Declining Interest Rates and Firm Dynamics: Falling Startups and Rising Concentration

Satyajit Chatterjee and Burcu Eyigungor

Federal Reserve Bank of Philadelphia

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Abstract

Bigger COMPUSTAT firms (by sales) have lower coefficient of variation of sales and higher leverage ratios. This pattern is explained using a model of firm growth and entry in which firms produce multiple varieties and borrow (with the option to default) against their future cash flow. A variety can die with a constant probability, implying that bigger firms (those with more varieties) have lower coefficient of variation of sales and higher leverage ratios. A lower risk-free rate benefits larger firms more as they are more able to borrow, leading to lower startup rates and greater concentration of sales in large firms.

Keywords: Startup rates, leverage, firm dynamics

JEL Codes: E22 E43 E44 G32 G33 G34

Satyajit.Chatterjee@phil.frb.org. Suggestions and comments from conference participants at the 2018 Spring Midwest Macro Meeting are gratefully acknowledged.

1 Introduction

In this paper, we document that among COMPUSTAT firms, leverage is increasing in firm size and volatility of sales growth is declining in firm size. We propose a model of firm dynamics with borrowing and default that is consistent with these facts and then explore the model's potential to explain the decline in the business startup rate and the rise in business concentration of US firms since the late 1990s.

In our model, the facts regarding leverage, sales volatility and firm size are explained in the following way. First, business owners are assumed to be more impatient than lenders and therefore seek to borrow against the future cash flow of their companies. Second, firms manage different numbers of varieties (of products) and each variety is assumed to be subject to an independent extinction shock. This setup implies that firms with larger numbers of varieties have a less volatile cash flow per variety and, therefore, can (and do) sustain a higher level of debt per variety.

Firm entry and firm growth are driven by the continual arrival of ideas for new varieties, with the number of ideas ocurring in a firm being proportional to the number of varieties owned by it. Ex-ante, all new ideas look equally profitable and the firm selects one idea for further consideration. If the realized match between the idea and the firm is good enough, the idea is bought by the firm implemented by it. If the match is not good enough, the idea leaves the firm and is independently implemented in a startup, along with all the other new ideas that occurred in the firm but were not considered for purchase. The key benefit of implementing an idea in an existing firm is that once the firm has successfully absorbed the new variety, it can borrow against a greater fraction of the future cash flow of the new variety compared to a startup.

In our explanation of the decline in the startup rate and of the rise in business concentration since the late 1990s, the driving force is the decline in the risk-free interest rate over the same period. In the model, a decline in the risk-free interest rate benefits bigger firms more than smaller firms, as bigger firms are more able to borrow against their future cash flow. This makes bigger firms more willing to buy new ideas and implies both a lower rate startup rate and a higher fraction of total sales in bigger firms. To illustrate the quantitative significance of this channel, we show that our model is capable of generating the decline in the startup rates between 1997 and 2015, given the decline in the risk-free rate over the same period and reasonable values of model parameters.

A crucial element in our explanation of the two trends is the responsiveness of leverage to firm size. We obtain this information for the set of COMPUSTAT firms by regressing a firm leverage (i.e., the ratio of net debt to total value of a firm) on the logarithm of firm sales (and other controls). Our base regression implies that the leverage ratio rises by 1.7 percentage points for every 100-fold increase in firm size. In the model, we match this degree of responsiveness of leverage to sales by choosing the probability of default permitted on a firm's debt. Because the cash flow of small firms is more volatile, this constraint curtails the borrowing of small firms more severely than large firms.

A second important element is the modeling of the match between a new idea and the firm in which it arises. While an idea has a fixed probability of success if it is implemented independently in a startup, its probability of success if implemented in the firm is drawn from a uniform distribution. A high probability of success can be thought of as a better match between the idea and the firm. In the model there will be a firm-size-dependent threshold probability of success above which the firm will purchase the idea and implement it. When the risk-free rate declines, this threshold declines as well and more of the new ideas are bought by firms. By choosing the dispersion of the distribution of the probability of success, we are able to match the observed decline in the startup rate between 1997 and 2015 given the observed decline in the risk-free rate over the same period.

An implication of our model is that the decline in startup rates should be accompanied by an increase in the survival rate of existing firms. A decline in interest rates causes incumbent firms to absorb more of the new varieties and, hence, lowers their probability of losing all varieties and exiting. As we document later in the paper, the survival probability conditional on firm age has generally increased over this period.

2 Literature Review (Incomplete)

Our paper is related to several strands of the recent literature on firm dynamics. First and foremost, it is related to the literature that has documented and studied the secular decline in the startup rate in the US (Decker, Haltiwanger, Jarmin, and Miranda (2014), Hathaway and Litan (2014b), among others). The reasons for the decline remains an active area of research with no settled answers. Hathaway and Litan (2014a) list several factors, including, slowing population growth, increasing business consolidation, and rising burden of regulation and taxes. Our focus is not the secular decline that has been going on since early 1980s but on the decline that has occurred since late 1990s. Still, in terms of this general categorization of causes, our paper falls in the second category, namely, rising business consolidation. What we add to this perspective is the role of the decline in the risk-free rate in encouraging the growth of existing firms at the expense of startups. Karahan, Pugsley, and Sahin (2016) make the case for the role of declining population growth and Neira and Singhania (2017) and Kaymak and Schott (2018) make the case for the role of changes in the corporate tax rate.

Next, Autor, Dorn, Katz, Patterson, and van Reenen (2017) show that in four out of the six major industries (Manufacturing, Finance, Services, and Utilities and Transportation) business concentration has risen sharply since the mid-to-late 1990s, when measured by the share of sales accounted by the top 4 (or top 20) firms relative to total sales in their 4-digit industry.[1] This rising concentration is most pronounced in sales but it is present for employment as well. Relatedly, there is work arguing that markups have risen in US industries (Gutierrez and Philippon (2017) and De Loecker and Eeckhout (2017)). Our paper does not speak to the rise in market power (or take a stand on whether it has occurred or not), only to the fact that market share of large firms have increased since the late 1990s.

Third, many papers have highlighted the decline in real interest rates and studied the possible reasons underlying this decline (Eichengreen (2015), Del Negro, Giannone, Giannoni, and Tambalotti (2017), among others). The general view that emerges from these studies is that the decline in real interest rates (as measured by the real yield on 10-yr U.S. Treasuries) began in the late 1990s. Del Negro, Giannone, Giannoni, and Tambalotti (2017) attribute the majority of the 2 percentage point decline to a rise in the convenience yield (i.e., a rise in the premium placed on safety and liquidity) that is unrelated to other determinants of real interest rate such as rate of growth of business sector productivity. Consistent with this, we treat the model decline in the risk-free rate as stemming from a change in the preferences of lenders, specifically, as change in their degree of impatience.

On the theory side, our model relates to papers that focus on an inventor's decision to sell a new idea to an existing firm or commercially develop the idea via a startup (Anton and Yao (2002), Anton and Yao (1994), Chatterjee and Rossi-Hansberg (2012), Zábojník (2016)). The focus of these studies is on the inability of the inventor to maintain property rights on his idea once the idea is revealed to buyers and the mechanisms that can develop to (imperfectly) circumvent this

[1] Business concentration has also risen in the remaining two industries – Retail and Wholesale – but the rise in these industries apppears to have begun in early 1980s.

problem and allow the sale of some (but not all) ideas. In contrast, we abstract from the difficulties that inhere in selling ideas and simply assume that such sale is possible. In return, we enrich the model by incorporating financial friction in the form of bankruptcy and bring out the implications of the bankruptcy friction for the sell/startup decision.

There is an extensive macroeconomic literature on firm entry and firm dynamics (Jovanovic (1982), Hopenhayn (1992), Cooley and Quadrini (2001), Luttmer (2007), among others) to which our paper is connected. In all these studies, a firm is identified with a technology so that a new idea, if it goes into production, is automatically a new firm. In contrast, in our model – as in Chatterjee and Rossi-Hansberg (2012) – new ideas occur to people and the inventor chooses the organizational form in which to implement his or her idea (through an incumbent firm or a startup).

Finally, our paper contributes to a nascent quantitative-theoretic literature on firm dynamics in the presence of borrowing constraints (Khan and Thomas (2013)) and equilibrium default risk (Arellano, Bai, and Kehoe (2016), Corbae and D'Erasmo (2017)). Khan and Thomas (2013) study how a large shock to the economy's financial sector can propagate through the economy. Arellano, Bai, and Kehoe (2016) examine how a positive shock to the volatility of sales can cause firms whose borrowings is subject to default risk to reduce production (so as to reduce default risk). Corbae and D'Erasmo (2017) examine how proposed alterations to the U.S. corporate bankruptcy law (Chapters 7 and 13) might impact the long run frequency (and efficiency) of firm bankruptcies. However, these papers neither focus on nor can they explain the positive relationship between leverage and firm size. Regarding the relationship between leverage and firm size in the COMPUSTAT data, Bates, Kahle, and Stulz (2009) and Khan and Thomas (2013) also find a positive relationship using different definitions.

3 Firm Leverage and Firm Size

The goal of this section is to document that large US firms tend also to be more leveraged. To so do so, we use the COMPUSTAT database for the years 1978-2014. Our sample consists of all nonfinancial and nonutilities firms that report in US dollars.

The top panel in Table 1 reports the observed variables taken directly from COMPUSTAT. The bottom panel describes the constructed variables that appear in our regressions. We define the market value of a firm to be the market value of its common equity outstanding plus its total liabilities. We experiment with two alternative measures of a firm's leverage ratio. In the first

Table 1: Variable Name and Description

Variable	Description
SALES/TURNOVER (Net)	Sales
LT	Total Liabilities
DLC	Debt in Current Liabilities - Total
DLTT	Long-Term Debt - Total
CSHO	Common Shares Outstanding
PRCC_F	Price Close - Annual Fiscal
CHE	Cash and Short-Term Investments
PPEGT	Plant, Property and Equipment - Total (Gross)
lnSale	ln(SALES/TURNOVER (Net))
Mkt_Val	CHSO*PRCC_F + LT
Leverage Ratio I	(LT - CHE)/Mkt_Val
Leverage Ratio II	(DLC + DLTT - CHE)/Mkt_Val
Cap_Ratio	PPEGT/Mkt_Val
VolSalesGrowth in t	|(SALES(t)-SALES(t-1)|/[SALES(t)+SALES(t-1)]

measure, we define leverage as the ratio of total liabilities minus cash and short-term investments to market value. In the second measure, we include only liabilities that arise as a result of the firm's active borrowing. Liabilities that arise arise simply from a firm selling to its customers (such as account payables and tax liabilities) are not included. We define the capital ratio of a firm to be the ratio of the value of its tangible capital to its market value. We define the volatility of sales growth as the ratio of the absolute difference between sales in two consecutive periods to the sum of sales in those periods. Finally, for each year of our sample, we limit attention to firms that had sales equivalent to at least $1 million in 2014 terms. This gave us an overall sample of a little under $180,000$ firm-year observations.

The second column of Table 2 reports the linear relationship between the first measure of the leverage ratio and firm size as measured by the logarithm of firm sales. Whenever possible, our regressions include both time and GIC subindustry fixed effects to absorb any time- or subindustry-specific variations in a firm's leverage ratio. The second column reports the results of this basic regression. The coefficient on size is strongly significant. In terms of magnitude, it implies that the difference in the leverage ratios of a million dollar firm and a billion dollar firm is 0.17. To put this in perspective, note that the average leverage ratio (for the first measure) is 0.28. The regression results reported in the next column adds in the firm's capital ratio as another explanatory variable. The coefficient on this is also strongly significant and positive indicating that tangibility of capital helps account for leverage. Adding this variable does not affect the coefficient on size but does improve R^2. The final column reports the results for the same regression but with a cap on firm size. Doing so, increases the coefficient on size somewhat. This reflects the fact the relationship between leverage and size flattens out for very large firms. Hence, when very large firms are excluded, the relationship between size and leverage becomes stronger.

Table 2:
Relationship Between Leverage Ratio (I) and Firm Size

Expl. Vars	Dep Var: Leverage Ratio (I)		
lnSale	0.025 (73.21)	0.025 (74.59)	0.030 (55.00)
Cap_Ratio	-	0.087 (68.02)	0.077 (54.21)
Subindustry Fixed Effects	Yes	Yes	Yes
Time Fixed Effects	Yes	Yes	Yes
Num of Obs	174,497	173,274	133,322
R^2	0.22	0.24	0.21
Adj. R^2	0.22	0.24	0.21
Sample	\$1 Mil $\leq$ Sales	\$1 Mil $\leq$ Sales	\$1 Mil $\leq$ Sales $\leq$ \$1 Bil

Table 3 reports the same set of regressions for the second measure of leverage. As is evident, the pattern of the results is similar but the coefficient on size is somewhat lower. For instance, for the second regression, the coefficient on size is 0.018 as opposed to the 0.025 reported in Table 2. Hence, for this measure, the difference in leverage between a million dollar firm and a billion dollar firm is, on average, 0.12. However, the average leverage ratio for this measure is significantly lower: 0.09 as compared to 0.28.

The goal of the regressions reported in Tables 4-7 is to document that the positive relationship between leverage and size is not driven by any particular industry. With the exception of one industry (GIC=45), the relationship between size and leverage is of the same order of magnitude as in Tables 2 and 3.

Table 3:
Relationship Between Leverage Ratio (II) and Firm Size

Expl. Vars	Dep Var: Leverage Ratio (II)		
lnSale	0.017 (58.11)	0.018 (59.68)	0.025 (51.31)
Cap_Ratio	-	.038 (33.50)	.033 (25.88)
Subindustry Fixed Effects	Yes	Yes	Yes
Time Fixed Effects	Yes	Yes	Yes
Num of Obs	174,224	173,023	133,129
R^2	0.19	0.19	0.18
Adj R^2	0.19	0.19	0.18
Sample	\$1 Mil $\leq$ Sales	\$1 Mil $\leq$ Sales $\leq$ \$1 Bil	\$1 Mil $\leq$ Sales

Table 4:
Relationship Between Leverage Ratio (I) and Firm Size for GIC Industries

Expl. Vars	Dep Var: Leverage Ratio I							
	GIC=10	GIC=15	GIC=20	GIC=25	GIC=30	GIC=35	GIC=45	GIC=50
lnSale	0.038	0.025	0.029	.024	0.020	0.030	0.015	0.010
	(36.71)	(26.02)	(39.04)	(30.11)	(18.27)	(33.23)	(19.15)	(5.65)
Cap_Ratio	0.038	0.142	0.155	0.045	0.210	0.273	0.245	0.119
	(20.10)	(32.46)	(38.07)	(18.55)	(28.77)	(37.83)	(49.22)	(13.16)
Subindustry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Num of Obs	12,859	13,061	33,361	38,985	10,442	22,742	37,460	4,320
R^2	0.19	0.24	0.16	0.11	0.19	0.24	0.15	0.15
Adj. R^2	0.19	0.23	0.16	0.11	0.18	0.24	0.15	0.14
Sample	\$1 Mil $\leq$ Sales							

Table 5:
Relationship Between Leverage Ratio (I) and Firm Size for GIC Industries

Expl. Vars	Dep Var: Leverage Ratio I							
	GIC=10	GIC=15	GIC=20	GIC=25	GIC=30	GIC=35	GIC=45	GIC=50
lnSale	0.054	0.038	0.033	.034	0.026	0.035	0.008	0.029
	(31.94)	(21.13)	(27.12)	(26.74)	(10.14)	(24.94)	(7.77)	(7.83)
Cap_Ratio	0.036	0.123	0.143	0.035	0.190	0.269	0.244	0.154
	(17.16)	(22.75)	(30.52)	(13.44)	(18.91)	(34.54)	(44.69)	(10.73)
Subindustry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Num of Obs	$9,523$	$8,148$	$25,106$	$29,744$	$5,895$	$19,430$	$32,855$	$2,619$
R^2	0.17	0.21	0.13	0.10	0.16	0.21	0.14	0.14
Adj. R^2	0.17	0.21	0.13	0.10	0.15	0.21	0.13	0.13
Sample	\$1 Mil $\leq$ Sales $\leq$ \$1 Bil							

Table 6:
Relationship Between Leverage Ratio (II) and Firm Size for GIC Industries

Expl. Vars	Dep Var: Leverage Ratio II							
	GIC=10	GIC=15	GIC=20	GIC=25	GIC=30	GIC=35	GIC=45	GIC=50
lnSale	0.02	0.015	0.016	0.018	0.015	0.025	0.013	.011
	(23.20)	(17.64)	(23.99)	(25.08)	(14.90)	(29.77)	(18.66)	(7.02)
Cap_Ratio	0.020	0.063	0.087	0.009	0.010	0.15	0.09	0.062
	(11.53)	(16.70)	(23.87)	(4.19)	(15.51)	(22.30)	(18.66)	(7.33)
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Subindustry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Num of Obs	12,857	13,050	33,361	38,851	10,427	22,736	37,424	4,317
R^2	0.11	0.13	0.11	0.10	0.10	0.18	0.11	0.08
Adj R^2	0.11	0.12	0.11	0.10	0.10	0.18	0.11	0.07
Sample	\$1 Mil $\leq$ Sales							

Table 7:
Relationship Between Leverage Ratio (II) and Firm Size for GIC Industries

Expl. Vars	Dep Var: Leverage Ratio II							
	GIC=10	GIC=15	GIC=20	GIC=25	GIC=30	GIC=35	GIC=45	GIC=50
lnSale	0.045	0.024	0.024	.028	0.020	0.032	0.009	0.037
	(29.15)	(15.62)	(21.79)	(24.99)	(8.99)	(24.99)	(0.0092)	(10.69)
Cap_Ratio	0.020	0.050	0.081	0.005	0.091	0.148	0.090	0.109
	(10.01)	(10.65)	(19.22)	(1.87)	(10.05)	(20.35)	(18.64)	(8.15)
Subindustry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Num of Obs	9,522	8,139	25,074	29,636	5,884	19,424	32,34	2,616
R^2	0.14	0.11	0.11	0.10	0.10	0.16	0.11	0.13
Adj. R^2	0.14	0.11	0.10	0.10	0.09	0.16	0.11	0.12
Sample	\$1 Mil $\leq$ Sales $\leq$ \$1 Bil							

Table 8:
Volatility of Sales Growth and Sales

Expl. Vars	Dep Var: VolSaleGrowth
lnSale	-0.022 (-126.77)
Cap_Ratio	-0.023 (-35.64)
Subindustry FE	Yes
Num of Obs	$158,426$
R^2	0.16
Adj. R^2	0.16
Sample	\$1 Mil $\leq$ Sales

Finally, Table 8 documents the negative relationship between volatility of sales and firm size. The difference in volatility of sales between a million dollar firm and a billion dollar firm is -0.15.

4 Model

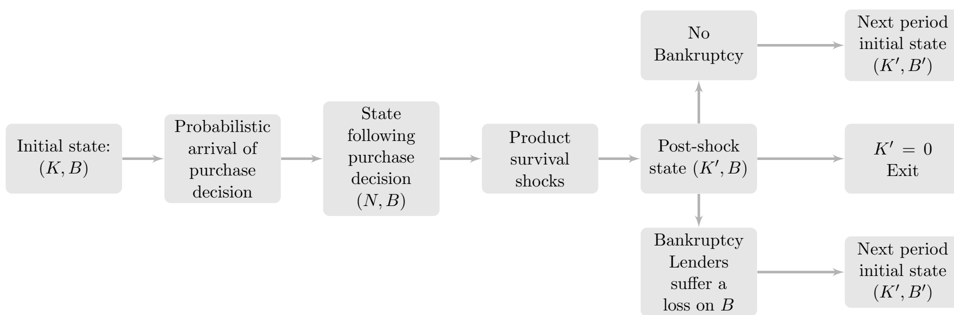
Time is discrete and a period should be thought of as a month. The state vector of a firm is the number of varieties owned by the firm at the start of a period, denoted $K \in \mathbb{N} = \{1, 2, 3, \ldots\}$, and the firm's net asset position, denoted B. B can be positive (savings) or negative (debt). We will treat B as discrete and assume it takes values in the set $\mathbb{B}$, a sufficiently fine discrete approximation of an interval on the real line.

Each period is composed of two subperiods. At the start of the first subperiod, with some positive probability that depends on K, the firm is confronted with the decision to purchase a new variety. If it chooses to purchase the new variety, with some probability s the new variety can be successfully produced. Denote the state of the firm at the end of the first subperiod by (N, B) – where N is either K or $K + 1$ – and the value in that state by $W(N, B)$.

At the start of the second subperiod, the firm is hit with the variety extinction shocks. Following these shocks, the firm is in state (K', B), where K' can be as high as N (none of its varieties is lost) or as low as 0 (all varieties are lost). We denote the value of the firm following the extinction shocks at the start of the second subperiod by $V(K', B)$. Following the shocks, the firm's current period cash flow is realized and the firm chooses it next period level of net assets. It is possible that even with the maximum level of revenue from new debt issuance, the firm cannot pay back all its existing debt. In that case, the firm enters bankruptcy and lenders suffer a loss on their debt. Otherwise, the firm pays off its obligations (if any). A firm that loses all its varieties ceases to exist: The owners pay nothing back if B is negative or consume B if it is nonnegative.

This timing of events is depicted in Figure 1.

Figure 1:
Timing of Events for an Existing Firm



The aggregate state of the economy is a distribution over $\mathbb{N} \times \mathbb{B}$. The mass at any point is the measure of firms on that point. We assume that the economy starts off in some initial period with a measure 1 of firms, which are distributed in some way over the (sub) state space $\mathbb{N} \times \{0\}$ (i.e., initially there are different sized firms all of which have zero net assets).

We assume that a measure $M > 0$ of ideas for new varieties arrive in the economy each period. These new ideas are distributed among existing firms in proportion to the number of varieties owned by each firm. Since the total number of varieties in the economy is endogenous, the probability that a variety begets a new idea in any period, ρ, is endogenous as well. However, each firm takes ρ as given. Since firms own finite numbers of varieties, the number of new ideas occurring in a firm is binomially distributed with "success" probability ρ and the number of "trials" equal to the number of varieties owned.

A firm has the option of purchasing ideas that occurs to its workers but it can consider only one new idea for purchase per period and this idea is selected randomly from the total number of new ideas generated in the firm in that period. Then, the probability that a firm with K owned varieties has purchase decision to make is simply the probability that at least one idea is generated in the firm. Specifically,

$$p(K) = [1 - (1 - \rho)^K].$$

The new idea that a firm is considering for purchase comes with a success probability of s. The value of s is drawn from a Uniform distribution with support $[s_{\min}, 1]$. The closer s is to 1, the more likely it is that the idea will succeed and add to the portfolio of varieties owned by the firm. We can view s as index of quality of the match between the idea and the firm. If a new idea is implemented in a start up (equivalently, in a spin-off) the probability of success is a constant σ.

There is a surplus from implementing the new variety in the incumbent firm if

$$s[W(K + 1, B) - W(K, B)] \geq \sigma W(1, 0).$$

The l.h.s. is the expected gain to the firm if it absorbs the new variety and the r.h.s. is the gain to spinning off. If the surplus is nonnegative, we assume that the new variety is absorbed by the incumbent and the inventor gets $\sigma W(0, 1)$. Therefore, post purchase, the value to the original owners of the firm rises by

$$sW(K + 1, B) + (1 - s)W(K, B) - \sigma W(1, 0).$$

Let $\mathbb{M}(K, B, s)$ be an indicator function that takes the value 1 if the surplus from an acquition is nonnegative and 0 otherwise.

Recall that a firm can get more than one idea but can only consider one of those ideas for purchase. Any new idea that is not selected to be considered for a purchase is assumed to spin off. Therefore, a firm that owns K varieties contributes, on average,

$$\sigma \sum_{J \geq 2}^{K} z(J, K; \rho)(J - 1)$$

"forced" startups in a period. Here $z(J, K; \rho)$ is the probability that a firm with K varieties generates J ideas and is given by

$$z(J, K; \rho) = \frac{K!}{J!(K - J)!} \rho^J (1 - \rho)^{K-J}.$$

Following the arrival of new ideas and merger decisions, each variety in existence receives a extinction shock with probability ϕ. We denote by $x(K', N)$, the probability that a firm that has N varieties at the end of the first subperiod ends up with $0 \leq K' \leq N$ varieties at the start of the second subperiod.

$$x(K', N) = \frac{N!}{K'!(N-K')!}(1-\phi)^{K'}\phi^{N-K'}.$$

If the firm retains at least one variety and repays its current obligations (if any), it chooses B' to solve

$$\begin{aligned}
V^R(K', B) = \max_{B'} \pi K' + B - q(K', B')B' + \beta\Big[&[1-p(K')]W(K', B') + \\
p(K')\int_{s_{\min}}^{1}&\{\max\{W(K', B'), sW(K'+1, B') + (1-s)W(K', B') - \sigma W(1,0)\}]\}\mathrm{U}(ds)\Big]
\end{aligned}$$

s.t.

$$\pi K' + B - q(K', B')B' \geq 0$$

$$d(K', B') \leq \theta.$$

Here π is the free cash flow from each variety and, hence, $\pi K'$ is the total free cash flow of the firm for the current period; for $B' < 0$, the function $q(K', B')$ gives the price of a unit of debt issued by the firm on which the firm might default next period; for $B' \geq 0$, it is understood that $q(B', K')$ is simply $1/(1+r)$; and $\mathrm{U}(ds)$ is short-hand for Uniform density over $[s_{\min}, 1]$. The probability of default next period is denoted by $d(K', B')$. The constraint that $d(K', B')$ cannot exceed θ will serve to capture the fact that small firms have limited access to capital markets.

A firm cannot repay its obligations if the nonnegativity constraint $\pi K' + B - q(K', B')B' \geq 0$ cannot be satisfied for any B' that respects the default probability constraint. In this case, the firm enters bankruptcy and there is default on debt. To formalize this, let $G(K')$ be the highest revenue from bond sales consistent with default probability constraint. That is,

$$G(K') = \max_{B'} q(K', B')(-B')$$

s.t.

$$d(K', B') \leq \theta.$$

Let $\overline{B}(K')$ solve:

$$\pi K' + \overline{B}(K') + G(K') = 0.$$

Then bankruptcy occurs if $B < \overline{B}(K')$ for then there cannot be any B' for which the firm will be able to pay back all its debts and still satisfy the default probability constraint. In the event of bankruptcy, the debt owed to creditors is reduced to $\overline{B}(K')$. The value of the firm under bankruptcy (default) is then given by:

$$\begin{aligned}
&V^D(K', B) = \max_{B'} \pi K' + \overline{B}(K') - q(K', B')B' + \beta\Big[[1 - p(K')]W(K', B') + \\
&\quad p(K') \int_{s_{\min}}^{1} \{\max\{W(K', B'), sW(K' + 1, B') + (1 - s)W(K', B') - \sigma W(1, 0)\}]\}\mathrm{U}(ds)\Big] \\
&\text{s.t.} \\
&\pi K' + \overline{B}(K') - q(K', B')B' \geq 0 \\
&d(K', B') \leq \theta.
\end{aligned}$$

It is worth noting that when there is a unique B' that attains $G(K)$ then it is also the B' that (trivially) solves the firms optimization problem under bankruptcy: It is the only choice that is available to the firm (any other choice will either violate the nonnegativity constraint or the default probability constraint). Furthermore, the firm's dividend payment during bankruptcy is exactly zero but owners continue to retain rights over the firms future cash flow. Thus bankruptcy in our model is a reorganization rather than a liquidation. In what follows, we will use $B(K')$ to denote the firm's optimal choice of debt. As we explain in Section 6, the firm will choose the same B' regardless of its current B and, hence, we do not need to index the choice of B' by B.

If a firm loses all its varieties then its continuation value becomes zero since it loses the ability to acquire new varieties ($p(0) = 0$). Hence,

$$V^R(0, B) = B \text{ if } B \geq 0 \text{ and } V^D(0, B) = 0 \text{ and } \overline{B}(0) = 0 \text{ if } B < 0.$$

Since a firm with 0 varieties stays permanently in that state, we treat $K' = 0$ as exit of the firm.

Let $\mathbb{D}(K', B)$ be an indicator function that takes the value 1 if (K', B) is a bankruptcy state and 0 otherwise. We can now state the expression for $W(N, B)$, the value at the end of the first subperiod (i.e., the value of the firm following the merger decision but before the realization of the extinction shocks).

$$W(N,B) = \sum_{K'=0}^{N} x(K',N)\left[[1-\mathbb{D}(K',B)]V^R(K',B) + \mathbb{D}(K',B)V^D(K',B)\right].$$

Next, we turn to the expression for the probability of default on bonds issued in the current period:

$$\begin{aligned} d(K',B') = {} & [p(K')\textstyle\int_{s_{\min}}^{1} \mathbb{M}(K',B',s)\mathrm{U}(ds)] \sum_{K''=0}^{K'+1} x(K'',K'+1)\mathbb{D}(K'',B') + \\ & [1-p(K')\textstyle\int_{s_{\min}}^{1} \mathbb{M}(K',B',s)\mathrm{U}(ds)] \sum_{K''=0}^{K'} x(K'',K')\mathbb{D}(K'',B'). \end{aligned}$$

The bracketed term multiplying the first summation term is the probability that the firm successfully adds to its product portfolio next period. The summation term is the probability of default conditional on having acquired a new variety. Correspondingly, the bracketed term multiplying the second summation term is the probability that it fails to do so and the summation term is the probability of default conditional on not having acquired a new variety.

We now turn to the two equilibrium conditions of the model. The first pertains to the pricing of bonds. We assume that lenders are risk neutral and lending is a competitive business. This implies that the price a bond, conditional on the amount borrowed and the number of varieties in possession of the borrowing firm, must fetch the risk-free rate in expectation. To give the resulting expression for bond prices, define

$$Q(K',B) = \begin{cases} 1 & \text{if } \mathbb{D}(K',B) = 0 \\ \frac{\overline{B}(K')}{B} & \text{if } \mathbb{D}(K',B) = 1. \end{cases}$$

Q is the "recovery rate" on each bond next period: In the event there is no default, the rate is 1; in the event of bankruptcy, the rate is the ratio of $\overline{B}(K')$ to B (all bonds are treated equally). With this definition of the recovery rate, the requirement that investors break even on their loans implies that bond price is given by:

$$\begin{aligned} q(K',B') = (1+r)^{-1} \Big[& p(K')\textstyle\int_{s_{\min}}^{1} \mathbb{M}(K',B',s)\mathrm{U}(ds) \sum_{K''=0}^{K'+1} x(K'',K'+1)Q(K'',B') + \\ & [1-p(K')\textstyle\int_{s_{\min}}^{1} \mathbb{M}(K',B',s)\mathrm{U}(ds)] \sum_{K''=0}^{K'} x(K'',K')Q(K'',B') \Big]. \end{aligned}$$

The second equilibrium condition pertains to the determination of ρ, the probability that a variety begets the idea of a new variety. To express this condition, let $H(K, B)$ denote the steady state distribution of firms over the state space. Then, the first requirement is that $H(K, B)$ exist and, second, that ρ be such that

$$M = \sum_{B \in \mathbb{B}} \left[\sum_{K=1}^{\infty} \sum_{J=0}^{K} z(J, K; \rho) H(K, B; \rho) \right],$$

where we have recognized that the distribution H is affected by the value of ρ as well.

5 Calibration

The model has two market parameters, r and θ, one preference parameter, β, and four technological parameters, M, ϕ, σ and $s_{\min}$.

Of these seven parameters, the three reported in Table 9 are set independently. The risk-free interest rate r is set to match the average real return on 3-month Treasury bills of 2.40 percent in 1997. The exact value of β is not very important for our results but it is important that firms be more impatient than lenders. For the baseline calibration, we set β to 0.95. The value of M can be set to any positive number since, in steady state, it affects only the total number of varieties in the economy. We set it to a numerically convenient value of 10.

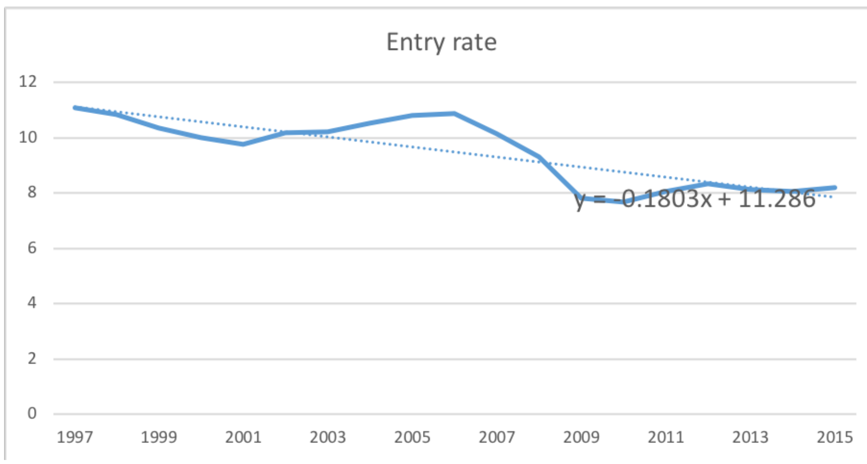
Table 10 reports the four remaining parameters that are set jointly to match four data moments. The moments (targets) are listed in the first column along with their observed values. The third column lists the corresponding parameter that most affects the target statistic and the final column reports the parameter values that (effectively) achieve the targets listed in the second column.

Table 9:
Parameters Set Independently

Parameter, Annualized	Value
r	0.024
β	0.95
M	10

The first target moment listed is the response of the leverage ratio to log sales reported earlier in the paper. We chose 0.025 as the target (see Table 2, column 3). In the model, for a firm with $B < 0$, leverage is measured as $(-B)/[V(K,B) + (-B)]$, i.e., it is the ratio of debt to the sum of equity plus debt and ln(Sales) is measured by $\ln(K)$. The responsiveness of model leverage to $\ln(K)$ is controlled largely by θ, the maximum default probability permitted on bonds: The higher θ the larger the responsiveness (the reason for this and other features of the equilibrium will be explained in the next section). The implied value of θ gives a maximum allowable annual default probability of 6.1 percent.

Figure 2:
Entry Rate of New Firms, 1997-2015



Next, we chose the annual rate of entry of new firms in 1997 as another target. As shown in Figure 2, the annual entry rate has fallen over the 1997-2014 period. The straight line in the figure is a linear trend fitted to the data series. We take the predicted value 0.111 in 1997 as our target (the value is very close to the actual reported entry rate). The model parameter that most affects the entry rate is σ, the success probability of spinoffs, with the entry rate increasing in σ. The implied value of σ is 0.98.

The third target is the survival rate of 1-year old firms, again in the year 1997. Figure 3 fit a trend line through the data and take the predicted value of 0.824 in 1997 as our target. The model paramater that most affects this target is the product extinction probability ϕ: The higher ϕ, lower is the survival rate. The implied value of ϕ is 0.192.

The final target is the decline in the entry rate between 1997 and 2014 evident in Figure 2. As discussed in the introduction to this paper, our goal is to relate this decline in the entry rate to the

Figure 3:
Survival Rate of 1-yr Old Firms, 1997-2015

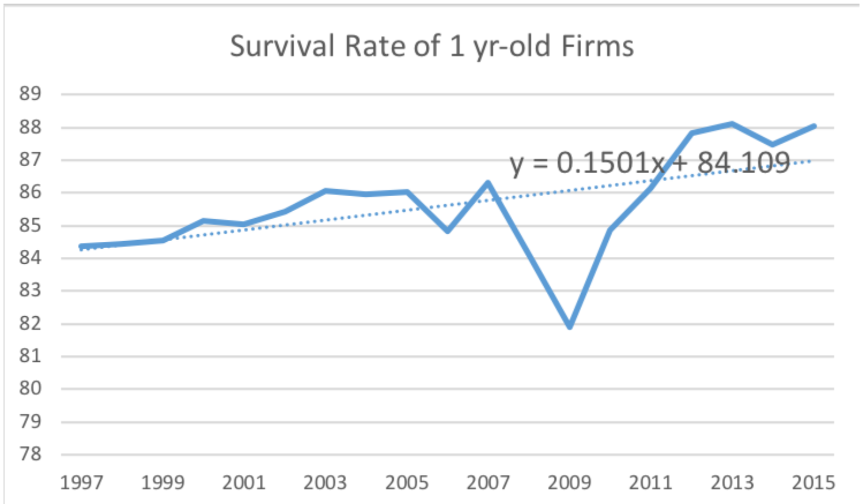


Table 10:
Parameters Set Jointly

Description of Target	Value	Parameter	Value
Response of leverage to size	0.025	θ	0.061
Annual entry rate of new firms	0.111	σ	0.98
Survival rate of 1-yr old firms	0.824	ϕ	0.192
Decline in the entry rate between 1997-2015	0.032	$s_{\min}$	0.964

decline in real interest rates over this same period. For the purposes of quantitatively exploring our proposed channel we assume, for now, that all of the decline in entry rates captured by the linear trend line is due to the observed decline in interest rates over the same period. In a later section, we take into account the impact of the slowdown in employment growth in the post-1997 period as well. The parameter in the model that most affects the sensitivity of the entry rates to interest rates is $s_{\min}$: The higher the value of $s_{\min}$ the greater the sensitivity. The implied value of $s_{\min}$ is 0.964.

Table 11 reports some relevant nontargeted data moments along with their model counterparts. The model's response of sales volatility (as defined earlier in the paper) is in line with the data, if

Table 11:
Nontargeted Moments: Data and Model

Moments	Data	Model
Response of sales volatility to size	-0.022	-0.028^*
Survival rate of 0-yr old firms	0.77	0.83
Survival rate of 2-yr old firms	0.87	0.85
Survival rate of 3-yr old firms	0.88	0.86
Survival rate of 4-yr old firms	0.90	0.87
Fraction of firms that declare bankruptcy	-	0.005
Prob. of a variety generating a new idea (ρ), annualized	-	0.219

*: Excludes 1 variety firms

we exclude all $K = 1$ firms.[2] Since COMPUSTAT firms are large firms, the exclusion is probably justified on data ground as well. The survival probabilities in the model rises with age, as it does in the data although the model slope is less steep. We don't have a data analog of all business bankruptcies but the rate is quite low in the model – about half a percentage point per year. Note that bankruptcy is not the same as exit: Many firms exit each year but relatively few go bankrupt. The probability of a variety getting a new idea is slightly higher than the probability of a variety becoming extinct.

6 Model Properties

Why Do Firms Borrow and How Much?

Firms borrow because owners discount the future more than lenders. Furthermore, since both owners and lenders are risk-neutral, owners will borrow as much as possible given the default probability constraint θ. To see this, note that a firm that issues an additional (infinitesimal) unit of bonds will obtain $q(K', B')$ in the current period and, next period, will owe 1 if there is no bankruptcy and something less than 1 if there is bankruptcy. But, in equilibrium, the value of $q(B', K')$ is exactly this expected repayment on the marginal debt discounted by $(1 + r)$. Thus,

[2]Such firms exit with high probability and, hence, their current period ln(sales) is often $-\infty$. If we include only surviving firms, then the volatility of sales growth is artificially depressed for 1-variety firms.

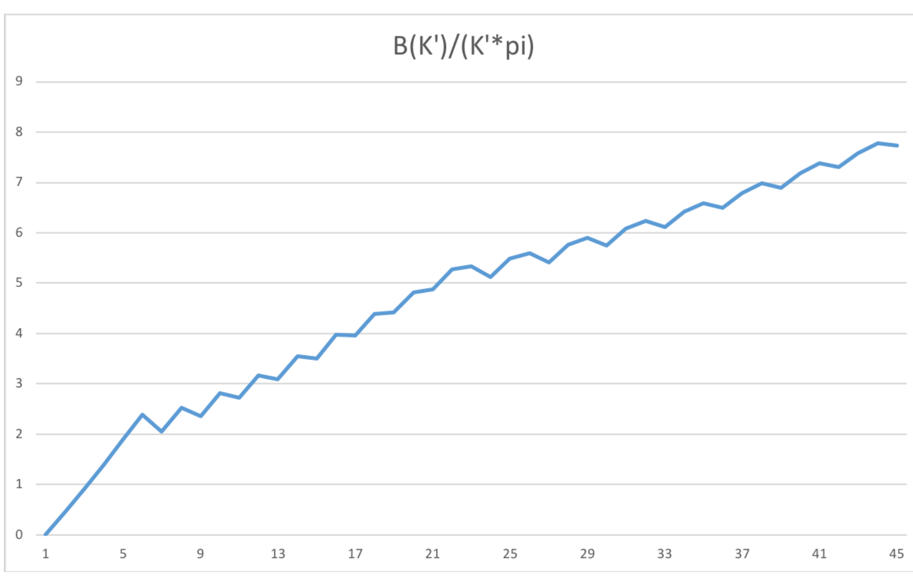
the future consumption cost of issuing an additional unit of debt is simply $\beta q(B', K')(1+r)$. Since $\beta(1+r) < 1$, firms always have a strict incentive to increase revenue from bond sales and, so, will issue the level of debt that maximizes current revenue subject to the default probability constraint. Note that this implies that a firm's optimal choice of debt always attains $G(K')$ (the value of the program that maximizes revenue subject to the default probability constraint) independent of whether the firm is in bankruptcy or not (in bankruptcy, all of this revenue plus current cash flow is handed over to creditors).

Why Does Leverage Rise with K?

For the purposes of understanding model mechanics, it is helpful to think of leverage as $B'(K')/\pi K'$. This ratio rises with K' because, on a per-variety basis, a firm with larger number of varieties has less variable cash flow than a firm with fewer number of varieties and, therefore, is able to borrow more subject to the default probability constraint. We explain this below.

Consider a single variety firm, i.e., a firm with $K' = 1$. The probability that this firm will end up with $K'' = 0$ in the next period is at least $(1-\rho)\phi$ (probability of the event that the firm does not get to make a purchase decision and then loses the one variety it has). Our calibration implies that this lower bound on the firm's exit probability is 0.15. Therefore, the probability of default on any amount of debt (no matter how little) is at least 0.15. Since this exceeds the calibrated value of $\theta = 0.061$, a single variety firm is shutout of the credit market and its equilibrium leverage is 0.

Figure 4:
Debt to Output Ratio and Firm Size



Firms with two or more varieties can borrow. For the lenders, the risk associated with leverage depends on the number of surviving varieties next period as a proportion of the number of varieties today. Ignoring for the moment the possibility of acquiring a new variety, a firm will, on average, have roughly $(1-\phi)$ of its current varieties next period and the variation around this proportion, as measured by the variance, is $[(1-\phi)\phi/N]$. Thus, the riskiness of the cash flow shrinks with N. Consequently, a bigger firm is able to borrow a higher proportion of its current period cash flow before running into the default probability constraint. Figure 4 displays $[-B'(K')/\pi K']$ against K'. As is evident, leverage is strongly increasing in K': A firm that owns a larger number of varieties is able to borrow a greater multiple of its current-period cash flow.

While the maximum default probability on debt is 6.1 percent and firms borrow as much as possible subject to this constraint, the average probability of default in our economy is 0.5 percent. This is because the equilibrium probability of default for any firm is generally substantially less than 6.1 percent. For instance, the default probability constraint makes it impossible for a one variety firm to borrow and, so, its equilibrium default probability is zero. To understand why this is the case more generally, it helps to see a firm's borrowing decision as tantamount to its choosing the number varieties below which it will go into bankruptcy next period. Let's call the number N^{def}. A firm pushes up this number as high as possible by leveraging up, subject to meeting the default probability constraint. Since there will be a discrete jump up in the probability of ending up with fewer than N^{def} varieties as N^{def} is incremented by a unit, the default probability constraint will almost always be reached when the equilibrium probability of default is less than θ.

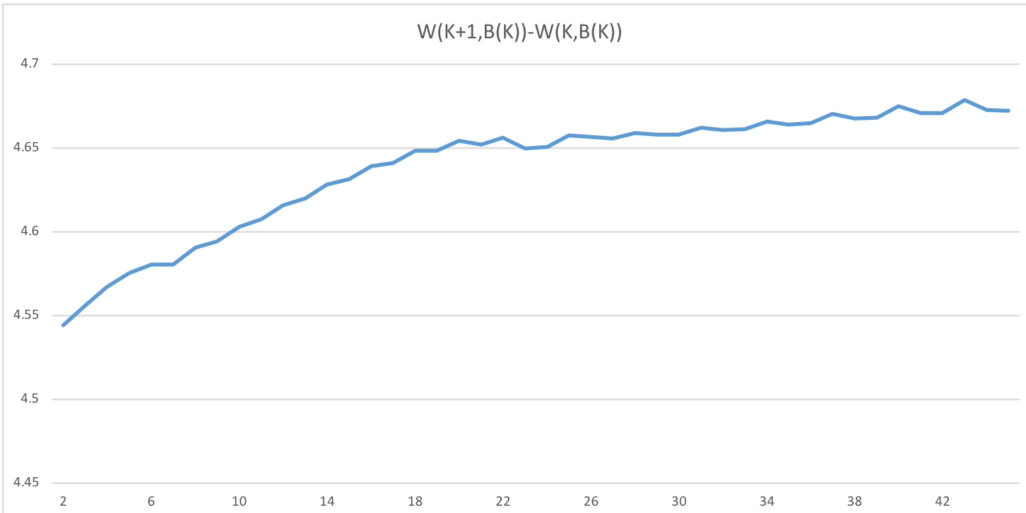
Why Are Larger Firms More Willing to Buy New Ideas?

New varieties are more valuable to larger firms because they are able to borrow a greater fraction of the present discounted value of the associated cash flow. This is the message of Figure 4 shown above. This point can be made more directly by plotting $[W(K+1,B'(K)) - W(K,B'(K))]$ against K, as done in Figure 5. The gain from absorbing a new variety is increasing in firm size. The implication is that the threshold level of s for which $s[W(K+1,B'(K))-W(K,B'(K))] = \sigma W(0,1)$ is falling in K: Larger firms are more willing to buy new varieties.

What is the key determinant of ρ?

The key determinant of ρ is ϕ, the extinction probability. To see this, consider the case where all new ideas are successful. Then, the measure of new varieties in the economy will be M per period. Let E denote the measure of varieties in the economy. Then measure of varieties exiting each period

Figure 5:
Firm Size and the Gain from Absorbing a New Variety



is ϕE. In the long run, the level of E will be determined by the condition that $M = \phi E$ and, hence, M/E must equal ϕ. But M/E is just ρ, which shows that in this case the key determinant of ρ is indeed ϕ.

The actual determination of ρ is somewhat more complicated because not all new varieties succeed and the success probability is partly endogenous: The varieties that are implemented as spin-offs have a success probability of σ while those that are implemented by incumbent firms depend on the associated value of s. However, in our calibrated model these success probabilities are close to 1, so $M \approx \phi E$ and, so, $\rho \approx \phi$.

7 Startups and Concentration in a Low Interest Rate World

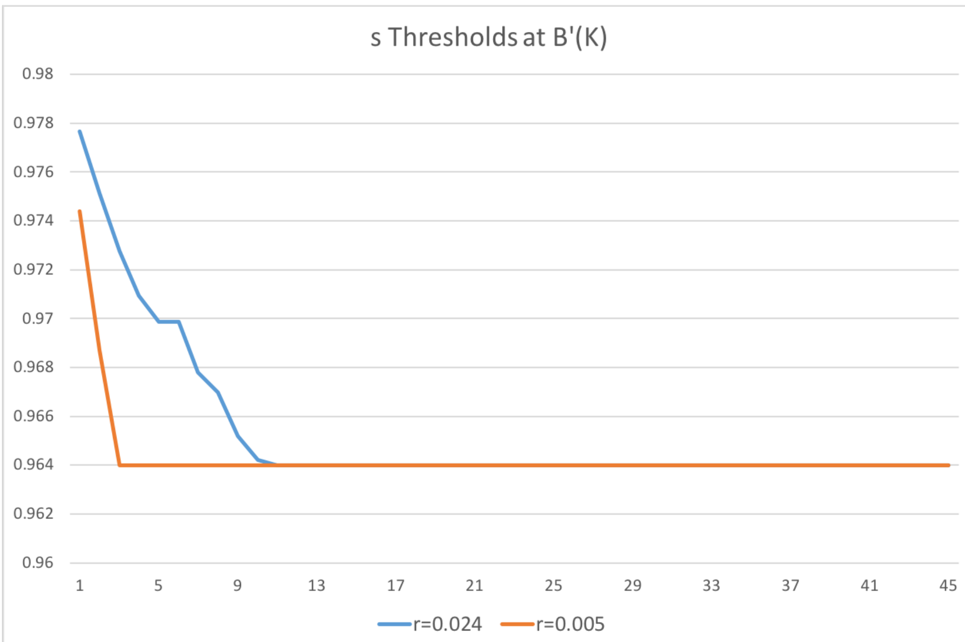
Our model implies a negative relationship between r and the steady state rate of startups.

A decline in r means that a firm sells its debt for a higher price. Since larger firms issue more debt against the future cash flow of their varieties (leverage increases with size), a decline in r increases $V(K', B)$ by more the larger is K'. Formally, holding all parameters constant, $V(K', B; r^{\text{low}}) - V(K', B; r^{\text{high}})$ is strictly positive and strictly increasing in K'. In turn, this property is inherited by the $W(N, B)$ function: $[W(N, B; r^{\text{low}}) - W(N, B; r^{\text{high}})]$ is also strictly positive and strictly increasing in N, provided all parameter, in particular ρ, are held fixed. Finally, this property of increasing differences implies

$$[W(N+1,B;r^{\text{low}})-W(N,B;r^{\text{low}})] > [W(N+1,B;r^{\text{high}})-W(N,B;r^{\text{high}})],$$

from which it follows that the threshold value of s above which a firm of size N purchases a new idea declines with a fall in r. Hence, a lower r leads to a larger fraction of new ideas being purchased by incumbent firms and a lower rate of startups.

Figure 6:
s Thresholds and Firm Size



To show this point explicitly, Figure 6 plots the threshold value of s above which an firm of size K will purchase an idea. The blue line shows these thresholds for the baseline model and the orange line shows it for the equilibrium with $r = 0.005$ (and no other changes in any parameters). Observe that for each K the threshold s is either unchanged or lower in the low interest rate equilibrium.

Table 12 reports the equilibrium effects of a drop in r from 2.40 percent to 0.5 percent. The top panel reports equilibrium effects for which we do not have data counterparts – either because data from all years was used to calibrate the model or because we do not have the data for the moment in question. The bottom panel reports model statistics for which we do have data counterparts for both the high and low interest rate environments.

Turning first to the top panel we see that there is a modest increase in the responsiveness of leverage to sales and a more rapid decline in the volatility of sales growth with size. One reason underlying these effects is the change in the distribution of firms (which shifts toward larger firms) and the fact that the relationship between leverage and volatility of sales growth and the logarithm of sales is not linear. In particular, the relationship is more steep for larger firms. The fact that the bankruptcy is higher in the low interest rate economy is also the result of the shift from small to larger firms. Recall that one-variety firms do not default at all and there are fewer of them in the low interest rate equilibrium.

Finally, the probability of a variety generating an idea, ρ also changes with a lower r. Because incumbent firms become less choosy about the new ideas they purchase (the s threshold falls), the fraction of purchases that are actually successful declines and, so, the rate of entry of new varieties (not just the entry rate of new *firms*) declines as well. In the long run equilibrium, the slower rate of entry of new varieties translates to a smaller equilibrium mass of varieties. Given the constancy of M, $\rho(= M/E)$ will be *higher* in the new steady state. This upward movement in ρ will affect the entry rate of new firms. But, as pointed out above, for our calibration the main determinant of ρ is ϕ (which is unchanged) and, so, general equilibrium effects on ρ are minor.

Turning now to the bottom panel of Table 12, the first line reports the change in entry rates in the model and in the data. Since the model is calibrated to match the drop in entry rates given the drop in interest rates, the match between data and model is perfect.[3] For the other moments, the direction of change in model-generated moments and the direction of change for the data give a sense of the explanatory power of our model. In the data, survival probability of 1 and more than 1-year old firms has risen over this period, while the survival rate of new firms has remained the same. Our model is consistent with this pattern. The reason why survival probabilities are generally rising in our model is because more new varieties are absorbed in existing firms and consequently these firms are less likely to lose all their varieties and exit.

We now turn to the prediction of our model regarding business concentration. Our explanation of the declining startup rate stresses the enhanced incentives of incumbent firms to purchase new ideas when interest rates fall, thus cutting into the rate of new firm creation. Importantly, in our model, the increased incentive to purchase new ideas is stronger for larger firms. Thus, our

[3] On the second line, the match between the model and the 1997 data value for the survival rate of 1-year old firms is also perfect because that moment was also used in the calibration.

Table 12:
Equilibrium Effects of Low Interest Rates

Moments		Baseline	Low r Eqbm	
Response of leverage to sales		0.025	0.028	
Response of sales volatility to size		-0.028^{*}	-0.033^{*}	
Fraction of firms that declare bankruptcy		0.005	0.008	
Prob. of a variety generating a new idea (ρ), ann.		0.219	0.219	
	Data 1997	Baseline	Low r Eqbm	Data 2015
Entry rate of new firms	0.11	0.11	0.08	0.08
Survival rate of 0-yr old firms	0.77	0.83	0.83	0.76
Survival rate of 1-yr old firms	0.84	0.84	0.85	0.87
Survival rate of 2-yr old firms	0.87	0.85	0.86	0.89
Survival rate of 3-yr old firms	0.88	0.86	0.87	0.91
Survival rate of 4-yr old firms	0.90	0.87	0.88	0.91

*: Excludes 1 variety firms

Table 13:
Effect on Business Concentration of Low Interest Rates

		Share of Output	
	Measure of firms	Baseline	Low r Eqbm
Top 1 percent by Size (K) in Baseline	0.21	0.008	0.009
Top 5 percent by Size (K) in Baseline	1.03	0.26	0.32
Top 10 percent by Size (K) in Baseline	2.06	0.38	0.51

explanation of declining startup rates also implies that larger firms should acquire proportionately more new varieties and account for a larger share of total output.

Table 13 quantifies this prediction of the model in the following way. For the baseline model, we use $H(K, B)$ to first determine the measure of firms for the biggest to the smallest sized firms in terms of the number of varieties owned. Then, starting with the firms with the largest number of varieties, we include firms with progressively fewer varieties until 1, 5 and 10 percent of the total measure of firms is included. The first column of numbers reports the resulting measures. Then, we compute the fraction of aggregate cash flow (our measure of output) accounted for each of the three measures of firms. These fractions are reported in the second column of numbers. Thus, in the baseline model, the top 1 percent of firms by size account for 0.8 percent of total output, the top 5 for 26 percent of output and the top 10 percent for 38 percent of output.

For the final column of numbers, we use $H(K, B)$ for the low interest rate equilibrium to once again determine the measure of firms for the biggest to the smallest sized firm in terms of varieties owned. Then we determine the share of output accounted for by the largest firms for the *measures reported in column 2.* Thus, the comparison between the last two columns holds fixed the number (in our case – the measure) of top firms. The comparison reveals that the low interest rate economy is substantially more concentrated: For instamce, for the top (by size) 1.03 and top 2.06 measures of firms, share of output rises by 6 and 13 percentage points, respectively.

8 The Role of Demographics

Between the late 1990s and now, the decline in the risk-free rate has also been accompanied by a decline in the rate of growth of the labor force. In our model, new ideas occur to workers and, so, a slowdown in the rate of the growth of the labor force implies a slow down in the rate of arrival of new ideas and, therefore, a decline in the startup rate. The goal of this section is to incorporate a positive growth rate of arrival of new ideas (workers) in the model and, then, to parse out the relative contributions of a decline in the rate of growth of employment and the decline in the risk-free rate on the startup rate, assuming that the observed decline in the startup rate is due solely to declines in employment growth and the risk-free rate.

The environment is the same as before, except that M (the measure of ideas for new varieties) is now assumed to grow at a constant rate $\gamma > 0$, i.e., $M' = (1+\gamma)M$. We will focus on steady state growth in which the measure of varieties, E, grows at the same rate as M and, thus, $M/E = \rho$ is constant. The decision problem of firm owners, inventors and lenders then remains exactly as described for the stationary model. The only difference is that the total measure of varieties $\sum_{B\in\mathbb{B}}\left[\sum_{K=1}^{\infty}\sum_{J=0}^{K} z(J,K;\rho)H(K,B;\rho)\right]$ is no longer constant. Thus, the equilbrium condition for ρ must now be stated in per-variety terms. Namely, ρ must satisfy:

$$\rho = \sum_{B\in\mathbb{B}}\left[\sum_{K=1}^{\infty}\sum_{J=0}^{K} z(J,K;\rho)h(K,B;\rho)\right],$$

where h is now the fraction of the total measure of varieties that is produced in a firm of type (K,B).

9 Conclusion

We presented a model in which firms manage collections of product varieties. The arrival into the economy of new varieties and the extinction of existing varieties are random events. Since firms manage collections of varieties, the random process of *product variety* entry and exit induces a stochastic process for the entry, growth and exit of *firms*. A firm's access to capital markets plays a key role in our theory of firm dynamics. Our theory implies that a decline in the risk-free rate will result in larger firms purchasing more of the new varieties entering the economy in any period, resulting in fewer startups and greater concentration of sales among top firms. Thus our paper causally connects the decline in real interest rates to the decline in the startup rate and to the rise of business concentration in the US.

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