2012, Optimal priority structure, capital structure, and investment, *Review of Financial Studies* pp. 747–796.
- Hennessy, Christopher A., and Toni M. Whited, 2005, Debt dynamics, *The Journal of Finance* 60, 1129–1165.
- , 2007, How costly is external financing? evidence from a structural estimation, *The Journal of Finance* 62, 1705–1745.
- Huynh, Kim P, Teodora Paligorova, and Robert J Petrunia, 2012, Capital structure decisions: Insights from private firms, in *Workshop on Productivity Determinants Within Firms and the Role*.
- Kalemli-Ozcan, Sebnem, Bent Sorensen, Carolina Villegas-Sanchez, Vadym Volosovych, and Sevcanyesiltas, 2015, How to construct nationally representative firm level data from the orbis global database, Discussion paper, National Bureau of Economic Research.
- Karaivanov, Alexanderr, and Robert Townsend, 2014, Dynamic financial constraints: Distinguishing mechanism design from exogenously incomplete regimes, *Econometrica* 82, 887–959.
- Karaivanov, Alexander, and Mark Wright, 2011, Distinguishing across models of international capital flows, Working Paper, Simon Fraser University.
- Korteweg, Arthur, 2010, The net benefits to leverage, *The Journal of Finance* 65, 2137–2170.
- Li, Shaojin, Toni Whited, and Yufeng Wu, 2016, Capital supply uncertainty, cash holdings, and investment, *Review of Financial Studies* pp. 1453–1500.
- Matvos, Gregor, 2013, Estimating the benefits of contractual completeness, *Review of Financial Studies* 25, 1248–1290.
- McKoen, Stephen B., 2015, Employee option exercise and equity issuance motives, Working Paper.
- Morellec, Erwan, Boris Nikolov, and Norman Schurhoff, 2012, *Journal of Finance* pp. 803–848.

- , 2016, Agency conflicts around the world, Working Paper.
- Moyen, Nathalie, 2004, Investment-cash flow sensitivities: Constrained versus unconstrained firms, *Journal of Finance* pp. 2061–2092.
- , 2007, How big is the debt overhang problem?, *Journal of Economic Dynamics and Control* pp. 433–472.
- Nikolov, Boris, Lukas Schmid, and Roberto Steri, 2016, Dynamic corporate liquidity, *Journal of Financial Economics* forthcoming.
- Quadrini, Vincenzo, 2004, Investment and liquidation in renegotiation-proof contracts with moral hazard, *Journal of Monetary Economics* 51, 861–885.
- Rampini, Adriano A., and S. Viswanathan, 2010, Collateral, risk management, and the distribution of debt capacity, *The Journal of Finance* 65, 2293–2322.
- , 2013, Collateral and capital structure, *Journal of Financial Economics* 109, 466–492.
- Ross, Sheldon M., 1983, *Recursive Methods in Economic Dynamics* (Academic Press: New York, NY).
- Schrijver, Alexander, 1998, *Theory of linear and integer programming* (John Wiley & Sons).
- Strebulaev, Ilya A, Toni M Whited, et al., 2012, Dynamic models and structural estimation in corporate finance, *Foundations and Trends® in Finance* 6, 1–163.
- Sun, Qi, and Mindy X. Zhang, 2016, Financing intangible capital, Working Paper.
- Taylor, Lucian A, 2010, Why are ceos rarely fired? evidence from structural estimation, *The Journal of Finance* 65, 2051–2087.
- , 2013, Ceo wage dynamics: Estimates from a learning model, *Journal of Financial Economics* 108, 79–98.

- Trick, Michael A., and Stanley E. Zin, 1993, A linear programming approach to solving stochastic dynamic programs, Working Paper.
- , 1997, Spline approximations to value functions: A linear programming approach, *Macroeconomic Dynamics* 1, 255–277.
- Verani, Stephane, 2016, Aggregate consequences of dynamic credit relationships, Working Paper, Board of Governors of the Federal Reserve.
- Vielma, Juan Pablo, and George L Nemhauser, 2011, Modeling disjunctive constraints with a logarithmic number of binary variables and constraints, *Mathematical Programming* 128, 49–72.
- Zhang, Mindy X., 2016, Who bears firm-level risk? implications for cash flow volatility, Working Paper.

# Appendix

## A. Solution by Mixed-Integer Programming

The following constraint generation algorithm converges to the unique fixed point of our Bellman problems.

1. solve the problem in (9) with an initial random subset of constraints for each state  $(k, u, z)$ ;
2. if all constraints  $a \in \Gamma^n(k, u, z)$ , for all  $(k, u, z)$ , are satisfied, terminate the algorithm (where  $\Gamma^n(k, u, z)$  is the set of feasible actions at iteration  $n$ );
3. for each state  $(k, u, z)$ , add the constraint  $a \in \Gamma^n(k, u, z)$  that generates the highest violation in (10) with respect to the current solution  $W^n(k, u, z)$ ;
4. solve the problem with the current set of constraints;
5. go back to step 2.

The separation oracles for the three problems are specified as follows.

### Definition 1 (Separation Oracle - Tradeoff)

$$\max_{a=\{k',b',d\}} d_{k,b',z} + \sum_{z'=1}^{nz} Q(z'|z) \frac{1}{1+r} W_{k'(a),b'(a),z'} - W_{k,b,z} \quad (\text{A1})$$

s.t.

$$0 \leq b' \leq \frac{\theta k'(1-\delta)}{1+r} \quad \forall z' \quad (\text{A2})$$

$$0 \leq p(i_k) \leq 1 \quad \forall i_k = 1, \dots, n_k \quad (\text{A3})$$

$$\sum_{i_k=1}^{n_k} p(i_k) = 1 \quad (\text{A4})$$

$$k' = \sum_{i_k=1}^{n_k} p(i_k) k^G(i_k) \quad (\text{A5})$$

$$d_{k,b',z} = (1-\tau)\pi(z, k) - k' + (1-\delta)k - \Psi(k', k) + \tau\delta k - (1+r(1-\tau))b + b' \quad (\text{A6})$$

$$d_{k,b',z} \geq 0$$

### Definition 2 (Separation Oracle - Limited Enforcement)

$$\max_{a=\{k',b(z'),d\}} d_{k,b(z'),z} + \sum_{z'=1}^{nz} Q(z'|z) \frac{1}{1+r} W_{k'(a),b'(a),z'} - W_{k,b,z} \quad (\text{A7})$$

s.t.

$$0 \leq b(z') \leq \frac{\theta k'(1-\delta)}{1+r} \quad \forall z' \quad (\text{A8})$$

$$0 \leq p(i_k) \leq 1 \quad \forall i_k = 1, \dots, n_k \quad (\text{A9})$$

$$\sum_{i_k=1}^{n_k} p(i_k) = 1 \quad (\text{A10})$$

$$k' = \sum_{i_k=1}^{n_k} p(i_k) k^G(i_k) \quad (\text{A11})$$

$$d_{k,b(z'),z} = (1-\tau)\pi(z, k) - k' + (1-\delta)k - \Psi(k', k) + \tau\delta k - (1+r(1-\tau))b + \sum_{z'=1}^{nz} Q(z'|z)b(z') \quad (\text{A12})$$

$$d_{k,b(z'),z} \geq 0$$

Equations (A2) and (A8) define the bounds for debt; equations (A3) , (A4), (A9) and (A10) define the variables  $p(i_k)$  that have the role to select a grid point for capital on the grid  $k^G(i_k)$  and linearize the term  $k'^\alpha$  in the adjustment cost function; equations (A5) and (A11) pick the grid point for the chosen capital stock from  $k^G(i_k)$ ; equations (A6) and (A12) defines dividends. The computation of the law of motion for future debt is obtained by interpolation with the logarithmic formulation of Vielma and Nemhauser (2011).

**Table 1**  
**Descriptive Statistics**

The table reports summary statistics for firms characteristics across the subsamples of firms in this paper. Data of public firms are from Compustat and cover an unbalanced panel of 59,143 firm/year observations from 1965 to 2015. Data on private firms are from Orbis and cover an unbalanced panel of 277,074 firm/year observations from 2003 to 2012. Subsamples of small vs large, profitable vs unprofitable, and high vs low leverage firms are obtained from Compustat. Firms are split according to firms characteristic in correspondence to the 20th and 80th percentile respectively. Size is measured as the value of properties, plants and equipment in million dollars, and profitability, dividends, and investment are scaled by total assets.  $\mu$  denotes means, and  $\sigma$  denotes standard deviations. All variables are winsorized at the 1 percent level.

	Profitability		Asset Growth		Leverage		Dividends		Log(Size)		Market/Book	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
COMPUSTAT	0.151	0.060	0.124	0.252	0.132	0.142	0.022	0.045	6.480	0.927	1.367	0.677
Large	0.155	0.045	0.090	0.178	0.187	0.124	0.021	0.040	8.281	0.726	1.215	0.494
Small	0.140	0.074	0.150	0.313	0.045	0.149	0.020	0.048	4.874	0.783	1.581	0.854
High Leverage	0.131	0.048	0.078	0.219	0.289	0.213	0.010	0.043	6.649	0.731	1.121	0.338
Low Leverage	0.171	0.066	0.117	0.230	-0.099	0.105	0.036	0.043	5.940	0.787	1.865	1.018
Profitable	0.249	0.046	0.111	0.156	0.053	0.112	0.052	0.056	6.437	0.819	2.044	0.914
Unprofitable	0.058	0.048	0.076	0.295	0.112	0.142	-0.000	0.034	6.304	0.826	1.009	0.445
Private	0.121	0.089	0.098	0.191	0.388	0.306	NA	NA	3.547	0.301	NA	NA

**Table 2**  
**Sensitivity of Moments to Technological Parameters**

The figure presents a heatmap based on the Jacobian matrix of average profitability, average investment, average leverage, and average payouts with respect to the technological parameters common to all models. TO denotes the tradeoff model, LE the limited enforcement model, and MH the model hazard model. For all models, parameters are set to the values of Figure 1. For each moment, green is associated with positive sensitivities, red with negative sensitivities, and yellow to zero sensitivities.

		$\alpha$	$f$	$\rho_z$	$\sigma_z$	$\delta$	$\psi$	$\eta$
Profitability	TO	-0.68	-0.01	-0.23	-0.13	+0.25	+0.00	+0.00
	LE	-0.34	+0.01	-0.20	-0.07	+0.15	+0.00	-0.01
	MH	-0.61	-0.01	-0.26	-0.09	+0.24	+0.00	-0.04
Investment	TO	+0.00	+0.00	+0.01	+0.00	+0.14	+0.00	+0.00
	LE	-0.56	-0.14	-0.38	-0.31	+0.03	-0.01	-0.04
	MH	+0.34	+0.00	-0.08	+0.12	+0.05	+0.00	-0.25
Leverage	TO	+0.28	+0.01	+0.23	+0.23	+0.12	+0.00	+0.06
	LE	-0.74	+0.00	-0.79	-0.30	+0.09	-0.05	+0.01
	MH	-0.31	+0.00	+0.10	-0.14	+0.08	+0.00	+0.24
Payouts	TO	-0.68	-0.01	-0.20	-0.13	+0.13	+0.00	+0.00
	LE	+0.96	-1.25	+0.88	+1.44	-0.88	+0.60	+0.90
	MH	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00



**Table 3**  
**Estimation: Large versus Small Firms**

The table reports parameter estimates for the tradeoff (TO), limited enforcement (LE), and moral hazard (MH) models using empirical policy functions. All models are estimated on the top and bottom terciles of firms sorted by size.  $\alpha$  denotes the curvature of the production function,  $f$  the fixed production cost,  $\rho_z$  the persistence of the profitability shock,  $\sigma_z$  the volatility of the profitability shock,  $\delta$  the depreciation rate,  $\psi$  the capital adjustment cost parameter,  $\eta$  the size of the iid shock to profits,  $\xi$  the recovery rate parameter in the tradeoff model,  $\theta$  the tangibility parameter in the limited enforcement model, and  $\lambda$  the diversion parameter in the moral hazard model. Obj. Fun. denotes the goodness of fit measures as the minimized criteria for the empirical policy function estimation. Standard errors are in parantheses.

	Large Firms			Small Firms		
	TO	LE	MH	TO	LE	MH
$\alpha$	0.760 (0.006)	0.808 (0.002)	0.741 (0.001)	0.764 (0.002)	0.764 (0.001)	0.764 (0.002)
$f$	0.848 (5.562)	0.776 (0.016)	0.670 (0.020)	0.623 (0.254)	0.796 (0.001)	0.630 (0.005)
$\rho_z$	0.834 (0.002)	0.779 (0.003)	0.834 (0.001)	0.774 (0.012)	0.774 (0.000)	0.774 (0.001)
$\sigma_z$	0.277 (0.014)	0.305 (0.012)	0.364 (0.004)	0.312 (0.004)	0.312 (0.000)	0.312 (0.001)
$\delta$	0.207 (0.005)	0.126 (0.009)	0.188 (0.004)	0.164 (0.014)	0.181 (0.001)	0.200 (0.001)
$\psi$	0.180 (0.191)	0.132 (0.024)	0.162 (0.006)	0.158 (0.048)	0.150 (0.001)	0.165 (0.002)
$\eta$	0.288 (0.009)	0.329 (0.017)	0.260 (0.008)	0.255 (0.024)	0.207 (0.001)	0.241 (0.002)
$\xi$	0.825 (0.001)			0.466 (0.015)		
$\theta$		0.727 (0.012)			0.647 (0.001)	
$\lambda$			0.039 (0.005)			0.021 (0.003)
Obj. Fun.	0.025	0.060	0.198	0.053	0.045	0.466

**Table 4**  
**Estimation: High versus Low Leverage Firms**

The table reports parameter estimates for the tradeoff (TO), limited enforcement (LE), and moral hazard (MH) models using empirical policy functions. All models are estimated on the top and bottom terciles of firms sorted by leverage.  $\alpha$  denotes the curvature of the production function,  $f$  the fixed production cost,  $\rho_z$  the persistence of the profitability shock,  $\sigma_z$  the volatility of the profitability shock,  $\delta$  the depreciation rate,  $\psi$  the capital adjustment cost parameter,  $\eta$  the size of the iid shock to profits,  $\xi$  the recovery rate parameter in the tradeoff model,  $\theta$  the tangibility parameter in the limited enforcement model, and  $\lambda$  the diversion parameter in the moral hazard model. Obj. Fun. denotes the goodness of fit measures as the minimized criteria for the empirical policy function estimation. Standard errors are in parentheses.

	High Leverage Firms			Low Leverage Firms		
	TO	LE	MH	TO	LE	MH
$\alpha$	0.735 (0.013)	0.700 (0.004)	0.585 (0.015)	0.796 (0.001)	0.717 (0.001)	0.719 (0.007)
$f$	1.250 (0.057)	1.000 (0.030)	1.659 (0.011)	1.170 (0.084)	0.974 (0.005)	1.152 (2.247)
$\rho_z$	0.794 (0.003)	0.882 (0.004)	0.860 (0.004)	0.749 (0.002)	0.750 (0.001)	0.750 (0.006)
$\sigma_z$	0.206 (0.017)	0.206 (0.004)	0.202 (0.015)	0.322 (0.009)	0.321 (0.006)	0.333 (0.017)
$\delta$	0.177 (0.015)	0.148 (0.011)	0.172 (0.006)	0.180 (0.011)	0.180 (0.004)	0.186 (0.027)
$\psi$	0.111 (0.206)	0.085 (0.010)	0.104 (0.000)	0.156 (0.014)	0.134 (0.006)	0.121 (0.000)
$\eta$	0.355 (0.024)	0.338 (0.023)	0.341 (0.018)	0.341 (0.005)	0.316 (0.004)	0.213 (0.017)
$\xi$	0.783 (0.130)			0.213 (0.002)		
$\theta$		0.783 (0.010)			0.227 (0.004)	
$\lambda$			0.127 (0.018)			0.200 (0.016)
Obj. Fun.	0.061	0.105	0.023	0.037	0.043	0.043

**Table 5**  
**Estimation: High versus Low Profitability Firms**

The table reports parameter estimates for the tradeoff (TO), limited enforcement (LE), and moral hazard (MH) models using empirical policy functions. All models are estimated on the top and bottom terciles of firms sorted by profitability.  $\alpha$  denotes the curvature of the production function,  $f$  the fixed production cost,  $\rho_z$  the persistence of the profitability shock,  $\sigma_z$  the volatility of the profitability shock,  $\delta$  the depreciation rate,  $\psi$  the capital adjustment cost parameter,  $\eta$  the size of the iid shock to profits,  $\xi$  the recovery rate parameter in the tradeoff model,  $\theta$  the tangibility parameter in the limited enforcement model, and  $\lambda$  the diversion parameter in the moral hazard model. Obj. Fun. denotes the goodness of fit measures as the minimized criteria for the empirical policy function estimation. Standard errors are in parantheses.

	High Profitability Firms			Low Profitability Firms		
	TO	LE	MH	TO	LE	MH
$\alpha$	0.790 (0.001)	0.750 (0.003)	0.790 (0.001)	0.575 (0.003)	0.569 (0.002)	0.575 (0.000)
$f$	0.492 (0.020)	0.124 (0.049)	0.378 (0.198)	5.937 (0.007)	5.357 (0.002)	6.388 (0.025)
$\rho_z$	0.778 (0.000)	0.427 (0.003)	0.778 (0.009)	0.548 (0.007)	0.498 (0.002)	0.512 (0.007)
$\sigma_z$	0.293 (0.065)	0.206 (0.003)	0.210 (0.021)	0.196 (0.011)	0.303 (0.002)	0.236 (0.026)
$\delta$	0.151 (0.080)	0.195 (0.007)	0.144 (0.008)	0.078 (0.008)	0.060 (0.004)	0.072 (0.009)
$\psi$	0.133 (0.020)	0.108 (0.009)	0.133 (0.006)	0.053 (0.029)	0.149 (0.004)	0.174 (0.217)
$\eta$	0.237 (0.078)	0.154 (0.009)	0.296 (0.049)	0.333 (0.020)	0.340 (0.005)	0.333 (0.085)
$\xi$	0.326 (0.000)			0.378 (0.008)		
$\theta$		0.408 (0.028)			0.350 (0.002)	
$\lambda$			0.188 (0.040)			0.149 (0.081)
Obj. Fun.	0.019	0.088	0.067	0.039	0.064	0.036

**Table 6**  
**Estimation: Public vs Private Firms**

The table reports parameter estimates for the tradeoff (TO), limited enforcement (LE), and moral hazard (MH) models using empirical policy functions. All models are estimated on the samples of public and private firms.  $\alpha$  denotes the curvature of the production function,  $f$  the fixed production cost,  $\rho_z$  the persistence of the profitability shock,  $\sigma_z$  the volatility of the profitability shock,  $\delta$  the depreciation rate,  $\psi$  the capital adjustment cost parameter,  $\eta$  the size of the iid shock to profits,  $\xi$  the recovery rate parameter in the tradeoff model,  $\theta$  the tangibility parameter in the limited enforcement model, and  $\lambda$  the diversion parameter in the moral hazard model. Obj. Fun. denotes the goodness of fit measures as the minimized criteria for the empirical policy function estimation. Standard errors are in parantheses.

	Public Firms			Private Firms		
	TO	LE	MH	TO	LE	MH
$\alpha$	0.797 (0.000)	0.797 (0.003)	0.776 (0.001)	0.642 (0.000)	0.635 (0.000)	0.625 (0.000)
$f$	0.950 (0.008)	0.813 (0.004)	0.849 (0.004)	8.808 (0.002)	8.972 (0.001)	7.929 (0.001)
$\rho_z$	0.712 (0.000)	0.721 (0.001)	0.863 (0.001)	0.499 (0.001)	0.500 (0.001)	0.502 (0.000)
$\sigma_z$	0.344 (0.001)	0.333 (0.004)	0.317 (0.005)	0.176 (0.001)	0.176 (0.001)	0.264 (0.001)
$\delta$	0.160 (0.001)	0.147 (0.006)	0.200 (0.003)	0.091 (0.001)	0.091 (0.001)	0.097 (0.001)
$\psi$	0.250 (0.006)	0.245 (0.003)	0.100 (0.001)	0.249 (0.005)	0.222 (0.001)	0.203 (0.028)
$\eta$	0.200 (0.021)	0.200 (0.006)	0.341 (0.001)	0.349 (0.002)	0.336 (0.002)	0.350 (0.003)
$\xi$	0.738 (0.000)			0.882 (0.001)		
$\theta$		0.736 (0.005)			0.882 (0.001)	
$\lambda$			0.098 (0.004)			0.129 (0.002)
Obj. Fun.	0.045	0.066	0.163	0.070	0.113	0.066

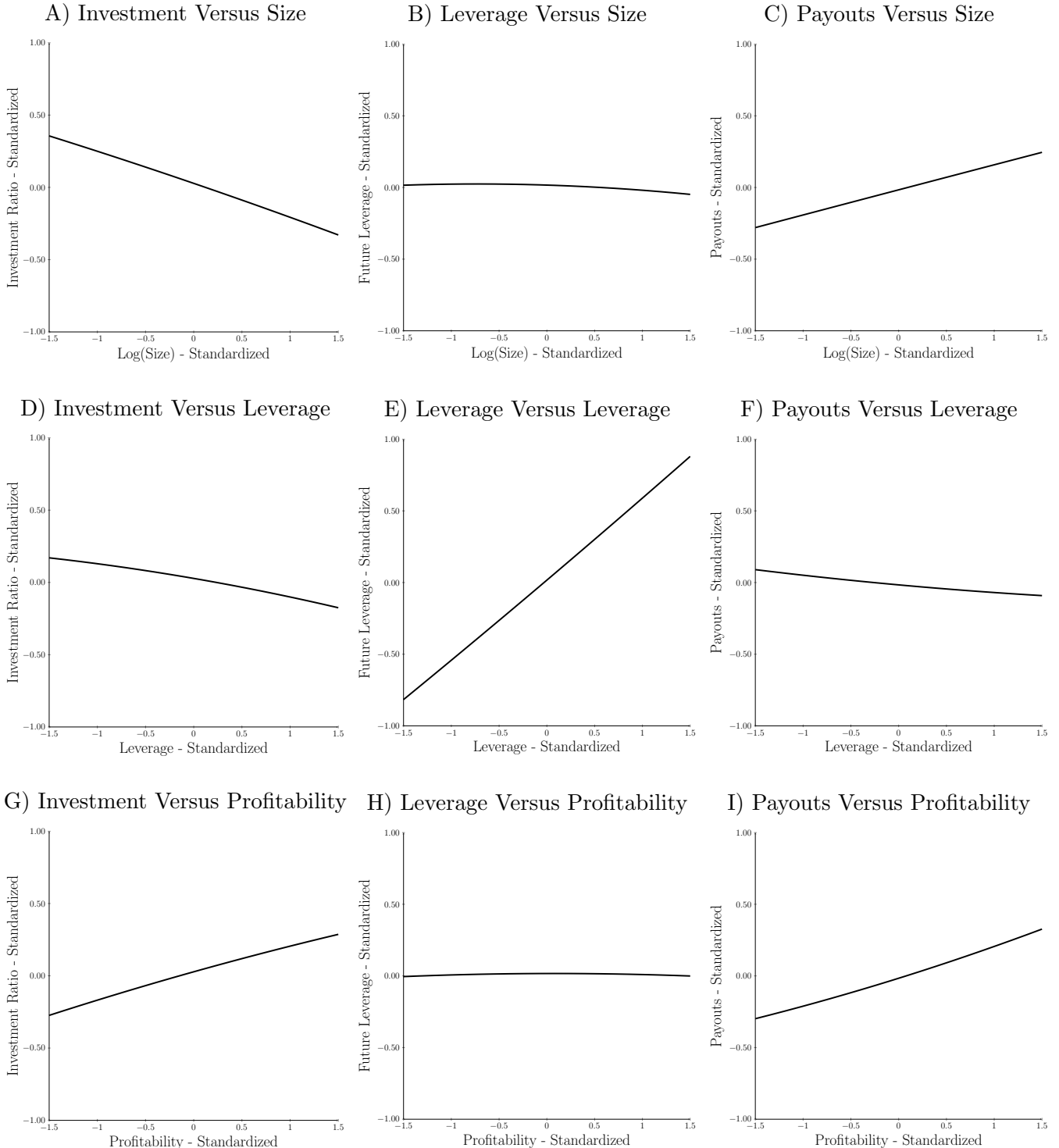
**Table 7**  
**Model Comparison**

The table reports the results of two-way and three-way statistical Wald tests to compare the tradeoff, limited commitment, and moral hazard model on different subsamples of firms. TO denotes the tradeoff model, LE denotes the limited enforcement model, and MH the moral hazard model. "Tie" indicates a non-statistically distinguishable performance between models. Data of public firms are from Compustat and cover an unbalanced panel of 59,143 firm/year observations from 1965 to 2015. Data on private firms are from Orbis and cover an unbalanced panel of 277,074 firm/year observations from 2003 to 2012. Subsamples of small vs large, high vs low profitability, and high vs low leverage firms are obtained from Compustat. Firms are split according to firms characteristic in correspondence to the top and bottom tercile respectively. Size is measured as total assets in million dollars, and profitability, dividends, and investment are scaled by total assets. All variables are winsorized at the one percent level.

	Comparison			Best Fit
	TO vs LE	TO vs MH	LE vs MH	
Small	LE	TO	LE	LE
Large	TO	TO	LE	TO
Low Leverage	TO	TO	Tie	TO
High Leverage	TO	MH	MH	MH
Profitable	TO	TO	ML	TO
Unprofitable	TO	MH	MH	MH
Public	TO	TO	LE	TO
Private	TO	MH	MH	MH

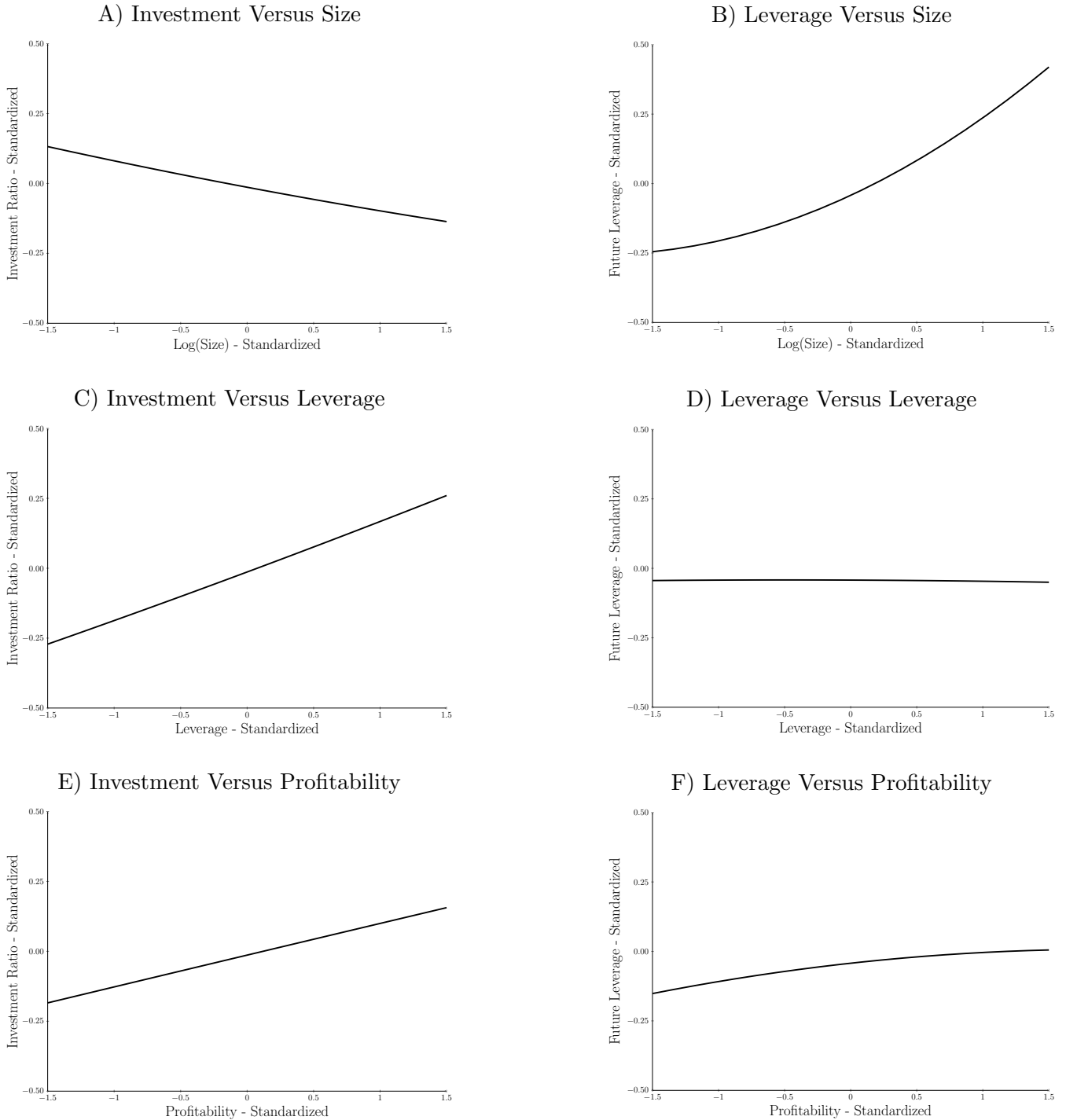
**Figure 1**  
**Empirical Policy Functions: Compustat Data**

The figure depicts the relation between the predicted empirical policy functions for the sample of Compustat data described in Section 4. Empirical policy functions are constructed as described in Section 3, where the transformed state variables that are not plotted are fixed to their average value in the sample.



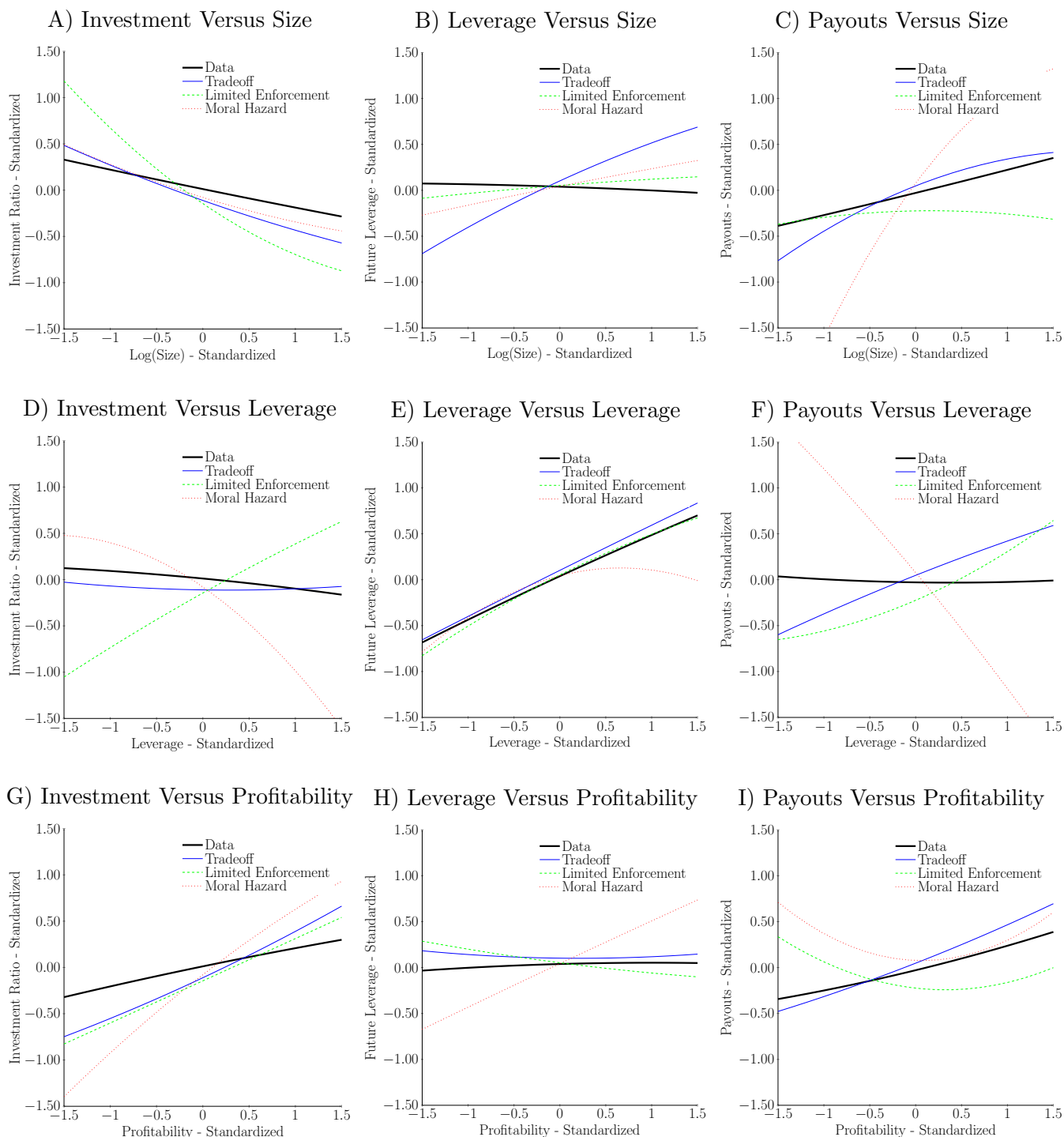
**Figure 2**  
**Empirical Policy Functions: Orbis Data**

The figure depicts the relation between the predicted empirical policy functions for the sample of Orbis data described in Section 4. Empirical policy functions are constructed as described in Section 3, where the transformed state variables that are not plotted are fixed to their average value in the sample.



**Figure 3**  
**Empirical Policy Functions: Large Public Firms**

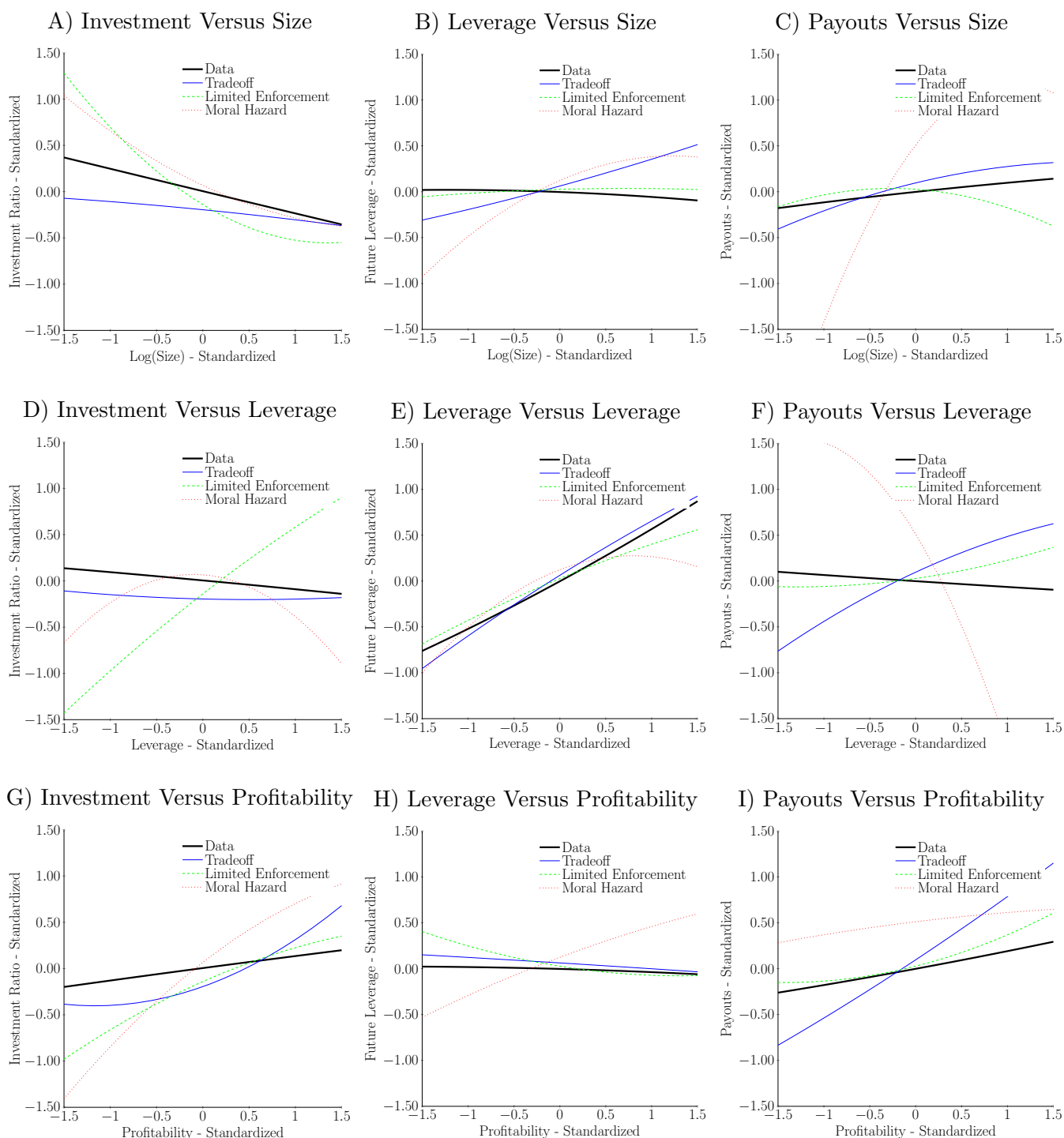
The figure depicts the relation between the predicted empirical policy functions for the sample of Compustat data described in Section 4. Empirical policy functions are constructed as described in Section 3, where the transformed state variables that are not plotted are fixed to their average value in the sample.





**Figure 4**  
**Empirical Policy Functions: Small Public Firms**

The figure depicts the relation between the predicted empirical policy functions for the sample of Compustat data described in Section 4. Empirical policy functions are constructed as described in Section 3, where the transformed state variables that are not plotted are fixed to their average value in the sample.



**Figure 5**  
**Empirical Policy Functions: Private Firms**

The figure depicts the relation between the predicted empirical policy functions for the sample of Orbis data described in Section 4. Empirical policy functions are constructed as described in Section 3, where the transformed state variables that are not plotted are fixed to their average value in the sample.

