Efficient and Incentive-Compatible Liver Exchange

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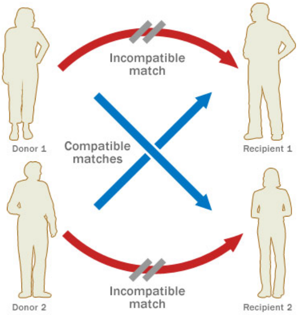
- **Kidney Exchange** became a wide-spread modality of transplantation within the last decade.
- About 600 patients a year receive kidney transplant in the US along through exchange, about 12% of all live-donor transplants.
- In theory **live donor organ exchange** can be utilized for any organ for which live donation is feasible.
- Liver is the second most transplanted organ after kidneys; moreover, live-donor lobar liver donation is feasible.



- Human organs cannot received or given in exchange for "valuable consideration" (US, NOTA 1984, WHO)
- However, **live donor kidney exchange** is not considered as "valuable consideration" (US NOTA amendment, 2007)

Literature

- **Kidney Exchange Literature**: Plenty...

Literature

- **Kidney Exchange Literature**: Plenty...
- **Liver Exchange Literature**:
  - Hwang et al. [10] proposed the idea and documented the practice in Korea since 03
  - Chen et al. [10] documented the program in Hong Kong
  - Dickerson & Sandholm [14] asymptotic gains from liver+kidney exchange over isolated liver exchange and kidney exchange
  - Ergin, Sönmez, & Ünver [17] proposed and modeled exchange for transplants each of that needs two live donors: lung, simultaneous liver+kidney, dual-graft liver

Contribution

- We model liver exchange as a matching problem – different from kidney exchange due to size-compatibility requirement.
- We find the structure of feasible two-way exchanges and a sequential algorithm to find an efficient matching for two patient/donor sizes.
- The requirement of size compatibility induces an incentive problem for the pair/donor to donate
    - the larger/riskier/easier to match right lobe or
    - the smaller/safer/more difficult to match left lobe
- For a continuum of patient/donor sizes, we propose a Pareto-efficient and incentive-compatible mechanism that elicits willingness to donate right lobe truthfully.
    - A new class of bilateral exchange mechanisms for vector-partial-order-induced weak preferences.

- Live donor liver donation is more common in Asian countries where deceased donation is at a minimum due to cultural reasons and legal non-recognition of "brain death".

Annual liver transplant activity per million population

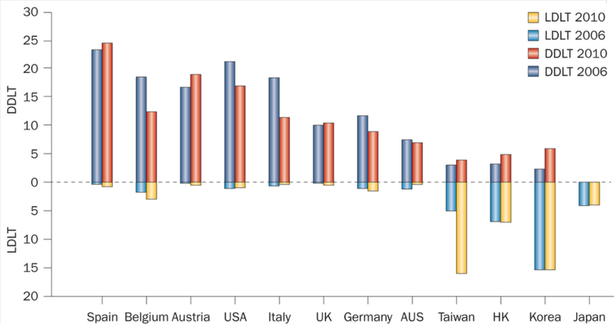
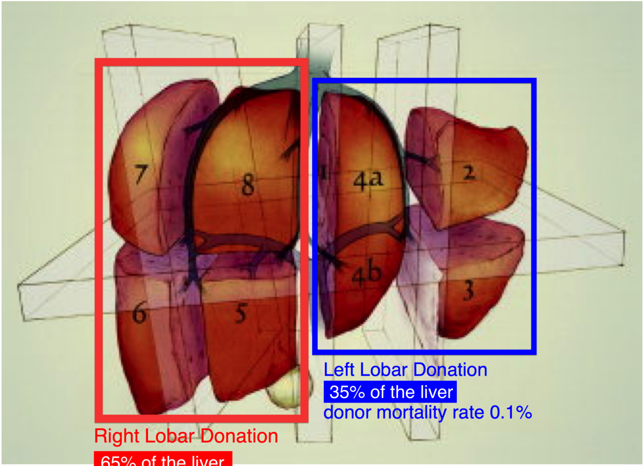


Figure from Chen et al Nature Reviews Gastroenterology & Hepatology 2013

Medical Background: Lobar Liver Donation



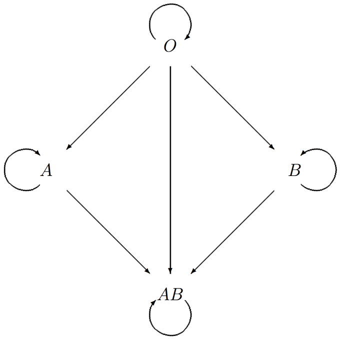
Right Lobar Donation

65% of the liver

donor mortality rate 0.5%

Medical Background: Compatibility

- Blood-type compatibility is required.



- Size compatibility is required unlike kidneys: A patient requires a graft relatively large to survive.
- Tissue-type compatibility is **not** required unlike kidneys.

Institutions: Right-Lobe Liver Transplant

- Right-lobe transplant has been utilized for size compatibility despite its heightened donor mortality risk.
  - Patient needs roughly at least 40% of his own liver size to survive.
  - Donor needs at least 30% remnant liver volume to survive.
  - Usually right lobe is ∼65%, left lobe is ∼35% of liver.
  - In many occasions, size compatibility is only satisfied through right-lobe donation.

Institutions: Living Donor Deaths

Donor deaths "definitely" related to donor hepatectomy			
11	2003	Japan	A mother in her late 40s donated a right lobe and died 9 months later from complications of hepatic failure.
12	2002	USA	A 57-year-old brother donated a right lobe and developed gastric gas gangrene and *Clostridium perfringens* infection 3 days after surgery and died.
13	2005	Brazil	A 31-year-old female right lobe donor of unknown relationship to the recipient died 7 days after surgery from a subarachnoid hemorrhage.
14	2003	India	A donor of unknown age and unknown relationship to the recipient donated an unknown lobe and died 10 days after surgery of unknown causes.
15	2003	India	A 52-year-old wife donated an unknown lobe and became comatose 48 hours after surgery from unknown causes and remains in chronic vegetative state.
16-18	1993	Germany	A 29-year-old mother donated a left lateral lobe and died of a pulmonary embolus 48 hours after surgery.
18, 19	2000	Germany	A 38-year-old father donated a right lobe, and 32 days after developing progressive hepatic failure, died during transplantation of acute cardiac failure. The cause of the donor's death was attributed to Berardinelli-Seip syndrome, a lipodystrophy syndrome characterized by loss of body fat, diabetes, hepatomegaly, and acanthosis nigricans.
18, 20	2000	France	A 32-year-old brother donated a right lobe and developed sepsis and multiple organ system failure 11 days after surgery and died of septic shock 3 days later.
18	2000	Europe	A 57-year-old wife donated a right lobe and died of sepsis and multiple organ system failure 21 days after surgery.
21, 22	1999	USA	A 41-year-old half-brother donated a right lobe and died of pancreatitis and sepsis 1 month later.
22, 23	1997	USA	A mother of unknown age donated an unknown lobe to a pediatric recipient and died 3 days after surgery of unknown causes.
24	2005	Asia	A 50-year-old mother donated a right hepatic lobe. She had no history of peptic ulcer disease and received a 2-week course of H2 antagonist. She died 10 weeks after surgery from an autopsy-proven duodenal ulcer with a duodenocaval fistula causing air embolism.
25	2006	Asia	A 39-year-old male "close relative" who donated an unknown lobe died of a myocardial infarction 4 days after donation. The patient reportedly had a preoperative electrocardiogram and treadmill test.
26	2005	Egypt	A brother of unknown age who donated a right lobe died of complications of sepsis from a bile leak 1 month after donation.
Donor deaths "possibly" related to donor hepatectomy			
27	2005	USA	A 35-year-old brother donated a right lobe and died of a self-induced drug overdose 23 months later.
27	2005	USA	A 50-year-old uncle donated a right lobe and died of a self-inflicted gunshot wound to the head 22 months after donation.

- Right-lobe donation is reported to be 5 times riskier in mortality rates than left-lobe donation (0.5% to 0.1%).
- A high profile death of a live right-lobe donor in the US in 2002 decreased live donation steadily not only for livers, but for other organs including kidneys in the US.
- Trotter et al. [06]: Documented living liver donor deaths due to donation.
- About half of live donations right lobe.

- **Liver exchange** first done in Korea, followed by Hong Kong and Turkey.
- Liver exchange can have two benefits:
  - It can **increase** the number of transplants.
  - It can **decrease** the share of right-lobe transplants (and increase donor safety) through matching with respect to size.

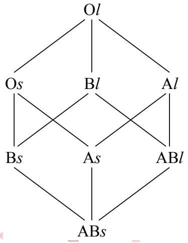
Living-Donor Liver Exchange

- Liver exchange differs from kidney exchange in two key ways:
  - The lack of tissue-type incompatibility, and
  - the presence of size incompatibility.
- In the absence of size incompatibility the scope for liver exchange would be very limited: The only viable exchange would be between
  - a blood type A patient with blood type B donor and
  - a blood type B patient with blood type A donor.

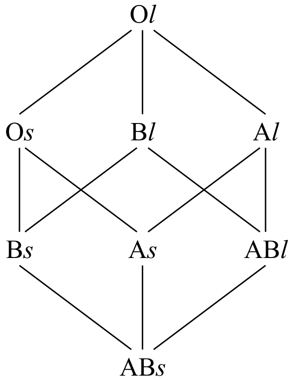
Liver Exchange Model: Two Sizes

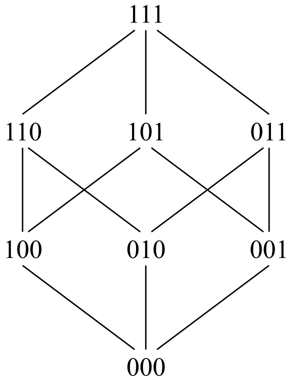
- $\underbrace{\{O, A, B, AB\}}_{\mathcal{B}} \times \underbrace{\{l, s\}}_{\mathcal{S}}$: Set of individual types
- Initially, we focus on live-donor left-lobe liver transplants.
- Left-Lobe Compatibility: A donor can donate to a patient if and only if
  (1) the patient is blood type compatible with the donor, and
  (2) the donor is not smaller than the patient.

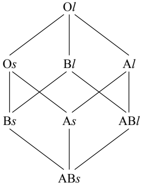
Liver Donation Partial Order $\trianglerighteq$ on $\mathcal{B} \times \mathcal{S}$

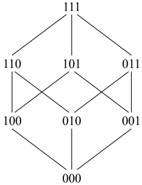


- Consider the following two partially ordered sets:
  (1) The liver donation partial order $\trianglerighteq$ on $\mathcal{B} \times \mathcal{S}$, and
  (2) the standard partial order $\geq$ over the corners of the three-dimensional cube $\{0,1\}^3$.









- Note that $(\mathcal{B} \times \mathcal{S}, \trianglerighteq)$ and $(\{0,1\}^3, \geq)$ are order isomorphic, where the order isomorphism associates each individual type $\tau \in \mathcal{B} \times \mathcal{S}$ with the following vector $X \in \{0,1\}^3$:

$$\begin{aligned} X_1 = 0 &\iff \tau \text{ has the } A \text{ antigen} \\ X_2 = 0 &\iff \tau \text{ has the } B \text{ antigen} \\ X_3 = 0 &\iff \tau \text{ is small} \end{aligned}$$

- For notational transparency, we will work with the equivalent representation $(\{0,1\}^3, \geq)$.

Liver Exchange Problem

- The type of a patient-donor pair is represented through the individual types of its patient and donor, respectively, as $X - Y \in \left(\{0,1\}^3\right)^2$.

Definition

A liver exchange problem is a list $\mathcal{E} = \{\mathcal{I}, \tau\}$ where $\mathcal{I} = \{1, 2, ..., I\}$ is a set of pairs, and for each $i \in \mathcal{I}$, $\tau(i) = X - Y$ is the type of pair i.

Possible *Left-Lobe-Only* Two-Way Liver Exchanges

- A (left-lobe) direct transplant consists of a single pair i of type $X - Y$ such that $Y \geq X$. Such pairs are called (left-lobe) compatible pairs.
  We represent it as $\{i\}$.
- A (left-lobe-only two-way) liver exchange consists of two pairs i of type $V - W$ and j of type $X - Y$ such that $Y \geq V$ and $W \geq X$.
  We represent it as $\{i, j\}$.
- There is embedded endowment bias.
- A matching is a collection of mutually exclusive exchanges and direct transplants such that if a pair is compatible, then it participates in a direct transplant.
  (individual rationality in embedded for now in definition).

Possible *Left-Lobe-Only* Two-Way Liver Exchanges

Observation

In any liver exchange problem, the only types that could be part of a two-way exchange are

$$X - Y \in \left(\{0,1\}^3\right)^2 \text{ such that } X \ngeq Y \text{ and } Y \ngeq X.$$

Possible *Left-Lobe-Only* Two-Way Liver Exchanges

- Value of a pair type $\underbrace{X_1X_2X_3}_{X} - \underbrace{Y_1Y_2Y_3}_{Y}$ is defined as $v(X-Y) = \sum_{k=1}^{3} Y_k - \sum_{k=1}^{3} X_k$.
- Waste of an exchange between pair types $V-W$ and $X-Y$ is defined as $v(V-W) + v(X-Y)$.
- All feasible exchanges have non-negative waste.
- Observation: All feasible exchanges are either Waste 0, 1, or 2.

Efficient *Left-Lobe-Only* Two-way Liver Exchanges

- A left-lobe-only two-way matching is (Pareto) efficient if and only if it maximizes the number of transplants.

  (follows from *matching matroid* - e.g. see Lawler [1976]).

- We propose a greedy (or sequential matching) algorithm to achieve an efficient matching.

  In general graphs, efficient two-way matchings cannot be found greedily (e.g. see Edmonds [1965])

Two-Size *Left-Lobe-Only* Sequential Exchange Alg

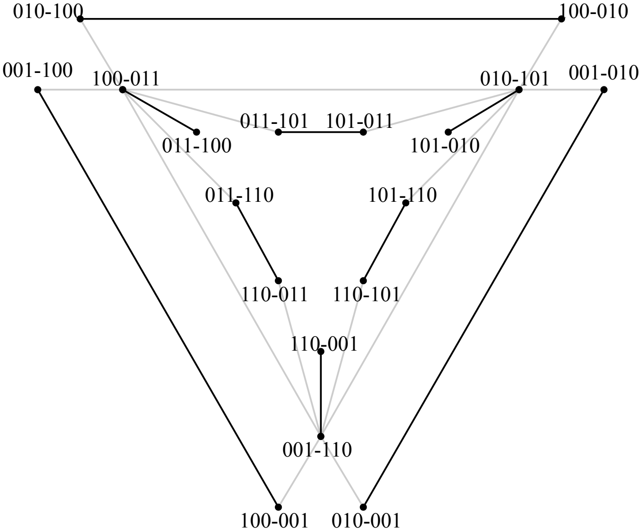
Step 0. Clear all feasible direct transplants.

Step 1. Clear Waste 0 Exchanges: Match the maximum number of $X - Y$ and $Y - X$ types for all $X, Y \in \{0, 1\}^3$.

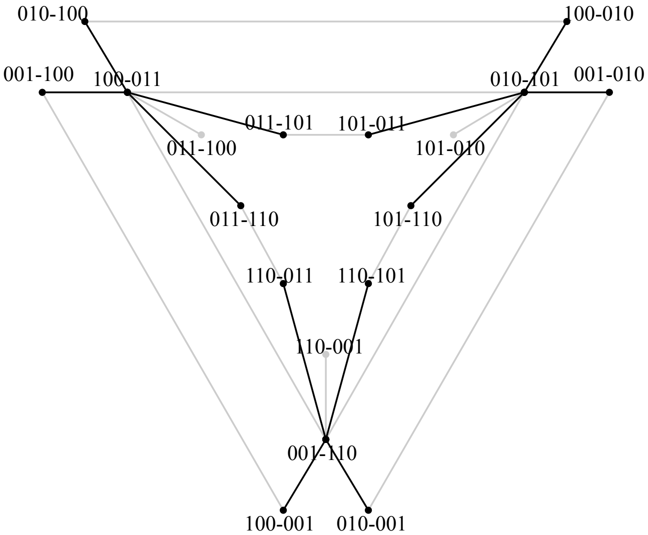
Step 2. Clear Waste 1 Exchanges: Match the maximum number of $100 - 011$, $010 - 101$, and $001 - 110$ types, *without matching them to each other*.

Step 3. Clear Waste 2 Exchanges: Match the maximum number of $100 - 011$, $010 - 101$, and $001 - 110$ types *among each other*.

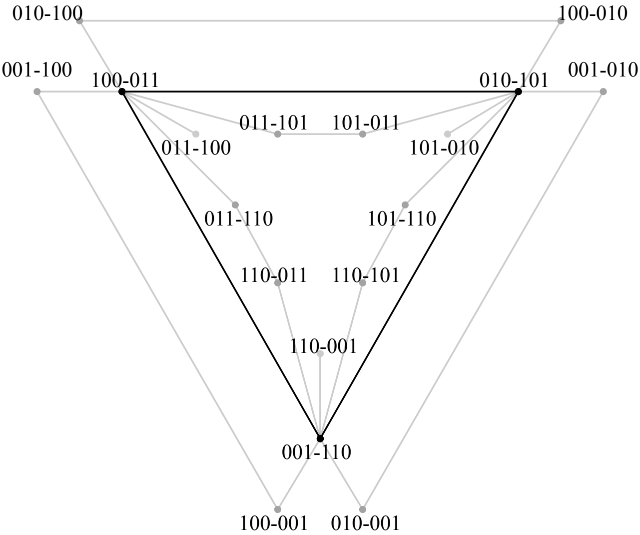
Algorithm Step 1: Waste 0 Exchanges



Algorithm Step 2: Waste 1 Exchanges



Algorithm Step 3: Waste 2 Exchanges



Theorem

Given a liver exchange problem, the two-size left-lobe-only sequential exchange algorithm maximizes the number of left-lobe-only two-way exchanges.

- Donation Possibilities:
  - Left-lobe donation: Less risky for the donor.
    Blood-type compatible donor should be at least as large as the patient.
  - Right-lobe donation: More risky for the donor.
    Blood-type compatible donor can donate to a larger patient.
- Pair Preferences:
  - Over donation
    - Donating left lobe is always preferable to donating right lobe or not donating at all
    - The pair may prefer donating right lobe to not donating at all: Type willing (w).
    - The pair may prefer not donating at all to donating right lobe: Type unwilling (u).
  - Over reception fixing the lobe donated:
    A pair prefers direct transplant to exchange.

Incentives and *Right-Lobe Donation*

Willing preferences R_i^w:

Incentives and *Right-Lobe Donation*

Willing preferences R_i^w:

Left Direct

Left Exchange

Willing preferences R_i^w:

Left Direct

Left Exchange

Right Direct

Right Exchange

Incentives and *Right-Lobe Donation*

Willing preferences R_i^w:

Left Direct

Left Exchange

Right Direct

Right Exchange

$\emptyset$

$\vdots$

Willing preferences R_i^w:

Left Direct
Left Exchange
Right Direct
Right Exchange
$\emptyset$
$\vdots$

Unwilling preferences R_i^u:

Incentives and *Right-Lobe Donation*

Willing preferences R_i^w:

Left Direct
Left Exchange
Right Direct
Right Exchange
$\emptyset$
$\vdots$

Unwilling preferences R_i^u:

Left Direct
Left Exchange

Incentives and *Right-Lobe Donation*

Willing preferences R_i^w:
Left Direct
Left Exchange
Right Direct
Right Exchange
$\emptyset$
$\vdots$

Unwilling preferences R_i^u:
Left Direct
Left Exchange
$\emptyset$
$\vdots$

Incentives and *Right-Lobe Donation*

- We focus only on individual rational exchanges:
  - A left-lobe compatible pair does not join in any exchange, but only in direct transplant.
  - A right-lobe-only compatible pair participates in an exchange only if its donor donates her left lobe; otherwise, it participates in direct transplant.

Incentives and *Right-Lobe Donation*

- We focus only on individual rational exchanges:
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  - A right-lobe-only compatible pair participates in an exchange only if its donor donates her left lobe; otherwise, it participates in direct transplant.
- Willingness type of a pair is private information.
- We inspect direct revelation mechanisms to elicit willingness types.

Exchanges under *Left&Right-Lobe Donation*

- Fix a willingness profile $R = (R_i)_{i \in \mathcal{I}}$ where all $R_i \in \{R_i^u, R_i^w\}$
- Type $X_1X_2X_3 - Y_1Y_20$ w is *treated like* $X_1X_2X_3 - Y_1Y_21$ when it donates right lobe. We refer to this as *transformation*.
- Hence, when type $X_1X_2X_3 - Y_1Y_21$ covers both native $X_1X_2X_3 - Y_1Y_21$ type pairs and *transformed* $X_1X_2X_3 - Y_1Y_20$ w type pairs, we refer to it as an auxiliary type.

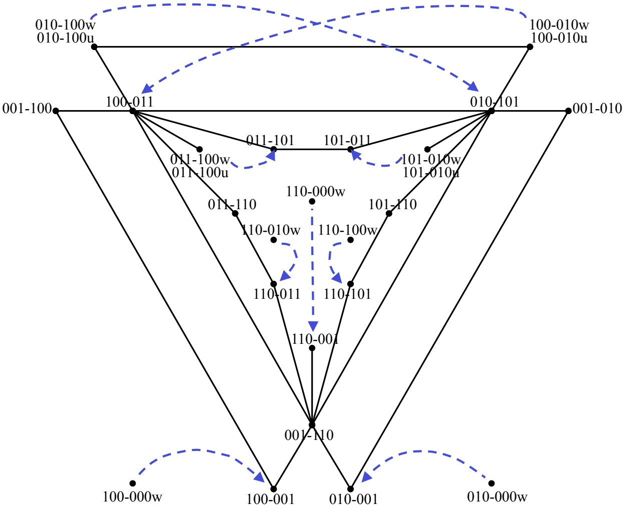
Exchanges under *Left&Right-Lobe Donation*

Lemma

Under left&right-lobe donation, pair types that can benefit from right-lobe donation are $X - Yw$ such that

- I $X \ngeq Y_1Y_21$ & $X > Y$, *which can only participate in exchange by donating right lobe* ($100 - 000w$, $010 - 000w$, $110 - 000w$, $110 - 100w$, $110 - 010w$);
- II $X \nleq Y_1Y_21$ & $X \ngtr Y$, *which can participate in exchange by donating left lobe or donating right lobe* ($010 - 100w$, $100 - 010w$, $011 - 100w$, $101 - 010w$);
- III $X = Y_1Y_21$, *which can donate right lobe to themselves and they do not participate in exchange* ($001 - 000w$, $101 - 100w$, $011 - 010w$, $111 - 110w$);
- IV $X < Y_1Y_21$, *which can only participate in exchange by donating left lobe and otherwise donate right lobe to themselves* ($001 - 100w$, $001 - 010w$, $001 - 110w$, $101 - 110w$, $011 - 110w$).

Possible *Left&Right-Lobe* Exchanges



Lemma (Observation about Efficiency and Incentive Compatibility)

Under right lobe donation, the pair types that can benefit from right-lobe donation are $X - Yw$ such that

- I $X \not\geq Y_1Y_21$ & $X > Y$, *which can only participate in exchange by donating right lobe*
  $\rightarrow$ *Transform at the beginning*
- II $X \not\leq Y_1Y_21$ & $X \not> Y$, *which can participate in exchange by donating left lobe or donating right lobe*
  $\rightarrow$ *Gradually transform as left-lobe exchanges are exhausted*
- III $X = Y_1Y_21$, *which can donate right lobe to themselves and they do not participate in exchange*
  $\rightarrow$ *Direct transplant at the beginning*
- IV $X < Y_1Y_21$, *which can only participate in exchange by donating left lobe and otherwise donate right lobe to themselves*
  $\rightarrow$ *Direct transplant at the end*

- A mechanism is a systematic procedure that finds a matching for each willingness type profile reported.
- A mechanism is incentive compatible if it is a weakly dominant strategy for each pair to reveal its willingness type truthfully.
- Any sequential algorithm using the above order of transformation for different category willing pairs and using an external fixed priority order in choosing among the same type of pairs will be incentive compatible as a mechanism.

Pareto Efficiency

- Sustaining Pareto efficiency is trickier than incentive compatibility.
- Pareto efficiency no longer implies transplant maximality.
- **Intuition from left-lobe-only exchanges**: Clear waste 0, waste 1,and then waste 2 exchanges, in this order, for efficiency.
- Use the same intuition, but *transform* willing pairs with small donors as soon as their left-lobe donation possibilities are exhausted, so incentives and efficiency are aligned.
- Restart clearing from waste 0 exchanges.
  Small caveat as the 2nd and 3rd/4th points can have a tradeoff.

Two-Size *Left&Right-Lobe* Sequential Exchange Mech

Fix a priority order over pairs for priority matching in each step.

Step 0. Direct transplant each left-lobe compatible and Category III w pair.

Step 1. *Transform* Category I w pairs.
Clear Waste 0 exchanges.

Step 2. Either $010 - 100$ or $100 - 010$ (or both) type pairs are depleted.
Assume wlog $100 - 010$ types are depleted.
(Otherwise proceed symmetrically.)
Clear Waste 1 exchanges of $100 - 010$.

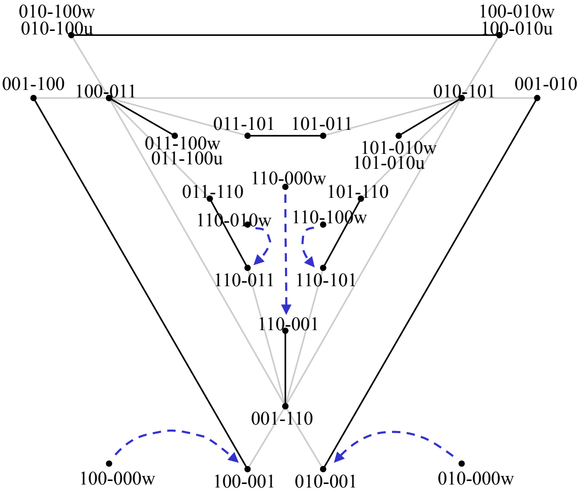
Step 3. *Transform* $010 - 100w$ pairs.
Clear Waste 0 exchanges.

Step 4. *Transform* $011 - 100w$ and $101 - 010w$ types.
Clear Waste 0 exchanges.

Step 5. Clear Waste 1 exchanges.

Step 6. Clear Waste 2 exchanges.

Step 7. Direct transplant each remaining Category IV w pair.


010-100w
010-100u
100-010w
100-010u
001-100
100-011
010-101
001-010
011-101
101-011
011-100w
011-100u
101-010w
101-010u
110-000w
011-110
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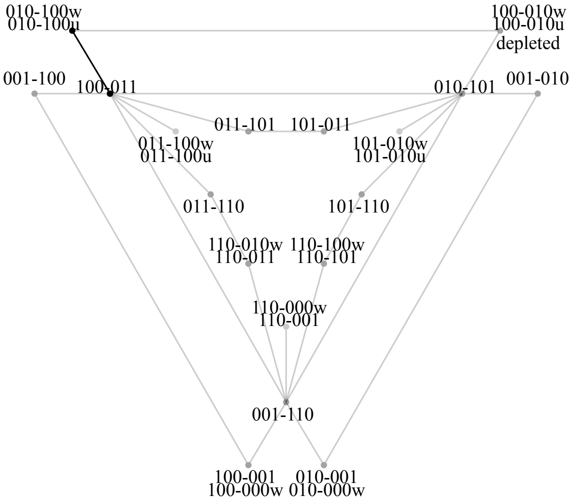
100-001
010-001
010-000w


010-100w
010-100u
100-010w
100-010u
depleted
001-100
100-011
010-101
001-010
011-101
101-011
011-100w
011-100u
101-010w
101-010u
011-110
101-110
110-010w
110-011
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110-101
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110-001
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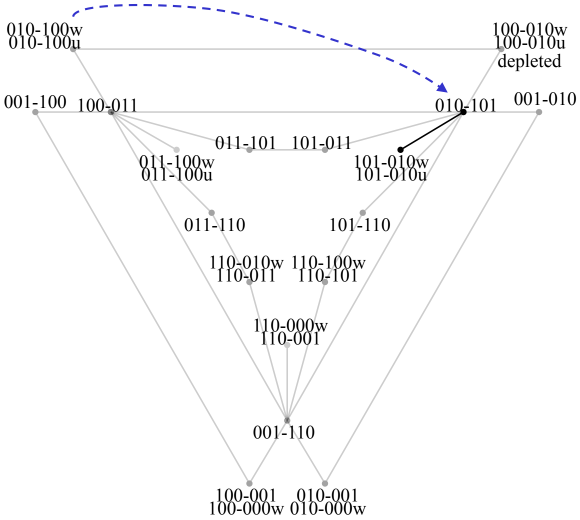
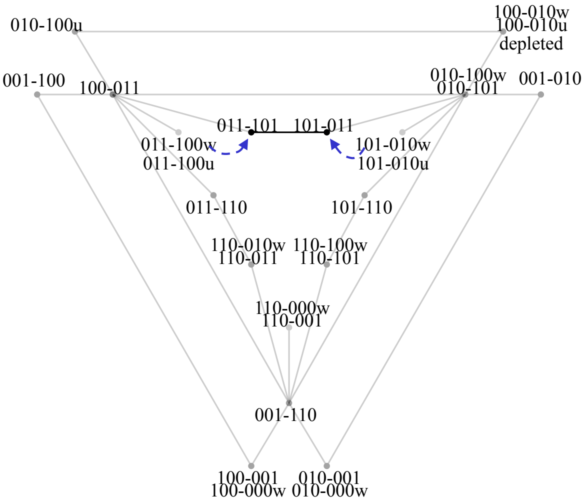
010-001
010-000w

Algorithm Step 3:

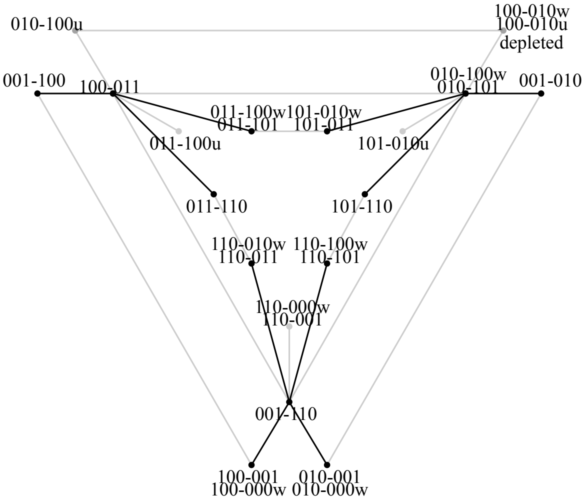


Algorithm Step 4:

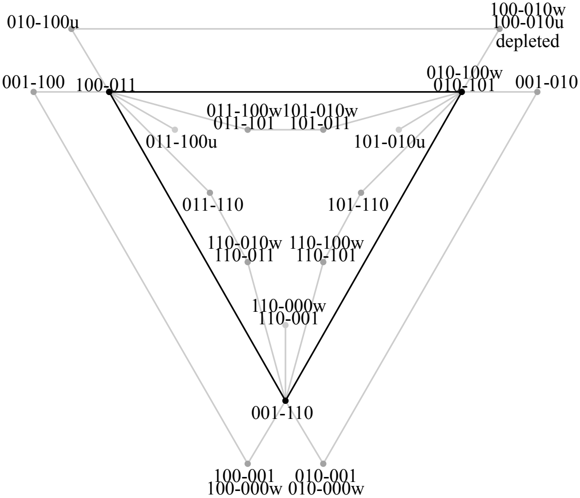


Precedence Order

Algorithm Step 5:



Algorithm Step 6:



Incentive Compatibility

Theorem

The two-size left&right-lobe sequential exchange mechanism is individually rational, Pareto-efficient, and incentive compatible for two sizes of individuals.

General Case: Continuum of Individual Sizes

Suppose $\mathcal{S} = \{0, 1, ..., S-1\}$ is the set of possible individual sizes (instead of just $\{0,1\}$) given a finite exchange pool $\mathcal{E} = (\mathcal{I}, \tau)$.

An individual type: $X = X_1X_2X_3 \in \{0,1\} \times \{0,1\} \times \mathcal{S}$

Pair type: $X - Y \in (\{0,1\}^2 \times \mathcal{S})^2$

Right-lobe donation function $\rho : \mathcal{S} \to \mathcal{S}$ such that it is non-decreasing, and $\rho(s) > s$ for all $s \in \mathcal{S} \setminus \{S-1\}$:
A donor of size s size can donate right lobe to any patient of size $s' \leq \rho(s)$.

Let for type $Y = Y_1Y_2Y_3$, let $\rho(Y) := Y_1Y_2\rho(Y_3)$.

1. Greedily committing to an exchange may be no longer possible even for just left-lobe-only exchange.
2. When right-lobe donation is also possible, the *transformation* order of Category II willing pairs is non-trivial.

Skip Diff. 1

Difficulties: 1. Left-Lobe-Only Non-Greedy Matching

A subset of pairs is matchable if there exists a matching that match each of them.

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Recursive augmentation on a graph can extend the matchable set and find an efficient matching using a non-greedy approach in polynomial time (Augment/Shrink-a-Blossom Algorithm of Edmonds, 1965).

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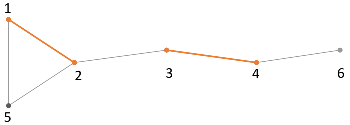

Matchable: $\{1, 2, 3, 4\}$

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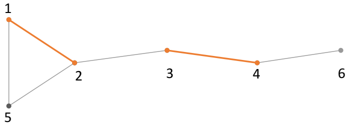
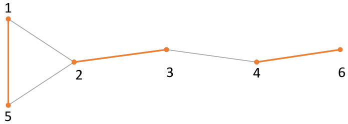
A subset of pairs is matchable if there exists a matching that match each of them.

Recursive augmentation on a graph can extend the matchable set and find an efficient matching using a non-greedy approach in polynomial time (Augment/Shrink-a-Blossom Algorithm of Edmonds, 1965).



Matchable: $\{1, 2, 3, 4\}$



Matchable: $\{1, 2, 3, 4, 5\}$

Difficulties: 2. Transformation Order of Category II Pairs

Suppose we want to use a priority approach based on matchability as the greedy approach may not work.
Transform Category II pairs after their left-lobe matchability option is exhausted, because of incentive compatibility. But in which priority order of pairs?

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Definition

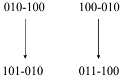
Define a directed graph on the set of Category II pair types ($X - Y$ such that $X \not\geq Y$ and $X \not\leq \rho(Y)$), that we will call the precedence digraph, where for any Category II pair types $X - Y$ and $U - V$:

$$X - Y \longrightarrow U - V \quad \Longleftrightarrow \quad X \leq V,\ U \not\leq Y\ \&\ U \leq \rho(Y).$$

If $X - Y \longrightarrow U - V$, we will also say that $X - Y$ precedes $U - V$.

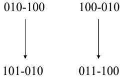
Difficulties: 2. Transformation Order of Category II Pairs

Precedence Partial Order for Two Sizes



▸ Two Sizes

Precedence Partial Order for Two Sizes



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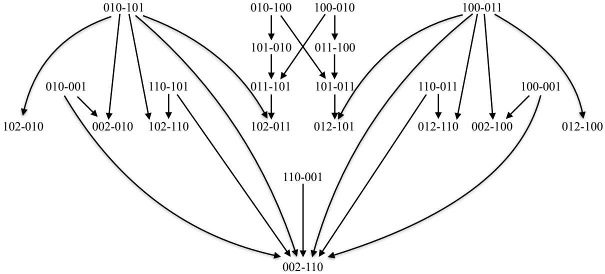
Precedence Partial Order for Three Sizes



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Lemma (from graph theory)

Given an acyclic digraph, there exists a linear order of all nodes, known as a topological order, L, that is consistent with the digraph:

$$x \to y \implies xLy$$

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Lemma

The precedence digraph on Category II pair types is acyclic.

Thus, a topological order of the precedence digraph of Category II pair types exists, and we can use this as our priority order over transformation.

Precedence-Order Induced Priority Mechanism

Fix a topological order over Category II pairs as $i_1, ..., i_K$ and a priority order over all pairs. Given a willingness profile R:

Step 0. Direct transplant left-lobe compatible and willing Category III pairs. *Transform* willing Category I pairs.

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Any matching in G^K that matches all pairs in $\mathcal{I}^{K+N}$ in exchanges together with the fixed direct transplants is the outcome.

Theorem

The precedence-order induced priority mechanism satisfies the following:

- *for given topological and priority orders and a willingness profile, all matchings that can be chosen by the mechanism are Pareto indifferent,*
- *individual rationality,*
- *Pareto efficiency, and*
- *incentive compatibility.*

Simulations

General Case Theorem

Intuition of the Proof.

Pareto indifference: We extend the matchable set in every step and only *transform* pairs that cannot be matched through left lobe in addition to the previously committed pairs. Thus those can only be matched through right lobe.

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Incentive compatibility: Acyclicity of the precedence digraph implies that *transformation* a willing Category II Pair i_k is independent of the willingness types of its lower-prioritized "graph neighbors." Thus, they cannot affect how i_k is matched by manipulating their own willingness types.

Incentive Compatibility and Maximizing Transplants

Proposition

There is no incentive compatible mechanism that maximizes the number of transplants or the number of left-lobe donations.

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Proof.

Consider an exchange pool with two sizes 0,1 such that

$n(101 - 011) \quad = 1, \qquad n(011 - 100w) = 2, \qquad n(100 - 011) \quad = 1.$

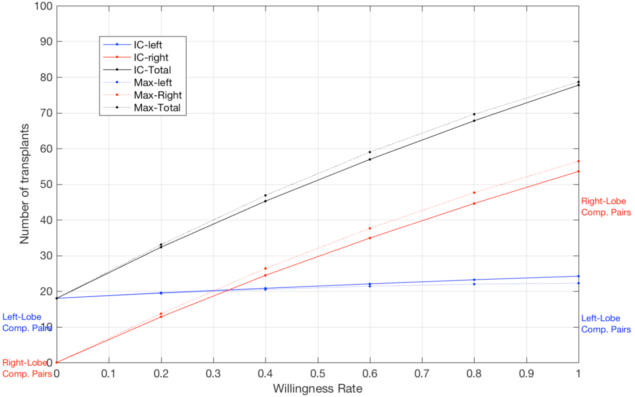
Any left-lobe donation or total transplant maximizing matching generates two exchanges with types

$100 - 011$ & $011 - 100$ (type 1) $\qquad 101 - 011$ & $011 - 100w$ (type 2),

resulting with 4 transplants and 3 left-lobe donations.

Observe that one of the willing pairs of type $011 - 100$ is matched to donate its left lobe (in type 1 exchange above) while the other one is matched to donate its right lobe (in type 2 exchange above). The one matched to donate its right lobe can manipulate. □

Simulated 100 pairs using South Korean patient/donor sizes and blood-type composition.



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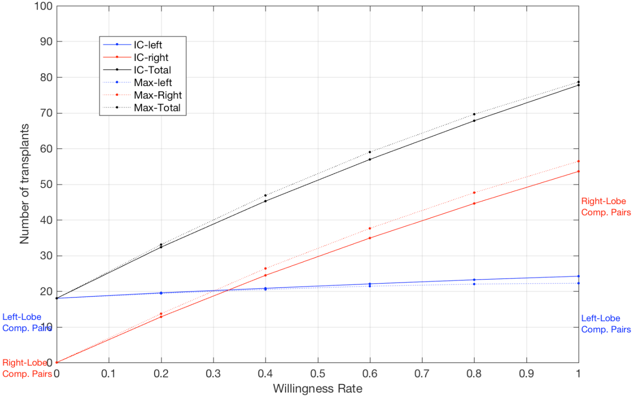

Very small cost of incentive compatibility!

Conclusion

- Model of live-donor lobar liver exchange as a market design problem. Information/incentive problems are modeled and solved through a PE + IC mechanism.
  Simulations: Relative cost of IC is less than 2%
- Size incompatibility increases the benefit from exchange, more gains plausible with respect to kidney exchange
- Off-the-shelf-implementable mechanism in Middle East and East Asia: Liver transplants are more complex, two-way may be the way to start the market design
- Implications for matching theory in general: A new class of bilateral exchange mechanisms for n-dimensional vector partial-order induced weak preferences:
  - Examples: vacation house exchanges, time/favor exchanges
  - Two-size model with three dimensions is of independent interest: Induces a fully-symmetric model where greedy mechanism design is possible.