The Term Structure of Credit Spreads with Dynamic Debt Issuance and Incomplete Information*

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Abstract

We investigate credit spreads and capital structure dynamics in a model in which management has private information regarding firm value and is able to issue *both* equity and debt to service existing debt. Rather than choosing to default, managers of investment-grade (IG) firms who receive bad private signals conceal this information by servicing existing debt via new debt issuance. As such, firms with IG-commensurate spreads have zero jump-to-default risk (and hence, command zero jump-to-default premium), at least until their debt capacity is fully utilized and spreads have increased to "fallen angel" status. These predictions match observation well.

JEL Classification Codes: G12; G32; G33
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"How did you go bankrupt?"

"Two ways. Gradually, then suddenly."

– Ernest Hemingway, The Sun Also Rises

1 Introduction

Most empirical studies of corporate bond yields report evidence of a "credit spread puzzle" in that it is difficult to explain observed spreads between corporate bond yields and Treasury yields in terms of expected losses and standard measures of risk. This credit spread puzzle is most striking for short-maturity investment-grade (IG) debt, since historical default rates for these bonds are extremely low. Several explanations for the credit spread puzzle have been suggested in the literature, including: (i) illiquidity premia, (ii) tax asymmetry (i.e., corporate bonds, but not Treasuries, are subject to state taxation) and (iii) jump-to-default (or credit-event) premia. This paper argues both theoretically and empirically that portfolio strategies which hold only investment-grade debt (that is, strategies that sell bonds once their spreads have increased to "fallen-angel" status) are subject to negligible jump-to-default risk, and hence command negligible jump-to-default premia. As such, short-maturity IG spreads must be mostly due to other channels.

The notion of jump-to-default (or credit-event) risk arises naturally in reduced-form models of default (e.g., Duffie and Singleton (1997); Jarrow, Lando, and Turnbull (1997)), in which default is modeled as an unpredictable jump event. If a sufficiently large premium is attributed to jump-to-default risk, then short-maturity spreads can be "explained" through this channel. In their seminal paper, Duffie and Lando (2001) (DL) provide an economic justification for reduced-form models. In particular, they investigate the optimal behavior of a manager of a firm that can issue *only equity* to service debt in place. They show that if this manager receives a sufficiently bad private signal, then it will be in the best interest of shareholders for the manager to declare default, rather than have them continue to service debt payments. From an outsider's information set, such a default will appear as an unexpected jump-to-default, which can be characterized by a default intensity process similar to those specified by reduced form models.

In this paper, we build on the insights of Duffie and Lando (2001) by investigating a framework in which a manager with private information can issue *new debt* to service existing debt, at least until the firm exhausts its debt capacity. In contrast to DL, our model predicts that IG firms will never jump to default due to a bad private signal, because the manager of an IG firm will maximize shareholders' value by concealing this bad signal, and issuing new debt to service debt in place. Indeed, our framework generates a prediction nearly opposite to that of DL: whereas in the DL framework, firms can jump-to-default even if the underlying asset value dynamics follows a diffusion process, in our setting, IG firms would not jump-to-default (at least, not immediately) even if the true asset value (known only to the manager) jumped below the default boundary. The implication of our model is that the relatively large spreads on short-maturity IG debt over risk-free securities cannot be explained by jump-to-default premia due to asymmetric information, and therefore other channels (e.g., asymmetric taxes, illiquidity, jumps in asset value due to public information) are needed to explain these large spreads. A jump to default due to asymmetric information is possible in our setting only after a firm exhausts its debt capacity.

To illustrate the implications of our model, we estimate empirical default rates as a function of both the firm's spread and its rating. In contrast to DL, which predicts a relatively flat term structure of default probabilities for horizons up to one year, our model better matches empirical observation in that the vast majority of IG firms that default within one year do so near the end of the year – that is, these firms first tend to diffuse toward "fallen angel" status prior to defaulting. As such, a portfolio consisting only of bonds with spreads that are commensurate with IG status is subject to virtually zero jump-to-default risk and, therefore, should not command a significant jump-to-default premium. Only after a firm drops to fallen angel status does jump-to-default become a possibility through the mechanism proposed by DL. This prediction is consistent with the empirical findings of Davydenko, Strebulaev, and Zhao (2013), who report that the default event of speculative-grade debt is associated with significant losses in firm value, consistent with the notion that the default event was indeed a surprise based on public information.[1]

Our paper contributes to two strands of literature. The first is the extensive body of

[1]Clark and Weinstein (1983), Lang and Stulz (1992) and Warner (1977) also report significant loss of equity and debt value at the time of bankruptcy announcement.

work that studies structural models of capital structure choice and leverage dynamics.[2] Unlike most papers in this literature, we allow for informational asymmetry between the manager and creditors. In our structural model, creditors are aware of the manager's information advantage, and price it rationally into the firms' claims.

The second strand of literature to which we contribute investigates credit spreads in reduced-form frameworks. In particular, we build on the body of work that focuses on decomposing credit spreads into components of expected loss, risk premia, liquidity premia and taxes.[3] In this literature, there is considerable disagreement on the magnitude of the jump-to-default premium. For example, Driessen (2005) estimates the ratio of risk-neutral to actual default intensity to be $\lambda^{\mathbb{Q}}/\lambda^{\mathbb{P}} = 2.3$, and infers a 31 bps spread due to the jump channel.[4] These estimates, however, are determined from residuals after all other market prices of risk are measured, and do not reflect how proxies of their pricing kernel covary with bond returns at jumps events. In contrast, Bai, Collin-Dufresne, Goldstein, and Helwege (2015) argue that, if jump-risk is priced due to a contagious response,[5] then the ratio $\lambda^{\mathbb{Q}}/\lambda^{\mathbb{P}}$ has an upper bound of approximately 1.1; hence, they conclude that spreads due to this channel cannot be much above expected losses. In our paper, we claim that $\lambda^{\mathbb{P}}$ is itself very small for IG firms. In particular, we find that historical one-year default rates over-estimate the jump-to-default intensity faced by an investment strategy that holds only IG corporate bonds.[6]

The combination of dynamic debt issuance and the presence of information asymmetry between the manager and creditors provides a framework that nests most of the models studied by the extant literature. We exploit this generality to compare the im-

[2]An incomplete list of contributions to this literature includes Merton (1974); Leland (1994); Goldstein, Ju, and Leland (2001); Hennessy and Whited (2007); Abel (2016, 2017); DeMarzo and He (2017); Admati, DeMarzo, Hellwig, and Pfleiderer (2017). Our work is also related to the previous literature that models firms' earnings, or assets, with complete information via jump-diffusion processes (e.g., Zhou (2001); Gorbenko and Strebulaev (2010)). These specifications, however, do not give rise to a stochastic intensity for default unless the only variation in asset levels is through jumps.

[3]See, e.g., Elton, Gruber, Agrawal, and Mann (2001), Longstaff, Mithal, and Neis (2005), Chen, Lesmond, and Wei (2007), Feldhütter and Schaefer (2017), and Culp, Nozawa, and Veronesi (2015).

[4]Similar results are reported by Saita (2006) and Berndt, Douglas, Duffie, Ferguson, and Schranz (2009).

[5]Note, however, that Bai et al (2015) do not consider the possibility of simultaneous default of a finite fraction of the economy.

[6]Our paper is also related to the vast literature that studies voluntary disclosure of managerial information (e.g., Shin (2003)), and the roll-over of short-maturity debt, market runs, and market freezes (e.g., Diamond and Dybvig (1983), Acharya, Gale, and Yorulmazer (2011), He and Xiong (2012), Schroth, Suarez, and Taylor (2014), Dang, Gorton, and Holmström (2015), and Carré (2016)).

plications of our model for optimal capital structure choice, credit spreads, and default frequencies with this literature. Specifically, by restricting the manager to issue only equity after date-0, our setting reverts back to Duffie and Lando (2001). If we assume that manager and creditors are equally informed, we obtain a model of optimal capital structure dynamics with complete information (e.g., Goldstein, Ju, and Leland (2001); Hennessy and Whited (2007); DeMarzo and He (2017)). By removing both the ability to issue debt and the presence of informational asymmetry, we recover a version of the Leland (1994) model.

Our analysis is most relevant for IG spreads of short-maturity debt where the "credit spread puzzle" is most prevalent. Indeed, a growing literature (e.g., Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010)) argues that IG spreads for maturities greater than a few years can be explained by combining pricing kernels that capture time varying Sharpe ratios over the business cycle with models that match the empirically observed clustering of defaults during recessions. In contrast, we show that incomplete information combined with debt issuance *lowers* short-term credit spreads of IG firms. Hence, our findings "deepen" the credit spread puzzle for IG firms at short maturities.

The rest of the paper is organized as follows. In Section 2, we present stylized facts on investment grade companies that motivate our analysis. Section 3 builds a model of corporate debt issuance and default decisions in the presence of asymmetric information between the manager and creditors. In Section 4 we present the implications of our model for optimal capital structure decisions and derive model-implied credit spreads and default rates. Section 5 concludes. Proofs are in Appendix A, while further details concerning the empirical analysis and the numerical solution of the model are in an Online Appendix.

2 Stylized facts

Here we discuss a few stylized facts about the corporate bond market that provide empirical foundations for our research.

Fact 1: IG companies dominate the bond market. Figure 1 tracks the percentage of investment-grade (IG) firms over time and compares it to the proportions of higher-and lower-quality speculative grade companies (labeled B and C).[7] Investment-grade companies are the majority among the firms with bonds monitored by credit rating agencies. The proportion is highest in the early part of the sample period, and fluctuates around 50% starting from the early 1990s. Higher-quality speculative grade companies make up the second largest group, while C-rated companies comprise less than 10% of the market.

Related, a large fraction of IG firms are net issuers of corporate debt. For instance, using the Mergent Fixed Income Security Database (FISD), Greenwood and Hanson (2013) document that on average 68% of all debt issuance in the period 1983–2008 are originated by IG firms. Using Moody's Bond Surveys, they show that 89% of all debt issuance in the 1926–1982 period originates from IG firms.

Fact 2: Firms with IG status rarely default. Table 1 shows average annualized default rates for companies in the IG, B, and C groups. Panels A and B concentrate on firms classified based on credit ratings issued by rating agencies over the periods 1985-2014 and 2001-2014. It is evident that defaults by IG firms are extremely rare. Panel A shows that on average only 0.11% of IG companies file for bankruptcy within a year of being assigned an IG classification. Defaults over the first month are even less frequent, with an annualized rate of 0.06%.

The likelihood of a default for an IG firm is even lower when we use market-based information to rate companies. Agencies do not continuously update their ratings to fully reflect the information available to market participants. Hence, we consider an alternative classification of firms into the same three creditworthiness groups that is based on CDS data (Appendix B). At horizons from one to three months, we find IG default rates that are virtually zero. In particular, we find a point estimate for the annualized rate equal to 0.01%, and statistically insignificant (Table 1, Panel C).

Overall, this evidence shows that defaults by investment grade firms rarely come as a total surprise to market participants. Indeed, even for higher-quality speculative grade firms, there is limited support for jumps-to-default. Rather, it is mostly the lowest-rated firms that file for bankruptcy. Even so, the fact that asset values (i.e., debt and equity) typically jump at the default event suggests that default does come at least as a partial surprise to market participants, providing empirical support for the Duffie-Lando (2001)

[7] Appendix A explains how we classify firms in the three categories, IG, B, and C.

mechanism. At the one-month horizon, the average default rate for C companies can exceed an annualized rate of 15% (Table 1, Panel A); beyond the first month, default rates for firms in the C group decline progressively but remain elevated. [8]

Even if we restrict our attention to firms that held IG status for at least one of the 12 months preceding the default event, we find that the great majority of these companies exhibit a considerable run-up in credit spreads for many months before they default. This provides investors with a signal that the credit worthiness of such companies has deteriorated below IG well before their bankruptcy. Figure 2 shows the difference between the average CDS premium on those firms and the CDX-IG index. This spread is very small 12 months prior to bankruptcy and then increases in the ensuing months as the firms drop out of the IG group and approach bankruptcy. This evidence suggests that an investment policy that (1) holds bonds issued by firms in the IG category, and (2) unwinds these positions when the firm loses IG status, faces virtually zero default risk. Hence, the jump-to-default premium for this portfolio should be negligible.[9]

3 Model

To explain the stylized facts of the previous section, we develop a model of corporate debt issuance and default decisions in the presence of asymmetric information. As in Duffie and Lando (2001) (DL hereafter) we assume that firms' creditors have less information than does management. However, two main features distinguish our model from DL. First, we allow firms to issue both debt and equity, a feature motivated by the fact that a large fraction of IG firms are net issuers of corporate debt. Second, we model asymmetric information by assuming that creditors can continuously observe the value of the firm's assets with a delay. This feature reflects the fact that it takes time for market participants to acquire the accounting information needed to accurately value a firm's assets.

[8] It is likely that CDS trading declines when the company is close to distress and the contract is in the money. In contrast, credit agencies are likely to update their ratings more frequently when conditions for a company deteriorate. Hence, since CDSs are less traded when default risk is high, it is not surprising that empirical default rates for companies assigned a C label based on CDS-implied rating underestimates the default rate obtained when companies are classified based on credit ratings (Table 1, Panel C vs. Panels A and B).

[9] Note that jump-to-default-risk is conceptually different from the risk that jump intensities increase in reduced-form models. Driessen (2005) estimates only moderate market prices of risk for exposure to shocks to jump intensities.

3.1 Setup

Following Leland (1994), we assume that the unlevered firm value V_t follows a geometric Brownian motion under the risk neutral measure $\mathbb{Q}$, that is,

$$\frac{dV_t}{V_t} = r\,dt + \sigma\,dB_t^{\mathbb{Q}}, \tag{1}$$

where r and σ denote the constant risk-free rate and asset volatility, and $dB_t^{\mathbb{Q}}$ denotes the increment of a standard Brownian motion under $\mathbb{Q}$. Defining $v_t \equiv \ln V_t$ and using Itô's lemma, we have

$$dv_t = m\,dt + \sigma\,dB_t^{\mathbb{Q}}, \quad \text{where} \quad m \equiv r - \frac{\sigma^2}{2}. \tag{2}$$

Insiders can observe the asset value V_t in real time, whereas creditors can observe V_t only with a time lag L. That is, at time t, creditors know only $V_{(t-L)}$. We define the lagged asset value observed by creditors as

$$\widehat{V}_t \equiv V_{(t-L)} \quad \text{and} \quad \hat{v}_t \equiv v_{(t-L)}. \tag{3}$$

Any default is observed immediately by both insiders and outside creditors.

After choosing an initial financing mix of debt and equity at time zero, we assume that firms can issue debt until the time in which they exhaust their "debt issuing capacity" at a future time t^*. After this time, the firm is forced to either issue equity to service the debt-in-place, or choose to default. Figure 3 provides an illustration of the model timeline. There are four relevant time regimes:

1. **Regime 1:** $0 \le t < t^*$. In this regime, firms have the capacity to raise new debt to service debt-in-place. Total debt outstanding at time t promises a perpetual and constant coupon payment, C_t, until the firm declares bankruptcy. Because equity holders do not need to infuse money into the firm, there is no reason for them to choose to default during this time regime. Therefore, this interval is characterized by a zero default intensity (and hence, command zero default-risk premia). We refer to firms in this regime as "Investment-Grade" firms.

2. **Regime 2:** $t = t^*_-$. At this point in time, the firm reaches debt capacity. Because the firm can no longer issue debt, if its asset value (which is known only

by the manager) falls below the default threshold (the value of which is publicly known), then the firm immediately defaults. Rather than a default intensity, this regime/instant of time is associated with a *finite* probability of default.

3. **Regime 3:** $t^* \leq t < t^* + L$. Firms can no longer issue debt and may default. Absent default, creditors infer that the firm's assets must have remained above the default threshold over the time interval $[t^*, t]$ (the duration of which is less than L). As we show below, in this regime, the default intensity depends on both lagged asset value $\hat{v}_t$ and the time interval since the firm reached debt capacity, $(t - t^*)$.

4. **Regime 4:** $t \geq t^* + L$. Firms can no longer issue debt and may default. Absent default, creditors infer that the firm's assets must have remained above the default threshold over the time interval $[t - L, t]$ of length L. As we show below, in this regime, the default intensity depends only on lagged asset value $\hat{v}_t$, independent of time.

To solve for the value of debt and equity under the information set of creditors, we first need to derive the optimal default boundary chosen by management, and then solve for the value of firm's securities backward in time, starting from Regime 4.

3.2 Shareholders' optimal default policy

Recall that, in our framework, it is in the shareholder's best interest for management to avoid default prior to the firm's debt capacity being exhausted (i.e., $t < t^*$). Hence, to determine the optimal default policy, we consider equity valuation at times after debt capacity is reached (i.e., $t > t^*$). In this regime, shareholders are committed to pay a continuous coupon $C = C_{t^*}$ to outstanding debt-holders. We assume that the tax rate is θ, and that coupon payments are tax deductible as long as the firm remains solvent. As demonstrated below, the coupon level $C = C_{t^*}$ will be endogenously determined as a function of both the evolution of firm value, and the firm's debt capacity.

As shown in Leland (1994), in this setting there exists a constant default threshold V_B such that it is in the shareholders' bests interests that management default on the debt and declare bankruptcy the first time asset value falls below V_B. Define $S(V_t)$ as the value of equity when the outstanding debt consists of a perpetual continuous coupon C. To determine the default boundary V_B, note that for asset values V_t above this default boundary, the equity claim satisfies:

$$S\left(V_t\right) = -(1-\theta)C\,dt + e^{-r\,dt}\,\mathbb{E}_t^{\mathbb{Q}}\left[S(V_{t+dt})\right]. \tag{4}$$

Intuitively, shareholders must pay out-of-pocket the after-tax coupon payment $(1-\theta)C\,dt$, and own the claim to $S\left(V_{t+dt}\right)$. Applying Itô's lemma to (4), we obtain that the equity value must satisfies the following ordinary differential equation

$$0 = -(1-\theta)C - rS + rVS_V + \frac{\sigma^2}{2}V^2 S_{VV}, \tag{5}$$

subject to the boundary conditions

$$\lim_{V_t\to\infty} S(V_t) = V_t - \frac{(1-\theta)C}{r}, \tag{6}$$

$$\lim_{V_t\to V_B} S(V_t) = 0, \tag{7}$$

$$\left.\frac{\partial}{\partial V_t}S(V_t)\right|_{V_t=V_B} = 0. \tag{8}$$

Condition (6) implies that, as asset value goes to infinity, the equity value converges to the default-free value. Condition (7) imposes that, at the default boundary, the equity value goes to zero. Condition (8) is the smooth pasting condition that guarantees the default decision is optimal for shareholders.

The solution of (5), subject to the boundary conditions (6)–(8), is given by

$$S(V_t) = V_t - \frac{(1-\theta)C}{r} - \left(\frac{V_t}{V_B}\right)^{-\frac{2r}{\sigma^2}}\left[V_B - \frac{(1-\theta)C}{r}\right], \tag{9}$$

where

$$V_B = \frac{C}{\beta}, \quad \text{with} \quad \beta \equiv \frac{2r+\sigma^2}{2(1-\theta)}. \tag{10}$$

Because all the parameters are common knowledge to all claim-holders, the default threshold (10) is known to both creditors and shareholders. However, whereas insiders observe firm value V_t at date t, creditors observe only the lagged value $\hat{V}_t = V_{(t-L)}$. An important scaling feature that we will use below is that the optimal default boundary V_B is linear in the size of the coupon C.

3.3 Debt valuation

To determine the value of debt from the creditors' perspective, we proceed by assuming that the default boundary (10) corresponds to a generic value of the cumulative coupon C, whose value C_{t^*} is discussed below. For convenience, we let $v_B \equiv \ln(V_B)$ and define the processes y_t and $\hat{y}_t$ as:

$$y_t \equiv v_t - v_B, \quad \text{and} \quad \hat{y}_t \equiv y_{(t-L)} = \hat{v}_t - v_B. \tag{11}$$

Debt-holders know the location of the default boundary v_B, and that equity holders will default the first time $\tau_d \geq t^*$ in which v_t falls below v_B.[10] Formally

$$\tau_d = \inf\{t \geq t^* : v_t \leq v_B\}. \tag{12}$$

In the event of default, debt-holders receive a fraction $(1-\alpha)$ of the firm's assets.

Let us denote by $D_t \equiv D(\hat{y}_t, t, C_t, \mathbf{1}_{\{\tau_d > t\}})$ the market value of firm's debt as assessed by creditors. Debt value for a firm that is not in default at time t depends on the lagged asset value $\hat{y}_t$, the time t, and the cumulative coupon level C_t. In general, we can express the value D_t recursively as the sum of the coupon payment flow and the discounted expected value of future debt, that is

$$D_t = \begin{cases} C_t \, dt + e^{-r\,dt}\, \mathbb{E}_t^{\mathbb{Q}} \left[\left(\dfrac{C_t}{C_{t+dt}} \right) D_{t+dt} \middle| \, \mathcal{F}_t \right] & \text{if } t < t^* \quad \text{(13a)} \\ \mathbb{E}_t^{\mathbb{Q}} \left[D_{t^*} \mathbf{1}_{\{y_{t^*} > 0\}} + (1-\alpha) e^{v_B + y_{t^*}} \mathbf{1}_{\{y_{t^*} < 0\}} \middle| \, \mathcal{F}_t \right], & \text{if } \ t = t^*_- \text{(13b)} \\ C_{t^*} \, dt + e^{-r\,dt}\, \mathbb{E}_t^{\mathbb{Q}} \left[D_{t+dt} \mathbf{1}_{\{\tau_d > (t+dt)\}} + (1-\alpha) e^{v_B} \, \mathbf{1}_{\{\tau_d \in (t, t+dt)\}} \middle| \, \mathcal{F}_t \right], & \text{if } \ t > t^* \text{ (13c)} \end{cases}$$

where the $\mathbf{1}_{\{\tau_d > t\}}$ indicates that default has not occurred by time t, and $\mathcal{F}_t$ is the information set of outsiders, which is described below for each regime. Expression (13a) shows that, because the firm issues debt before time t^*, existing debt-holders claims are diluted. The quantity $\left(\frac{C_t}{C_{t+dt}} \right) D_{t+dt}$ represents the time-$(t+dt)$ debt value accruing to time-t debt-holders. Note that there is no default before t^*, and thus, the default intensity is zero for dates $(t < t^*)$. Expression (13b) defines the debt value at time t^*_-, just as the firm reaches debt capacity, and equals the probability-weighted sum of: (i) the value of debt at time t^* if the firm survives, and (ii) the recovery value, otherwise.

[10] We note that, if firm value drops below v_B at dates $(t < t^*)$, default will not occur, as the manager will keep this private information secret, and new debt issuances will service debt in place.

Expression (13c) differs from (13a) along two dimensions: (i) the firm is no longer issuing debt, and hence existing debt-holders are not diluted; (ii) the firm defaults if v_t reaches v_B within the next interval dt, in which case debt-holders receive the recovery value $(1-\alpha)e^{v_B}$.

To solve for the bond price, we work backwards in time, starting with the time interval $t \geq t^*$ (which is similar in spirit to Duffie and Lando (2001)). We further break this regime into two separate sub intervals, depending on whether $t \in [t^*, t^* + L)$ or $t \in [t^* + L, \infty)$. In the next subsection we formally characterize the solution for each time regime.

3.3.1 Debt value in Regime 4, $t \in [t^* + L, \infty)$

If default has not yet occurred by date $t \in [t^* + L, \infty)$, then debt-holders, aware that the firm had previously exhausted its debt capacity at date t^*, infer that the value of the firm's assets must have remained above the threshold level v_B during the time interval (t^*, t):

$$\min_{s\in[t^*,t]} \{v_s\} > v_B. \tag{14}$$

Also at time t, debt-holders know the lagged asset value $\hat{v}_t = v_{(t-L)}$. Now, because the unlevered firm value process is one-factor Markov, it follows that the information inferred from equation (14) over the interval $(t^*, (t-L))$ is redundant. Hence, the debt-holders' useful information set in this regime is

$$\mathcal{F}_t = \left\{ \hat{v}_t = v_{(t-L)};\ \min_{s\in[t-L,t]} \{v_s\} > v_B \right\}, \quad t \geq t^* + L. \tag{15}$$

The following proposition shows that the information set characterized by equation (15) generates a time invariant setting in which the price of debt and the default intensity depend only on surivival up to date-t and the lagged firm value $\hat{v}_t = v_{(t-L)}$ (or equivalently, $\hat{y}_t = \hat{v}_t - v_B$).

Proposition 1 *For dates $t \geq (t^* + L)$, the price of debt $D_4^C(\mathbf{1}_{\{\tau_d > t\}}, \hat{y}_t = y_{(t-L)})$ with claim to coupon C is time invariant, and given by the sum of two components:*

$$D_4^C(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) \;=\; D_{4,1}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) + D_{4,2}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}), \tag{16}$$

where

1. $D_{4,1}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)})$ *is the present value of claims to coupon subject to no default*

$$D_{4,1}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) \;=\; C\int_t^\infty dT\, D^T_{4,1}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}), \tag{17}$$

with $D^T_{4,1}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)})$ *the date-t price of a claim that pays \$1 at date-T if and only if* $\tau_d > T$ *obtained in (A.9), and*

2. $D_{4,2}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)})$ *equals the claim to recovery* $(1-\alpha)V_B$ *at the default event:*

$$D_{4,2}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) \;=\; (1-\alpha)V_B\int_t^\infty dT\; D^T_{4,2}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) \tag{18}$$

with $D^T_{4,2}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)})$ *the date-t price of a claim that pays \$1 at date-T if and only if* $\tau_d = T$ *derived in (A.11).*

In the proof of the proposition, we show that the bond price in Regime 4 has no explicit time dependence, and is function only of the state vector $(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)})$.

Corollary 1 *Consistent with Duffie and Lando (2001), the default intensity* $\lambda^Q_{4,d}(\hat{y}_t)$ *satisfies*

$$\begin{aligned}\lambda^Q_{4,d}(\hat{y}_t) &= \frac{\sigma^2}{2}\frac{\partial}{\partial y_t}\pi^Q_4(y_t|\tau_d > t, \hat{y}_t)\Big|_{y_t=0} \\ &= \frac{\hat{y}_t}{\sqrt{2\pi\sigma^2 L^3}}\left(\frac{e^{-\left(\frac{1}{2\sigma^2 L}\right)\left(\hat{y}_t+mL\right)^2}}{\pi^Q_4\left(\tau_d > t|\, y_{(t-L)}, \tau_d > (t-L)\right)}\right)\mathbf{1}_{\{\hat{y}_t>0\}},\end{aligned} \tag{19}$$

where the conditional density $\pi^Q_4(y_t|\tau_d > t, \hat{y}_t)$ *is defined in equation (A.1) of Lemma 1.*

Note, however, that the bond price D^C_4 *does not* satisfy the standard pricing ODE:

$$0 \;\neq\; C - \left(r + \lambda^Q_{4,d}(\hat{y}_t)\right)D_4 + mD^C_{4,\hat{y}} + \frac{\sigma^2}{2}D^C_{4,\hat{y}\hat{y}} + (1-\alpha)e^{v_B}\lambda^Q_4(\hat{y}_t). \tag{20}$$

This is because the expected growth rate of dy conditional upon the information set of outsiders is larger than the unconditional growth rate m:[11]

$$\mu_{\hat{y}}(\hat{y}_t) \;\equiv\; \left(\frac{1}{dt}\right)\mathrm{E}^Q\left[dy_{(t-L)}\;\Big|\; y_{(t-L)}, \tau_d > (t-L)\right] > m. \tag{21}$$

[11]To understand this result intuitively, consider a gambler with initial wealth W_0 who wagers each period (of duration Δt) a bet of size $\$\sqrt{\Delta t}$ on a fair coin for $N = \left(\frac{T}{\Delta t}\right)$ periods, and who must

Indeed, we find that

$$\begin{aligned}\lim_{\hat{y}\Rightarrow 0}\ \mu_{\hat{y}}(\hat{y}_t) &= \infty \\ \lim_{\hat{y}\Rightarrow 0}\ D^C_{4,\hat{y}}(\hat{y}_t) &= 0,\end{aligned} \tag{22}$$

but that the product $\left(\mu_{\hat{y}}(\hat{y}_t) D^C_{4,\hat{y}}(\hat{y}_t)\right)$ remains finite in the limit.

3.3.2 Debt value in Regime 3, $t \in [t^*, (t^* + L))$

Using as input the value of debt $D^C_4(\hat{y}_{(t^*+L)} = y_{t^*}, \mathbf{1}_{\{\tau_d > (t^*+L)\}})$ identified in the previous section, we now determine the value of debt at earlier dates $t \in [t^*, t^* + L)$, which we refer to as Regime 3. An important special case is the price of debt at t^*, which will be used as an input to determine bond prices in earlier regimes. As such, in this section, we focus on this case, leaving the more general case to Appendix B.

Note that during Regime 3, assuming default has not occurred by date-t, outsiders know only lagged firm value $\hat{v}_t = v_{(t-L)}$ (or equivalently, $y_{(t-L)} = (v_{(t-L)} - v_B)$), and that firm value has remained above the default boundary from dates $s \in (t^*, t)$. In particular, it is possible that there existed dates $(s < t^*)$ for which firm value was below the default threshold (i.e., $v_s < v_B$), but that, because management concealed this low firm value by issuing new debt to service existing debt, default did not occur under these circumstances. The debt-holders information structure in this regime is thus:

$$\mathcal{F}_t = \left\{\hat{v}_t = v_{(t-L)};\ \min_{s\in[t^*,t]}\{v_s\} > v_B\right\}, \quad \forall\, t \in (t^*, (t^* + L)). \tag{23}$$

In particular, for the special case $t = t^*$, we have

$$\mathcal{F}_{t^*} = \left\{\hat{v}_{t^*} = v_{(t^*-L)}; v_{t^*} > v_B\right\}. \tag{24}$$

stop gambling if she ever becomes bankrupt. That is, each bet pays off $+\$\sqrt{\Delta t}$ ($-\$\sqrt{\Delta t}$) if the coin flip returns "Heads" ("Tails"). If we are told only that the gambler never went bankrupt, then the conditional probability that the first coin flip was "Heads" is higher than 50%. Moreover, this probability increases the lower is W_0. Indeed, for the special case $W_0 = \sqrt{\Delta t}$, the conditional probability that the first coin flip was heads equals 100% (because, had it been tails, the gambler would have gone bankrupt immediately), and the conditional expected change in wealth per period is $\mu_{\hat{y}}(\hat{y}_t) = \left(\frac{\sqrt{\Delta t}}{\Delta t}\right) = \left(\frac{1}{\sqrt{\Delta t}}\right)$, which explodes in the continuous time limit.

For convenience, we will use interchangeably $(v_{t^*} > v_B)$ and $(\tau_d > t^*)$. Hence, this information set can be equivalently expressed as:

$$\mathcal{F}_{t^*} = \left\{ \hat{y}_{t^*} = y_{(t^*-L)}; \tau_d > t^* \right\}. \tag{25}$$

Comparing to the information set in equation (15), we note that bondholders have less information in Regime 3 than in Regime 4, in that, in Regime-4, the agent knows that firm value remained above the default boundary for the time interval L. In contrast, for the special case $(t = t^*)$, bondholders know only that firm value is above the default boundary at a single instant (t^*). In Appendix B, we show that the price of debt and the default intensity depend both on $\hat{y}_t$ and date-t, in contrast to its time-independent values derived for Regime 4 debt prices in Proposition 1. Here, however, we focus only on the pricing of debt at date t^*.

Proposition 2 *The date-t^* value of debt in Regime 3, $D_3^C(t^*, \mathbf{1}_{\{\tau_d > t^*\}}, y_{(t^*-L)})$, with claim to coupon C, is the sum of three components:*

1. *For dates $T \in (t^*, (t^* + L))$, the claim to coupon payment $C\,dT$ at date-T if default has not yet occurred (i.e., $\tau_d > T$), given by*

$$D_{3,1}(t^*, \mathbf{1}_{\{\tau_d > t^*\}}, y_{(t^*-L)}) \quad = \quad C \int_{t^*}^{(t^*+L)} dT\, D_{3,1}^T(t^*, \mathbf{1}_{\{\tau_d > t^*\}}, y_{(t^*-L)}), \tag{26}$$

   where the date-t^*-price $D_{3,1}^T(t^*, \mathbf{1}_{\{\tau_d > t^*\}}, y_{(t^*-L)})$ of a claim that pays \$1 at date-T if and only if $\tau_d > T$ is derived in equation (A.24);

2. *For dates $T \in (t^*, (t^* + L))$, the claim to recovery if default occurs at date-T (i.e., $\tau_d = T$), given by*

$$D_{3,2}(t^*, \mathbf{1}_{\{\tau_d > t^*\}}, y_{(t^*-L)}) \quad = \quad (1-\alpha)V_B \int_{t^*}^{(t^*+L)} dT\, D_{3,2}^T(t^*, \mathbf{1}_{\{\tau_d > t^*\}}, y_{(t^*-L)}), \tag{27}$$

   where the date-t^*-price $D_{3,2}^T(t^*, \mathbf{1}_{\{\tau_d > t^*\}}, y_{(t^*-L)})$ of a claim that pays \$1 at date-T if and only if $\tau_d = T$ is derived in equation (A.26); and

3. *The claim $D_{3,3}(t^*, \mathbf{1}_{\{\tau_d > t^*\}}, y_{(t^*-L)})$ to the bond value $D_4(y_{t^*}, \tau_d > (t^*+L))$ if default occurs later than $(t^* + L)$ (i.e., $\tau_d > (t^* + L)$), derived in equation (A.28).*

Hence, the time-t^* value of the debt in Region 3 is given by

$$D^C_3(t^*, \mathbf{1}_{\{\tau_d > t^*\}}, y_{(t^*-L)}) \;=\; D_{3,1}(t^*, \mathbf{1}_{\{\tau_d > t^*\}}, y_{(t^*-L)}) + D_{3,2}(t^*, \mathbf{1}_{\{\tau_d > t^*\}}, y_{(t^*-L)}) + D_{3,3}(t^*, \mathbf{1}_{\{\tau_d > t^*\}}, y_{(t^*-L)}). \quad (28)$$

Consistent with DL, the default intensity $\lambda^Q_{3,d}(t^*, y_{(t^*-L)}, \tau_d > t^*)$ satisfies

$$\begin{aligned}\lambda^Q_{3,d}(t^*, y_{(t^*-L)}, \tau_d > t^*) &= \frac{\sigma^2}{2}\frac{\partial}{\partial y_t}\pi_3(y_{t^*}|y_{(t^*-L)}, \tau_d > t^*)\Big|_{y_t=0} \\ &= \left(\frac{1}{2}\right)\frac{\left(y_{(t^*-L)} + mL\right)}{\sqrt{2\pi\sigma^2 L^3}}\left(\frac{e^{-\left(\frac{1}{2\sigma^2 L}\right)\left(y_{(t^*-L)}+mL\right)^2}}{N\left(\frac{y_{(t^*-L)}+mL}{\sqrt{\sigma^2 L}}\right)}\right). \quad (29)\end{aligned}$$

In Appendix B we derive the value of the debt for each date $t \in (t^*, t^* + L)$.

3.3.3 Debt value in Regime 2, $t = t^*_-$

The instant t^* at which the firm exhausts its debt capacity is defined as the first time the ratio of the cumulative coupon level C_t to the lagged unlevered firm value $\widehat{V}_t$ reaches a threshold $\overline{\Psi}$ that has been exogenously specified in the covenants of all previously issued bonds. Formally,

$$t^* = \inf\left\{t : \frac{C_t}{\widehat{V}_t} = \overline{\Psi}\right\}. \quad (30)$$

For all dates $t < t^*$, the only information debt-holders receive about the firm is (i) the amount of debt outstanding, which is characterized by the level of coupon C_t, and (ii) the lagged unlevered firm value $\widehat{V}_t = V_{(t-L)}$, that is,

$$\mathcal{F}_t = \left\{\hat{v}_t = v_{(t-L)}; C_t\right\}, \quad t \le t^*. \quad (31)$$

Because the unlevered firm dynamics follow a diffusion process, under the information set (31) the event in which debt capacity is exhausted is therefore predictable by both debt-holders and the manager.

Given the process (2) for the unlevered log asset value, at time t^*, the probability density of the current value $y_{t^*} = v_{t^*} - v_B$, conditional on the lagged value $\hat{y}_{t^*} = y_{(t^*-L)}$ is normal with mean $(\hat{y}_{t^*} + mL)$ and volatility $\sigma\sqrt{L}$, that is,

$$\pi\left(y_{t^*}|\hat{y}_{t^*}\right) = \frac{1}{\sqrt{2\pi\sigma^2 L}}\, e^{-\left(\frac{1}{2\sigma^2 L}\right)\left[y_{t^*}-(\hat{y}_{t^*}+mL)\right]^2}. \quad (32)$$

It follows that the probability of a time-t^* default equals the probability that $y_{t^*} < 0$, or

$$\pi_2\left(\tau_d = t^* | \hat{y}_{t^*}\right) = \int_{-\infty}^{0} \pi\left(y_{t^*} | \hat{y}_{t^*}\right) dy_{t^*} = N\left(\frac{-\hat{y}_{t^*} - mL}{\sqrt{\sigma^2 L}}\right), \tag{33}$$

and the probability of survival is

$$\pi_2\left(\tau_d > t^* | \hat{y}_{t^*}\right) = N\left(\frac{\hat{y}_{t^*} + mL}{\sqrt{\sigma^2 L}}\right). \tag{34}$$

From the recursive pricing equation (13b), the debt value at t^* is a sum of two terms: (i) the expected value of debt $D_3(t^*,\, \hat{y}_{t^*})$ if no default occurs at t^* and (ii) an expectation of recovery in the event of default. Using the above conditional probabilities we can then write the value of debt $D_{t^*_-} = D_2(\hat{y}_{t^*}, t^*, C_{t^*})$ as follows:

$$\begin{aligned} D_2(\hat{y}_{t^*}, t^*, C_{t^*}) \;&=\; \mathbb{E}_t^{\mathbb{Q}}\left[D_{t^*} \mathbf{1}_{\{y_{t^*}>0\}} + (1-\alpha) e^{v_B + y_{t^*}} \mathbf{1}_{\{y_{t^*}<0\}} \middle| \; \hat{y}_{t^*} \right] \\ &= \pi_2\left(\tau_d > t^* | \hat{y}_{t^*}\right)\; D_3(\hat{y}_{t^*}, t^*, C_{t^*}) + \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma^2 L}}\; e^{-\left(\frac{1}{2\sigma^2 L}\right)\left[y_{t^*} - (\hat{y}_{t^*} + mL)\right]^2} (1-\alpha) e^{v_B + y_{t^*}}\, dy_{t^*} \\ &= N\left(\frac{\hat{y}_{t^*} + mL}{\sqrt{\sigma^2 L}}\right)\; D_3(\hat{y}_{t^*}, t^*, C_{t^*}) + (1-\alpha)\, e^{v_B + \hat{y}_{t^*} + (m + \frac{\sigma^2}{2})L}\; N\left(-\frac{\hat{y}_{t^*} + (m+\sigma^2)L}{\sqrt{\sigma^2 L}}\right), \end{aligned} \tag{35}$$

where the debt value $D_3(\hat{y}_{t^*}, t^*, C_{t^*})$ is obtained in Proposition 2 and the last equality uses the expressions from the densities (33) and (34).

3.3.4 Debt value in Regime 1, $t \in [0, t^*)$

At time 0, the firm issues a perpetuity with promised cash flows comprised of a continuous coupon $C_0\, dt$ until it optimally decides to default at time τ_d, where C_0 determines the initial capital structure as discussed in Section 3.6 below.

Because the firm does not generate any intermediate cash flows, we assume it continuously issues new *pari-passu* debt in order to service the debt-in-place until it exhausts its debt capacity at time t^*. Because shareholders are not required to contribute fresh cash to the firm (i.e., no equity issuances) it is optimal for them to avoid default for any $t < t^*$. Therefore, the default intensity is zero in this regime. After exhausting debt capacity, we assume that covenants restrict any future debt issuances, that is, for dates $t > t^*$, the firm has to either issue new equity to service debt-in-place or default.

The dynamics dC_t for the size of the coupon payment is determined endogenously by identifying how much future cash flow must be promised to new bondholders in order to entice them to service the current debt due. Define by $D_1(\hat{y}_t, C_t)$ the value of all debt in-place at date t prior to receiving the coupon payment.[12] At date t, the firm needs to raise $C_t\,dt$. However, due to tax-deductibility of interest on debt, a fraction $\theta C_t\,dt$ is covered by the government, where θ is the effective tax rate. Hence, the firm needs to raise only $(1-\theta)C_t\,dt$ to service the old debt. The present value of the new debt issuance must then equal $(1-\theta)C_t\,dt$. Because all debt is pari passu, the fraction of debt owned by the new owners, determined at date t, is

$$\pi_{\text{new}} = \left(\frac{C_{t+dt} - C_t}{C_{t+dt}}\right), \tag{36}$$

whereas the fraction of debt owned by previous owners is

$$\pi_{\text{old}} = \left(\frac{C_t}{C_{t+dt}}\right). \tag{37}$$

It follows that C_{t+dt}, and hence, the dynamics dC_t, can be determined by equating the value of the new debt claim to the amount new debt-holders pay for this claim:

$$(1-\theta)C_t\,dt = [D_1(\hat{y}_t, C_t) - C_t\,dt]\left(\frac{C_{t+dt} - C_t}{C_{t+dt}}\right). \tag{38}$$

Using the fact that $C_{t+dt} = C_t + dC_t$, as $dt \to 0$ equation (38) simplifies to

$$dC_t = (1-\theta)\left(\frac{C_t^2}{D_1(\hat{y}_t, C_t)}\right)dt, \tag{39}$$

which specifies the coupon dynamics in terms of the bond price D_1.

To determine the bond price $D_1(\hat{y}_t)$ recall that, from the recursive pricing equation (13a), we have

$$D_1(\hat{y}_t, C_t) = C_t\,dt + e^{-r\,dt}\,\mathbb{E}_t^{\mathbb{Q}}\left[\left(\frac{C_t}{C_{t+dt}}\right)D_1(\hat{y}_{t+dt}, C_{t+dt})\right]. \tag{40}$$

The above equation states that the present value of the debt-in-place at date t is equal to the value of the coupon, $C_t\,dt$, and the fraction $\pi_{\text{old}} = \left(\frac{C_t}{C_{t+dt}}\right)$ of next period's debt

[12]Note that, in continuous-time, the value of debt is the same before and after coupon payment, since the coupon payment is linear in dt.

claim, whose date-t present value is determined by risk-neutral discounting. Applying Itô's lemma to equation (40) we obtain that the value of the debt for $t < t^*$ solves the PDE[13]

$$\begin{aligned}
0 &= C_t - \left(r + (1-\theta)\frac{C_t}{D_1}\right) D_1 + (1-\theta)\left(\frac{C^2}{D_1}\right) D_{1,C} + mD_{1,\hat{y}} + \frac{\sigma^2}{2} D_{1,\hat{y}\hat{y}} \\
&= -rD_1 + \theta C_t + (1-\theta)\left(\frac{C_t^2}{D_1}\right) D_{1,C} + mD_{1,\hat{y}} + \frac{\sigma^2}{2} D_{1,\hat{y}\hat{y}}, \qquad (41)
\end{aligned}$$

subject to the boundary conditions:

$$\lim_{\hat{y}_t \to \infty} D_1(\hat{y}_t, C_t) = \frac{C_t}{r}, \qquad (42)$$

$$\lim_{\hat{y}_t \to \log\left(\frac{C_t}{\Psi V_B}\right)} D_1(\hat{y}_t, C_t) = D_2(\hat{y}_t, t, C_t) \qquad (43)$$

$$\lim_{C_t \to 0} D_1(\hat{y}_t, C_t) = \frac{C_t}{r}, \qquad (44)$$

where $D_2(\hat{y}_{t^*}, t^*, C_{t^*})$ is defined in (35), and where $\lim\left(\hat{y}_t \to \log\left(\frac{C_t}{\Psi V_B}\right)\right)$ is equivalent to $\lim(t \to t^*)$.

3.4 Creditors' valuation of equity

Let $\widehat{S}_t \equiv \widehat{S}_t(\hat{y}_t, C_t, \mathbf{1}_{\{\tau_d > t\}})$ be the equity value conditional on the creditors' information set, and $S_t \equiv S_t(y_t, C_t, \mathbf{1}_{\{\tau_d > t\}})$ be the equity value conditional on the manager's information set. The equity value $\widehat{S}_t$ can be expressed recursively as follows:

$$\widehat{S}_t = \begin{cases} \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(t^*-t)}\widehat{S}_{t^*} | \mathcal{F}_t\right] & \text{if } t < t^* \qquad (45a) \\ \int_0^\infty S_t(y_t, C_{t^*}, \mathbf{1}_{\{\tau_d > t\}})\, \pi^{\mathbb{Q}}(y_t, t | \hat{y}_t, \tau_d > t)\ dy_t & \text{if } t \geq t^* \qquad (45b) \end{cases},$$

where S_t in (45b) is the solution to equations (9)–(10) of the ODE (5) with boundary conditions (6)–(7). The following proposition characterizes the equity value $\widehat{S}_t$ for $t \geq t^*$.

[13]Note that $C_{t+dt} = C_t + dC_t = C_t + (1-\theta)\frac{C_t^2}{D_t}dt$ by equation (39). Hence, it follows that $\frac{C_t}{C_{t+dt}} = \frac{1}{1+(1-\theta)\frac{C_t}{D_t}dt} = \left[1 - (1-\theta)\frac{C_t}{D_t}dt\right]$.

Proposition 3 *The equity value $\widehat{S}_{t^*}$ is*

$$\begin{aligned}\widehat{S}_{t^*} &= e^{\hat{y}_{t^*}+v_B}\left\{e^{mL+\frac{\sigma^2 L}{2}}\, N\left[\frac{\log\left(\frac{\beta}{\overline{\Psi}}\right)+mL+\sigma^2 L}{\sqrt{\sigma^2 L}}\right]-\frac{(1-\theta)\overline{\Psi}}{r}\, N\left[\frac{\log\left(\frac{\beta}{\overline{\Psi}}\right)+mL}{\sqrt{\sigma^2 L}}\right]\right.\\ &\qquad \left.-\left(\frac{\beta}{\overline{\Psi}}\right)^{-\frac{2r}{\sigma^2}}\left(\frac{\overline{\Psi}}{\beta}-\frac{(1-\theta)\overline{\Psi}}{r}\right) e^{rL}\, N\left[\frac{\log\left(\frac{\beta}{\overline{\Psi}}\right)+mL-2rL}{\sqrt{\sigma^2 L}}\right]\right\}. \qquad (46)\end{aligned}$$

The equity value $\widehat{S}_t$ for $t > t^*$ is

$$\widehat{S}_t = \begin{cases}\displaystyle\int_0^\infty \pi_3(y_t \mid \tau_d > t,\, \hat{y}_t)\, S(y_t, C_{t^*})\, dy_t, & \text{if } t^* < t < t^*+L \qquad (47a)\\ \displaystyle\int_0^\infty \pi_4(y_t \mid \tau_d > t,\, \hat{y}_t)\, S(y_t, C_{t^*})\, dy_t, & \text{if } t \geq t^*+L\ , \qquad (47b)\end{cases}$$

where $\pi_3(y_t \mid \tau_d > t,\, \hat{y}_t)$ is given in equation (A.4), $\pi_4(y_t|\tau_d > t,\, \hat{y}_t)$ is given in equation (A.1) and $S_t(y_t, C_{t^*})$ is the solution (9)–(10) of the ODE (5) with boundary conditions (6)–(7).

With the value of $\widehat{S}_{t^*}$ obtained in equation (46), we can determine the outsiders' valuation of equity $\widehat{S}_t$ for $t < t^*$ by solving the expectation in equation (45a). Intuitively, this expectation captures the fact that there are no cash flows for equity over the interval $t \in (0, t^*)$ and hence, the equity value is given by its risk-neutral discounted value at $t = t^*$. To solve this expectation we note that $e^{-rt}\widehat{S}(\hat{y}_t, C_t) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-rt^*}\widehat{S}_{t^*}\right]$ is a $\mathbb{Q}$-martingale. Applying Itô's lemma and using the coupon dynamics (39) we obtain that $\widehat{S}_t$ satisfies the PDE

$$0 = -r\widehat{S} + m\widehat{S}_{\hat{y}} + \frac{\sigma^2}{2}\widehat{S}_{\hat{y}\hat{y}} + \widehat{S}_C(1-\theta)\left(\frac{C^2}{D(\widehat{y}, C)}\right), \qquad (48)$$

with boundary conditions

$$\lim_{\hat{y}_t \to \hat{y}_{t^*}} \widehat{S}_t(\hat{y}_t, C_t) = \widehat{S}_{t^*} \qquad (49)$$

$$\lim_{\hat{y}_t \to \infty} \widehat{S}_t(\hat{y}_t, C_t) = e^{(\hat{y}_t + v_B + mL + \frac{\sigma^2}{2}L)} - \frac{(1-\theta)C_t}{r}, \qquad (50)$$

$$\lim_{C_t \to 0} \widehat{S}_t(\hat{y}_t, C_t) = e^{(\hat{y}_t + v_B + mL + \frac{\sigma^2}{2}L)} - \frac{(1-\theta)C_t}{r}, \qquad (51)$$

where $\widehat{S}_{t^*}$ is given in equation (46).

3.5 Scaling property

The valuation expressions for debt and equity that we have obtained above exhibit an important scaling property. From equation (10), the default boundary is $V_B = C_{t^*}/\beta$. Hence, the state variable $\hat{y}_t = \log(\widehat{V}_t/V_B)$ is unaffected if the lagged asset value $\widehat{V}_t$ and cumulative coupon C_{t^*} are scaled by a constant factor. From the characterization of debt values for $t \geq t^*$ in Propositions 1, 2, and equation (13b), we note that the debt values are linear in the coupon level C, that is

$$D_t(\hat{y}_t, t, C) = C \times D_t(\hat{y}_t, t, 1), \quad t \geq t^*. \tag{52}$$

The following proposition shows that the value of debt and equity at any time $t \leq t^*$ are homogeneous of degree one in the state vector $\left(C_t, \widehat{V}_t\right)$. This allows us to reduce the dimensionality of the problem, and express (scaled) debt and equity as a function only of the debt-to-assets ratio $\Psi_t \equiv C_t/\widehat{V}_t$. It is convenient to rewrite the value of debt using $\hat{V}_t$ as a state variable instead of $\hat{y}_t$. We therefore define $D^*(\widehat{V}_t, C_t) = D(\widehat{y}_t, C_t)$. We claim:

Proposition 4 *Let $\widehat{V}_t$ be the lagged asset value (1), V_B the default boundary (10) for a generic cumulative coupon C_{t^*} and $\Psi_t = C_t/\widehat{V}_t$ the debt-to-assets ratio. Then the debt value $D^*(\widehat{V}_t, C_t)$ for $t < t^*$ is*

$$D^*(\widehat{V}_t, C_t) = \widehat{V}_t\, G\left(\Psi_t\right), \tag{53}$$

where $G(\Psi_t)$ solves the PDE

$$\theta\Psi - rG + (1-\theta)\Psi^2 \frac{G_\Psi}{G} + r\left(G - \Psi G_\Psi\right) + \frac{\sigma^2}{2}\Psi^2 G_{\Psi\Psi} = 0, \tag{54}$$

subject to the boundary conditions

$$G(0) \;=\; 0 \tag{55}$$

$$G(\overline{\Psi}) \;=\; \overline{\Psi}\, D_2\left(\log\left(\frac{\beta}{\overline{\Psi}}\right), t^*, 1\right), \tag{56}$$

where $\overline{\Psi} = C_{t^*}/\widehat{V}_{t^*}$ is the debt capacity constraint in equation (30), and $D_2(\hat{y}_{t^*}, t^*, C_{t^*})$ is defined in (35).

Note that the boundary condition (56) implies that the bond value $G(\Psi_t)$ does not depend on the actual level of coupon C_{t^*}. This is an implication of the scaling property (52) and of the definitions of maximum debt capacity (30), $\widehat{V}_{t^*} = C_{t^*}/\overline{\Psi}$, and default boundary (10), $V_B = \left(\frac{C_{t^*}}{\beta}\right)$. Using these definitions in $\hat{y}_{t^*} \equiv \log\left(\frac{\widehat{V}_{t^*}}{V_B}\right)$, we obtain $\hat{y}_{t^*} = \log\left(\frac{\beta}{\overline{\Psi}}\right)$, as in equation (56).

A similar scaling property also holds for the value of equity $\widehat{S}_t(\hat{y}_t, C_t)$ defined in equations (45a)–(45b). Just as we did for debt, it is convenient to rewrite the value of equity using $\hat{V}_t$ as a state variable instead of $\hat{y}_t$. We therefore define $\hat{S}^*(\widehat{V}_t, C_t) = \hat{S}(\widehat{y}_t, C_t)$. We claim:

Proposition 5 *Let $\widehat{V}_t$ be the lagged asset value (1), V_B the default boundary (10) for a generic cumulative coupon C_{t^*} and $\Psi_t = \left(\frac{C_t}{\widehat{V}_t}\right)$ the debt-to-assets ratio. Then the equity value $\widehat{S}^*(\widehat{V}_t, C_t)$ for $t \leq t^*$ can be expressed as:*

$$\widehat{S}^*\left(\widehat{V}_t, C_t\right) = \widehat{V}_t\, H\left(\Psi_t\right), \tag{57}$$

where $H(\Psi_t)$ solves the ODE

$$-r\Psi H_{\Psi} + \frac{\sigma^2}{2}\Psi^2 H_{\Psi\Psi} + \frac{H_{\Psi}}{G(\Psi)}\,(1-\theta)\Psi^2 = 0, \tag{58}$$

subject to the boundary conditions

$$\begin{aligned} H(\overline{\Psi}) &= e^{mL+\frac{\sigma^2 L}{2}}\, N\left[\frac{\log\left(\frac{\beta}{\overline{\Psi}}\right) + mL + \sigma^2 L}{\sqrt{\sigma^2 L}}\right] - \frac{(1-\theta)\overline{\Psi}}{r}\, N\left[\frac{\log\left(\frac{\beta}{\overline{\Psi}}\right) + mL}{\sqrt{\sigma^2 L}}\right] \\ &\quad - \left(\frac{\beta}{\overline{\Psi}}\right)^{-\frac{2r}{\sigma^2}} \left(\frac{\overline{\Psi}}{\beta} - \frac{(1-\theta)\overline{\Psi}}{r}\right) e^{rL}\, N\left[\frac{\log\left(\frac{\beta}{\overline{\Psi}}\right) + mL - 2rL}{\sqrt{\sigma^2 L}}\right] \qquad (59) \\ H(0) &= 1. \qquad (60) \end{aligned}$$

Here, we have defined $\overline{\Psi} = \left(\frac{C_{t^*}}{\widehat{V}_{t^*}}\right)$ as the credit constraint in equation (30), and $G(\Psi_t) = \left(D^*(\hat{V}_t, C_t)/\hat{V}_t\right)$ is defined in Proposition 4.

The boundary condition (59) follows directly from equation (46) in Proposition 3.

3.6 Optimal capital structure and debt dynamics

We use Propositions 4 and 5 to solve for the firm's optimal capital structure at time 0. For tractability, we assume that the manager does not have an informational advantage at this time and that, like the creditors, she observes only the lagged unlevered asset value $\widehat{V}_0 = V_{-L}$.

To determine the firm's optimal capital structure, we exploit the scaling property of the problem and, without loss of generality, set $\widehat{V}_0 = 1$ in Propositions 4 and 5. This implies $\Psi_0 = C_0$, $D_0 = G(\Psi_0)$ and $\widehat{S}_0 = H(\Psi_0)$. The firm value is then $G(\Psi_0) + H(\Psi_0)$, and therefore the optimal capital structure Ψ_0^{opt} at time 0 is given by:

$$\Psi_0^{\text{opt}} = \arg\max_{\Psi_0} \{G(\Psi_0) + H(\Psi_0)\}. \tag{61}$$

Note that choosing the initial capital structure Ψ_0^{opt} is equivalent to selecting an initial coupon level C_0^{opt} for a firm with asset size $\widehat{V}_0$. In general, the initial coupon will be $C_0^{\text{opt}} = \Psi_0^{\text{opt}}\,\widehat{V}_0$.

After choosing its initial capital structure, the firm continuously issues debt at a rate dC_t given by equation (39), until it exhausts its debt capacity. Applying Itô's lemma to $\Psi_t \equiv \left(C_t/\widehat{V}_t\right)$ we obtain the following dynamics for Ψ_t:

$$d\Psi_t = \left[(1-\theta)\frac{\Psi_t^2}{G(\Psi_t)} - (r-\sigma^2)\Psi_t\right] dt - \sigma\Psi_t\, dB_t^{\mathbb{Q}}, \quad \Psi_0 = \Psi_0^{\text{opt}} = C_0^{\text{opt}}. \tag{62}$$

By construction, at the random time t^*, $\Psi_{t^*} = \overline{\Psi}$, where $\overline{\Psi}$ is an exogenous parameter representing a firm's debt capacity. At any prior time ($t \leq t^*$), the cumulative coupon C_t for a firm of size $\widehat{V}_0 = 1$ is given by

$$C_t^{(\widehat{V}_0=1)} = \Psi_t\, \widehat{V}_t^{(\widehat{V}_0=1)}, \tag{63}$$

where $\widehat{V}_t^{(\widehat{V}_0=1)}$ is the lagged value of assets at time $(t-L)$, V_{t-L} obtained from the dynamics (1) with initial condition $V_{(-L)} = 1$. Because of the scaling property discussed in Section 3.5, the cumulative coupon of a firm with initial asset size $\widehat{V}_0 = \nu$ is simply $\nu \times C_t^{(\widehat{V}_0=1)}$.

4 Results

There are two defining features of our model that, taken together, set it apart from previous contributions. First, we allow for information asymmetry between the firm's manager and creditors. Second, the firm continues to issue debt until it reaches its debt capacity. It is useful to organize the discussion of our model's implications along these two elements, so as to more easily draw a comparison with the previous literature.

The key model coefficients associated with information asymmetry and debt issuance are L and $\overline{\Psi}$. In the baseline case, we assume that it takes creditors six months to learn the true value of the firm's assets, i.e., they observe V with a $L = 0.5$ delay. Furthermore, we use $\overline{\Psi}$ to generate a leverage at time t^* of approximately 75%, in line with leverage values of firms that recently transitioned to "fallen angel" status.

Other special cases are also relevant. For instance, when $(L = 0, \overline{\Psi} = C_0)$, the manager and creditors share the same information set, and the firm is permitted to issue debt only at time 0. This case is similar to the Leland (1994) setting. Another special case is $(L > 0, \overline{\Psi} = C_0)$—which is closely related to the economy of Duffie and Lando (2001) in which a better-informed manager chooses the optimal mix of debt and equity at time zero, but is prevented from issuing debt in the future. Finally, the case in which $(L = 0, \overline{\Psi} > C_0)$ falls within the literature on optimal capital structure dynamics with complete information (e.g., Goldstein, Ju, and Leland (2001), Hennessy and Whited (2007), DeMarzo and He (2017)).

Table 2 reports the rest of the model coefficients for the baseline calibration. We normalize the total coupon payment at one, $C_{t^*} = 1$. The risk-neutral asset dynamics in equation (1) are governed by a 0.5% riskfree rate[14] and 30% volatility per year. In the model, the capital structure choice is driven by the trade-off between debt tax shield and bankruptcy cost. In this respect, we assume that corporate profits are taxed at a $\theta = 25\%$ rate, while the bankruptcy cost parameter is set to $\alpha = 0.4$.

4.1 Capital structure

Figure 4 shows the optimal firm capital structure at time $t = 0$ as a function of the information lag L. The blue line, labeled 'BGG', portrays the solution for our model,

[14] In a more general setting in which the firm has a constant payout ratio δ, the risk-neutral drift of firm value is $(r - \delta)$ rather than r in our paper. Our calibration choice of $r = 0.5\%$ reflects this.

computed as in Section 3.6. The red line shows similar results for a firm that is restricted to issue equity at time $t = 0$ only, i.e., $\overline{\Psi} = C_0$. We label this case as "Duffie-Lando".

In our baseline calibration ($L = 0.5$), the optimal initial leverage is 31.73%, compared to 40.40% in the Duffie-Lando case. In our model, the firm continues to borrow after time 0 to service debt in place, and its leverage increases to 77.33% by the time the credit constraint becomes binding at time t^*. Hence, while the ability to issue debt guarantees that default cannot occur in the "short term" (i.e., prior to t^*), compared to a firm in which the level of outstanding debt is fixed (as in DL), these future debt issuances significantly increase the probability of default at longer maturities. This in turn makes it optimal for the firm to choose a lower initial leverage in our framework compared to that of DL.

Furthermore, Figure 4 shows that as the information asymmetry between the manager and creditors increases, the firm issues less debt initially. For instance, when creditors observe the value of the assets with a one-year delay, optimal initial leverage decreases to 31.16% in our model; a similar drop occurs in Duffie-Lando, where optimal initial leverage is 39.73%. At the other extreme, when $L \to 0$, both manager and creditors observe the true value of the assets without a lag. In this case, the Duffie-Lando case collapses into the Leland model, with initial optimal leverage peaking at 41.35%. In our model, the ability to issue additional debt in the future increases the riskiness of the initial debt in place, which in turn causes the optimal initial leverage to be lower (32.30%).

Next, we illustrate the sensitivity of the firm's optimal initial capital structure to the credit constraint coefficient $\overline{\Psi}$. The left panels of Figures 5 report both the optimal leverage at time 0 (the triangles) and debt capacity leverage at time-t^* (the stars) as a function of $\overline{\Psi}$ when information asymmetry is (i) low ($L = 0.01$, top panel), (ii) medium ($L = 0.5$, center), and (iii) high ($L = 1$, bottom). In all cases, we find that the optimal initial leverage ratio is a decreasing function of $\overline{\Psi}$. Intuitively, this occurs because, when $\overline{\Psi}$ is decreased, original debtholders know that the firm's debt capacity will be exhausted at lower levels of leverage, which in turn decreases the probability of default at "medium" maturities. This reduction in default rates will increase the value of the original debt issuance, which in turn will increase the optimal initial leverage chosen by the manager. As $\overline{\Psi}$ drops further to approach C_0, initial leverage converges to the

Duffie-Lando capital structure solution that we discussed in the previous paragraph.

In the right panels, we show how the degree of information asymmetry, captured by varying the lag parameter L, interacts with the credit constraint in determining a firm's financing decisions. Both at time 0 and t^*, a larger information lag between creditors and the manager increases the cost of debt issuance, and therefore reduces optimal initial leverage.

4.2 Term structure of credit spreads

We use the pricing formulas for perpetual debt, derived in Section 3, to simulate model-implied default times as detailed in the Online Appendix. We then compute the term-structure of defaultable bond spreads as in Duffie and Lando (2001). Specifically, in each simulated path for asset values, we price a defaultable zero coupon bond with maturity T whose payoff at maturity is either \$1, if there is no default, or a recovery value of $\$(1-\alpha)$ if default occurs at any time $\tau_d \leq T$. Figure 6 shows the model-implied credit spreads curve as a function of the information lag L. In the baseline case, we identify a typical investment-grade company with credit spreads of about 65 bps at the five-year horizon. We find such firm to have leverage of 32% at time 0; to facilitate comparisons with the other cases, when we change L we also adjust the initial amount of debt issued to keep leverage at the same level. As the information lag L increases, debt becomes riskier and credit spreads go up. However, in all cases short-maturity credit spreads remain very low. This is because in our model IG companies are subject to negligible jump-to-default risk; hence, they command little or no jump-to-default premium, consistent with the stylized facts discussed in Section 2.

Figures 7 and 8 document the sensitivity of credit spreads to changes in $\overline{\Psi}$ and σ when keeping initial leverage at the same 32% level. As $\overline{\Psi}$ increases, firms arrive at t^* with a larger stock of debt. Hence, creditors expect the firm to reach its default boundary sooner than in the baseline case, and therefore price debt lower. Even in this case, however, the impact of a higher $\overline{\Psi}$ is mostly visible in longer-dated spreads. In contrast, short-maturity debt largely remains safe, as the IG company can avoid default at short horizons by accessing its available debt capacity. A similar pattern is evident in Figure 8: as asset volatility increases, debt becomes riskier and spreads go up. However, the increase is mostly visible at longer maturities, while short-term IG spreads stay low.

In contrast, Figure 9 shows model-implied spreads for a fallen-angel company that has reached a leverage of 80%. In the complete information case ($L \approx 0$), spreads are small and close to zero at short maturities. As the degree of asymmetric information between managers and creditors increases, spreads rise considerably. For instance, an information lag of $L = 1$ year produces spreads of 750 basis points at the one-month horizon. This happens because after the firm reaches its debt capacity at time t^*, it behaves similar to a firm described by the Duffie-Lando economy. In particular, as leverage increases jumps to default are likely and priced in the firm's debt.

4.3 Default rates

Table 3 shows model-implied expected default rates for firms in different credit-rating groups. We simulate a sample of 10,000 firms, and track each of them until their eventual default (details on the simulation scheme are in the Online Appendix). For each firm and at each point of its simulated life span, we record the time to the company's default and use the firm's leverage as a proxy for credit worthiness. In particular, we assign firms with leverage no higher than 65% to the IG group. Companies with leverage between 65% and 75% are in the B group, while the rest are given a C label. We then compute the proportion of the firms in a rating group that default at various time horizons, and report the annualized default rate in the table.

Model-implied default rates are close to the empirical estimates in Table 1. Just as in the data, IG companies hardly ever go bankrupt; at short horizons, default rates are virtually zero and they increase progressively over time. Failures remain infrequent among B firms, though in this case the annualized default rate exceeds 2% at the 9–12 months horizon. Firms in the C category behave instead in a way that is consistent with the possibility of jumps to defaults. At the one-month horizon, the annualized default rate is approximately 15%, a number that matches closely empirical default rates for companies that are rated C by the three main rating agencies. Beyond the first month, default rates decline progressively, though they remain elevated, like we have have found in the data.

5 Conclusion

We provide theoretical and empirical support for the notion that IG firms face virtually zero jump-to-default risk, and therefore their short-maturity bonds command virtually zero jump-to-default premium. We develop a model in which the manager has superior information about the value of the firm's assets relative to creditors, and can access the debt markets if the firm's debt capacity has not been fully utilized. In this framework, a manager of an IG firm will maximize shareholder value by concealing any bad private signal and servicing existing debt via additional borrowing. This strategy permits IG firms to avoid jumping to default, at least until their debt capacity has been used up and the firm has dropped down to "fallen angel" status with speculative-grade spreads. Creditors are aware of the manager's information advantage and price it rationally into the firms' claims. Since firms with IG-level spreads do not face jump-to-default risk, their bond yields do not command a jump-to-default premium.

By allowing for dynamic debt issuance, our model extends the seminal work of Duffie and Lando (2001) in a way that greatly helps to characterize the way a vast portion of corporate bond issuers fund themselves. We acknowledge, however, that our analysis abstracts from some important features of credit markets. For instance, shareholders in our model do not receive any cash dividend payout, and the only reason for issuing debt is to repay debt in place. Adding a dividend payout will accelerate the time in which debt capacity is exhausted but will not qualitatively alter the key insights from our analysis. Further, in our model, firms continue to issue debt until its debt capacity has been fully utilized. In reality, conditions might improve after a firm reaches its debt capacity. For instance, the asset value could grow significantly to bring leverage down back to IG level, thus allowing the firm to tap into the bond market again. We can allow for this possibility in our model, however at significant costs in the computations and exposition.

We note that our framework generates a prediction nearly opposite to DL. By precluding the manager from issuing debt after date 0, DL show that firms can jump-to-default due to asymmetric information even if the underlying asset value dynamics follows a diffusion process. In contrast, by allowing the manager of IG firms to issue debt, we show that, in the presence of asymmetric information, such firms will not jump to default even if the underlying asset value dynamics follows a jump process. A jump to default due to asymmetric information is possible in our setting only when firms become "fallen angels." The implication of our model is that the relatively large spreads on short-maturity IG debt over risk-free securities cannot be explained by jump-to-default premia due to asymmetric information, and therefore implies that other channels (e.g., asymmetric taxes, illiquidity, rare disasters) are needed to explain these large spreads.

A Appendix: Proofs

Lemma 1 *The density* $\pi_4^Q(y_t|\tau_d > t,\, y_{(t-L)})$, $t \geq t^* + L$ *is given by*

$$\pi_4^Q(y_t|\tau_d > t,\, y_{(t-L)}) \quad = \quad \frac{\pi^Q(y_t, \tau_d > t|y_{(t-L)}, \tau_d > (t-L))}{\pi^Q\left(\tau_d > t|\, y_{(t-L)}, \tau_d > (t-L)\right)}, \tag{A.1}$$

where the numerator and denominator are

$$\begin{aligned} \pi^Q(y_t, \tau_d > t|y_{(t-L)}, \tau_d > (t-L)) \quad &= \quad \mathbf{1}_{\{y_t>0\}}\, \mathbf{1}_{\{y_{(t-L)}>0\}} \times \\ &\left[\begin{array}{l} \frac{1}{\sqrt{2\pi\sigma^2 L}} \exp\left\{ \left(\frac{-1}{2\sigma^2 L}\right) \left[y_t - y_{(t-L)} - mL\right]^2 \right\} \\ -e^{-\frac{2y_{(t-L)}m}{\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2 L}} \exp\left\{ \left(\frac{-1}{2\sigma^2 L}\right) \left[y_t + y_{(t-L)} - mL\right]^2 \right\} \end{array}\right] \end{aligned} \tag{A.2}$$

$$\pi^Q\left(\tau_d > t|\, y_{(t-L)}, \tau_d > (t-L)\right) \quad = \quad \left[N\left(\frac{\hat{y}_t + mL}{\sqrt{\sigma^2 L}}\right) - e^{-\frac{2\hat{y}_t m}{\sigma^2}} N\left(\frac{-\hat{y}_t + mL}{\sqrt{\sigma^2 L}}\right)\right] \mathbf{1}_{\{\hat{y}_t>0\}}, \tag{A.3}$$

and $N(\cdot)$ *denotes the cumulative standard normal distribution.*

Proof. From Proposition 8.1, p. 11 in Harrison (1985) the density $\pi^Q(y_t, \tau_d > t|y_{(t-L)}, \tau_d > (t-L))$ in the numerator of (A.1) is characterized by the "free solution" minus an "image solution" whose initial location $(-\hat{y}_t \equiv -y_{(t-L)})$ is the same distance from the default boundary as is the actual initial location, that is, $(\hat{y}_t \equiv y_{(t-L)})$. Hence, the numerator of (A.1) is given by

$$\begin{aligned} \pi^Q(y_t, \tau_d > t|y_{(t-L)}, \tau_d > (t-L)) \quad &= \quad \mathbf{1}_{\{y_t>0\}}\, \mathbf{1}_{\{y_{(t-L)}>0\}} \times \\ &\left[\begin{array}{l} \frac{1}{\sqrt{2\pi\sigma^2 L}} \exp\left\{ \left(\frac{-1}{2\sigma^2 L}\right) \left[y_t - y_{(t-L)} - mL\right]^2 \right\} \\ -e^{-\frac{2y_{(t-L)}m}{\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2 L}} \exp\left\{ \left(\frac{-1}{2\sigma^2 L}\right) \left[y_t + y_{(t-L)} - mL\right]^2 \right\} \end{array}\right] \end{aligned}$$

By integrating over y_t we obtain the denominator of (A.1), that is,

$$\pi^Q\left(\tau_d > t|\, y_{(t-L)}, \tau_d > (t-L)\right) \quad = \quad \left[N\left(\frac{\hat{y}_t + mL}{\sqrt{\sigma^2 L}}\right) - e^{-\frac{2\hat{y}_t m}{\sigma^2}} N\left(\frac{-\hat{y}_t + mL}{\sqrt{\sigma^2 L}}\right)\right] \mathbf{1}_{\{\hat{y}_t>0\}},$$

where $N(\cdot)$ denotes the cumulative standard normal density.

$\blacksquare$

Lemma 2 *The density* $\pi_3^Q(y_t|\tau_d > t,\, y_{(t-L)})$, $t \in (t^*, t^*+L)$ *is given by*

$$\pi_3^Q(y_t|\tau_d > t,\, y_{(t-L)}) \;=\; \frac{\pi^Q(y_t, \tau_d > t|y_{(t-L)}, \tau_d > (t-L))}{\pi^Q\left(\tau_d > t|\, y_{(t-L)}, \tau_d > (t-L)\right)}, \tag{A.4}$$

where the numerator and denominator are

$$\begin{aligned}
\pi^Q(y_t, \tau_d > t|y_{(t-L)}, \tau_d > (t-L)) \;&=\; \mathbf{1}_{\{y_t>0\}} \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2(t-t^*)}} \\
&\times \left[e^{-\left(\frac{1}{2\sigma^2(t-t^*)}\right)[y_t - y_{t^*} - m(t-t^*)]^2} - e^{-\frac{2y_{t^*}m}{\sigma^2}} e^{-\left(\frac{1}{2\sigma^2(t-t^*)}\right)[y_t + y_{t^*} - m(t-t^*)]^2} \right] \\
&\times \frac{1}{\sqrt{2\pi\sigma^2(t^*-(t-L))}}\; e^{-\left(\frac{1}{2\sigma^2(t^*-(t-L))}\right)[y_{t^*} - \hat{y}_t - m(t^*-(t-L))]^2}\; dy_{t^*},
\end{aligned} \tag{A.5}$$

$$\begin{aligned}
\pi^Q\left(\tau_d > t|\, y_{(t-L)}, \tau_d > (t-L)\right) \;&=\; \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2(t^*-(t-L))}}\; e^{-\left(\frac{1}{2\sigma^2(t^*-(t-L))}\right)[y_{t^*} - \hat{y}_t - m(t^*-(t-L))]^2} \\
&\times \left\{ N\left[\frac{y_{t^*} + m(t-t^*)}{\sqrt{\sigma^2(t-t^*)}}\right] - e^{-\frac{2y_{t^*}m}{\sigma^2}}\; N\left[\frac{-y_{t^*} + m(t-t^*)}{\sqrt{\sigma^2(t-t^*)}}\right] \right\} dy_{t^*}.
\end{aligned} \tag{A.6}$$

Proof. By definition

$$\begin{aligned}
\pi_3(y_t, \tau_d > t \,|\, \hat{y}_t) \;&\equiv\; \pi(y_t, \min\{y_s\} > 0\; \forall s \in (t^*, t) \,|\, \hat{y}_t) \\
&= \int_{-\infty}^{\infty} \pi(y_{t^*}, y_t, \min\{y_s\} > 0\; \forall s \in (t^*, t) \,|\, \hat{y}_t)\; dy_{t^*} \\
&= \int_{-\infty}^{\infty} \pi(y_t, \min\{y_s\} > 0\; \forall s \in (t^*, t) \,|\, y_{t^*})\;\; \pi(y_{t^*} \,|\, \hat{y}_t)\; dy_{t^*} \\
&= \mathbf{1}_{\{y_t>0\}} \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2(t-t^*)}} \\
&\times \left[e^{-\left(\frac{1}{2\sigma^2(t-t^*)}\right)[y_t - y_{t^*} - m(t-t^*)]^2} - e^{-\frac{2y_{t^*}m}{\sigma^2}} e^{-\left(\frac{1}{2\sigma^2(t-t^*)}\right)[y_t + y_{t^*} - m(t-t^*)]^2} \right] \\
&\times \frac{1}{\sqrt{2\pi\sigma^2(t^*-(t-L))}}\; e^{-\left(\frac{1}{2\sigma^2(t^*-(t-L))}\right)[y_{t^*} - \hat{y}_t - m(t^*-(t-L))]^2}\; dy_{t^*},
\end{aligned} \tag{A.7}$$

where the third line holds because, when conditioning on $(y_{t^*}, \hat{y}_t)$, y_{t^*} is a sufficient statistic.

By integrating the joint density (A.7) over $y_t \in (0, \infty)$, we find

$$\begin{aligned}
\pi_3(\tau_d > t \,|\, \hat{y}_t) &\equiv \pi(\min\{y_s\} > 0 \; \forall s \in (t^*, t) \,|\, \hat{y}_t) \\
&= \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2(t^* - (t-L))}} \; e^{-\left(\frac{1}{2\sigma^2(t^*-(t-L))}\right)[y_{t^*} - \hat{y}_t - m(t^* - (t-L))]^2} \\
&\quad \times \left\{ N\left[\frac{y_{t^*} + m(t - t^*)}{\sqrt{\sigma^2(t - t^*)}}\right] - e^{-\frac{2y_{t^*} m}{\sigma^2}} \; N\left[\frac{-y_{t^*} + m(t - t^*)}{\sqrt{\sigma^2(t - t^*)}}\right]\right\} dy_{t^*}. \qquad \text{(A.8)}
\end{aligned}$$

■

Proof of Proposition 1

Let us define $D^T_{4,1}(\mathbf{1}_{\{y_t>0\}}, y_{(t-L)})$ as the date t price of a claim that pays \$1 at date-T if and only if $\tau_d > T$. Its value is obtained as follows:

$$\begin{aligned}
D^T_{4,1}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) &= e^{-r(T-t)} \, \mathbf{1}_{\{\tau_d>t\}} \, \mathrm{E}^Q\left[\mathbf{1}_{\{\tau_d>T\}} | \tau_d > t, y_{(t-L)}\right] \\
&= e^{-r(T-t)} \mathbf{1}_{\{\tau_d>t\}} \, \pi^Q\left(\tau_d > T | \tau_d > t, y_{(t-L)}\right) \\
&= e^{-r(T-t)} \mathbf{1}_{\{\tau_d>t\}} \int_0^\infty dy_t \, \pi^Q\left(\tau_d > T, \, y_t | \tau_d > t, y_{(t-L)}\right) \\
&= e^{-r(T-t)} \mathbf{1}_{\{\tau_d>t\}} \int_0^\infty dy_t \, \pi^Q\left(\tau_d > T | \tau_d > t, y_t\right) \pi^Q_4\left(y_t | \tau_d > t, y_{(t-L)}\right) \\
&= e^{-r(T-t)} \mathbf{1}_{\{\tau_d>t\}} \int_0^\infty dy_t \left[N\left(\frac{y_t + m(T-t)}{\sqrt{\sigma^2(T-t)}}\right) - e^{-\frac{2y_t m}{\sigma^2}} N\left(\frac{-y_t + m(T-t)}{\sqrt{\sigma^2(T-t)}}\right)\right] \\
&\qquad \times \pi^Q_4\left(y_t | \tau_d > t, y_{(t-L)}\right), \qquad \text{(A.9)}
\end{aligned}$$

where $\pi^Q_4\left(y_t | \tau_d > t, y_{(t-L)}\right)$ is given in (A.1).

It therefore follows that the date-t price of a perpetuity that pays $C \, dT \mathbf{1}_{\{\tau_d>T\}}$ for all dates-T is

$$D_{4,1}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) = C \int_t^\infty dT \, D^T_{4,1}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}). \qquad \text{(A.10)}$$

Somewhat analogously, let us define $D^T_{4,2}(\mathbf{1}_{\{y_t>0\}}, y_{(t-L)})$ as the date-t price of a claim that pays \$1 at date-T if and only if $\tau_d = T$. Its value is obtained as follows

$$
\begin{aligned}
& D^T_{4,2}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) = \\
& \quad = \; e^{-r(T-t)}\, \mathbf{1}_{\{\tau_d>t\}}\; \mathrm{E}^Q\left[\delta(\tau_d - T)|\tau_d > t, y_{(t-L)}\right] \\
& \quad = \; e^{-r(T-t)}\, \mathbf{1}_{\{\tau_d>t\}}\; \pi^Q\left(\tau_d = T|\tau_d > t, y_{(t-L)}\right) \\
& \quad = \; e^{-r(T-t)}\, \mathbf{1}_{\{\tau_d>t\}} \int_0^\infty dy_t\; \pi^Q\left(\tau_d = T, y_t|\tau_d > t, y_{(t-L)}\right) \\
& \quad = \; e^{-r(T-t)}\, \mathbf{1}_{\{\tau_d>t\}} \int_0^\infty dy_t\; \pi^Q\left(\tau_d = T|\tau_d > t, y_t\right)\; \pi^Q_4\left(y_t|\tau_d > t, y_{(t-L)}\right) \\
& \quad = \; e^{-r(T-t)}\, \mathbf{1}_{\{\tau_d>t\}} \int_0^\infty dy_t\; \pi^Q\left(\tau_d = T|\tau_d > t, y_t\right)\; \pi^Q_4\left(y_t|\tau_d > t, y_{(t-L)}\right) \\
& \quad = \; e^{-r(T-t)}\, \mathbf{1}_{\{\tau_d>t\}} \int_0^\infty dy_t\; \pi^Q_4\left(y_t|\tau_d > t, y_{(t-L)}\right) \frac{y_t}{\sqrt{2\pi\sigma^2(T-t)^3}}\; e^{-\left(\frac{1}{2\sigma^2(T-t)}\right)\left(y_t+m(T-t)\right)^2},
\end{aligned}
\tag{A.11}
$$

where $\delta(\cdot)$ denotes the Dirac's delta function. Finally, the claim to $(1-\alpha)V_B$ at the time of default equals

$$
D_{4,2}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) \;=\; (1-\alpha)V_B \int_t^\infty dT\; D^T_{4,2}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}). \tag{A.12}
$$

Therefore, the claim to debt in Regime 4 is the sum:

$$
D_4(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) \;=\; D_{4,1}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) + D_{4,2}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}). \tag{A.13}
$$

Note that by change of variables $s = (T-t)$, it becomes clear that the bond price in Regime 4 has no explicit time dependence, and is function only of the state vector $(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)})$. ∎

Proof of Corollary 1

We define the risk-neutral default intensity as

$$
\begin{aligned}
\lambda^Q_{4,d}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) \;&=\; \mathbf{1}_{\{\tau_d>t\}} \lim_{dt\Rightarrow 0}\left(\frac{1}{dt}\right)\pi^Q\left(\tau_d < (t+dt)|\tau_d > t, y_{(t-L)}\right) \qquad \text{(A.14)} \\
&=\; \mathbf{1}_{\{\tau_d>t\}} \lim_{dt\Rightarrow 0}\left(\frac{1}{dt}\right)\int_0^\infty dy_t\,\pi^Q\left(\tau_d < (t+dt),\, y_t|\tau_d > t, y_{(t-L)}\right) \\
&=\; \mathbf{1}_{\{\tau_d>t\}} \lim_{dt\Rightarrow 0}\left(\frac{1}{dt}\right)\int_0^\infty dy_t\,\pi^Q\left(\tau_d < (t+dt)|\tau_d > t, y_t\right)\pi^Q\left(y_t|\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}\right).
\end{aligned}
$$

Note that only values of $y_t = \alpha\sigma\sqrt{dt}$ contribute to the determination of the default intensity. To see why this is the case, recall that, from a binomial model, $dy = \pm\sigma\sqrt{dt}$. Therefore, only if y_t is a few standard deviations away from the boundary at date-t is it possible for default to occur by $(t+dt)$. Changing the integration variables $y_t = \alpha\sigma\sqrt{dt}$, $dy_t = d\alpha\,\sigma\sqrt{dt}$, we find

$$
\begin{aligned}
\pi^Q\left(\tau_d < (t+dt)|\mathbf{1}_{\{\tau_d>t\}}, y_t = \alpha\sigma\sqrt{dt}\right) \;=\; & \\
&= \left[1 - \pi^Q\left(\tau_d > (t+dt)|\mathbf{1}_{\{\tau_d>t\}}, y_t = \alpha\sigma\sqrt{dt}\right)\right] \\
&= \left[1 - \left\{N\left(\frac{\alpha\sigma\sqrt{dt} + mdt}{\sqrt{\sigma^2\,dt}}\right) - e^{-\frac{2\alpha\sigma\sqrt{dt}m}{\sigma^2}} N\left(\frac{-\alpha\sigma\sqrt{dt} + mdt}{\sqrt{\sigma^2\,dt}}\right)\right\}\right] \\
&= \left[1 - \left\{N\left(\alpha + \frac{m}{\sigma}\sqrt{dt}\right) - e^{-\frac{2\alpha\sigma\sqrt{dt}m}{\sigma^2}} N\left(-\alpha + \frac{m}{\sigma}\sqrt{dt}\right)\right\}\right] \\
&\overset{dt\Rightarrow 0}{=} 1 - \{N(\alpha) - N(-\alpha)\} \\
&= 2N(-\alpha)\,. \qquad \text{(A.15)}
\end{aligned}
$$

(It might be surprising that this result is not of order $\mathcal{O}(dt)$, but this is because we are considering only those values of $y_t = \alpha\sigma\sqrt{dt}$ that are "very close" to the default boundary.)

Now, from eq. (A.1), we have

$$
\pi_4(y_t = \alpha\sigma\sqrt{dt}|\tau_d > t,\, y_{(t-L)}) \;=\; \frac{\pi(y_t = \alpha\sigma\sqrt{dt}, \tau_d > t|y_{(t-L)}, \tau_d > (t-L))}{\pi\left(\tau_d > t|\, y_{(t-L)}, \tau_d > (t-L)\right)} \qquad \text{(A.16)}
$$

Taylor expanding with respect to $y_t = \alpha\sigma\sqrt{dt}$, and using the fact that the density goes zero at $y_t = 0$, we get

$$
\begin{aligned}
\pi(y_t = \alpha\sigma\sqrt{dt}|\tau_d > t,\, y_{(t-L)}) \;=\; & \\
&= \frac{\pi(y_t = 0, \tau_d > t|y_{(t-L)}, \tau_d > (t-L)) + \alpha\sigma\sqrt{dt}\,\pi_y(y_t = 0, \tau_d > t|y_{(t-L)}, \tau_d > (t-L))}{\pi\left(\tau_d > t|\, y_{(t-L)}, \tau_d > (t-L)\right)} \\
&= \frac{\alpha\sigma\sqrt{dt}\,\pi_y(y_t = 0, \tau_d > t|y_{(t-L)}, \tau_d > (t-L))}{\pi\left(\tau_d > t|\, y_{(t-L)}, \tau_d > (t-L)\right)}. \qquad \text{(A.17)}
\end{aligned}
$$

Combining these results, we find:

$$\lambda^Q_{4,d}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) = \lim_{dt\Rightarrow 0}\left(\frac{1}{dt}\right)\int_0^\infty \sigma\sqrt{dt}\, d\alpha\, 2N(-\alpha)\frac{\alpha\sigma\sqrt{dt}\;\pi_y(y_t=0,\tau_d>t|y_{(t-L)},\tau_d>(t-L))}{\pi\left(\tau_d>t|\,y_{(t-L)},\tau_d>(t-L)\right)}$$
$$= \left(\frac{2\sigma^2\,\pi_y(y_t=0,\tau_d>t|y_{(t-L)},\tau_d>(t-L))}{\pi\left(\tau_d>t|\,y_{(t-L)},\tau_d>(t-L)\right)}\right)\int_0^\infty d\alpha\;\alpha N(-\alpha) \tag{A.18}$$

Using integration by parts, can show that

$$\int_0^\infty d\alpha\;\alpha N(-\alpha) = \frac{1}{4}. \tag{A.19}$$

Hence,

$$\lambda^Q_{4,d}(\mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) = \left(\frac{\sigma^2}{2}\right)\frac{\partial}{\partial y_t}\left(\frac{\pi(y_t=0,\tau_d>t|y_{(t-L)},\tau_d>(t-L))}{\pi\left(\tau_d>t|\,y_{(t-L)},\tau_d>(t-L)\right)}\right)$$
$$= \left(\frac{\sigma^2}{2}\right)\frac{\partial}{\partial y_t}\pi_4(y_t|y_{(t-L)},\tau_d>t)\Big|_{y_t=0}. \tag{A.20}$$

Expression (19) for the default intensity follows by differentiation of the above expression.

∎

Proof of Proposition 2

If default has not occurred during Regime-2 at date-t^*, it follows that outsiders information set is

$$\mathcal{F}(t^*) = \left(y_{(t^*-L)}, y_{t^*}>0\right). \tag{A.21}$$

An important probability that we need for pricing the bond in this regime is $\pi^Q_3\left(y_{t^*}\,|\,y_{t^*}>0, y_{(t^*-L)}\right)$. To derive it, we use the identity

$$\pi\left(y_{t^*},\, y_{t^*}>0\,|\,y_{(t^*-L)}\right) = \pi^Q_3\left(y_{t^*}\,|\,y_{t^*}>0, y_{(t^*-L)}\right)\,\pi\left(y_{t^*}>0\,|\,y_{(t^*-L)}\right)$$
$$= \pi\left(y_{t^*}>0\,|\,y_{t^*},\, y_{(t^*-L)}\right)\pi\left(y_{t^*}\,|\,y_{(t^*-L)}\right). \tag{A.22}$$

Combining both RHS of the previous equation, we find

$$
\begin{aligned}
&\pi_3^Q\left(y_{t^*}\,|\,y_{t^*}>0, y_{(t^*-L)}\right)\\
&\qquad = \frac{\pi\left(y_{t^*}>0\,|\,y_{t^*},\,y_{(t^*-L)}\right)\pi\left(y_{t^*}\,|\,y_{(t^*-L)}\right)}{\pi\left(y_{t^*}>0\,|\,y_{(t^*-L)}\right)} \qquad \text{(A.23)}\\
&\qquad = \mathbf{1}_{\{y_{t^*}>0\}}\,\frac{\pi\left(y_{t^*}\,|\,y_{(t^*-L)}\right)}{\pi\left(y_{t^*}>0\,|\,y_{(t^*-L)}\right)}\\
&\qquad = \left(\frac{\mathbf{1}_{\{y_{t^*}>0\}}}{N\left(\frac{y_{(t^*-L)}+mL}{\sqrt{\sigma^2 L}}\right)}\right)\frac{1}{\sqrt{2\pi\sigma^2 L}}\exp\left[-\left(\frac{1}{2\sigma^2 L}\right)\left(y_{t^*}-y_{(t^*-L)}-mL\right)^2\right].
\end{aligned}
$$

In Regime 3, the bond has 3 claims: (1) For dates T in the interval $T\in(t^*,(t^*+L))$, the claim to coupon payment $C\,dT$ at date-T if default has not yet occurred (i.e., $\tau_d>T$); (2) For dates T in the interval $T\in(t^*,(t^*+L))$, the claim to recovery if default occurs at date-T (i.e., $\tau_d=T$); and (3) the claim to $D_4(y_{t^*},\tau_d>(t^*+L))$ if default occurs later than (t^*+L) (i.e., $\tau_d>(t^*+L)$). We price each claim separately.

Let us define $D_{3,1}^T(\mathbf{1}_{\{y_{t^*}>0\}},y_{(t^*-L)})$ as the date-t^* price of a claim that pays \$1 at date-T if and only if $\tau_d>T$ (Here, we assume $(T>t^*)$). Its value is

$$
\begin{aligned}
D_{3,1}^T(\mathbf{1}_{\{y_{t^*}>0\}},y_{(t^*-L)}) \;&=\; e^{-r(T-t^*)}\,\mathbf{1}_{\{y_{t^*}>0\}}\,\mathrm{E}^Q\left[\mathbf{1}_{\{\tau_d>T\}}|y_{t^*}>0,y_{(t^*-L)}\right]\\
&= e^{-r(T-t^*)}\mathbf{1}_{\{y_{t^*}>0\}}\,\pi^Q\left(\tau_d>T|y_{t^*}>0,y_{(t^*-L)}\right)\\
&= e^{-r(T-t^*)}\mathbf{1}_{\{y_{t^*}>0\}}\int_0^\infty dy_{t^*}\,\pi^Q\left(\tau_d>T,\,y_{t^*}|y_{t^*}>0,y_{(t^*-L)}\right)\\
&= e^{-r(T-t^*)}\mathbf{1}_{\{y_{t^*}>0\}}\int_0^\infty dy_{t^*}\,\pi^Q\left(\tau_d>T|y_{t^*}\right)\,\pi_3^Q\left(y_{t^*}|y_{t^*}>0,y_{(t^*-L)}\right)\\
&= e^{-r(T-t^*)}\mathbf{1}_{\{y_{t^*}>0\}}\int_0^\infty dy_{t^*}\left[N\left(\frac{y_{t^*}+m(T-t^*)}{\sqrt{\sigma^2(T-t^*)}}\right)-e^{-\frac{2y_{t^*}m}{\sigma^2}}N\left(\frac{-y_{t^*}+m(T-t^*)}{\sqrt{\sigma^2(T-t^*)}}\right)\right]\\
&\qquad\times\pi_3^Q\left(y_{t^*}|y_{t^*}>0,y_{(t^*-L)}\right),. \qquad \text{(A.24)}
\end{aligned}
$$

It therefore follows that the date-t^* price of a perpetuity that pays $C\,dT\mathbf{1}_{\{\tau_d>T\}}$ for all dates-$T \in (t^*, (t^*+L))$ is

$$D_{3,1}(\mathbf{1}_{\{y_{t^*}>0\}}, y_{(t^*-L)}) \;=\; C \int_{t^*}^{(t^*+L)} dT\, D^T_{3,1}(\mathbf{1}_{\{y_{t^*}>0\}}, y_{(t^*-L)}). \tag{A.25}$$

Somewhat analogously, let us define $D^T_{3,2}(\mathbf{1}_{\{y_{t^*}>0\}}, y_{(t^*-L)})$ as the date-t^* price of a claim that pays \$1 at date-T if and only if $\tau_d = T$ (Here, we assume $(T > t^*)$). Defining $\delta(\cdot)$ as the Dirac delta function, the value of this claim is

$$\begin{aligned}
&D^T_{3,2}(\mathbf{1}_{\{y_{t^*}>0\}}, y_{(t^*-L)}) = \\
&\quad= e^{-r(T-t^*)}\,\mathbf{1}_{\{y_{t^*}>0\}}\,\mathrm{E}^Q\left[\delta(\tau_d = T)|y_{t^*}>0, y_{(t^*-L)}\right] \\
&\quad= e^{-r(T-t^*)}\,\mathbf{1}_{\{y_{t^*}>0\}}\,\pi^Q\left(\tau_d = T|y_{t^*}>0, y_{(t^*-L)}\right) \\
&\quad= e^{-r(T-t^*)}\,\mathbf{1}_{\{y_{t^*}>0\}}\int_0^\infty dy_{t^*}\;\pi^Q\left(\tau_d = T, y_{t^*}|y_{t^*}>0, y_{(t^*-L)}\right) \\
&\quad= e^{-r(T-t^*)}\,\mathbf{1}_{\{y_{t^*}>0\}}\int_0^\infty dy_{t^*}\;\pi^Q\left(\tau_d = T|y_{t^*}\right)\;\pi^Q_3\left(y_{t^*}|y_{t^*}>0, y_{(t^*-L)}\right) \\
&\quad= e^{-r(T-t^*)}\,\mathbf{1}_{\{y_{t^*}>0\}}\int_0^\infty dy_{t^*}\;\pi^Q_3\left(y_{t^*}|y_{t^*}>0, y_{(t^*-L)}\right) \\
&\qquad\qquad\times\frac{y_{t^*}}{\sqrt{2\pi\sigma^2(T-t^*)^3}}\,e^{-\left(\frac{1}{2\sigma^2(T-t^*)}\right)\left(y_{t^*}+m(T-t^*)\right)^2}.
\end{aligned} \tag{A.26}$$

Hence, the claim to $(1-\alpha)V_B$ at the time of default equals

$$D_{3,2}(\mathbf{1}_{\{y_{t^*}>0\}}, y_{(t^*-L)}) \;=\; (1-\alpha)V_B \int_{t^*}^{(t^*+L)} dT\; D^T_{3,2}(\mathbf{1}_{\{y_{t^*}>0\}}, y_{(t^*-L)}). \tag{A.27}$$

Third, let us define $D_{3,3}(\mathbf{1}_{\{y_{t^*}>0\}}, y_{(t^*-L)})$ as the date-t^* price of a claim that pays $D^C_4(\mathbf{1}_{\{\tau_d>(t^*+L)\}}, y_{t^*})$ at date (t^*+L) if $\tau_d > (t^*+L)$.

$$\begin{aligned}
&D_{3,3}(\mathbf{1}_{\{y_{t^*}>0\}}, y_{(t^*-L)}) \;=\; e^{-rL}\,\mathbf{1}_{\{y_{t^*}>0\}}\,\mathrm{E}^Q\left[\mathbf{1}_{\{\tau_d>(t^*+L)\}}\,D^C_4(\mathbf{1}_{\{\tau_d>(t^*+L)\}}, y_{t^*})|y_{t^*}>0, y_{(t^*-L)}\right] \\
&= e^{-rL}\mathbf{1}_{\{y_{t^*}>0\}}\int_0^\infty dy_{t^*}\;\pi^Q\left(\tau_d>(t^*+L), y_{t^*}\mid y_{t^*}>0, y_{(t^*-L)}\right)D^C_4(\mathbf{1}_{\{\tau_d>(t^*+L)\}}, y_{t^*}) \\
&= e^{-rL}\mathbf{1}_{\{y_{t^*}>0\}}\int_0^\infty dy_{t^*}\;\pi^Q\left(\tau_d>(t^*+L)\mid y_{t^*}\right)\;\pi^Q_3\left(y_{t^*}\mid y_{t^*}>0, y_{(t^*-L)}\right)\;D^C_4(\mathbf{1}_{\{\tau_d>(t^*+L)\}}, y_{t^*}) \\
&= e^{-rL}\mathbf{1}_{\{y_{t^*}>0\}}\int_0^\infty dy_{t^*}\;\pi^Q_3\left(y_{t^*}\mid y_{t^*}>0, y_{(t^*-L)}\right)\;D^C_4(\mathbf{1}_{\{\tau_d>(t^*+L)\}}, y_{t^*}) \\
&\qquad\times\left[N\left(\frac{y_{t^*}+mL}{\sqrt{\sigma^2L}}\right)-e^{-\frac{2y_{t^*}m}{\sigma^2}}N\left(\frac{-y_{t^*}+mL}{\sqrt{\sigma^2L}}\right)\right].
\end{aligned} \tag{A.28}$$

Therefore, the claim to debt in regime 3 is the sum:

$$D^C_3(\mathbf{1}_{\{y_{t^*}>0\}}, y_{(t^*-L)}) \;=\; D_{3,1}(\mathbf{1}_{\{y_{t^*}>0\}}, y_{(t^*-L)}) + D_{3,2}(\mathbf{1}_{\{y_{t^*}>0\}}, y_{(t^*-L)}) + D_{3,3}(\mathbf{1}_{\{y_{t^*}>0\}}, y_{(t^*-L)}).$$

∎

Proof of Proposition 3

To find $\widehat{S}_t$ for $t < t^*$, we first determine

$$\widehat{S}(\hat{y}_{t^*},\, C_{t^*}) = \mathbb{E}^{\mathbb{Q}}_{t^*}\left[S(y_{t^*},\, C_{t^*}) \mid \hat{y}_{t^*},\, C_{t^*}\right]. \tag{A.29}$$

Using the fact that, by (8), $V_B = \frac{C_{t^*}}{\beta}$, and by (30), $\frac{C_{t^*}}{\widehat{V}_{t^*}} = \overline{\Psi}$, we obtain

$$\hat{y}_{t^*} \equiv \log\left(\frac{\widehat{V}_{t^*}}{V_B}\right) = \log\left(\frac{\beta}{\overline{\Psi}}\right). \tag{A.30}$$

Using (9) and recalling the definition of $y_{t^*} = \log\left(\frac{V_{t^*}}{V_B}\right)$ and $\hat{y}_{t^*} = \log\left(\frac{\widehat{V}_{t^*}}{V_B}\right)$, we can re-write the equity value S_t under the manager's information set as follows

$$\begin{aligned} S(y_{t^*}, C_{t^*}) \;=\; & e^{\hat{y}_{t^*}+v_B}\, \mathbf{1}_{\{y_{t^*}>0\}} \left\{ \left(e^{y_{t^*}-\hat{y}_{t^*}} - \frac{(1-\theta)\overline{\Psi}}{r} \right) - \right. \\ & \left. e^{-\frac{2r}{\sigma^2}(y_{t^*}-\hat{y}_{t^*})} \left(\frac{\beta}{\overline{\Psi}}\right)^{-\frac{2r}{\sigma^2}} \left(\frac{\overline{\Psi}}{\beta} - \frac{(1-\theta)\overline{\Psi}}{r} \right) \right\}. \end{aligned} \tag{A.31}$$

From (2) we have that the distribution of y_{t^*} conditional on $\hat{y}_{t^*}$, under the risk-neutral measure $\mathbb{Q}$ is

$$\pi^{\mathbb{Q}}\left[y_{t^*} \mid \hat{y}_{t^*}\right] = \frac{1}{\sqrt{2\pi\sigma^2 L}} e^{-\frac{(y_{t^*}-\hat{y}_{t^*}-mL)^2}{2\sigma^2 L}}. \tag{A.32}$$

Therefore the equity value $\widehat{S}_{t^*}$ under the creditors' information set is

$$\begin{aligned} \widehat{S}(\hat{y}_{t^*},\, C_{t^*}) \;=\; & \mathbb{E}^{\mathbb{Q}}_t\left[S(y_{t^*}, C_{t^*}) \middle|\, \hat{y}_{t^*}\right] \\ = e^{\hat{y}_{t^*}+v_B} & \left\{ e^{mL+\frac{\sigma^2 L}{2}}\, N\left[\frac{\log\left(\frac{\beta}{\overline{\Psi}}\right) + mL + \sigma^2 L}{\sqrt{\sigma^2 L}}\right] - \frac{(1-\theta)\overline{\Psi}}{r}\, N\left[\frac{\log\left(\frac{\beta}{\overline{\Psi}}\right) + mL}{\sqrt{\sigma^2 L}}\right] \right. \\ & \left. - \left(\frac{\beta}{\overline{\Psi}}\right)^{-\frac{2r}{\sigma^2}} \left(\frac{\overline{\Psi}}{\beta} - \frac{(1-\theta)\overline{\Psi}}{r}\right) e^{rL}\, N\left[\frac{\log\left(\frac{\beta}{\overline{\Psi}}\right) + mL - 2rL}{\sqrt{\sigma^2 L}}\right] \right\}. \end{aligned} \tag{A.33}$$

To determine the value of equity $\widehat{S}_t$ for $t > t^*$, note that, using the change of variable $e^{y_t} = \left(\frac{V_t}{V_B}\right)$ we can write the value of equity S_t in the manager's information set (9) as

$$S(y_t) \;=\; V_B e^{y_t} - \frac{(1-\theta)C}{r} - e^{\frac{-2ry_t}{\sigma^2}}\left[V_B - \frac{(1-\theta)C}{r}\right], \tag{A.34}$$

The value of equity in the creditors' information set for $t > t^*$ is then obtained by integrating (A.34) over y_t after using the conditional densities (A.1), for $t \geq t^* + L$, and (A.4), for $t^* < t < t^* + L$. This yields

$$\widehat{S}_t = \begin{cases} \displaystyle\int_0^\infty \pi_3(y_t \,|\, \tau_d > t,\, \hat{y}_t)\, S(y_t, C_{t^*})\, dy_t, & \text{if } \; t^* < t < t^* + L \quad \text{(A.35a)} \\ \displaystyle\int_0^\infty \pi(y_t|\tau_d > t,\, \hat{y}_t)\, S(y_t, C_{t^*})\, dy_t, & \text{if } \; t \geq t^* + L\;. \quad \text{(A.35b)} \end{cases}$$

∎

Proof of Proposition 4

Note that if both $\widehat{V}_t$ and C are scaled by a factor ν, $V_B = C/\beta$, would also scale by ν, and therefore $\hat{y}_t = \log(\widehat{V}_t/V_B)$ would be unaffected by this scaling. This implies that $D_3(t^*,\, \hat{y}_{t^*})$ would scale by the factor ν. Plugging these scaled factors into equation (35), we see that the bond price $D_2(t^*, \hat{y}_{t^*})$ would also scale by the same factor ν. To emphasize this dependence, we express the debt value as $D_2(\hat{y}_{t^*}, t^*, C_{t^*}) = C_{t^*} \times D_2(\hat{y}_{t^*}, t^*, 1)$. Indeed, even though (35) represents the value of debt in Regime 2 for all values of $\hat{y}_{t^*}$, in fact only one value is relevant, namely, the value for which $C_{t^*} = \overline{\overline{\Psi}}\widehat{V}_{t^*}$, and thus, $\hat{y}_{t^*} = \log\left(\frac{C_{(t^*)}}{\overline{\overline{\Psi}}V_B}\right) = \log\left(\frac{\beta}{\overline{\overline{\Psi}}}\right)$. That is, the only value we use below is

$$D_2\left(\log\left(\frac{\beta}{\overline{\overline{\Psi}}}\right), t^*, 1\right). \tag{A.36}$$

Expressing the bond value for $t < t^*$ in (13a) as a function of the asset value, $D(\widehat{V}_t, C_t)$, from Itô's lemma and the law of motion of V_t in (1) and C_t in (39), we have that D satisfies the following PDE:

$$\begin{aligned} 0 \;&=\; C_t - \left(r + (1-\theta)\frac{C_t}{D_1}\right) D + (1-\theta)\frac{C^2}{D} D_C + r\widehat{V} D_{\widehat{V}} + \frac{\sigma^2}{2}\widehat{V}^2 D_{\widehat{V}\widehat{V}} + D_t \\ &=\; -rD + \theta C + (1-\theta)\frac{C^2}{D} D_C + r\widehat{V} D_{\widehat{V}} + \frac{\sigma^2}{2}\widehat{V}^2 D_{\widehat{V}\widehat{V}}, \end{aligned} \tag{A.37}$$

subject to the boundary conditions

$$
\begin{aligned}
\lim_{\widehat{V}\to\infty} D(\widehat{V}, C) &= \frac{C}{r} \\
\lim_{C/\widehat{V}\to\overline{\Psi}} D(\widehat{V}, C) &= D_2\left(\log\left(\frac{\beta}{\overline{\overline{\Psi}}}\right), t^*, C_{t^*}\right). \qquad \text{(A.38)}
\end{aligned}
$$

Given the scaling property of debt in Regimes 2–4, we guess a solution of the form:

$$
D(\widehat{V}_t, C_t) = \widehat{V}_t\, G\left(\Psi_t\right), \quad \text{where } \Psi_t \equiv \frac{C_t}{\widehat{V}_t}. \qquad \text{(A.39)}
$$

The partial derivatives in (A.37) can be expressed as:

$$
\begin{aligned}
D_C &= G_\Psi \\
D_{\widehat{V}} &= G - \Psi G_\Psi \\
D_{\widehat{V}\widehat{V}} &= \left(\frac{\Psi^2}{\widehat{V}}\right) G_{\Psi\Psi} \\
D_t &= \widehat{V} G_t. \qquad \text{(A.40)}
\end{aligned}
$$

Substituting into (A.37), and dividing through by $\widehat{V}$, we see that our guess is self-consistent and that the PDE reduces to:

$$
0 = \theta\Psi - rG + (1-\theta)\Psi^2 \frac{G_\Psi}{G} + r\left(G - \Psi G_\Psi\right) + \frac{\sigma^2}{2}\Psi^2 G_{\Psi\Psi}. \qquad \text{(A.41)}
$$

The boundary conditions simplify to:

$$
\begin{aligned}
G(0) &= \frac{\Psi_t}{r} \qquad \text{(A.42)} \\
G(\overline{\Psi}) &= \overline{\Psi} D_2\left(\log\left(\frac{\beta}{\overline{\overline{\Psi}}}\right), t^*, 1\right). \qquad \text{(A.43)}
\end{aligned}
$$

∎

Proof of Proposition 5

Expressing the equity value for $t < t^*$ in (45a) as a function of the asset value, $\widehat{S}(\widehat{V}_t, C_t)$, from Itô's lemma and the law of motion of V_t in (1) and C_t in (39), we have that $\widehat{S}$ satisfies the following ODE:

$$
0 = -r\widehat{S} + r\widehat{V}\widehat{S}_{\widehat{V}} + \frac{\sigma^2}{2}\widehat{V}^2\widehat{S}_{\widehat{V}\widehat{V}} + \widehat{S}_C(1-\theta)\left(\frac{C^2}{D(\widehat{V}, C)}\right), \qquad \text{(A.44)}
$$

subject to the boundary conditions

$$\lim_{C/\widehat{V}\to\overline{\Psi}} \widehat{S}(\widehat{V},C) = \widehat{S}_{t^*} \tag{A.45}$$

$$\lim_{\widehat{V}\to 0} D(\widehat{V},C) = 0, \tag{A.46}$$

where $\widehat{S}_{t^*}$ is given in Equation (46) of Proposition 3. Notice that, from (9), the equity value $S(V_t,C)$ satisfies the scaling property $S(V_t,C) = V_t \times S(V_t/C,1)$. Hence we guess a solution for equation (A.44) of the form

$$\widehat{S}\left(\widehat{V}_t, C_t\right) \equiv \widehat{V}_t\, H\left(\Psi_t\right) \quad \text{where } \Psi_t \equiv \frac{C_t}{\widehat{V}_t}. \tag{A.47}$$

Substituting this guess in (A.44)–(A.46) we find that our scaling assumption is correct, and that H satisfies the ODE:

$$0 = -r\Psi H_\Psi + \frac{\sigma^2}{2}\Psi^2 H_{\Psi\Psi} + \frac{H_\Psi}{G(\Psi)}\,(1-\theta)\Psi^2, \tag{A.48}$$

subject to the boundary conditions

$$\begin{aligned} H(\overline{\Psi}) &= e^{mL+\frac{\sigma^2 L}{2}}\, N\left[\frac{\log\left(\frac{\beta}{\overline{\Psi}}\right)+mL+\sigma^2 L}{\sqrt{\sigma^2 L}}\right] - \frac{(1-\theta)\overline{\Psi}}{r}\, N\left[\frac{\log\left(\frac{\beta}{\overline{\Psi}}\right)+mL}{\sqrt{\sigma^2 L}}\right] \\ &\quad - \left(\frac{\beta}{\overline{\Psi}}\right)^{-\frac{2r}{\sigma^2}} \left(\frac{\overline{\Psi}}{\beta} - \frac{(1-\theta)\overline{\Psi}}{r}\right) e^{rL}\, N\left[\frac{\log\left(\frac{\beta}{\overline{\Psi}}\right)+mL-2rL}{\sqrt{\sigma^2 L}}\right] \qquad \text{(A.49)} \\ H(0) &= 1, \qquad \text{(A.50)} \end{aligned}$$

∎

B Pricing Bonds in Regime 3 for dates $t \in (t^*, (t^*+L))$

In the text, we focused Regime-3 bond pricing on only the date $t = t^*$. Here, we identify bond prices for all dates $t \in (t^*, (t^*+L))$. We will use interchangeably the terms $\min_{s\in(t^*,t)}\{y_s\} > 0$ and $(\tau_d > t)$.

The claim to debt in regime 3 is the sum of three components: i) For dates T in the interval $T \in (t^*, (t^*+L))$, the claim to coupon payment $C\,dT$ at date-T if default has not yet occurred (i.e., if $\tau_d > T$), ii) For dates T in the interval $T \in (t^*, (t^*+L))$, the claim to recovery if default occurs at date-T (i.e., if $\tau_d = T$), and iii) at date $(t^* + L)$, the claim to $D_4^C(\mathbf{1}_{\{\tau_d > (t^*+L)\}}, y_{t^*})$ if default has not yet occurred (i.e., if $\tau_d > (t^* + L)$):

$$D_3^C(t, \mathbf{1}_{\{\tau_d > t\}}, y_{(t-L)}) \;=\; D_{3,1}(t, \mathbf{1}_{\{\tau_d > t\}}, y_{(t-L)}) + D_{3,2}(t, \mathbf{1}_{\{\tau_d > t\}}, y_{(t-L)}) + D_{3,3}(t, \mathbf{1}_{\{\tau_d > t\}}, y_{(t-L)}).$$

The date-t price of a perpetuity that pays $C\,dT\,\mathbf{1}_{\{\tau_d > T\}}$ for all dates-$T \in (t^*, (t^* + L))$ is

$$D_{3,1}(t, \mathbf{1}_{\{\tau_d > t\}}, y_{(t-L)}) \;=\; C \int_t^{(t^*+L)} dT\, D_{3,1}^T(t, \mathbf{1}_{\{\tau_d > t\}}, y_{(t-L)}), \tag{B.1}$$

where:

$$\begin{aligned}
D_{3,1}^T(t, \mathbf{1}_{\{\tau_d > t\}}, y_{(t-L)}) \;&=\; e^{-r(T-t)}\,\mathbf{1}_{\{\tau_d > t\}}\, \mathrm{E}^Q\left[\mathbf{1}_{\{\tau_d > T\}} \mid \tau_d > t,\, y_{t-L}\right] \\
&= e^{-r(T-t)}\,\mathbf{1}_{\{\tau_d > t\}}\, \pi^Q\left[\tau_d > T \mid \tau_d > t,\, y_{(t-L)}\right] \\
&= e^{-r(T-t)}\,\mathbf{1}_{\{\tau_d > t\}} \int_0^\infty dy_{t^*} \int_0^\infty dy_t\, \pi^Q\left[\tau_d > T, y_{t^*}, y_t \mid \tau_d > t,\, y_{(t-L)}\right] \\
&= e^{-r(T-t)}\,\mathbf{1}_{\{\tau_d > t\}} \int_0^\infty dy_{t^*} \int_0^\infty dy_t\, \pi^Q\left[\tau_d > T \mid \tau_d > t,\, y_t\right] \\
&\qquad \times \pi^Q\left[y_t \mid \tau_d > t,\, y_{t^*}\right]\, \pi^Q\left[y_{t^*} \mid \tau_d > t,\, y_{(t-L)}\right],
\end{aligned} \tag{B.2}$$

and where

$$\pi^Q\left[\tau_d > T \mid \tau_d > t,\, y_t\right] \;=\; \left[N\left(\frac{y_t + m(T-t)}{\sqrt{\sigma^2(T-t)}}\right) - e^{-\frac{2y_t m}{\sigma^2}} N\left(\frac{-y_t + m(T-t)}{\sqrt{\sigma^2(T-t)}}\right)\right]\mathbf{1}_{\{y_t > 0\}}$$

$$\pi^Q\left[y_t \mid \tau_d > t,\, y_{t^*}\right] \;=\; \frac{\pi^Q\left[y_t,\, \tau_d > t \mid y_{t^*}\right]}{\pi^Q\left[\tau_d > t \mid y_{t^*}\right]} \tag{B.3}$$

$$\pi^Q\left[y_t,\, \tau_d > t \mid y_{t^*}\right] \;=\; \mathbf{1}_{\{y_t > 0\}}\,\mathbf{1}_{\{y_{t^*} > 0\}} \times \tag{B.4}$$

$$\left[\begin{array}{l} \frac{1}{\sqrt{2\pi\sigma^2(t-t^*)}} \exp\left\{\left(\frac{-1}{2\sigma^2 L}\right)\left[y_t - y_{t^*} - m(t-t^*)\right]^2\right\} \\ -e^{-\frac{2y_{t^*} m}{\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2(t-t^*)}} \exp\left\{\left(\frac{-1}{2\sigma^2 L}\right)\left[y_t + y_{t^*} - m(t-t^*)\right]^2\right\} \end{array}\right] \tag{B.5}$$

$$\pi^Q\left(\tau_d > t \middle| y_{t^*}\right) \;=\; \left[N\left(\frac{y_{t^*} + m(t-t^*)}{\sqrt{\sigma^2(t-t^*)}}\right) - e^{-\frac{2\hat{y}_{t^*} m}{\sigma^2}} N\left(\frac{-y_{t^*} + m(t-t^*)}{\sqrt{\sigma^2(t-t^*)}}\right)\right]\mathbf{1}_{\{y_{t^*} > 0\}} \tag{B.6}$$

$$\pi^Q\left[y_{t^*} \mid \tau_d > t,\, y_{(t-L)}\right] \;=\; \frac{\pi^Q\left[\tau_d > t \mid y_{t^*}\right]\, \pi^Q\left[y_{t^*} \mid y_{(t-L)}\right]}{\int_0^\infty dy_{t^*}\, \pi^Q\left[\tau_d > t \mid y_{t^*}\right]\, \pi^Q\left[y_{t^*} \mid y_{(t-L)}\right]}. \tag{B.7}$$

Second, the date-t price of a perpetuity that pays $(1-\alpha)V_B\ \delta(\tau_d - T)$ for all dates-$T \in (t^*, (t^*+L))$ is

$$D_{3,2}(t, \mathbf{1}_{\{\tau_d > t\}}, y_{(t-L)}) \;=\; (1-\alpha)V_B \int_t^{(t^*+L)} dT\, D^T_{3,2}(t, \mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}), \qquad \text{(B.8)}$$

where:

$$\begin{aligned} D^T_{3,2}(t, \mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) \;&=\; e^{-r(T-t)}\,\mathbf{1}_{\{\tau_d>t\}}\, \mathrm{E}^Q\left[\delta(\tau_d - T) \mid \tau_d > t,\, y_{t-L}\right] \\ &= e^{-r(T-t)}\,\mathbf{1}_{\{\tau_d>t\}}\, \pi^Q\left[\tau_d = T \mid \tau_d > t,\, y_{(t-L)}\right] \\ &= e^{-r(T-t)}\,\mathbf{1}_{\{\tau_d>t\}} \int_0^\infty dy_{t^*} \int_0^\infty dy_t\, \pi^Q\left[\tau_d = T, y_{t^*}, y_t \mid \tau_d > t,\, y_{(t-L)}\right] \\ &= e^{-r(T-t)}\,\mathbf{1}_{\{\tau_d>t\}} \int_0^\infty dy_{t^*} \int_0^\infty dy_t\, \pi^Q\left[\tau_d = T \mid \tau_d > t,\, y_t\right] \\ &\qquad \times \pi^Q\left[y_t \mid \tau_d > t,\, y_{t^*}\right]\, \pi^Q\left[y_{t^*} \mid \tau_d > t,\, y_{(t-L)}\right], \qquad \text{(B.9)} \end{aligned}$$

and where

$$\pi^Q\left[\tau_d = T \mid \tau_d > t,\, y_t\right] \;=\; \frac{y_t}{\sqrt{2\pi\sigma^2(T-t)^3}}\ \exp\left[-\left(\frac{1}{2\sigma^2(T-t)}\right)(y_t + m(T-t))^2\right]. \qquad \text{(B.10)}$$

Third, define $v \equiv (t^* + L - t)$ and $D_{3,3}(t, \mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)})$ as the date-t price of a claim that pays $D_4(\mathbf{1}_{\{\tau_d > (t^*+L)\}}, y_{t^*})$ at date (t^*+L) if $\tau_d > (t^*+L)$.

$$\begin{aligned} D_{3,3}(t, \mathbf{1}_{\{\tau_d>t\}}, y_{(t-L)}) \;&=\; e^{-rv}\,\mathbf{1}_{\{\tau_d>t\}}\, \mathrm{E}^Q\left[\mathbf{1}_{\{\tau_d>(t^*+L)\}}\, D_4(\mathbf{1}_{\{\tau_d>(t^*+L)\}}, y_{t^*}) | \tau_d > t, y_{(t-L)}\right] \\ &= e^{-rv}\mathbf{1}_{\{\tau_d>t\}} \int_0^\infty dy_{t^*} \int_0^\infty dy_t\, D_4(\mathbf{1}_{\{\tau_d>(t^*+L)\}}, y_{t^*})\, \pi^Q\left[\tau_d > (t^*+L), y_{t^*}, y_t \mid \tau_d > t,\, y_{(t-L)}\right] \\ &= e^{-rv}\mathbf{1}_{\{\tau_d>t\}} \int_0^\infty dy_{t^*} \int_0^\infty dy_t\, D_4(\mathbf{1}_{\{\tau_d>(t^*+L)\}}, y_{t^*})\, \pi^Q\left[\tau_d > (t^*+L) \mid \tau_d > t,\, y_t\right] \\ &\qquad \times \pi^Q\left[y_t \mid \tau_d > t,\, y_{t^*}\right]\, \pi^Q\left[y_{t^*} \mid \tau_d > t,\, y_{(t-L)}\right], \qquad \text{(B.11)} \end{aligned}$$

and where

$$\pi^Q\left[\tau_d > (t^*+L) \mid \tau_d > t,\, y_t\right] \;=\; \left[N\left(\frac{y_t + mv}{\sqrt{\sigma^2 v}}\right) - e^{-\frac{2y_t m}{\sigma^2}} N\left(\frac{-y_t + mv}{\sqrt{\sigma^2 v}}\right)\right] \mathbf{1}_{\{y_t>0\}}. \qquad \text{(B.12)}$$

Table 1: **Empirical Defaults Rates.** Each month, we classify firms as investment grade (IG) higher-quality speculative grade (B), and lower-quality speculative-grade firms (C). In Panels A and B, the classification is based on credit ratings issued by the three main rating agencies (Moody's, Standard and Poor's, and Fitch). In Panel C, the classification is implied by the price of CDS contracts written on debt issued by the firms. Panel A shows average annualized default rates from 1985 to 2014 for firms in each rating category that have defaulted in the next 12 months, while Panels B and C show default rates for the 2001-2014 period. Heteroskedasticity- and autocorrelation-robust (Newey-West) standard errors are in parentheses.

Rating	Annualized Default Rates						
	0-1M	1-2M	2-3M	3-6M	6-9M	9-12M	0-12M
Panel A: Classification based on credit ratings, 1985-2014							
IG	0.06	0.07	0.07	0.08	0.11	0.15	0.10
	(0.02)	(0.03)	(0.03)	(0.02)	(0.03)	(0.04)	(0.02)
B	0.20	0.34	0.48	0.63	0.84	1.02	0.71
	(0.05)	(0.08)	(0.10)	(0.13)	(0.16)	(0.20)	(0.13)
C	14.46	13.66	12.54	11.10	9.14	7.47	10.31
	(1.41)	(1.45)	(1.21)	(1.08)	(0.92)	(0.75)	(0.94)
Panel B: Classification based on credit ratings, 2001-2014							
IG	0.07	0.09	0.09	0.09	0.12	0.17	0.11
	(0.03)	(0.04)	(0.04)	(0.03)	(0.04)	(0.05)	(0.03)
B	0.21	0.37	0.50	0.58	0.75	0.84	0.63
	(0.07)	(0.10)	(0.13)	(0.15)	(0.18)	(0.21)	(0.15)
C	12.94	12.12	11.03	9.74	7.76	6.18	8.93
	(1.64)	(1.65)	(1.34)	(1.22)	(0.93)	(0.73)	(0.98)
Panel C: Classification based on CDS-implied ratings, 2001-2014							
IG	0.01	0.01	0.01	0.04	0.04	0.05	0.03
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.01)
B	0.29	0.35	0.34	0.42	0.51	0.51	0.44
	(0.11)	(0.12)	(0.13)	(0.15)	(0.19)	(0.20)	(0.14)
C	3.30	3.12	3.12	3.05	2.94	2.81	3.00
	(0.91)	(0.95)	(0.82)	(0.80)	(0.73)	(0.61)	(0.60)

Table 2: **Baseline Model Coefficients.** Below are the values of the model coefficients in the baseline calibration.

Parameter	Symbol	Value
Cumulative coupon at time t^*	C_{t^*}	1
Annual risk-free rate	r	0.5%
Annual asset volatility	σ	0.3
Corporate tax rate	θ	0.25
Loss given default	α	0.4
Maximum debt capacity	$\overline{\Psi}$	0.03
Creditors' information delay (in years)	L	0.5

Table 3: **Average Model-Implied Default Rates.** We simulate a history of 10,000 firms from our model and track them from inception through their default date. For each firm and at any point in time of the simulations we record the time to default and classify the observation as investment grade (IG) if the firm's leverage is below 65%. We classify as higher-quality speculative grade (B category) firms with leverage between 65% and 75%. Lower-quality speculative-grade firms (C category) have leverage in excess of 75%. The table shows average default rates across firms in the simulated sample.

	Average annualized default rates						
	0-1M	1-2M	2-3M	3-6M	6-9M	9-12M	0-12M
IG	0.00	0.00	0.00	0.00	0.01	0.02	0.01
B	0.07	0.17	0.31	0.71	1.51	2.41	1.20
C	15.51	15.44	15.32	14.96	14.19	13.23	14.45

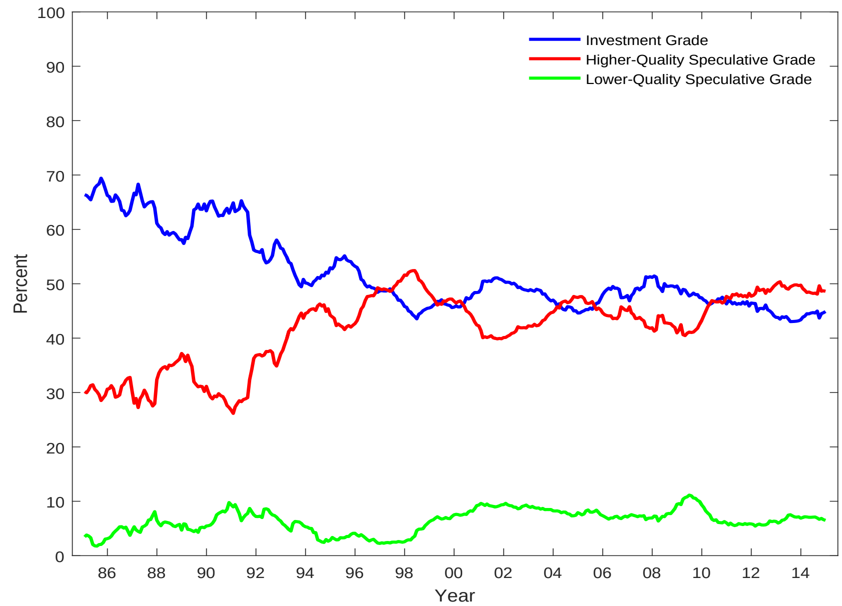

Figure 1: **Percentage of Firms by Credit Ratings.** The plots show the percentage of firms in each of the three rating categories: investment grade (the IG category), higher-quality speculative grade (the B category), and lower-quality speculative grade (the C category). The sample period goes from 1985 to 2014. Source: Mergent database.

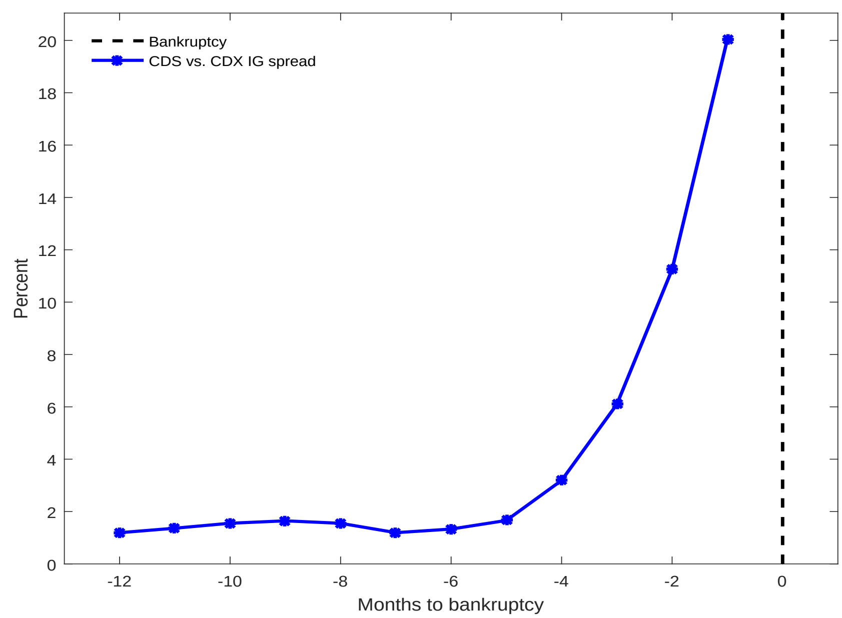

Figure 2: **Average CDS Premium on Investment Grade Firms up to Bankruptcy.** Among the firms that went bankrupt from 2001 to 2014, we classify as investment grade those that had CDS contracts trading at a premium no higher than 100 basis points of the CDX Investment Grade Index for at least one of the 12 months preceding the bankruptcy date. The plot shows the average CDS premium, in excess of the CDX Investment Grade Index, on those investment grade firms in the 12 months leading up to their bankruptcy.

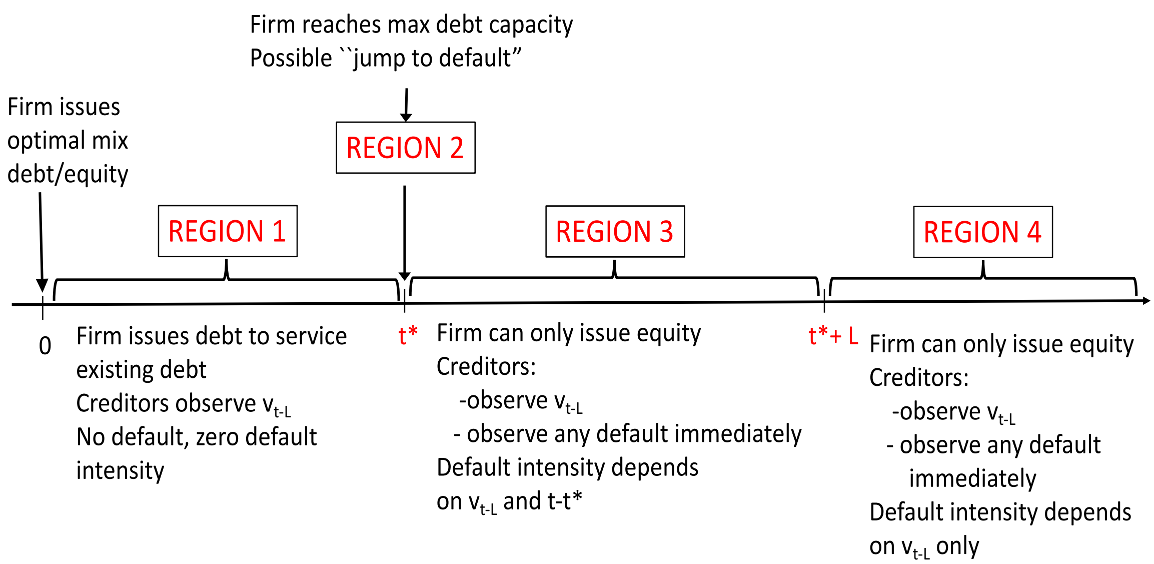

Figure 3: **Model Timeline**

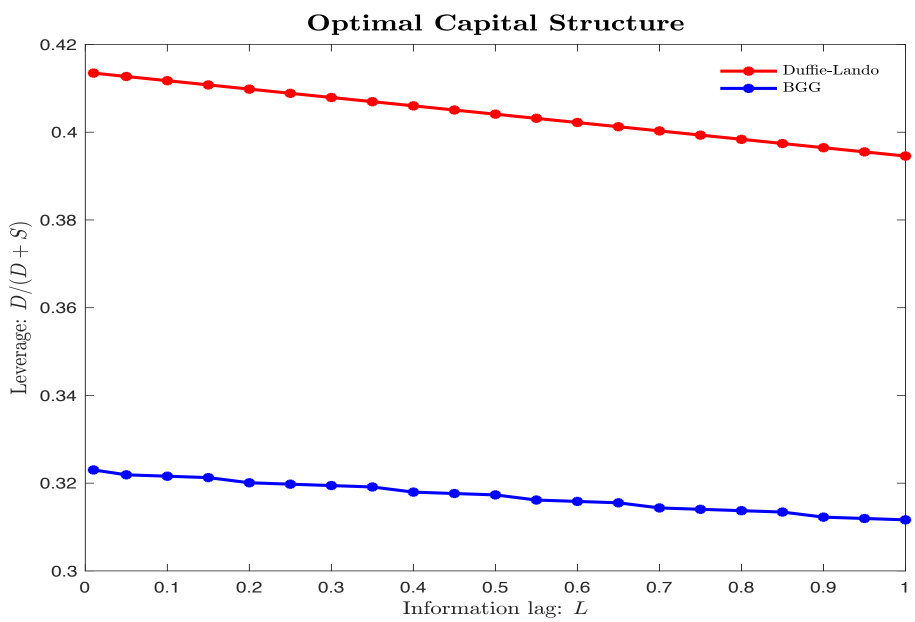

Figure 4: **Optimal Capital Structure.** The plots show the optimal time-0 capital structure as a function of the information lag L between creditors and the manager, where L ranges from 0 to 1 year, $0 \leq L \leq 1$. The ‘BBG’ line denotes our baseline model in which the manager can issue debt till borrowing capacity is reached; the ‘Duffie-Lando’ line corresponds to a firm that can only issue equity to service debt in place. Parameter values are in Table 2.

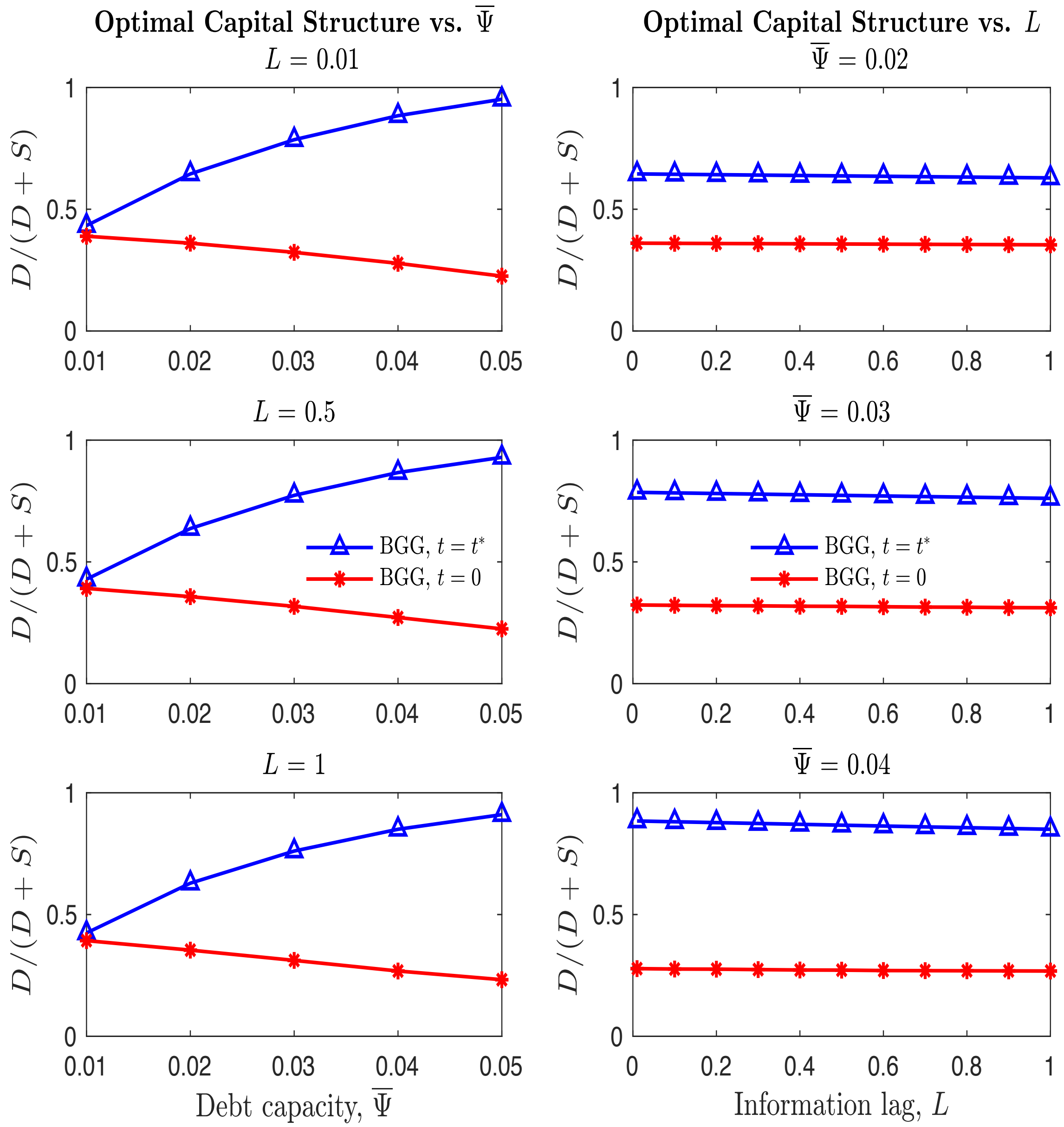

Figure 5: **Optimal Capital Structure and Credit Constraints.** The plots show the optimal leverage at times $t = 0$ and $t = t^*$ as a function of the credit constraint parameter $\overline{\Psi}$ and the information lag L. The left panels show optimal leverage as a function $\overline{\Psi}$ for three values of L, $L = 0.01, 0.5$ and 1 year. The right panels show optimal leverage as a function of L for three values of $\overline{\Psi}$, $\overline{\Psi} = 0.02, 0.03$ and 0.04. Parameter values are in Table 2.

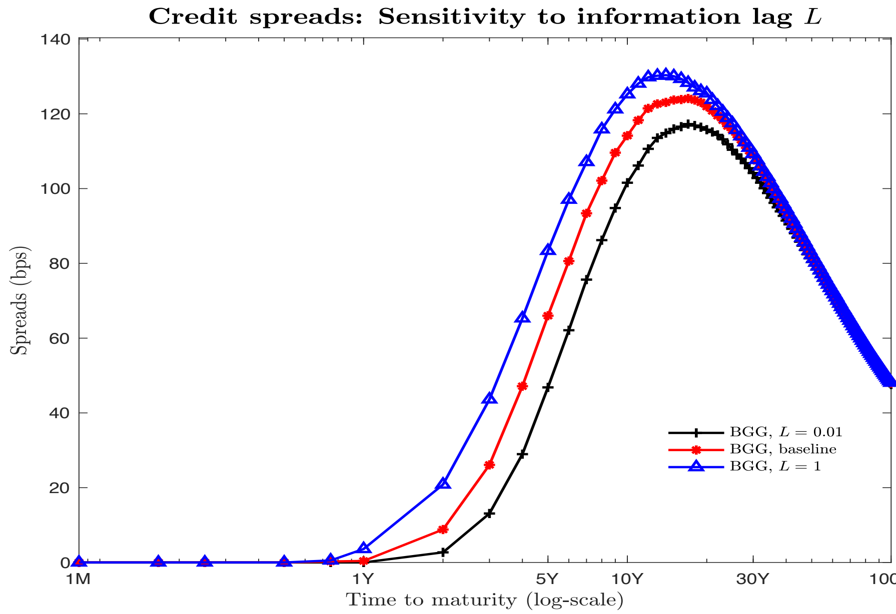

Figure 6: **Credit Spreads and Information Asymmetry.** The plots illustrate the sensitivity of the credit spreads curve to the information gap parameter L. In all cases, the initial leverage is fixed at 32%, so as to match the typical 65 bps five-year spread of an investment-grade firm in the baseline BGG model. Parameter values are in Table 2.

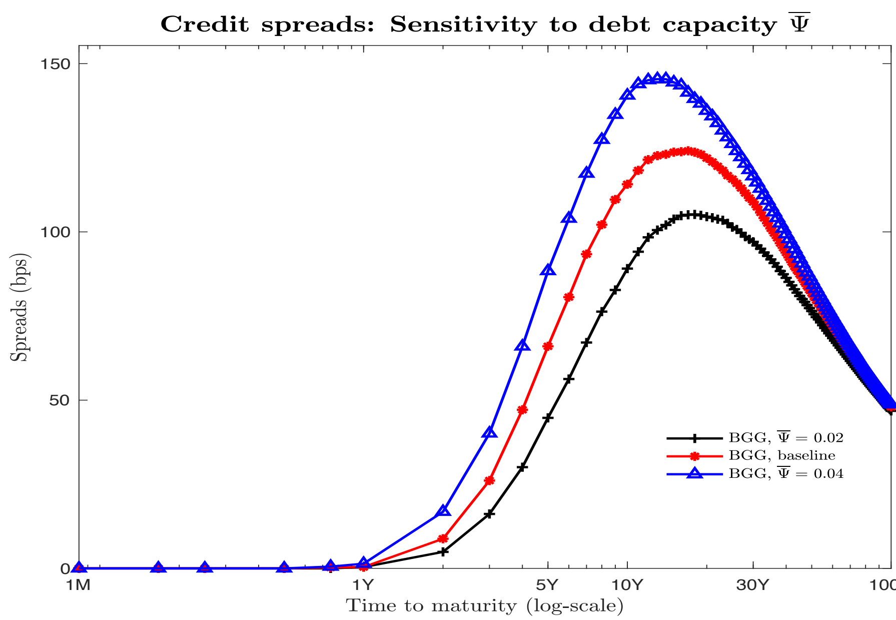

Figure 7: **Credit Spreads and Credit Constraints.** The plots illustrate the sensitivity of the credit spreads curve to the credit constraint parameter $\overline{\Psi}$. In all cases, the initial leverage is fixed at 32%, so as to match the typical 65 bps five-year spread of an investment-grade firm in the baseline BGG model. Parameter values are in Table 2.

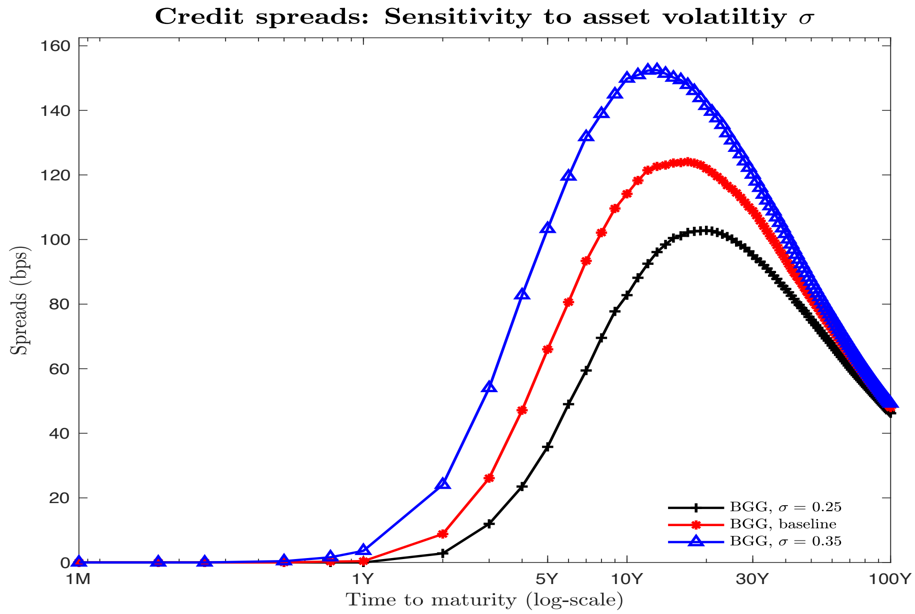

Figure 8: **Credit Spreads and Assets' Volatility.** The plots illustrate the sensitivity of the credit spreads curve to the asset volatility parameter σ. In all cases, the initial leverage is fixed at 32%, so as to match the typical 65 bps five-year spread of an investment-grade firm in the baseline BGG model. Parameter values are in Table 2.

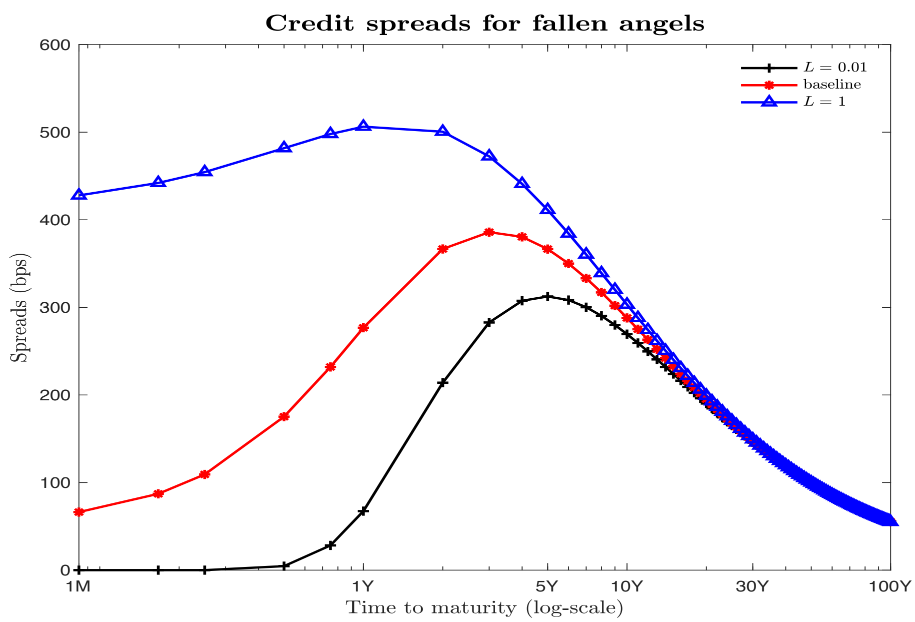

Figure 9: **Credit Spreads for Fallen Angels.** The plots shows credit spreads for a fallen angel company that has reached leverage of 80%. The three lines contrast the complete information case ($L \approx 0$) to the cases in which the information lags between creditors and the manager are $L = 0.5$ and $L = 1$ years. Parameter values are in Table 2.

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Incomplete Information, Security Issuance, and Default Dynamics

Online Appendix

Luca Benzoni, Lorenzo Garlappi, and Robert S. Goldstein

Here we provide more details on the data that we use to illustrate the stylized facts in Section 2.

A Classifying firms based on bond ratings

We collect the entire history of credit ratings given by the three main U.S. rating agencies (Moody's Investor Services, Standard & Poor's Ratings Services, and Fitch Ratings) from the Mergent database. While Mergent contains ratings going back to the early part of the 20$^{\text{th}}$ century, ratings are limited to a very small number of debt issues through the mid 1980s. Hence, here we focus on the sample period from 1985 to 2014.

Mergent provides ratings specific to particular bond issues, rather than an overall company rating. Hence, for a given issuer, each month we collect all ratings awarded on that month to any of its outstanding bonds and use that information to classify the company.[15] We divide individual bond ratings in three categories: investment grade (IG), higher-quality speculative grade (B), and lower-quality speculative grade (C and lower), where the last two categories, B and C, together comprise the universe of speculative grade ratings. We then assign the company to one of the three categories when the majority of the company's bond ratings are in that category. When no fresh ratings are given by any agency to the outstanding bonds of that company, we classify the firm based on ratings collected in the previous month. If no new ratings were issued the previous month, we go further back, up to 12 months. In case no new ratings are available in the entire 12 month period, we do not classify the firm. This approach mitigates the problem of classifying companies based on stale ratings.

Figure 1 shows the percentage of firms in each of the three rating categories: investment grade (the IG category), higher-quality speculative grade (the B category), and lower-quality speculative grade (the C category). The figure reflects the changing nature of debt markets and the evolution of the rating agencies' services. Prior to the 1980s, rating agencies were mostly focusing on blue chip industrial firms. This is consistent with a preponderance of IG ratings in the early part of the sample period. Over time,

[15] We exclude ratings on government agencies' bonds (e.g., U.S. Treasury, U.S. and foreign agencies, municipalities).

financial disintermediation and capital markets development allowed a broader variety of firms to raise funds in the bond market. Along the way, rating agencies expanded their coverage of lower-quality issues. These changes are reflected in an increased proportion of speculative-grade firms. Higher-quality speculative issues display an increasing trend through the 1990s. The proportion of lower-quality ratings remains mostly stable over the sample period, but increases during recessions; for instance, the percentage of firms in the C category peaks in 1991, 2001, and 2009.

To document default rates among rated firms, we obtain the entire history of bankruptcy filings starting from 1985 (also available through the Mergent database). Each month, after we classify firms in the three rating categories, we identify those that filed for bankruptcy over the next 12 months. We record the number of months that have elapsed between the month of the classification and the bankruptcy date. Then we count bankruptcies that occurred within the first, second, and third month of the classification (0-1, 1-2, and 2-3 months), the second, third, and fourth quarters (3-6, 6-9, and 10-12 months), and the entire year (0-12 months). For ease of comparison across periods of different length, we annualize all count variables.[16]

B Classifying firms based on CDS premia

In the previous section, we have classified firms based on credit ratings that are up to 12 months old. Such ratings might not fully reflect the information available to market participants at the time of the classification. Hence, here we consider an alternative classification of firms into the same three rating categories that is based on CDS data.

CDS contracts provide insurance in case of credit events that affect the value of a reference entity (such as the bond issued by a company that files for bankruptcy). Therefore, CDS premia reflect market participants' assessment of default risk for the company that issues the reference bond. The CDS market is generally liquid. Thus, CDS contract are a useful source of real-time information about a company's credit

[16]We multiply the count variables for the 0-1, 1-2, and 2-3 periods by 12, and those for the quarterly periods by 4.

worthiness.

To translate CDS premia into a proxy for a company's credit rating, we compare the cost of insuring bonds issued by that company with that of insuring portfolios of investment-grade and high-yield bonds (the CDX-IG and CDX-HY indices constructed by Markit Financial Information Services). Each month from 2001 to 2014 we aggregate daily five-year CDS premia from the Markit database into an average monthly CDS premium.[17] Similarly, we compute monthly averages of daily five-year CDX-IG and CDX-HY premia. If the CDS premia on a firm's bonds do not exceed the CDX-IG index by more than 100 basis points (bps), then we classify that firm as investment grade (the IG category). We use the 100 bps threshold to avoid excluding creditworthy companies whose CDS premia lie slightly above the CDX-IG level, i.e., the average IG premium. In unreported results, we find the analysis to be robust to the choice of the threshold value. In contrast, when CDS premia on a firm's bonds exceed the CDX-HY premium we classify that firm as lower-quality speculative grade (the C category). Finally, if CDS premia lie in between the IG and C thresholds, then we classify the company as higher-quality speculative grade (the B category).

Figure B.1 shows the proportion of firms in each category based on CDS-implied ratings. The trading of CDS contracts on IG companies is predominant throughout the sample period, especially in the early 2000s when the CDS market was in its infancy and trading concentrated in high-quality big names. Over time, the proportion of CDS contracts on IG firms fluctuates around a downward trend, with drops in 2001-2002 and 2008-2009 at the depth of two recessions, and peaks during the subsequent recovery periods.

The proportion of CDS contracts on higher-quality speculative-grade firms generally increases throughout the sample. Further, CDS trading in the B category exhibits peaks during recessions and declines in the expansions that follow, a pattern that is the direct opposite of the fluctuations in the proportion of IG CDS contracts. This is consistent with both (1) a reshuffling in CDS trading across rating categories over the business

[17] Prior to analysis, we exclude CDS contracts written on bonds issued by Government and sovereign entities.

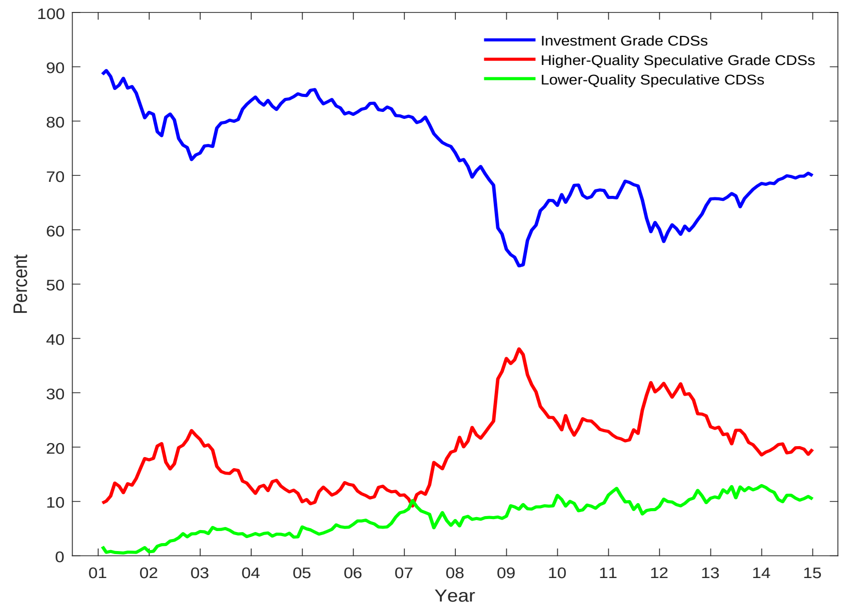

Figure B.1: **Percentage of Firms by CDS-Implied Ratings** The plots show the percentage of firms in each of the three rating categories (IG, B, and C) based on ratings implied by CDS data. The sample period goes from 2001 to 2014. Source: Markit databases.

cycle and (2) an increase in CDS premia for IG firms during recessions combined with a decline in CDS premia of B firms during expansions that shift firms from one rating category to the other.

Finally, Figure B.1 shows that little high-yield trading takes place in the early years of the CDS market. That changes over time, with a proportion of CDS contracts on low-grade speculative firms steadily increasing over the sample period.

Similar to Appendix A, each month we identify firms that filed for bankruptcy within the following 12 months. We then compute the average annualized default rates within the first, second, and third month of the classification (0-1, 1-2, and 2-3 months), the second, third, and fourth quarters (3-6, 6-9, and 10-12 months), and the entire year (0-12 months).

B.1 Robustness Checks

In the paper, we classify firms in the IG, B, and C rating categories over the period 1985–2014 using the Mergent dataset. We then compute empirical default rates for companies that have experienced bankruptcy within a year of the classification. Here we check the robustness of our findings using data from the Moody's Default and Recovery Database. There are two advantages to this dataset. First, the database spans a longer sample period starting from 1920. Second, Moody's reports a default flag that captures not only bankruptcies but also other credit events such as missed payments beyond the grace period and debt restructuring that reduces the value of the bondholder claim.

While the Moody's data go as far back as 1920, we find many early ratings to be stale. For instance, Table 5 shows that for most IG defaults occurred over 1920–1940 Moody's did not update the IG rating past the default event. Hence, in Panel B of Table 4 we compute empirical default rates from 1940 to 2014. The results are similar to those reported in the paper over the shorter 1985–2014 window relying on the stricter bankruptcy classification flag. For completeness, panels C and D show results for the 1985–2014 and 2001–2014 windows, which are directly comparable to those reported in the main text.

Table 4: **Empirical Defaults Rates.** Each month, we classify firms as investment grade (IG), higher-quality speculative grade (B), and lower-quality speculative-grade firms (C) based on ratings issued by Moody's Investors' Services. The panels show average annualized default rates computed over various periods for firms in each rating category that have defaulted in the next 12 months. Heteroskedasticity- and autocorrelation-robust (Newey-West) standard errors are in parentheses. Source: Moody's Default and Recovery Database.

Rating	Annualized Default Rates						
	0-1M	1-2M	2-3M	3-6M	6-9M	9-12M	0-12M
				Panel A: 1920–2014			
IG	0.09	0.10	0.11	0.13	0.16	0.18	0.14
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)
B	1.56	1.78	1.91	2.11	2.35	2.51	2.18
	(0.24)	(0.25)	(0.26)	(0.28)	(0.30)	(0.32)	(0.27)
C	8.39	7.58	7.11	6.47	5.58	4.85	6.15
	(1.22)	(1.11)	(1.04)	(0.89)	(0.73)	(0.62)	(0.79)
				Panel B: 1940–2014			
IG	0.03	0.04	0.04	0.07	0.08	0.10	0.07
	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
B	1.16	1.45	1.61	1.88	2.20	2.41	1.97
	(0.20)	(0.23)	(0.25)	(0.29)	(0.33)	(0.36)	(0.29)
C	10.33	9.25	8.66	7.81	6.60	5.73	7.39
	(1.62)	(1.47)	(1.37)	(1.17)	(0.94)	(0.80)	(1.02)

Table 4, continued

Rating	Annualized Default Rates						
	0-1M	1-2M	2-3M	3-6M	6-9M	9-12M	0-12M
				Panel C: 1985–2014			
IG	0.03	0.04	0.05	0.08	0.11	0.13	0.09
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)
B	1.30	1.68	1.90	2.24	2.65	2.92	2.36
	(0.23)	(0.27)	(0.29)	(0.33)	(0.38)	(0.42)	(0.33)
C	14.36	12.85	12.00	10.76	9.11	7.88	10.20
	(1.83)	(1.66)	(1.55)	(1.29)	(0.99)	(0.84)	(1.05)
				Panel D: 2001–2014			
IG	0.05	0.06	0.07	0.11	0.14	0.16	0.11
	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)
B	0.53	0.90	1.07	1.32	1.72	1.98	1.46
	(0.15)	(0.23)	(0.27)	(0.33)	(0.45)	(0.50)	(0.35)
C	14.33	13.10	12.46	11.36	9.65	8.43	10.68
	(2.26)	(2.04)	(1.91)	(1.58)	(1.17)	(0.99)	(1.26)

Table 5: **IG Defaults: 1920-1940.** The table lists corporate defaults from 1920-1940 for companies that held an investment-grade rating as of January 1 of the default year. Also reported are the date the investment-grade rating was issued for the purpose of the January 1 classification, and the date when the company lost its investment grade status. Source: Moody's Default and Recovery Database.

Company name	Default date	Date IG rating issued	End of IG classification period
	1920		
Pensacola Electric Company	Jan 01, 1920	May 01, 1919	Jul 01, 1920
Colorado Springs Electric Company	Apr 20, 1920	Jul 01, 1919	Jul 01, 1920
Gardner, Westminster & Fitchburg St. Railway	Feb 01, 1920	May 01, 1919	Jul 01, 1920
TOLEDO, FREMONT AND NORWALK R.R.	Jan 01, 1920	May 01, 1919	Jul 01, 1925
SAGINAW VALLEY TRACTION	Feb 01, 1920	May 01, 1919	Jul 01, 1920
Gaston, Williams & Wigmore, Inc.	Mar 16, 1920	May 01, 1919	May 01, 1921
Lockport & Olcott Railway	Jul 01, 1920	May 01, 1919	Jul 01, 1920
Houston Gas Company	Mar 01, 1920	May 01, 1919	Jul 01, 1923

Table 5 , continued

Company name	Default date	Date IG rating issued	End of IG classification period
1921			
REPUBLIC MOTOR TRUCK CO.	Nov 01, 1921	May 01, 1920	May 01, 1921
HABIRSHAW ELECTRIC CABLE CO.	Sep 01, 1921	May 01, 1920	May 01, 1922
Fort Worth & Denver City Ry. Co.	Dec 01, 1921	Oct 01, 1919	Sep 01, 1924
Marlborough & Westborough Street Railway	Jul 01, 1921	Jul 01, 1920	Jul 01, 1925
GREEN STAR STEAMSHIP CORP.	Feb 01, 1921	May 01, 1920	Mar 11, 1921
Moline Plow Company, Inc.	Sep 01, 1921	May 01, 1919	May 01, 1921
ATLANTIC FRUIT CO.	Dec 01, 1921	Jul 01, 1920	Jul 01, 1921
1922			
Astoria Veneer Mills & Dock Company	Jan 01, 1922	Jul 01, 1921	Jul 01, 1922
Denver & Rio Grande Railroad Co.	Feb 01, 1922	Oct 01, 1919	Sep 01, 1932
Amalgamated Petroleum Corp.	Jul 01, 1922	Jul 01, 1921	Jul 01, 1923
Riordon Pulp & Paper Co.	Jan 01, 1922	May 01, 1921	May 01, 1922
New York, Lake Erie & Western Coal & Railroad Co.	May 01, 1922	Oct 01, 1919	Sep 01, 1926
TEMTOR CORN & FRUIT CO.	Feb 01, 1922	May 01, 1921	May 01, 1922
AMES-HOLDEN-MCCREADY, LTD.	Apr 01, 1922	Jul 01, 1921	Jul 01, 1922
Cass Avenue & Fair Grounds Railway	Jul 01, 1922	Jul 01, 1920	Jul 01, 1925
DISTILLERS SECURITIES	Apr 01, 1922	May 01, 1919	May 01, 1922
PORTLAND FLOURING MILLS, CO.	Aug 01, 1922	May 01, 1921	May 01, 1922

Table 5 , continued

Company name	Default date	Date IG rating issued	End of IG classification period
1923			
LUCEY MFG. CO.	Aug 01, 1923	May 01, 1921	May 01, 1923
Compton Heights, Union Depot & Merchants Terminal Railroad	Oct 01, 1923	Jul 01, 1920	Jul 01, 1925
GOFF (D.) & SONS	Jan 01, 1923	May 01, 1920	May 01, 1923
Davis Sewing Machine Company (The)	May 07, 1923	May 01, 1920	May 01, 1923
HYDRAULIC STEEL	Nov 01, 1923	May 01, 1921	May 01, 1923
1924			
Wilson (Hj) Co Inc	Dec 01, 1924	May 01, 1923	Jun 01, 1924
Texas & Pacific Railway Company	Jul 01, 1924	Sep 01, 1922	Aug 18, 1941
Atlantic & Yadkin Railway Co.	Mar 21, 1924	Jul 01, 1919	Jul 01, 1949
1925			
CHICAGO, MILWAUKEE AND ST. PAUL RY.	Mar 01, 1925	Oct 01, 1919	Sep 01, 1925
Marquette, Houghton & Ontonagon Railroad	Apr 01, 1925	Sep 01, 1924	Sep 01, 1929
SALT LAKE & UTAH RAILROAD	Jul 24, 1925	Jul 01, 1923	Jul 01, 1925
GREAT NORTHERN POWER CO.	May 01, 1925	Jul 01, 1924	Jul 01, 1936
Cape Electric Tramways, Ltd.	Jul 01, 1925	Jul 01, 1922	Dec 30, 1930
Union Traction Co. (Pa.)	Jan 01, 1925	May 01, 1919	Jul 01, 1927
Kinloch-Bloomington Telephone Company (Ill.)	Jul 15, 1925	Jul 01, 1923	Jul 01, 1928

Table 5 , continued

Company name	Default date	Date IG rating issued	End of IG classification period
	1926		
West Philadelphia Passenger Railway	May 01, 1926	Jul 01, 1922	Aug 01, 1926
Raleigh & Augusta Air Line Railroad	Jan 01, 1926	Sep 01, 1923	Feb 15, 1932
Clinton Street Railway	Apr 01, 1926	Jul 01, 1922	Aug 01, 1926
Seaboard & Roanoke Railroad	Jul 01, 1926	Sep 01, 1923	Feb 15, 1932
	1927		
Philadelphia & Darby Railway	May 01, 1927	May 01, 1919	Jan 15, 1927
	1928		
	1929		
Atlantic City Railroad Co.	May 01, 1929	Jul 01, 1920	Jul 01, 1932
UTICA BELT LINE STREET RAILWAY	Nov 01, 1929	Jul 01, 1920	Jul 01, 1929
Johnstown Passenger Railway	Dec 01, 1929	Dec 30, 1928	Dec 30, 1930
	1930		
INDIANAPOLIS TRACTION AND TERMINAL	Apr 01, 1930	Jul 01, 1925	Jul 01, 1930
United Rys. of the Havana & Regla Whses., Ltd.	Jun 30, 1930	Sep 01, 1928	Aug 01, 1930

Table 5 , continued

Company name	Default date	Date IG rating issued	End of IG classification period
	1931		
Wabash Rr	Dec 01, 1931	Aug 01, 1930	Sep 01, 1932
Arizona Edison Co., Inc.	Dec 01, 1931	Dec 30, 1930	Nov 03, 1931
Mogyana Railways Co. (Brazil)	Sep 01, 1931	Sep 01, 1925	Aug 01, 1931
Cincinnati, Hamilton & Dayton Railway Co.	Jan 01, 1931	Sep 01, 1929	Jul 01, 1942
Chilian Northern Railway Company, Ltd.	Dec 31, 1931	Sep 01, 1926	Aug 01, 1931
HOLLAND AMERICAN LINE	Nov 01, 1931	Jun 01, 1926	Jun 01, 1931
	1932		
Norfolk & Southern R R	Jul 28, 1932	Aug 01, 1931	Feb 29, 1932
Cairo Bridge Company	Jan 01, 1932	Sep 01, 1929	Sep 01, 1938
INTERNATIONAL MATCH CORP.	May 01, 1932	Jun 01, 1931	Jan 01, 1932
Florida Central & Peninsular Railroad Co.	Jan 01, 1932	Sep 01, 1928	Jan 19, 1932
Carolina Central Railroad Co.	Jan 01, 1932	Sep 01, 1923	Sep 01, 1932
UNION ELECTRIC LIGHT AND POWER CO.	Oct 01, 1932	Aug 01, 1926	Jul 01, 1937
Raleigh & Cape Fear Railroad	Sep 01, 1932	Aug 01, 1930	Sep 01, 1932
Paulista Railways Co.	Sep 15, 1932	Sep 01, 1922	Feb 15, 1932
ATLANTIC & NORTH CAROLINA RR	Jul 01, 1932	Aug 01, 1930	Sep 01, 1932
Raleigh & Gaston Railroad	Jan 01, 1932	Sep 01, 1928	Sep 01, 1932

Table 5 , continued

Company name	Default date	Date IG rating issued	End of IG classification period
1933			
Pennsylvania Electric Company	Aug 01, 1933	Jul 01, 1932	Apr 19, 1979
Burmeister & Wain, Ltd.	Jan 01, 1933	Jun 01, 1932	Jun 01, 1933
Rochester & Lake Ontario Water Co.	Mar 01, 1933	Jul 01, 1932	Jul 01, 1936
Sulzer-Unternehmungen A.G.	Jul 01, 1933	Jun 01, 1931	Jun 01, 1933
Bush Terminal Company	Apr 01, 1933	Jun 01, 1932	Mar 21, 1933
New Jersey & New York Railroad	Jan 01, 1933	Feb 15, 1932	Sep 01, 1933
NORTH MOUNTAIN WATER SUPPLY CO.	Jul 01, 1933	Jul 01, 1932	Jul 01, 1933
St. Louis, Iron Mountain & Southern Railway, River & Gulf Div.	May 01, 1933	Aug 01, 1930	Apr 11, 1933
St. Paul, Minneapolis & Manitoba Railway	Jun 01, 1933	Sep 01, 1932	Jul 01, 1940
1934			
Laclede Gas Company	Apr 01, 1934	Jul 01, 1933	Jul 01, 1934
Minnesota & South Dakota Railroad	Dec 20, 1934	Sep 01, 1932	Sep 01, 1934
Chicago Great Western Railroad Company	Aug 01, 1934	Sep 01, 1929	Sep 01, 1936
Iowa, Minnesota & North Western Railway	Dec 20, 1934	Sep 01, 1932	Sep 01, 1934
Kansas City, Memphis & Birmingham Railroad Co.	Mar 01, 1934	Sep 01, 1932	Sep 01, 1936

Table 5 , continued

Company name	Default date	Date IG rating issued	End of IG classification period
	1935		
Colorado & Southern Ry Co	May 01, 1935	Sep 01, 1932	Sep 01, 1935
CHICAGO, MILWAUKEE AND ST. PAUL RY.	Jul 01, 1935	Sep 01, 1934	Sep 01, 1935
Rio Grande Junction Railway Company	Dec 01, 1935	Sep 01, 1921	Sep 01, 1936
Harlem River & Port Chester RR	Nov 01, 1935	Sep 01, 1934	Sep 01, 1936
Dutchess County Railroad	Dec 01, 1935	Sep 01, 1924	Mar 02, 1937
Long Dock Company	Oct 01, 1935	Aug 01, 1931	Jun 16, 1938
Baldwin Locomotive Works	Mar 01, 1935	Jun 01, 1934	Jun 01, 1935
Housatonic Railroad	Nov 01, 1935	Sep 01, 1933	Sep 01, 1936
Naugatuck Railroad	Nov 01, 1935	Sep 01, 1933	Mar 02, 1937
	1936		
New York & New England Railroad Company	Oct 01, 1936	Sep 01, 1933	Sep 01, 1937
Old Colony Railroad Co.	Jun 01, 1936	Sep 01, 1933	Mar 02, 1937
New York, Providence & Boston Railroad	Oct 01, 1936	Sep 01, 1934	Sep 01, 1938
	1937		
Sharon Railway (Pa.)	Jan 01, 1937	Sep 01, 1936	Sep 01, 1937
Paterson Extension Railroad	Jun 01, 1937	Sep 01, 1933	Jun 29, 1937
Providence Terminal Co.	Mar 01, 1937	Sep 01, 1936	Sep 01, 1937
New York, Ontario & Western Railway Co.	Mar 01, 1937	Sep 01, 1924	Jan 07, 1938

Table 5 , continued

Company name	Default date	Date IG rating issued	End of IG classification period
	1938		
Erie Railroad	Jan 18, 1938	Sep 01, 1929	Sep 01, 1938
Baltimore & Ohio Railroad Company	Sep 02, 1938	Sep 01, 1922	Jan 07, 1938
Buffaloe Rochester & Pitts Ry Co	Nov 01, 1938	Sep 01, 1937	Jan 07, 1938
Pittsburgh Railways Company	May 10, 1938	Jul 01, 1937	Jul 01, 1938
Genesee River Railroad	Jan 01, 1938	Sep 01, 1924	Sep 01, 1938
Erie & Jersey Railroad	Jan 01, 1938	Sep 01, 1924	Sep 01, 1938
Pacific Railroad of Missouri	Jul 01, 1938	Sep 01, 1935	Jun 16, 1938
New York & Erie Railroad	Mar 01, 1938	Apr 04, 1933	Jun 16, 1938
McKesson & Robbins, Incorporated	Dec 08, 1938	Jun 01, 1935	Jun 01, 1939
	1939		
Boston Terminal Company	Nov 01, 1939	Jun 16, 1938	Aug 03, 1939
Raritan River Railroad Co.	Jan 01, 1939	Jan 07, 1938	Jul 01, 1939
	1940		
Paris-Orleans Railroad Co.	Jul 01, 1940	Jan 07, 1938	Dec 01, 1945
Midi Railroad Co. (France)	Jul 01, 1940	Sep 01, 1928	Jul 01, 1940