June 7, 2018

Tail-Risk Dumping

Kenichi Ueda, The University of Tokyo *

ABSTRACT

This paper analyzes a banking crisis and related policy issues from a viewpoint of equilibrium contracts by providing a micro-foundation of typical macro models with financial frictions. Namely, it characterizes equilibrium contracts and their consequences in a simple one-period general equilibrium model with ex ante identical agents who endogenously choose to be depositors, borrowers, and bankers. Assuming costly state verification, the equilibrium loan and deposit contracts become a standard debt type. Limited liability with a simple asset seizure rule makes the equilibrium bank capital to be positive, creating a sizable banking sector. In equilibrium, income is shared incompletely among depositors, borrowers, and bankers, unlike "big household" assumption in typical macro models. In particular, when a large negative productivity shock hits, borrowers and bankers walk away with predetermined retained assets. Then, depositors have to assume all the tail risk (*tail-risk dumping*). Here, bank bailouts that insure deposits by consumption tax contingent on ex post shocks are welfare improving. This optimality of bailouts relies on the assumption that borrowers can walk away easily from their debts. However, a simple and speedy bankruptcy procedure is itself optimal if otherwise a debt overhang problem emerges.

JEL Classification Numbers: E44, G21, G28
Keywords: Bailout, prudential regulation, deposit insurance, insolvency, tail risk, endogenous banking sector

*I benefited a lot from discussions with Grace Li. I would like to thank for helpful comments from Nobu Kiyotaki, Tom Sargent, Rob Townsend, and Ivan Werning as well as participants of seminars at the FRB Boston, FRB Richmond, Georgetown, MIT, and Paris School of Economics and of conferences, Econometric Society Asia Meeting at Chinese University Hong Kong and Summer Workshop on Economic Theory at Hokkaido University.

I. INTRODUCTION

To analyze a banking crisis, a general equilibrium perspective is important since systemic importance has been stressed as a major reason for bank bailouts in many countries. Also, because regulations and bailout expectations affect the banking sector size, policy implications are better to be studied recognizing endogeneity of the banking sector size. However, in typical general equilibrium macroeconomic models with financial frictions, income risks are assumed to be shared among bankers, borrowers, and depositors, with population of each type exogenously given (e.g., Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Gertler (2010)).[2] Although such a fictitious "big household" assumption is convenient to track macroeconomic dynamics, it is not usual in reality that typical households share their income with bankers. Also, the big household assumption is not theoretically consistent with financial frictions that govern the lending activities in these models.

To understand a banking crisis and related policy issues from a viewpoint of equilibrium contracts, this paper provides a micro-foundation of typical macro models with financial frictions. Namely, it characterizes equilibrium contracts and their consequences in a simple one-period general equilibrium model with depositors, borrowers, and bankers. Assuming costly state verification, the equilibrium loan and deposit contracts become a standard debt type. Limited liability with a simple asset seizure rule, which is often assumed in the literature, makes income sharing incomplete among depositors, borrowers, and bankers, unless the "big household" assumption were made. In particular, when a large negative productivity shock hits, borrowers and bankers walk away with predetermined outputs. Then, depositors have to assume all the tail risk (*tail-risk dumping*).

In this paper, ex ante identical agents choose to work either in the production sector or the banking sector in the beginning of the period. As a result, the expected utility of bankers should equate with those of entrepreneurs in the production sector. Entrepreneurs with a high talent borrows capital from bankers while those with a low talent deposits. Loan and deposit contracts determine how risks are shared among three types of agents. In equilibrium, the bankers own positive capital with which they provide a partial insurance for depositors against aggregate shocks. As the insurance premium, the bankers obtain income from a spread between the deposit and loan rates.

Here, bank bailouts that insure deposits by consumption tax contingent on ex post shocks are welfare improving, even ex ante. This optimality of bailouts relies on the limited liability with a simple asset seizure rule. This is a typically-made assumption in the literature, which allows borrowers to walk away easily from their debts. However, this paper also argues that such a simple and speedy bankruptcy procedure is itself optimal if otherwise a debt overhang problem emerges.

In summary, this paper identifies a reason why many governments ended up to bail out many banks in crises. Moreover, ex ante optimality validates to set up a resolution fund permanently. It is a way to protect depositors against large negative aggregate shocks when the legally allowed retained assets by borrowers and bankers are unconditional on the aggregate output levels. When a tail risk event hits, defaulted borrowers and bankers can still enjoy consumption levels protected by the limited liability but depositors are not (*tail-risk dumping*). By transferring the goods from defaulters to depositors using consumption tax, a government can mitigate the incomplete risk sharing arrangement embedded in the bankruptcy procedure that is efficient in normal times.

---

[2] Often banks are not well separated from firms in the literature.

The literature so far identified a few different justifications for bank bailouts. In many theoretical models, as in this paper, distinction between bank depositors and other creditors are not well delineated. Hence, theories to support deposit insurance can also support bailouts. A seminal paper for the need of deposit insurance is Diamond and Divbig (1983). Because a bank borrows short-term funds (deposits) but lends to long-term projects, it cannot repay to all the depositors at once if all of them asked to do so. Knowing this, if depositors expect a bank run, they would try to withdraw their deposits as fast as possible. In this sense, a bank run becomes a self-fulfilling equilibrium. Deposit insurance can eliminate depositors' incentive to withdraw deposits even if they know many other depositors would do so. He and Xiong (2012) extended this analysis to the market based funding. Depending on parameter values, they support public protection of investors in a short-term funding market. However, such protection is never used in equilibrium.

Another reason to bail out banks is its ex post optimality due to a (reduced form) cost of bankruptcy and debt overhang in theory (e.g., Chari and Kehoe, 2013).[3] Indeed, several empirical papers (e.g., Ashcraft, 2005; Peek and Rosengren, 2000) found sizable aggregate costs stemming from banks' bankruptcies. For those mechanisms that thin capital is the source of problem, the recapitalization or other form of bank restructuring would be beneficial to the economy. However, theoretically, the recapitalization by bank itself is often blocked by the shareholders because it benefits mostly debt holders and dilute shareholders' values (Landier and Ueda, 2009). In particular, when the default is imminent, public recapitalization may be worth to pursue (Philippon and Schnabl, 2013). Empirically, some argues that the Japanese lost decade is a result of reluctant government involvement on decisive recapitalization (Hoshi and Kashyap, 2010). The situation may be similar in the recent southern European countries (IMF, 2013). Although numerous papers implicitly or explicitly argue the needs for government intervention to bank restructuring in crisis, to the best of my knowledge, theoretical argument (i.e., better sharing of tail risks) proposed by this paper is a novel one, not articulated before in the literature.[4]

This paper also endogenizes the banking sector size. In most of the macroeconomic models with financial frictions, bank defaults are absent and the capital ratio is not determined endogenously. If the banking sector size were exogenously given in my model, then it would be difficult to identify a full scale of distortions. For example, the capital adequacy ratio requirement would create higher monopoly rents for bankers in an exogenously given banking sector, but such rents would dissipate with endogenous entry of bankers. Only a few papers have investigated the endogenous nature of the financial sector size. The U.S. financial sector has grown over time with increased bankers' wage that compensates increased bankers' income risk (Phillippon, 2008). In an occupational choice model, Bolton, Santos, and Scheinkman (2011) argues that the financial traders attract too many talents due to profitable opportunities in the opaque OTC market. Their papers apparently bring important arguments but have little to say about policies towards deposit-taking banks.

Because it relies on specific financial frictions, this paper obviously misses other important issues relating to bank bailouts, in particular, the moral hazard related to making low efforts or diverting

---

[3] A deeper argument of debt overhang problem is as follows. If it is close to bankruptcy, a bank may not lend to profitable projects (Myers, 1977) or it may lend to highly risky projects only (Jensen and Meckling, 1976).

[4] Green (2010) is related. In a model similar to Diamond and Dibvig (1983) but with production by firms (no banks in his model), the only available contract is subject to limited liability. A better contract in terms of incentive to induce higher production is the contract without limited liability. The difference needs to be fill in by the government's tax-subsidy system.

funds by bank mangers. This does not imply that the moral hazard is not important. Rather, when making an actual policy decision, all the issues on bailouts, including the moral hazard, should be carefully considered. Indeed, since Karecken and Walles (1977), the moral hazard of too much risk taking by banks due to deposit insurance and other forms of government protections have been well recognized and called for the regulations. Moreover, Calomiris and Gorton (1991) argue, based on historical evidences, that the bank runs often associated with insolvency and not characterized as random events that models with self-fulfilling equilibria would imply. Demirgüç-Kunt and Detrageache (2002) show more formally that crisis probability is not reduced by the presence of deposit insurance in their regression analysis. Again, the model explored in this paper should be considered as finding one more aspect of the multifaceted nature of the bank bailouts. Still, this paper also shows that, with an explicit bailout expectation, welfare is improved ex ante and more risks are taken but without need for a prudential regulation. This prediction departs from previous literature, which often admits ex post optimality of bailouts but recommend ex ante intervention by regulations to control bank risk taking.

II. Model Setup

A. Demography, Utility, and Technology

I analyze a simple one-period model to understand the basic characteristics of a simple default procedure in allocating factors among depositors, borrowers, and bankers. A continuum of ex ante identical agents lives in the interval of $[0, 1]$and endowed with the same initial capital $k_0 > 0$. An agent chooses to become an entrepreneur or a banker endogenously. An entrepreneur then becomes either a depositor or a borrower depending on his talent. I denote bankers' population by μ and entrepreneurs' by $1 - \mu$. I assume bankers locate from 0 to μ on the unit line, as if indexed by subscript h. A half of the remaining $1 - \mu$ people are depositors, as if indexed by i and the rest are borrowers indexed by j.

Bankers intermediates the capital market in this paper. In a Walrasian equilibrium, in tradition of Arrow and Debreu (1954), there would be an auctioneer who offers price and matches demand and supply. This paper departs from the Walrasian setup in a few ways.[5] First, instead of an auctioneer, there is a continuum of nonatomic bankers who intermediate capital markets. Second, a banker offers a more general form of "price" in the capital market, that is, deposit and loan repayment schedules. Given the deposit and loan contracts that specify repayment schedules, entrepreneurs decide the amounts of deposits and loans. After the production takes place, goods are allocated among borrowers, depositors, and bankers in accordance with the repayment schedules.

Both entrepreneurs and bankers have the same utility function from consumption. Each agent maximizes the expected utility $E[u(c)]$. For the sake of simplicity, I assume the constant relative risk aversion, that is, $u(c) = c^{1-\sigma}/(1 - \sigma)$ with positive relative risk aversion parameter $\sigma > 0$. Note that, the utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is increasing $u' > 0$ and concave $u'' < 0$ and satisfies the Inada conditions.

[5] The equilibrium concept can be regarded as a variant of Prescott and Townsend (1984 a,b) or Ueda (2013).

Once an agent becomes an entrepreneur, he observes his talent shock e to carry out production.[6] After observing his talent e, he makes an investment decision on endowed capital $k_0 \in \mathbb{R}_{++}$. The capital reallocation is assumed to be intermediated by bankers. Depositor i decides to make deposits $s_i \in [0, k_0]$, which is the sum of deposits in each bank s_{hi}. Borrower j decides to take loans $l_j \in \mathbb{R}_+$, which is the sum of loans in each bank l_{hj}. An entrepreneur then invest capital and produce outputs. While producing goods, entrepreneurs are hit by aggregate and idiosyncratic productivity shocks.

In total, there are three types of shocks that each entrepreneur faces: the idiosyncratic talent shock e from the cumulative distribution $F(e) : [\underline{e}, \overline{e}] \to [0, 1]$ with mean one and $\underline{e} > 0$; the idiosyncratic productivity shock ϵ from the distribution $H(\epsilon) : [\underline{\epsilon}, \overline{\epsilon}] \to [0, 1]$ also with mean one and $\underline{\epsilon} > 0$; and the aggregate productivity shock A from the cumulative distribution $G(A) : [\underline{A}, \overline{A}] \to [0, 1]$ with mean greater than one and $\underline{A} > 0$. Without loss of generality, I assume only two levels of talent, e^U and e^D (i.e., up or down) with equal probability $1/2$. Note that $e^U > 1 > e^D > 0$ and $(e^U + e^D)/2 = 1$.

The production function is Cobb-Douglas with capital share $0 < \alpha < 1$ as in a standard macroeconomic model. One unit of labor is assumed to be inelastically supplied by each agent to his own project. The output is then expressed as

$$\begin{aligned} y_i^D &= y(s_i, e^D, A, \epsilon) = \epsilon_i A e^D (k_0 - s_i)^\alpha \quad \text{for those make deposits } s_i; \\ y_j^U &= y(l_j, e^U, A, \epsilon) = \epsilon_j A e^U (k_0 + l_j)^\alpha \quad \text{for those take loans } l_j. \end{aligned} \tag{1}$$

The production function exhibits diminishing marginal returns to capital. Entrepreneurs who received the high talent e^U have higher expected marginal returns on the endowed capital k_0 than the expected loan repayment. Accordingly, they would like to borrow capital l from bankers until the expected marginal returns equate to the expected loan repayment. From a banker's point of view, she lends L to firms.

On the other hand, entrepreneurs with low talent e^D have lower expected marginal returns on the endowed capital k_0 than the effective deposit rate. They will deposit some of their endowed capital s to bankers and operate more productive activities in a smaller scale. From a banker's point of view, her deposit intake is S. A depositor's expected marginal return from own business should become equal to the expected deposit return.

Note that, with a risk averse utility function, deposit and loan amounts are affected by risk sharing considerations and so do the expected deposit return and loan repayment in equilibrium. With a positive spread between the deposit and loan rates, some entrepreneurs might not engage in transactions with banks. However, in the case with two talents, a sufficient difference between the two are assumed so that the high talent type always become borrowers and the low type always become depositors for a reasonable range of the spread.

After he finishes producing goods, borrower j transfers outputs to banker h according to a potentially nonlinear loan repayment schedule, $R^L(l_{hj}, A, \epsilon) : \mathbb{R}_+ \times [\underline{A}, \overline{A}] \times [\underline{\epsilon}, \overline{\epsilon}] \to \mathbb{R}_+$, which is a gross return rate to lenders and potentially depends on the borrower's loan amount. The function space from which R^L is chosen is denoted by Λ. If there is a flat rate portion in the repayment schedule, the return rate is $(1 + \rho^L)$, where $\rho^L \in \mathbb{R}_+$ is called as a promised loan rate. For the sake of

[6] This e can be also regarded as a business idea.

simplicity and without loss of generality, I assume that capital inputs depreciate 100 percent after they are used in the production process.[7]

A banker is assumed not to price-discriminate among borrowers so that the repayment function offer $\tilde{R}^L$ must be the same for all borrowers.[8] She can still specify the loan amount in her offer $\tilde{l}_{hj} \in \mathbb{R}_+ \times \{N.S.\}$, where $\{N.S.\}$ denotes the non-specified option.[9] With denoting the loan amount offer to all possible borrowers $\tilde{l}_h = \tilde{l}_{h1} \times \tilde{l}_{h2} \times \cdots$, banker h offers a loan contract $(\tilde{R}^L, \tilde{l}_h)$ from $\overline{\Lambda} \equiv \Lambda \times (\mathbb{R}_+ \times \{N.S.\})^{\frac{1-\mu}{2}}$, which is the strategy space covering for all possible loan amounts for each borrower from a banker with one loan rate.[10] In particular, it is taken for granted that a banker always pools idiosyncratic shocks by allocating loans equally among all borrowers. This implies that deposit repayment from a banker depends only on the aggregate shock A.

A banker transfers outputs to a depositor according to a potentially nonlinear deposit repayment schedule, $R^D(S, A) : \mathbb{R}_+ \times [\underline{A}, \overline{A}] \to \mathbb{R}_+$, which is a gross return rate to depositors and potentially depends on the bank's deposit intake. The function space from which R^D is chosen is denoted by Δ. If there is a flat rate portion, the return rate is $(1 + \rho^D)$, where $\rho^D \in \mathbb{R}_+$ denotes a promised deposit rate. Again, a banker is assumed not to price-discriminate among depositors so that the repayment function offer $\tilde{R}^D$ must be the same for all depositors. She can specify the deposit amount in her offer $\tilde{s}_{hi} \in \mathbb{R}_+ \times \{N.S.\}$. With denoting the deposit amount offer to all possible depositors $\tilde{s}_h = \tilde{s}_{h1} \times \tilde{s}_{h2} \times \cdots$, banker h offers a deposit contract $(\tilde{R}^D, \tilde{s}_h)$ from $\overline{\Delta} \equiv \Delta \times (\mathbb{R}_+ \times \{N.S.\})^{\frac{1-\mu}{2}}$, which is the strategy space covering for all possible deposit amounts for each depositor to a banker with one deposit rate.

At the end of the period, a borrower repays gross loan rate R^L for loan l_j from his own output y_j^U. A depositor consumes the return from his returned deposits $R^D s_i$ and his own product y_i^D. Note that a banker may not repay the promised deposit rate $1 + \rho^D$ in full and gross return R^D can be even less than one ex post (i.e., the net return can be negative). In the worst case, a depositor receives nothing from a banker, i.e., $R^D = 0$.

For the sake of simplicity, I focus on a symmetric equilibrium, in which the equal market shares in the deposit and loan markets are achieved by those bankers who offer the best contracts.[11] More specifically, borrower j's total loan l_j is equal to μl_{hj} in a symmetric equilibrium as each banker give loans to all borrowers to pool the idiosyncratic risks. As for the deposit market, a depositor is assumed to choose one bank to deposit all of his s_i—this assumption is not restrictive as long as the equal deposit market share is satisfied. Still, in an off-equilibrium, heterogeneous offers of loan and deposit contracts by bankers are allowed. I denote $\Omega \in \overline{\Lambda}^{\mu}$ as the set of loan contract offers from all the bankers and $\Psi \in \overline{\Delta}^{\mu}$ as the all deposit contract offers.

[7]More generally, any depreciation rate can be assumed in the model. Theoretical implications would still go through, as long as the retained outputs plus assets are larger than the remained capital net of depreciation.

[8]I could assume that borrowers can have side contracts among themselves to arbitrage any loan rate differentials.

[9]I denote a generic loan by l_{hj}, the optimal offer (i.e., the supply) by a bank by $\tilde{l}_{hj}$, the optimal choice by a firm (i.e., the demand) by l_{hj}^*, and the equilibrium loan by L_{hj} (see below). I use similar notations for deposits.

[10]Superscript $1 - \mu/2$ means $1 - \mu/2$-time Cartesian product of $(\mathbb{R}_+ \times \{N.S.\})$.

[11]If the number of bankers is finite and possibly attracts different numbers of depositors and borrowers, strategic actions in both deposit and loan markets could produce a complex equilibrium (Ueda, 2013).

Consumption of a high talent entrepreneur is determined by the budget constraint.It is expressed as a function, if selecting loan amount l_j and loan contracts $R_j^L(l_j, A, \epsilon_j) \in \Omega$ from all available offers, $c^U : \mathbb{R}_+ \times \Lambda \times [\underline{A}, \overline{A}] \times [\underline{\epsilon}, \overline{\epsilon}] \to \mathbb{R}_+$.

$$c^U(l_j, R_j^L, A, \epsilon_j) = y(l_j, e^U, A, \epsilon_j) - \tau 1_N - R_j^L(l_j, A, \epsilon_j)l_j \quad \text{for those take loans;} \tag{2}$$

Note that the loan repayment $R_j^L(l_j, A, \epsilon_j)l_j$ is representing $R_j^L(l_j, A, \epsilon_j)l_{hj}(1-\mu)/(2\mu)$ in a symmetric equilibrium. Although loans are given by each bank l_{hj}, the loan repayment schedule, in particular default, is conditional on the borrower j's total loans l_j.

Consumption of a low talent entrepreneur is expressed similarly.

$$c^D(s_i, R_i^D, A, \epsilon_i) = y(s_i, e^D, A, \epsilon_i) + R_i^D(S_h, A)s_i \quad \text{for those make deposits.} \tag{3}$$

Note that the deposit repayment schedule, in particular default, is conditional on the bank h's deposit intake, not by depositor i's deposits in a symmetric equilibrium.

More discussions on costs are given in the next section but, without loss of generality, a borrower ends up paying the verification cost τ when he needs to do so. In (2), the cost payment is denoted by an indicator function $1_N \in \{0, 1\}$, i.e., it is $1_N = 1$ if paid, otherwise $1_N = 0$. I also assume that cost τ is small relative to endowed capital k_0 so that it can be always paid. This assumption will be clarified later. Note that (2) and (3) also represent the budget constraints with nonlinear price of loans and deposits.

A banker lends loan l_{hj} to a borrower funded by her own capital $k_0^B = k_0$ and the deposits that she collected, i.e., $s_h = s_{hi}(1-\mu)/(2\mu)$. She then earns, and consumes, the spread income. Banker's consumption is determined by the budget constraint and expressed as a function of her own loan and deposit contract offers given all offers and aggregate shock, i.e., $c^B : [\underline{A}, \overline{A}] \times \Lambda^\mu \times \Delta^\mu \to \mathbb{R}_+$, if entrepreneurs choose her contracts among other offers.

$$c^B(A, R_h^L, l_h, R_h^D, s_h) = \int_{\underline{\epsilon}}^{\overline{\epsilon}} R_h^L(l_j, A, \epsilon_j)l_h dH(\epsilon) - R_h^D(s_h, A)s_h. \tag{4}$$

In summary, there are three stages within one period.

- Stage I: Each agent chooses occupation, either a banker or an entrepreneur.
  - Each banker offers deposit and loan contracts.
  - Each entrepreneur submits deposit supply and loan demand for the offered contracts, contingent on talent shock realizations.
  - Deposit and loan contracts are agreed among bankers and entrepreneurs.
- Stage II: Talent is revealed. Entrepreneurs are sorted to depositors or borrowers. Deposits and loans are made according to the agreed contracts.
- Stage III: Production takes place. Repayments are made according to the agreed contracts. Agents consume what they have at the end of the period.

B. Equilibrium Definition

Before specifying the institutional setup on financial intermediation, a decentralized equilibrium can be defined generally as follows.

Definition 1. *An equilibrium is the number of bankers μ, the capital allocation s_{hi} and l_{hj}, and the consumption allocations represented by deposit contract $R^D(s_h, A)$ and loan contract $R^L(l_j, A, \epsilon_j)$, that satisfy the following conditions:*

- *Given loan offers Ω, a high talent entrepreneur (i.e., a borrower) chooses the best loan contract $R^{L*} \in \Omega$ and simultaneously decides the loan amount $l_j^* \in \mathbb{R}_+$ to maximize his expected utility,*

$$V^U(k_0) = \max_{R^L \in \Omega, l_j \in \mathbb{R}_+} \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} u\left(c^U(l_j, R^L, A, \epsilon_j)\right) dH(\epsilon) dG(A). \tag{5}$$

  subject to budget constraint (2). Here, a borrower choses the loan repayment schedule R^{L*} from Ω, a set of $\tilde{R}_h^L$. He may pick several contracts, which should be the same repayment schedule R^{L*}, and then decides l_j^* to be the sum of picked l_{hj}^*.

- *Given deposit offers Ψ, a low talent entrepreneur (i.e., a depositor) chooses the best deposit contract $R^{D*} \in \Psi$ and simultaneously decides the deposit amount $s_i^* \in \mathbb{R}_+$ to maximize his expected utility,*

$$V^D(k_0) = \max_{R^D \in \Psi, s_i \in \mathbb{R}_+} \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} u\left(c^D(s_i, R^D, A, \epsilon_i)\right) dH(\epsilon) dG(A). \tag{6}$$

  subject to budget constraint (3).

- *A banker offers her deposit contract $\tilde{R}_h^D \in \Lambda$ and loan contract $\tilde{R}_h^L \in \Delta$ to maximize her expected utility, given the reaction functions of entrepreneurs (i.e., deposit supply and loan demand functions) and other bankers' loan offers Ω and deposit offers Ψ,*

$$V^B(k_0) = \max_{R^L \in \Lambda, R^D \in \Delta} \int_{\underline{A}}^{\overline{A}} u\left(c^B(A, R^L, L_h, R^D, S_h)\right) dG(A). \tag{7}$$

  subject to budget constraint (4). Note that L_h and S_h in (7) are the equilibrium values, which depends not only by bank h's offers but also by reaction functions of entrepreneurs and other bankers' offers (see below).

- *Before the production takes place, demand and supply for each deposit and loan are met, that* *is,*[12]

$$\begin{aligned} S_{hi} = \tilde{s}_{hi} \le s^*_{hi} &\quad \textit{if banker } h\textit{'s loan offer } \tilde{R}^L_{hj} \textit{ is weakly preferred to other offers in } \Psi_{-h}\textit{, and} \\ = 0 &\quad \textit{if banker } h\textit{'s loan offer} \tilde{R}^L_{hj} \textit{ is less preferred to other offers in } \Psi_{-h}\ . \end{aligned} \tag{8}$$

Here, a depositor is assumed to pick the best deposit offers randomly if he faces equally preferred offers. So, bank's equilibrium deposit intake S_h *becomes a function of its offer (*$\tilde{R}^D_h$*,* $\tilde{s}_{hi}$*) as well as all other banks' offers* Ψ_{-h}*, that is,* S_h *is a function of all offers* Ψ*. And, it obviously depends on the reaction function described above.*

As for loans, bankers prefer to diversify investments and pool risks and thus,

$$\begin{aligned} L_{hj} = \tilde{l}_{hj} \le \frac{1}{\mu} l^*_{hj} &\quad \textit{if banker } h\textit{'s loan offer } \tilde{R}^L_{hj} \textit{ is equally preferred to other offers in } \Omega_{-h}, \\ = \tilde{l}_{hj} \le l^*_{hj} &\quad \textit{if banker } h\textit{'s loan offer } \tilde{R}^L_{hj} \textit{ is strictly preferred to other offers in } \Omega_{-h}\textit{, and} \\ = 0 &\quad \textit{if banker } h\textit{'s loan offer } \tilde{R}^L_{hj} \textit{ is less preferred to other offers in } \Omega_{-h}. \end{aligned} \tag{9}$$

- *Aggregate capital market clears. In a symmetric equilibrium, a representative banker* h *takes deposits* S_h *and make loans* L_h *from a representative depositor* i *and borrower* j*, respectively. She also invests her own capital* $k^B_0 = k_0$ *as a part of loans to high talent entrepreneurs. Adjusting the relative size, the resource constraint in the capital market for the representative bank is expressed as,*

$$\frac{1-\mu}{2\mu} L_j = L_h = S_h + k^B_0 = \frac{1-\mu}{2\mu} S_i + k^B_0. \tag{10}$$

- *Bankers offer the same deposit and loan contracts faced by entrepreneurs in a symmetric equilibrium (i.e., fixed point conditions)*

$$\tilde{R}^L_h = R^{L*}_j = R^L, \quad \tilde{R}^D_i = R^{D*}_i = R^D. \tag{11}$$

- *After the production takes place, consumption goods market clears for any realization of aggregate shock* $A \in [\underline{A}, \overline{A}]$*,*

$$\begin{aligned} &\frac{1-\mu}{2} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left(c^U(L_j, R^L, A, \epsilon_j) + c^D(S_i, R^D, A, \epsilon_i) \right) dH(\epsilon) + \mu c^B(A, R^L, L_h, R^D, S_h) \\ =& \frac{1-\mu}{2} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left(y(e^U, A, \epsilon_j) + y(e^D, A, \epsilon_i) \right) dH(\epsilon_i). \end{aligned} \tag{12}$$

- *The ex ante arbitrage condition for occupational choice to become a banker or an*

[12] More generally, double competition in both deposit and loan markets can be analyzed for possibly a different selection by depositors and borrowers (e.g., Ueda, 2013). Here, however, for the sake of simplicity, I assume a symmetric equilibrium in which a banker can lend the same amount of her collected deposits as long as a banker offers the equilibrium deposit and loan contracts.

entrepreneur before observing talent $e = e^U$ or e^L holds,

$$V^B(k_0) = V^E(k_0) \equiv \frac{1}{2}V^U(k_0) + \frac{1}{2}V^D(k_0). \tag{13}$$

Note that, in accounting, the bank balance sheet is reported differently from the banker's budget constraint (4) as well as the resource constraint (10). In equilibrium, a deposit takes a form of debt contract, which will be proved in Section III. B below. In other words, the deposit contract has a flat payment portion (i.e., a promised payment) and a default region. A typical accounting standard uses the market values for assets but the face values for liabilities. Then, the ex ante accounting balance sheet of the representative bank in a symmetric equilibrium is expressed as

$$\int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} R^L(L_j, A, \epsilon_i) L_h dH(\epsilon) dG(A) = (1+\rho^D)S_h + w(k_0^B, S_h), \tag{14}$$

where $w(k_0^B, S_h)$ is the accounting valuation of the net worth. Ex post, depending on the realization of the aggregate shock, the net worth can become tiny as a bank needs to repay deposits in full if not default.

III. Financial Intermediation

A. Institutional Assumptions

Because this paper discusses bankruptcy-related issues, the model's institutional setup on financial intermediation needs to allow debt-type contracts with bankruptcy in equilibrium. This paper essentially follows the costly state verification setup introduced by Townsend (1979), who shows that costly state verification gives rise to a debt contract, which has a flat payment (i.e., honoring the face value) for good realizations of shocks and state-contingent payments (i.e., default and partial recovery) for bad realizations of shocks. Townsend (1979) introduces one friction in the complete state contingent securities market. That is, information regarding which state is realized can be verified only with a cost. Hence, the state contingent return of a security includes additional endogenous contingency, i.e., its return when state is verified and its return otherwise. Apparently, the return becomes insensitive to state realizations when state is not verified. However, in Townsend (1979), the state contingent return in case verification occurs is prescribed in the security at its issuance as in the Arrow-Debreu market. Townsend (1979) also assumes (implicitly) the enforcement of contracts is free, again as in the Arrow-Debreu market. In summary, both explicit and implicit assumptions of Townsend (1979) on the financial market can be summarized as follows.

Assumption 1. ***[CSV - Contingent Contract Regime]***
(a) [Costly State Verification] A specific realization of combined productivity shock ϵA is private information to a borrower but can be verified by the borrower with cost τ.
(b-0) [Free Contingent Contracts] It is free to write ex ante contingent loan contracts for all possible states supposing the realized state is verified.
(c-0) [Free Enforcement] Repayment by a borrower is enforced freely according to the contract, including default cases.

Note that, in the literature, the verification cost is sometimes assumed to be paid by borrowers (e.g. hiring an accounting firm) or by a bank (e.g., examining documents), who would however charge the cost to the borrower in equilibrium. In either case, the cost ends up to be deducted from the borrower's income and assets in equilibrium.

While Townsend (1979) and many followers assume such a partially state-contingent contract can be written ex ante and enforced freely ex post, the financial crisis literature has been concerned about costs associated with bankruptcy, (e.g.Chari and Kehoe, 2016).[13] Here, I introduce a cost associated with bankruptcy in a simple way. I assume costly negotiation into the original Townsend Regime as follows.

Assumption 2. ***[CSV - Costly Negotiation Regime]***
(a-1) [Costly State Verification] A specific realization of combined productivity shock ϵA is private information to a borrower but can be verified by the borrower with cost τ to his lender only.
(b-1) [Incomplete Contract] There is a prohibitive cost for agents to write ex ante contingent loan contracts for any possible states.
(c-I) [Costly Negotiation] Once the shock realization is verified, loan repayment by a borrower is decided by Nash bargaining after the negotiation cost T is subtracted from the output. More specifically, for any realization of triple (l, A, ϵ), a borrower-bank pair maximizes its joint surplus, given a bargaining power parameter ξ^L,[14]

$$\max u(c^U)^{\xi^L} u(c^B)^{(1-\xi^L)}. \tag{15}$$

Assumption (c-1), costly negotiation, is the major assumption I introduce here. Assumption (b-1), incomplete contract, is naturally associated with ex post negotiations because writing contingent contracts ex ante for states that will be verified would make ex post negotiation redundant.[15] Assumption (a-1), costly state verification, is almost the same as in the CSV-Contingent Contract Regime, as it needs to make debt-type contracts exist in equilibrium. A slight clarification is added that the verified information remains private to the borrower and the lender who obtained the information. Note that, under Assumption 1, in the original Townsend (1979) model, it does not matter much if the verified information remains to be private for these two parties or becomes public. This is because contingent contracts can be written ex ante for the disclosed information and enforced freely. However, if the information on the realized state becomes public, a third party could involve ex post in the distressed asset market freely. It is not only inconsistent to Assumptions (b-1) and (c-1) but also to the costs paid (and profits enjoyed) by the real world specialists such as loan servicers and vulture funds.[16]

---

[13]The bankruptcy cost could be considered as a part of debt overhang cost. The bankruptcy cost could have a spillover effect (e.g., Igan, et. al., 2011). However, this paper assumes away the spill over effect for the sake of simplicity.

[14]The cost of negotiation can be defined as a part of c^L, borrower's consumption by the same way as the cost of verification.

[15]Even if any promise is not honored and requires negotiation ex post, without assumption (b-1), a contingent contract for verified states would be written as a negotiation-proof contract. Hence, if contingent contracts can be written freely, it avoids costly negotiation. But, then, the model misses the important friction that focuses on.

[16]I do not model the distressed asset specialists in this paper. However, as long as they have to pay the costs of verification (and negotiation), they can be considered as one function of bankers in my model. Indeed, Diamond and Rajan (2001) argues that the loan collection skill is a key feature of a bank.

Note that incomplete contract literature (e.g., Hart, 1995), stresses prohibitive costs of specifying all the states and writing contracts for all the contingencies. Although Hart (1995) points out importance of firm ownership (i.e., equity contracts), this paper focuses on debt contracts. Incompletely prescribed contingent returns on verified states naturally calls for ex post negotiation on splitting outputs and assets of the borrowers. In reality, indeed, disputes and lengthy negotiations for a bankrupt entity between creditors and borrowers often occur (e.g., Djankov, et. al., 2008). Such costs associated with disputes and lengthy negotiations, including any real and opportunity losses, should be considered as a bankruptcy cost, and a part of debt overhang costs.

Regarding the equilibrium allocations, the CSV-Costly Negotiation Regime is *not* different from the CSV-Contingent Contract Regime. When he needs to do so, a borrower verifies the realized state of the world. Then, he negotiates with the banker to split the borrower's assets and outputs by Nash bargaining, which is by definition Pareto optimal, as it would be written in the contingent contract in the CSV-Contingent Contract Regime. And, this whole process can be done with cost $\tau + T$. Therefore, the goods allocation in the CSV-Costly Negotiation Regime must be the same as in the CSV-Contingent Contract Regime, except for a cost difference by T in case of default. And, it is obvious that the bank bailouts (or any policy interventions), which introduce a different goods allocation than the market, would not improve the welfare because Townsend (1979) already shows that the general equilibrium allocation in his model, i.e., what I call the CSV-Contingent Contract Regime, is Pareto optimal.

However, the CSV-Costly Negotiation Regime, however, still misses an important friction that the real world faces. It is a simple and speedy bankruptcy procedure, which is increasingly adopted in the world since around 2000 and especially after the global financial crisis of 2008. For example, Germany and Japan changed its insolvency related laws and precedents at least a few times to adopt US Chapter 11 like debt restructuring law for private entities (e.g., 1999, 2005, and 2012 in Germany; 2000, 2003, and 2006 in Japan). Before such changes, when a firm becomes insolvent in these countries, liquidation with lengthy court process was the norm. And, to avoid it, insolvencies often ignited private negotiations, without involving a court, between creditors and a borrower.[17] Moreover, in the aftermath of the Global Financial Crisis, even more simpler and speedier debt restructuring schemes are adopted in many countries (IMF, 2013). For example, the U.S. adopted the Home Affordable Modification Program in 2009 to expedite massive mortgage bankruptcy cases to settle smoothly. The International Monetary Fund also recommends crisis-hit countries to adopt such simple and speedy bankruptcy procedures (Claessens, et. al, 2014). Indeed, the law-and-finance literature suggests that a speedy bankruptcy regime is growth and efficiency enhancing (e.g., cross-country panel regression studies by Djankov, et. al., (2008) and by Claessens, Ueda, and Yafeh (2014)).

The simple and speedy allocation of outputs and assets in case of default is often assumed in the literature of macroeconomics with financial frictions (e.g. Gertler and Kiyotaki, 2010). The literature often assume that the defaulter can retain a fixed portion of his assets and the rest is given to the creditors. This assumption is not so much differentfrom the relatively new bankruptcy schemes

[17] Chapter 11-type debt restructuring procedure is often characterized not only by a simple rule but also debtor in possession, which allows debtors to keep key assets to run a firm as a going concern or, in household bankruptcy cases (Chapter 13 in the US), to keep key assets such as a house to ensure a minimum consumption level. However, the allowed retained asset values can be large. For example, in the State of Florida, the value of the primary residence that can be retained by a defaulted borrower is almost unlimited as long as it is less than half acre.

adopted in many countries in the recent past. Therefore, I assume such a bankruptcy regime in this paper as follows.

Assumption 3. ***[CSV-Speedy Bankruptcy Regime]***
(a-1) [Costly State Verification] A specific realization of combined productivity shock ϵA is private information to a borrower but can be verified by the borrower with cost τ to his lender only.
(b-1) [Incomplete Contract] There is a prohibitive cost for agents to write ex ante contingent loan contracts for any possible states.
(c-2) [Simple Debt Restructuring Rule] When a borrower declares default, his business is seized by a banker except for the outputs that he is allowed to retain. It is simply assumed worth $\lambda > 0$ portion of his invested capital, λk_0.[18]
(d) [Small Cost] $\lambda k_0 + \tau \leq \underline{\epsilon A} e^U k_0^\alpha$.

Assumption (c-2), simple debt restructuring rule is the one I introduce here. This rule creates a potentially non-optimal allocations, but it allows the creditors and the borrowers to save the costs associated with lengthy negotiations, represented by T.[19] It is a simplest way to represent the *raison d'etre* of a speedy bankruptcy scheme. Note that Assumption (b-1), incomplete contract, is the same as in the CSV-Costly Negotiation Regime, and is naturally associated with the simple debt restructuring rule because writing contingent contracts in case of state verification, which is Pareto optimal, would make the simple but potentially non-optimal rule redundant. Assumptions (a-1) and (b-1) are the same as in the CSV-Costly Negotiation Regime. Under Assumption (b-1), costly negotiations could occur, and thus it lays the basis for a cost advantage of adopting a speedy bankruptcy rule. For the sake of simplicity, Assumption (d) is made so that the retained outputs and the verification cost can be always secured by the borrower's outputs.

Below, I focus on the CSV-Speedy Bankruptcy Regime. Note that the regime is so far defined for the loan market but, in a later section, essentially the same setup will be also assumed for the deposit market, between depositors and bankers.

B. Loan Contract

Assumption 3 (a-1) makes a loan contract to be possibly contingent on each borrower's combined shock if verified, but never on the aggregate shock alone. The assumption implies that the aggregate shock is not public information if no borrower declares default. Moreover, that the verified information remains private to a borrower and his lender implies that a borrower cannot recognize for sure the other borrowers' combined shocks ϵA and thus the aggregate shock A either.

Lemma 1. ***[Townsend]*** *There is a unique non-stochastic default threshold θ^L defined for combined shocks ϵA. It is determined by borrowers for any given loan rate ρ^L and for any repayment schedules that give a defaulted borrower to retain income that is non-decreasing in shock realization.*

---

[18]If the model were to incorporate a dynamic setup, it would make sense to assume that a defaulted borrower retains his capital to continue production as a going concern or keeps his own house.

[19]Without the simple rule, the optimal contract with CSV implies a potentially non-linear risk sharing between two parities when a default occurs. The literature has so far shown that it becomes a straight line when at least one party is risk neutral (see a review by Fulghieri and Goldman, 2008).

The proof follows Townsend (1979) and is omitted. Essentially, the borrowers would not want to pay the cost to verify (and negotiate) for all realizations. To minimize the cost payment, they would do so only when necessary. But, not verifying states implies the repayment schedule to be non-contingent on state, i.e., there is a flat portion on repayment schedule, which can be viewed as the promised fixed interest rate. The borrowers want to verify states only when they cannot repay the promised interest rate., i.e., when they need to declare default.

Lemma 1 and Assumption 3 restricts the equilibrium loan repayment schedule $R^L(L_j, A, \epsilon_j)$ as follows.

- Above threshold θ^L, a borrower repays in full, $(1 + \rho^L)$.
- Below threshold θ^L, a borrower defaults with retaining assets λk_0, and thus a defaulter's repayment schedule has an intercept term and a linearly increased portion with respect to the realized (combined) productivity shock $\epsilon_j A$:

$$\frac{\epsilon_j A e^U (k_0 + L_j)^\alpha - \lambda k_0 - \tau}{L_j}. \tag{16}$$

Accordingly, a borrower consumes, if default,

$$c^U(L_j, A, \epsilon_j) = \lambda k_0, \quad \text{if} \quad \epsilon_j A \in [\underline{\epsilon A}, \theta^L], \tag{17}$$

otherwise

$$c^U(L_j, A, \epsilon_j) = \epsilon_j A e^U (k_0 + L_j)^\alpha - (1 + \rho^L) L_j, \quad \text{if} \quad \epsilon_j A \in [\theta^L, \overline{\epsilon A}]. \tag{18}$$

At the threshold, the rational borrower equates consumption from defaulting and that from not-defaulting. This decision is made after the production takes place given loan size L_j.

$$\theta^L e^U (k_0 + L_j)^\alpha - (1 + \rho^L) L_j = \lambda k_0 + \tau. \tag{19}$$

i assume away a corner solution $\theta^L = \underline{\epsilon A}$ because a borrower never defaults in this case, which is not interesting. In other words, $\underline{\epsilon_j A}$ is assumed to be small enough so that the smallest output is smaller than the retained assets λk_0. I also assume away another corner solution $\theta^L = \overline{\epsilon A}$, with which the loan contract becomes an equity. In other words, $\overline{\epsilon A}$ is assumed to be large enough so that the largest output exceeds the retained output λk_0.

C. Deposit Contract

Regarding defaults of bankers in the deposit market, institutional assumptions on bankruptcy is made in a manner similar to the loan market. However, for the deposit market, even without costly state verification assumption, the equilibrium deposit contract becomes debt type due to the equilibrium loan repayment schedule.

Lemma 2. *There is a flat portion in an equilibrium deposit repayment schedule.*

Proof. As long as a banker transacts with a measurable set of borrowers, banker's consumption is not affected by idiosyncratic shocks of borrowers. By Lemma 1, the loan default threshold and the loan rate is not contingent on the aggregate shocks. Therefore, when all borrowers repay loans in full, bank revenue would be flat, non-contingent on any shocks. Hence, even if a depositor has an equity-type claim on bank revenue, the deposit return shows a flat portion above a certain threshold level of aggregate shock realization. *Q.E.D.*

Other assumptions regarding the bankruptcy regime remains the same.

Assumption 4. *[Speedy Bankruptcy for Bankers]*
(b-1') [Incomplete Contract] There is a prohibitive cost for agents to write ex ante contingent deposit contracts for all possible states.
(c-2') [Simple Debt Restructuring Rule] When a banker declares default (i.e., does not pay the flat deposit rate as promised), her business is immediately seized by depositors except for the assets that she is allowed to retain. It is assumed worth $\lambda > 0$ portion of her invested capital, λk_0.

In a region where a borrower does not default (i.e., $\epsilon A \geq \theta^L$), repayment from the borrower is constant $(1 + \rho^L)$. On the other hand, in a region where a borrower defaults (i.e., $\epsilon_j A < \theta^L$), repayment from the borrower is increasing with aggregate shock A with possible zero return.

A banker's gross income from all borrowers, denoted by B, is expressed as a function of the aggregate shocks given the loan market outcomes (i.e., the loan repayment schedule, loans per bank, and loans per firm):[20]

$$\begin{aligned} B(A|R^L, L_h, L_j) &= \int_{\underline{\epsilon}}^{\overline{\epsilon}} R^L(A, \epsilon_j) L_h dH(\epsilon) \\ &= \left(1 - H\left(\frac{\theta^L}{A}\right)\right)(1 + \rho^L) L_h \\ &+ \int_{\underline{\epsilon}}^{\frac{\theta^L}{A}} \left(\epsilon_j A e^U (k_0 + L_j)^\alpha - \lambda k_0 - \tau\right) \frac{L_h}{L_j} dH(\epsilon). \end{aligned} \tag{20}$$

Note that $B(A|R^L, L_h, L_j) \geq 0$ because of Assumption 3 (d). Also note that $\partial B/\partial A > 0$, that is, the

[20] In an overall symmetric equilibrium, $L_j = L_h\mu/(1-\mu)$ and $L_h = S_h + k_0^B$, that is, loan market outcomes are dictated by the deposit market outcome. See below the section on the general equilibrium. Note that, for a deposit market partial equilibrium, the loan market outcomes are taken as given.

banker's income is increasing in the aggregate shock.

$$\begin{aligned}
&\frac{\partial B}{\partial A}\\
=&(1+\rho^L)L_h\frac{\theta^L}{A^2}h-\left(\frac{\theta^L}{A}Ae^U(k_0+L_j)^\alpha-\lambda k_0-\tau\right)\frac{L_h}{L_j}\frac{\theta^L}{A^2}h+\int_{\underline{\epsilon}}^{\frac{\theta^L}{A}}\epsilon_j e^U(k_0+L_j)^\alpha\frac{L_h}{L_j}dH(\epsilon)\\
=&\left\{(1+\rho^L)L_h-\left(\theta^L e^U(k_0+L_j)^\alpha-\lambda k_0-\tau\right)\frac{L_h}{L_j}\right\}\frac{\theta^L}{A^2}h+\int_{\underline{\epsilon}}^{\frac{\theta^L}{A}}\epsilon_j e^U(k_0+L_j)^\alpha\frac{L_h}{L_j}dH(\epsilon)\\
=&\int_{\underline{\epsilon}}^{\frac{\theta^L}{A}}\epsilon_j e^U(k_0+L_j)^\alpha\frac{L_h}{L_j}dH(\epsilon)\\
>&0,
\end{aligned} \tag{21}$$

where pdf h is evaluated at θ^L/A. Note that the brace term in the penultimate line is zero because the full loan repayment $1+\rho^L$ is equal to the all the outputs plus net-of-retained portion of depreciated capital of a borrower at the default threshold of a loan (i.e., ϵA is at θ^L) as shown in (19).

Therefore, Assumption 4, in particular, (c-2'), implies that a banker optimally choses a unique threshold θ^D above which a banker does not default and below which she defaults. If not default, the banker enjoys consumption from the income after repaying the full obligation to depositors. The deposit repayment schedule is

$$R^D(S_h,A)=1+\rho^D,\quad \text{if } A\in[\theta^D,\overline{A}]. \tag{22}$$

Below θ^D, the banker needs to settle a low consumption level. A defaulted banker retains λ portion of their book capital, if possible. The deposit repayment function reflects revenues from borrowers net of retainment assets, correcting for the relative size of the banking sector:

$$R^D(S_h,A)=\frac{B(A|R^L,L_h,L_j)-\lambda k_0^B}{S_h},\quad \text{if } A\in[\theta^B,\theta^D). \tag{23}$$

where θ^B is the minimum aggregate shock level with which a banker has enough revenue to retain λk_0^B.

In the worst case, a defaulted banker do not obtain sufficient revenue to cover allowed retained assets, the return to depositors inevitably becomes zero,

$$R^D(S_h,A)=0,\quad \text{if } A\in[\underline{A},\theta^B). \tag{24}$$

In summary, the deposit repayment schedule is simply but optimally chosen by the bankers facing a specific bankruptcy regime under Assumption 4.

Lemma 3. *The optimal deposit repayment schedule takes a form of a standard debt-type contract. The deposit repayment schedule $R^D(S_h,A)$ has a flat portion with full-pay deposit rate ρ^D above the threshold θ^D defined for aggregate shock A. Below θ^D is the bank default region, and the repayment is contingent on the realization of aggregate shock A (i.e., (23) and (24)).*

D. Bankers' Choice and Consumption

In the beginning of the period, bankers offer deposit and loan rates by equating their expected utility from the spread income to the reservation utility, which is equal to the expected income of an entrepreneur. When banks offer deposit and loan rates, they rationally expect the possibility of defaults of both borrowers and themselves. A banker's net income is the gross income net of (size-corrected) repayments.

If every borrower repays in full and a banker can do so, a banker enjoys the maximum income from the spread between loan and deposit rates,

$$c^B(A, R^L, L_h, R^D, S_h) = (\rho^L - \rho^D)S_h + (1+\rho^L)k_0^B, \quad \text{if} \quad A \in [\frac{\theta^L}{\underline{\epsilon}}, \overline{A}]. \tag{25}$$

Even if some borrowers cannot repay back the promised loan returns to a banker, a banker can still repay deposits in full to depositors using her own capital buffer,

$$c^B(A, R^L, L_h, R^D, S_h) = B(A|R^L, L_h, L_j) - (1+\rho^D)S_h, \quad \text{if} \quad A \in [\theta^D, \frac{\theta^L}{\underline{\epsilon}}]. \tag{26}$$

In case a banker defaults and can retain allowed amounts at hand,

$$c^B(A, R^L, L_h, R^D, S_h) = \lambda k_0^B, \quad \text{if} \quad A \in [\theta^B, \theta^D). \tag{27}$$

In the worst case, a defaulted banker's revenue is less than the allowed retained amounts, $B(A) < \lambda k_0^B$, and his consumption becomes equal to his revenue,

$$c^B(A, R^L, L_h, R^D, S_h) = B(A|R^L, L_h, L_j), \quad \text{if} \quad A \in [\underline{A}, \theta^B). \tag{28}$$

IV. Equilibrium Allocation

A. Illustrative Explanation of Equilibrium

I explain illustratively here what are the income and consumption of borrowers, depositors, and bankers. First, consider hypothetical contingent contracts for idiosyncratic shock ϵ and aggregate shock A in an economy with borrowers and depositors but without bankers. And, keep assuming that the talent shock e is not insured. Then, the contracts are written essentially to exchange capital to arbitrage the returns. Figure 1 shows the consumption schedules for a representative borrower and depositor with and without capital exchange. For the illustrative purpose, $\epsilon = 1$ case is shown.

The capital exchange means that a low talented entrepreneur invests part of his capital to a high type until equating the marginal product of capital. It allows a higher output for a high talent entrepreneur and a lower output for a low type. Hence, these two output levels (the dotted lines) diverge from the autarkic levels (the dashed lines). However, the low type receives the returns from the high type, the consumption levels improves from the autarkic level (the solid line). Also, accepting this contract means that the high type consumption after repaying dividends is still better than the autarkic level (the solid line).

Next, still assuming only borrowers and depositors exist, but suppose available contracts are those characterized as a simple debt type that does *not* allow default. Figure 2 shows the consumption schedules for a representative borrower and depositor in this case. From the output levels after the capital exchange (the dotted line), consumption of the borrower shifts down by the flat loan repayments regardless of states as default is assumed away here, while that of the depositor shits up as much as the flat deposit return (the solid lines). Compared to the equity contract in Figure 1, the borrower (the high type) gains upside potentials but suffers from downside risks.[21]

To limit the downside risks for borrowers, assume now that the debt contract allows default with retaining some assets. The borrower's consumption is bounded by below at the allowed retained assets (see Figure 3, the solid line). Then, when a large negative aggregate shock (i.e., a tail risk) is realized, the consequences are mostly assumed by depositors (*tail risk dumping*).

The situation is a bit improved by introducing bankers into this economy. Figure 4 adds banker's consumption (the additional solid line). Both borrowers and bankers would keep minimum guaranteed consumption from their assets, while depositors cannot do so. However, the bankers provide some insurance using their capital buffers for depositors to mitigate the tail-risk dumping problem. Because of this reason, banks emerges in equilibrium in this paper.

Below I explain more formally what are the consumption allocations, or equivalently the loan repayment and deposit repayment functions, in equilibrium. Although the model assumes standard utility and production functions, which are often used macro models with financial frictions, it allows the deposit and loan repayment schedules to have kinks (Figure 4) and investigate the equilibrium thoroughly. For example, a natural question may arise on existence and uniqueness of equilibrium with kinked repayment functions. One way to analyze is to allow lotteries (i.e., correlated or mixed strategies) to convexify the kinks à la Prescott and Townsend (1984 a, b). This approach could make sure the existence of equilibrium, but with complex contracts traded in the market without banks. However, in this paper, I would like to analyze banking sector policies, especially bailouts. So, I write a model with bankers who strategically designing contracts, intermediate capital, and possibly default by themselves. I also focus on pure strategies with identifying some restrictions on parameter values under which a unique equilibrium is supported, and provides analytical characterizations of the equilibrium and associated policy implications. Still, whenever possible, I explain similarities and differences from a general contract approach by Prescott and Townsend (1984 a, b). I show the analysis below by a constructive manner on (i) the partial equilibrium in the loan market, (ii) the partial equilibrium in the deposit market, and (iii) the general equilibrium.

Note that rights to consume a portion of the banker's consumption allocation c^B would be called "equity shares" of a bank. In the perfect world, everyone has incentive to sell such equities and hold other people's share (i.e., perfect cross-share holdings) so that "big household" assumption would prevail. However, as stressed at the beginning, the purpose of this paper is to investigate how consumption and investment allocations would be characterized and whether any policy interventions can improve welfare in the segregated household economy without perfect risk sharing. For this focus, the model assumes away the equity issuance or cross-share holdings by bankers or entrepreneurs.[22] However, the non-availability of such equity-type contracts is consistent with the

[21] The capital exchange itself are not likely the same as under the equity contracts.

[22] If bankers' income were shared among people perfectly, no demonstrations would have occurred against the bank failure and bailouts in the aftermath of the global financial crisis.

informational assumptions that make debt-type contracts to be chosen optimally. In other words, just assuming equity ownership of banks and firms in an ad hoc manner would not be consistent with the model presented in this paper.

B. Loan Market Partial Equilibrium

Given the deposit repayment schedule, bankers supply loans up to the deposit amounts for a profitable loan repayment schedule as long as the expected income is non-negative. Therefore, given the deposit market partial equilibrium, the loan market is essentially determined by borrowers' decisions. There are two decisions: one for how much to borrow before the production; and the other for either default or not after the production.

A borrower at the repayment stage has one decision to make, either repaying in full or verifying the state to pay state-contingent amounts. The default threshold is determined by the borrower as shown by Lemma 1. For any given amount of loans $l|j$, the *iso-loan default curve* can be drawn on the θ^L-ρ^L plane (Figure 5) based on the default condition (19):

$$1 + \rho^L = \theta^L e^U \frac{(k_0 + l_j)^\alpha}{l_j} - \lambda \frac{k_0}{l_j}. \tag{29}$$

Lemma 4. *For any given amount of loans l_j, the iso-loan default curve on the θ^L-ρ^L plane is monotonically increasing in default threshold θ^L. It shifts down and becomes flatter with a larger loan l_j on the θ^L-ρ^L plane.*

Proof. Increasing in θ^L on the θ^L-ρ^L plane is easy to see because the slope of the iso-loan default function (29) with respect to θ^L is positive:

$$e^U \frac{(k_0 + l_j)^\alpha}{l_j} > 0. \tag{30}$$

To see the effect of larger loans l_j, I investigate changes in the slope and the intercept of this iso-loan default curve. The slope of (29) is decreasing with a larger loan l_j since its derivative with respect to l_j is

$$\alpha e^U \frac{(k_0 + l_j)^{\alpha-1}}{l_j} - e^U \frac{(k_0 + l_j)^\alpha}{l_j^2} = e^U \frac{(k_0 + l_j)^{\alpha-1}}{l_j^2}((\alpha - 1)l - k_0) < 0. \tag{31}$$

Also, the intercept k_0/l_j of (29) on the θ^L-ρ^L plane is decreasing with a larger l_j. Therefore, the iso-loan default curve becomes flatter and shifts down. *Q.E.D.*

Knowing his own behavior at the repayment stage, a borrower choses his demand for a loan at the borrowing stage. Let $\eta_j \equiv \epsilon_j A$ denote the combined shock with the cdf $M \equiv G \circ H$. The first order condition for the borrower's problem to ask for loans (5) is

$$\int_{\theta^L}^{\overline{\epsilon A}} \left(\alpha \eta_j e^U (k_0 + l_j)^{\alpha-1} - (1 + \rho^L)\right) u'(c^L) dM(\eta) = 0. \tag{32}$$

This is essentially the optimal leverage problem for a limited-liability entrepreneur. An entrepreneur borrows capital until the expected marginal productivity of capital equals to the loan rate but only for the non-default region because, if defaulted, the borrower would consume only the retained initial wealth regardless of the loan amount.

The *iso-loan demand curve* of the loan rate ρ^L with respect to default threshold θ^L given loan amount l_j is expressed as an implicit function of the borrower's first order condition (32),

$$\chi(\theta^L, \rho^L) \equiv \int_{\theta^L}^{\overline{\epsilon A}} \left(\alpha \eta_j e^U (k_0 + l_j)^{\alpha-1} - (1 + \rho^L)\right) u'(c^L) dM(\eta) = 0. \tag{33}$$

Lemma 5. *For any given amount of optimally chosen loans l_j^*, the iso-loan demand curve on the θ^L-ρ^L plane is monotonically increasing in default threshold θ^L. It shifts up-left with a larger loan l_j^*.*

The proof is provided in Appendix. On the θ^L-ρ^L plane, the *iso-loan demand function* can be drawn like Figure 5.

Lemma 4 implies that ex post default decision pins down the relationship between the default threshold θ^L and the loan rate ρ^L as the iso-default curve. However, this relation must be consistent with the pair (θ^L, ρ^L) in the loan contract when a borrower decides to take loans before the production. The latter relation is represented by the iso-loan demand curve. Therefore, the cross point of those two curves are the consistent choice. A question may arise if a unique pair of loan rate and default threshold is determined by a borrower given a loan amount. Indeed, this is true under reasonable parameter values with a not-so-tight restriction on (equilibrium) banker population.

Assumption 5. *[Restrictions on parameter values]*

- *Maximum difference in talents: there exists Z such that*

$$\frac{e^U}{e^D} \leq Z^{1-\alpha}. \tag{34}$$

- *Maximum curvatures of utility and production functions (i.e., σ and α) relative to banker population μ,*

$$\sigma + \alpha \leq \frac{2Z}{(Z+1)\mu + Z - 1}. \tag{35}$$

In the extreme case, where I assume any talent difference is possible, i.e, $Z = \infty$, then the assumption becomes[23]

$$\sigma + \alpha \leq \frac{2}{1 + \mu}. \tag{36}$$

Even this tighter condition allows most of reasonable parameter values assumed in the macroeconomic literature. For example, the relative risk aversion parameter $\sigma = 1.2$ and the capital share $\alpha = 0.3$ can allow the equilibrium banker population up to $1/3$, at which banker population

[23] The same condition appears if I assume possibility of the maximum loan equaling to the whole capital owned by depositors (i.e., there is no production by depositors) plus banker's capital. In this case, $l = k_0 + 2\mu k_0/(1 - \mu)$.

equals to the population of borrowers (or depositors) in a symmetric equilibrium. If I can focus on more reasonable range of the banker population, say up to 10 percent, then, $\sigma = 1.5$ and $\alpha = 0.3$ are consistent with this tighter assumption.

If we allow higher banker population, $\mu \leq 1/3$ and assume potentially any α up to 0.5, then the tighter restriction (36) becomes $\sigma \leq 1$. This is quite restrictive compared to usual assumptions of σ between one and two. Hence, in general, we also need to focus a reasonable range of the talent difference.

Indeed, for more general cases for parameter values of (σ, α, μ), it is sufficient to restrict the maximum talent difference as (34) in Assumption 5. It is not so restrictive in the real world application. Take, for example, $Z = 2$, which means $e^U\sqrt{2}e^D$, that is, a little more than 40 percent difference in productivity. In this case, $\sigma = 2$ and $\alpha = 0.5$ can allow the equilibrium banker population up to 20 percent, which covers almost all the countries, even financial centers like Switzerland.[24]

Proposition 1. *In a partial equilibrium of the loan market, under Assumption 5, for each loan amount l_j, there exists a unique loan contract (loan repayment schedule), characterized by a unique pair of loan rate and default threshold (ρ^L, θ^L).*

See the proof in Appendix. Call this (ρ^L, θ^L) pair as an *admissible loan contract.*

Corollary 1. *Both the loan rate ρ^L and the default threshold θ^L of admissible loan contracts becomes higher with a larger loan amount. In other words, the optimal loan choice by a borrower l_j^* is uniquely determined given a representative loan contract offer (ρ^L, θ^L) chosen from the set of admissible loan contracts.*

The proof for Proposition 1 shows that the iso-loan default curve is always steeper than the iso-loan demand curve. With a larger loan amount, the iso-default curve shifts down-right (Lemma 4) and the iso-loan demand curve shifts up-left (Lemma 5). Hence, the cross point must shift up-right as the loans increase (see Figure 5).

C. Credible Deposit Demand by Bankers

When a person decides to be a banker, he offers a deposit contract, which specifies a deposit repayment schedule. As long as the spread income is positive, a banker is happy to take as much deposits as possible. That is, the deposit demand by a bank is inelastic to any given profitable pair of positive spread and default thresholds.[25]

---

[24] Just before the Global Financial Crisis, in a height of a credit boom, financial sector GDP to total GDP ratio of Switzerland is around 12 percent and the employment share of the financial sector is around 6 percent (IMF, 2008).

[25] In more general double competition in deposit and loan markets, bank competition brings zero rents at least when banks compete for deposits first (Stahl (1988), Ueda (2013)). In this paper, no rents implies that bankers earn the same expected income as entrepreneurs. When deciding a deposit market strategy, a banker takes into account his reservation utility by switching jobs. And hence there should exist positive expected profits from loan and deposit repayment schedules in equilibrium.

A banker is supposed to repay deposits in full, $(1 + \rho^D)$, under a deposit contract $R^D(S_h, A)$ as long as the aggregate shock A is above the default threshold θ^D. However, this repayment schedule is credible only if a banker chooses the default threshold in his own behalf. Default means for a banker to give up his revenue to depositors and thus higher default threshold θ^D for a given spread does not necessarily provide a banker a higher profit. Indeed, given loan rate ρ^L and deposit rate ρ^D (or spread $\pi = \rho^L - \rho^D$), a banker can maximize his utility by choosing threshold θ^D such that consumption under default (27) is equal to consumption under full deposit repayment (26) for any level of deposits demand $\tilde{s}_h$:[26]

$$\lambda k_0^B = B(\theta^D | R^L, L_h, L_j) - (1 + \rho^D)\tilde{s}_h. \tag{37}$$

Given initial capital k_0^B, banker population μ, and loan market variables (R^L, L_i, L_j), this constraint (37) should hold and appears as *credible deposit contract offer curve* $s^d(\theta^D, \rho^D)$ on the θ^D-ρ^D plane for each deposit demand $\tilde{s}_h$ as (see Figure 6),

$$(1 + \rho^D) = \frac{B(\theta^D | R^L, L_h, L_j) - \lambda k_0^B}{\tilde{s}_h}. \tag{38}$$

Note that I am not requiring the bank balance sheet condition $\tilde{s}_h = L_h + k_0^B$ for now in the deposit market partial equilibrium.

Lemma 6. *Given banker population and loan market variables, the credible deposit contract offer curve is strictly increasing in θ^D on the θ^D-ρ^D plane.*

The proof is provided in the Appendix.

Corollary 2. *With larger deposits $\tilde{s}_h$, given default threshold θ^D, the credible deposit contract offer curve shifts down (i.e., lower ρ^D) on the θ^D-ρ^D plane. On the other hand, given deposit rate ρ^D, it shifts right (i.e., higher θ^D). Overall, the larger deposits make the curve to shift lower right.*

This result directly follows the signs of derivatives of (38), that is, $\partial\rho^D/\partial\tilde{s}_h < 0$ and $\partial\theta^D/\partial\tilde{s}_h > 0$ (see (21) for changes in banker's income with respect to θ^D).

D. Deposit Supply by Depositors

Proposition 2 (Optimal Deposit Size)**.** *Given a deposit contract $R^D(S_h, A)$, a depositor determines deposit amount s_i^* uniquely to maximize his utility (6).*

---

[26]The first order condition for (7) with respect to θ^D is

$$u(\lambda k_0^B) = u(B(\theta^D | R^L, L_h, L_j) - (1 + \rho^D)\tilde{s}_h),$$

which is simplified to (37).

Note that

$$c^D(s_i, A, \epsilon_i) = \epsilon A e^D (k_0 - s_i)^\alpha + R^D(S_h, A)s_i, \tag{39}$$

where $R^D(S_h, A)$ is defined in (22) to (24). The first order condition (FOC) for depositor's problem (6) with respect to deposits s_i is

$$\begin{aligned} 0 = &- \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} u'(c^D(s_i, A, \epsilon_i))\alpha\epsilon_i A e^D (k_0 - s_i)^{\alpha-1} dH(\epsilon)dG(A) \\ &+ \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} R^D(S_h, A)u'(c^D(s_i, A, \epsilon_i))dH(\epsilon)dG(A) \\ &\equiv \Phi(\theta^D, \rho^D). \end{aligned} \tag{40}$$

See the rest of the proof in Appendix, which shows that the second order condition is valid, that is,

$$\frac{\partial \Phi}{\partial s_i} < 0. \tag{41}$$

For given initial capital k_0 and parameter values of the production and utility functions as well as the deposit repayment schedule R^D, the utility level is determined in equilibrium by optimally chosen deposit s_i^*. On the other hand, equation (40) shows the relation between the deposit rate ρ^D and the default threshold θ^D given a specific level of deposits s_i^* and loan market variables. This relation can be drawn on θ^D-ρ^D plane as the *iso-deposit supply curve*.

Given a FOC-satisfying deposit level s_i^* fixed, the slope of the iso-deposit supply curve on the θ^D-ρ^D plane is determined by the first order condition (40). For the same level of optimal deposits s_i^*, the slope of the iso-deposit supply curve is given by the implict function theorem applied to the FOC (40):[27]

$$\frac{d\rho^D}{d\theta^D} = -\frac{\partial \Phi(\theta^D, \rho^D)/\partial \theta^D}{\partial \Phi(\theta^D, \rho^D)/\partial \rho^D}. \tag{42}$$

Lemma 7. *The iso-deposit supply curve on the θ^D-ρ^D plane has zero slope in the neighborhood of the credible deposit contract offer curve. Its slope is positive on the right of the default threshold θ^D and negative on the left side, creating a parabola-like figure.*[28]

---

[27]The derivative changes at default threshold θ^D depending on whether depositors expect default or not by banks at the threshold. The derivative is taken from the right side for the non-default neighborhood and from the left side for the default neighborhood.

[28]The parabola-like shape for given deposit comes from the fact that, for the same deposit rate ρ^D, higher default threshold θ^D enables depositors on the right of the original default threshold to seize bankers' assets that has higher value than the deposit rate. Also, for the same deposit rate ρ^D, lower default threshold θ^D makes depositors to receive larger return than seizing the assets on the left of the original default threshold. Therefore, for the same deposit rate, depositors gain more by either higher or lower default threshold than the one on the credible deposit contract curve. To keep the same utility (deposits?), then the deposit rate needs to be lower, i.e., the spread π needs to be higher. Hence, the parabola-like shape emerges.

Proof. The numerator of (42) is expressed as

$$\frac{\partial \Phi(\theta^D, \rho^D)}{\partial \theta^D} = \left\{ \left(B(\theta^D | R^L, L_i, L_j) - \lambda k_0^B \right) - (1+\rho^D) s_i^* \right\} g(\theta^D) \int_{\underline{\epsilon}}^{\overline{\epsilon}} u'(c^D(s_i^*, \theta^D, \epsilon_i)) dH(\epsilon), \tag{43}$$

where $g(A)$ is pdf for cdf $G(A)$.

The term inside the brace is zero if θ^D is also satisfying the credible deposit contract offer (38), that is, $s_i^* = \sum_h \tilde{s}_{hi}$. For a larger θ^D given ρ^D, the brace term is positive and vice versa. Recall that $\partial B > \partial A$ as shown in (21). *Q.E.D.*

Corollary 3. *In the neighborhood of the credit deposit contract offer curve, given ρ^D, the iso-deposit supply curve is insensitive in θ^D to a change in s_i^*. Its slope become steeper on the right and left of the default threshold θ^D. On the other hand, given θ^D, it shifts up (i.e., higher ρ^D) with larger s_i^*.*

Proof. Although the set of deposit rate and default threshold together affects the deposit supply, the deposit supply turns out insensitive to a change in default threshold alone at the value satisfying the credible deposit contract offer (43). The implicit function theorem with (41) and (43) shows that

$$\frac{\partial s_i}{\partial \theta^D} = -\frac{\partial \Phi / \partial \theta^D}{\partial \Phi / \partial s_i} = 0. \tag{44}$$

On the right of default threshold θ^D, the numerator is positive (Lemma 7) and thus the slope is positive (i.e., $\partial s / \partial \theta^D > 0$). The opposite is true for the left side. Hence, overall, the iso-deposit supply curve becomes steeper with larger s_i though the slope remains at zero in the neighborhood of the credit deposit contract offer curve.

Next, noting that $\partial c^D / \partial \rho^D = s_i$ for $A \in [\theta^D, \overline{A}]$ and $u'' < 0$,

$$\begin{aligned} \frac{\partial \Phi}{\partial \rho^D} &= -\int_{\overline{A}}^{\theta^D} \int_{\underline{\epsilon}}^{\overline{\epsilon}} s_i u'' \alpha \epsilon_i A e^D (k_0 - s_i)^{\alpha - 1} dH(\epsilon) dG(A) \\ &+ \int_{\overline{A}}^{\theta^D} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left\{ (1+\rho^D) s_i + 1 \right\} u' dH(\epsilon) dG(A) \\ &> 0. \end{aligned} \tag{45}$$

Hence,

$$\frac{\partial s_i}{\partial \rho^D} = -\frac{\partial \Phi / \partial \rho^D}{\partial \Phi / \partial s_i} > 0. \tag{46}$$

Q.E.D.

E. Deposit Market Partial Equilibrium

Proposition 3. *Given loan market variables, the deposit market equilibrium—deposits (s_i, s_h) and deposit repayment schedule R^D represented by deposit rate ρ^D and bank default threshold θ^D—is uniquely determined.*

Proof. Given an equilibrium deposit amount, on the θ^D-ρ^D plane, the iso-deposit supply curve has a zero slope only in the neighborhood of the credible deposit contract offer curve with parabola-like U shape (Lemma 7). The credible deposit contract offer curve is strictly increasing (Lemma 6) and thus crosses the parabola-like iso-deposit supply curve at the bottom, where the slope is zero (see Figure X).

Given a crossing point at the bottom of the iso-deposit supply curve, two curves never crosses again. To see this, suppose on the contrary that the iso-deposit supply curve once again crosses the deposit contract offer curve in the upper right region of the original crossing point, as the credible deposit contract offer curve is strictly increasing. However, Lemma 7 states that the iso-deposit supply curve has a zero slope at the crossing points. This contradicts to the characteristics of the iso-deposit supply curve in Lemma 7: it has a positive slope in the upper right region of the original crossing point at the bottom.

Now, suppose there are possibly different levels of equilibrium deposits and associated deposit repayment schedules. Corollary 3 implies that the bottom of the iso-deposit supply curve shifts straight up with larger deposits s_i. Apparently, for any deposit level, the crossing point, i.e., the equilibrium, must be on this path. On the other hand, Corollary 2 states that the credible deposit contract offer curve shifts down right with larger deposits s_h. With the equilibrium condition $s_i = \sum_h s_h$ and the restriction on the crossing point, the fact that one curve goes up and the other goes down with larger deposits implies that these two curves meet only under one pair of deposits (s_i, s_h). And, at the same time, only one deposit repayment schedule (ρ^D, θ^D) lies at the crossing point of two curves. *Q.E.D.*

F. Bank Size and Capital Ratio in General Equilibrium

A sizable banking sector can exist to provide an insurance to mitigate the tail-risk dumping to depositors. A depositor faces shock-contingent income up to threshold θ^D and then flat income ρ^D from full deposit repayment. With large enough variations in the aggregate shocks, there are sizable chances that deposits are not repaid in full. Then, the depositors would prefer a more insured contract that provides a same expected return with a lower deposit rate but also with a lower default risk. A banker also prefers to provide this insurance contract, implying that he will have strictly positive capital.

Proposition 4. *In the general equilibrium, the banking sector is sizable $\mu^* > 0$. This implies that the capital ratio k_0^B/L_h and the spread $\pi = \rho^L - \rho^D$ are strictly positive.*

Proof. Suppose that the banking sector size μ is measure zero. Then, by construction, the capital ratio becomes (almost) zero, and the expected loan and deposit repayment schedules becomes (almost) the same by arbitrage. Specifically, there is one threshold $\hat{\theta}^D = \hat{\theta}^L/\underline{\epsilon}$ below which borrowers start to default and bankers default. Then, the full-pay deposit and loan rate becomes (almost) equal and the spread is zero, $\hat{\pi} = \hat{\rho}^L - \hat{\rho}^D = 0$.

Consider a slightly different deposit contract with positive capital buffer, while keeping the same loan contract. I can show that, even if deposit per bank falls, a banker and a depositor strictly prefer the new contract—the lower expected value with less volatile deposit repayments—given prevailing loan contracts $R^{L*}(l, A, \epsilon)$ and the amount of deposit $\hat{s}$ per depositor. For illustrative purpose, I draw the original and new deposit repayment schedules with the aggregate shock realization on the x-axis and the repayment on the y-axis (see Figure 8). Let $\underline{\theta}^D$ denote the new threshold of default by the banker.

A depositor prefers the deposit contract which gives the same expected return with less volatile repayment. Here, less volatile means a lower full repayment (i.e., lower deposit rate $\underline{\rho}^D$) but with lower threshold and higher repayment in case of banker's default. This contract (partially) insures depositors' income for the combined idiosyncratic and aggregate productivity shocks and thus it is preferred by a risk averse depositor.

Think about a bank default case. Repayment to a depositor is the seized banker's income (23) and (24), multiplied by relative depositor size, $\hat{w} = (1-\mu)/2\mu$. Under the measure zero banking sector, this is equal to zero. So, the depositor's gain from the new contract with the positive capital buffer is the difference between the default region under two contracts weighted by the probability of realization of combined shocks. That is, the amount that shifts up linearly the recovery rate of deposit contract by the capital buffer in the default region in Figure 8. As long as the probability becomes decreasing for the lower tail, the depositor's gain has the lower bound: the rectangle consists of the height of the capital buffer and the range between the new default threshold $underline\theta^D$ and the threshold depleting the capital buffer $\underline{\theta}^B$:

$$Gain > \underline{Gain} = \int_{\underline{\theta}^B}^{\underline{\theta}^D} \hat{w}\lambda k_0^B dH = \hat{w}\lambda k_0^B \left(H(\underline{\theta}^D) - H(\underline{\theta}^B) \right). \tag{47}$$

Next, this partially insured contracts can be designed so that the overall repayment has the same expected values. Note that the depositor's expected utility is larger with the same expected return but more insured income risk. A banker can make such a contract by limiting deposits and loans given his endowed capital. Essentially he uses his capital as a buffer to depositors, so that the default threshold θ^D will be lowered. Because this new contract is assumed to give the same expected repayments to depositors, the banker is offering a lower deposit rate $\underline{\rho}^D$ for the same loan rate $\hat{\rho}^L$. This change of the deposit rate is denoted by the increase in the spread $\triangle\pi$ from zero.

The depositor's loss is strictly lower than the upperbound of the loss, which is measured by the rectangle made by the change in the spread and the cumulative probability above the new threshold in Figure 8. That is,

$$Loss < \overline{Loss} = \triangle\pi(1 - H(\underline{\theta}^D)). \tag{48}$$

Because the new contract is assumed to have the same expected returns for a depositor, the gain and the loss must be the same. Therefore, the lowerbound of the gain must be strictly lower than the upperbound of the loss: That is,

$$\triangle\pi > \hat{w}\lambda k_0^B \frac{H(\underline{\theta}^D) - H(\underline{\theta}^B)}{1 - H(\underline{\theta}^D)}. \tag{49}$$

Because the current banking sector size is almost zero, I evaluate this at the limit $\hat{\mu} \to 0$ (i.e., $\hat{w} \to 0$) to see the profit of stemming from a new contract only slightly different from the current one. The limit value is strictly positive:

$$\lim_{\hat{\mu}\to 0} \triangle\pi > 0. \tag{50}$$

This implies that the increase in the spread by offering a insuring contract is strictly positive near $\mu = 0$. The spread is literary an insurance premium for the strictly positive value of the capital buffer as an insurance for depositors. With the new contract, a banker will have a higher spread income in addition to a lower default probability.

By limiting loans and deposits, the total income of a banker could become less because the total income is affected by the spread times deposits. But, recall that the spread under the original contract is zero. Thus, the total income also increases from zero to a positive sum with a slightly different contract.

In summary, both a depositor and a banker prefer a new contract, given the same loan contract (i.e., the same utility for borrowers). Therefore, in equilibrium, the banker size μ (and the capital buffer) must be positive. Associated is the positive spread ($\pi > 0$). *Q.E.D.*

G. General Equilibrium Existence and Uniqueness

First, I characterize how the small changes in banker population μ affects the deposit and loan market partial equilibrium in Lemmas 8 and 9 below (proofs are shown in Appendix). I focus on a probable case where the equilibrium results of an increase of banker population through the deposit contract term (ρ^D, θ^D) is not strong.

Assumption 6. *The deposit supply elasticity to banker population in an equilibrium is not more than one:*

$$\frac{\partial s_i}{\partial \mu}\frac{\mu}{s_i} \leq 1 \tag{51}$$

Lemma 8. *In the deposit-market partial equilibrium, given loan market variables and under Assumption 6, larger banker population μ brings lower deposits per banker, i.e., $\partial s_h/\partial\mu < 0$. It leads to a higher deposit rate, $\partial\rho^D/\partial\mu > 0$, and a lower default threshold, $\partial\theta^D/\partial\mu < 0$. This implies that the deposits per depositor increase $\partial s_i/\partial\mu > 0$ even though the deposits per banker decrease.*

Lemma 9. *Given a loan amount, the loan-market partial equilibrium contract is not affected by a change in banker population μ, i.e., $\partial\rho^L/\partial\mu = 0$ and $\partial\theta^L/\partial\mu = 0$.*

Now, I analyze the effects of banker population μ on the utility level of each type of agents.

Lemma 10. *The ex ante entrepreneur's utility $V^E = V^L/2 + V^D/2$ is strictly increasing with higher banker population μ, i.e., $dV^E/d\mu > 0$.*

Proof. For a depositor, higher bank population brings a better deposit term (ρ^D, θ^D) by Lemma 8, and thus apparently $dV^D/d\mu > 0$. For a borrower, there is no direct effects in the lending term $(\rho^L.\theta^L)$ due to a change in banker population μ by Lemma 9, and thus $dV^D/d\mu = 0$. Overall, $dV^E/d\mu > 0$. *Q.E.D.*

Lemma 11. *With larger banker population μ, the utility of a banker, V^B, strictly decreases, i.e., $dV^B/d\mu < 0$.*

See the proof in Appendix.

Finally, here is the general equilibrium result.

Proposition 5. *The banker population μ is determined uniquely in the decentralized equilibrium.*

Proof. As banker population become smaller $\mu \to 0$, entrepreneur's utility V^E decreases but bounded by the finite value achieved in the first best allocation, denoted by $\overline{V}^E$ (see next section). In this case, a tiny number of bankers. Let π_0 denote the equilibrium spread near $\mu = 0$. It is positive by Proposition 4. Then, a banker's income when banker population is tiny at $\underline{\mu}$ reaches quite high,$\pi_0/\underline{\mu}$. Hence, as $\mu \to 0$, $V^B > V^E$.

On the other hand, as $\mu \to 1$, only a tiny portion of people produce outputs and thus the banker's spread income is close to zero, $V^B \to 0$. The entrepreneur's consumption is bounded below by the autarkic level, $V^E_{aut} > 0$ and thus $V^B < V^E$.

Both banker's utility V^B and entrepreneur's utility V^E are apparently continuous functions on banker population μ. Lemmas 10 and 11 state that V^E is strictly increasing in banker population μ while V^B is strictly decreasing in μ. Both V^B and V^E are apparently continuous functions with respect to μ.

Consider a function $(V^B - V^E)$, which is a continuous function of μ, mapping from compact domain $[0, 1]$ to compact range $[-V^E_{aut}, \overline{V}^E/\underline{\mu}]$. It is strictly decreasing by Lemmas 10 and 11. By the contraction mapping theorem, there is a unique fixed point μ^* at which $V^B = V^E$. *Q.E.D.*

H. Walrasian Equilibrium as the Limit

For a reference, in this section I analyze a complete market case as the limit of the model. Here, the verification and negotiation costs are close to zero, or equivalently all the information is almost public. Accordingly, I assume here no retained assets. I can still keep assuming prohibitive cost of writing ex ante contingent contracts but with ex post Nash bargaining occurs without costs. the allocations are equivalent between the economies with and those without writing ex ante contracts. In this limit, the loan and deposit contracts take a form of equity contract as the limit of debt contracts.[29]

Proposition 6. *As the verification and negotiation costs becomes close to zero, deposit and loan contracts become complete, equity-type, contingent contracts. The equilibrium mimics the Warlasian competitive equilibrium and is the first best. Specifically, the equilibrium deposit and loan repayment schedules are the same in the limit and so do the deposit and loan amounts. A small number of (i.e., measure zero) banks intermediate the capital. Accordingly, the optimal capital ratio of banks are (almost) zero. As a result, consumption is almost perfectly shared equally among all households.*

[29]This result follows in spirit that of Townsend (1978) on costly bilateral exchange.

Proof. (Sketch). The positive spread between deposit and loan rates is a friction to the production sector. The smaller the spread, the larger is the outputs. Thus, a smallest number of banks should intermediate capital from the social planner's point of view. In the limit, the deposit-loan rate spread becomes zero with the same deposit and loan size. With zero spread and compete contracts, the idiosyncratic shocks are shared perfectly among all agents. As Proposition 7 still applies, this allocation is also supported by a decentralized equilibrium. *Q.E.D.*

V. Policy Implications

A. Welfare Theorem

The institutional assumptions restrict the shape of the loan and deposit repayment schedules R^L and R^D, which agents in this economy as well as the social planner obey. Because R^L and R^D also dictates the consumption allocation, the constrained social planner can determine the consumption allocation indirectly by choosing the loan and deposit repayment schedules.

The social planner's problem can be expressed as maximizing a representative banker's utility, similar to (7), but also with controls μ, l, and s in addition to R^L and R^D:

$$\max_{\mu,l,s,R^L,R^D} \int_{\underline{A}}^{\overline{A}} u\left(c^B(A, \tilde{R}^L, \Omega, \tilde{R}^D, \Psi)\right) dG(A), \tag{52}$$

subject to the representative borrower's utility maximization, i.e., an incentive constraint, (5), the representative depositor's utility maximization, i.e., another incentive constraint, (6), the occupational choice, i.e., yet another incentive constraint, (13), and resource constraints (10) and (12). Note that the social planner can set the all the loan and deposit contracts to be the same $\Omega = R^{L\mu}$ and $\Psi = R^{D\mu}$, i.e., μ-Cartesian products of R^L and R^D, respectively.[30]

Definition 2. *The constrained social optimal allocation is the solution to the social planner's problem in which the social planner faces the same restrictions as the private agents.*

Proposition 7. *The decentralized equilibrium achieves the constrained social optimum. That is, it achieves the constrained social optimal allocation given banker's population μ, which then is determined optimally.*

The proof is straightforward and sketched here. Inside the social planning problem, the incentive constraints for borrowers (5), for depositors (6), and for occupational choice (13) are exactly the same problems that are solved in the decentralized equilibrium under the same resource constraints and Assumptions. Only difference is the problem for bankers. In decentralized equilibrium, a banker maximizes his utility by choosing loan and deposit repayment schedules given the reaction functions (i.e., loan demand and deposit supply functions) from borrowers and depositors. The social planner achieves the social optimum by maximizing the representative banker's utility by choosing loan and

[30]The social planner problem can be set up as maximizing the social weight weighted sum of utilities $\mu V^B + (1-\mu)V^E$ subject to the occupation arbitrage condition $V^B = V^E$. However, by substituting the occupation arbitrage condition into the weighted sum of utilities, the objective becomes simply to maximize V^E (or V^B) with the condition $V^B = V^E$. This is the same the formulation in (52).

deposit repayment schedules and also by selecting loans and deposits. However, those are chosen from the sets restricted by incentive constraints. The restrictions are the same as reaction functions in the decentralized equilibrium.

Moreover, the occupational arbitrage constraint implies that banker's spread income plus any rents from selling contracts are equated with the income of entrepreneurs. The extra rents to sell contracts are uniquely determined at zero. And, there is no distinction between social and private solutions. Therefore, the social planner's problem and the decentralized equilibrium brings the same allocations.

Note that there is no externality to break the link between the decentralized equilibrium and the social optimum in the model. Because the occupational arbitrage equates the banker's and entrepreneur's utility and everyone is fully employed, there is no externality associated with the number of bankers in Proposition 5. The capital buffer acts like insurance for depositors and so it looks like an externality. However, banks charge the insurance service by collecting spread income, that is, internalizing the externality, if any.

However, the allocation of assets and outputs are not the same as in the CSV-contingent contract regime (Townsend ,1979) or the CSV-costly negotiation regime with zero negotiation cost. And, with additional friction of the simple bankruptcy rule, the welfare is apparently lower than in the CSV-contingent contract regime.[31] This is not because the linear nature of the rule but the one-fits-all rule of splitting the assets. And, under the CSV-speedy bankruptcy regime, the social planner is assumed not to be able to escape from such institutional setup.

B. Bank Bailouts

So far, the government (or the social planner) is assumed to face the same constraints as the private agents. This assumption supports the welfare theorem, Proposition 7.

Now, instead, I assume that the government has an extra power compared to the private agents. This assumption, not surprisingly, creates a room for a government intervention. In particular, the government can make those who defaulted to contribute to the bailout expenditure. The debt contract with limited liability implies that the borrowers and bankers are well insured for a very low realization of aggregate shock while the depositors absorb the whole tail risk. If there is a way to redistribute the borrowers' retained assets to the depositors, the ex ante overall welfare can improve. Given the limited liability laws, one of a few ways is to use the tax system. Note that, for a private agent, it is legally difficult to collect funds from those who defaulted.

A bailout policy is defined as guaranteeing a banker's income in case that a banker would default without the bailout policy in order to enable a banker to repay deposits in full. This description represents actual bailouts (see e.g., Landier and Ueda, 2009). The bank bailout funds are often financed by government bonds, which the government repays over time, for example, by

[31] In the current CSV-speedy bankruptcy regime, if the borrower ask to give back some portion after the banker seize her assets and observe the state, the borrower and the banker would have to go through verification of the state and negotiation of splitting assets with cost τ. If they agree to do this ex ante and can commit, this regime becomes essentially the CSV-contingent contract regime. However, a banker cannot commit and has no incentive to give back a portion of assets to the borrower after seizing them.

consumption tax.[32] To avoid additional distortion in the model, I focus on a system of consumption tax and lump-sum transfers.

The government is still assumed to follow the essentially same information structure of the model. However, it seems natural to assume that the government can know at least the aggregate shocks when it bails out bankers. Still, it is likely that only bankers know idiosyncratic shocks of client firms through costly verification. Otherwise (i.e., under good shock realizations), it cannot know the aggregate shock. In other words, this paper does *not* naively assume that the government can do any redistribution policy with perfect information.

Assumption 7. ***[CSV-Speedy Bankrupty Regime with Bank Bailout]***
In addition to Assumption 3,
(e) [Government Power to Tax] The government can collect tax (e.g., consumption tax) even from those who defaulted.
(f) [Government Information upon Bailout] The government can know (only) the aggregate shocks when bailing out bankers.

The bailout can be designed so that the bankers would not benefit from them directly.

Definition 3. *A "transparent" bailout transfers funds to depositors via bankers without benefiting bankers directly, while an "untransparent" bailout benefits bankers directly.*

Note that the bankers may benefit indirectly through the general equilibrium effect, but not directly. Hence, if looking at the direct effects, a transparent bailout serves as an insurance for depositors at the cost of borrowers. Depositors can have the perfectly constant deposit repayment under this bailout policy for any realizations of the shocks. Everyone needs to pay contingent consumption tax κ per person ex post to finance the bailouts. The consumption changes to C^D_{BO}, C^U_{BO}, andC^B_{BO} for depositors, borrowers, and bankers, respectively, and they are defined as the original consumption C^D, C^U, and C^B, respectively, minus consumption tax κ plus bank bailout transfers.

Under a transparent bailout, when a banker faces default, the government transfers funds to a banker just to repay the deposit in full so that the banker's consumption schedule c^B remains unchanged. The transfer occurs only when the aggregate shock is lower than the banker's default threshold, $A < \theta^D_{BO}$. In this case, per depositor transfer is the difference between the banker's retained asset (27) and what a banker would consume without retained asset (26), correcting for relative population of bankers to depositors. As the consumption tax is assumed not discriminatory to anyone, everyone pays $\kappa(A)$ to the government under this scheme. Then, the government transfers $2\kappa(A)/(1-\mu)$ to a depositor via bankers.

$$\begin{aligned} \frac{2}{1-\mu}\kappa(A) &= \frac{2\mu}{1-\mu}\left((1+\rho^D)S_h - B(A|R^L, L_h, L_j)\right), \quad \text{if} \quad A \in [\underline{A}, \theta^D_{BO}]; \\ \kappa(A) &= 0, \quad \text{otherwise.} \end{aligned} \tag{53}$$

To see the informational restriction (f) in Assumption 7, consider the case in which the government can even access to the idiosyncratic shocks upon bank bailouts.

[32] Other examples include inflation tax on monetary assets or income tax on human capital, though they are not modeled in this paper.

Lemma 12. *If the government could access to the all the information that banks possess when bails them out, and tailor the tax rate conditional on idiosyncratic shocks of defaulted borrowers, which notationally can be written as $\hat{\kappa}(A, \epsilon)$, then, the ex ante designed transparent bailout scheme can bring a Pareto superior allocation than in the economy without such scheme.*

Proof. If a government can tax each defaulted borrower at a rate conditional on realized aggregate and idiosyncratic shocks, then the government can relax a constraint, the simple debt restructuring rule, of the social planner's problem. And, the optimal government tax-transfer should mimic the equilibrium allocation in the CSV-contingent contract regime. *Q.E.D.*

However, the tax-transfer system based on idiosyncratic shocks is ruled out under (f) in Assumption 7, since it is natural to assume so as discussed above. Hence, I focus on a conventional tax policy, which is not discriminatory among people and can be conditional only on the aggregate shocks. Transfers then become conditional on the aggregate shocks only but can be targeted, for example, to depositors in the case of deposit insurance. Under this assumption, the optimality of bailout is less obvious.

Bank bailout, when formulated ex post, it is often the case that bank shareholders need to approve any capital injections (Landier and Ueda, 2009). Hence, in the model, I require the following bank's participation constraint to the bailout scheme:

$$V^B_{Bailout} \geq V^B_{NoBailout}. \tag{54}$$

Obviously, this is also a condition for Pareto superiority of the bank bailout compared with the no-bailout equilibrium.

Lemma 13. *A sudden, unexpected transparent bank bailout is welfare improving from the ex ante point of view.*

Proof. The repayment function R^D and R^L were not reoptimized with this unexpectd transparent bailout. When the ex post bank participation constraint (54) is met with equality, bankers do not gain or lose. The depositors and borrowers would share the cost of bank defaults. Without the bailout scheme, the borrowers would not share the cost. The depositors would be better off but the borrowers would be worse off at the time of bailout.

However, from the ex ante viewpoint, with the bailout policy, the entrepreneurial risk is reduced. That is, the consumption volatility stemming from uncertainty for talent shocks that makes an entrepreneur either a borrower or a depositor. In particular, this risk sharing occurs at the low consumption level with high marginal utility (i.e., $\lim_{c \to 0} u'(c) = \infty$). Hence, better sharing the risk between a depositor and a borrower for a large negative aggregate shock is welfare improving for an entrepreneur, when assessed ex ante using an equal-weight utilitarian social welfare function, i.e.,

$$V^E_{Bailout} > V^E_{NoBailout}. \tag{55}$$

Q.E.D.

However, an unexpected bailout, announced ex post suddenly, is not supported by everyone. In the model, the borrowers are worse off at the time of bailout ex post, although they would agree from the ex ante point of view. Then, it may be better to institutionalize (and commit) bank bailouts ex ante, for example, establishing a resolution fund. On the other hand, many people argue that expectation of bank bailouts would induce distorted behavior by banks and increase the probability of bank defaults. Indeed, if the bailout is ex ante designed and expected, it is the case in the new equilibrium as shown in Proposition 8 below.

Still, the policy should be evaluated by the welfare it brings. Proposition 8 shows that the welfare is improved with transparent bailouts, which makes the economy closer to the CSV-contingent contract regime (i.e., the original Townsend (1979) economy). The institutionalized bailout policy allows more optimal risk taking by entrepreneurs and bankers due to a better risk sharing.[33]

Proposition 8. *Institutionalizing a transparent bank bailout scheme ex ante improves the social welfare, even though the expectation for bank bailouts may cause more bank defaults.*

Proof. The government can adopt the tax-transfer system to mimic the consumption allocation in the CSV-contingent contract regime, except for idiosyncratic shock adjustment for the borrowers. This policy is Pareto superior to the no-bailout policy case because the additional constraints by the simple debt restructuring rules (c-2 in Assumption 3 and c-2' in Assumption 4) are relaxed in the economy.

With better tail-risk sharing, the depositors' demand for bank capital buffer declines. This leads to lower banker population μ with higher bank leverage. *Q.E.D.*

Here, a question may be the choice of tax-transfer system (CSV-speedy bankruptcy regime with baiiout regime) or the contingent-claim market (CSV-contingent contract regime). Again, however, the government intervention is necessary in the case where the contingent-claims cannot be written. And, also ex post efficient bargaining is costly (CSV-costly negotiation regime) so that the simple debt restructuring rule can prevail. In other words, when speedy bankruptcy is called for, it is also better to institutionalize bank bailouts.

C. Deposit Insurance and Double Liability

A deposit insurance can be defined as a protection for depositors' income in case that a banker would default. It is usually financed by taxing the bankers, ex ante. This "taxing bankers ex ante" is the difference from the bailout, which is "taxing everyone ex post."

Consider a case that the depositors will not lose the face value of the deposit, that is, a full coverage deposit insurance.

Claim. *The full coverage deposit insurance with ex ante fees does not improve welfare.*

---

[33] Obstfeld (1994) also shows in a different model setup (i.e., perfect information with Epstein-Zin preference) that a better risk sharing makes people to invest higher-risk-higher-return projects optimally and improves welfare.

Here is the sketch of the proof. The full coverage deposit insurance with ex ante fee is essentially the same as restricting the bankers' offer of deposit contracts to be very safe, close to zero default $\theta^D \approx 0$, associated with high spread π to pay the insurance fee. This restriction on the bankers' offers of deposit contracts is an obvious distortion to the economy and the associated social planner's problem. Therefore, such a scheme cannot improve the social welfare.

Note that a partial, but substantial, coverage deposit insurance—which covers the full amount down to the government-set threshold θ_G^D—with ex ante fees would create the similar distortion as the full coverage version. Essentially, the bankers are constrained to choose the deposit repayment schedule and thus the welfare decreases.

The unlimited liability or "double liability" of bankers as in the pre-Great Depression in the U.S. would not work well either.[34] Under the double liability regime, in essence, bankers always had to pay deposits in full, otherwise they were jailed (i.e., their consumption level is almost zero). In this regime, bankers were the ones that assumed all the tail risks. This would not be the optimal risk sharing among different types of agents, and thus is not socially optimal. The key friction is not the limited liability itself, but rather the limited liability being non-contingent to the aggregate shocks. A transparent bailout scheme can fine-tune the limited liability, making it contingent to the aggregate shocks.

D. Prudential Regulations

Following the discussion on the optimal bank capital level in Section F, a natural policy implication can be made as a corollary to Proposition 4.

Corollary 4. *Introduction of the capital adequacy ratio regulation as defined in (56) below is either redundant or welfare decreasing in an economy without a possibility of bailouts (i.e., without Assumption 7).*

$$q \equiv \frac{k_0^B}{D} \geq \hat{q}. \tag{56}$$

Proof. Proposition 4 says the optimal capital ratio is positive in an economy with debt contracts. By Proposition 5, bankers hold strictly positive capital by themselves in an equilibrium which is the constrained social optimum. Therefore, the capital adequacy ratio requirement is either binding (i.e., welfare decreasing) or not binding (i.e., redundant). *Q.E.D.*

Note that there are two sources of inefficiency. First, with the capital ratio requirement, there will be more bankers with less customer base and a higher spread to compensate less customer base. Second, with a sizable positive spread and resulting wedge between the loan and deposit rates, the marginal product of capital would be less equated between the borrowers and depositors. When $\tau = 0$ (complete market case), with the capital adequacy ratio requirement, the economy cannot reach the limit that mimics the first best, Walrasian equilibrium.

Corollary 5. *When the capital adequacy ratio requirement is introduced to the economy with the bailout policy with tax system $\kappa(A)$ defined in (53), there will be more bankers with a higher capital*

[34] See a history and theory paper by Kane and Wilson (1998) and an empirical work by Grossman (2001).

ratio and a lower spread but also with a higher probability of bank bailouts. The overall welfare becomes worse.

Proof. The bailout scheme alone takes care of tail risks. With a binding capital adequacy ratio requirement, banker's leverage becomes lower than the optimal level. Then, a banker needs a higher spread to satisfy the same default threshold and to keep the same utility. The banker's deposit demand $s^d(\theta^D, \pi)$ shifts upward and flattens as a result. Even if there were no changes in the loan market, the deposit market equilibrium would change so that deposit rate ρ^D becomes lower with uncertain movement on threshold θ^D. This leads to a lower utility for depositors. *Q.E.D.*

E. Bad Bailouts and Income Shifting

So far, I have focused on the optimal bailout scheme in a realistic institutional setup and find that a bank bailout, if designed well, is welfare improving. And, there is no role for other policies such as prudential regulations. However, in the real world, there can be a bad bailout. In particular, the literature (and newspaper articles) often discuss about corruption and other problems like moral hazard.[35] In other words, ex post "looting" opportunity may also be available for banks if banks can seize a part of bailout funds. In this case, bailouts are not transparent as defined in Definition 3, but include some hidden subsidies to banks. I call this *untransparent bank bailouts*. Some of them may be necessary to persuade bank owners to agree on bailouts (e.g., Landier and Ueda, 2009) but others may well be a result of political influence by bank lobby (e.g., Igan, Mishra, and Tressel, 2011). Here, I do not attempt to theorize the underlying mechanism of such practices in this paper but characterize the implications of this bad bailout scheme.

Under an *untransparent* bailout policy, the banker's consumption increases by extra transfer $\underline{\kappa}$ for low realizations of aggregate shock, $A < \hat{\theta}^D_{BO}$, where the bank default threshold $\hat{\theta}^D_{BO}$ could be higher than θ^D_{BO} under a transparent bailout scheme, because a bank has a little more incentive to declare default to receive extra transfer $\underline{\kappa}$. The overall transfer is simply shifted upwards by this extra transfer:

$$\begin{aligned} \frac{2}{1-\mu}\kappa(A) &= \frac{2\mu}{1-\mu}\left((1+\rho^D)S_h - B(A|R^L, L_h, L_j) + \underline{\kappa}\right), \quad \text{if} \quad A \in [\underline{A}, \hat{\theta}^D_{BO}]; \\ \kappa(A) &= \underline{\kappa}, \quad \text{otherwise.} \end{aligned} \tag{57}$$

Since bankers are enriched by bailouts, bailout expectations will create distorted incentives for people to become bankers rather than productive entrepreneurs. As a result, there will be too many

---

[35] Distortions in the presence of the government protection in the financial system has been discussed mostly in a partial equilibrium framework. For example, the risk shifting problem induced by deposit insurance requires prudential regulations such as a capital adequacy ratio requirement in Kareken and Wallace (1978), Keeley (1991), and Allen and Gale (2007). The moral hazard problem from expected bailouts requires prudential regulations in Chari and Kehoe (2009) or tax in Kocherlakota (2010) although Chari and Kehoe (2009) admit the bailout of firms via banks is ex post efficient to avoid assumed fixed costs associated with bankruptcy. In a general equilibrium framework, Van den Heuvel (2008) argues that the capital adequacy ratio requirement is costly as it limits the liquidity available in the general equilibrium. Related issue is the effect of competition policy as regulations such as capital adequacy ratio requirement reduces competition. Some argue that risk taking becomes too excessive under freer competition (Allen and Gale, 2000) because monopolistic rents limit the banks' risk taking behavior. The others argue the opposite (Boyd and De Nicolo, 2005) because bank's higher monopolistic rents implies firms' lower rents that lead to higher risk taking at the firm level.

bankers and too little production. Lower production implies lower entrepreneurs' utility, and so is bankers' utility through occupational arbitrage in a general equilibrium. I call this *income shifting* problem.[36]

Almost tautologically, this problem requires a policy to limit bank profits so as not to attract too many people to become bankers. Introducing the capital adequacy ratio can mitigate a unnecessarily high incentive to become a banker by lowering bankers' utility. Introducing a bank levy to lower the (present value of) transfer works as well. With these regulations to countervail bad transfers $\underline{\kappa}$, an untransparent bank bailout could still become an optimal response to tail-risk events in the presence of a simple debt restructuring rule.

VI. CONCLUSION

I develop a simple macro model with realistic financial frictions in bank lending, namely, costly state verification and limited liability with simple and speedy bankruptcy procedure. These frictions prohibit perfect income risk sharing among bankers, borrowers, and depositors. Moreover, I assume endogenous bank size—a banker is an occupation under a strict assumption of not sharing income (i.e., ownership). Ex ante, banker's income is equated in expectation with entrepreneur's in expectation. Entrepreneurs are further sorted to either borrowers or depositors depending on the idea shocks they draw. Overall outputs are affected by financial frictions and possible policy distortions. Inefficiency can appear in the labor allocation (i.e., the occupational choice) and the capital allocation (i.e., deposits and loans), which are affected by the spreads, set endogenously by bankers.

I show that the optimal loan and deposit contracts take a form of a standard debt contract, following costly state verification literature. The optimal bank capital is shown to be positive to provide a buffer to depositors and bankers themselves. And, the banking sector is sizable. However, when a large negative shock hits, both borrowers and bankers would walk away with retained assets because of limited liability protection. The depositors would assume all the tail risk. This *tail-risk dumping* problem creates too risky consumption profile ex post and thus make the occupational choice too risky ex ante. This is the new perspective to the existing literature on macro models with financial frictions.

A government can play some role in the economy for the tail-risk dumping. Once the government is allowed to tax on consumption, it can de facto relax the limited liability constraint and the simple asset split rule. The government can then make trasfers to be contingent on the aggregate shocks. This transparent bank bailout can mitigate the *tail-risk dumping* problem and improve the social welfare. The deposit insurance, if funded ex post by tax, can mimic such a transparent bailout. In other words, however, a bailout is welfare improving only when banks and borrowers are too protected by limited liability. As such, *bail-in* of defaulters is called for in case of a tail event. Or, if negotiation costs is negligible, the simple rule should be abandoned because the ex post split of assets are done optimally through Nash Bargaining. However, in reality, negotiation is often costly, which many authors suggests a source of the debt overhang costs. Therefore, in the presence of sizable verification and negotiation costs, the simple and speedy bankruptcy procedure is preferred to the costly negotiation, and in this case the transparent bank bailout is welfare improving.

---

[36]This income shifting problem in a general equilibrium setup has not been much, if any, referred in the literature so far.

In summary, this paper provides a more solid microfoundation for the macroeconomic models with financial frictions by looking at the incomplete consumption sharing among different types of households as the logical consequence of financial frictions. With this new framework, the financial sector policies—bank bailouts, deposit insurance, and regulations—can be examined as redistribution policies, which could be welfare improving.

References

Akerlof, George A., and Paul M. Romer, 1993, “Looting: The Economic Underworld of Bankruptcy for Profit,” *Brooking Papers on Economic Activity*, Vol. 2, pp. 1–73.

Allen, Franklin, and Douglas Gale, 2000, “Financial Contagion,” *Journal of Political Economy*, Vol. 108, pp. 1–33.

______, and ______, 2007, *Understanding Financial Crises* (Oxford: Oxford University Press).

Ashcraft, Adam, 2005, “Are banks Really Special? New Evidence from the FDIC-Induced Failure of Healthy Banks,” Vol. 95, No. 5, pp. 1713–32.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist, “The Financial Accelerator in a Quantitative Business Cycle Framework,” 1999, in John B. Taylor, and Michael Woodford, eds., *Handbook of Macroeconomics*, Vol. 1C (Amsterdam: Elsevier), chapter 21, pp. 1341–1393.

Bolton, Patrick, Tano Santos, and Jose A. Scheinkman, 2011, “Creme Skimming in Financial Markets,” *NBER Working Paper*, No. 16804.

Boyd, John H., and Gianni De Nicolo, 2005, “The Theory of Bank Risk Taking and Competition Revisited,” *Journal of Finance*, Vol. 60, No. 3 (June), pp. 1329–1343.

Calomiris, Charles W., and Gary Gorton, “The Origins of Banking Panics: Models, Facts, and Bank Regulation,” 1991, in R. Glenn Hubbard, ed., *Financial Markets and Financial Crises*, (University of Chicago Press), chapter 4, pp. 109–174.

Chari, V. V., and Patrick J. Kehoe, 2013, “Baiouts, Time Inconsistency, and Optimal Regulation,” *Federal Reserve Bank of Minneapolis Research Department Staff Report 481.*

Chrsitiano, Lawrence, Roberto Motto, and Massimo Rostagno, 2010, “Financial Factors in Economic Fluctuations,” *ECB Working Paper*, No. 1192.

Demirgüç-Kunt, Asli, and Enrica Detrageache, 2002, “Does Deposit Insurance Increase Banking System Stability? An Empirical Investigation,” *Journal of Monetary Economics*, Vol. 49, pp. 1373-1406.

Diamond, Douglas W., and Philip H. Dibvig, 1983, “Bank Runs, Deposit Insurance, and Liquidity,” *Journal of Political Economy*, Vol. 91, No. 3, pp. 401–419.

Fulghieri, Paolo, and Eitan Goldman, “The Design of Debt Contracts,” 2008, in Anjan V. Thakor, and Arnoud W.A. Boot, eds., *Handbook of Corporate Finance, Vol 3: Financial Intermediation and Banking*, (North-Holland).

Gale, Douglas, and Martin F. Hellwig, 1985, “Incentive-Compatible Debt Contracts: The One-Period Problem,” *Review of Economic Studies*, Vol. 52, No. 4 (October), pp. 647–663.

Green, Edward J., 2010, “Bailouts,” *Federal Reserve Bank of Richmond Economic Quarterly*, Vol. 96, No. 1.

Grossman, Richard S., 2001, “Double Liability and Bank Risk Taking,” *Journal of Money, Credit and Banking*, Vol. 33, No. 2, pp. 143–159.

He, Zhiguo, and Wei Xiong, 2012, "Dynamic Debt Runs," *Review of Financial Studies*, Vol. 25, No. 6, pp. 1799–1843.

Hoshi, Takeo, and Anil K. Kashyap, 2010, "Will the U.S. Bank Recapitalization Succeed? Eight LLessons from Japan," *Journal of Financial Economics*, Vol. 97, pp. 398–417.

Igan, Deniz, Prachi Mishra, and Thierry Tressel, "A Fistful of Dollars: Lobbying and the Financial Crisis," 2011, in Daron Acemoglu, and Michael Woodford, eds., *NBER Macroeconomic Annual*, (MIT Press).

International Monetary Fund, "Assessing Policies to Revive Credit Markets," 2013, in "Global Financial Stability Report," October, chapter 2.

Kane, Edward J., and Berry K. Wilson, 1998, "A Contracting-Theory Interpretation of the Origins of Federal Deposit Insurance," *Journal of Money, Credit and Banking*, Vol. 30, No. 3, pp. 573–595.

Kareken, John H., and Neil Wallace, 1978, "Deposit Insurance and Bank Regulation: A Partial-Equilibrium Exposition," *Journal of Business*, Vol. 51, No. 3, pp. 413–438.

Keeley, Michael C., 1990, "Deposit Insurance, Risk, and Market Power in Banking," *American Economic Review*, Vol. 80, No. 5 (December), pp. 1183–1200.

Kiyotaki, Nobuhiro, and John Moore, 1997, "Credit Cycles," *Journal of Political Economy*, Vol. 105, No. 2, pp. 211–248.

Kocherlakota, Narayana, 2010, "Taxing Risk and the Optimal Regulation of FInancial Institutions," *FRB Minneapolis Economic Policy Paper*, Vol. 10, No. 3.

Landier, Augustin, and Kenichi Ueda, 2009, "The Economics of Bank Restructuring: Understanding the Options," *IMF Staff Position Note 09/12*.

Peek, Joe, and Erick S. Rosengren, 2000, "Collateral Damage: Effects of the Japanese Bank Crisis on Real Activity in the United States," *American Economic Review*, Vol. 90, No. 1, pp. 30–45.

Philippon, Thomas, 2008, "The Evolution of the US Financial Industry from 1860 to 2007: Theory and Evidence." NYU: mimeo.

———, and Philipp Schnabl, 2013, "Efficient Recapitalization," *Journal of Finance*, Vol. 68, No. 1, pp. 1–42.

Reinhart, Carmen M., and Kenneth S. Rogoff, 2009, *This Time is Different* (Princeton: Princeton University Press).

Townsend, Robert M., 1978, "Intermediation with Costly Bilateral Exchange," *Review of Economic Studies*, Vol. 45, No. 3 (October), pp. 417–425.

———, 1979, "Optimal Contracts and Competitive Markets with Costly State Verification," *Journal of Economic Theory*, Vol. 21, No. 2 (October), pp. 265–293.

Ueda, Kenichi, 2013, "Banks as Coordinators of Economic Growth and Stability: Microfoundation for Macroeconomy with Externality," *Journal of Economic Theory*, Vol. 148.

Van den Heuvel, Skander J., 2008, "The Welfare Cost of Bank Capital Requirements," *Journal of Monetary Economics*, Vol. 55 (December), pp. 298–320.

Figure 1. Capital Exchange with Complete Contracts

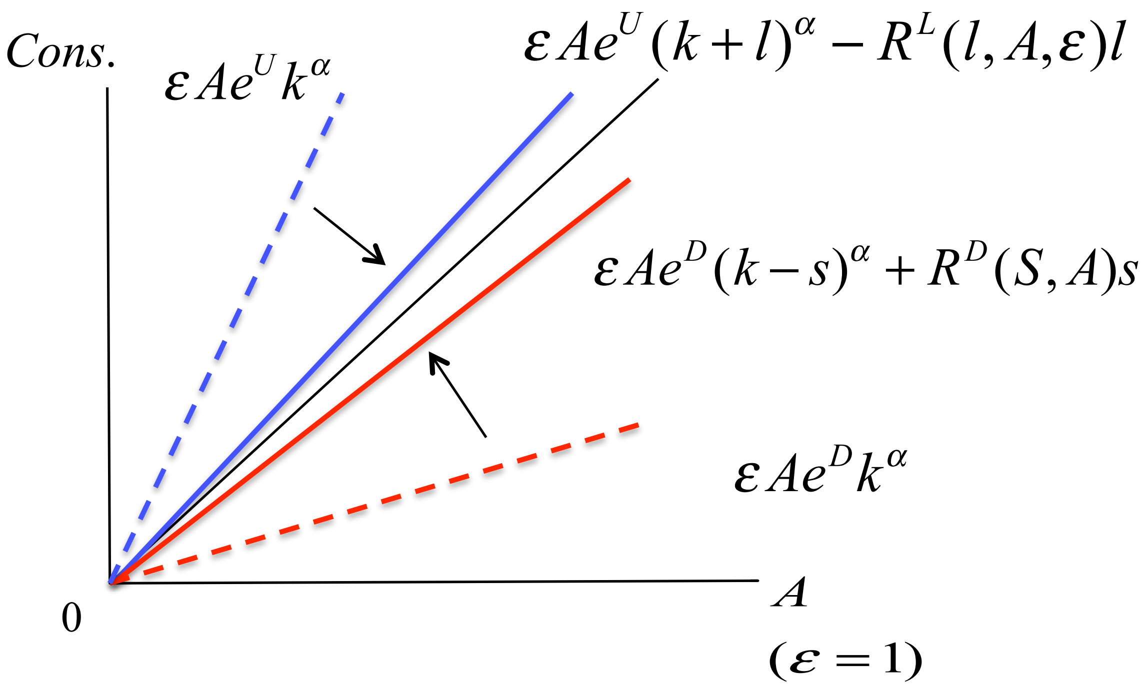


Figure 2. Capital Exchange with Simple Debt Contracts

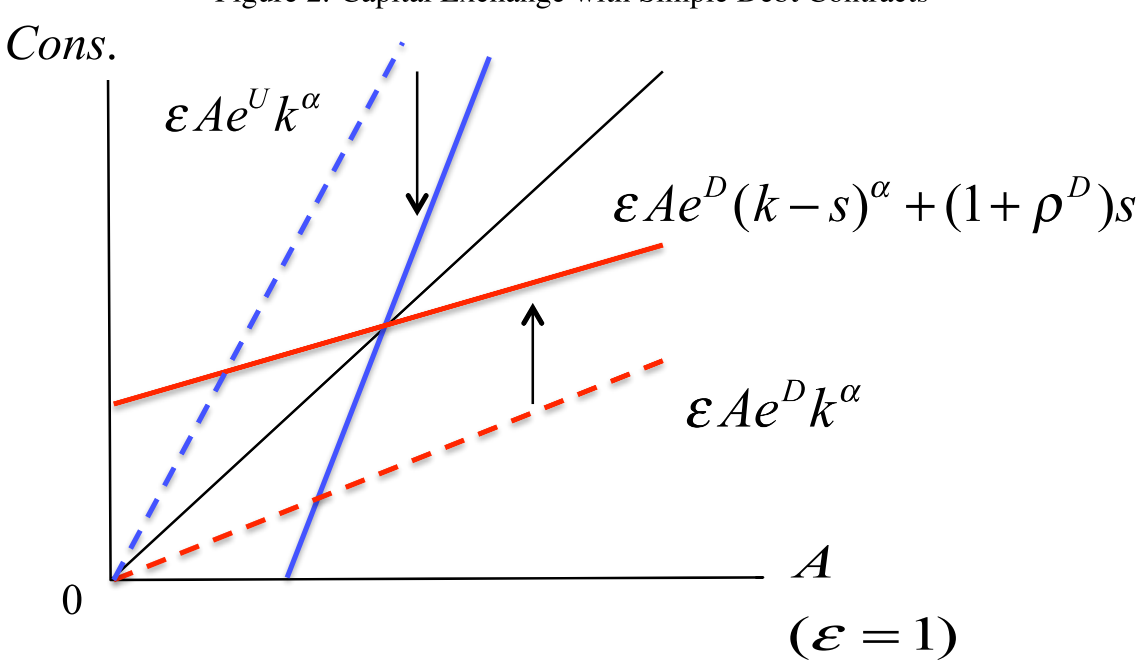


Figure 3. Debt Contracts with Limited Liability

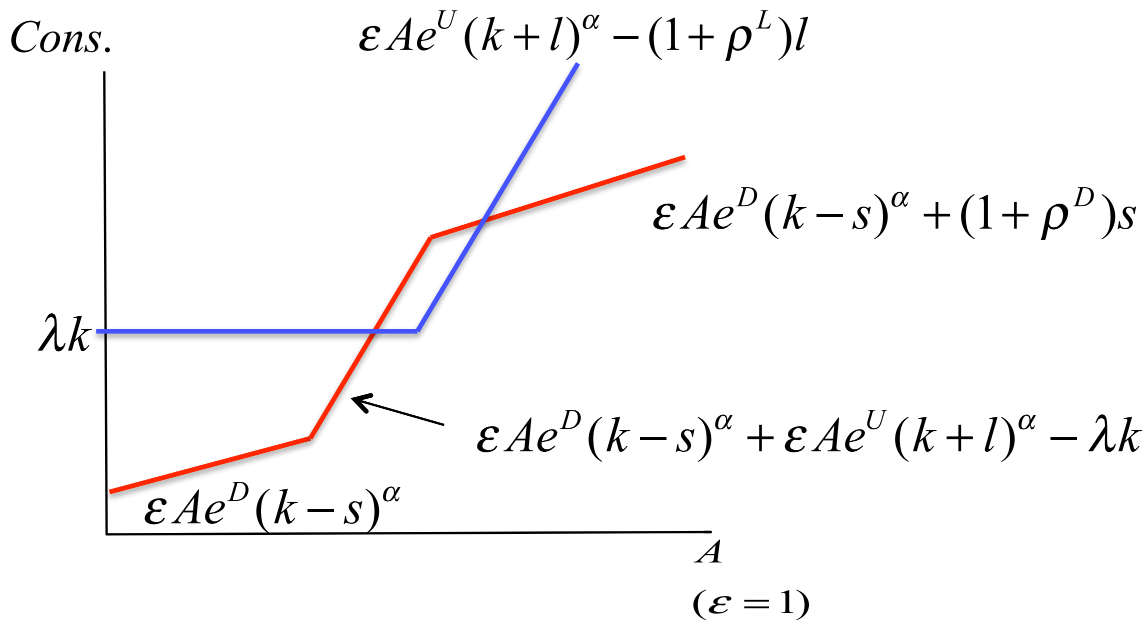


Figure 4. Debt Contracts and Banks

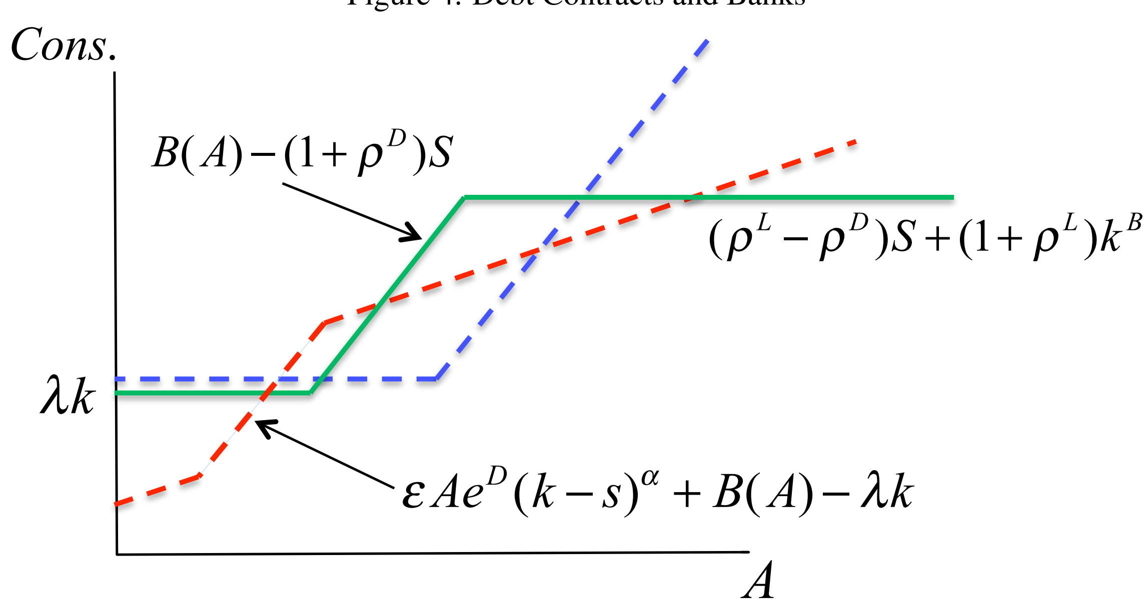


Figure 5. Loan Market Partial Equilibrium

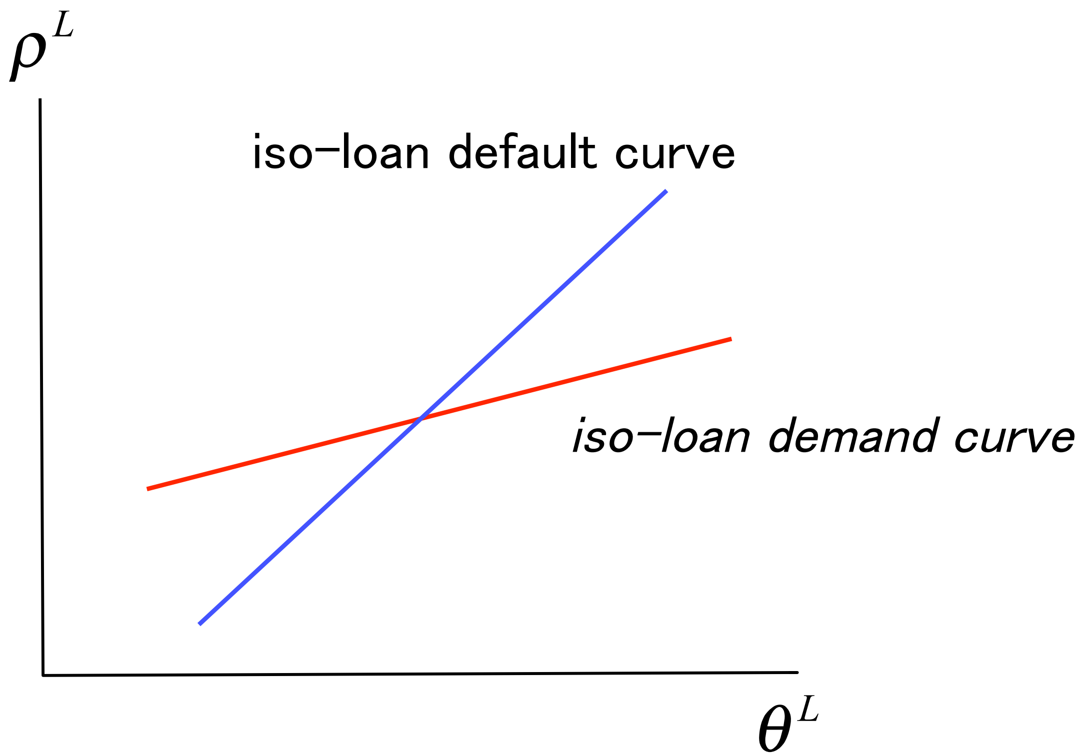


Figure 6. Deposit Market Partial Equilibrium

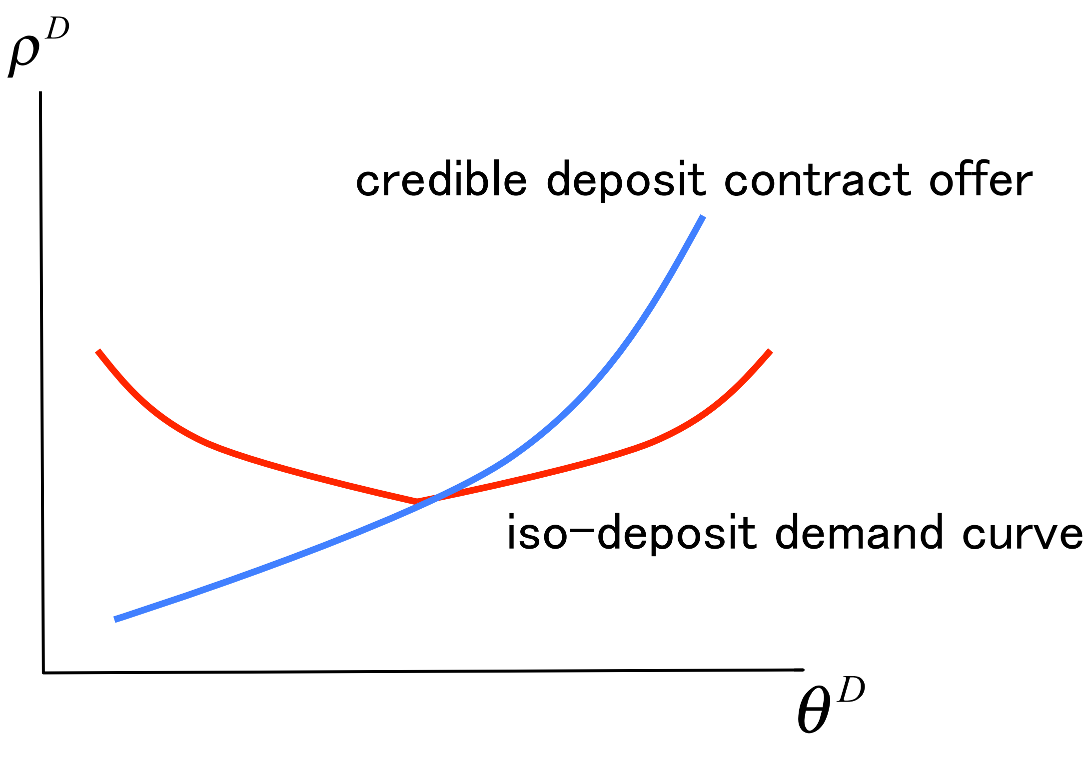


Figure 7. Unique Bank Population in General Equilibrium

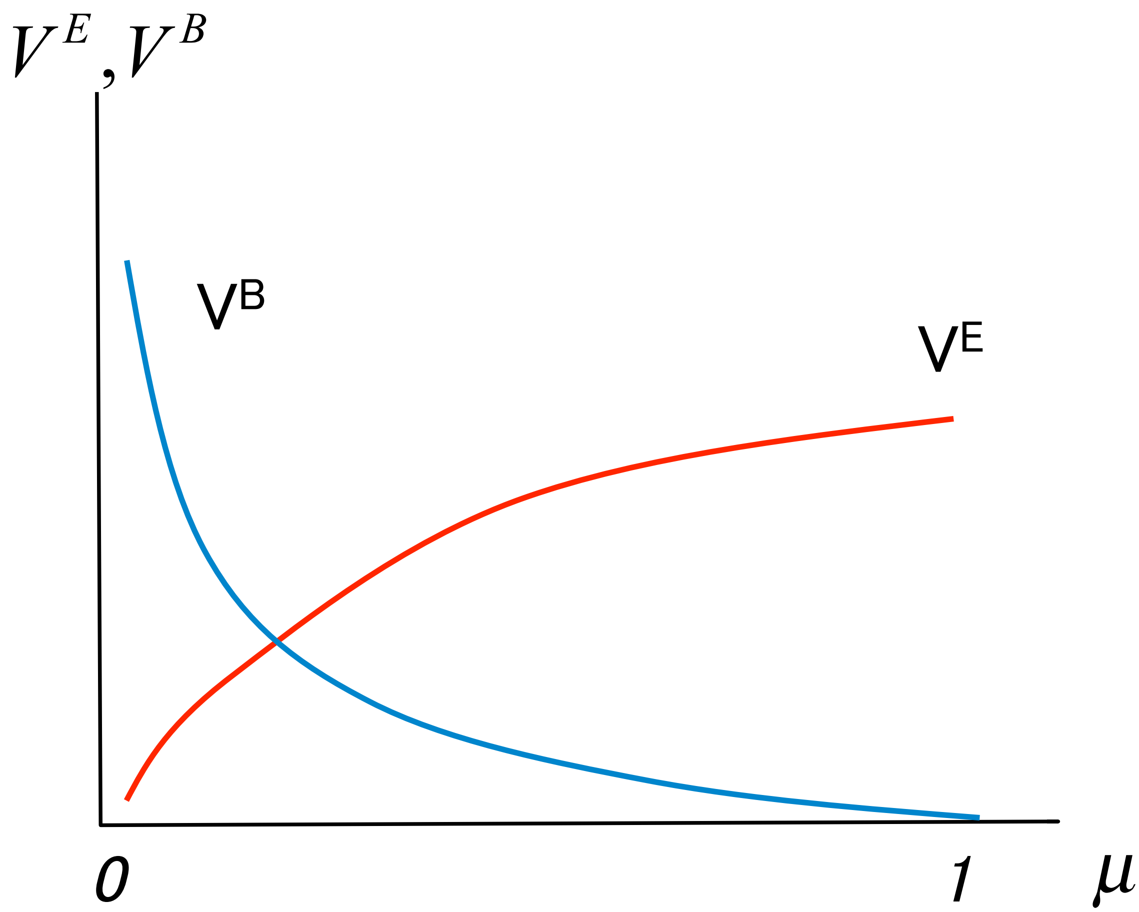


Figure 8. Deposit Return per Depositor

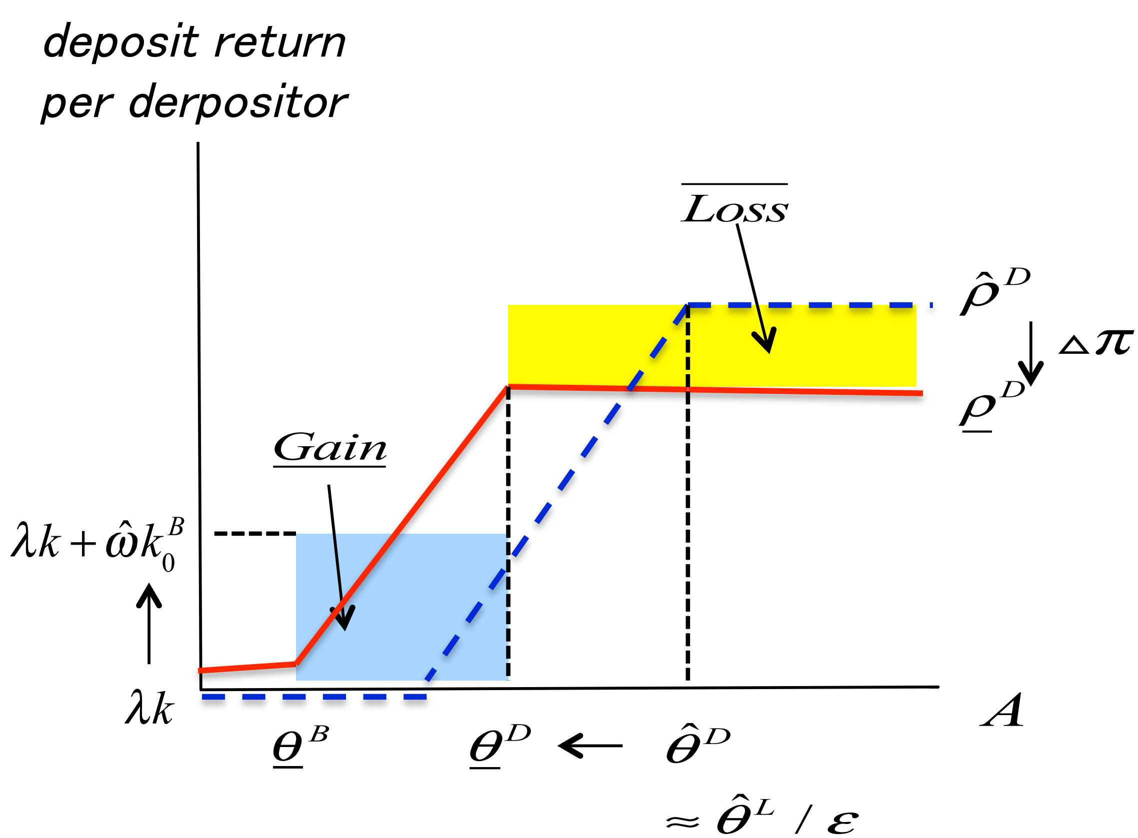
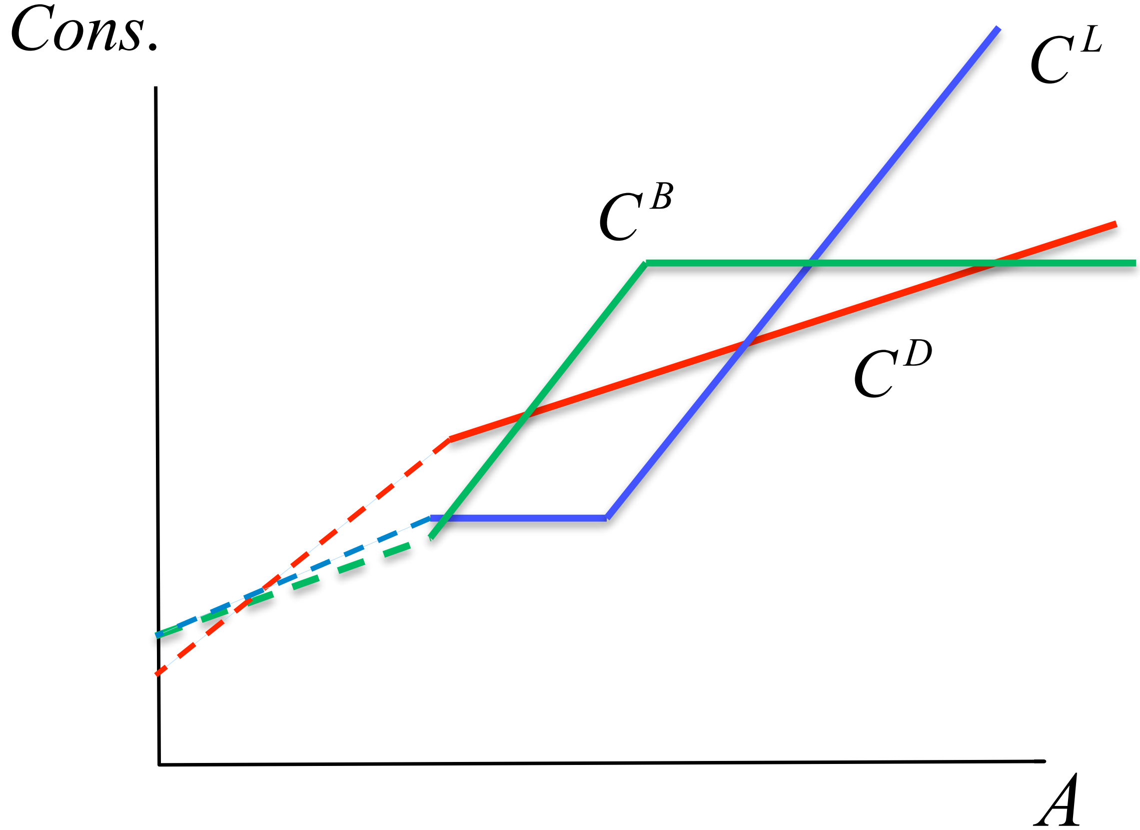


Figure 9. Optimal Tax-Transfer System



Appendix I. Proofs

A. Proof for Lemma 5

Proof. The derivative of the iso-loan demand function with respect to θ^L is expressed as

$$\frac{\partial \chi(\theta^L, \rho^L)}{\partial \theta^L} = \left((1+\rho^L) - \alpha \theta^L e^U (k_0 + l_j)^{\alpha-1}\right) u'(\underline{c}^U) m(\theta^L) > 0, \tag{A1}$$

where $\underline{c}^U$ is evaluated at $\eta = \theta^L$ and m is the probability density function. This is positive because the first order condition (32) implies that the loan rate $(1+\rho^L)$ is on average equal to the marginal product of capital conditional on not defaulting. Hence, the loan rate $(1+\rho^L)$ must be higher than the marginal product of capital with the lowest non-default shock θ^L. Note that the second term in the parenthesis is the marginal product of capital with shock θ^L.

The derivative with respect to ρ^L is

$$\begin{aligned}\frac{\partial \chi(\theta^L, \rho^L)}{\partial \rho^L} &= \int_{\theta^L}^{\overline{\epsilon A}} \left(-u'(c^U) - (\alpha \eta_j e^U (k_0 + l_j)^{\alpha-1} - (1+\rho^L)) l u''(c^U)\right) dM \\ &= -\int_{\theta^L}^{\overline{\epsilon A}} u'(c^U) dM + \sigma l_j^* \int_{\theta^L}^{\overline{\epsilon A}} (\alpha \eta_j e^U (k_0 + l_j)^{\alpha-1} - (1+\rho^L)) \frac{u'(c^U)}{c^U} dM \\ &< 0.\end{aligned} \tag{A2}$$

Note that inside the second integral in the penultimate line has a "weight" of u'/c^U, which has higher weights for the lower realization of shocks and lower weights for the higher realization of shocks compared to the weight u' in the first order condition (33). Because the second integral is different only in this "weight" from the borrower's first order condition (33), the second integral must be negative.

In summary,

$$\frac{d\rho^L}{d\theta^L} = -\frac{\partial \chi / \partial \theta^L}{\partial \chi / \partial \rho^L} > 0. \tag{A3}$$

Moreover,

$$\begin{aligned}\frac{\partial \chi(\theta^L, \rho^L)}{\partial l_j^*} &= \int_{\theta^L}^{\overline{\epsilon A}} (\alpha - 1) \alpha \eta e^U (k_0 + l_j^*)^{\alpha-2} - (1+\rho^L) u'(c^L) dM(\eta) \\ &+ \int_{\theta^L}^{\overline{\epsilon A}} \left(\alpha \eta e^U (k_0 + l_j^*)^{\alpha-1} - (1+\rho^L)\right)^2 u''(c^L) dM(\eta) \\ &< 0.\end{aligned} \tag{A4}$$

Hence by the implicit function theorem, with a larger loan, the loan rate increases,

$$\frac{d\rho^L}{dl_j^*} > 0, \tag{A5}$$

that is, the iso-loan demand curve shifts up on the θ^L-ρ^L plane. At the same time, with a larger loan, the default threshold decreases,

$$\frac{d\theta^L}{dl_j^*} < 0, \tag{A6}$$

that is the iso-loan demand curve shifts left on the θ^L-ρ^L plane. *Q.E.D.*

B. Proof for Proposition 1

Proof. On the θ^L-ρ^L plane, the iso-loan default curve is increasing (Lemma 4) and the iso-loan default curve is also increasing (Lemma 5).

As $\theta^L \to 0$, the iso-loan default curve converges to the intercept: $-1 - \lambda k_0/l_j$, which is lower than -1. For the iso-loan demand curve, the first order condition (32) implies$(1 + \rho^L) > 0$ no matter what the level of θ^L. Hence, the intercept at $\theta^L = 0$ is bigger than -1 on the θ^L-ρ^L plane.

Now, I show that the slope of the iso-loan default curve is always steeper than that of the iso-loan demand curve. This means that the iso-loan default curve crosses the iso-loan demand curve from below only once on the θ^L-ρ^L plane. Therefore, for each loan amount l_j, the pair (θ^L, ρ^L) is determined uniquely in an equilibrium of the loan market.

First, I simplify the term inside the second integral of (A2), as follows

$$\begin{aligned}
&\alpha\eta_j e^U (k_0 + l)^{\alpha-1} - (1 + \rho^L) \\
<&\eta_j e^U (k_0 + l_j)^{\alpha-1} - (1 + \rho^L) \\
<&\eta_j e^U (k_0 + l_j)^{\alpha-1} - (1 + \rho^L)\frac{l_j}{k_0 + l_j} \\
=&\frac{c^U}{k_0 + l_j} \quad \text{for} \quad \eta_j = \epsilon_j A \geq \theta^L.
\end{aligned} \tag{A7}$$

This implies

$$\begin{aligned}
-\frac{\partial\chi(\theta^L, \rho^L)}{\partial\rho^L} &> \int_{\theta^L}^{\overline{\epsilon A}} u'(c^L) dM - \sigma l_j \int_{\theta^L}^{\overline{\epsilon A}} \frac{c^L}{k_0 + l_j} \frac{u'(c^L)}{c^L} dM \\
&= \left(1 - \frac{\sigma l_j}{k_0 + l_j}\right) \int_{\theta^L}^{\overline{\epsilon A}} u'(c^L) dM.
\end{aligned} \tag{A8}$$

Hence, the slope of iso-loan demand curve is

$$\begin{aligned}\frac{d\rho^L}{d\theta^L} &= -\frac{\partial\chi/\partial\theta^L}{\partial\chi/\partial\rho^L} \\ &< \frac{\left((1+\rho^L) - \alpha\theta^L e^U (k_0+l_j)^{\alpha-1}\right) u'(\underline{c}^L) m(\theta^L)}{\left(1 - \frac{\sigma l_j}{k_0+l_j}\right)\int_{\theta^L}^{\overline{\epsilon A}} u'(c^L) dM} \\ &< \frac{\left((1+\rho^L) - \alpha\theta^L e^U (k_0+l_j)^{\alpha-1}\right)}{1 - \frac{\sigma l_j}{k_0+l_j}}.\end{aligned} \tag{A9}$$

Note that the last line uses the apparent relation

$$u'(\underline{c}^L) m(\theta^L) < \int_{\theta^L}^{\overline{\epsilon A}} u'(c^L) dM. \tag{A10}$$

Hence, the slope of the iso-loan default curve is always steeper than that of the iso-loan demand curve if

$$\frac{\left((1+\rho^L) - \alpha\theta^L e^U (k_0+l_j)^{\alpha-1}\right)}{1 - \frac{\sigma l_j}{k_0+l_j}} < \frac{e^U (k_0+l_j)^{\alpha}}{l_j}, \tag{A11}$$

or equivalently,

$$\frac{\left((1+\rho^L) - \alpha\theta^L e^U (k_0+l_j)^{\alpha-1}\right)}{k_0 + l_j - \sigma l_j} < \frac{e^U (k_0+l_j)^{\alpha-1}}{l_j}. \tag{A12}$$

Here, I first show that the numerator of the left-hand side is smaller than that of the right-hand side. Note that the first order condition (32) can be expressed as

$$1+\rho^L = \frac{\int_{\theta^L}^{\overline{\epsilon A}} \left(\alpha\eta_j e^U (k_0+l_j)^{\alpha-1}\right) u'(c^L) dM(\eta)}{\int_{\theta^L}^{\overline{\epsilon A}} u'(c^L) dM(\eta)}. \tag{A13}$$

Because the marginal product of capital is increasing in productivity shock η while the marginal utility is decreasing in the same shock, covariance of these two terms are negative and hence[37]

$$\begin{aligned}1+\rho^L &< \frac{\left\{\int_{\theta^L}^{\overline{\epsilon A}} \alpha\eta_j e^U (k_0+l_j)^{\alpha-1} dM(\eta)\right\}\left\{\int_{\theta^L}^{\overline{\epsilon A}} u'(c^L) dM(\eta)\right\}}{\int_{\theta^L}^{\overline{\epsilon A}} u'(c^L) dM(\eta)} \\ &= \int_{\theta^L}^{\overline{\epsilon A}} \alpha\eta_j e^U (k_0+l_j)^{\alpha-1} dM(\eta).\end{aligned} \tag{A14}$$

Hence, to prove the numerator of the left hand side of (A12) is smaller than that of the right hand

[37]Recall that, for any two random variables ξ and ν, $E[\xi,\nu] = E[\xi]E[\nu] + cov(\xi,\nu)$.

side, it suffices to show

$$\int_{\theta^L}^{\overline{\epsilon A}} \alpha\eta_j e^U (k_0 + l_j)^{\alpha-1} dM(\eta) - \alpha\theta^L e^U (k_0 + l_j)^{\alpha-1} < e^U (k_0 + l_j)^{\alpha-1}, \tag{A15}$$

or equivalently,

$$\alpha \left(\int_{\theta^L}^{\overline{\epsilon A}} \eta_j dM(\eta) - \theta^L \right) = \alpha \left(1 - M(\theta^L) - \theta^L\right) < 1. \tag{A16}$$

But, because the inside of the parenthesis is less than 1, this is satisfied for any $\alpha \in [0, 1]$.

Let $\zeta \equiv \alpha(1 - M(\theta^L) - \theta^L)$. Then, the condition (A12) becomes,

$$\begin{aligned} k_0 + l_j - \sigma l_j &\geq \zeta l_j \\ k_0 &\geq (\sigma - (1 - \zeta))l_j \\ \frac{k_0}{l_j} &\geq \sigma - (1 - \zeta) \equiv \tilde{\sigma}. \end{aligned} \tag{A17}$$

Consider the largest capital exchange (l^{FB}, s^{FB}), which occurs under the first best allocation. In this case, the marginal product of capital equates in each state, i.e.,

$$\alpha\eta_j e^U (k_0 + l_j^{FB})^{\alpha-1} = \alpha\eta_j e^D (k_0 - s_i^{FB})^{\alpha-1}. \tag{A18}$$

This can be simplified to

$$\frac{k_0 + l_j^{FB}}{k_0 - s_i^{FB}} = \frac{e^U}{e^D}^{\frac{1}{1-\alpha}} = Z, \tag{A19}$$

where Z is defined in Assumption 5. Using $l_j^{FB} = s_i^{FB} + 2\mu k_0^B/(1 - \mu)$, this can be expressed as

$$(Z + 1)l_j^{FB} = (Z - 1)k_0 + \frac{2\mu}{1 - \mu} Z k_0^B, \tag{A20}$$

and then, because $k_0^B = k_0$,

$$\frac{k_0}{l_j^{FB}} = \frac{Z + 1}{\frac{1+\mu}{1-\mu} Z - 1}. \tag{A21}$$

By construction $l_j \leq l_j^{FB}$, to prove (A17), it is sufficient to show $k_0/l_j^{FB} \geq \tilde{\sigma}$, that is,

$$\frac{Z + 1}{\frac{1+\mu}{1-\mu} Z - 1} \geq \tilde{\sigma} = \sigma - (1 - \zeta). \tag{A22}$$

This is simplified to

$$\sigma + \zeta \leq \frac{2Z}{(Z + 1)\mu + Z - 1}. \tag{A23}$$

Because $\zeta < \alpha$, it suffices to show

$$\sigma + \alpha \leq \frac{2Z}{(Z+1)\mu + Z - 1}. \tag{A24}$$

This is true under Assumption 5. In other words, the slope of the iso-loan default curve is always steeper than that of the iso-loan demand curve under Assumption 5. *Q.E.D.*

C. Proof for Lemma 6

Proof. Multiply both sides of (38) by s and take a derivative of the right hand side with respect to θ^D:

$$\begin{aligned}
&(1+\rho^L)L_h \frac{\theta^L}{\theta^{D2}} h - \left(\frac{\theta^L}{\theta^D} \theta^D e^U (k_0 + L_j)^\alpha - \lambda k_0 - \tau \right) \frac{L_h}{L_j} \frac{\theta^L}{\theta^{D2}} h + \int_{\underline{\epsilon}}^{\frac{\theta^L}{\theta^D}} \epsilon_j e^U (k_0 + L_j)^\alpha \frac{L_h}{L_j} dH(\epsilon) \\
=&\left\{ (1+\rho^L)L_h - \left(\theta^L e^U (k_0 + L_j)^\alpha - \lambda k_0 - \tau \right) \frac{L_h}{L_j} \right\} \frac{\theta^L}{\theta^{D2}} h + \int_{\underline{\epsilon}}^{\frac{\theta^L}{\theta^D}} \epsilon_j e^U (k_0 + L_j)^\alpha \frac{L_h}{L_j} dH(\epsilon) \\
=&\int_{\underline{\epsilon}}^{\frac{\theta^L}{\theta^D}} \epsilon_j e^U (k_0 + L_j)^\alpha \frac{L_h}{L_j} dH(\epsilon) \\
>&0,
\end{aligned} \tag{A25}$$

where pdf h is evaluated at θ^L/θ^D. This is positive and thus the credible deposit contract offer curve is strictly increasing. Note that the last line follows the same way as in (21). *Q.E.D.*

D. Proof for Proposition 2

Proof. The first order condition (40) essentially is the optimal portfolio problem of allocating capital so as to equate the internal marginal product from own business (*MPK*) to the outside opportunity, which is the deposit to banks, weighted by the marginal utility, *u'*, that is,

$$0 = -\int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} (MPK) u' dH(\epsilon) dG(A) + \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} R^D(S_h, A) u' dH(\epsilon) dG(A) \tag{A26}$$

To secure the uniqueness, I show below that the second order condition with respect to deposit s is negative.

$$\begin{aligned}
\frac{\partial \Phi}{\partial s_i} &- \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} u' \frac{\partial MPK}{\partial s_i} dH(\epsilon) dG(A) - \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \frac{\partial u'}{\partial s_i} MPK dH(\epsilon) dG(A) \\
&+ \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} R^D(S_h, A) \frac{\partial u'}{\partial s_i} dH(\epsilon) dG(A).
\end{aligned} \tag{A27}$$

Note that
$$\frac{\partial MPK}{\partial s_i} = -(\alpha-1)\alpha\epsilon_i A e^D (k_0 - s_i)^{\alpha-2} = \frac{1-\alpha}{k_0 - s_i} MPK, \tag{A28}$$
and
$$\frac{\partial u'}{\partial s_i} = u'' \frac{\partial c^D}{\partial s_i} = u'' \left(-MPK + R^D(S_h, A)\right). \tag{A29}$$
Then, the second order condition (A27) becomes

$$\begin{aligned}
&-\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} \frac{1-\alpha}{k_0-s_i}(MPK)u' dH(\epsilon)dG(A) + \int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} (-MPK)(-MPK+R^D)u'' dH(\epsilon)dG(A)\\
&+\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} R^D(-MPK+R^D)u'' dH(\epsilon)dG(A)\\
=&-\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} \frac{1-\alpha}{k_0-s_i}(MPK)u' dH(\epsilon)dG(A) + \int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} (R^D - MPK)^2 u'' dH(\epsilon)dG(A)\\
<&0.
\end{aligned} \tag{A30}$$

Q.E.D.

E. Proof for Lemma 8

Proof. Higher banker population μ means lower per bank deposits $\partial s_h/\partial\mu < 0$, if there is no change in deposits per depositor s_i. Then, the higher deposit rate ρ^D and lower default threshold θ^D are immediate results from Corollary 2.

However, with these better deposit contract term, deposits per depositor should increase. Still, despite of an increase in per depositor deposits s_i, per bank deposits s_h decrease. Here is the proof. As $s_h = (1-\mu)s_i/(2\mu)$, each line below is equivalent:

$$\begin{aligned}
\frac{\partial s_h}{\partial \mu} &< 0,\\
\frac{1-\mu}{2\mu}\frac{\partial s_i}{\partial \mu} - \frac{1}{2\mu^2}s_i &< 0,\\
(1-\mu)\frac{\partial s_i}{\partial \mu} &< \frac{s_i}{\mu},\\
\frac{\partial s_i}{\partial \mu}\frac{\mu}{s_i} &< \frac{1}{1-\mu}.
\end{aligned} \tag{A31}$$

This is true under Assumption 6. *Q.E.D.*

F. Proof for Lemma 9

Proof. The iso-loan supply function (29) does not have any term involving banker population μ and thus is not affected by a change in μ. This is because the loan market contracts are offered by banks based solely on the optimization taking into account the financial frictions, not on their own capital amount or on the size of the banking sector.

Similarly, the iso-loan demand function (33) is not affected by a change in μ either. This is because the borrowers do not care about the balance sheet condition of the lenders.

Therefore, for any given loan amount l, the equilibrium loan contract (ρ^L, θ^L) is not affected by a change in μ. *Q.E.D.*

G. Proof for Lemma 11

Proof. Take the derivative of consumption of a banker (25) – (28) with respect to banker population μ. If every bank defaults, the effect of banker population to consumption of a banker is zero: For $A \in [\underline{A}, \theta^D]$, based on (27),

$$\frac{dC^B(A, R^L, L_h, R^D, S_h)}{d\mu} = 0. \tag{A32}$$

If every bank honors the deposit contract and every borrowers repays in full to banks, a larger number of bankers implies less profits per banker: For $A \in [\frac{\theta^L}{\underline{\epsilon}}, \overline{A}]$, based on (25),

$$\frac{dC^B(A; R^L, L_h, R^D, S_h)}{d\mu} = \frac{d\pi S_h}{d\mu} < 0. \tag{A33}$$

Here, the inequality is validated by Lemmas 8 and 9. Specifically, Lemma 8 states that the per bank deposits S_h decreases with banker population μ. Lemma 8 also states that the deposit rate ρ^D increases, while Lemma 9 predicts te loan rate ρ^L is inelastic to banker population μ. Hence, the spread $\pi = \rho^L - \rho^D$ decreases with banker population μ.

In between, for $A \in [\theta^D, \frac{\theta^L}{\underline{\epsilon}}]$, bank honors the deposit contract but some borrowers default. Consumption in this case (26) is apparently lower than the case (25). Moreover, in (26) case, because $\partial B/\partial\mu < 0$, the total income is decreasing compared to the constant full income in case of (25), while paying the full deposit rate remains the same. Therefore,

$$\frac{dC^B(A; R^L, L_h, R^D, S_h)}{d\mu} < \frac{d\pi S_h}{d\mu} < 0. \tag{A34}$$

In the last, worst case, for $A \in [\underline{A}, \theta^B)$, based on (28),

$$\frac{dC^B(A; R^L, L_h, R^D, S_h)}{d\mu} = \frac{\partial B}{\partial \mu} < 0. \tag{A35}$$

Note that the external margins by the change in the default threshold due to a change in μ are canceled out because the consumption just above and just below the threshold are the same. *Q.E.D.*