Firm Wages in a Frictional Labor Market[*]

Leena Rudanko

Federal Reserve Bank of Philadelphia

June 25, 2018

Preliminary!

Abstract

This paper studies a labor market with search frictions and directed search, where firms employ multiple workers and follow a firm-wage policy: a firm pays all its (equally productive) workers the same. The policy introduces a tension into the static firm problem, between setting a high wage to attract more new workers versus a low one to economize on labor costs on existing ones. The policy also introduces a time-inconsistency into the dynamic firm problem that affects equilibrium allocations. A firm with commitment plans on higher wages in the future than in the short run, where the firm takes advantage of its existing workers with a low wage. I study labor market outcomes when firms cannot commit to future wages, and show that one can, despite the time-inconsistency, analyze Markov-perfect equilibria using a standard Euler equation approach. The model generates endogenous real wage rigidity as firms raising wages to increase hiring in an expansion must raise them for all workers instead of only new hires. The commitment problem also gives a motive for firms to adjust wages only infrequently, as observed. An equilibrium where firms adjust wages infrequently can be better for welfare, especially that of workers.
JEL Codes: E24; E32; J41; J64.
Keywords: Labor Market Search; Business Cycles; Wage Rigidity; Competitive Search; Limited Commitment.

1 Introduction

The work of Truman Bewley (1999), based on interviews of managers in corporate America, sketches a view of labor markets where employee compensation within firms is determined by formal internal pay structures. These structures seek to balance the dual goals of providing incentives and maintaining equity among a firm's employees. The structure is described as emerging as a managerial tool in a situation where employee productivity cannot be perfectly measured, but the pay of a large number of employees within the firm must be determined by their respective managers in a mutually consistent way, seeking to avoid favoritism between individuals. Managers believe their employees to be aware of pay differences, even if salaries are not made public, and inequity to antagonize and embitter employees. Such structures are, in fact, reminiscent of the ones imposed on their members by labor unions, with employee pay determined in grades according to the job and jobholder characteristics.[1] To shed light on the implications of such structures for labor market dynamics, this paper develops a macroeconomic theory of multi-worker firms in a frictional labor market that incorporates a notion of firm wages, and studies the consequences.

I study a labor market with search frictions and competitive search, where firms employ a measure of workers and must pay their (equally productive) workers the same. I begin by showing, in the context of a static model, that the equal treatment constraint changes the tradeoffs firms face in choosing a wage to offer. In a standard model of competitive search, firms set wages trading off the increased wage costs associated with offering higher wages against the increased hiring rates resulting from such wages attracting more job seekers. Here, higher wages increase the wage costs associated with the firm's existing workers as well as new hires, leading firms to choose lower wages than in the standard model. At the same time, this also means that when firms start raising wages to increase hiring in an expansion, the incentives to do so are curbed by the growing wage costs associated with their existing workers. The equal treatment constraint thus gives rise to a mechanism generating wage rigidity over the business cycle, relative to the standard competitive search model.

I then show, in the context of a dynamic infinite horizon model, that the firm's wage set-

---

[1]In a related effort seeking to shed light on the underpinnings of wage rigidities, Blinder and Choi (1990) also surveyed businesses on their wage setting practices. They found managers to believe workers to be concerned with how their wages compare to other workers, with 84 percent of managers agreeing that workers want to maintain a hierarchy of wages, and resist wage reductions for fear of interfering with this hierarchy. Card, Mas, Moretti, and Saez (2012), Bracha, Gneezy, and Loewenstein (2015) and Breza, Kaur, and Shamdasani (2017) provide recent evidence that relative pay concerns enter worker preferences, and affect effort and output: workers appear to prefer equal treatment, unless productivity differences are sufficiently large and evident.

ting problem involves a time-inconsistency, an observation that appears new to the literature on competitive search. The time-inconsistency arises due to the equal treatment constraint, which ties together the wages of different cohorts of workers within the firm. The constraint limits the ability of the firm to adjust the timing of wage payments a worker receives over the course of a long-term employment relationship, and to a degree that the timing of those payments is pinned down uniquely for a given path of market tightness. To see this, note that, as usual in these models, there is a one-to-one mapping between the market tightness and present value of wages in each period. With future wages given, the way the firm controls the current present value – with the equal wage constraint in place – is via the current wage. Hence, a path for the market tightness pins down a unique path of per-period wages.

Due to the linkages between the wages of different cohorts of workers, the dynamic firm problem differs from the standard one in a non-trivial way. I show that if the firm has commitment to future wages, this does not affect the firm's optimality conditions beyond the initial period, however, meaning the firm is able to get round the constraints to a sufficient degree. In the initial period this is not the case, however, as the firm sets the initial wage to simultaneously optimize on new hiring and minimize labor costs associated with existing workers, with no way to independently control different cohorts. As a consequence, the initial wage is set lower than in the standard model, leading to a different allocation as well. In the standard competitive search model, this issue does not arise because firms are able to offer different cohorts of workers different wages.[2]

The path of wages characterized above requires commitment on the part of the firm to future wages, because if the firm were to reoptimize at a later date, it would generally depart from its plan by setting lower wages in the reoptimization period than planned.[3] To consider outcomes when firms cannot commit to future wages, I study Markov-perfect equilibria. In particular, I show that in this environment it can be profitable for firms to fix wages for a period of time, because doing so allows the firm to get around the commitment problem it faces. However, I also show that if all firms do so, the equilibrium shifts toward higher wages in a way that makes workers better off. Thus, in this model infrequent wage adjustment can

---

[2]The standard competitive search framework does not determine firm size, hence whether employed workers work for the same or different firms makes no difference. Moreover, in the standard competitive search framework, whether the firm can commit to future wages matters for the path of wages during an employment relationship, but not allocations. With no commitment the firm pays the worker a signing bonus up front, and after that a lower wage making the worker indifferent between staying in the job and unemployment. With commitment, the path of wages within the employment relationship is not pinned down, as many paths are consistent with the same allocation. See Section 3 for a discussion.

[3]The firm problem resembles the problem of optimal capital taxation (Chamley 1986, Judd 1985) in the sense that the firm would like to treat its initial stock of workers differently from those in later periods, with an implied time-inconsistency of plans.

be profitable for firms to adopt, as well as welfare improving for workers. While the socially optimal outcome would have flexible wages, in a second best world where the duration of wage commitments is the only instrument a social planner could use, also the planner would prefer longer commitments.[4]

In addition to developing a model of labor market dynamics, this paper contributes by offering a tractable approach to solving for Markov perfect equilibria in an environment with a time-inconsistency. Such environments are typically difficult to analyze because the decision-maker's objective does not coincide with maximizing his/her value function, meaning that standard dynamic programming arguments cannot be directly applied. An approach that has been developed in the literature for characterizing differentiable Markov perfect equilibria involves deriving a generalized Euler equation, which spells out the tradeoffs faced by the decision-maker, as well as serves as a basis for solving the problem numerically. Solving the generalized Euler equation remains challenging, however, due to the dependence of the equation on the derivative of choice variables with respect to the state.[5] In the environment of the present paper solving for time-consistent outcomes is substantially simplified because the firm's decision problem is independent of the relevant state – its size – and thus a standard Euler equation approach can be used. In addition to simplifying solving the model, this also allows incorporating stochastic shocks into the model without difficulty.

In terms of empirical evidence, the observation that wages are adjusted only intermittently is discussed by John Taylor (1999, 2016), with evidence going back to his own study on union wage contracts (Taylor 1983). Documenting infrequent wage adjustment, he found that only 15 percent of workers saw contract adjustments each quarter, and only 40 percent each year. Recently, Barattieri, Basu, and Gottschalk (2014) have revisited this question with broader data from the Survey of Income and Program Participation. They document a quarterly frequency of wage adjustment ranging from 12 to 27 percent, which implies an average duration of wages of 4-8 quarters. For European countries, Lamo and Smets (2009) summarize related results based on data collected by the Wage Dynamics Network of the European Central Bank. They report that 60 percent of firms surveyed changed wages once a year, and 26 percent less frequently, with an average duration of wages of 15 months. From a theoretical perspective such infrequent adjustment comes across as somewhat puzzling, as

---

[4]The equity constraints render the competitive search equilibrium allocation inefficient, both in the static and dynamic context, as shown in Sections 2 and 3, departing from the typical efficiency result (Rogerson, Shimer, and Wright 2005).

[5]In the literature time-inconsistencies appear in multiple contexts, either due to preferences directly or the economic environment, such as in problems of optimal fiscal or monetary policy. See Klein, Krusell, and Rios-Rull (2008) for a discussion on characterizing Markov-perfect equilibria in problems with time-inconsistency, in the context of a study of optimal government spending.

it should be costly for firms in a changing world.

Related Literature A set of papers have introduced multi-worker firms into search and matching models to study the impact on employment dynamics. Such models with random search include Smith (1999), Cooper, Haltiwanger, and Willis (2007), Hawkins (2011), Elsby and Michaels (2013), Acemoglu and Hawkins (2014), and Fujita and Nakajima (2016), and in the context of competitive/directed search Hawkins (2013), Kaas and Kircher (2015), and Schaal (2017). In these papers production technologies feature decreasing returns to scale, with a focus on how such technologies affect labor market dynamics. The most closely related paper in this literature is Kaas and Kircher (2015), where firms also post long-term wage contracts to attract workers. The key difference is that here the wages of different cohorts of workers are explicitly linked, because the currently prevailing wage applies to all cohorts today, imposing an additional restriction. At the same time, the present paper abstracts from decreasing returns in technology, something that leads to significant simplification in solving the dynamic model, as discussed in Section 3.

One recent study that also emphasizes the role of equity in pay is Gertler and Trigari (2009), who study employment dynamics in a model where multi-worker firms pay their workers the same and only rebargain wages when a Calvo-draw allows it. This leads to amplification in the response of hiring to aggregate shocks, because firms that have not adjusted to a shock offer pre-shock wages also to new hires, affecting vacancy creation. To support the assumption of equal pay, Gertler and Trigari (2009) also offer new empirical evidence that the wages of new hires and existing workers appear equally procyclical. A conventional view has held that the wages of job movers are more cyclical than those of stayers (Bils 1985, Haefke, Sonntag, and van Rens 2013), but the authors argue that these findings are driven by cyclical variation in job quality, and vanish with proper controls – an idea they pursue further in Gertler, Huckfeldt, and Trigari (2017). In a sense, the present paper offers a microfoundation for the assumption of infrequent wage adjustment adopted by these authors.[6]

Firm wages appear also in the framework of Burdett and Mortensen (1998) with on the job search, and its dynamic extensions such as Moscarini and Postel-Vinay (2013, 2016). The latter consider equilibria where firms have full commitment to future wages, while Coles

---

[6]Pay equity has been emphasized also by Snell and Thomas (2010), who propose a non-search model of labor markets where equity across workers and the motive of risk neutral firms to insure risk averse workers by smoothing their wages combine to result in real wage rigidity. The classic contribution of Akerlof and Yellen (1990) also considered the implications of fairness concerns for labor market outcomes, formally connecting a worker's effort to the fairness of pay.

(2001) considers the implications of relaxing this assumption, revealing the complexity of the problem. In a related contribution, Menzio (2005) develops a framework where firms bargain with their workers in the presence of private information about the firm's productivity, and offers a microfoundation for firm wages by showing that they can emerge as an endogenous outcome of the bargaining game.

The role of commitment has also been emphasized in the competitive search models of Rudanko (2009), who studies labor market dynamics when firms insure workers through wage contracts, and Menzio and Moen (2010), who consider a related problem where firms commit to wages but not employment. Krusell and Rudanko (2016) emphasize a time-inconsistency and related commitment problem arising when a union sets wages in a frictional labor market.

This paper is organized as follows. Section 2 begins with a one-period model, to illustrate the static tradeoffs involved with the firm-wage policy. Section 3 turns to a dynamic infinite horizon model, to illustrate the time-inconsistency. Section 4 extends the basic model to allow longer wage commitments. Section 5 considers the implications for wage rigidity and infrequent wage adjustment in a quantitative experiment. The appendix contains proofs, a two period model demonstrating the time-inconsistency in a simpler setting, as well as details on the parametrization.

2   Static Model

This section considers a one period model, before proceeding to an infinite horizon model in Section 3. The one period model illustrates the static tradeoffs involved in introducing firm wages into a competitive search model of frictional labor markets.

Consider a labor market with measure one workers, and a large number I firms each with n_i existing workers for all $i \in I$. In the beginning of the period there are thus $N = \sum_{i \in I} n_i$ workers that are employed and $1 - N$ workers looking for work. Each firm has access to a linear production technology with output z per worker. Those workers who do not find work have access to a home production technology with output b.

Search frictions in the labor market are formalized with a constant returns to scale matching function. I denote the market tightness, or ratio of vacancies to job seekers, by θ. The probability a worker finds a job is denoted $\mu(\theta)$ and the probability a vacancy is filled $q(\theta)$, where $\mu(\theta) = \theta q(\theta)$. Posting v vacancies is associated with a convex cost $\kappa(v, n) = f(v/n)n$, where n is the producer's existing workforce and $f' > 0, f'' > 0$.[7]

---

[7]The convexity in the vacancy cost is introduced to help ensure the multi-worker producer's first order

In a competitive search equilibrium, optimizing job seeker behavior ensures that for any wage w_i offered by a firm, the corresponding market tightness θ_i satisfies the equation

$$U = \mu(\theta_i)(w_i - b), \tag{1}$$

where U is the equilibrium value of search. The tightness is assumed to adjust such that job seekers derive the same utility from applying for all wages offered, with the utility given by the probability of getting hired times the gain from being employed. The equation implies that if multiple wages are offered in equilibrium, a firm offering a higher wage attracts more applicants per vacancy.

Each firm chooses the wage to offer and measure of vacancies to maximize profit:

$$\max_{w_i, v_i} (n_i + q(\theta_i)v_i)(z - w_i) - \kappa(v_i, n_i), \tag{2}$$

taking as given n_i and the condition (1). The firm has n_i existing workers and hires $q(\theta_i)v_i$ new ones, all of whom produce z unit of output and are paid the wage w_i. The new workers are also associated with a vacancy cost, $\kappa(v_i, n_i)$.

The key constraint the firm wage policy implies here is that all workers are paid the same. In a multi-worker firm extension of the competitive search equilibrium (Kaas and Kircher 2015), the wages offered to workers hired at different points in time are independent of each other. The corresponding firm problem in this case would read

$$\max_{w_i, \theta_i, v_i} n_i(z - w^a) + q(\theta_i)v_i(z - w_i) - \kappa(v_i, n_i), \tag{3}$$

where the firm again takes as given the constraint (1) on the wages of new hires, while w^a is the average wage of existing workers. Note that in this case the hiring wage can be chosen freely to maximize profits on new hires, while the wages of existing workers naturally affect profits.[8]

Returning to problem (2), note that if we define the firm's rate of vacancy creation as $x_i := v_i/n_i$, we can scale the firm problem by n_i to arrive at the scale-independent problem:

$$\max_{w_i, \theta_i, x_i} (1 + q(\theta_i)x_i)(z - w_i) - \kappa(x_i), \tag{4}$$

---

conditions characterize optimizing behavior. The form is consistent with Kaas and Kircher (2015).

[8]One aspect of the equal pay constraint is that in some circumstances the firm might prefer to opt out of hiring altogether, and pay its existing workers the minimum to keep them, by setting $v_i = 0, w_i = b$. In what follows I focus on the situation where the firm does not find it optimal to opt out, but ultimately one must compare profits attained via such interior solutions with this corner solution, to ensure optimality.

taking as given (1).[9] This means that heterogeneity in initial size across firms does not translate into differences in wages offered, or the vacancy rate, so the firms will behave identically. Of course, because vacancies are proportional to initial size, larger firms hire more new workers as well, preserving any initial differences in size. The same holds for the case without the firm wage policy. Hence, I will drop the firm indexes on w_i, θ_i, x_i here on.[10]

Proposition 1. *Firms' posted wages and hiring rates are independent of firm size, and vacancy creation is proportional to firm size:* $w_i = w$, $\theta_i = \theta$, $v_i = xn_i$, $\forall i$.

The first order condition for vacancy creation reads:

$$\kappa_v(x) = q(\theta)(z - w). \tag{5}$$

This says that the firm creates vacancies to a point where the marginal cost equals the expected profits from filled vacancies.

The first order condition for the optimal market tightness involves the tradeoff the firm faces when setting the wage: a higher wage increases the hiring rate but also implies greater wage costs. For thinking about this tradeoff, denote by $\theta = g(w; U)$ the relationship between wage and tightness defined by the unemployment value constraint (1). The implied decline in tightness from an increase in the wage is given by the derivative $g_w(w; U) = -\mu(\theta)/(\mu'(\theta)(w - b))$. From the expression for firm profits then, the optimal choice of wage is such that the decline in profits due to the higher wage costs equals the gains from increased hiring:

$$1 + q(\theta)x = q'(\theta)x(z - w)g_w(w; U) \tag{6}$$

The corresponding optimality conditions for the firm without the firm wage policy in (3) include the same condition for optimal vacancy creation (5), together with the condition for the optimal wage-market tightness tradeoff:

$$q(\theta)x = q'(\theta)x(z - w)g_w(w; U) \tag{7}$$

While both firms thus face a tradeoff when setting the wage between the increased hiring rate involved with raising the wage and the coinciding increase in wage costs, the increase

---

[9]Note that $\kappa(v, n)/n, \kappa_v(v, n), \kappa_n(v, n)$ are all functions of $x = v/n$ only, so with a light abuse of notation, in what follows I denote them $\kappa(x), \kappa_v(x), \kappa_n(x)$, respectively.

[10]If production technologies had decreasing returns, such scale independence would no longer hold. Multi-worker firm models such as Kaas and Kircher (2015) focus precisely on the implications of relaxing constant returns in technology and the ability of that framework to explain firm-level observations. I abstract from decreasing returns in order to focus on the impact of firm wages. In the context of the dynamic model doing so leads to a significant simplification in solving the model, as discussed in section 3.

in wage costs is greater with the firm wage policy because the increase in wage applies also to its existing workers without a corresponding impact of hiring. Thus, the firm with the firm wage policy sets a lower wage than the unconstrained firm. Optimal vacancy creation then implies that the firm also creates more vacancies per existing worker, as workers are less expensive for the firm.

Definition 1. *A competitive search equilibrium with firm wages is an allocation $\{w, \theta, x\}$ and value of unemployment U such that the allocation and value solve the problem (4), and that each job seeker applies to one firm: $1 - \sum_i n_i = \sum_i x n_i / \theta$.*

If all producers behave the same, then the equilibrium condition becomes $x = \theta(1-N)/N$. Due to this equilibrium condition, the allocation will generally depend on the aggregate measure of existing workers N, even if firm choices do not depend on individual producer size. This dependence also leads to an intuition that the firm wage policy will lead to wage rigidity over the business cycle. As in the standard competitive search model if market productivity increases, it will lead to an increase in wages as firms seek to attract more workers by offering higher wages. In the model with the firm wage policy the incentives in setting wages depend on the share of new hires relative to existing workers, with a larger share of existing workers pushing down wages. This suggests that the firm wage policy will reduce the procyclicality of wages by reducing wages in expansions relative to recessions.

Planner Problem With the dynamic model of the next section in mind, it is useful to connect market outcomes to the planner problem. Firm wages break the typical efficiency of the competitive search equilibrium discussed for example in Rogerson, Shimer, and Wright (2005).

The planner problem maximizes output, taking into account the costs of search:

$$\max_{\theta_i, v_i} \sum_i [(n_i + q(\theta_i)v_i)z - \kappa(v_i, n_i)] + [1 - \sum_i (n_i + q(\theta_i)v_i)]b, \tag{8}$$

such that $\sum_i v_i/\theta_i = 1 - \sum_i n_i$. Here the first term reflects the gains from a total of $n_i + q(\theta_i)v_i$ matches, while the second deducts the costs of search involved with creating new matches. Each production unit attracts v_i/θ_i searching workers, the sum of whom must equal the measure searching.

The first order conditions for the planner problem read

$$\kappa_v(x_i) + \frac{\lambda}{\theta_i} = q(\theta_i)(z - b) \tag{9}$$

$$\frac{\lambda}{\theta_i^2} = -q'(\theta_i)(z - b). \tag{10}$$

This says that the planner creates vacancies to a point where the marginal cost equals the expected gains from the workers working in the market rather than not. The planner's choice of market tightness reflects the efficient tightness to apply in hiring, taking into account the frictions inherent in the matching function and the gains from having workers work with the market technology rather than at home. Note that producer size does not enter into this calculation. Combining the two equations and dropping the producer subscript, we have that the optimal allocation is characterized by the optimality condition

$$\kappa_v(x) = \mu'(\theta)(z - b), \tag{11}$$

where $\sum_i x n_i/\theta = 1 - \sum_i n_i$, or $xN/\theta = 1 - N$.

Proposition 2. *The efficient hiring rates are independent of producer size and the efficient vacancy creation is proportional to producer size: $\theta_i = \theta$, $v_i = x n_i$, $\forall i$.*

Going back to the firm problem, it turns out that if one substitutes out wages using equation (1), the firm problem becomes

$$\max_{\theta, x} -\frac{U}{\mu(\theta)} + (1 + q(\theta)x)(z - b) - \kappa(x) - U(\frac{x}{\theta} + 1), \tag{12}$$

where the firm takes U as given. This problem is nearly equivalent to the producer's contribution to the planner problem (8) above, with the value of unemployed workers standing in for the planner's Lagrange multiplier.[11] The only difference is that the firm problem involves one additional term $-\frac{U}{\mu(\theta)}$ involving θ, which means the firm's choice of market tightness will not be efficient. And consequently, other equilibrium allocations are expected to be distorted also.

With this, the firms first order conditions can alternatively be written as

$$\kappa_v(x) + \frac{U}{\theta} = q(\theta)(z - b), \tag{13}$$

$$\frac{U}{\theta^2}[1 + \frac{\mu'(\theta)\theta^2}{x\mu(\theta)^2}] = -q'(\theta)(z - b). \tag{14}$$

These equations are equivalent to the planner's, short of the difference in the condition for the optimal tightness (14), where the firm-wage policy leads to a distortion due to the influence of the existing workers. For the firm problem without the firm wage policy on the other hand, they coincide with the planner's optimality conditions.

---

[11]The planner's objective is equivalent to the objective $\sum_i n_i[q(\theta_i)x_i(z - b) - \kappa(x_i) - \lambda(\frac{x_i}{\theta_i} + 1)]$, where λ is the Lagrange multiplier.

Proposition 3. *The competitive search equilibrium with firm wages is inefficient.*

The next section extending the model to a dynamics setting will begin with a planner problem, and use the same relationship between the firm and planner problems to analyze the problem.

3   Dynamic Model

This section endogenizes the measure of existing workers by extending the model to a dynamic infinite horizon setting, at the same time revealing the time-inconsistency in the firm's wage-setting problem. For a benchmark, I begin with a planner problem. This efficient allocation corresponds to market outcomes when firms are not constrained to use firm wages, as shown at the end of the section. I consider the role of commitment with and without firm wages, in the context of equilibrium outcomes.

Consider a dynamic extension of the static environment of the previous section. Time is discrete and the horizon infinite. All agents are rational and discount the future at rate β. Employment relationships are long-term, and end with probability δ at the end of each period, with the worker returning to the pool of job seekers.

Planner Problem   To characterize efficient hiring, consider the planner problem:

$$\max_{\{\theta_{it}, v_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \Big[\sum_i [(n_{it} + q(\theta_{it})v_{it})z_t - \kappa(v_{it}, n_{it})] + [1 - \sum_i (n_{it} + q(\theta_{it})v_{it})]b\Big] \quad (15)$$

$$\text{s.t. } n_{it+1} = (1 - \delta)(n_{it} + q(\theta_{it})v_{it}), \ \forall t \geq 0$$

$$\sum_i v_{it}/\theta_{it} = 1 - \sum_i n_{it}, \ \forall t \geq 0$$

with initial employment n_{i0} given for all i. The planner maximizes the present discounted value of market and home output, net of the costs of vacancy creation, taking into account the law of motion for employment for each producer. In addition, the planners choices of θ_{it}, v_{it} must be consistent each period, in the sense that the measure of job seekers allocated to each producer v_{it}/θ_{it} must add up to the total measure of job seekers in period t.

The first order conditions for the planner's choice of vacancies and tightness v_{it}, θ_{it} are

$$\kappa_v(x_{it}) + \frac{\lambda_t}{\theta_{it}} = q(\theta_{it})E_t[z_t - b + \sum_{k=1}^{\infty} \beta^k(1 - \delta)^k(z_{t+k} - b - \kappa_n(x_{it+k}) - \lambda_{t+k})] \quad (16)$$

$$\frac{\lambda_t}{\theta_{it}^2} = -q'(\theta_{it})E_t[z_t - b + \sum_{k=1}^{\infty} \beta^k(1 - \delta)^k(z_{t+k} - b - \kappa_n(x_{it+k}) - \lambda_{t+k})] \quad (17)$$

where λ_t is the Lagrange multiplier reflecting the value of job seekers in period t. The first determines optimal vacancy creation such that the marginal cost of an additional vacancy together with the value of job seekers implied by the chosen tightness θ equal the resulting returns to the marginal increase in employment relationships: the value of having the workers working with the market technology rather than at home, together with the implied decrease in vacancy costs from the increase in employment, and net of the value of keeping those workers from applying for job. The second determines the optimal market tightness such that the marginal value of additional unemployed workers per chosen measure of vacancies equals the same.

Note that these are independent across producers, and in particular independent of firm size. As a consequence, in what follows I drop the firm subscripts, for convenience.

Proposition 4. *Efficient hiring rates are independent of producer size, and efficient vacancy creation is proportional to producer size: $\theta_{it} = \theta_t$, $v_{it} = x_t n_{it}$, $\forall i, t$.*

The optimality condition for vacancy creation (16) implies

$$\frac{\kappa_v(x_t) + \frac{\lambda_t}{\theta_t}}{q(\theta_t)} = z_t - b + \beta(1-\delta)E_t\Big[\frac{\kappa_v(x_{t+1}) + \frac{\lambda_{t+1}}{\theta_{t+1}}}{q(\theta_{t+1})} - \kappa_n(x_{t+1}) - \lambda_{t+1}\Big], \tag{18}$$

where the Lagrange multiplier can be expressed, combining the conditions (16) and (17), as $\lambda_t = \kappa_v(x_t)\frac{\mu(\theta_t) - \mu'(\theta_t)\theta_t}{\mu'(\theta_t)}$.

Taken together, this yields the Euler equation:

$$\frac{\kappa_v(x_t)}{\mu'(\theta_t)} = z_t - b + \beta(1-\delta)E_t\Big[(1 - \mu(\theta_{t+1}) + \mu'(\theta_{t+1})\theta_{t+1})\frac{\kappa_v(x_{t+1})}{\mu'(\theta_{t+1})} - \kappa_n(x_{t+1})\Big]. \tag{19}$$

The equation sets the marginal cost of a new employment relationship today, $\kappa_v(x_t)/\mu'(\theta_t)$, equal to the current period gains $z_t - b$ involved, together with the expected value of a relationship tomorrow, $\kappa_v(x_{t+1})/\mu'(\theta_{t+1})$. The expected value of the relationship takes into account the probability of a separation, and that an increase in hires today reduces hires per vacancy tomorrow, due to fewer job seekers.

In all, the planner's allocation is thus characterized by the Euler equation (19), together with the law of motion $N_{t+1} = (1-\delta)(1 + q(\theta_t)x_t)N_t$ and the constraint $x_t = \theta_t(1 - N_t)/N_t$. Any initial heterogeneity in firm sizes persists over time, but aggregate employment follows this system with identical growth across producers.

Firm Wages To think about a competitive search equilibrium with firm wages, begin by considering the behavior of unemployed workers. The unemployed are assumed to be aware

of the wages offered by all firms, as well as the corresponding market tightness. In the context of this infinite horizon model, this means workers are aware of the present discounted value of wages the firm will pay during the employment relationship, together with the corresponding market tightness.

With this, the utility value of workers that enter period t unemployed, and apply for a job with firm i, can be written as

$$U_t = \mu(\theta_{it})E_t\Big[\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(w_{it+k} + \beta\delta U_{t+1+k}) - b - \beta U_{t+1}\Big] + b + \beta E_t U_{t+1}. \qquad (20)$$

In the competitive search equilibrium the assumption is that for any present discounted value of wages a firm might consider offering, the flow of job applicants will adjust such that the market tightness leaves workers indifferent between applying with this firm and other firms. The firm takes as given the value of unemployment determined by the market $\{U_t\}_{t=0}^{\infty}$, but can consider alternative combinations of present value of wages and market tightness (W_{it}, θ_{it}) satisfying (20), where $W_{it} \equiv E_t \sum_{k=0}^{\infty} \beta^k(1-\delta)^k w_{it+k}$.

For notational convenience, I define auxiliary variables $X_t := U_t - b - \beta E_t U_{t+1}$ and $Y_t := E_t\beta\delta \sum_{k=0}^{\infty} \beta^k(1-\delta)^k U_{t+1+k} - b - \beta E_t U_{t+1}$. Note that these are also taken as given by the firms, as they derive from $\{U_t\}_{t=0}^{\infty}$. With these, equation (20) becomes

$$X_t = \mu(\theta_{it})(W_{it} + Y_t). \qquad (21)$$

Commitment   With this, the firm problem with commitment to future wages can be written

$$\max_{\{w_{it}, \theta_{it}, v_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t[(n_{it} + q(\theta_{it})v_{it})(z_t - w_{it}) - \kappa(v_{it}, n_{it})] \qquad (22)$$

$$\text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})v_{it}), \ \forall t \geq 0,$$

$$X_t = \mu(\theta_{it})E_t\Big(\sum_{k=0}^{\infty} \beta^k(1-\delta)^k w_{it+k} + Y_t\Big), \ \forall t \geq 0, \qquad (23)$$

with n_{i0} and $\{X_t, Y_t\}_{t=0}^{\infty}$ given. The firm maximizes the present discounted value of profits, revenue net of wage costs and vacancy costs, taking into account the law of motion for the firm's workers and the constraint reflecting optimal worker behavior each period.

Proposition 5. *Problem* (22) *is equivalent to problem* (24) *if the firm participates in the labor market each period.*

To make progress on solving this problem, I proceed to substitute out the wages that feed back into each period's market tightness θ_t through the present values in the worker constraint (23) and simultaneously appear in the firm profits, with the wages of different cohorts of workers linked via the firm wage policy. Appendix A shows that one can do so using equation (23), arriving at the firm problem:

$$\max_{\{\theta_{it},v_{it}\}_{t=0}^{\infty}} -\frac{X_0 n_{i0}}{\mu(\theta_{i0})} + E_0 \sum_{t=0}^{\infty} \beta^t[(n_{it} + q(\theta_{it})v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - X_t(\frac{v_{it}}{\theta_{it}} + n_{it})] \quad (24)$$

$$\text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})v_{it}), \ \forall t \geq 0,$$

with n_{i0} and $\{X_t\}_{t=0}^{\infty}$ given. Note that in this problem the firm simply chooses sequences of market tightness and vacancies, reducing the dimensionality of the space of choice variables significantly, while making sure the firm-wage policy is guaranteed to hold in the underlying problem with wages.

The first order condition for vacancy creation reads, for $t \geq 0$,

$$\kappa_v(x_{it}) + \frac{X_t}{\theta_{it}} = q(\theta_{it})E_t[z_t - b + \sum_{k=1}^{\infty} \beta^k(1-\delta)^k(z_{t+k} - b - \kappa_n(x_{it+k}) - X_{t+k})], \quad (25)$$

while the condition for the market tightness reads, for $t > 0$

$$\frac{X_t}{\theta_{it}^2} = -q'(\theta_{it})E_t[z_t - b + \sum_{k=1}^{\infty} \beta^k(1-\delta)^k(z_{t+k} - b - \kappa_n(x_{it+k}) - X_{t+k})]. \quad (26)$$

and for the first period, $t = 0$,

$$\frac{X_0}{\theta_{i0}^2}[1 + \frac{\mu'(\theta_{i0})\theta_{i0}^2}{x_{i0}\mu(\theta_{i0})^2}] = -q'(\theta_{i0})E_0[z_0 - b + \sum_{k=1}^{\infty} \beta^k(1-\delta)^k(z_k - b - \kappa_n(x_{ik}) - X_k)]. \quad (27)$$

Note that these optimality conditions are identical across firms and in particular independent of firm size n_{it} for any period. In what follows I will thus drop the firm indexes, considering symmetric equilibria. The firm's posted wages are also independent of size in this context, because condition (23) implies that identical tightnesses across firms imply identical present values of wages across firms, which means the current period wage is also.

Proposition 6. *Firms' posted wages and hiring rates are independent of firm size, and vacancy creation is proportional to firm size:* $\theta_{it} = \theta_t$, $v_{it} = x_t n_{it}$, $\forall i, t$.

The condition for vacancy creation (25) implies

$$\frac{\kappa_v(x_t) + \frac{X_t}{\theta_t}}{q(\theta_t)} = z_t - b + \beta(1-\delta)E_t[\frac{\kappa_v(x_{t+1}) + \frac{X_{t+1}}{\theta_{t+1}}}{q(\theta_{t+1})} - \kappa_n(x_{t+1}) - X_{t+1}], \quad (28)$$

where the value of job seekers satisfies

$$X_t = \kappa_v(x_t) \frac{\mu(\theta_t) - \mu'(\theta_t)\theta_t}{\mu'(\theta_t)} > 0 \text{ for } t > 0, \tag{29}$$

$$X_0 = \kappa_v(x_0) \frac{\mu(\theta_0) - \mu'(\theta_0)\theta_0}{\mu'(\theta_0)} \frac{q(\theta_0)x_0}{1 + q(\theta_0)x_0} > 0 \text{ for } t = 0. \tag{30}$$

Note that the expressions suggest this value X_t is lower in the initial period than later periods, for the same θ, x.

Combining, these yield the inter-temporal Euler equation

$$\frac{\kappa_v(x_t)}{\mu'(\theta_t)} = z_t - b + \beta(1 - \delta)E_t[(1 - \mu(\theta_{t+1}) + \mu'(\theta_{t+1})\theta_{t+1})\frac{\kappa_v(x_{t+1})}{\mu'(\theta_{t+1})} - \kappa_n(x_{t+1})], \tag{31}$$

for $t > 0$ and, for the initial period,

$$\frac{\kappa_v(x_0)}{\mu'_0}[1 - \frac{(1 - \mu'_0\theta_0/\mu_0)}{1 + q_0 x_0}] = z_0 - b + \beta(1 - \delta)E_0\Big[\big(1 - \mu_1 + \mu'_1\theta_1\big)\frac{\kappa_v(x_1)}{\mu'_1} - \kappa_n(x_1)\Big]. \tag{32}$$

These optimality conditions coincide with the planner's after the initial period, but in the initial period they differ. This is not surprising noting the close relation between the firm problem and the planner problem, with the value of job seekers X_t standing in for the Lagrange multiplier . The producer's contribution to the planner problem (15)[12] coincides with the firm objective in (24) with X_t standing in for the Lagrange multiplier, except for the additional term in the firm problem in period zero, $-X_0 n_{i0}/\mu(\theta_{i0})$. This additional term includes θ_0 and thus distorts the initial period choice of θ_0, something that feeds into the equilibrium allocations in the initial period. After the initial period the allocation follow the same dynamics, however.

Here the expression for X_0 clearly differs from the planner's shadow value λ_0, and the left hand side of the Euler equation reflects that difference. The expression for X_0 includes a multiplier on the right which is less than one, suggesting that the equilibrium valuation of searching workers is lower in the market outcome than what the planner problem. This would mean that the firm chooses a wage which is lower than what the planner would choose, saving on labor costs on its existing workers, but also attracting fewer applicants per vacancy. Because the measure of total applicants at time $t = 0$ is given, the latter also means that firms will be creating more vacancies.

The firm's behavior lines up with the planner's after the initial period, in that the firm chooses an efficient balance of using higher wages versus more vacancy creation to attract

---

[12] $E_0 \sum_{t=0}^{\infty} \beta^t[(n_{it} + q(\theta_{it})v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - \lambda_t(\frac{v_{it}}{\theta_{it}} + n_{it})]$

workers. The difference in the initial period is that there the firm takes the existing stock of workers as fixed, and thus has an incentive to lower wages to economize on the wage bill of those existing workers.

Definition 2. *A competitive search equilibrium with firm wages is an allocation $\{w_t, \theta_t, x_t\}_{t=0}^{\infty}$ and job seeker value $\{X_t\}_{t=0}^{\infty}$ such that the allocation and value solve the problem (24), and that each searching worker applies to one firm: $1 - \sum_i n_{it} = \sum_i x_t n_{it}/\theta_t, \forall t$.*

Relationship between market outcomes and efficient allocations:

Proposition 7. *The competitive search equilibrium with firm wages is inefficient.*

Limited Commitment What if the firm reoptimizes each period? To think about this case, I write the firm problem recursively, taking into account the fact that the firm objective has an additional term corresponding to the period when the firm reoptimizes, relative to the continuation value. I consider Markov-perfect equilibria where the current aggregate state is $S := (N, z)$, and the firm takes as given the equilibrium aggregate $X(S)$.

The recursive firm problem can be written:

$$\max_{\theta, v} -\frac{X(S)n}{\mu(\theta)} + (n + q(\theta)v)(z - b) - \kappa(v, n) - X(S)(\frac{v}{\theta} + n) + \beta E_S V(n'; S') \qquad (33)$$
$$\text{s.t. } n' = (1 - \delta)(n + q(\theta)v),$$

where

$$V(n; S) = (n + q(\theta)v)(z - b) - \kappa(v, n) - X(S)(\frac{v}{\theta} + n) + \beta E_S V(n'; S') \qquad (34)$$

and the firm takes as given the aggregate $X(S)$. Note that this firm problem is the no-commitment version of the sequence problem (22), with the distortion term appearing in the period of reoptimization. The distortion term only includes the choice variable θ and not v, so the only tradeoff directly affected by it is the tradeoff between wages and market tightness.

Scaling by size, and defining $\hat{V}(S) := V(n; S)/n$, the firm problem becomes size independent:

$$\max_{\theta, x} -\frac{X(S)}{\mu(\theta)} + (1 + q(\theta)x)(z - b + \beta(1 - \delta)E_S \hat{V}(S')) - \kappa(x) - X(S)(\frac{x}{\theta} + 1) \qquad (35)$$

where

$$\hat{V}(S) = (1 + q(\theta)x)(z - b + \beta(1 - \delta)E_S \hat{V}(S')) - \kappa(x) - X(S)(\frac{x}{\theta} + 1). \qquad (36)$$

Because the firm problem (35) is size independent, so should the firm decisions be as well. The first order conditions characterising firm behavior read

$$\kappa_v(x) + \frac{X(S)}{\theta} = q(\theta)(z - b + \beta(1-\delta)E_S\hat{V}(S')), \tag{37}$$

$$\frac{X(S)}{\theta^2}[1 + \frac{\mu'(\theta)\theta^2}{x\mu(\theta)^2}] = -q'(\theta)(z - b + \beta(1-\delta)E_S\hat{V}(S')). \tag{38}$$

To arrive at an intertemporal Euler equation, note that one can combine equations (36) and (37) to arrive at the same equation (28) as in the commitment problem.[13]

Meanwhile, from (37) and (38), we have that the value of job seekers in this case satisfies

$$X_t = \kappa_v(x_t)\frac{\mu(\theta_t) - \mu'(\theta_t)\theta_t}{\mu'(\theta_t)}\frac{q(\theta_t)x_t}{1 + q(\theta_t)x_t}. \tag{39}$$

With this, the intertemporal Euler equation becomes (combining (28) and (39)):

$$\frac{\kappa_v(x_t)}{\mu'_t}[1 - \frac{(1 - \mu'_t\theta_t/\mu_t)}{1 + q_tx_t}] \tag{40}$$

$$= z_t - b + \beta(1-\delta)E_t\Big[\frac{\kappa_v(x_{t+1})}{\mu'_{t+1}}\big(1 - \mu_{t+1} + \mu'_{t+1}\theta_{t+1} - (1 - \mu_{t+1})\frac{(1 - \mu'_{t+1}\theta_{t+1}/\mu_{t+1})}{1 + q_{t+1}x_{t+1}}\big) - \kappa_n(x_{t+1})\Big].$$

Note that despite the time-inconsistency, this Euler equation takes the form of a standard Euler equation, instead of the generalized Euler equations that typically appear in problems with time-inconsistencies. Such generalized Euler equations generally involve a derivative of the choice variable with respect to a state, something that makes the Euler equation a more complicated object to solve than standard Euler equations. The scale-independence of the problem plays a key role in explaining this difference, because the derivative would appear if the firm's decisions depended on the size of the firm explicitly.[14] Allowing for such dependence would make for a richer problem to analyze, but its absence also serves to make the problem significantly more tractable, and allows using standard solution methods.

In a (symmetric) equilibrium, it must also hold that $x_t = \theta_t(1 - N_t)/N_t$. The equilibrium is thus characterized by the Euler equation (40), this equilibrium condition, and the law of motion $N_{t+1} = (1 - \delta)(N_t + \mu(\theta_t)(1 - N_t))$. This makes for a simple system of equations, where it is easy to verify uniqueness of steady-state and the saddle-point stability of the system.

---

[13]Note that $\kappa(x) = x\kappa_v(x) - \kappa_n(x)$.

[14]For example due to decreasing returns in technology.

Definition 3. *A competitive search equilibrium with firm wages is an allocation $\{w_t, \theta_t, x_t\}_{t=0}^{\infty}$ and job seeker value $\{X_t\}_{t=0}^{\infty}$ such that the allocation and value solve the problem (35), and that each job seeker applies to one firm: $1 - \sum_i n_{it} = \sum_i x_t n_{it}/\theta_t$, $\forall t$.*

Note that each firm in this equilibrium would, by construction, find it profitable to commit to future wages over reoptimizing each period. The commitment wage would generally involve a fixed – higher – level of wages, after the initial period distortion, than the no-commitment wage. But in the absence of the ability to commit to arbitrary wage paths, the firm might also find it profitable to adopt a simple wage rule of fixing wages and only resetting them periodically. This is an experiment I consider in the next section. Note, however, that even though individual firms might prefer this strategy, if all firms implement it, the level of wages in the labor market will rise and the outcome may be favorable for workers as well. I return to this in Section 4.

Without Firm Wages   Finally, it is useful to contrast outcomes with the standard competitive search benchmark, where firms offer independent contracts to workers hired at different points in time. In this case one can write the problem of a firm hiring in period t as follows

$$\max_{\{w_{t+k}^t\}_{k=0}^{\infty}, \theta_t, v_t} E_t[q(\theta_t)v_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (z_{t+k} - w_{t+k}^t) - \sum_{k=0}^{\infty} \beta^k \kappa(v_{t+k}, n_{t+k})] \tag{41}$$

$$\text{s.t. } X_t = \mu(\theta_{it}) E_t (\sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k}^t + Y_t), \tag{42}$$

where n_{t+k} follows the law of motion for all k with n_t given and optimal hiring for $t+k$, $k > 0$. The first term in the objective represents the present value of output net of wages associated with workers hired in period t, given vacancies v_t and hiring rate $q(\theta_t)$, taking into account turnover. The second term represents the vacancy costs affected by this hiring: the costs of posting vacancies in period t together with the reduced vacancy costs associated with having more workers in future.

Returning to consider the role of commitment in this case, note that wages enter the problem only through their present value $W_t = \sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k}$. If the firm does not have commitment to future wages, after a match has been created the firm would, in each period, set this present value as low as possible while sustaining the match.[15] The firm makes the worker indifferent between remaining in the match and quitting into unemployment by setting $W_{t+k} + Y_{t+k} = 0$, or $w_{t+k} = b + \beta(1-\delta)X_{t+k+1}$, for all $k > 0$.

---

[15]Note that if the worker walks away the firm value from the match is zero.

Maintaining the assumption that the firm is able to commit to the posting-period payment to the worker, and assuming the firm can choose this payment in an unconstrained way, it will nevertheless be able to credibly offer job seekers any present value W_t despite the constraints later on in the match. To determine the present value the firm chooses to offer, one can again substitute out wages in the profit expression using the constraint (42). With that, the firm problem in (41) becomes

$$\max_{\theta_t, v_t} E_t[q(\theta_t)v_t[z_t - b + \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (z_{t+k} - b - X_{t+k})] - \frac{X_t v_t}{\theta_t} - \sum_{k=0}^{\infty} \beta^k \kappa(v_{t+k}, n_{t+k})] \quad (43)$$

where n_{t+k} follows the law of motion for all k with n_t given and optimal hiring for $t + k$, $k > 0$. The optimality conditions read:

$$\kappa_v(x_t) + \frac{X_t}{\theta_t} = q(\theta_t)E_t[z_t - b + \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (z_{t+k} - b - \kappa_n(x_{t+k}) - X_{t+k})] \quad (44)$$

$$\frac{X_t}{\theta_t^2} = -q'(\theta_t)E_t[z_t - b + \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (z_{t+k} - b - \kappa_n(x_{t+k}) - X_{t+k})] \quad (45)$$

These optimality conditions coincide with the planner's, with X_t representing the shadow value of job seekers. The firm thus attains the efficient allocation in this case, despite having limited commitment to future wages. The wage contract achieving the planner allocation involves a signing bonus in the first period of employment, designed to provides the appropriate present value to the worker to attract the efficient measure of job seekers per vacancy. In subsequent periods, on the other hand, the worker is paid a wage just enough to prevent him/her from quitting into unemployment.

If the firm does have commitment to future wages, on the other hand, it is left indifferent across a variety of wage contracts, each of which offer the same present value which achieves the efficient allocation.

4 Infrequent Wage Adjustment

The commitment problem discussed in previous sections suggests that firms would prefer to fix wages for a period of time rather than reoptimizing each period. To allow for longer horizons of wage commitment, this section extends the firm problem to a setting where firms set wages for a probabilistic period of time. I consider two experiments. In the first, a single firm sets its wage for a probabilistic period in an equilibrium where other firms reoptimize wages each period. I consider the impact of the duration of wages on firm profitability in

particular. In the second, I consider equilibrium outcomes when all firms adjust infrequently and how equilibrium outcomes change as the duration of wages changes.

In what follows, I begin with a setting where firms face purely idiosyncratic shocks to their productivity, reflecting the empirical prevalence of firm level risk, before returning to the setting with aggregate shocks.

Equilibrium with Firm Heterogeneity Consider an environment where firms face idiosyncratic shocks to their productivity. In a stationary equilibrium with firm heterogeneity, the aggregate measure of matches N and value of job seekers X remain constant, while firm shocks lead to reallocation of labor across firms over time.

The firm problem in this context reads:

$$\max_{\theta,v} -\frac{Xn}{\mu(\theta)} + (n + q(\theta)v)(z - b) - \kappa(v, n) - X(\frac{v}{\theta} + n) + \beta E_z V(n', z') \tag{46}$$
$$\text{s.t. } n' = (1 - \delta)(n + q(\theta)v),$$

where the continuation value satisfies

$$V(n, z) = (n + q(\theta)v)(z - b) - \kappa(v, n) - X(\frac{v}{\theta} + n) + \beta E_z V(n', z'). \tag{47}$$

Scaling by size, these equations again yield the size-independent problem:

$$\max_{\theta,x} -\frac{X}{\mu(\theta)} + (1 + q(\theta)x)(z - b + \beta(1 - \delta)E_z \hat{V}(z')) - \kappa(x) - X(\frac{x}{\theta} + 1) \tag{48}$$

where

$$\hat{V}(z) = (1 + q(\theta)x)(z - b + \beta(1 - \delta)E_z \hat{V}(z')) - \kappa(x) - X(\frac{x}{\theta} + 1). \tag{49}$$

The implied intertemporal Euler equation characterising firm behavior remains unchanged from equation (40), while the intratemporal optimality condition (39) is simplified by the value of job seekers remaining constant over time.

Definition 4. *A stationary competitive search equilibrium with firm wages is an allocation $\{w_{it}, \theta_{it}, x_{it}\}_{t=0}^{\infty}$ $\forall i$ and job seeker value X such that the allocation and value solve the problem (48-49), and that each job seeker applies to one firm: $1 - \sum_i n_{it} = \sum_i x_{it} n_{it}/\theta_{it}$, $\forall t$.*

The stationary equilibrium is characterized by heterogeneity across firms in size and productivity, with individual firms growing and shrinking over time in response to changes in their productivity. Note that the size-independence of the firm problem implies that

wages and corresponding firm growth rates are independent of size, but firms differing in productivity will generally offer different wages and have different growth rates, which leads to differences in size in equilibrium. Thus, in the cross section of firms, one would expect wages and size to be correlated.

Single Firm Deviation to Longer Wage Commitment Consider introducing into the above equilibrium an individual firm, small relative to the size of the market, that today makes a wage commitment for a probabilistic period of time, returning to equilibrium behavior once the commitment expires. Is the commitment profitable for the firm?

The deviating firm chooses a wage w, expecting each period going forward to revert to equilibrium behavior with probability α and to maintain the wage with probability $1 - \alpha$. To connect the per-period wage to the market tightness, note that the equilibrium firms' market tightnesses imply these firms offer their workers specific present values of wages for each z, due to the job seeker constraint. Taking these equilibrium values as given, one can solve for the present value of wages for the deviating firm as a function of the wage w and productivity z, denoted below as $\phi(w, z)$.[16] Finally, the job seeker constraint gives the implied tightness: $X = \mu(\theta)(\phi(w, z) + Y)$.

In the period the firm deviates to the longer wage commitment, it chooses a wage w and vacancy creation v, to solve the problem

$$\max_{w,v} -\frac{Xn}{\mu(\theta)} + (n + q(\theta)v)(z - b) - \kappa(v, n) - X(\frac{v}{\theta} + n) + \beta E_z(\alpha V(n', z') + (1 - \alpha)V^f(n', w, z'))$$

$$\text{s.t. } n' = (1 - \delta)(n + q(\theta)v),$$

$$X = \mu(\theta)(\phi(w, z) + Y),$$

given n, z. Here the firm expects to revert to equilibrium behavior in the following period with probability α, implying the continuation value $V(n', z')$, and to maintain the wage commitment otherwise, implying the continuation value $V^f(n', w, z')$, discussed below.

In periods when the firm maintains the wage commitment, it only chooses vacancies, to solve the problem:

$$\max_{v} -\frac{Xn}{\mu(\theta)} + (n + q(\theta)v)(z - b) - \kappa(v, n) - X(\frac{v}{\theta} + n) + \beta E_z(\alpha V(n', z') + (1 - \alpha)V^f(n', w, z'))$$

$$\text{s.t. } n' = (1 - \delta)(n + q(\theta)v),$$

---

[16]Denote the vector of equilibrium present values of wages across z as $\mathbf{W}$ and that of the deviating firm as $\mathbf{W}^f(w)$. We have that $\mathbf{W}^f(w) = w\mathbf{i} + \beta(1 - \delta)[\alpha \Pi \mathbf{W} + (1 - \alpha)\Pi \mathbf{W}^f(w)]$, where Π is the transition matrix for the productivity process and $\mathbf{i}$ a vector of ones. This gives the deviating firm's present values as $\mathbf{W}^f(w) = (I - \beta(1 - \delta)(1 - \alpha)\Pi)^{-1}(w\mathbf{i} + \beta(1 - \delta)\alpha \Pi \mathbf{W})$. I denote the components of this vector in the text by $\phi(w, z)$. Note that the derivative of the value satisfies $\phi_w(w, z) = (1 - \beta(1 - \delta)(1 - \alpha))^{-1}$.

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w, z) + Y)$. The continuation value $V^f(n', w, z')$ satisfies

$$V^f(n, w, z) = (n + q(\theta)v)(z - b) - \kappa(v, n) - X(\frac{v}{\theta} + n) + \beta E_z(\alpha V(n', z') + (1 - \alpha)V^f(n', w, z')).$$

The deviating firm's problems can also be scaled to arrive at size-independent problems. Defining $\hat{V}^f(w, z) := V^f(n, w, z)/n$, the deviating firm chooses w, x to solve

$$\max_{w,x} -\frac{X}{\mu(\theta)} + (1 + q(\theta)x)(z - b + \beta(1 - \delta)E_z(\alpha\hat{V}(z') + (1 - \alpha)\hat{V}^f(w, z'))) - \kappa(x) - X(\frac{x}{\theta} + 1)$$

s.t. $X = \mu(\theta)(\phi(w, z) + Y)$.

In periods when the firm maintains the commitment to w, it chooses x to solve

$$\max_{x} -\frac{X}{\mu(\theta)} + (1 + q(\theta)x)(z - b + \beta(1 - \delta)E_z(\alpha\hat{V}(z') + (1 - \alpha)\hat{V}^f(w, z'))) - \kappa(x) - X(\frac{x}{\theta} + 1),$$

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w, z) + Y)$. The continuation value satisfies

$$\hat{V}^f(w, z) = (1 + q(\theta)x)(z - b + \beta(1 - \delta)E_z(\alpha\hat{V}(z') + (1 - \alpha)\hat{V}^f(w, z'))) - \kappa(x) - X(\frac{x}{\theta} + 1).$$

The two problems yield the same first order condition for vacancy creation

$$\kappa_v(x) + \frac{X}{\theta} = q(\theta)(z - b + \beta(1 - \delta)E_z(\alpha\hat{V}(z') + (1 - \alpha)\hat{V}^f(w, z'))), \tag{50}$$

for the deviation period and periods when the commitment is maintained. Meanwhile, the deviating firm's first order condition characterizing the wage-tightness tradeoff reads

$$\frac{X}{\theta^2}[1 + \frac{\mu'(\theta)\theta^2}{x\mu(\theta)^2}] = -q'(\theta)[z - b + \beta(1 - \delta)E_z[\alpha\hat{V}(z') + (1 - \alpha)\hat{V}^f(w, z')]] \tag{51}$$
$$- \beta(1 - \delta)(1 - \alpha)(1 + q(\theta)x)/x E_z\hat{V}_w^f(w, z')/\theta_w,$$

where the derivative of θ with respect to w is $\theta_w = -\mu(\theta)^2/(\mu'(\theta)X(1 - \beta(1 - \delta)(1 - \alpha)))$, while the derivative of the continuation value satisfies

$$\hat{V}_w^f(w, z) = xq'(\theta)[z - b + \beta(1 - \delta)E_z[\alpha\hat{V}(z') + (1 - \alpha)\hat{V}^f(w, z')]]\theta_w$$
$$+ \frac{xX}{\theta^2}\theta_w + \beta(1 - \delta)(1 - \alpha)(1 + q(\theta)x)E_z\hat{V}_w^f(w, z').$$

Note that while the deviating firm holds its per-period wage fixed, the corresponding tightness is expected to vary (as long as shocks are not iid). The tightness is determined by the present value of wages, and if firm productivity changes toward a greater expected present value of wages from reoptimizing tomorrow, this raises the present value of wages today, bringing with it a lower θ and greater probability of filling vacancies $q(\theta)$. Of course the firm also generally adjusts vacancy creation to changes in productivity as well.

Equilibrium with Infrequent Adjustment   If a longer wage commitment is profitable for the deviating firm, it becomes interesting to consider an equilibrium where all firms follow a strategy of infrequent adjustment. How do wage commitments affect equilibrium outcomes? Do firms continue to benefit from the commitment, and do they do so at the expense of workers?

To think about these questions, suppose all firms reoptimize their wage w each period with probability α and maintain their existing wage commitment with probability $1 - \alpha$. To connect the per-period wage to the corresponding market tightness, one can again solve for the present value of wages as a function of the wage w and productivity z, denoted $\phi(w, z)$.[17]

In this case, firms reoptimizing wages solve

$$\max_{w,v} - \frac{Xn}{\mu(\theta)} + (n + q(\theta)v)(z - b) - \kappa(v, n) - X(\frac{v}{\theta} + n) + \beta E_z(\alpha V^r(n', z') + (1 - \alpha)V^f(n', w, z'))$$

$$\text{s.t. } n' = (1 - \delta)(n + q(\theta)v),$$

$$X = \mu(\theta)(\phi(w, z) + Y),$$

where the implied continuation value satisfies

$$V^r(n, z) = (n + q(\theta)v)(z - b) - \kappa(v, n) - X(\frac{v}{\theta} + n) + \beta E_z(\alpha V^r(n', z') + (1 - \alpha)V^f(n', w, z')),$$

and firms holding the wage commitment fixed solve

$$\max_{v} - \frac{Xn}{\mu(\theta)} + (n + q(\theta)v)(z - b) - \kappa(v, n) - X(\frac{v}{\theta} + n) + \beta E_z(\alpha V^r(n', z') + (1 - \alpha)V^f(n', w, z'))$$

$$\text{s.t. } n' = (1 - \delta)(n + q(\theta)v),$$

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w, z) + Y)$, the continuation value $V^f(n', w, z')$ satisfies

$$V^f(n, w, z) = (n + q(\theta)v)(z - b) - \kappa(v, n) - X(\frac{v}{\theta} + n) + \beta E_z(\alpha V^r(n', z') + (1 - \alpha)V^f(n', w, z')).$$

Once again, the problems can be scaled. Thus, firms reoptimizing wages solve

$$\max_{w,x} - \frac{X}{\mu(\theta)} + (1 + q(\theta)x)(z - b + \beta E_z(\alpha \hat{V}^r(z') + (1 - \alpha)\hat{V}^f(w, z'))) - \kappa(x) - X(\frac{x}{\theta} + 1)$$

---

[17]Denote the vector of equilibrium present values of wages for a reoptimizing firm across z as $\mathbf{W}^r$ and that of a firm maintaining wage commitment w as $\mathbf{W}^f(w)$. We have that $\mathbf{W}^f(w) = w\mathbf{i} + \beta(1 - \delta)[\alpha \Pi \mathbf{W}^r + (1 - \alpha)\Pi \mathbf{W}^f(w)]$, where Π is the transition matrix for the productivity process and $\mathbf{i}$ a vector of ones. This gives the deviating firm's present values as $\mathbf{W}^f(w) = (I - \beta(1 - \delta)(1 - \alpha)\Pi)^{-1}(w\mathbf{i} + \beta(1 - \delta)\alpha \Pi \mathbf{W}^r)$. I denote the components of this vector in the text by $\phi(w, z)$.

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w,z)+Y)$, and the implied continuation value satisfies

$$\hat{V}^r(z) = (1+q(\theta)x)(z-b+\beta E_z(\alpha\hat{V}^r(z')+(1-\alpha)\hat{V}^f(w,z'))) - \kappa(x) - X(\frac{x}{\theta}+1),$$

and firms holding the wage commitment fixed solve

$$\max_x -\frac{X}{\mu(\theta)} + (1+q(\theta)x)(z-b+\beta E_z(\alpha\hat{V}^r(z')+(1-\alpha)\hat{V}^f(w,z'))) - \kappa(x) - X(\frac{x}{\theta}+1)$$

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w,z)+Y)$, the continuation value $V^f(w,z')$ satisfies

$$\hat{V}^f(w,z) = (1+q(\theta)x)(z-b+\beta E_z(\alpha\hat{V}(z')+(1-\alpha)\hat{V}^f(w,z'))) - \kappa(x) - X(\frac{x}{\theta}+1).$$

The first order conditions for the firms' choice of wage and vacancy creation rate coincide with those for the deviating firm (50-51), with the continuation values $\hat{V}^r(z)$ and $\hat{V}^f(w,z)$ as characterized above. Each state z is associated with a corresponding reoptimization wage w, tightness and vacancy rate. Once set, this wage remains fixed while the commitment is maintained, despite changes in productivity. If productivity does vary during this time, the tightness and firm's probability of filling vacancies does vary as well, however, as does the firm's vacancy creation rate itself. Once the commitment finally expires, the firm adopts the equilibrium wage consistent with the then prevailing state, with corresponding tightness and vacancy creation rate.

Aggregate Shocks The above experiments can be implemented also in the preceding setting where firms face aggregate shocks. Firms are ex post heterogeneous also in this version of the model, because wages adjust to shocks in a staggered manner, leading to heterogeneous growth rates across firms. The equations characterizing this alternative specification of infrequent adjustment are spelled out in Appendix D.

5 Quantitative Illustration

This section uses the model developed in the previous sections to study the implications of firm wages for labor market outcomes. I consider in particular: i) how firm wages affect wage setting and hiring over the business cycle, and ii) the profitability and equilibrium impact of infrequent wage adjustment in this context.

5.1 Parameterizing and Solving the Model

Parametrization I begin with a benchmark parametrization for the standard competitive search model, before proceeding to a comparable one for the firm wage model.[18] I adopt a monthly frequency, and set the discount rate to $\beta = 1.05^{-1/12}$. To be consistent with an average duration of employment of 2.5 years, I set the separation rate to $\delta = 0.033$. To be consistent with an average unemployment rate of 5 percent, when the steady-state unemployment rate in the model is $\mu(\theta)(1-\delta)/(\mu(\theta) + \delta - \mu(\theta)\delta)$, requires a steady-state job-finding rate of $\mu(\theta) = 0.388$. I adopt the matching function $m(v,u) = vu/(v^\ell + u^\ell)^{1/\ell}$ for this discrete time model, as in den Haan, Ramey, and Watson (2000), and target a steady-state level of θ of 0.43 as in Kaas and Kircher (2015). To fit the above job finding probability then requires $\ell = 1.85$.

Labor productivity is normalized to $z = 1$ and for the vacancy cost I follow Kaas and Kircher (2015) in setting $\kappa(v,n) = \frac{\kappa_0}{1+\gamma}(v/n)^\gamma v$ with $\gamma = 2$.[19] For this benchmark, I further follow Shimer (2005) in adopting the value $b = 0.4$ and setting κ_0 to ensure the corresponding Euler equation holds in steady state. The implied value of κ_0 implies an average cost of an additional vacancy of 1.8.

To arrive at a comparable parametrization of the firm wage model, I seek to maintain the basic labor market transition rates described above unchanged. To this end I hold the values of δ, ℓ unchanged, which ensures that $\theta, \mu(\theta)$ as well as the vacancy rate x and unemployment rate remain unchanged across models.

The firm wage model tends to have lower wages than the standard model, so to set the remaining parameters in a way that holds steady-state levels fixed I adjust the values of b, κ_0. From the steady-state equations, it turns out that for the two models to yield identical levels for wages and firm profit rate, in addition to the transition rates above, one must hold the value of κ_0 fixed between models (see Appendix C). I thus hold κ_0 fixed, and allow b to adjust so as to guarantee the firm wage model's Euler equation holds. Doing so involves raising the value of b relative to the standard model, to bring wages in the firm wage model to their levels in the standard model, to $b = 0.89$. Wages are higher in the model with infrequent wage adjustment, leading to $b = 0.76$.

---

[18]Details can be found in Appendix C.

[19]Note that $\kappa(v,n)/n = \frac{\kappa_0}{1+\gamma}(v/n)^{1+\gamma}$ and $\kappa_v(v,n) = \kappa_0(v/n)^\gamma$, $\kappa_n(v,n) = -\frac{\kappa_0\gamma}{1+\gamma}(v/n)^{1+\gamma}$ are all functions of $x = v/n$ only.

Solution Approach   Despite the time-inconsistency, the model is relatively straightfor-ward to solve. For the baseline model with aggregate shocks described in Section 3, one can use Dynare to produce solutions to the Euler equation (40), corresponding intratemporal optimality condition (39) and equilibrium condition $x_t = \theta_t(1 - N_t)/N_t$.

For the model with firm heterogeneity, I use both Dynare as well as solving the non-linear dynamic firm problem on a finite grid for productivity directly. The latter uses the feature that the firm problem has effectively no endogenous state variable, depending only on current productivity. For the model without probabilistic wages, solving the model this way involves solving a nonlinear system of equations in the equilibrium firm choices of $\{\theta_z, x_z\}$ for each productivity realization $z \in Z$. For the model with probabilistic wages, the set of unknowns is larger because a wage commitment chosen in state z can prevail also in other states z' before wages are reoptimized. Finally, the model must be simulated to find the job seeker value X consistent with the equilibrium condition aggregating across firms.[20]

Finally, in solving the model I also check second order conditions for the firm problem, as well as that the interior solution prescribed by the first order conditions yields greater value to the firm than the corner solution of creating zero vacancies and paying existing workers the value of unemployment.

Next, I turn to describing the results.

5.2   Firm Wages over the Business Cycle

As discussed, firm wages pose an added constraint on firms setting wages over the business cycle, by requiring any raises or cuts in wages aimed at attracting more or less new workers to apply also to the firm's existing workers. In an expansion firms would like to raise wages to attract more workers, in addition to creating more vacancies. Intuition suggests that when this wage increase must carry over to the firm's existing workers as well, it becomes more costly for firms to implement, leading them to curb the wage increase in favor of a larger increase in vacancies instead. Thus, one would expect firm wages to reduce cyclical variation in wages, and amplify those of labor market flows. Is this what the firm wage model implies?

A side-by-side comparison of the standard competitive search model and the firm wage model, parameterized to maintain the levels of unemployment, wages and profits the same as described, indicates that the firm wage model features more rigid wages over the business cycle. To illustrate, Figure 1 plots impulse responses to a one percent increase in labor

---

[20]The model with infrequent adjustment with aggregate shocks is solved by linearizing and aggregating up across firms, as in Gertler and Trigari (2009) (see Appendix D).
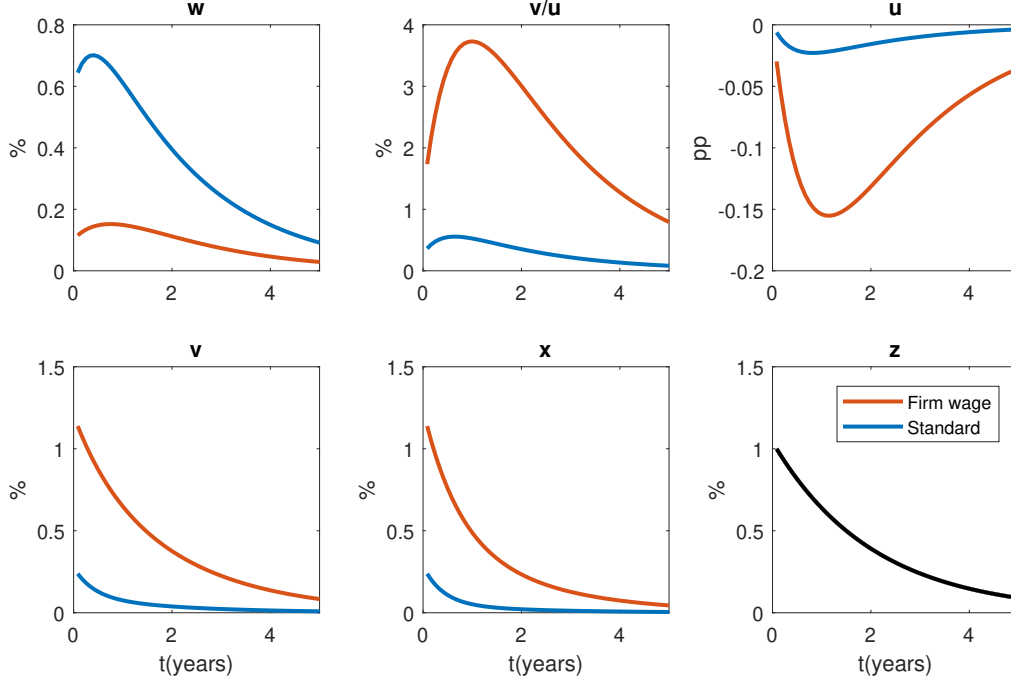
Figure 1: Impulse Responses in Firm Wage vs Standard Model

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and the standard competitive search model without firm wages. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.96$ and standard deviation $\sigma_z = 0.02$. The two models compared have the same steady-state levels of wage, tightness, unemployment, as described in the section on parametrization. The plotted vacancy-unemployment ratio is its model counterpart, which differs slightly from θ due to timing.

productivity in the two models. As the figure shows, while the wage increase in the standard model is almost identical to the increase in productivity, the corresponding increase in the firm wage model is only about a quarter of that. This means that while in the standard model the increase in wages absorbs a large share of the increase in productivity, leaving limited room for the profitability of hiring to increase, in the firm wage model the profitability of hiring rises more. This leads to an increase in the vacancy-unemployment ratio that is nearly nine times greater, coinciding with equally significant amplification in the increase in vacancies and reduction in unemployment. This degree of amplification is also non-trivial in magnitude: As discussed in Shimer (2005), the standard model would require a ten fold increase in the volatility of the tightness to be consistent with measured volatility in the same.[21,22]

---

[21]The model in Shimer (2005) produces more variability that the standard model considered here, however, because it has a linear vacancy cost, while the convex vacancy cost works to dampen fluctuations here.

[22]A higher value of b in itself generally amplifies responses to shocks in the Mortensen-Pissarides model.
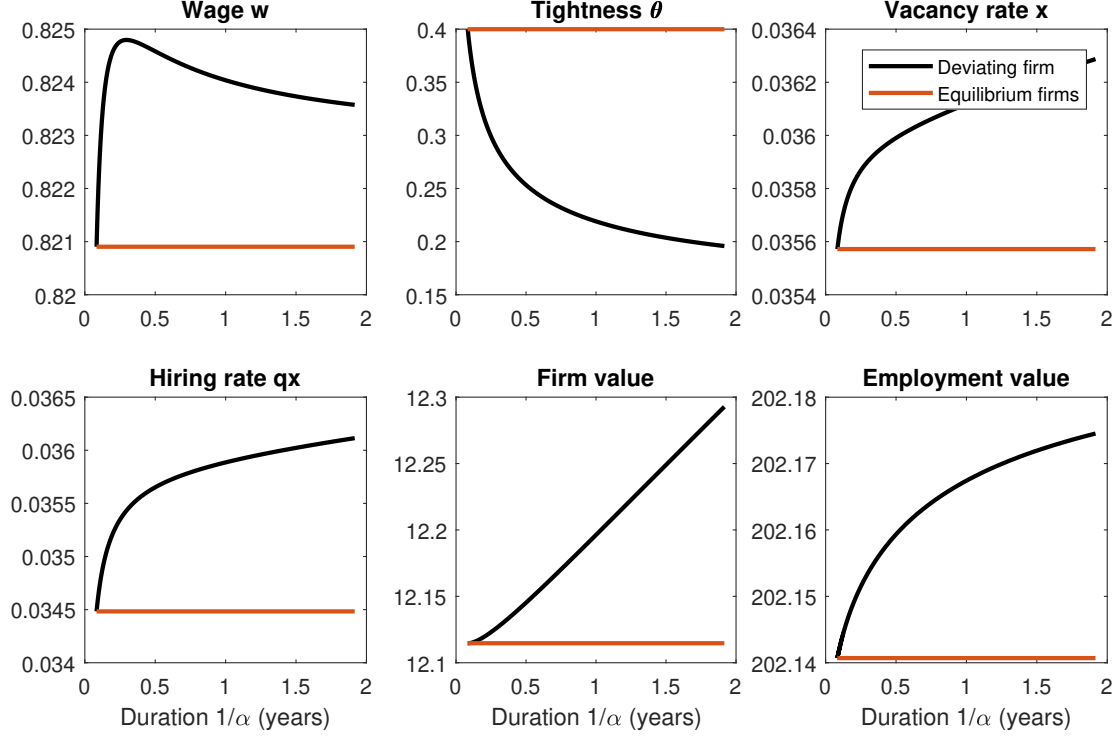
Figure 2: Single Firm Deviating to Longer Wage Commitment

Notes: The figure displays the steady-state values of a number of variables in the stationary equilibrium with firm wages, along with the corresponding values for an individual firm in that equilibrium that is able to set a wage commitment for a probabilistic period of time. The latter are plotted as a function of $1/\alpha$, the expected duration of the wage commitment. The firm value plotted is the scaled firm value per existing match.

The firm wage policy thus appears to translate into rigidity in wages that can be substantial in magnitude, and amplify labor market flows to a degree that helps bridge the gap between model and data discussed in the literature.

5.3   Infrequent Wage Adjustment

It is intuitive that firms facing a commitment problem would prefer to be able to commit to future wages. In this section I show that this mechanism can support a simple rule of committing to wages for a period of time as a policy that is profitable for firms, despite the costs associated with not responding to shocks in the interim.

The firm wage model does generate wage rigidity and amplification in the vacancy-unemployment ratio even if the parameters are held fixed between the two models, but doing so affects the levels significantly. See Figure 7 in Appendix C.

The Profitability of a Wage Commitment   To isolate the role of the wage commitment, I begin with a version of the stationary equilibrium with firm heterogeneity discussed in Section 4 where the firm-level shocks have been shut down. In this setting firms may differ in size in the cross-section, but choose identical wages and vacancy rates independent of their size, which are constant over time and leave firm sizes unchanged over time.[23]

I then consider an individual firm in this equilibrium that is fixing its wage for a probabilistic period of time. Figure 2 shows how the deviating firm compares to the equilibrium firms in terms of wages, market tightness, vacancy rate, hiring rate, and firm value, as a function of the expected duration of the wage commitment $1/\alpha$. As the figure shows, the deviating firm sets higher wages and creates more vacancies, thus hiring more than the equilibrium firms and growing over time while the deviation lasts.[24] In particular, the deviation clearly increases firm value. Wage commitments are desirable for firms in the model because of the commitment problem, something that is not true in the standard competitive search model where firms do not face this issue.[25]

But fixing wages involves also costs for firms in an environment where firm productivity varies over time. To show that the profitability of wage commitments survives these costs in this context, I now return to the specification of the stationary equilibrium with firm heterogeneity that includes firm-level shocks, discussed in Section 4.

To demonstrate how individual firms behave in this environment, Figure 9 produces impulse responses to such firm-level shocks. An increase in productivity causes the firm to raise wages, which attracts more job seekers per vacancy, as well as to increase vacancy creation. Both contribute to an increase in the firm growth rate, causing firm employment to rise over time. Firm growth rates are independent of firm size in the model, but because more productive firms will tend to grow to be larger, firm size and wages are positively correlated in the cross-section of firms. The figure also shows that the firm wage model tends to feature more rigid wages than the standard competitive search model also in response to firm-level, as well as aggregate, shocks. This wage rigidity translates to more variable hiring

---

[23]In anticipation of incorporating large and persistent firm-level shocks into this model with linear production technologies, I adjust the calibration slightly to increase the magnitude of firm adjustment costs. Specifically, I lower the target tightness to 0.4, implying $\ell = 2.67$, $b = 0.81$ and an average cost of vacancies of 3.8.

[24]Note that the deviation must be short enough that the firm remains small relative to the market. If the deviating firm grows at rate g such that $1+g = (1+qx)(1-\delta) > 1$, and its size remains fixed after it reverts to equilibrium behavior, then the expected firm size in period t is $[\alpha + (1-\alpha)\alpha(1+g) + \ldots + (1-\alpha)^t(1+g)^t]n_1 = [\alpha(1 + (1-\alpha)(1+g) + \ldots + (1-\alpha)^{t-1}(1+g)^{t-1}) + (1-\alpha)^t(1+g)^t]n_1$, where $n_1 = (1+qx)(1-\delta)n_0$ is size after the initial deviation period. Firm size remains bounded as t grows iff $(1-\alpha)(1+g) < 1$.

[25]For reference, Figure 8 in Appendix C produces the corresponding figure for the standard competitive search model.
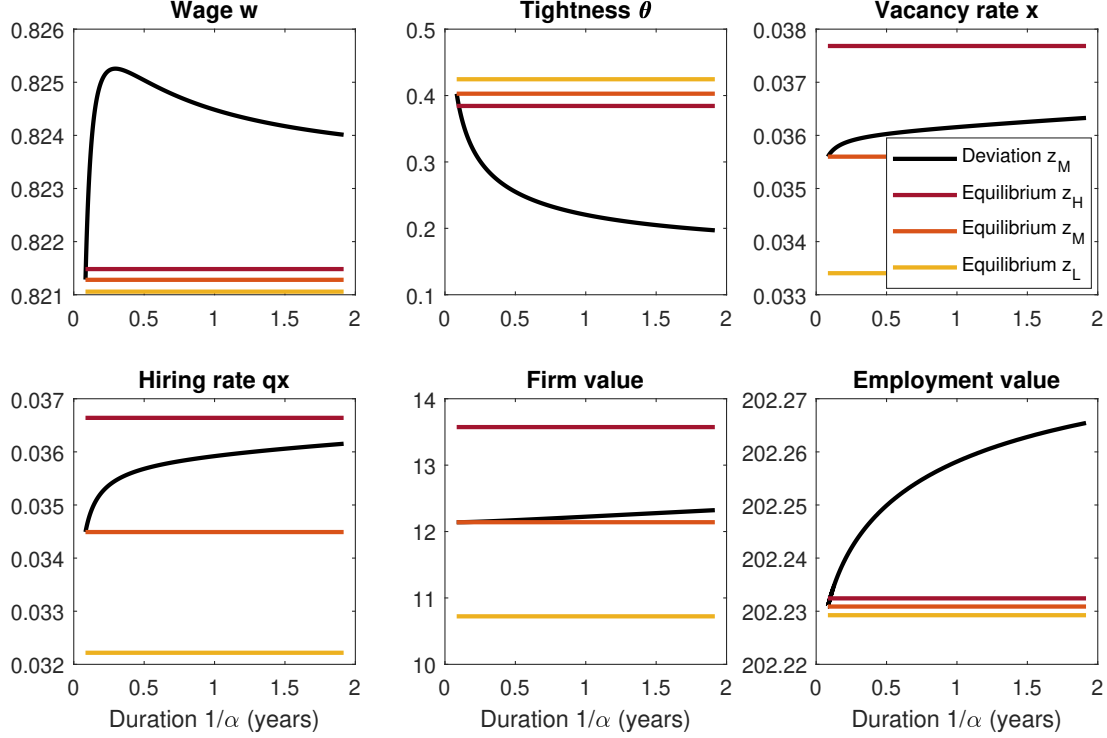
Figure 3: Single Firm Deviating to Longer Wage Commitment with Firm-Level Shocks

Notes: The figure displays the equilibrium values of a number of variables in the stationary equilibrium with firm wages, along with the corresponding values for an individual firm in that equilibrium that is able to set a wage commitment for a probabilistic period of time. The model is solved on three state grid for productivity, approximating an $AR(1)$ with autocorrelation $\rho_z = 0.9$ and standard deviation $\sigma_z = 0.1$ based on the Rouwenhorst method. The deviating firm is in the intermediate productivity state and its choices are plotted as a function of $1/\alpha$, the expected duration of the wage commitment. The firm value plotted is the scaled firm value per existing match.

and employment in the firm wage model than the standard model, in this model comparison.

Returning to the impact of fixing wages, Figure 3 again shows how the deviating firm compares to equilibrium firms in terms of wages, market tightness, vacancy rate, hiring rate, and firm value, now in an environment with large firm-level shocks. For purposes of illustration, the model is solved on a grid where productivity takes on three values: high, intermediate and low. The figure shows the equilibrium firms' values of wages, market tightness, vacancy rate, hiring rate, and firm value, for each of these three states separately. In line with the impulse responses, higher productivity is associated with a higher wage, lower tightness, higher vacancy rate and greater firm value. The figure then shows how a firm in the intermediate productivity state that deviates from equilibrium by fixing its wage for a probabilistic period of time compares to these equilibrium firms. The figure reflects similar

Figure 4: Equilibrium with Longer Wage Commitments

Notes: The figure displays the steady-state values of a number of variables in the stationary equilibrium with firm wages and infrequent adjustment, as a function of $1/\alpha$, the expected duration of wages. The firm value plotted is the scaled firm value per existing match. The figure also shows the corresponding values in the planner's allocation.

level effects as in the deterministic case: the deviating firm offers higher wages, attracts more job seekers per vacancy, creates more vacancies and hires more, than other firms with similar productivity. At the same time, the figure also shows that as the productivity shocks firms face are large, prevailing productivity is also quite important for hiring outcomes. However, despite this uncertainty, it remains true that firm value increases in the wage commitment.

Equilibrium with Wage Commitments   Given the profitability of wage commitments, it becomes interesting to consider also the equilibrium effects of firms systematically applying such rules. To that end, I solve for the equilibrium with infrequent adjustment, and consider how the duration of wage commitments affects the average level of wages, tightness, vacancy creation, employment, as well as firm and worker value. For simplicity, I again abstract from shocks in this exercise. Figure 4 plots the results. As expected, longer commitments raise the level of wages, helping the firm overcome its desire to cut wages ex post on its existing workforce. This makes the firm more attractive to job applicants, thus reducing

31

market tightness, but also reduces vacancy creation as the profitability of hiring falls. As a consequence, employment falls from the heightened levels associated with the firm wage equilibrium. Overall, these changes bring the equilibrium closer to efficient allocations, improving outcomes in that sense.

It is not the firms that benefit from this change, however, as in the case of the single deviating firm. The higher wages are associated with the equilibrium shifting in a way that is favorable to workers over firms, making both unemployed and employed workers better off, but meanwhile reducing the profitability of firms. While longer wage commitments thus improve on the overall allocation of resources, bringing the allocation closer to efficiency, it is to the benefit of workers over firms.

In sum, the model thus helps explain the observation of infrequent wage adjustment by offering a natural environment where longer wage commitments can be optimal for firms. At the same time, it suggests that such infrequent adjustment can be good for welfare rather than purely costly, especially for workers.

5.4   Infrequent Wage Adjustment over the Business Cycle

The findings carry over to the setting with aggregate shocks to productivity also. To demonstrate, Figure 5 first considers a setting where a measure of identical firms reoptimize their wages each period in the face or aggregate shocks, as in Section 5.2, and a single firm in that equilibrium considers a deviation to a fixed wage for a probabilistic period of time. While the deviation lasts, the firm behaves differently from the others in terms of the level of wage, and consequently its growth, as well as responses to shocks. The longer wage commitment tends to make the firm more forward-looking in setting the wage, leading to a higher wage and promoting firm growth. At the same time, the wage commitment tends to also make the firm more responsive to shocks, in terms of its hiring and profits, as the wage does not respond to shocks.

To gauge the net effect on firm value, as well as demonstrating the effect on the level of wages and growth, Figure 5 plots the mean values of wages, market tightness, vacancy rate, and firm growth, for a firm deviating for a given duration. The figure also shows the impact on the welfare of workers in the firm, as well as the impact on firm value, at the time of the deviation. The values plotted come from averaging over a simulation of the model, which compares at each instant the values of the then deviating firm to those of the equilibrium firms. The figure mirrors the corresponding one with firm-level shocks, and in particular shows that deviating tends to raise firm value, despite the added volatility implied.
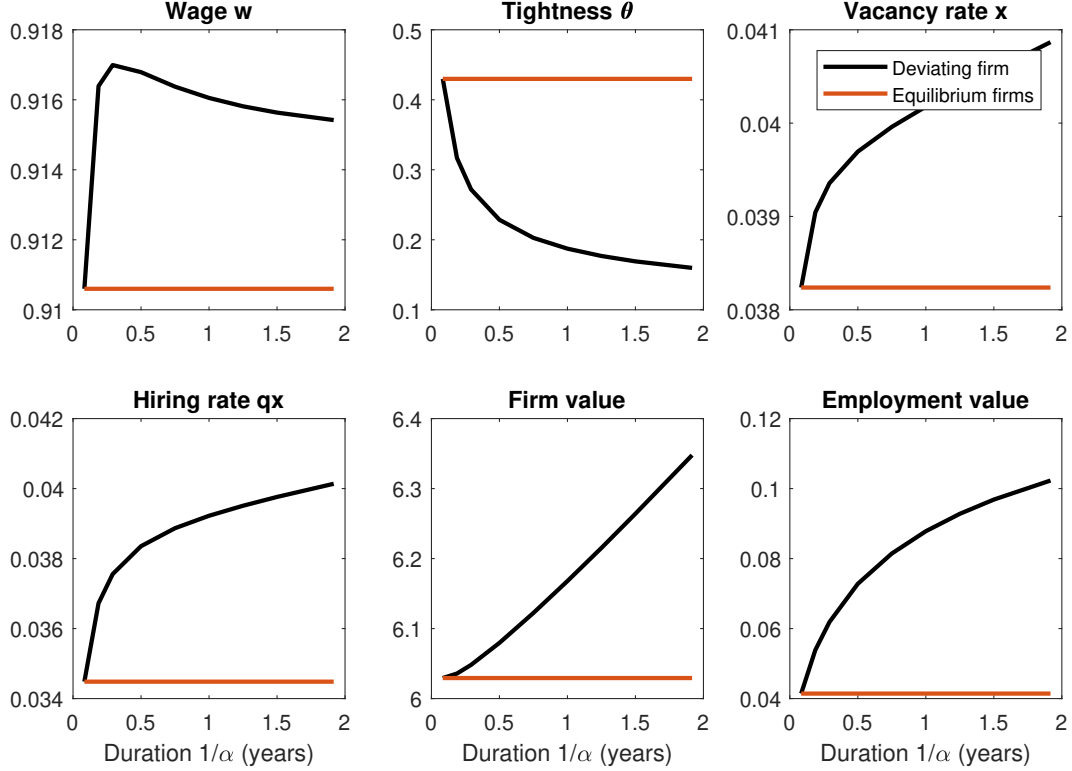
32

Figure 5: Deviation to Longer Wage Commitment with Aggregate Shocks

Notes: The figure displays simulation means in an equilibrium where wages are reoptimized each period, together with those chosen by a firm deviating each period, as a function of $1/\alpha$, the expected duration of wages. Labor productivity follows an AR(1) with autocorrelation $\rho_z = 0.96$ and standard deviation $\sigma_z = 0.02$. The firm value plotted is firm value per existing match.

To then demonstrate the effects of longer wage commitments on labor market outcomes, in a setting where all firms set wages probabilistically and in a staggered manner, Figure 6 plots simulation averages as a function of the duration of wages. Again, the presence of aggregate shocks does not change the conclusions, including that workers are better off with longer wage commitments – despite added labor market volatility.

Finally, one can revisit the impulse responses of Section 5.2 with this model of infrequent adjustment. Adopting an annual horizon of wage commitment, a calibration holding the level of profits fixed now brings the calibrated value of b down to 0.76, because the longer planning horizon raises the level of wages in the model. The inertia in wages implied by the longer wage commitments generates significant amplification in labor market flows nevertheless. Figure 10 demonstrates that the resulting amplification in labor market flows is similar in magnitude to the calibration of the firm wage model in Figure 1.
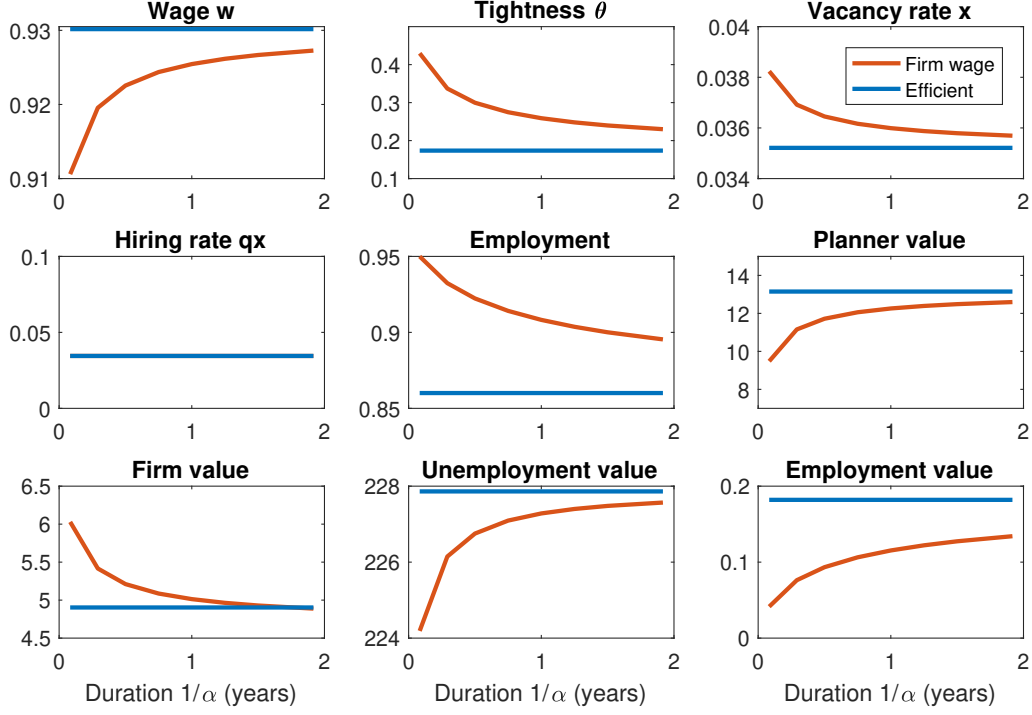
Figure 6: Equilibrium with Longer Wage Commitments with Aggregate Shocks

Notes: The figure displays simulation means in the firm wage equilibrium with infrequent adjustment, as a function of $1/\alpha$, the expected duration of wages. Labor productivity follows an AR(1) with autocorrelation $\rho_z = 0.96$ and standard deviation $\sigma_z = 0.02$. The firm value plotted is firm value per existing match. The figure also shows the corresponding values in the planner's allocation.

6 Conclusions

TBW

References

ACEMOGLU, D., AND W. B. HAWKINS (2014): "Search with multi-worker firms," *Theoretical Economics*, 9, 583–628.

AKERLOF, G., AND J. YELLEN (1990): "The Fair Wage-Effort Hypothesis and Unemployment," *Quarterly Journal of Economics*, 105, 255–283.

AMADOR, M., I. WERNING, AND G.-M. ANGELETOS (2006): "Commitment vs. Flexibility," *Econometrica*, 74, 365–396.

ATHEY, S., A. ATKESON, AND P. J. KEHOE (2005): "The Optimal Degree of Discretion in Monetary Policy," *Econometrica*, 73, 1431–75.

BARATTIERI, A., S. BASU, AND P. GOTTSCHALK (2014): "Some Evidence on the Importance of Sticky Wages," *American Economic Journal: Macroeconomics*, 6, 70–101.

BEWLEY, T. (1999): *Why Wages Don't Fall During a Recession*. Harvard University Press, Cambridge, Massachusetts.

BILS, M. (1985): "Real Wages over the Business Cycle: Evidence from Panel Data," *Journal of Political Economy*, 93, 666–689.

BLINDER, A., AND D. CHOI (1990): "A Shred of Evidence on Theories of Wage Stickiness," *Quarterly Journal of Economics*, 105, 1003–15.

BRACHA, A., U. GNEEZY, AND G. LOEWENSTEIN (2015): "Relative Pay and Labor Supply," *Journal of Labor Economics*, 33, 297–315.

BREZA, E., S. KAUR, AND Y. SHAMDASANI (2017): "The Morale Effects of Pay Inequality," *Quarterly Journal of Economics*, forthcoming.

BURDETT, K., AND D. MORTENSEN (1998): "Wage Differentials, Employer Size and Unemployment," *International Economic Review*, 39, 257–273.

CARD, D., A. MAS, E. MORETTI, AND E. SAEZ (2012): "Inequality at Work: The Effect of Peer Salaries on Job Satisfaction," *American Economic Review*, 102, 2981–3003.

CHAMLEY, C. (1986): "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 54, 607–622.

COLES, M. (2001): "Equilibrium Wage Dispersion, Firm Size and Growth," *Review of Economic Dynamics*, 4, 159–187.

COOPER, R., J. HALTIWANGER, AND J. L. WILLIS (2007): "Search Frictions: Matching Aggregate and Establishment Observations," *Journal of Monetary Economics*, 54, 56–78.

DEN HAAN, W. J., G. RAMEY, AND J. WATSON (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, 90, 482–98.

ELSBY, M., AND R. MICHAELS (2013): "Marginal Jobs, Heterogeneous Firms, and Unemployment Flows," *American Economic Journal: Macroeconomics*, 5, 1–48.

FUJITA, S., AND M. NAKAJIMA (2016): "Worker Flows and Job Flows: A Quantitative Investigation," *Review of Economic Dynamics*, 22, 1–20.

GERTLER, M., C. HUCKFELDT, AND A. TRIGARI (2017): "Unemployment Fluctuations with Staggered Nash Wage Bargaining," Unpublished manuscript, New York University.

GERTLER, M., AND A. TRIGARI (2009): "Unemployment Fluctuations with Staggered Nash Wage Bargaining," *Journal of Political Economy*, 117, 38–86.

HAEFKE, C., M. SONNTAG, AND T. VAN RENS (2013): "Wage Rigidity and Job Creation," *Journal of Monetary Economics*, 60, 887–899.

HALAC, M., AND P. YARED (2014): "Fiscal Rules and Discretion under Persistent Shocks," *Econometrica*, 82, 1557–1614.

HAWKINS, W. B. (2011): "Do Large-Firm Bargaining Models Amplify and Propagate Aggregate Productivity Shocks?," Unpublished manuscript, University of Rochester.

——— (2013): "Competitive Search, Efficiency and Multiworker Search," *International Economic Review*, 54, 219–251.

JUDD, K. (1985): "Redistributive taxation in a simple perfect foresight model," *Journal of Public Economics*, 28, 59–83.

KAAS, L., AND P. KIRCHER (2015): "Efficient Firm Dynamics in a Frictional Labor Market," *American Economic Review*, 105, 3030–60.

KLEIN, P., P. KRUSELL, AND V. RIOS-RULL (2008): "Time-Consistent Public Policy," *Review of Economic Studies*, 75, 789–808.

KRUSELL, P., AND L. RUDANKO (2016): "Unions in a Frictional Labor Market," *Journal of Monetary Economics*, 80, 35–50.

LAMO, A., AND F. SMETS (2009): "Wage Dynamics in Europe: Final Report of the Wage Dynamics Network (WDN)," European Central Bank research report.

MENZIO, G. (2005): "High Frequency Wage Rigidity," Unpublished manuscript, University of Pennsylvania.

MENZIO, G., AND E. MOEN (2010): "Worker Replacement," *Journal of Monetary Economics*, 57, 623–636.

MOSCARINI, G., AND F. POSTEL-VINAY (2013): "Stochastic Search Equilibrium," *Review of Economic Studies*, 80, 1545–81.

———— (2016): "Wage Posting and Business Cycles: a Quantitative Exploration," *Review of Economic Dynamics*, 19, 135–60.

ROGERSON, R., R. SHIMER, AND R. WRIGHT (2005): "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature*, 18, 959–988.

RUDANKO, L. (2009): "Labor Market Dynamics under Long-Term Wage Contracting," *Journal of Monetary Economics*, 56, 170–83.

SCHAAL, E. (2017): "Uncertainty, Productivity and Unemployment during the Great Recession," *Econometrica*, forthcoming.

SHIMER, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95, 25–49.

SMITH, E. (1999): "Search, Concave Production and Optimal Firm Size," *Review of Economic Dynamics*, 2, 456–471.

SNELL, A., AND J. THOMAS (2010): "Labor Contracts, Equal Treatment and Wage-Unemployment Dynamics," *American Economic Journal: Macroeconomics*, 2, 98–127.

TAYLOR, J. B. (1983): "Union Wage Settlements During a Disinflation," *American Economic Review*, 73, 981–993.

———— (1999): "Staggered Price and Wage Setting in Macroeconomics," in *The Handbook of Macroeconomics*, ed. by J. B. Taylor, and M. Woodford, chap. 1. Elsevier Science, Amsterdam.

———— (2016): "The Staying Power of Staggered Wage and Price Setting Models in Macroeconomics," in *The Handbook of Macroeconomics*, ed. by J. B. Taylor, and H. Uhlig, chap. 25. Elsevier Science, Amsterdam.

Appendix

A Proofs and Details

Proof of Proposition 5 For convenience, let $y_t := Y_t - \beta(1-\delta)E_t Y_{t+1}$. We have $y_t = E_t[\beta\delta U_{t+1} - b - \beta U_{t+1} + \beta(1-\delta)(b+\beta U_{t+2})] = E_t[-b - \beta(1-\delta)(U_{t+1} - b - \beta U_{t+2})]$, meaning that $y_t = -b - \beta(1-\delta)E_t X_{t+1}$.

First, the firm objective in (22) can be rewritten as

$$E_0[n_0 \sum_{t=0}^{\infty} \beta^t(1-\delta)^t(z-w_t) + \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t-1}(1-\delta)^{t-k}q(\theta_k)v_k(z-w_t)], \tag{52}$$

using that $n_t + q(\theta_t)v_t = (1-\delta)^t n_0 + \sum_{k=0}^{t-1}(1-\delta)^{t-k}q(\theta_k)v_k$.

The first term in (52) can then be rewritten as

$$E_0 n_0 \sum_{t=0}^{\infty} \beta^t(1-\delta)^t(z-w_t) = n_0[Z_0 + Y_0 - \frac{X_0}{\mu(\theta_0)}], \tag{53}$$

using that the job seeker value constraint (23) implies $W_0 = X_0/\mu(\theta_0) - Y_0$.

The second term in (52) can be rewritten as

$$E_0 \sum_{k=0}^{\infty} \beta^k q(\theta_k)v_k \sum_{t=k}^{\infty} \beta^{t-k}(1-\delta)^{t-k}(z_t - w_t)$$

$$= E_0 \sum_{k=0}^{\infty} \beta^k[q(\theta_k)v_k \sum_{t=k}^{\infty} \beta^{t-k}(1-\delta)^{t-k}(z_t + y_t) - \frac{v_k}{\theta_k} \sum_{t=k}^{\infty} \beta^{t-k}(1-\delta)^{t-k}X_t]$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t-1}(1-\delta)^{t-k}[q(\theta_k)v_k(z_t + y_t) - \frac{v_k}{\theta_k}X_t], \tag{54}$$

where the first equality follows from rearranging terms, and the second uses the job seeker value constraint to substitute out the present value of wages.

Combining the terms in (53) and (54) and rearranging, the firm objective becomes

$$E_0 n_0[\sum_{t=0}^{\infty} \beta^t(1-\delta)^t(z_t + y_t) - \frac{X_0}{\mu(\theta_0)}]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t-1}(1-\delta)^{t-k}[q(\theta_k)v_k(z_t + y_t) - \frac{v_k}{\theta_k}X_t] - E_0 \sum_{t=0}^{\infty} \beta^t \kappa(v_t/n_t, n_t)$$

$$= -\frac{X_0 n_0}{\mu(\theta_0)} + E_0 \sum_{t=0}^{\infty} \beta^t[(n_t + q(\theta_t)v_t)(z_t + y_t) - \frac{X_t v_t}{\theta_t} - \kappa(v_t/n_t, n_t)]. \tag{55}$$

Using that $y_t = -b - \beta(1-\delta)E_t X_{t+1}$, and rearranging, the firm objective can be written as

$$-\frac{X_0 n_{i0}}{\mu(\theta_{i0})} + n_{i0} X_0 + E_t \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it})v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - X_t(\frac{v_{it}}{\theta_{it}} + n_{it})]. \qquad (56)$$

Note that the term $n_{i0}X_0$ is independent of the firm's actions, so may be omitted in writing the firm problem as in (24) in the text.

Opting Out of the Labor Market Note that because the firm begins with a stock of existing workers, it could potentially find it optimal to, instead of following the interior solution characterized by the first order conditions, not hire at all in the first period and instead set a wage that is so low as to make those existing workers indifferent between remaining with the firm and unemployment. The latter would mean that $W_0 + Y_0 = 0$ and no hiring that $v_0 = 0$. How would this change firm value?

In the derivation above, it would mean that the expression in (53) would reduce to $n_{i0}[Z_0 + Y_0]$, and the expression in (54) would have $v_{i0} = 0$, such that θ_{i0} no longer appears. Firm value, as in (56), would then become

$$n_{i0}X_0 + E_t \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it})v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - X_t(\frac{v_{it}}{\theta_{it}} + n_{it})]. \qquad (57)$$

with $v_{i0} = 0$. With commitment, after this initial period the firm problem becomes equivalent to the planner problem, and hence hiring should be consistent with efficient allocations and interior as long as standard conditions are met (z sufficiently above b). In the initial period, one would want to check that this value does not dominate the equilibrium value. Note that due to the size-independence of the firm problem, if one firm prefers to deviate, all firms will.

In the context of no commitment, if a firm in any period were to deviate to this non-hiring option, its value would be, instead of that in (33):

$$n(z - b) - X(S)n + \beta E_S V(n'; S') \qquad (58)$$
$$\text{s.t. } n' = (1 - \delta)n,$$

where the continuation value $V(n; S)$ follows (34). In solving the model using first order conditions, one would want to make sure this deviation value does not exceed equilibrium values, something that can restrict parameter values. In practice high aggregate levels of existing matches tend to make deviating more attractive, so one would choose parameters such that the desired steady-state measure of matches is sufficiently below this range, keeping the economy below a range where deviating becomes attractive.

Second Order Conditions For the sequence problem, denoting the firm objective as g, second order conditions read, for $t > 0$: $g_{x_t x_t} = -\kappa''(x_t) < 0$, $g_{\theta_t \theta_t} = q''(\theta_t)x_t(z_t - b + \beta(1-\delta)E_t\hat{V}_{t+1}) - \frac{2X_t x_t}{\theta_t^3} < 0$, and $det = g_{x_t x_t}g_{\theta_t \theta_t} - g_{x_t \theta_t}^2 > 0$, where $g_{x_t \theta_t} = q'(\theta_t)(z_t - b + \beta(1-\delta)E_t\hat{V}_{t+1}) + \frac{X_t}{\theta_t^2} = 0$. For the initial period: $g_{x_0 x_0} = -\kappa''(x_0)$, $g_{\theta_0 \theta_0} = \frac{X_0 \mu''(\theta_0)}{\mu(\theta_0)^2} - \frac{2X_0 \mu'(\theta_0)^2}{\mu(\theta_0)^3} + q''(\theta_0)x_0(z_0 - b + \beta(1-\delta)E_0\hat{V}_1) - \frac{2X_0 x_0}{\theta_0^3}$ and $det = g_{x_0 x_0}g_{\theta_0 \theta_0} - g_{x_0 \theta_0}^2 > 0$, where $g_{x_0 \theta_0} = q'(\theta_0)(z_0 - b + \beta(1-\delta)E_0\hat{V}_1) + \frac{X_0}{\theta_0^2}$. The periods separate when calculating second order conditions.

For the no commitment case, again denoting the firm objective as g, second order conditions read: $g_{xx} = -\kappa''(x) < 0$, $g_{\theta\theta} = \frac{X\mu''(\theta)}{\mu(\theta)^2} - \frac{2X\mu'(\theta)^2}{\mu(\theta)^3} + q''(\theta)x(z - b + \beta(1-\delta)E\hat{V}) - \frac{2Xx}{\theta^3} < 0$, and $det = g_{xx}g_{\theta\theta} - g_{x\theta}^2 > 0$, where $g_{x\theta} = q'(\theta)(z - b + \beta(1-\delta)E\hat{V}) + \frac{X}{\theta^2}$.

Proof of Proposition 6 The firm problem (22) is equivalent to the problem

$$\max_{\{w_{it},\theta_{it},x_{it}\}} E_0 \sum_{t=0}^{\infty} \beta^t(1-\delta)^t \prod_{k=0}^{t-1}(1 + q(\theta_{ik})x_{ik})[(1 + q(\theta_{it})x_{it})(z_t - w_{it}) - \kappa(x_{it})] \tag{59}$$

$$\text{s.t. } X_t = \mu(\theta_{it})(E_t \sum_{k=0}^{\infty} \beta^k(1-\delta)^k w_{it+k} + Y_t), \ \forall t \geq 0,$$

which does not depend on n_{i0}. This can be seen by expressing the profits in problem (22) in each period t scaled by size n_{it}, and using the law of motion to adjust the discounting for this scaling. Finally, normalizing the firm problem with initial size n_{i0} yields the expression above.

B   Two Period Model

Consider a deterministic two period version of the dynamic problem considered in Section 3.

Planner Problem   The planner problem reads:

$$\max_{\{\theta_{it}, v_{it}\}_{t=1}^{2}} \sum_{t=1}^{2} \beta^t \Big[\sum_i [(n_{it} + q(\theta_{it})v_{it})z_t - \kappa(v_{it}, n_{it})] + (1 - \sum_i (n_{it} + q(\theta_{it})v_{it})b) \Big] \quad (60)$$

$$\text{s.t. } n_{i2} = (1-\delta)(n_{i1} + q(\theta_{i1})v_{i1}),$$

$$\sum_i v_{it}/\theta_{it} = 1 - \sum_i n_{it}, \text{ for } t = 1, 2, \quad (61)$$

with n_{i1} given for all i. The planner maximizes the present discounted value of output produced by employed workers with the market technology and by unemployed workers with the home technology, net of the costs of vacancy creation. The planner takes as given the law of motion for employment relationships, as well as a constraint (61) that imposes that the planner's choices of vacancies and market tightness across markets must be consistent with the total measure of job seekers in each period. In what follows, the latter constraint is associated with a Lagrange multiplier λ_t for $t = 1, 2$, reflecting the planner's shadow value of job seekers.

The first order conditions for the planner's choice of v_{it}, θ_{it} for $t = 1, 2$, read

$$\kappa_v(x_{i2}) + \frac{\lambda_2}{\theta_{i2}} = q(\theta_{i2})(z_2 - b), \quad (62)$$

$$\frac{\lambda_2}{\theta_{i2}^2} = -q'(\theta_{i2})(z_2 - b), \quad (63)$$

$$\kappa_v(x_{i1}) + \frac{\lambda_1}{\theta_{i1}} = q(\theta_{i1})[z_1 - b + \beta(1-\delta)(z_2 - b - \kappa_n(x_{i2}) - \lambda_2)], \quad (64)$$

$$\frac{\lambda_1}{\theta_{i1}^2} = -q'(\theta_{i1})[z_1 - b + \beta(1-\delta)(z_2 - b - \kappa_n(x_{i2}) - \lambda_2)]. \quad (65)$$

Note that these are independent of firm size, and in what follows I hence drop the producer index i to consider symmetric allocations.

Taken together, the optimality conditions imply that the Lagrange multipliers satisfy

$$\lambda_t = \kappa_v(x_t)\frac{\mu(\theta_t) - \mu'(\theta_t)\theta_t}{\mu'(\theta_t)} \quad (66)$$

for $t = 1, 2$. Using this, the optimality conditions can be written in terms of allocations as

$$\frac{\kappa_v(x_2)}{\mu'(\theta_2)} = z_2 - b, \tag{67}$$

$$\frac{\kappa_v(x_1)}{\mu'(\theta_1)} = z_1 - b + \beta(1 - \delta)(\frac{\kappa_v(x_2)}{\mu'(\theta_2)}(1 - \mu(\theta_2) + \theta_2\mu'(\theta_2)) - \kappa_n(x_2)). \tag{68}$$

In addition, the planner's allocation must also satisfy the constraint (61), $x_t = \theta_t(1 - N_t)/N_t$ for $t = 1, 2$, where the total measure of existing relationships satisfies the law of motion $N_2 = (1 - \delta)(1 + q(\theta_1)x_1)N_1$.

Firm Wages The worker value of entering period $t = 1, 2$ unemployed satisfies

$$U_2 = \mu(\theta_{i2})w_{i2} + (1 - \mu(\theta_{i2}))b, \tag{69}$$

$$U_1 = \mu(\theta_{i1})(w_{i1} + \beta(1 - \delta)w_{i2} + \beta\delta U_2) + (1 - \mu(\theta_{i1}))(b + \beta U_2). \tag{70}$$

For convenience, define $X_2 \equiv U_2 - b$, $X_1 \equiv U_1 - b - \beta U_2$ and $Y_2 \equiv -b$, $Y_1 \equiv -b - \beta(1-\delta)U_2$. With this, the worker value constraints can be written as

$$X_t = \mu(\theta_{it})(W_{it} + Y_t), \tag{71}$$

for $t = 1, 2$.

Commitment Assuming the firm can commit to future wages, the firm problem reads

$$\max_{\{w_{it},\theta_{it},v_{it}\}_{t=1}^2} \sum_{t=1}^{2} \beta^t[(n_{it} + q(\theta_{it})v_{it})(z_t - w_{it}) - \kappa(v_{it}, n_{it})], \tag{72}$$

$$\text{s.t. } n_{i2} = (1 - \delta)(n_{i1} + q(\theta_{i1})v_{i1}),$$

$$X_t = \mu(\theta_{it})(\sum_{k=1}^{2} \beta^k(1 - \delta)^k w_{it+k} + Y_t), \text{ for } t = 1, 2, \tag{73}$$

with n_{i1} given for all i. The firm maximizes the present discounted value of profits, taking into account the law of motion for employment relationships, as well as the constraint reflecting job seeker behavior each period, where the firm takes the market-determined values of X_t, Y_t as given.

Using the constraints (73), the firm problem can be rewritten as

$$\max - \frac{X_1}{\mu(\theta_{i1})} + X_1 + (1 + q(\theta_{i1})x_{i1})(z_1 - b) - \kappa(x_{i1}) - X_1(\frac{x_{i1}}{\theta_{i1}} + 1)$$

$$+ \beta(1 - \delta)(1 + q(\theta_{i1})x_{i1})[(1 + q(\theta_{i2})x_{i2})(z_2 - b) - \kappa(x_{i2}) - X_2(\frac{x_{i2}}{\theta_{i2}} + 1)]. \tag{74}$$

Note that this problem is independent of scale, and hence in what follows the producer-level indicator is dropped.

The first order conditions for optimality in the second period read[26]

$$\kappa_v(x_2) + \frac{X_2}{\theta_2} = q(\theta_2)(z_2 - b),$$ (75)

$$\frac{X_2}{\theta_2^2} = -q'(\theta_2)(z_2 - b).$$ (76)

Taken together, these imply that $X_2 = \kappa_v(x_2)\frac{\mu(\theta_2) - \mu'(\theta_2)\theta_2}{\mu'(\theta_2)}$ and $\frac{\kappa_v(x_2)}{\mu'(\theta_2)} = z_2 - b$. Note that these correspond to the planner's optimality conditions, with X_2 corresponding to the Lagrange multiplier reflecting the shadow value of job seekers.

With the second period allocation θ_2, x_2, the second period firm value normalized by size is

$$\hat{V}_2 = -\frac{X_2}{\mu(\theta_2)} + X_2 + (1 + q(\theta_2)x_2)(z_2 - b) - \kappa(x_2) - X_2(\frac{x_2}{\theta_2} + 1).$$ (77)

while the continuation value of the firm is

$$\hat{V}_2^c = (1 + q(\theta_2)x_2)(z_2 - b) - \kappa(x_2) - X_2(\frac{x_2}{\theta_2} + 1).$$ (78)

Using the optimality conditions, these can be written as

$$\hat{V}_2 = -\frac{X_2}{\mu(\theta_2)} + z_2 - b - \kappa_n(x_2)$$ (79)

and

$$\hat{V}_2^c = z_2 - b - \kappa_n(x_2) - X_2.$$ (80)

The first order conditions for optimality in the first period are

$$\kappa_v(x_1) + \frac{X_1}{\theta_1} = q(\theta_1)[z_1 - b + \beta(1 - \delta)\hat{V}_2^c],$$ (81)

$$\frac{X_1}{\theta_1^2}[1 + \frac{\mu'(\theta_1)\theta_1^2}{x_1\mu(\theta_1)^2}] = -q'(\theta_1)[z_1 - b + \beta(1 - \delta)\hat{V}_2^c].$$ (82)

___

[26]Denoting the firm objective as g, second order conditions read: $g_{x_2 x_2} = -\kappa''(x_2) < 0$, $g_{\theta_2 \theta_2} = q''(\theta_2)x_2(z_2 - b) - \frac{2X_2 x_2}{\theta_2^3} < 0$, and $det = g_{x_2 x_2}g_{\theta_2 \theta_2} - g_{x_2 \theta_2}^2 > 0$, where $g_{x_2 \theta_2} = q'(\theta_2)(z_2 - b) + \frac{X_2}{\theta_2^2} = 0$, and $g_{x_1 x_1} = -\kappa''(x_1)$, $g_{\theta_1 \theta_1} = \frac{X_1 \mu''(\theta_1)}{\mu(\theta_1)^2} - \frac{2X_1 \mu'(\theta_1)^2}{\mu(\theta_1)^3} + q''(\theta_1)x_1(z_1 - b + \beta(1 - \delta)E_1 \hat{V}_2) - \frac{2X_1 x_1}{\theta_1^3}$ and $det = g_{x_1 x_1}g_{\theta_1 \theta_1} - g_{x_1 \theta_1}^2 > 0$, where $g_{x_1 \theta_1} = q'(\theta_1)(z_1 - b + \beta(1 - \delta)E_1 \hat{V}_2) + \frac{X_1}{\theta_1^2} < 0$. The periods separate when calculating second order conditions.

Taken together, they imply that

$$X_1 = \kappa_v(x_1)\frac{\mu(\theta_1) - \mu'(\theta_1)\theta_1}{\mu'(\theta_1)}\frac{q(\theta_1)x_1}{1 + q(\theta_1)x_1},\tag{83}$$

and

$$\frac{\kappa_v(x_1)}{\mu'(\theta_1)}[1 - \frac{(1 - \mu'(\theta_1)\theta_1/\mu(\theta_1))}{1 + q(\theta_1)x_1}] = z_1 - b + \beta(1 - \delta)\left[\frac{\kappa_v(x_2)}{\mu'(\theta_2)}(1 - \mu(\theta_2) + \mu'(\theta_2)\theta_2) - \kappa_n(x_2)\right].\tag{84}$$

Given allocations and the continuation value of the firm, and using the optimality conditions, the normalized firm value in the first period can be written

$$\hat{V}_1 = -\frac{X_1}{\mu(\theta_1)} + z_1 - b + \beta(1 - \delta)\hat{V}_2^c - \kappa_n(x_1),\tag{85}$$

while the corresponding continuation value would read

$$\hat{V}_1^c = z_1 - b + \beta(1 - \delta)\hat{V}_2^c - \kappa_n(x_1) - X_1.\tag{86}$$

Limited Commitment   If firms do not have commitment to future wages, the firm problem is solved recursively.

In the second period, firms maximize the firm value

$$\max -\frac{X_2}{\mu(\theta_2)} + X_2 + (1 + q(\theta_2)x_2)(z_2 - b) - \kappa(x_2) - X_2(\frac{x_2}{\theta_2} + 1)].\tag{87}$$

The first order conditions for optimality read

$$\kappa_v(x_2) + \frac{X_2}{\theta_2} = q(\theta_2)(z_2 - b),\tag{88}$$

$$\frac{X_2}{\theta_2^2}[1 + \frac{\mu'(\theta_2)\theta_2^2}{x_2\mu(\theta_2)^2}] = -q'(\theta_2)(z_2 - b).\tag{89}$$

They imply

$$X_2 = \kappa_v(x_2)\frac{\mu(\theta_2) - \mu'(\theta_2)\theta_2}{\mu'(\theta_2)}\frac{q(\theta_2)x_2}{1 + q(\theta_2)x_2},\tag{90}$$

and

$$\frac{\kappa_v(x_2)}{\mu'(\theta_2)}[1 - \frac{(1 - \mu'(\theta_2)\theta_2/\mu(\theta_2))}{1 + q(\theta_2)x_2}] = z_2 - b.\tag{91}$$

44

Given allocations, the normalized firm value satisfies

$$\hat{V}_2 = -\frac{X_2}{\mu(\theta_2)} + (1 + q(\theta_2)x_2)(z_2 - b) - \frac{X_2 x_2}{\theta_2} - \kappa(x_2) = -\frac{X_2}{\mu(\theta_2)} + z_2 - b - \kappa_n(x_2) \quad (92)$$

and the corresponding continuation value

$$\hat{V}_2^c = z_2 - b - \kappa_n(x_2) - X_2. \quad (93)$$

The first order conditions for optimality in the first period read[27]

$$\kappa_v(x_1) + \frac{X_1}{\theta_1} = q(\theta_1)[z_1 - b + \beta(1 - \delta)\hat{V}_2^c], \quad (94)$$

$$\frac{X_1}{\theta_1^2}[1 + \frac{\mu'(\theta_1)\theta_1^2}{x_1\mu(\theta_1)^2}] = -q'(\theta_1)[z_1 - b + \beta(1 - \delta)\hat{V}_2^c]. \quad (95)$$

They imply

$$X_1 = \kappa_v(x_1)\frac{\mu(\theta_1) - \mu'(\theta_1)\theta_1}{\mu'(\theta_1)}\frac{q(\theta_1)x_1}{1 + q(\theta_1)x_1}. \quad (96)$$

and

$$\frac{\kappa_v(x_1)}{\mu'(\theta_1)}[1 - \frac{(1 - \mu'(\theta_1)\theta_1/\mu(\theta_1))}{1 + q(\theta_1)x_1}] \quad (97)$$

$$= z_1 - b + \beta(1 - \delta)\Big[\frac{\kappa_v(x_2)}{\mu'(\theta_2)}\Big(1 - \mu(\theta_2) + \mu'(\theta_2)\theta_2 - (1 - \mu(\theta_2))\frac{(1 - \mu'(\theta_2)\theta_2/\mu(\theta_2))}{1 + q(\theta_2)x_2}\Big) - \kappa_n(x_2)\Big].$$

Given allocations and continuation values, firm value in the first period equals

$$\hat{V}_1 = -\frac{X_2}{\mu(\theta_2)} + z_2 - b + \beta(1 - \delta)\hat{V}_2^c - \kappa_n(x_2). \quad (98)$$

Whether or not firms have commitment, equilibrium requires allocations to be optimal for firms, as well as the total measure of job seekers allocated to firms to be consistent with the measure of job seekers in the market: $\frac{x_t N_t}{\theta_t} = 1 - N_t$ for $t = 1, 2$.

---

[27]Denoting the firm objective as g, second order conditions read: $g_{x_2 x_2} = -\kappa''(x_2) < 0$, $g_{\theta_2 \theta_2} = \frac{X_2\mu''(\theta_2)}{\mu(\theta_2)^2} - \frac{2X_2\mu'(\theta_2)^2}{\mu(\theta_2)^3} + q''(\theta_2)x_2(z_2 - b) - \frac{2X_2 x_2}{\theta_2^3} < 0$, and $det = g_{x_2 x_2}g_{\theta_2 \theta_2} - g_{x_2 \theta_2}^2 > 0$, where $g_{x_2 \theta_2} = q'(\theta_2)(z_2 - b) + \frac{X_2}{\theta_2^2} < 0$, and $g_{x_1 x_1} = -\kappa''(x_1)$, $g_{\theta_1 \theta_1} = \frac{X_1\mu''(\theta_1)}{\mu(\theta_1)^2} - \frac{2X_1\mu'(\theta_1)^2}{\mu(\theta_1)^3} + q''(\theta_1)x_1(z_1 - b + \beta(1 - \delta)E_1\hat{V}_2) - \frac{2X_1 x_1}{\theta_1^3}$ and $det = g_{x_1 x_1}g_{\theta_1 \theta_1} - g_{x_1 \theta_1}^2 > 0$, where $g_{x_1 \theta_1} = q'(\theta_1)(z_1 - b + \beta(1 - \delta)E_1\hat{V}_2) + \frac{X_1}{\theta_1^2} < 0$. The periods separate when calculating second order conditions.

C   Calibration Details

The law of motion for matches implies steady-state unemployment:

$$u = 1 - N - \mu(\theta)(1 - N) = \frac{\mu(\theta)(1 - \delta)}{\mu(\theta) + \delta - \mu(\theta)\delta}, \tag{99}$$

and if δ is given, a target for steady-state u determines $\mu(\theta)$.

Given a target for the tightness θ, the matching function parameter γ is then pinned down (uniquely) from $\mu(\theta) = \theta/(1 + \theta^\ell)^{1/\ell}$. This also determines steady-state values of $x = \theta(1 - N)/N = \delta\theta/((1 - \delta)\mu(\theta))$ and $\mu'(\theta)$.

These labor market flows must also be consistent with the Euler equation (28):

$$\frac{\kappa_v(x) + \frac{X}{\theta}}{q(\theta)} = z - b + \beta(1 - \delta)[\frac{\kappa_v(x) + \frac{X}{\theta}}{q(\theta)} - \kappa_n(x) - X], \tag{100}$$

where the value of job seekers X satisfies

$$X = \kappa_v(x)\frac{\mu(\theta) - \mu'(\theta)\theta}{\mu'(\theta)} \tag{101}$$

in the standard competitive search model and

$$X = \kappa_v(x)\frac{\mu(\theta) - \mu'(\theta)\theta}{\mu'(\theta)}\delta \tag{102}$$

in the firm wage model. Note that X/κ_0 is pinned down by the flows above for both models, as are $\kappa_v(x)/\kappa_0$ and $\kappa_n(x)/\kappa_0$. It follows that the Euler equation pins down a unique value of $(z - b)/\kappa_0$ for each model, that allows the equation to hold with the flows chosen. This still allows alternative combinations of b, κ_0 consistent with any such value, however.

To consider the implications for wages and profits, note that (from equation (21)) the present value of wages satisfies $W = X/\mu(\theta) - Y$, where $Y = -(b + \beta(1 - \delta)X)/(1 - \beta(1 - \delta))$.[28] The Euler equation can be rewritten using this expression for Y as

$$\frac{\kappa_v(x) + \frac{X}{\theta}}{q(\theta)} = z + \beta(1 - \delta)[\frac{\kappa_v(x) + \frac{X}{\theta}}{q(\theta)} - \kappa_n(x)] + (1 - \beta(1 - \delta))Y, \tag{103}$$

or in terms of wages as

$$\frac{\kappa_v(x)}{q(\theta)} + W = z + \beta(1 - \delta)[\frac{\kappa_v(x)}{q(\theta)} + W - \kappa_n(x)]. \tag{104}$$

---

[28]Appendix A shows that $y_t = -b - \beta(1 - \delta)X_{t+1}$, and by definition $Y = y/(1 - \beta(1 - \delta))$.

This equation determines the steady-state wage $w = W(1 - \beta(1 - \delta))$ as

$$w = z - \frac{\kappa_v(x)}{q(\theta)} + \beta(1 - \delta)[\frac{\kappa_v(x)}{q(\theta)} - \kappa_n(x)]. \tag{105}$$

It follows that for both models to have the same steady-state wage, both models must have the same κ_0.

If this is the case, firm profits are also the same across models, as firm profit per worker equals

$$\frac{(n + q(\theta)v)(z - w) - \kappa(v, n)}{n + q(\theta)v} = \frac{(1 + q(\theta)x)(z - w) - \kappa(x)}{1 + q(\theta)x}. \tag{106}$$

The above reasoning suggests a calibration approach where one first picks a b for the standard model, with κ_0 set to satisfy the corresponding Euler equation. To arrive at a comparable parametrization of the firm wage model, one adopts the same κ_0 to keep the steady-state wage and profit rate unchanged across models, with b set to satisfy the Euler equation for that model.

Probabilistic wages The calibration strategy again pins down values for θ, x via steady state labor market flows. The optimality condition for vacancy creation and accounting equation for firm value imply the same Euler equation (100) and wage expression (105) in steady state. The latter again implies that a steady state target for the wage requires holding κ_0 fixed. The optimality condition for the wage, together with the accounting equation for firm value and its derivative with respect to the wage imply:

$$X = \kappa_v(x)\frac{\mu(\theta) - \mu'(\theta)\theta}{\mu'(\theta)}\frac{\delta}{1 - \beta(1 - \delta)(1 - \alpha)}. \tag{107}$$

Holding the wage unchanged, this implies a corresponding value of $Y = X/\mu(\theta) - W$, and further an implied value of b such that $Y = -(b + \beta(1 - \delta)X)/(1 - \beta(1 - \delta))$.

D   Infrequent Adjustment with Aggregate Shocks

This section lays out the equations characterising the impact of infrequent adjustment in the presence of aggregate shocks.

Deviating Firm   Take an equilibrium with aggregate shocks where firms reoptimize each period, and consider a single deviating firm in that environment. The equilibrium determines

$X(S), Y(S), \hat{V}(S), W(S)$ as well as $\Lambda(S) = E_S \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-\alpha)^k \beta(1-\delta)\alpha W(S^{k+1})$.
The deviating firm's choice of wage is characterized by the first order condition

$$\frac{X(S)}{\theta^2}[1 + \frac{\mu'(\theta)\theta^2}{x\mu(\theta)^2}] = -q'(\theta)[z - b + \beta(1-\delta)E_S[\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w, S')]]$$
$$- \beta(1-\delta)(1-\alpha)(1 + q(\theta)x)/xE_S\hat{V}_w^f(w, S')/\theta_w,$$

where the tightness coinciding with the wage is determined by $X(S) = \mu(\theta)(\phi(w, S) + Y(S))$ where $\phi(w, S) = w/(1 - \beta(1-\delta)(1-\alpha)) + \Lambda(S)$, and the derivative of θ with respect to w is $\theta_w = -\mu(\theta)^2/(\mu'(\theta)X(S)(1 - \beta(1-\delta)(1-\alpha)))$.

The first order condition for vacancy creation reads

$$\kappa_v(x) + \frac{X(S)}{\theta} = q(\theta)(z - b + \beta(1-\delta)E_S(\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w, S'))),$$

for the deviation period and periods when the commitment is maintained.

The continuation value satisfies

$$\hat{V}^f(w, S) = (1 + q(\theta)x)(z - b + \beta(1-\delta)E_S(\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w, S'))) - \kappa(x) - X(S)(\frac{x}{\theta} + 1),$$

and the derivative of the continuation value

$$\hat{V}_w^f(w, S) = xq'(\theta)[z - b + \beta(1-\delta)E_z[\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w, S')]]\theta_w$$
$$+ \frac{xX(S)}{\theta^2}\theta_w + \beta(1-\delta)(1-\alpha)(1 + q(\theta)x)E_S\hat{V}_w^f(w, S').$$

Equilibrium with Infrequent Adjustment The equilibrium determines $X(S), Y(S), \hat{V}(S), W(S)$ as well as $\Lambda(S) = E_S \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-\alpha)^k \beta(1-\delta)\alpha W(S^{k+1})$.

The reoptimizing firm's choice of wage is characterized by the first order condition

$$\frac{X(S)}{\theta^2}[1 + \frac{\mu'(\theta)\theta^2}{x\mu(\theta)^2}] = -q'(\theta)[z - b + \beta(1-\delta)E_S[\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w, S')]]$$
$$- \beta(1-\delta)(1-\alpha)(1 + q(\theta)x)/xE_S\hat{V}_w^f(w, S')/\theta_w,$$

where the tightness coinciding with the wage is determined by $X(S) = \mu(\theta)(\phi(w, S) + Y(S))$ where $\phi(w, S) = w/(1 - \beta(1-\delta)(1-\alpha)) + \Lambda(S)$, and the derivative of θ with respect to w is $\theta_w = -\mu(\theta)^2/(\mu'(\theta)X(S)(1 - \beta(1-\delta)(1-\alpha)))$. The firm that holds the wage fixed, on the other hand, has tightness pinned down by the condition $X(S) = \mu(\theta)(\phi(w, S) + Y(S))$.

For both types of firms, the first order condition for vacancy creation reads

$$\kappa_v(x) + \frac{X(S)}{\theta} = q(\theta)(z - b + \beta(1-\delta)E_S(\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w, S'))),$$

with the corresponding wage and tightness (as discussed above).

The fixed-wage firms' continuation value satisfies

$$\hat{V}^f(w,S) = (1 + q(\theta)x)(z - b + \beta(1-\delta)E_S(\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w,S'))) - \kappa(x) - X(S)(\frac{x}{\theta} + 1),$$

and the derivative of the continuation value

$$\begin{aligned}
\hat{V}^f_w(w,S) =& xq'(\theta)[z - b + \beta(1-\delta)E_S[\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w,S')]]\theta_w \\
& + \frac{xX(S)}{\theta^2}\theta_w + \beta(1-\delta)(1-\alpha)(1 + q(\theta)x)E_S\hat{V}^f_w(w,S').
\end{aligned}$$

The reoptimizing firms' value satisfies

$$\hat{V}(S) = (1 + q(\theta)x)(z - b + \beta(1-\delta)E_S(\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w,S'))) - \kappa(x) - X(S)(\frac{x}{\theta} + 1),$$

where the tightness and wage are the reoptimized values in the prevailing state.

Solving for Equilibrium with Infrequent Adjustment The model is solved by linearization, following Gertler and Trigari (2009).

Given a wage w, we have the present value of wages:

$$W(w) = \frac{w}{1 - \beta(1-\delta)(1-\alpha)} + \beta(1-\delta)\alpha\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(1-\alpha)^kW_{t+k+1}$$

For short, let $\Lambda_t = \beta(1-\delta)\alpha\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(1-\alpha)^kW_{t+k+1}$, which satisfies the dynamic equation

$$\frac{\Lambda_t}{\beta(1-\delta)\alpha} = W_{t+1} + \beta(1-\delta)(1-\alpha)\frac{\Lambda_{t+1}}{\beta(1-\delta)\alpha}$$

First, I solve for a linear approximation to the firm continuation value when the wage is fixed: $V^f_t(w) - \bar{V} = V^0_t + V^1_t(w - \bar{w})$.

While a firm's wage w is fixed, the present value of wages at the firm follows:

$$W_t(w) - \bar{W} = (w - \bar{w})/(1 - \beta(1-\delta)(1-\alpha)) + \Lambda_t - \bar{\Lambda}$$

where the equilibrium contracting wages (not the wage held fixed w) determine Λ_t according to

$$(\Lambda_t - \bar{\Lambda})/\beta(1-\delta)\alpha = W_{t+1} - \bar{W} + \beta(1-\delta)(1-\alpha)(\Lambda_{t+1} - \bar{\Lambda})/\beta(1-\delta)\alpha.$$

The present value of wages $W_t(w)$ determines the tightness according to:

$$X_t - \bar{X} = \mu'(\bar{\theta})(\theta_t(w) - \bar{\theta})[\bar{W} + \bar{Y}] + \mu(\bar{\theta})[W_t(w) - \bar{W} + Y_t - \bar{Y}]$$

as a linear function $\theta(w, S) - \bar{\theta} = A_t + B(w - \bar{w})$ with

$$B = -\mu(\bar{\theta})/(\mu'(\bar{\theta})[\bar{W} + \bar{Y}](1 - \beta(1 - \delta)(1 - \alpha)))$$

$$A_t = (X_t - \bar{X} - \mu(\bar{\theta})(\Lambda_t - \bar{\Lambda} + Y_t - \bar{Y}))/(\mu'(\bar{\theta})[\bar{W} + \bar{Y}])$$

The firm's choice of x follows:

$$\kappa''(\bar{x})(x_t - \bar{x}) + \frac{X_t - \bar{X}}{\bar{\theta}} - \frac{\bar{X}}{\bar{\theta}^2}(\theta_t - \bar{\theta}) = q'(\bar{\theta})(\theta_t - \bar{\theta})(\bar{z} - b + \beta(1 - \delta)\bar{V}))$$
$$+ q(\bar{\theta})(z_t - \bar{z} + \beta(1 - \delta)(\alpha(V_{t+1} - \bar{V}) + (1 - \alpha)(V^0_{t+1} + V^1_{t+1}(w - \bar{w}))))$$

Substituting in for $\theta_t(w)$, this gives the hiring rate x as a linear function $x_t(w) - \bar{x} = \hat{A}_t + \hat{B}_t(w - \bar{w})$, where

$$\hat{B}_t = \frac{B\bar{X}}{\kappa''(\bar{x})\bar{\theta}^2} + \frac{Bq'(\bar{\theta})(\bar{z} - b + \beta(1 - \delta)\bar{V})}{\kappa''(\bar{x})} + \frac{q(\bar{\theta})}{\kappa''(\bar{x})}\beta(1 - \delta)(1 - \alpha)V^1_{t+1}$$

$$\hat{A}_t = -\frac{X_t - \bar{X}}{\kappa''(\bar{x})\bar{\theta}} + \frac{\bar{X}A_t}{\kappa''(\bar{x})\bar{\theta}^2} + \frac{q'(\bar{\theta})}{\kappa''(\bar{x})}(\bar{z} - b + \beta(1 - \delta)\bar{V})A_t + \frac{q(\bar{\theta})}{\kappa''(\bar{x})}(z_t - \bar{z} + \beta(1 - \delta)(\alpha(V_{t+1} - \bar{V}) + (1 - \alpha)V^0_{t+1})$$

Finally, the dynamic equation for the value $V^f(w, S)$ implies that for all such w we have:

$$V^0_t + V^1_t(w - \bar{w})$$
$$= z_t - \bar{z} + \beta(1 - \delta)(\alpha(V_{t+1} - \bar{V}) + (1 - \alpha)(V^0_{t+1} + V^1_{t+1}(w - \bar{w}))) + \bar{x}\kappa''(\bar{x})(x_t(w) - \bar{x}) - (X_t - \bar{X})$$

Using the expression for $x_t(w)$, the expression yields equations for the constant and coefficient on w for this equation to hold.

The coefficient on w:

$$V^1_t = \beta(1 - \delta)(1 - \alpha)V^1_{t+1} + \frac{\bar{x}\bar{X}B}{\bar{\theta}^2}$$
$$+ \bar{x}q'(\bar{\theta})(\bar{z} - b + \beta(1 - \delta)\bar{V})B + \bar{x}q(\bar{\theta})\beta(1 - \delta)(1 - \alpha)V^1_{t+1}$$

Note that this is an unstable equation with constant coefficients, implying the coefficient V^1_t is a constant. Further, $\hat{B}_t$ is also then a constant.

The constant:

$$V^0_t = z_t - \bar{z} + \beta(1 - \delta)(\alpha(V_{t+1} - \bar{V}) + (1 - \alpha)V^0_{t+1}) - (X_t - \bar{X})$$
$$- \frac{\bar{x}(X_t - \bar{X})}{\bar{\theta}} + \frac{\bar{x}\bar{X}A_t}{\bar{\theta}^2} + \bar{x}q'(\bar{\theta})(\bar{z} - b + \beta(1 - \delta)\bar{V})A_t$$
$$+ \bar{x}q(\bar{\theta})(z_t - \bar{z} + \beta(1 - \delta)(\alpha(V_{t+1} - \bar{V}) + (1 - \alpha)V^0_{t+1}))$$

This is a dynamic equation that is also unstable, but with coefficients that can vary over time. Add this equation into model system, to determine the coefficients (they enter into the system).

Second, proceed to solve for equilibrium.

Firms that are optimizing this period, choose a wage according to:

$$
\frac{X_t - \bar{X}}{\bar{\theta}^2} - 2\frac{\bar{X}}{\bar{\theta}^3}(\theta_t - \bar{\theta}) + \frac{\mu'(\bar{\theta})}{\bar{x}\mu(\bar{\theta})^2}(X_t - \bar{X})
$$
$$
- \frac{\mu'(\bar{\theta})\bar{X}}{\bar{x}^2\mu(\bar{\theta})^2}(x_t - \bar{x}) + \frac{\bar{X}}{\bar{x}}\frac{\mu(\bar{\theta})^2\mu''(\bar{\theta}) - 2\mu(\bar{\theta})(\mu'(\bar{\theta}))^2}{\mu(\bar{\theta})^4}(\theta_t - \bar{\theta})
$$
$$
= -q''(\bar{\theta})(\theta_t - \bar{\theta})(\bar{z} - b + \beta(1-\delta)\bar{V})
$$
$$
- q'(\bar{\theta})(z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)(V_{t+1}^0 + V_{t+1}^1(w_t - \bar{w}))))
$$
$$
- \beta(1-\delta)(1-\alpha)[q'(\bar{\theta})(\theta_t - \bar{\theta})\bar{V}^1/\bar{\theta}_w - (x_t - \bar{x})\bar{V}^1/(\bar{\theta}_w\bar{x}^2)
$$
$$
+ (1 + q(\bar{\theta})\bar{x})(V_{t+1}^1 - \bar{V}^1)/(\bar{\theta}_w\bar{x}) - (1 + q(\bar{\theta})\bar{x})\bar{V}^1(\theta_{wt} - \bar{\theta}_w)/(\bar{\theta}_w^2\bar{x})]
$$

with $\theta_t(w) = A_t + B(w - \bar{w})$, $x_t(w) = \hat{A}_t + \hat{B}_t(w - \bar{w})$ from above and

$$
(X_t - \bar{X})\frac{\bar{\mu}'}{\bar{\mu}^2}\bar{\theta}_w + \frac{\bar{\mu}'}{\bar{\mu}^2}\bar{X}(\theta_{wt} - \bar{\theta}_w) + \frac{\bar{\mu}^2\bar{\mu}'' - 2\bar{\mu}(\bar{\mu}')^2}{\bar{\mu}^4}\bar{X}\bar{\theta}_w(\theta_t - \bar{\theta}) = 0
$$

The rest of firms apply a previously set wage, and the cross-firm average wage follows: $\hat{w}_t = \alpha w_t + (1-\alpha)\hat{w}_{t-1}$.

The cross-firm average tightness and vacancy rate are: $\hat{\theta}_t = A_t + B(\hat{w}_t - \bar{w})$, $\hat{x}_t = \hat{A}_t + \hat{B}_t(\hat{w}_t - \bar{w})$.

The average firm size follows the law of motion:

$$
\hat{n}_{t+1} - \bar{n} = (1-\delta)((1 + q(\bar{\theta})\bar{x})(\hat{n}_t - \bar{n}) + \bar{n}q(\bar{\theta})(\hat{x}_t - \bar{x}) + \bar{n}q'(\bar{\theta})\bar{x}(\hat{\theta}_t - \bar{\theta}))
$$

Finally, the equilibrium adding up constraint reads:

$$
\frac{\bar{n}}{\bar{\theta}}(\hat{x}_t - \bar{x}) + \frac{\bar{x}}{\bar{\theta}}(\hat{n}_t - \bar{n}) - \frac{\bar{x}\bar{n}}{\bar{\theta}^2}(\hat{\theta}_t - \bar{\theta}) = -(\hat{n}_t - \bar{n})
$$

System equations read (in linear form)

$$(\Lambda_t - \bar{\Lambda})/\beta(1-\delta)\alpha = W_{t+1} - \bar{W} + \beta(1-\delta)(1-\alpha)(\Lambda_{t+1} - \bar{\Lambda})/\beta(1-\delta)\alpha.$$

$$W_t - \bar{W} = (w_t - \bar{w})/(1 - \beta(1-\delta)(1-\alpha)) + \Lambda_t - \bar{\Lambda}$$

$$(X_t - \bar{X})\frac{\bar{\mu}'}{\bar{\mu}^2}\bar{\theta}_w + \frac{\bar{\mu}'}{\bar{\mu}^2}\bar{X}(\theta_{wt} - \bar{\theta}_w) + \frac{\bar{\mu}^2\bar{\mu}'' - 2\bar{\mu}(\bar{\mu}')^2}{\bar{\mu}^4}\bar{X}\bar{\theta}_w(\theta_t - \bar{\theta}) = 0$$

$$\frac{X_t - \bar{X}}{\bar{\theta}^2} - 2\frac{\bar{X}}{\bar{\theta}^3}(\theta_t - \bar{\theta}) + \frac{\mu'(\bar{\theta})}{\bar{x}\mu(\bar{\theta})^2}(X_t - \bar{X})$$

$$- \frac{\mu'(\bar{\theta})\bar{X}}{\bar{x}^2\mu(\bar{\theta})^2}(x_t - \bar{x}) + \frac{\bar{X}}{\bar{x}}\frac{\mu(\bar{\theta})^2\mu''(\bar{\theta}) - 2\mu(\bar{\theta})(\mu'(\bar{\theta}))^2}{\mu(\bar{\theta})^4}(\theta_t - \bar{\theta})$$

$$= -q''(\bar{\theta})(\theta_t - \bar{\theta})(\bar{z} - b + \beta(1-\delta)\bar{V})$$

$$- q'(\bar{\theta})(z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)(V^0_{t+1} + V^1_{t+1}(w_t - \bar{w}))))$$

$$- \beta(1-\delta)(1-\alpha)[q'(\bar{\theta})(\theta_t - \bar{\theta})\bar{V}^1/\bar{\theta}_w - q(\bar{\theta})(x_t - \bar{x})\bar{V}^1/(\bar{\theta}_w x^2)$$

$$+ (1 + q(\bar{\theta})\bar{x})(V^1_{t+1} - \bar{V}^1)/(\bar{\theta}_w\bar{x}) - (1 + q(\bar{\theta})\bar{x})\bar{V}^1(\theta_{wt} - \bar{\theta}_w)/(\bar{\theta}_w^2\bar{x})]$$

$$V^0_t = z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1}) - (X_t - \bar{X})$$

$$- \frac{\bar{x}(X_t - \bar{X})}{\bar{\theta}} + \frac{\bar{x}\bar{X}A_t}{\bar{\theta}^2} + \bar{x}q'(\bar{\theta})(\bar{z} - b + \beta(1-\delta)\bar{V})A_t$$

$$+ \bar{x}q(\bar{\theta})(z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1}))$$

$$\hat{A}_t = -\frac{X_t - \bar{X}}{\kappa''(\bar{x})\bar{\theta}} + \frac{\bar{X}A_t}{\kappa''(\bar{x})\bar{\theta}^2} + \frac{q'(\bar{\theta})}{\kappa''(\bar{x})}(\bar{z} - b + \beta(1-\delta)\bar{V})A_t + \frac{q(\bar{\theta})}{\kappa''(\bar{x})}(z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1}))$$

$$A_t = (X_t - \bar{X} - \mu(\bar{\theta})(\Lambda_t - \bar{\Lambda} + Y_t - \bar{Y}))/(\mu'(\bar{\theta})[\bar{W} + \bar{Y}])$$

$$\hat{n}_{t+1} - \bar{n} = (1-\delta)((1 + q(\bar{\theta})\bar{x})(\hat{n}_t - \bar{n}) + \bar{n}q(\bar{\theta})(\hat{x}_t - \bar{x}) + \bar{n}q'(\bar{\theta})\bar{x}(\hat{\theta}_t - \bar{\theta}))$$

$$\frac{\bar{n}}{\bar{\theta}}(\hat{x}_t - \bar{x}) + \frac{\bar{x}}{\bar{\theta}}(\hat{n}_t - \bar{n}) - \frac{\bar{x}\bar{n}}{\bar{\theta}^2}(\hat{\theta}_t - \bar{\theta}) = -(\hat{n}_t - \bar{n})$$

$$\hat{w}_t = \alpha w_t + (1-\alpha)\hat{w}_{t-1}$$

$$\theta_t - \bar{\theta} = A_t + B(w_t - \bar{w})$$

$$x_t - \bar{x} = \hat{A}_t + \hat{B}(w_t - \bar{w})$$

$$\hat{\theta}_t - \bar{\theta} = A_t + B(\hat{w}_t - \bar{w})$$

$$\hat{x}_t - \bar{x} = \hat{A}_t + \hat{B}(\hat{w}_t - \bar{w})$$

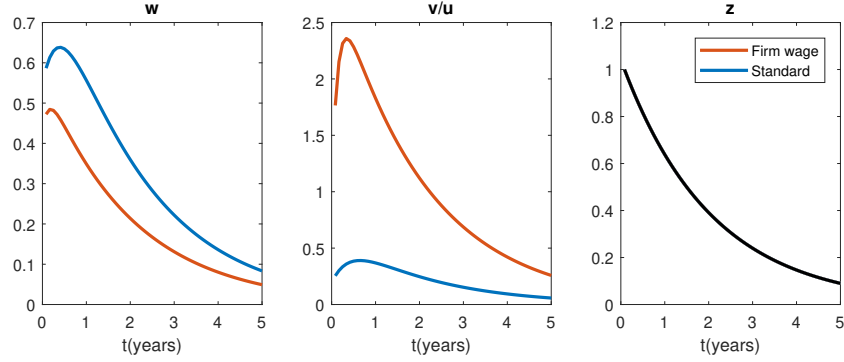$$V_t - \bar{V} = V^0_t + V^1(w_t - \bar{w})$$

E   Additional Figures

Figure 7: Impulse Responses with Identical Parameters

Notes: The figure plots the level responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and the standard competitive search model without firm wages. Productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.96$ and standard deviation $\sigma_z = 0.02$. The two models compared are parameterized identically. The plotted vacancy-unemployment ratio is its model counterpart, which differs slightly from θ due to timing.
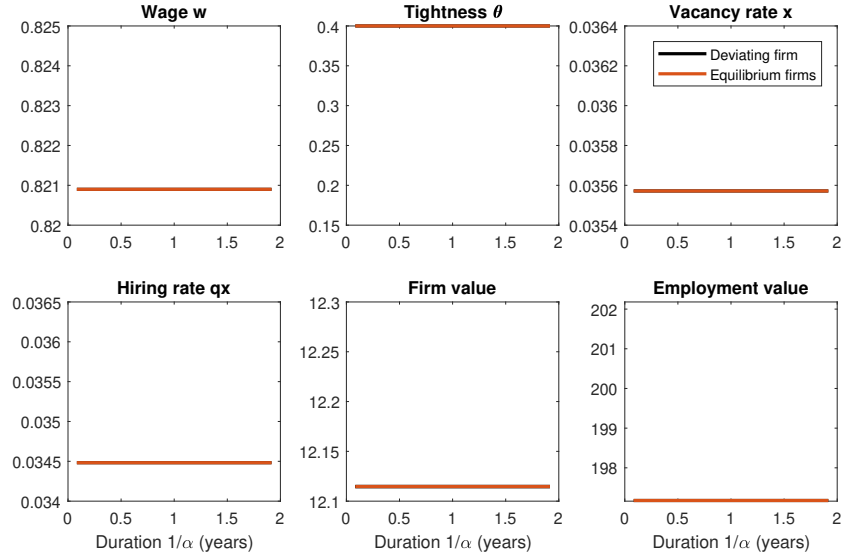

Figure 8: Single Firm Deviating in the Standard Competitive Search Model

Notes: The figure displays the steady-state values of a number of variables in a stationary equilibrium with competitive search, along with the corresponding values for an individual firm in that equilibrium that is able to set a wage commitment for a probabilistic period of time. The latter are plotted as a function of $1/\alpha$, the expected duration of the wage commitment. The firm value plotted is the scaled firm value per existing match.
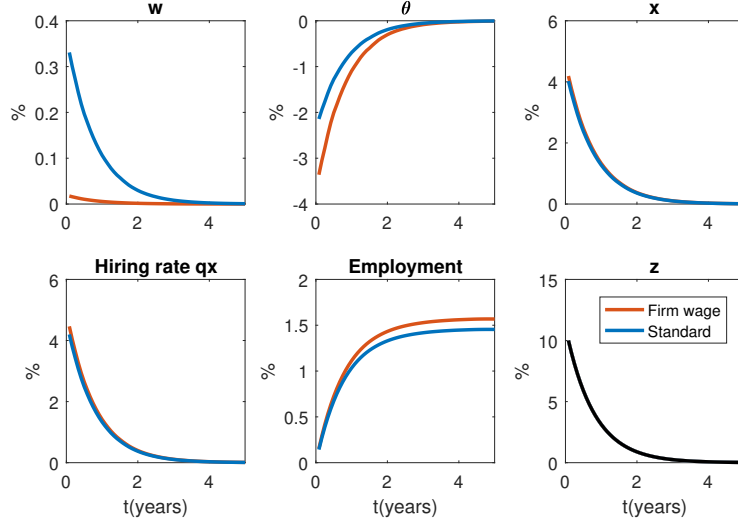
Figure 9: Impulse Responses in Firm Wage vs Standard Model

Notes: The figure plots the percentage responses of model variables to a one percent increase in firm-level labor productivity in the firm wage model and the standard competitive search model without firm wages. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.9$ and standard deviation $\sigma_z = 0.1$. The response is based on a quadratic approximation, produced with Dynare. The two models compared have the same steady-state levels of wage, tightness, unemployment.
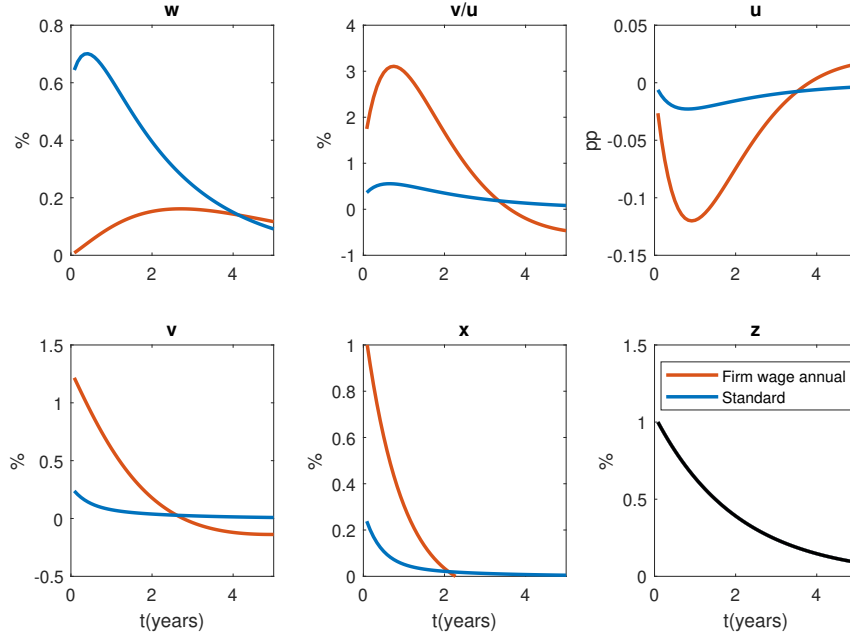

Figure 10: Impulse Responses with Annually Reset Wages vs Standard Model

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model with annual wage adjustment and the standard competitive search model without firm wages. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.96$ and standard deviation $\sigma_z = 0.02$. The two models compared have the same steady-state levels of wage, tightness, unemployment, as described in the section on parametrization. The plotted vacancy-unemployment ratio is its model counterpart, which differs slightly from θ due to timing.