Financial Vulnerability and Monetary Policy

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Financial vulnerability: Amplification mechanisms in the financial sector

Two questions are hotly debated

1. Does monetary policy impact the degree of financial vulnerability?

2. Should monetary policy take financial vulnerability into account?
Financial Variables Predict Tail of Output Gap Distribution

Based on “Vulnerable Growth” by Adrian, Boyarchenko and Giannone (AER, 2018)
Conditional Mean-Volatility Line for Output Gap Growth

\[ \text{Mean} = 0.67 - 1.15 \text{Volatility} + \varepsilon \]
Conditional Mean-Volatility Relation for Inflation
Patterns Hold in Panel of Countries

Advanced Economies

[1973-1990] Mean = 0.29 - 0.97 Vol + \( \varepsilon \)
[1991-2016] Mean = 1.67 - 5.49 Vol + \( \varepsilon \)

Emerging Economies

[1973-1990] Mean = 0.29 - 0.97 Vol + \( \varepsilon \)
[1991-2016] Mean = 1.67 - 5.49 Vol + \( \varepsilon \)

Conditional Mean

Conditional Vol

1973-1990
1991-2016
Fitted 1973-1990
Fitted 1991-2016

1991-1995
1996-2016
Fitted 1991-1995
Fitted 1996-2016
Overview of Microfounded Non-Linear Model

- Firm optimization gives standard New Keynesian Phillips Curve
- Households are as in New Keynesian model but
  - Cannot finance firms directly
  - Can trade any financial assets (stocks, riskless deposits, etc.) with banks
- Banks
  - Finance firms
  - Trade financial assets among themselves and with households
  - Have a preference (risk aversion) shock
  - Subject to Value-at-Risk constraint
- Financial markets are complete but prices are distorted
Price of Risk and No Arbitrage

- Single source of risk: Browninan motion $Z_t$
- Real risk-free rate is $R_t$
- A state price density (SPD) is a process with $Q_0 \equiv 1$ and

$$\frac{dQ_t}{Q_t} \equiv -R_t dt - \eta_t dZ_t$$

such that for all assets $j$

$$S_{j,t} = \frac{1}{Q_t} \mathbb{E}_t \left[ \int_t^\infty Q_s D_{j,s} ds \right]$$

where $\eta_t$ is the “market price of risk”
- Expected excess returns $\mu_t$, volatility $\sigma_t$ and $\eta_t = \sigma_t^{-1} \mu_t$
The Intermediation Sector Setup

- Each “bank” solves a standard Merton portfolio choice problem augmented by a Value-at-Risk constraint and preference shocks.

\[
V(X_t) = \max_{\{\theta_t, \delta_t\}} \mathbb{E}_t \left[ \int_t^\infty e^{-\beta(u-t)} e^{\zeta_u} \log(\delta_u X_u) \, du \right]
\]

subject to

\[
\frac{dX_t}{X_t} = (R_t - \delta_t + \theta_t \mu_t) \, dt + \theta_t \sigma_t \, dZ_t
\]

\[
\text{VaR}_{\tau,\alpha}(X_t) \leq a \sqrt{X_t}
\]

\[
d\zeta_t = -\frac{1}{2} s_t^2 \, dt - s_t \, dZ_t
\]

\[
ds_t = -\kappa(s_t - \bar{s}) + \sigma_s \, dZ_t
\]
The Intermediation Sector Setup

\[ V(\mathbb{X}_t, t) = \max_{\{\theta_t, \delta_t\}} E^\text{bank}_t \left[ \int_t^\infty e^{-\beta(u-t)} \log (\delta_u \mathbb{X}_u) \, du \right] \]

s.t.

\[ \frac{d\mathbb{X}_t}{\mathbb{X}_t} = (R_t - \delta_t + \theta_t \mu_t - \theta_t \sigma_t s_t) \, dt + \theta_t \sigma_t \, dZ^s_t \]

\[ \text{VaR}_{\tau, \alpha} (\mathbb{X}_t) \leq a_V \mathbb{X}_t \]

\[ ds_t = -\kappa (s_t - \bar{s}) + \sigma_s \, dZ^s_t \]
The Banks’ VaR Constraint and Amplification

- Let $\hat{X}_t$ be projected wealth with fixed portfolio weights from $t$ to $t + \tau$
- $\text{VaR}_{\tau, \alpha} (X_t)$ is the $\alpha^{th}$ quantile of the distribution of $\hat{X}_{t+\tau}$ conditional on time-$t$ information
Intermediation Sector

Optimal Portfolio and Dividends

The optimal portfolio is characterized by

\[ \theta_t = \frac{1}{\gamma_t} \left( \frac{\mu_t}{\sigma_t^2} - \frac{s_t}{\sigma_t} \right) \]

\[ \delta_t = u(\gamma_t) \beta \]

\[ \gamma_t \in (1, \infty) \text{ such that: } \text{VaR}_{\tau, \alpha}(X_t) = X_t a_V \]

or \[ \gamma_t = 1 \text{ if VaR does not bind} \]
State-Price Density of Intermediaries

- Lagrange multiplier of VaR is increasing in $\eta_t$ and $\gamma_t$

$$\lambda_{VaR,t} = G(\eta_t, \gamma_t, s_t)$$

- The marginal value of one unit of wealth is

$$Q_{bank}^t = e^{\zeta_t} e^{-\beta t} (\delta_t X_t)^{-1} (1 - \lambda_{VaR,t})$$
$$= e^{\zeta_t} e^{-\beta t} (\delta_t X_t)^{-1} (1 - G(\eta_t, \gamma_t, s_t))$$
Representative Household

Household solves

$$\max_{\{C_t, N_t, \omega_t\}} \mathbb{E}_t \left[ \int_t^\infty e^{-\beta(u-t)} \left( \frac{C_u^{1-\sigma}}{1-\sigma} - \frac{N_u^{1+\varsigma}}{1+\varsigma} \right) du \right]$$

subject to

$$d(P_t F_t) = W_t N_t dt - P_t C_t dt + \omega_t d(P_t S_t)$$

$$\omega_{\text{goods},t} = 0$$
Households and Intermediaries Agree on Pricing

➤ The household’s SPD is

\[ Q_{house}^t = e^{-\beta t} C_t^{-\gamma} \]

➤ The household’s Euler equation and market clearing \((C_t = Y_t)\) give the IS equation

\[ d \log Y_t = \frac{1}{\gamma} \left( i_t - \pi_t - \beta + \frac{1}{2} \eta_t^2 \right) dt + \frac{\eta_t}{\gamma} dZ_t \]

where \(i_t\) is the nominal interest rate
Households and Intermediaries Agree on Pricing

- Banks and households trading in complete markets means marginal utilities agree $Q_t^{house} = Q_t^{bank}$
- Matching the volatility of $Q_t^{house}$ and $Q_t^{bank}$

$$\eta_t = \eta(\gamma_t, s_t) = \eta(V_t, s_t)$$

where

$$V_t \equiv \text{VaR}_{\tau, \alpha} (dy_t)$$

$$= -\tau \mathbb{E}_t[dy_t/dt] - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} \text{Vol}_t(dy_t/dt)$$
Power of Continuous Time

- Can solve banks’ VaR problem in closed form (even if markets were incomplete)
- Linearizing drift and stochastic parts retains time variation in risk premium

\[ dy_t = \frac{1}{\gamma} (i_t - r - \pi_t) \, dt + \xi (V_t - s_t) \, dZ_t \]

\[ V_t = -\tau (i_t - r - \pi_t) / \gamma - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} \xi (V_t - s_t) \]

- Need at least 3rd order approximation in discrete time
Central bank solves

\[ L = \min_{\{y_s, \pi_s, i_s\}} \mathbb{E}_t \int_t^\infty e^{-s\beta} (y_s^2 + \lambda \pi_s^2) ds \]

subject to

**Dynamic IS:** \[ dy_t = \frac{1}{\gamma} (i_t - r - \pi_t) dt + \xi (V_t - s_t) dZ_t \]

**NKPC:** \[ d\pi_t = (\beta \pi_t - \kappa y_t) dt \]

**Vulnerability:** \[ V_t = -\tau \mathbb{E}_t [dy_t/dt] - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} Vol_t(dy_t/dt) \]

**Bank shocks:** \[ ds_t = -\kappa (s_t - \bar{s}) + \sigma_s dZ_t \]
The Optimal Monetary Policy

- Augmented Taylor

\[ i_t = \phi_0 + \phi_\pi \pi_t + \phi_y y_t + \phi_v V_t \]
\[ = \Phi(V_t) \]

- Or flexible inflation targeting

\[ \pi_t = \psi_0 + \psi_y y_t + \psi_v V_t + \psi_s s_t \]

- Coefficients \( \phi \) and \( \psi \) are a function of structural parameters that govern vulnerability
Optimal Monetary Policy

Risk-Taking Channel of Monetary Policy

- Fixed prices for simplicity ($\pi = 0$)
- Using the IS equation

$$dy_t = \frac{1}{\gamma} (i_t - r) \, dt + \xi (V_t - s_t) \, dZ_t$$

- We can plug

$$\mathbb{E}_t[dy_t/dt] = \frac{1}{\gamma} (i_t - r)$$

$$Vol_t(dy_t/dt) = \xi (V_t - s_t)$$

into

$$V_t = -\tau \mathbb{E}_t[dy_t/dt] - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} \, Vol_t(dy_t/dt)$$

to see that $V_t$ and $i_t$ are one-to-one: The risk-taking channel of monetary policy
Output Gap Mean-Volatility Tradeoff

- Eliminating $i_t$, dynamics of the economy are

$$dy_t = \xi \left( M V_t + \frac{N^{-1}(\alpha)}{\sqrt{\tau}} s_t \right) dt + \xi (V_t - s_t) dZ_t$$

where

$$M \equiv -\frac{\xi + N^{-1}(\alpha) \sqrt{\tau}}{\tau \xi}$$

is the slope of the mean-volatility line for output gap

$$\mathbb{E}_t \left[ \frac{dy_t}{dt} \right] = M Vol_t \left( \frac{dy_t}{dt} \right) - \frac{1}{\tau} s_t$$
Recall Mean-Vol Line for Output Gap Growth

\[ \text{Mean} = 0.67 - 1.15 \text{ Volatility} + \varepsilon \]
Model Produces Mean-Vol Line for Output Gap Growth
Model Produces Mean-Vol Line for Output Gap Growth
Model Produces Mean-Vol Line for Output Gap Growth
Calibration

- Use standard New Keynesian parameters when possible
- For parameters relating to vulnerability, match empirical and model-based regression of the conditional mean on the conditional volatility of output gap growth
- Intercept, slope, standard deviation and AR(1) coefficient of residuals identify all new parameters
Interest Rate Response to Vulnerability

The graph illustrates the relationship between the interest rate (R) and financial vulnerability (V). The blue line represents the optimal policy, while the red line represents the Taylor rule. The shaded area indicates the frequency of financial vulnerabilities, with darker shades representing higher frequencies.

LAW

Clean up

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Welfare Gains: Steady State Distribution of Output Gap
Conclusion

- The NK model can be augmented by
  - A financial sector that intermediates subject to a Value-at-Risk constraint
  - Shocks to financial sector
- Matches the stylized fact that conditional upper GDP quantiles are constant, while lower GDP quantiles move with financial conditions
- Mathematically tractable
- Optimal monetary policy always depends on vulnerability
  - Optimal monetary policy conditions on vulnerability
  - Vulnerability responds to monetary policy
  - LAW or clean up after crisis depending on vulnerability
  - Magnitudes are potentially large quantitatively