The Lost Capital Asset Pricing Model

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Beta

Expected return (bps)

Savor and Wilson (2014)

- Announcement days
- Non-announcement days

Hendershott, Livdan, and Rösch (2018)

- Close-to-Open (Night)
- Open-to-Close (Day)

CAPM widely used in the industry (Graham and Harvey, 2001; Barber, Huang, and Odean, 2016; Berk and Van Binsbergen, 2016)

Why is the Securities Market Line flat?

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Why is the Securities Market Line flat?
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- CAPM widely used in the industry (Graham and Harvey, 2001; Barber, Huang, and Odean, 2016; Berk and Van Binsbergen, 2016)

- Why is the Securities Market Line flat?
\[ \mathbb{V}[R] = \mathbb{E}[\mathbb{V}[R | \mathcal{F}]] + \mathbb{V}[\mathbb{E}[R | \mathcal{F}]] \]
\[ \text{V}[R] = \mathbb{E} \left[ \text{V}[R | \mathcal{F}] \right] + \text{V} \left[ \mathbb{E}[R | \mathcal{F}] \right] \]

Measured by
the econometrician
\[ \mathbb{V}[R] = \mathbb{E}[\mathbb{V}[R | \mathcal{F}]] + \mathbb{V}[\mathbb{E}[R | \mathcal{F}]] \]

Measured by the econometrician

Equilibrium model
\[ V[R] = \mathbb{E}[V[R \mid \mathcal{F}]] + V[E[R \mid \mathcal{F}]] \]

Measured by the econometrician

Equilibrium model

\[ V[R] = \widehat{V}[R] + V[\widehat{E}[R]] \]
\[ \mathbb{V}[R] = \mathbb{E}(\mathbb{V}[R | \mathcal{F}]) + \mathbb{V}(\mathbb{E}[R | \mathcal{F}]) \]

Measured by the econometrician

Equilibrium model

\[ \mathbb{V}[R] = \mathbb{V}[\hat{R}] + \mathbb{V}[\hat{\mathbb{E}}[R]] \]

\[ \text{Cov}[R, R_{\bar{M}}] = \text{Cov}[R, R_{\bar{M}}] + \text{Cov}[\hat{\mathbb{E}}[R], \hat{R}_{\bar{M}}] \]
\[ \Sigma[R] = \text{E}[\Sigma[R | \mathcal{F}]] + \text{V}[\text{E}[R | \mathcal{F}]] \]

Measured by the econometrician

Equilibrium model

\[ \Sigma[R] = \hat{\Sigma}[R] + \Sigma[\hat{\text{E}}[R]] \]

\[ \text{Cov}[R, R^{-}_{M}] = \hat{\text{Cov}}[R, R^{-}_{M}] + \text{Cov}[\hat{\text{E}}[R], \hat{\text{E}}[R^{-}_{M}]] \]

⇒ true betas  ⇒ distortion

The CAPM holds unconditionally, but fails empirically (Type I error)

The CAPM looks “flat”

The empiricist observes a stronger CAPM on announcement days (when the market risk premium is higher) or at night (when there is less mispricing and less informational trading)
\[ \nabla[R] = \mathbb{E}[\nabla[R|\mathcal{F}]] + \nabla[\mathbb{E}[R|\mathcal{F}]] \]

Measured by the econometrician

Equilibrium model

\[ \nabla[R] = \hat{\nabla}[R] + \nabla[\hat{\mathbb{E}}[R]] \]

\[ \text{Cov}[R, R_{\mathcal{M}}] = \hat{\text{Cov}}[R, R_{\mathcal{M}}] + \text{Cov}[\hat{\mathbb{E}}[R], \hat{\mathbb{E}}[R_{\mathcal{M}}]] \]

⇒ true betas ⇒ distortion

The CAPM holds unconditionally, but fails empirically (Type I error)
\[ \mathbb{V}[R] = \mathbb{E}[\mathbb{V}[R \mid \mathcal{F}]] + \mathbb{V} [\mathbb{E}[R \mid \mathcal{F}]] \]

Measured by
the econometrician

Equilibrium
model

\[ \mathbb{V}[R] = \hat{\mathbb{V}}[R] + \mathbb{V}[\hat{\mathbb{E}}[R]] \]

\[ \text{Cov}[R, R_\hat{M}] = \hat{\text{Cov}}[R, R_\hat{M}] + \text{Cov}[\hat{\mathbb{E}}[R], \hat{\mathbb{E}}[R_\hat{M}]] \]

⇒ true betas
⇒ distortion

- The CAPM holds unconditionally, but fails empirically (Type I error)
- The CAPM looks “flat”
\[ V[R] = \mathbb{E} [V[R \mid \mathcal{F}]] + \mathbb{V} [E[R \mid \mathcal{F}]] \]

- Measured by the econometrician
- Equilibrium model

\[ V[R] = \hat{V}[R] + \mathbb{V} [\hat{E}[R]] \]
\[ \text{Cov}[R, R_{\tilde{M}}] = \hat{\text{Cov}}[R, R_{\tilde{M}}] + \text{Cov}[\hat{E}[R], \hat{E}[R_{\tilde{M}}]] \]
\[ \Rightarrow \text{true betas} \quad \Rightarrow \text{distortion} \]

- The CAPM holds unconditionally, but fails empirically (Type I error)
- The CAPM looks “flat”
- The empiricist observes a stronger CAPM on announcement days (when the market risk premium is higher) or at night (when there is less mispricing and less informational trading)
This is not an argument that the unconditional CAPM does not hold
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Merton (1987), Black (1972), Frazzini and Pedersen (2014), Cohen, Polk,
and Vuolteenaho (2005), Hong and Sraer (2016), Kumar (2009), Bali,
Cakici, and Whitelaw (2011), Antoniou, Doukas, and Subrahmanyam
(2015), Campbell, Giglio, Polk, and Turley (2012), Baker, Bradley, and

This is not an argument that the CAPM holds conditionally, but not
unconditionally (Hansen and Richard, 1987; Jagannathan and Wang, 1998)
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This is not an argument that the market proxy is incorrect (Roll, 1977;
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This is not an argument that the market proxy is incorrect (Roll, 1977; Stambaugh, 1982; Dybvig and Ross, 1985; Roll and Ross, 1994)
Payoffs:

\[
\begin{bmatrix}
D_1 \\
D_2 \\
\vdots \\
D_N
\end{bmatrix} = \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_N
\end{bmatrix} F + \begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\vdots \\
\epsilon_N
\end{bmatrix}
\]
Payoffs:

\[
\begin{bmatrix}
D_1 \\
D_2 \\
\vdots \\
D_N \\
\end{bmatrix}
= \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_N \\
\end{bmatrix}
\begin{bmatrix}
F + \\
\vdots \\
F + \\
\end{bmatrix}
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\vdots \\
\epsilon_N \\
\end{bmatrix}
\]

Information:

Private: \( V_i = F + v_i \)

Public: \( G = F + v \)
• Payoffs:

\[
\begin{bmatrix}
D_1 \\
D_2 \\
\vdots \\
D_N \\
\end{bmatrix} = \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_N \\
\end{bmatrix} F + \begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\vdots \\
\epsilon_N \\
\end{bmatrix}
\]

• Information:

Private: \( V_i = F + v_i \)

Public: \( G = F + \nu \)

• Supply of assets \( [M_1, M_2, \ldots M_N]' \) with mean \( \bar{M} \equiv [\frac{1}{N}, \frac{1}{N}, \ldots \frac{1}{N}]' \)
Payoffs:

\[
\begin{bmatrix}
D_1 \\
D_2 \\
\vdots \\
D_N
\end{bmatrix} = \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_N
\end{bmatrix} F + \begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\vdots \\
\epsilon_N
\end{bmatrix}
\]

Information:

Private: \( V_i = F + v_i \)

Public: \( G = F + v \)

Supply of assets \( \left[ M_1, M_2, \ldots, M_N \right]' \) with mean \( \overline{M} \equiv \left[ \frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N} \right]' \)

Proposition 1 (Equilibrium):

\[
\begin{bmatrix}
P_1 \\
\vdots \\
P_N
\end{bmatrix} = \begin{bmatrix}
\alpha_1 \\
\vdots \\
\alpha_N
\end{bmatrix} F + \begin{bmatrix}
g_1 \\
\vdots \\
g_N
\end{bmatrix} G + \begin{bmatrix}
\xi_{11} & \cdots & \xi_{1N} \\
\vdots & \ddots & \vdots \\
\xi_{N1} & \cdots & \xi_{NN}
\end{bmatrix} \begin{bmatrix}
M_1 \\
\vdots \\
M_N
\end{bmatrix}
\]
Each agent observes: private signal, public signal, prices
All agents have the same precision of information:
\[ \tau \equiv \nabla[F | \mathcal{F}^i]^{-1} \]
Each agent observes: private signal, public signal, prices
All agents have the same precision of information:

$$\tau \equiv \nabla[F \mid \mathcal{F}^i]^{-1}$$

The econometrician observes: realized returns
Each agent observes: private signal, public signal, prices
All agents have the same precision of information:

\[ \tau \equiv \nabla [F \mid \mathcal{F}^i]^{-1} \]

The econometrician observes: realized returns

Informational distance:

\[ \nabla [R] = \hat{\nabla} [R] + \nabla \left[ \hat{E} [R] + \Phi \frac{\tau_v}{\tau} v_i \right] \]
Each agent observes: private signal, public signal, prices
All agents have the same precision of information:

\[ \tau \equiv \nabla[F | \mathcal{F}^i]^{-1} \]

The econometrician observes: realized returns

Informational distance:

\[ \nabla[R] = \hat{\nabla}[R] + \nabla \left[ \hat{\mathbb{E}}[R] + \Phi \frac{\tau_v}{\tau} v_i \right] \]

Lemma 1:

\[ \text{Cov}[R, R_M] = \left( 1 + \frac{\gamma^2}{\tau_M \tau_\epsilon} \right) \hat{\text{Cov}}[R, R_M] + \frac{\Phi \kappa}{\tau} \Phi \]

where

\[ \kappa \equiv \frac{\gamma^2}{\tau_M} \left( \frac{1}{\tau_\epsilon} + \frac{\Phi' \Phi}{\tau} \right) + \frac{\tau_v}{\tau} > 0 \]
True unconditional CAPM:

\[
\mathbb{E}[R] = \frac{\hat{\text{Cov}}[R, R_M]}{\hat{\text{V}}[R_M]} \mathbb{E}[R_M] \\
\beta
\]
True unconditional CAPM:

$$\mathbb{E}[R] = \frac{\hat{\text{Cov}}[R, R_M]}{\hat{\text{V}}[R_M]} \mathbb{E}[R_M]$$

Theorem 1 (Econometrician’s CAPM):

$$\mathbb{E}[R_n] = \frac{\delta}{1 + \delta} \mathbb{E}[R_M] + \frac{\text{Cov}[R, R_M]}{\text{V}[R_M]} \frac{\mathbb{E}[R_M]}{1 + \delta}$$

where

$$\delta \equiv \frac{1}{N \text{V}[R_M]} \left[ \frac{\gamma^2}{\tau_M \tau_\epsilon} \left( \frac{1}{\tau_\epsilon} + \frac{\Phi' \Phi}{\tau} \right) + \frac{\tau_v}{\tau \tau_\epsilon} \right] > 0$$
Theorem 1 (Relationship between betas):

\[ \tilde{\beta}_1 - \beta_1 = (1 + \delta)(\beta - \beta_1) \]

Expected Return

\[ \mathbb{E}[R_{\bar{M}}] \]

Expected Return vs. Beta

\[ \tilde{\beta}_1, \beta_1, 1, \beta_2, \tilde{\beta}_2 \]

- - - True SML
Theorem 1 (Relationship between betas):

$$\tilde{\beta}_1 - 1 = \left(1 + \delta\right) \left(\beta - 1\right)$$
Theorem 1 (Relationship between betas):

\[ \bar{\beta} - 1 = (1 + \delta)(\beta - 1) \]
Betting Against Beta (BAB)

▶ Empiricist’s SML:

\[
\mathbb{E}[R_n] = \frac{\delta}{1+\delta} \mathbb{E}[R_M] + \tilde{\beta}_n \frac{\mathbb{E}[R_M]}{1+\delta}
\]

Long low-beta stocks \((\tilde{\beta}_L < 1)\), leveraged to a beta of one:

\[
\mathbb{E}[R_L] = \frac{1}{1+\delta} \mathbb{E}[R_M] + \mathbb{E}[R_M] \frac{1}{1+\delta}
\]

Short high-beta stocks \((\tilde{\beta}_H > 1)\), de-leveraged to a beta of one:

\[
-\mathbb{E}[R_H] = -\frac{1}{1+\delta} \mathbb{E}[R_M] - \mathbb{E}[R_M] \frac{1}{1+\delta}
\]

⇒ BAB earns positive expected excess returns:

\[
\mathbb{E}[R_{BAB}] = \left(1 - \frac{\tilde{\beta}_L - 1}{\tilde{\beta}_H - 1}\right) \frac{\delta}{1+\delta} \mathbb{E}[R_M] + \mathbb{E}[R_M] \frac{1}{1+\delta} > 0
\]

▶ Betting against measured beta really is betting on true beta.
Betting Against Beta (BAB)

Empiricist’s SML:

\[
\mathbb{E}[R_n] = \frac{\delta}{1 + \delta} \mathbb{E}[R_M] + \tilde{\beta}_n \frac{\mathbb{E}[R_M]}{1 + \delta}
\]

1. Long low-beta stocks \((\tilde{\beta}_L < 1)\), leveraged to a beta of one:

\[
\mathbb{E}[R_L] = \frac{1}{\tilde{\beta}_L} \frac{\delta}{1 + \delta} \mathbb{E}[R_M] + \frac{\mathbb{E}[R_M]}{1 + \delta}
\]

2. Short high-beta stocks \((\tilde{\beta}_H > 1)\), de-leveraged to a beta of one:

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Betting Against Beta (BAB)

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1. Long low-beta stocks ($\tilde{\beta}_L < 1$), leveraged to a beta of one:

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\mathbb{E}[R_L] = \frac{1}{\tilde{\beta}_L} \frac{\delta}{1 + \delta} \mathbb{E}[R_M] + \frac{\mathbb{E}[R_M]}{1 + \delta}
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▶ ⇒ BAB earns positive expected excess returns:

\[
\mathbb{E}[R_{BAB}] = \left( \frac{1}{\tilde{\beta}_L} - \frac{1}{\tilde{\beta}_H} \right) \frac{\delta}{1 + \delta} \mathbb{E}[R_M] > 0
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Betting Against Beta (BAB)

- Empiricist's SML:

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\mathbb{E}[R_n] = \frac{\delta}{1+\delta} \mathbb{E}[R_M] + \tilde{\beta}_n \frac{\mathbb{E}[R_M]}{1+\delta}
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- ⇒ BAB earns positive expected excess returns:

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\mathbb{E}[R_{BAB}] = \left( \frac{1}{\tilde{\beta}_L} - \frac{1}{\tilde{\beta}_H} \right) \frac{\delta}{1+\delta} \mathbb{E}[R_M] > 0
\]

- Betting against measured beta really is betting on true beta
A dynamic setup with periodic public announcements

\[ t - T \rightarrow \ldots \rightarrow t - 1 \rightarrow t \rightarrow t + 1 \rightarrow \ldots \rightarrow t + T \]

A-day

\[ a \]

Expected return

\[ \text{True CAPM} \]

\[ \text{Observed CAPM} \]
A dynamic setup with periodic public announcements

A-day

\[ t - T \]
\[ t - 1 \]
\[ t \]
\[ t + 1 \]
\[ t + T \]

Announcement cycle

A-day

Expected return

\( a \)

True CAPM

\( b \)

Observed CAPM

The Lost Capital Asset Pricing Model
A dynamic setup with periodic public announcements

\[ t - T \rightarrow \cdots \rightarrow t - 1 \rightarrow t \rightarrow t + 1 \rightarrow \cdots \rightarrow t + T \]

Announcement cycle

(a) True CAPM

(b) Observed CAPM

Expected return vs. Beta

**True CAPM**
- Red line: A-days
- Blue dashed line: N-days

**Observed CAPM**
- Red line: A-days
- Blue dashed line: N-days
Plausible values for the distortion $\delta$

$$\text{Cov}[R, R_M] = \widehat{\text{Cov}}[R, R_M] + \text{Cov}[\widehat{\mathbb{E}}[R], \widehat{\mathbb{E}}[R_M]]$$
Plausible values for the distortion $\delta$

$$\text{Cov}[R, R_M^\text{\tiny M}] = \hat{\text{Cov}}[R, R_M^\text{\tiny M}] + \text{Cov}[\hat{\text{E}}[R], \hat{\text{E}}[R_M^\text{\tiny M}]]$$

“Expected returns vary over time” = “Returns are predictable”
Plausible values for the distortion $\delta$

\[ \text{Cov}[R, R_{\bar{M}}] = \hat{\text{Cov}}[R, R_{\bar{M}}] + \text{Cov}[\hat{\mathbb{E}}[R], \hat{\mathbb{E}}[R_{\bar{M}}]] \]

“Expected returns vary over time” = “Returns are predictable”

\[ R^2 = \frac{\text{Var}[\hat{\mathbb{E}}[R_{\bar{M}}]]}{\text{Var}[R_{\bar{M}}]} \]
Plausible values for the distortion $\delta$

$$\text{Cov}[R, R_M] = \text{Cov}[\hat{\mathbb{E}}[R], \hat{\mathbb{E}}[R_M]] + \text{Cov}[\hat{\mathbb{E}}[R], \hat{\mathbb{E}}[R_M]]$$

“Expected returns vary over time” = “Returns are predictable”
Further work: Type II error
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(a) Multiple factors

(b) Heterogeneous sizes

\[ E[R_n] = \delta E[R_M] + \tilde{\beta}n + f(\delta) \]

Expected returns

Beta

Beta

The Lost Capital Asset Pricing Model
Further work: Type II error

(a) Multiple factors

\[ E[R_n] = \frac{\delta E[R_{\bar{M}}]}{1+\delta} + \frac{E[R_{\bar{M}}]}{1+\delta} \tilde{\beta}_n + f(\delta) Size_n + g(\delta) Price_n \]

(b) Heterogeneous sizes


References II


