Information and Market Power*

Dirk Bergemann† Tibor Heumann‡ Stephen Morris§

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Abstract

We analyze demand function competition with a finite number of agents and private information. We show that the nature of the private information determines the market power of the agents and thus price and volume of equilibrium trade.

We establish our results by providing a characterization of the set of all joint distributions over demands and payoff states that can arise in equilibrium under any information structure. In demand function competition, the agents condition their demand on the endogenous information contained in the price.

We compare the set of feasible outcomes under demand function to the feasible outcomes under Cournot competition. We find that the first and second moments of the equilibrium distribution respond very differently to the private information of the agents under these two market structures. The first moment of the equilibrium demand, the average demand, is more sensitive to the nature of the private information in demand function competition, reflecting the strategic impact of private information. By contrast, the second moments are less sensitive to the private information, reflecting the common conditioning on the price among the agents.

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†Department of Economics, Yale University, New Haven, CT 06520, U.S.A., dirk.bergemann@yale.edu.

‡Department of Economics, Yale University, New Haven, CT 06520, U.S.A., tibor.heumann@yale.edu.

§Department of Economics, Princeton University, Princeton, NJ 08544, U.S.A. smorris@princeton.edu.

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1 Introduction

1.1 Motivation and Results

Models of demand function competition (or equivalently, supply function competition) provide a cornerstone to the analysis of markets in industrial organization and finance.[1] As an important, descriptive as well as normative, model of competition it is useful to analyze market behavior. The model also has well-known difficulties: (i) under complete information there may be multiple equilibria that yield radically different outcomes, (ii) under incomplete information the equilibrium prediction is sensitive to the assumed information structures.

In this paper we attempt to provide an answer to the following two distinct, but related questions: (i) can we select any of the equilibria that arise under complete information by considering small amounts of incomplete information? (ii) can we provide predictions that hold across all information structure?

We answer these questions in a setting with a finite number of agents that compete via demand function competition for a divisible asset. Traders have linear-quadratic preferences over their asset holdings, and the marginal utility of an agent is determined by a payoff shock. We restrict attention to symmetric environments (in terms of payoff shocks and information structures) and symmetric linear Nash equilibria.

The first main result of our paper shows that every outcome that can arise as a Nash equilibrium under complete information can also arise as the outcome of the unique linear Nash equilibrium under some small amount of incomplete information. A direct corollary of this result is that, for any number of agents, all outcomes — from price-taking behavior to a complete market shutdown — can be rationalized as the unique equilibrium for some information structure that is close to complete information. The result shows that all outcomes that can arise under complete information can be selected as being the unique outcome that arise under a small perturbations of complete information.

The reason that asymmetric information has an impact on the equilibrium outcome is that it changes the market power of agents. That is, it changes how much an agent changes the equilibrium price by changing the quantity he demands. We present two results as to how asymmetric information interacts with the equilibrium market power of agents. First, we show that changes

[1] There are two basic motivations to study demand function competition: (i) it is an accurate description of divisible good auctions (e.g. treasury auctions), and (ii) it is one of the few static models of competition, besides Cournot competition and Bertrand competition.

in the market power can be caused by arbitrarily small amounts of incomplete information. We show that market power is determined by residual uncertainty rather than by some absolute level of uncertainty. Hence, markets that apparently do not suffer from large amounts of uncertainty, may still be strongly impacted by asymmetric information. Second, we show that market power can range from zero to infinity, regardless of the details of the payoff environment (e.g. regardless of the number of agents). Hence, the bounds on market power in terms of the number of agents and the correlation of the payoff shocks that arise when studying a specific class of signals are not necessarily robust to the nature of the private information.

Given the sharp indeterminacy in the level of market power induced by the information structure, it is natural to ask whether there are any predictions at all that hold across all information structures. Clearly, it is essentially impossible to provide any prediction on the level of market power that is robust to the choice of the information structure. Nevertheless, the level of market power is only one of the possible measures that allow us to understand an equilibrium outcome. We establish that it is possible to provide robust predictions on the price volatility and the dispersion in the quantities bought by agents. These are two alternative measures that can help us understand the performance of trading mechanism under incomplete information.

The second main result of our paper shows that the price volatility is always (that is, regardless of the information structure) at most equal to the variance of the average shock across agents. Moreover, the dispersion in the quantities bought by agents is always (that is, regardless of the information structure) at most four times the dispersion of the payoff shocks. Hence, we show that it is possible to provide sharp bounds on some equilibrium statistics, which hold across all information structures.

As a by-product of the volatility bounds, we develop a methodology that helps us study the set of outcomes that can arise for all information structures. This serves two purposes. First, we can fully characterize the set of outcomes that can be achieved in demand function competition in terms of necessary and sufficient conditions. This allow us to immediately show that the volatility bounds we find are in fact tight. That is, the volatility bounds we provide can be achieved for some information structure.

The methodology used to study volatility bounds can also be used to compare the set of outcomes of different trading mechanisms across all information structures. We define a distribution of outcomes as the joint distribution of quantities, payoff shocks and price that is induced by an equilibrium outcome. A distribution of outcomes provides a description of the outcome of demand function competition that allows to abstract from the strategies used in equilibrium and the precise description of the information structure. The key conceptual innovation is to describe the outcomes of the demand function completion game not in terms of the strategies used by the agents (that is, the demand functions), but instead, in terms of the induced quantities (purchased quantity and price) and payoff shocks.

A critical advantage of the focus on the distribution of outcomes is that it can be easily compared with the distribution of outcomes induced by any other trading mechanisms. In the paper we focus our analysis in comparing demand function competition with Cournot competition, as a particular instance of what we call fixed slope mechanism. The set of possible first moments under demand function competition has one more degree of freedom than under Cournot competition, while the set of possible second moments under demand function competition has one less degree of freedom than under Cournot competition. This apparently abstract description of the two mechanisms allow us to conclude that price volatility is bounded by the size of aggregate shocks in demand function competition, while in Cournot competition price volatility cannot be bounded by the size of the aggregate shocks. By contrast, the first moment, the market power, or the average volume of trade is uniquely determined in the Cournot competition, but larger than the positive real line with demand function competition.

1.2 Related Literature

Our paper is closely related to two strands of the literature studying demand function competition. The first strand is the literature studying equilibrium refinements in demand function competition. The second strand is the literature studying the impact of asymmetric information on the equilibrium outcome. Finally, our paper is methodologically related to a strand of the literature in game theory studying the impact of asymmetric information in games.

The existence of multiple equilibria in demand function under complete information dates back at least to Grossman (1981) and Hart (1985). In a seminal contribution, Klemperer and Meyer (1989) show that a small perturbation to the exogenous supply of asset reduces the set of equilibria. In a linear environment like ours, the perturbation studied by Klemperer and Meyer (1989) yields a unique equilibrium. Our results show that any of the equilibria that arise under complete information can be selected by considering a small perturbation to the complete information setting. We interpret the equilibrium selected by Klemperer and Meyer (1989) as the equilibrium that can arise in a model with uncertainty and private values. Yet, we highlight that a critical aspect of their equilibrium selection argument is the assumption of private values, which may not be hold anymore even when the amount of incomplete information is small.

Vives (2011) pioneered the study of asymmetric information under demand function competition in the same setting of linear-quadratic payoffs and interdependent values that we investigate. He studied a particular class of information structures where each trader observes a noisy signal of his own payoff types. Our results strengthen in some directions his results. We show that the impact of asymmetric information on the equilibrium market power can even be larger than the ones derived from the one-dimensional signals studied in Vives (2011). In other directions, our results overturns some of the comparative statics and bounds that are found using a specific class of one-dimensional signal structures. In particular, market power can be large even when any of the following conditions is satisfied: (*i*) the amount of asymmetric information is small, (*ii*) the number of players is large, or (*iii*) payoff shocks are independently distributed.

Our paper is also related to a recent literature studying games under incomplete information, but without specifying the information structure. Bergemann and Morris (2016) provide a solution concept — Bayes correlated equilibrium —- which allows to study the set of outcomes of a game under all information structures.[2]

2 Model

Payoff Environment There are N agents that demand a divisible good. The utility of agent $i \in \{1, ..., N\}$ who buys $q \in \mathbb{R}$ units of the good at price $p \in \mathbb{R}$ is given by:

$$u_i(\theta_i, q_i, p) \triangleq \theta_i q_i - p \cdot q_i - \frac{1}{2} q_i^2,$$

where $\theta_i \in \mathbb{R}$ is a payoff shock. The payoff shocks are symmetrically and normally distributed and for any $i, j \in N$:

$$\begin{pmatrix} \theta_i \\ \theta_j \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \rho_{\theta\theta}\sigma_\theta^2 \\ \rho_{\theta\theta}\sigma_\theta^2 & \sigma_\theta^2 \end{pmatrix} \right)$$

where $\rho_{\theta\theta}$ is the correlation coefficient between θ_i and θ_j. By symmetry, and for notational convenience, we omit the subscripts i and j in the description of the moments (thus, e.g. μ_θ instead of μ_{θ_i}).

[2]This has been used in subsequent work to study a variety of games. See, for example, ...

With the symmetry of the payoff states across agents, a useful and alternative representation of the environment is obtained by decomposing the random variable into a common and an idiosyncratic component, ω and τ_i respectively:

$$\theta_i \triangleq \omega + \tau_i. \tag{1}$$ cco

Provided that the correlation coefficient $\rho_{\theta\theta}$ is nonnegative, the common and idiosyncratic components are independent from each other, and the joint distribution is therefore given by:

$$\begin{pmatrix} \omega \\ \tau_i \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ 0 \end{pmatrix}, \begin{pmatrix} \rho_{\theta\theta}\sigma_\theta^2 & 0 \\ 0 & (1-\rho_{\theta\theta})\,\sigma_\theta^2 \end{pmatrix} \right).$$

There is an exogenous supply of the good with an inverse supply function given by:

$$p(q) = \alpha + \beta \cdot q. \tag{2}$$ kolp

infostr **Information Structure** We assume that each agent i observes J signals:

$$s_i \triangleq (s_{i1}, ..., s_{iJ}).$$

We assume that the joint distribution of signals and payoff shocks ($s_1, ..., s_N, \theta_1, ..., \theta_N$) is symmetrically and normally distributed. For now we keep the description of the information structure abstract. We study specific examples in the following sections.

Demand Function Competition Agents compete via demand function competition. Each agent submits a demand function $x_i : \mathbb{R}^{J+1} \to \mathbb{R}$ that specifies the demanded quantity as a function of the received signal s_i and the market price p, denoted by $x_i(s_i, p)$. The Walrasian auctioneer sets a price p^* such that the market clears:

$$p^* = \alpha + \beta \cdot \sum_{i \in N} x_i(s_i, p^*) \tag{3}$$ mk2

for every signal realization s.

We study the Nash equilibrium of the demand function competition game. The strategy profile $(x_1^*, ..., x_N^*)$ forms a Nash equilibrium if:

$$x_i^* \in \underset{\{x_i:\mathbb{R}^{J+1}\to\mathbb{R}\}}{\arg\max}\ \mathbb{E}\left[\theta_i \cdot x_i(s_i, p^*) - p^* \cdot x_i(s_i, p^*) - \frac{x_i(s_i, p^*)^2}{2}\right],$$

where

$$p^* = \alpha + \beta \cdot (x_i(s_i, p^*) + \sum_{j \neq i} x_j^*(s_i, p^*))$$

We say a Nash equilibrium $(x_1^*, ..., x_N^*)$ is linear and symmetric if there exists $(c_0, ..., c_J, m) \in \mathbb{R}^{J+2}$ such that for all $i \in N$:

$$x_i\left(s_i, p\right) = c_0 + \sum_{j \in J} c_j \cdot s_{ij} + m \cdot p.$$

Equilibrium Statistics: Market Power and Price Volatility We will frequently summarize the equilibrium outcome in one of two distinct statistics. We define the (expected) *equilibrium market power* by:

$$l \triangleq \frac{1}{N}\mathbb{E}\left[\frac{\sum_{i \in N}\left(\frac{\partial u_i(\theta_i, q_i, p)}{\partial q_i} - p\right)}{p}\right] = \frac{1}{N}\mathbb{E}\left[\frac{\sum_{i \in N} \theta_i - q_i - p}{p}\right]. \tag{4}$$ mpde

The market power l is defined as the ratio of the difference between the marginal utility and the price paid for the good relative to the equilibrium price, and averaged across agents. The measure l of market power is a version of the Lerner index. As we consider a demand function game, the index captures the difference between the marginal utility and the price, rather than the more conventional notion based on the difference between price and marginal cost.

If agents were price takers, then the market power would be $l = 0$. If agents were to compete in Cournot competition, then the market power would be $l = 1/N$.

A second equilibrium statistic of interest is *price volatility*, the variance of the equilibrium price, which we denote by σ_p^2.

motexa

3 The Main Results Visualized

The aim of this section is to illustrate visually the main results of this paper. We will present all the associated analytic results in the subsequent sections. For the moment, we shall restrict ourselves to display the sensitivity of the equilibrium outcome to the private information of the agents in terms of the equilibrium statistics introduce a moment ago, *market power* and *price volatility.*

We shall consider three different classes of information structures: (i) noise-free one-dimensional signals, (ii) noisy one-dimensional signals and (iii) noisy multi-dimensional signals and now briefly describe them.

The *noise-free one-dimensional* signal is given by

$$s_i = \tau_i + (1+\gamma)\,\omega, \tag{5}$$ nf

with $\gamma \in \mathbb{R}_+$. Thus for $\gamma = 0$, the signal s_i informs agent i perfectly about his payoff state θ_i. But for $\gamma > 0$, the signal overweighs the common component of his payoff shock relative to his idiosyncratic component. In either case, agent i can use his signal to update his beliefs about the payoff states over the other agents. We refer to it as noise-free as the signal does not contain any extraneous shocks or noise terms.

The *noisy one-dimensional* signal is given by

$$s_i = \tau_i + \omega + \varepsilon_i, \tag{6}$$ no

where the noise term ε_i is normally distributed with variance σ_ε^2 and independent across agents. Thus for $\sigma_\varepsilon^2 = 0$, the signal s_i informs agent i again perfectly about this payoff state, and for $\sigma_\varepsilon^2 > 0$, the signal is a noisy version of the payoff state of agent i. In contrast to the noise-free signal, the idiosyncratic and common component always enter the signal with their "true" weight.

The *noisy multi-dimensional signal* gives each agent a separate noisy signal about the idiosyncratic and the common components in the payoff state, and thus each agent i observes $N+1$ signals:

$$s_i^i = \tau_i, \quad s_i^j = \tau_j + \varepsilon_i^j, \ \forall j \neq i \in N,$$

and

$$s_i^{N+1} = \omega + \varepsilon_i.$$

We assume that all the noise terms $\{\varepsilon_i^j\}$ are normally distributed and independent of each other. The environment is symmetric and the variance of the noise terms are given by:

$$\mathrm{var}(\varepsilon_i^j) \triangleq \widehat{\sigma}_\varepsilon^2\ ,\ \mathrm{var}(\varepsilon_i^{N+1}) \triangleq \sigma_\varepsilon^2.$$

That is, each agent i knows his own idiosyncratic component τ_i and receives a noisy signal about the idiosyncratic component of the other agents, as well as about the common component. We observe that for $\gamma = 0$ and σ_ε^2 respectively, each agent i knows his owns payoff state, but remains uncertain about the payoff state over the other agents.

In the following illustration, we show how market power, l, and price volatility, σ_p^2, are affected by the nature of the private information. Across all information structures, we keep the payoff environment fixed, and hence, any changes in the market power and price volatility stem only from the differences in the information structure. The payoff environment consists of three competitors, $N = 3$, with positively correlated types, $\rho_{\theta\theta} = 1/2$ and variance $\sigma_\theta^2 = 2$. The supply function is specified with $\alpha = 0$ and $\beta = 3$.

In Figure 1 we plot how the market power changes with the private information of the agents. As we consider noise-free as well as noisy information structure, we parametrize the information received by the agent through the variance of the conditional expectation $E\left[\theta_i \left| s_i \right.\right]$, or conditional variance:

$$\sigma^2_{\theta_i|s_i} \triangleq \operatorname{var}\left(\theta_i \left| s_i \right.\right).$$

For all three classes of information structures, the variance starts at 0 with $\gamma = 0$ or $\sigma_\varepsilon^2 = 0$, and then increases with an increase in γ or σ_ε^2 respectively. We display the multi-dimensional signal in two versions, with a low and high variance, $\widehat{\sigma}_\varepsilon^2$ of the noise term appearing in the signals regarding the idiosyncratic terms. Thus the low variance version of the multi-dimensional signal is close to the complete information environment for $\sigma_\varepsilon^2 \approx 0$. After all, if $\sigma_\varepsilon^2 \approx 0$ and $\hat{\sigma}_\varepsilon^2 \approx 0$, every agent observes every component of the payoff state almost without noise.

In Figure 1 we can see that an increase in the noise of the individual signal can *either* increase *or* decrease the equilibrium market power. Moreover, even restricting attention to a specific class of information structures, say the noisy multi-dimensional signal, the market power does not necessarily display a monotonic behavior in the conditional variance.

As we consider the limit as $\sigma^2_{\theta_i|s_i} \to 0$, we observe that the market power approaches to a common limit. The limit is precisely the market power induced by the equilibrium selected using the selection criteria proposed by Klemperer and Meyer (1989). As $\sigma^2_{\theta_i|s_i} \to 0$, every agent i can perfectly predict his own payoff shock using only his private information. This is the limit in which agents have private (and correlated) values.

Importantly, as long as $\hat{\sigma}_\varepsilon^2$ is kept fixed, an agent still remains largely uncertain about the realization of the payoff shock of other agents. Hence, even in the limit $\sigma_\varepsilon^2 \to 0$ the information structure is still substantially different than the complete information environment.

If we compare one-dimensional information structures with multi-dimensional information structures, we can see several differences. First, with one-dimensional signals the level of market power is monotonic in the conditional variance. By contrast, in the multi-dimensional environment the level of market power is non-monotonic in the size of the noise term.

In Figure 2 we display the impact that the private information has on price volatility. As with market power, we find that, locally as well as globally, an increase in the noise of the signal does

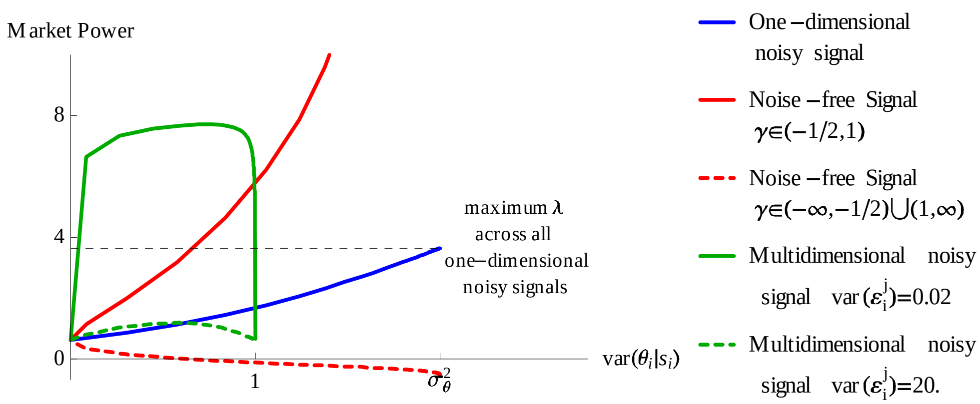

Figure 1: Information and Market Power mp1

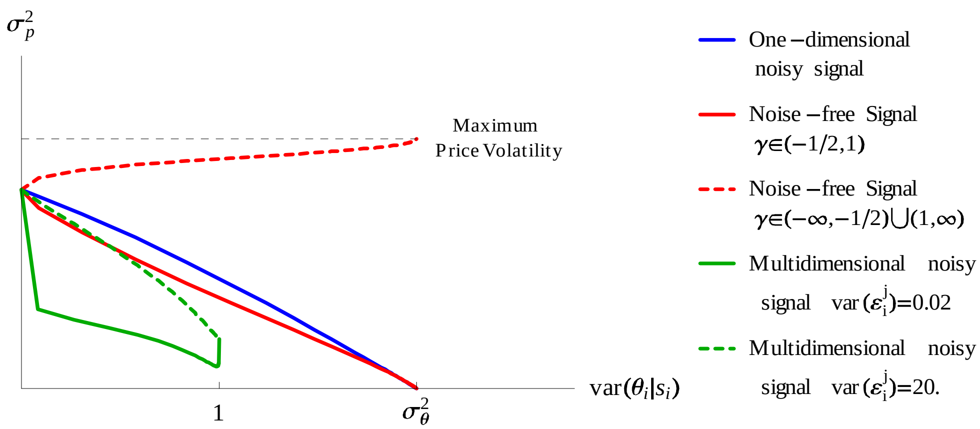

Figure 2: Information and Price Volatility pvol

not necessarily lead to a decrease in the price volatility. But in contrast to the behavior of the market power, we find that the price volatility remains within a small range. Indeed, Proposition 4 establishes a sharp upper bound on the price volatility in terms of underlying volatility of the payoff state.

ci 4 Market Power Close to Complete Information

We begin the analysis of the demand function competition with environments "close to" complete information. In fact, we first analyze demand function competition with complete information in Section 4.1. In the linear-quadratic environment, we find that under complete information there is a continuum of linear equilibria. This results, of course, echoes the findings in the earlier literature that stressed the multiplicity of outcomes under demand function competition. The complete information setting serves as benchmark. In Section 4.2, we then show that with incomplete information, anyone of the multiple complete information equilibrium can arise as a unique linear equilibrium with an arbitrarily small amount of incomplete information. We explicitly construct the information structure that achieves the unique equilibrium outcome. It is a noise-free signal of the format appearing in the previous section.

4.1 Complete Information

We begin by studying demand function under complete information. That is, every agent i observes the realization of all payoff shocks $\{\theta_i\}_{i\in N}$. We characterize the set of linear Nash equilibria. To this end, we denote the average (realized) payoff shock across agents by:

$$\bar{\theta} \triangleq \frac{1}{N}\sum_{i\in N}\theta_i \tag{7}$$

In an analogous way, any variable with an over-bar represents the average across agents.

Proposition 1 (Continuum of Complete Information Equilibria)
For every $\lambda \geq -1/2$, there exists an equilibrium in which agent i submits a linear demand function:

$$x_i(p) = \frac{1}{1+\lambda}\left(\theta_i - (1-\widehat{\gamma})\cdot\overline{\theta}\right) - \frac{\left(\frac{(\lambda+1)}{N-1}\left(\frac{1}{\beta}-\frac{1}{\lambda}\right)+1\right)}{(\lambda+\beta\cdot N+1)}\alpha - \frac{1}{N-1}\left(\frac{1}{\lambda}-\frac{1}{\beta}\right)\cdot p, \tag{8}$$

with:

$$\widehat{\gamma}(\lambda) \triangleq \frac{(\lambda+1)(\beta N-\lambda)}{\lambda(N-1)(\beta N+\lambda+1)}. \tag{9}$$

Proposition 1 characterizes a class of equilibria that are parametrized by a one-dimensional parameter $\lambda \in [-1/2, \infty)$. For every agent i, the *slope* of the demand function is independent of the realization of the payoff shock. By contrast, the intercept of the demand function depends on the individual payoff shock θ_i and the aggregate payoff shock $\overline{\theta}$ through the term

$$\frac{1}{1+\lambda}\left(\theta_i - (1-\widehat{\gamma})\cdot\overline{\theta}\right), \tag{10}$$

which represents a weighted difference between the individual and the aggregate payoff shock.

In equilibrium λ is equal to the equilibrium level of market power (as defined in (4)). To see this it is convenient to analyze the residual supply faced by agent i. We define the *residual supply* of agent i as:

$$r_i(p) \triangleq \frac{p-\alpha-\beta\sum_{j\neq i}x_j(p)}{\beta}. \tag{11}$$

If agent i submits demand $x_i(p)$, then the equilibrium price p^* is chosen to satisfy $x_i(p^*) = r_i(p^*)$. That is, $r_i(p)$ is how much quantity agent i can buy at price p given the demands submitted by all other agents. It is easy to check that:

$$\frac{\partial r_i(p)}{\partial p} = \frac{1}{\lambda}.$$

This implies that, if agent i increases the quantity he buys by Δq_i, the equilibrium price will increase by $\Delta q_i \cdot \lambda$. Hence, λ is the price impact of agent i, or how much agent i changes the realized price by changing his demand. This is also the level of market power (as defined in (4)).

It is important to highlight that the precise slope and intercept of the demand submitted by agent i is irrelevant from his own perspective. All agent i cares about is the intercept between his own demand $x_i(p)$ and the residual supply he faces $r_i(p)$. Nevertheless, there are multiple affine functions with different slopes that lead to the same intercept with $r_i(p)$. The slope of the demand submitted by agent i only changes the slope of the residual supply faced by other agents. Hence, the price impact of other agents.[3] By changing the slope of the demands agent submit, it is possible to generate different equilibria that lead to different outcomes.

It is useful to understand some extreme cases. For this, we note that the slope of the demand of agent i in equilibrium λ is given by:

$$m = \frac{1}{N-1}\left(\frac{1}{\lambda} - \frac{1}{\beta}\right). \tag{12}$$

This slope is the number that makes the market power λ consistent with the slope of the demand functions submitted by agents. We explain in detail how three equilibrium outcomes can arise: (i) price taking behavior ($\lambda \approx 0$), (ii) the Cournot competition outcome ($\lambda = \beta$), (iii) monopsony quantity ($\lambda = \beta \cdot N$).

First, we describe how agents can play an equilibrium in which each agent behaves "as if" he was a price taker ($\lambda \approx 0$). This equilibrium can be supported under demand function competition if every agent submits very elastic demands ($m \approx \infty$). In this case, each agent behaves as a price taker: any change in the quantity agent i buys is offset by the demand of other agents. Hence, agent i does not have an impact on the realized price. That is, if agent i withdraws demand to decrease the price, then this would change the quantity demanded by other agents, but the equilibrium price would not change.

[3] This has a similar intuition as how a Nash equilibrium in mixed strategies is determined in any game. In a mixed-strategy Nash equilibrium an agent is indifferent between the strategies over which he randomizes. Yet, he chooses the weights to leave the other agents indifferent. Similarly, in demand function competition, an agent chooses the slope so that the price impact of other agents is equal to the conjectured market power.

Second, we consider the Cournot competition outcome. If agents submit perfectly inelastic demands ($m = 0$), then agents are effectively submitting a fixed quantity. This is the equivalent behavior to Cournot competition. In this case, the market power of an agent is determined by the slope of the exogenous supply function ($\lambda = \beta$).

Finally, we show how the monopsony outcome can be realized with $\lambda = \beta \cdot N$. This is achieved most transparently in the symmetric case when the payoff shocks of all agents are equal ($\theta_1 = ... = \theta_N$).[4] If every agent submits a demand with a slope $m = -1/(\beta \cdot N)$, each agent commits to buying a large quantity of asset if the price is lower than the conjectured Nash equilibrium price. Note that each agent behaves as a supplier in the sense that they buy a higher quantity when the price is higher (or alternatively, they submit upward sloping demands). In this case, each agent incorporates that increasing the quantity that he buys will also induce other agents to buy more. This way, an agent internalizes the pecuniary externality that his demand has on the profits of other agents. Hence, the monopsony quantity is implemented.

The level of market power λ is has a profound impact on the equilibrium outcome. The equilibrium price p^* is equal to:

$$p^* = \frac{\beta \cdot N \cdot \bar{\theta} + (1 + \lambda) \cdot \alpha}{(1 + \lambda + \beta \cdot N)} \tag{13}$$ out1

and the aggregate equilibrium demand, q^*, and the individual equilibrium demand q_i^* are given by:

$$q^* = \frac{N\left(\bar{\theta} - \alpha\right)}{(1 + \lambda + \beta \cdot N)} \quad \text{and} \quad q_i^* = \frac{\bar{\theta} - \alpha}{(1 + \lambda + \beta \cdot N)} + \frac{\theta_i - \bar{\theta}}{1 + \lambda} \tag{14}$$ out2

If agents behave as price takers then we attain the competitive outcome with $\lambda = 0$. As the market power λ increases, the agents withdraw demand relative to the competitive outcome. This implies that the price is lower (as demand is less). This also implies that the dispersion in the quantities bought by agents ($q_i - q_j$) decreases. As the market power λ increases, agents have more price impact, and hence their demands become less responsive to their payoff shocks.

ii 4.2 Incomplete Information

We now study demand function competition with incomplete information. In this subsection, we shall restriction our attention to a class of $(N + 1)$ −dimensional noise free signals for every agent. Towards this end, we decompose the individual payoff shock as the sum of two independent shocks:

$$\theta_i \triangleq \eta_i + \phi_i,$$

[4] It is easy to check that, if $\theta_1 = ... = \theta_N$, then (out1 13) maximize $\sum_{i \in N} u(\theta_i, q_i, p)$.

where the sets of payoff shocks $\{\eta_i\}_{i \in N}$ and $\{\phi_i\}_{i \in N}$ are independent across the sets. These shocks are all symmetrically and normally distributed. The distribution of shocks η_i and ϕ_i must satisfy the following conditions:

$$\sigma_\theta^2 = \sigma_\eta^2 + \sigma_\phi^2,$$

and

$$\rho_{\theta\theta} \cdot \sigma_\theta^2 = \rho_{\eta\eta} \cdot \sigma_\eta^2 + \rho_{\phi\phi} \cdot \sigma_\phi^2,$$

to guarantee that the joint distribution of $\{\eta_i + \phi_i\}_{i \in N}$ is the same as the joint distribution of payoff shocks $\{\theta_i\}_{i \in N}$ which define the payoff environment.

We assume that the realization of the shocks

$$\{\eta_i\}_{i \in N}$$

are common knowledge among the agents, and they form the first N signals for every agent. That is, every agent observes the realization of all shocks $\{\eta_i\}_{i \in N}$. Additionally, agent i observes a signal that represent a weighted difference between his idiosyncratic and the common payoff shock ϕ_i:

$$s_i = \phi_i - (1 - \gamma)\bar{\phi}. \tag{15}$$

nois

Signal s_i is a noise-free signal. By observing s_i, agent i cannot perfectly infer the realization of ϕ_i. Instead, if agent i could observe all signals $\{s_i\}_{i \in N}$, then agent i would be able to perfectly infer ϕ_i (and hence, θ_i). We call the parameter γ the *confounding* parameter, as it measures how much the signal confounds the payoff shock of agent i (ϕ_i) with the payoff shock of all other agents (aggregated in $\bar{\phi}$).

For the results in this subsection, we will frequently consider the case when the variance of the second component, σ_ϕ^2, (and the mean μ_ϕ) is arbitrarily close to zero, or

$$\mu_\phi \approx 0, \sigma_\phi^2 \approx 0. \tag{16}$$

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That is, the component shock ϕ_i is arbitrarily small, and thus the payoff shock θ_i is almost completely represented by the component η_i, or $\theta_i \approx \eta_i$. If we assume that (16) is satisfied, then we have that:

$$\text{var}(\theta_i | s_i, \{\eta_k\}_{k \in N}) \approx 0 \text{ and } \text{var}(\theta_j | s_i, \{\eta_k\}_{k \in N}) \approx 0.$$

That is, agent i knows almost perfectly the realization of all the payoff shocks $\{\theta_k\}_{k \in N}$ just by observing his private $N + 1$ dimensional signals. While (16) is useful to interpret the results, this is not used in any of the proofs.

Proposition 2 (Unique Linear Equilibrium with Noise-Free Signals)
For every $\lambda \geq -1/2$, if agents observe a $N+1$ dimensional noise-free signal with $\gamma = \widehat{\gamma}$, then there is a unique linear equilibrium in which the equilibrium outcome is attained by (13).

Proposition 2 shows that the outcome of every linear Nash equilibrium that arises under complete information is also the outcome of the unique linear Nash equilibrium under a noise-free information structure. Noteworthy, this information structure can be arbitrarily close to complete information. That is, the realization of the payoff shocks $\{\theta_i\}_{i \in N}$ can be arbitrarily close to complete information, nevertheless the outcomes of the demand function competition game can be radically different.

There is a mechanic way of understanding why incomplete information allows us to select any equilibrium that arises under complete information. By providing the agents with a noise-free signal, we give the agents information that is consistent with only one of the intercepts that arises under the multiple complete information equilibria. Hence, incomplete information shrinks the set of available strategies of the agents, by restricting the set of possible intercepts they can submits. To be more specific, it restricts the measurability of the intercept of the demand function with respect to the realization of the payoff shocks of the agents. Once agents are restricted to submit one specific intercept, they also adjust the slope to be the one that would be consistent under one of the equilibria under complete information. In equilibrium, agent i submits a demand function:

$$x_i(p) = \frac{1}{1+\lambda}\left(s_i + \eta_i - (1-\gamma)\cdot\bar{\eta}\right) - \frac{\left(\frac{(\lambda+1)}{N-1}\left(\frac{1}{\beta}-\frac{1}{\lambda}\right)+1\right)}{(\lambda+\beta\cdot N+1)}\alpha - \frac{1}{(N-1)}\left(\frac{1}{\lambda}-\frac{1}{\beta}\right)\cdot p. \tag{17}$$

Note that (17) is the same as (8), in fact:

$$s_i + \eta_i - (1-\gamma)\cdot\bar{\eta} = \theta_i - (1-\gamma)\cdot\bar{\theta}.$$

This is the unique equilibrium under complete information in which the strategy of agent i is measurable with respect to agent i's signal (s_i).

The example in Section 3 showed that there are information structures close to complete information in which the market power can be any number in $\lambda \in [\lambda_{KM}, \beta \cdot N]$. Proposition 2 generalizes this intuition. It shows that in fact any market power that is consistent with being an outcome under complete information can also be an outcome under a small amount of incomplete information. Note that in Section 3 we only discussed the market power in an information structure close to complete information, but Proposition 2 generalizes the result by showing that in fact the entire set of outcome is the same as under complete information. Thus, the noise free signals provide a set of information structures in which the entire set of outcomes is identical to the set of outcomes under complete information.

5 Equilibrium Restrictions Across All Signals

The analysis of the complete information environment, and a small class of nearby incomplete information structures, showed a wide range of possible equilibrium outcomes. The complete information environment displayed a continuum of equilibrium outcomes, and thus could only provide very weak equilibrium predictions. The analysis of nearby incomplete information environment showed that a possible cause of the weak equilibrium predictions is the sensitivity of equilibrium behavior to the nature of the private information. Thus, with private information we established uniqueness of the (linear) equilibrium, but small changes in the signal structure allowed us to recover the entire set of complete information equilibrium.

Yet, as we only investigated a small class of signal structure, noise free and almost complete information environment, the question remains how much larger is the set of all possible equilibrium outcomes under all possible multi-dimensional (normal) information structures. In other words, are there any equilibrium restrictions that we establish across all information structures. This is the subject of this section. We now allow for arbitrary multi-dimensional information structure but retain the normality and symmetry across agents. We initially provide a description of the set of outcomes in terms of the induced joint distribution of payoff shocks, quantities and price in Section 5.1. We then provide necessary conditions on the joint distribution in order for it to be consistent with a linear Nash equilibrium. We distinguish between purely statistical restrictions, presented in Section 5.2, and equilibrium restriction, presented in Section 5.3. We use these restrictions to jointly provide robust predictions on the set of outcomes that hold across all information structures in Section 5.4. In Section ?? we then show that the these necessary conditions are also sufficient conditions, and hence completely characterize the set of all equilibrium outcomes.

5.1 Distribution of Outcomes

We provide a description of the equilibrium outcomes from an ex ante perspective. We say that the joint distribution of payoff shocks $(\theta_i, \bar{\theta})$ and outcome quantities (q_i, p) form a joint outcome distribution $(\theta_i, \bar{\theta}, q_i, p)$ of the demand function competition if the distribution is induced by an equilibrium outcome. The advantage of the description in terms of distributions of equilibrium outcomes is that it does not depend on the detail description of the signals. That is, two information structures may induce different beliefs and may induce different realization over outcomes ex post, but as long as the distribution of outcomes ex ante is the same, these two information structures will be indistinguishable in terms of outcomes.

We continue to restriction attention to multi-dimensional Gaussian distributions. The joint distribution is hence completely characterized by the first and second moments:

$$\begin{pmatrix} \theta_i \\ \bar{\theta} \\ q_i \\ p \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ \mu_\theta \\ \mu_q \\ \mu_p \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \rho_{\bar{\theta}\theta}\sigma_\theta\sigma_{\bar{\theta}} & \rho_{q\theta}\sigma_\theta\sigma_q & \rho_{p\theta}\sigma_\theta\sigma_p \\ \rho_{\theta\bar{\theta}}\sigma_\theta\sigma_{\bar{\theta}} & \sigma_{\bar{\theta}}^2 & \rho_{q\bar{\theta}}\sigma_{\bar{\theta}}\sigma_q & \rho_{p\bar{\theta}}\sigma_{\bar{\theta}}\sigma_p \\ \rho_{q\theta}\sigma_\theta\sigma_q & \rho_{q\bar{\theta}}\sigma_{\bar{\theta}}\sigma_q & \sigma_q^2 & \rho_{qp}\sigma_q\sigma_p \\ \rho_{p\theta}\sigma_\theta\sigma_p & \rho_{p\bar{\theta}}\sigma_{\bar{\theta}}\sigma_p & \rho_{qp}\sigma_q\sigma_p & \sigma_p^2 \end{pmatrix}\right). \tag{18}$$

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Some of the coefficients are part of the distribution of payoff shocks, and hence, they are exogenously determined: (i) the expected payoff shock of every agent (μ_θ), (ii) the expected average payoff shock $(\mu_{\bar{\theta}})$, (iii) the variance of the payoff shock of an agent σ_θ^2, (iv) the variance of the average payoff shock $\sigma_{\bar{\theta}}^2$, and (v) the correlation between the payoff shock of an agent and the average payoff shock $(\rho_{\theta\bar{\theta}})$. The rest of the coefficients are endogenously determined by the equilibrium outcome.

The joint distribution of outcomes thus contains nine endogenous variables: (i) the mean quantity bought by agent (μ_q) , (ii) the mean price (μ_p), (iii) the variance of the quantity bought by an agent (σ_q^2), (iv) the price volatility (σ_p^2), (v) the correlation between the price and the payoff shock of agent i $(\rho_{p\theta})$, (vi) the correlation between the price and the average payoff shock $(\rho_{p\bar{\theta}})$, (vii) the correlation between the quantity bought by an agent and the payoff shock of this agent $(\rho_{q\theta})$, $(viii)$ the correlation between the quantity bought by an agent and the average payoff shock $(\rho_{q\bar{\theta}})$, (ix) the correlation between the quantity bought by an agent and the price (ρ_{qp}).

sro 5.2 Statistical Restrictions on Outcomes

It is convenient to first provide the restrictions on the set of outcomes that are simply statistical conditions from the law of iterated expectations. We begin by providing conditions on the exogenous variables.

lemmexo **Lemma 1 (Statistical Conditions on Exogenous Variables)**
Every distribution of payoff shocks must satisfy:

$$\mu_\theta = \mu_{\bar{\theta}}, \quad \rho_{\theta\bar{\theta}}\sigma_\theta = \sigma_{\bar{\theta}}. \tag{19}$$

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Lemma 1 provides restrictions on the distribution of payoff shocks. These conditions are purely statistical and are implied by the fact that payoff shocks are symmetrically distributed. In particular, the expected payoff shock of agent i (μ_θ) is equal to the expected average payoff shock ($\mu_{\bar{\theta}}$). Similarly, the variance of the payoff shocks, the variance of the average payoff shock, and the correlation between the payoff shock and the average payoff shock must satisfy a consistency requirement.

We now provide the respective statistical conditions on the endogenous variables.

Lemma 2 (Statistical Conditions on Endogenous Variables)
Every distribution of outcomes must satisfy:

$$\mu_p = \alpha + \beta \cdot N \cdot \mu_q, \quad \rho_{p\theta} = \rho_{p\bar{\theta}}\rho_{\theta\bar{\theta}}, \quad \rho_{q\bar{\theta}} = \rho_{p\bar{\theta}} \cdot \rho_{qp}, \quad \rho_{qp} \cdot \sigma_q \cdot \beta \cdot N = \sigma_p. \tag{20}$$

The four equations in (20) are derived exclusively from the assumption that the payoff shocks and quantities $(\theta_1, ..., \theta_N, q_1, ..., q_N)$ are symmetrically distributed and that the price is a linear function of the quantities (see (??)). Thus, the endogenous coefficients $(\mu_p, \sigma_q, \sigma_p, \rho_{p\bar{\theta}}, \rho_{q\theta})$ and the restrictions imposed by (20) are sufficient to fully identify an outcome distribution.

5.3 Equilibrium Restrictions on Outcomes

We now provide necessary conditions for a joint distribution given by (18) to be consistent with equilibrium. We begin with conditions for the first moments.

Lemma 3 (Distribution of Outcomes: Mean)
The first moments of any linear Nash equilibrium must satisfy:

$$\mu_p = \frac{1}{1 + \beta N + \lambda} \cdot (N \cdot \beta \cdot \mu_\theta + (1 + \lambda) \cdot \alpha), \tag{21}$$

for some $\lambda \geq -1/2$.

The first moment of the equilibrium distribution of outcomes, in terms of price, and through the linear supply function, also the quantities are thus determined exclusively by the market power λ. Interestingly, this implies by Proposition 1 that all the possible first moments that can be achieved by any information structure can also be achieved by an equilibrium under complete information. In other words, the analysis of the complete information environment is sufficient to characterize all the first moments that can be achieved in any incomplete information environment.

To complete describe the second moments, it is useful to define the idiosyncratic components in the payoffs shocks and the demanded quantities:

$$\Delta\theta_i \triangleq \theta_i - \overline{\theta} \text{ and } \Delta q_i \triangleq q_i - \overline{q}.$$

The variable $\Delta\theta_i$ is the difference between the payoff shock of agent i and the average payoff shock (and analogously Δq_i). The correlation $\rho_{\Delta q \Delta\theta}$ is an economically important quantity. It is a measure of how efficiently the good is allocated across agents. More precisely, given a dispersion in the quantities bought by agents $\{\Delta q_i\}_{i\in N}$, $\rho_{\Delta q\Delta\theta}$ measures how much of this dispersion is caused by fundamental shocks and how much is caused by noise.

polol2 **Lemma 4 (Distribution of Outcomes: Variance)**
The second moments of any linear Nash equilibrium must satisfy:

$$\sigma_p^2 = \rho_{p\bar{\theta}}^2 \cdot \left(\frac{1}{1+\lambda+\beta\cdot N}\right)^2 \cdot (\beta\cdot N)^2 \cdot \sigma_{\bar{\theta}}^2, \tag{22}$$

$$\sigma_q^2 = \rho_{p\bar{\theta}}^2 \cdot \left(\frac{1}{1+\lambda+\beta\cdot N}\right)^2 \cdot \sigma_{\bar{\theta}}^2 + \rho_{\Delta q\Delta\theta}^2 \cdot \left(\frac{1}{1+\lambda}\right)^2 \cdot (\sigma_\theta^2 - \sigma_{\bar{\theta}}^2), \tag{23}$$

and the correlation coefficient must satisfy:

$$\rho_{\Delta q\Delta\theta} = \frac{(\rho_{q\theta} - \rho_{p\bar{\theta}}\rho_{pq}\rho_{\theta\bar{\theta}})}{\sqrt{(1-\rho_{qp}^2)(1-\rho_{\theta\bar{\theta}}^2)}}, \tag{24}$$

with $\lambda \in [-1/2, \infty)$.

Proposition 4 characterizes the variance of the quantity bought by an agent (σ_q^2) and the price volatility (σ_p^2). The variance of the price is determined by three term: (i) the variance of the average payoff shock of agents ($\sigma_{\bar{\theta}}^2$), (ii) the market power (λ), and (iii) the correlation between the price and the common shock ($\rho_{\bar{\theta}p}$). The price co-moves with the average payoff shock. Hence, if the average payoff shock is more volatile, this induces a higher price volatility. Similarly, an increase in the correlation between the price and the average payoff shock implies that the price is more responsive to the common shock and less responsive to idiosyncratic and noise terms. Hence, this also increases the price volatility. Finally, the market power λ just decreases the amount each agent trades, and hence, an increase in market power decreases the price volatility.

The total variance of the quantity q_i bought by agent i can be explained by two different quantities:

$$\text{var}(q_i) = \text{var}(\bar{q}) + \text{var}(\Delta q_i),$$

namely the variance of the average quantity bought by the agents and the dispersion across agents in the quantities bought. The variance of the average quantity bought by the agents equals:

$$\sigma_{\bar{q}}^2 = \rho_{p\bar{\theta}}^2 \cdot \left(\frac{1}{1+\lambda+\beta\cdot N}\right)^2 \cdot \sigma_{\bar{\theta}}^2.$$

As the average quantity $\bar{q}$ is collinear with p, with a constant of proportionality equal to $\beta \cdot N$, the variance of the average quantity is explained in the same way as the price volatility. The dispersion across agents of the quantities bought by the agents is equal to:

$$\sigma_{\Delta q}^2 = \rho_{\Delta q \Delta\theta}^2 \cdot \left(\frac{1}{1+\lambda}\right)^2 \cdot \sigma_{\Delta\theta}^2.$$

The dispersion across agents of the quantities bought by the agents is determined by three terms: (i) the dispersion of the agents' payoff shocks ($\sigma_{\Delta\theta}^2$), (ii) the market power (λ), and (iii) the correlation between the dispersion in the quantities bought by agents and the dispersion of the payoff shocks ($\rho_{\Delta q\Delta\theta}$). This allows us to characterize the restrictions on the correlations of the joint distribution.

Lemma 5 (Distribution of Outcomes: Correlation)
For every linear equilibrium the correlations of the distribution must satisfy:

$$\rho_{p\bar{\theta}} \in [0,1], \tag{25}$$

and

$$\rho_{\Delta q\Delta\theta} \in [0,1]. \tag{26}$$

Given our original description of the joint distribution in (18) and the statistical restrictions in (20), we finally consider two correlations that are left to be determined: $(\rho_{p\bar{\theta}}, \rho_{q\theta})$. Condition (25) indicates that the correlation between the price and the average payoff shock must be positive. Condition (26) provides an implicit restriction $\rho_{q\theta}$ that depends on $\rho_{p\bar{\theta}}$, ρ_{pq}, $\rho_{\theta\bar{\theta}}$. In turn, the correlation ρ_{pq} is determined by the variances σ_p and σ_q (as in (20)). To interpret condition (26) we recall the expression for $\rho_{\Delta q\Delta\theta}$ in (24). It follows that the restrictions for $\rho_{\Delta\theta\Delta q}$ and $\rho_{p\bar{\theta}}$ are symmetric and independent of each other. (25) and (26) state that the quantities and the price must be positively correlated with fundamentals. We thus realize that the equilibrium outcome is determined exclusively by three endogenous variables.

Proposition 3 (Determining the Distribution of Outcomes)
For any $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q\Delta\theta}) \in (-1/2, \infty) \times [0,1] \times [0,1]$*, the distribution of outcomes (18) is completely determined by (20) - (23).*

Proposition all 3 characterizes the set of all outcome distribution in terms of three parameters $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q \Delta\theta})$ which will be determined by the signal structure. While we provide intervals which restrict the set of values of these three parameters, we have not yet provided any joint restriction on them. A priori, it is not clear whether any set of endogenous parameters $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q \Delta\theta}) \in [-1/2, \infty) \times [0,1] \times [0,1]$ are consistent with being a outcome distribution for some information structure. Yet, in the following section, we show that this is true. That is, for any parameters $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q \Delta\theta}) \in (-1/2, \infty) \times [0,1] \times [0,1]$ there exists an information structure that has a linear Nash equilibrium that induces a distribution of outcomes given by these three parameters.

rp 5.4 Robust Predictions

We use the restrictions on the distribution of outcomes to derive predictions that hold across all information structures. We begin by providing bounds on the price volatility:

prp **Proposition 4 (Price Volatility)**
In any linear Nash equilibrium, the price volatility must satisfy:

$$\sigma_p^2 \leq \left(\frac{\beta \cdot N}{1/2 + \beta \cdot N}\right)^2 \cdot \sigma_{\bar{\theta}}^2.$$

Proposition prp 4 provides a bound on the maximum volatility that can be achieved for any information structure. The most interesting thing to highlight is that the price volatility is bounded by the size of the average shock across agents. That is, if $\sigma_{\bar{\theta}}^2 \to 0$, then $\sigma_p^2 \to 0$.

We now study the dispersion of the quantities. For this, we characterize the bounds on the variance of the idiosyncratic component in the demand, $\Delta q_i = q_i - \overline{q}$. The variance of Δq_i is a measure of how dispersed the quantities bought by agents are.

prq **Proposition 5 (Dispersion of Quantities)**
In any linear Nash equilibrium, the dispersion of quantities must satisfy:

$$\sigma_{\Delta q}^2 \leq \frac{1}{4}\sigma_{\Delta\theta}^2.$$

As in the case of price volatility, the dispersion in the quantities bought by agents is bounded by the dispersion of the payoff shocks, and if $\sigma_{\Delta\theta}^2 \to 0$, then $\sigma_{\Delta q}^2 \to 0$.

6 Informational Decentralization

We now establish that for any outcome distribution that satisfies (20) - (26) there exists an information structure that realizes this distribution as a Nash equilibrium. Thus, the conditions on the distribution of outcomes we previously characterized are not only necessary, but also sufficient. In particular, the volatility bounds that we provided in the previous section are in fact sharp and can be achieved for some information structure.

We provide two different decentralizations. We first show that all distribution of outcomes can be implemented by selecting an equilibrium when agents observe only public signals. We then show that the distribution of outcomes can also be implemented as a unique linear Nash equilibrium when agents observe one-dimensional signals. These two distinct decentralizations allow us to provide different intuitions on how the equilibrium outcomes can be achieved and how the information structure determines the equilibrium outcome.

6.1 Multi-Dimensional Public Signals

We assume that all agents observe the same set of signals. In contrast to Section 4, agents observe the payoff shocks through noisy signals. Every agent observes N signals labeled $\{s_j\}_{j\in N}$, where s_i is given by:

$$s_i = \theta_i + \varepsilon_i. \tag{27}$$

The term ε_i is a noise term that is independent of all payoff shocks, has a variance of σ_ε^2, and a correlation across signals of $\rho_{\varepsilon\varepsilon}$ (that is, $\mathrm{corr}(\varepsilon_i, \varepsilon_j) = \rho_{\varepsilon\varepsilon}$).

Proposition 6 (Equilibrium Outcomes with Multi-Dimensional Signals)
For every distribution (18) that satisfies (20)- (26), there exists a set of public signals that have a linear Nash equilibrium that induces this distribution.

Thus every distribution of outcomes can be decentralized by providing agents with public signals under some linear Nash equilibrium. As any distribution of outcomes is determined by three coefficients $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q \Delta\theta}) \in (-1/2, \infty) \times [0,1] \times [0,1]$, it follows that we map the primitives of the information structure and the equilibrium selection into these three parameters.

The market power λ is determined by the equilibrium selection. This follows the same intuition as the equilibrium selection under complete information (see Section 4). On the other hand, the correlations $\rho_{p\bar{\theta}}$ and $\rho_{\Delta q \Delta \theta}$ are determined by the noise terms in (27). In particular,[5]

$$\rho_{p\bar{\theta}} = \text{corr}(\mathbb{E}[\bar{\theta}|\{s_i\}_{i\in N}], \bar{\theta}) \quad \text{and} \quad \rho_{\Delta q \Delta \theta} = \text{corr}(\mathbb{E}[\Delta\theta_i|\{s_i\}_{i\in N}], \Delta\theta_i). \tag{28}$$

That is, $\rho_{p\bar{\theta}}$ is determined by how precise the collection of all signals $\{s_i\}_{i\in N}$ is about the average shock $\bar{\theta}$. On the other hand, $\rho_{\Delta q \Delta \theta}$ is determined by how precise the collection of all signals $\{s_i\}_{i\in N}$ is about the orthogonal component of the payoff shock of agent i, $\Delta\theta_i$.

6.2 One-Dimensional Private Signals

We now consider a restricted class of one-dimensional signals. We assume that agent i observes a one-dimensional signal s_i given by:

$$s_i = \theta_i + \varepsilon_i + (\gamma - 1)(\bar{\theta} + \bar{\varepsilon}). \tag{29}$$

The term ε_i is a noise term that is independent of all payoff shocks $\{\theta_i\}_{i\in N}$, has a variance of σ_ε^2, and a correlation $\rho_{\varepsilon\varepsilon}$ across signals. There are two changes with respect to the information structure in which agents observe public signals (as in the previous section). First, agent i observes only a one-dimensional signal s_i, which is a private signal. Second, each signal has an additional component that is weighted by the confounding parameter γ. The confounding parameter γ would be innocuous if agent i could observe all signals $\{s_j\}_{j\in N}$ in (29). Since by taking the average of these signals, an agent would learn:

$$\frac{1}{N}\sum_{i\in N} s_i = \gamma(\bar{\theta} + \bar{\varepsilon}).$$

Hence, agent i could learn $\bar{\theta} + \bar{\varepsilon}$ by simply adding the signals regardless of the level of $\gamma \neq 0$. It is only when agent i observes only his one-dimensional private signal s_i that the confounding parameter will have an impact on the equilibrium outcome.

Proposition 7 (Equilibrium Outcomes with One-Dimensional Signals)
For every distribution (18) that satisfies (20) - (26), there exists a one-dimensional signal given by (29) that has a unique linear Nash equilibrium that induces this distribution.

[5]To compute the correlations in (28) in terms of the variances it is easy to check that:

$$corr(\mathbb{E}[\bar{\theta}|\{s_i\}_{i\in N}], \bar{\theta}) \;=\; \sqrt{\frac{\sigma_{\bar{\theta}}^2}{\sigma_{\bar{\theta}}^2 + \sigma_{\bar{\varepsilon}}^2}} \quad ; \quad corr(\mathbb{E}[\Delta\theta_i|\{s_i\}_{i\in N}], \Delta\theta_i) = \sqrt{\frac{\sigma_\theta^2 - \sigma_{\bar{\theta}}^2}{\sigma_\theta^2 - \sigma_{\bar{\theta}}^2 + \sigma_\varepsilon^2 - \sigma_{\bar{\varepsilon}}^2}}.$$

Proposition 7 shows that every distribution of outcomes can be decentralized as the unique Nash equilibrium when agents observe one-dimensional signals. Hence, we can map the primitives of the information structure and the equilibrium selection into the three parameters that determine an equilibrium outcome $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q \Delta\theta}) \in (-1/2, \infty) \times [0,1] \times [0,1]$. The correlations $\rho_{p\bar{\theta}}$ and $\rho_{\Delta q\Delta\theta}$ are determined by (28). This is determined the same way as with public signals.

To understand how the equilibrium market power is determined, we define:

$$\varphi_i \triangleq \mathbb{E}[\theta_i | s_1, ..., s_N].$$

That is, φ_i is the expected value of θ_i conditional on all the signals. Now, note that (29) can be written as follows:

$$s_i \propto (\varphi_i + (\tilde{\gamma} - 1)\bar{\varphi}), \tag{30}$$

where

$$\tilde{\gamma} \triangleq \gamma \cdot \frac{\sigma^2_{\bar{\theta}} + \sigma^2_{\bar{\varepsilon}}}{\sigma^2_{\bar{\theta}}} \cdot \frac{\sigma^2_{\theta} - \sigma^2_{\bar{\theta}}}{\sigma^2_{\theta} - \sigma^2_{\bar{\theta}} + \sigma^2_{\varepsilon} - \sigma^2_{\bar{\varepsilon}}}. \tag{31}$$

We can see that (30) has the same form as (15). Hence, $\tilde{\gamma}$ is the right confounding parameter once we incorporate the fact that the collection of all signals does not allow to perfectly predict θ_i, but that the best possible prediction is φ_i.

The market power λ is determined by γ and the noise terms as follows. Let $\widehat{\gamma}^{-1}(\cdot)$ be the inverse function of $\widehat{\gamma}(\lambda)$ (as defined in (9)) with values greater or equal than $-1/2$. The inverse restricted to values greater or equal than -1/2 is unique. Then the equilibrium market power is given by $\widehat{\gamma}^{-1}(\tilde{\gamma})$. Hence, the market power is determined the same way as in the case with noise-free signals, but taking into account the best prediction of θ_i conditional on all the signals.

7 Beyond Demand Function Competition

In the analysis of demand function competition we found that the equilibrium outcome can be rather sensitive to the private information. Yet, this did not imply that there were no equilibrium restriction. We found that will the second moments of the equilibrium could be located in a narrow range, and within a sharp upper bound. By contrast, we found that there was almost no restriction on the first moment of the equilibrium outcome. This naturally raises the question whether other market mechanism may impose more or perhaps different kind of restrictions on the equilibrium outcome. More generally, we might ask whether our methods of analyzing all information structure at once could be applicable to other market mechanisms. In this section we take a modest step in this direction and analyze a class of fixed slope mechanisms that includes as special or limit case, Cournot and Bertrand competition, respectively. In Section 7.1 we show that every fixed slope mechanism has a unique complete information equilibrium. Moreover, the first moment is the same across all possible information structures with incomplete information. Thus, fixed slope mechanisms allow much sharper predictions regarding the first moments of the equilibrium distribution. In Section 7.2 we then consider the second moment restriction for one specific fixed slope mechanism, the Cournot competition. By contrast to the first moment, we find that Cournot competition leads to a much larger set of second moments of the equilibrium. This would suggest that in the choice of market mechanism in the presence of concerns for robustness, there is trade-off across market mechanism in which some trading mechanism may offer favorable bounds on first moments, other trading mechanism favorable bounds on second moments of the equilibrium distribution.

7.1 Market Mechanisms

We study a class of mechanisms described by a single parameter $\kappa \in \mathbb{R}_+$ which leads to a fixed slope in the demand quantity of agent i. In the mechanism each agent i is asked to submit a bid $p_i \in \mathbb{R}_+$. The equilibrium price is then determined by the average bid:

$$p^* = \frac{1}{N}\sum_{i \in N} p_i, \tag{32}$$

and the resulting quantity allocation is given by

$$q_i = \frac{p^*}{\beta N} + \frac{1}{\kappa N}(p_i - p^*) \tag{33}$$

The price is determined by the average price submitted by all agents. Given an equilibrium price p^*, market clearing dictates that agents must buy on average a quantity $p^*/(\beta \cdot N)$. The quantity bought by agent i is equal to the average quantity bought by agents plus an additional term that depends on the difference between the price submitted by agent i and the average realized price $(p_i - p^*)$. The quantity purchased by agent i is determined by the difference between the bid p_i and the price p^* and the rate is determined by κ. If $\kappa \approx 0$, then a small difference between the equilibrium price and the bid p_i offered by agent i leads to a large difference between the individual quantity q_i and the average quantity bought by all agents. If $\kappa \to \infty$, then all agents buy similar quantities, regardless of the differences in the bids submitted.

This class of mechanisms contains the Cournot and Bertrand competition as special cases. If $\kappa = \beta$, then we are in Cournot competition. If $\kappa \to 0$, then we approach Bertrand competition. We characterize the unique equilibrium of this game when agents have complete information.

Proposition 8 (Equilibrium of Fixed-Slope Competition)
The fixed-slope game with parameter κ has a unique equilibrium in which the outcome is given by

$$p^* = \frac{\beta \cdot N \cdot \bar{\theta} + (1+\lambda) \cdot \alpha}{(1+\lambda+\beta \cdot N)} \tag{34}$$

and the aggregate equilibrium demand, q^*, and the individual equilibrium demand q_i^* are given by:

$$q^* = \frac{N\left(\bar{\theta} - \alpha\right)}{(1+\lambda+\beta \cdot N)} \quad and \quad q_i^* = \frac{\bar{\theta} - \alpha}{(1+\lambda+\beta \cdot N)} + \frac{\theta_i - \bar{\theta}}{1+\lambda} \tag{35}$$

with $\lambda = N \cdot \kappa \cdot \beta / (\kappa + (N-1)\beta)$.

Proposition 8 thus establishes that any of the multiple equilibria that can arise in the demand function competition game, see Proposition 1 and the equilibrium outcomes described in (13) and (14), can arise as unique equilibria of a specific fixed slope mechanism. The set of mechanisms (parametrized by κ) correspond to a class of games in which agents submit demand functions, but they cannot choose the slope of the demand function they submit. To see this, note that (32) and (33) imply the following:

$$p_i = (\kappa \cdot N)\left(q_i - (\frac{1}{\beta N} - \frac{1}{\kappa \cdot N})p^*\right).$$

That is, the bid of agent i determines a linear combination between the equilibrium price (p^*) and the quantity q_i that agent i buys. By changing κ we change this linear combination. This is equivalent to changing the slope of the demand function submitted by agents.

The intuition of the result is as follows. In demand function competition there is a continuum of equilibria. In each of these equilibria agents are required to submit a demand function that specifies a slope and an intercept. The intercept must be a function of the realized payoff shocks. As long as they can choose these two objects freely, there will be multiple equilibria. This is because from the perspective of an individual agent for a fixed residual supply, multiple combinations of a slope and a demand give rise to the same equilibrium price and quantities. By fixing the slope of the demand function they submit, we can select any of these equilibria.

A fixed-slope mechanism corresponds to mechanically shrinking the set of available strategies to agents. This implies that only a subset of the equilibrium strategies are left for the agents, and hence only one equilibrium of the demand function competition game is consistent with the set of available strategies in the fixed slope mechanism. Note that, in general, by shrinking the set of available strategies one can trivially reduce the amount of Nash equilibria that a game has. Yet, it is not always the case that shrinking the strategy space of agents does not also add additional Nash equilibria. This is because in general we are also shrinking the set of available deviations for agents, which a priori, could reduce the number of Nash equilibria. Yet, as we showed in Proposition mechpr 8, in the fixed-slope mechanism the set of Nash equilibria is a proper subset of the set of Nash equilibria that arise under demand function competition.

An alternative interpretation of mechanism can be provided in terms of the quantities demanded. We define

$$q_i \triangleq p_i/(\kappa \cdot N).$$

We can write (mech1 32) and (mech2 33) as follows:

$$p^* = \kappa \sum_{i \in N} q_i, \tag{36}$$ mech

and

$$q_i^* = q_i + \frac{1}{N}\left(\frac{\kappa}{\beta} - 1\right) \sum_{i \in N} q_i \tag{37}$$ mech

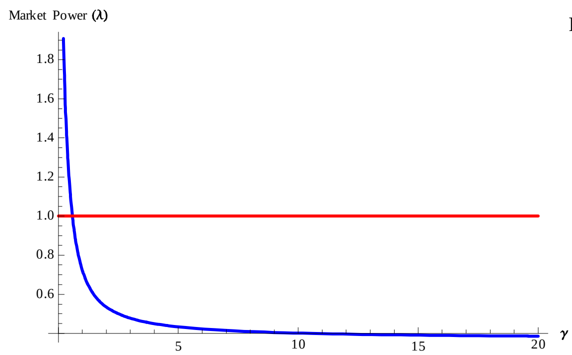
Under this alternative formulation the price is proportional to the bids q_i submitted by all agents. The quantity bought by agent i is related to the bid submitted by agent i, q_i, but it is demeaned in an amount that is propositional to $(\kappa/\beta - 1)$.

cdf 7.2 Cournot vs. Demand Function Competition

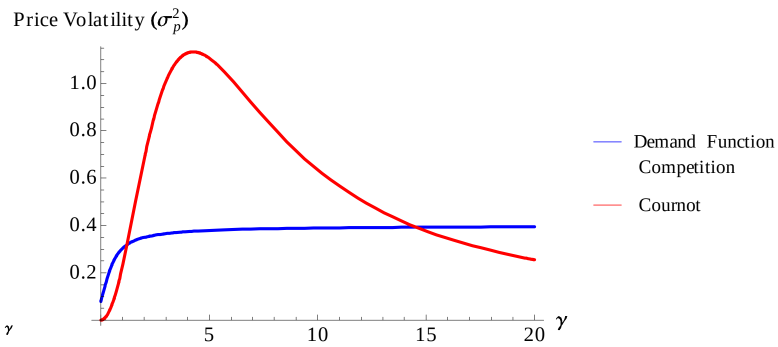
We now compare Cournot competition with demand function competition. We begin by comparing the equilibrium outcomes for a class of one one-dimensional signals:

$$s_i = \theta_i + (\gamma - 1)\bar{\theta}.$$

In Figure aasdd ?? and Figure fpv 7.2 we compare the equilibrium market power and the induced price volatility respectively under Cournot and demand function competition.



Comparison between Demand Function and Cournot Competition: Market Power



Comparison between Demand Function and Cournot Competition: Volatility

There are two stark differences. First, the level of market power is constant in γ for Cournot competition, but it is decreasing in γ for demand function competition. The second stark difference is regarding the level of price volatility. In demand function competition the price volatility is increasing in γ and it is bounded. In Cournot competition the price volatility may be much higher than under demand function competition. In fact, under Cournot competition, we have that price volatility (σ_p^2) can grow without bounds, even if $\sigma_{\bar{\theta}} \to 0$ (see Bergemann, Heumann, and Morris (2015))). For example, if we consider independent shocks ($\rho_{\bar{\theta}\theta} = 0$) and a large market ($N \to \infty$), then price volatility will be equal to 0 in demand function competition, while price volatility can be proportional to the size of the variance of the payoff shocks in Cournot competition.

The study of both forms of market competition under noise free signals suggests that both mechanisms respond very differently to the degree of asymmetric information. In fact, we can generalize this comparison, and compare both mechanisms across all information structures. In earlier work, (Bergemann, Heumann, and Morris (2015)) we analyzed Cournot competition and characterized the restrictions on the outcomes across all information structures. Here, we provide a brief discussion on how the distribution of outcomes under Cournot competition is different than in demand function competition.

In Cournot competition the first moment of the distribution is independent of the information structure. In particular, the expected price is always equal to (21), with $\lambda = \beta$. In contrast, the set of feasible second moments is a three dimensional object. In particular, for any $(\rho_{p\bar{\theta}}, \rho_{\Delta q \Delta\theta}, \rho_{qq}) \in [0,1]^3$, there exists an information structure that induces a distribution of outcomes under Cournot competition with correlations $(\rho_{p\bar{\theta}}, \rho_{\Delta q \Delta\theta}, \rho_{qq})$.

In contrast, in demand function competition, the set of feasible first moments is a one-dimensional object. This can be seen from the fact that the distribution of the price is determined by λ, with any $\lambda \geq -1/2$. Yet, for a fixed first moment, the set of possible second moments is a two dimensional object. In particular, in the limit $\beta \to 0$, we have that:

$$\rho_{qq} = \frac{\rho^2_{p\bar{\theta}} \cdot (\frac{1}{N-1} + \rho_{\theta\theta}) - \rho^2_{\Delta q \Delta \theta} \cdot (1 - \rho_{\theta\theta}) \frac{1}{(N-1)}}{\rho^2_{p\bar{\theta}} \cdot (\frac{1}{N-1} + \rho_{\theta\theta}) + \rho^2_{\Delta q \Delta \theta} \cdot (1 - \rho_{\theta\theta})}.$$

Hence, for any $(\rho_{p\bar{\theta}}, \rho_{\Delta q \Delta \theta}) \in [0,1]^2$, ρ_{qq} is uniquely determined.[6]

The extra degree of freedom that demand function competition has on the first moment is a reflection of the fact that market power is endogenously determined. The extra degree of freedom that Cournot competition has in the second moments is reflection of the fact that agents cannot condition the quantity bought on the equilibrium price. Hence, there is less information that disciplines the quantities bought by agents. In Cournot competition, the price volatility and the volatility in the quantity demanded by the agents are not determined separately (as σ_p^2 and σ_q^2 in (22) and (23)) but rather there is a single equation that jointly determines the volatility in the quantities demanded by agents. This implies that the price volatility can increase with the absolute level of uncertainty about payoff shocks, σ_θ^2, and not only with the uncertainty about the average payoff shock $\sigma_{\bar{\theta}}^2$.

8 Conclusions

In this paper we study demand function competition. Our results provide positive and negative results regarding our ability to make predictions in this form of market microstructure. On the one hand, we showed that any market power is possible— from $-1/2$ to infinity. Considering small amounts of incomplete information does not allow us to provide any form of sharper predictions, unless one is able to make more restrictive assumptions regarding the nature of the incomplete information. On the other hand, we showed that we can provide many substantive predictions on the outcome of demand function competition that are robust to the choice of the information structure. Hence, a bit surprisingly, market power is the only quantity of the market outcome that is indeterminate.

[6] Away form the limit $\beta \to 0$, the correlation ρ_{qq} depends on $(\rho_{p\bar{\theta}}, \rho_{\Delta q \Delta \theta}) \in [0,1]^2$ and the level of market power λ. Yet, once we have fixed the first moment, we also fix the market power. Hence, for a fixed first moments, the set of correlations is a two-dimensional object.

The analysis in our paper provides a way of thinking about demand function competition in a more abstract way. In particular, we analyze directly quantities and payoff shocks, abstracting from the specific demands that are submitted in equilibrium. While this allows us to analyze demand function competition, it may also be helpful to analyze other forms of market microstructure, and perhaps more interestingly, to compare between them. We believe this may a fruitful direction for future work.

Appendix

We begin with the following lemma that gives a complete characterization of the set of linear Bayes Nash equilibria.

Lemma 6 (Characterization of Linear Nash Equilibrium)
The demand function $x(s_i, p) = c_0 + \sum_{j \in J} c_j s_j - m \cdot p$ is a linear Nash equilibrium if and only if:

$$x(s_i, p) = c_0 + \sum_{j \in J} c_j s_{ij} - m \cdot p = \frac{\mathbb{E}[\theta_i | p, \{s_{ij}\}_{j \in J}] - p}{1 + \lambda}, \tag{38}$$

where λ is given by:

$$\lambda = \frac{\beta}{1 + \beta \cdot m \cdot (N-1)}, \tag{39}$$

and the expectation $\mathbb{E}[\theta_i | p, \{s_{ij}\}_{j \in J}]$ is computed assuming that p is distributed as follows:

$$p = \frac{\alpha + \beta(N \cdot c_0 + c_1 \sum_{i \in N} \sum_{j \in J} s_{ij})}{1 + m \cdot \beta \cdot N}.$$

Proof. First, note that if all agents submit demand function as in (38), then market clearing implies that:

$$p^* = \alpha + \beta(N \cdot c_0 + \sum_{i \in N} \sum_{j \in J} c_j s_{ij}) - N \cdot m \cdot p^*.$$

Hence, the equilibrium price is given by:

$$p^* = \frac{\alpha + \beta(N \cdot c_0 + c_1 \sum_{i \in N} \sum_{j \in J} s_{ij})}{1 + m \cdot \beta \cdot N}.$$

Now fix an agent i. Given the demands submitted by other agents $\{x_j(p)\}_{j \neq i}$, agent i maximizes:

$$\max_{x_i(p) \in \mathcal{C}(\mathbb{R})} \quad \mathbb{E}[\theta_i \cdot x_i(p^*) - p^* \cdot x_i(p^*) - \frac{x_i(p^*)^2}{2}].$$

$$\text{such that } \alpha + \beta \sum_{\ell \in N} x_\ell(p^*) = p^*.$$

We conjecture a linear equilibrium in which:

$$x_j = c_0 + \sum_{j \in J} c_j s_j - m \cdot p.$$

Agent i faces a residual supply:

$$r_i(p) = \frac{p - \alpha - \beta \sum_{\ell \neq i} x_\ell(p)}{\beta}.$$

That is, if agent i submits a demand $x_i(p)$, then the equilibrium price is chosen to satisfy $x_i(p^*) = r_i(p^*)$.

We first solve the optimal quantity for agent i if he knew his residual supply. If agent i knows his residual supply, then he chooses a quantity q_i such that:

$$\max_{q_i \in \mathbb{R}} \mathbb{E}[\theta_i | r_i(p), \{s_{ij}\}_{j \in J}] q_i - r^{-1}(q_i) \cdot q_i - \frac{1}{2} q_i, \tag{40}$$

where $r_i^{-1}(\cdot)$ is the inverse function of r_i. Note that the residual supply of agent i (r_i) a priori contains information about θ_i, and hence, this is added as a conditioning variable. That is, in a linear Nash equilibrium the intercept of the residual supply $r_i(p)$ is measurable with respect to:

$$\sum_{\ell \neq i} \sum_{j \in J} c_j s_{\ell j}.$$

Hence, agent i can use the intercept of $r_i(p)$ as additional information on θ_i. Taking the first order condition:

$$\mathbb{E}[\theta_i | r_i, \{s_{ij}\}_{j \in J}] - r^{-1}(q_i^*) - q_i^* \frac{\partial r^{-1}(q_i^*)}{\partial q_i^*} - q_i^* = 0$$

With a minor abuse of notation, we define:[7]

$$\lambda \triangleq \frac{\beta}{1 + \beta \cdot m \cdot (N-1)},$$

and note that:

$$\frac{\partial r^{-1}(q_i)}{\partial q_i} = \left(\frac{\partial r_i(p)}{\partial p} \right)^{-1} = \lambda.$$

Note that the second order conditions is satisfied if and only if $\lambda \geq -1/2$. If $\lambda < -1/2$, then the objective function in (40) is a convex function of q_i, and hence, a maximum does not exist.

If agent i knows his residual demand, then the first order condition can be written as follows:

$$q_i^* = \frac{\mathbb{E}[\theta_i | r_i, \{s_{ij}\}_{j \in J}] - r^{-1}(q_i^*)}{1 + \lambda}.$$

Note that $r^{-1}(q_i^*)$ is the equilibrium price:

$$p^* = r^{-1}(q_i^*).$$

Hence, we can write the first order condition of agent i as follows:

$$q_i^* = \frac{\mathbb{E}[\theta_i | p^*, \{s_{ij}\}_{j \in J}] - p^*}{1 + \lambda}.$$

[7] There is an abuse of notation with respect to the definition of λ in (4). Nevertheless, in a linear Nash equilibrium both definitions coincide.

Note that the equilibrium price p^* is informationally equivalent to the intercept of the residual supply faced by agent i. This is because p^* is computed using r_i and the demand function submitted by agent i. Hence, for agent i, conditioning on the residual supply or the equilibrium price is informationally equivalent. Hence, we replace it as a conditioning variables.

Of course, in demand function competition agent i does not know his residual supply. Nevertheless, agent i submits a whole demand schedule. If agent i submits demand schedule:

$$x(p) = \frac{\mathbb{E}[\theta_i | p, \{s_{ij}\}_{j\in J}] - p}{1+\lambda}, \tag{41}$$

then he will buy the same quantity as if he knew his residual supply. The expectation $\mathbb{E}[\theta_i | p, \{s_{ij}\}_{j\in J}]$ is computed the same way as if p was the equilibrium price. That is, for any residual supply $r_i(p)$, if agent i submits demand function (41), then p^* is chosen to satisfy $x(p^*) = r_i(p^*)$. Hence, agent i buys a quantity:

$$q_i^* = \frac{\mathbb{E}[\theta_i | r_i, \{s_{ij}\}_{j\in J}] - p}{1+\lambda},$$

which is the optimal quantity as if he knew his residual supply.

Hence, a linear Nash equilibrium is determined by constants $(c_0, ..., c_J, m)$ such that:

$$c_0 + \sum_{j\in J} c_j s_j - m \cdot p = \frac{\mathbb{E}[\theta_i | p, \{s_{ij}\}_{j\in J}] - p}{1+\lambda},$$

where λ is given by:

$$\lambda = \frac{\beta}{1 + \beta \cdot m \cdot (N-1)},$$

and where expectation $\mathbb{E}[\theta_i | p, \{s_{ij}\}_{j\in J}]$ is computed the same way as if p was the equilibrium price. Hence, we prove the result. ■

Proof of Proposition 1. The equilibrium price is determined to satisfy:

$$\alpha + \beta \sum_{i\in N} x_i(p^*) = p^*$$

If agents submit demand functions as in (8), it is easy to check that the equilibrium price will be given by:

$$p^* = \frac{(1+\lambda)\alpha + \beta \cdot N \cdot \bar{\theta}}{1 + \lambda + \beta \cdot N}.$$

We now note that:

$$-\frac{(1-\widehat{\gamma}) \cdot \bar{\theta}}{1+\lambda} - \frac{\left(\frac{(\lambda+1)}{N-1}\left(\frac{1}{\beta} - \frac{1}{\lambda}\right) + 1\right)}{(\lambda + \beta \cdot N + 1)}\alpha - \frac{1}{N-1}(\frac{1}{\lambda} - \frac{1}{\beta}) \cdot p^* + \frac{1}{1+\lambda} p^* = 0.$$

Hence, we trivially have that:

$$\frac{\theta_i}{1+\lambda} - \frac{(1-\widehat{\gamma})\cdot\bar{\theta}}{1+\lambda} - \frac{\left(\frac{(\lambda+1)}{N-1}\left(\frac{1}{\beta}-\frac{1}{\lambda}\right)+1\right)}{(\lambda+\beta\cdot N+1)}\alpha - \frac{1}{N-1}(\frac{1}{\lambda}-\frac{1}{\beta})\cdot p^* + \frac{1}{1+\lambda}p^* = \frac{\theta_i}{1+\lambda}.$$

Hence, we can write (8) as follows:

$$x_i(p) = \frac{\mathbb{E}[\theta_i|\theta_i,\bar{\theta},p]-p}{1+\lambda}.$$

Additionally, note that if agents submit demand functions as in (8), then

$$m = \frac{1}{N-1}(\frac{1}{\lambda}-\frac{1}{\beta}).$$

Hence,

$$\lambda = \frac{\beta}{1+\beta\cdot m\cdot(N-1)}.$$

Hence, using Lemma 6, this is a linear Nash equilibrium. ■

Proof of Proposition 2. In any linear Nash equilibrium, the equilibrium price must be a linear function of the shocks $\{\eta_i\}_{i\in N}$ and the signals $\{s_i\}_{i\in N}$. Using the symmetry of the conjectured equilibrium, we have that in any symmetric linear Nash equilibrium, there exists constants $\hat{c}_0, \hat{c}_1, \hat{c}_2$ such that the equilibrium price satisfies:

$$p^* = \hat{c}_0 + \hat{c}_1\cdot\bar{\phi} + \hat{c}_2\cdot\bar{\eta}.$$

Regardless of the value of the constants, we have that:

$$\mathbb{E}[\theta_i|\{\eta_i\}_{i\in N}, s_i, p^*] = \theta_i.$$

That is, agent i can infer perfectly θ_i using the realization of the shocks $\{\eta_i\}_{i\in N}$, the signal s_i and the equilibrium price. This is because agent i can infer $\bar{\phi}$ from p^*, which in addition to s_i, allows agent i to perfectly infer ϕ_i. Using Lemma 6, agent i submits demand function:

$$x_i(p) = \frac{\mathbb{E}[\theta_i|\{\eta_i\}_{i\in N}, s_i, p^*]-p}{1+\lambda},$$

for some $\lambda \geq -1/2$. Hence, agent i in equilibrium buys a quantity:

$$q_i^* = \frac{\theta_i - p^*}{1+\lambda},$$

for some $\lambda \geq -1/2$. Using the market clearing condition, we must have that the equilibrium price is given by:

$$p^* = \frac{(1+\lambda)\alpha + \beta \cdot N \cdot \bar{\theta}}{1+\lambda+\beta \cdot N}, \tag{42}$$

for some $\lambda \geq -1/2$. Hence, the price must be measurable with respect to $\bar{\theta}$. That is, we have that the equilibrium price, must satisfy that $\hat{c}_1 = \hat{c}_2$. It is important to clarify that the linearity and symmetry of the conjectured equilibrium guarantees that the price is an affine function of $\bar{\eta}$ and $\bar{\phi}$. Yet, since the equilibrium price plus the private signals observed by agent i allow agent i to infer θ_i, the quantity bought by agent i is measurable with respect to θ_i. Hence, using the linearity and the symmetry, the price must be a linear function of $\bar{\theta}$.

Given the equilibrium price in (42) (as a function of λ), we can find an expression for the expected value of θ_i conditional on the private information of agent i and the equilibrium price. We first note that:

$$(1-\gamma)\left(\frac{p^*}{\beta \cdot N} \cdot (1+\lambda+\beta \cdot N) - \frac{(1+\lambda)\alpha}{\beta \cdot N} - \bar{\eta}\right) = (1-\gamma)\bar{\phi}.$$

Hence, the expectation cane written as follows:

$$\mathbb{E}[\theta_i | p^*, s_i, \{\eta_i\}_{i\in N}] = s_i + \eta_i + (1-\gamma)\left(\frac{p^*}{\beta \cdot N} \cdot (1+\lambda+\beta \cdot N) - \frac{(1+\lambda)\alpha}{\beta \cdot N} - \bar{\eta}\right) = \theta_i.$$

Remember that in equilibrium agent i submits demand function:

$$x_i(p) = \frac{\mathbb{E}[\theta_i | p^*, s_i, \{\eta_i\}_{i\in N}] - p}{1+\lambda}.$$

Hence, the slope of the demand submitted by agent i is equal to:

$$m = -\frac{\partial x_i(p)}{\partial p} = \frac{1-(1-\gamma)\frac{1}{\beta \cdot N} \cdot (1+\lambda+\beta \cdot N)}{1+\lambda}.$$

Yet, λ is determined by (39). Hence, we have that λ is the root to a quadratic equation:

$$\lambda = \frac{1}{2}\left(-1 - N \cdot \beta \cdot \frac{\gamma(N-1)-1}{\gamma(N-1)+1} \pm \sqrt{\left(N \cdot \beta \cdot \frac{\gamma(N-1)-1}{\gamma(N-1)+1}\right)^2 + 2 \cdot N \cdot \beta + 1}\right). \tag{43}$$

Only the positive root is a valid solution as the negative root yields a $\lambda < -1/2$, and hence, this is not an equilibrium. Hence, for a fixed γ, there is a unique symmetric linear Nash equilibrium. Additionally, inverting (43) (using the positive solution), we have that γ as a function of λ is given by (9). Hence, if γ is given by (9) there is a unique linear Nash equilibrium in which the equilibrium market power is λ. ■

Proof of Lemma 1. Remember that:

$$\bar{\theta} = \frac{1}{N}\sum_{i\in N}\theta_i.$$

Taking expectations, we get:

$$\mathbb{E}[\bar{\theta}] = \frac{1}{N}\sum_{i\in N}\mathbb{E}[\theta_i].$$

Using that the payoff shocks are symmetrically distributed, we get $\mu_{\bar{\theta}} = \mu_\theta$.

Note $\sigma^2_{\bar{\theta}} = \text{cov}(\bar{\theta},\bar{\theta})$. Using the collinearity of the covariance:

$$\sigma^2_{\bar{\theta}} = \frac{1}{N}\sum_{i\in N}\text{cov}(\theta_i,\bar{\theta}).$$

Using the symmetry assumption:

$$\sigma^2_{\bar{\theta}} = \text{cov}(\theta_i,\bar{\theta}) = \rho_{\theta\bar{\theta}}\sigma_{\bar{\theta}}\sigma_\theta.$$

Hence, $\sigma_{\bar{\theta}} = \rho_{\theta\bar{\theta}}\sigma_\theta$. Hence, we prove the result. ■

Proof of Lemma 2. (**First equation**) The market clearing condition is given by:

$$p = \alpha + \beta\sum_{i\in N} q_i.$$

Taking expectations of this equation:

$$\mathbb{E}[p] = \alpha + \beta\sum_{i\in N}\mathbb{E}[q_i].$$

Using that by symmetry $\mathbb{E}[q_i] = \mathbb{E}[q_j]$ we get:

$$\mu_p = \alpha + \beta\cdot N\cdot\mu_q.$$

(**Second equation**) By symmetry:

$$\text{cov}(\theta_i,p) = \text{cov}(\theta_j,p).$$

Hence, we have that:

$$\text{cov}(\theta_i,p) = \frac{1}{N}\sum_{j\in N}\text{cov}(\theta_j,p) = \text{cov}(\bar{\theta},p).$$

Hence,

$$\rho_{\theta p}\sigma_\theta\sigma_p = \rho_{\bar{\theta}p}\sigma_{\bar{\theta}}\sigma_p.$$

Using that $\sigma_{\bar{\theta}} = \rho_{\theta\bar{\theta}}\sigma_\theta$, we get:

$$\rho_{\theta p} = \rho_{\bar{\theta}p}\rho_{\theta\bar{\theta}}.$$

(Third equation) By symmetry:

$$\mathrm{cov}(q_i, \bar{\theta}) = \mathrm{cov}(q_j, \bar{\theta}).$$

Hence, we have that:

$$\mathrm{cov}(q_i, \bar{\theta}) = \frac{1}{N}\sum_{j\in N}\mathrm{cov}(q_j, \bar{\theta}) = \mathrm{cov}(\bar{q}, \bar{\theta}).$$

Hence,

$$\rho_{q\bar{\theta}}\sigma_q\sigma_{\bar{\theta}} = \rho_{\bar{q}\bar{\theta}}\sigma_{\bar{q}}\sigma_{\bar{\theta}}.$$

We have that $\sigma_{\bar{q}} = \rho_{q\bar{q}}\sigma_q$ (this can be proven the same way as we proved that $\sigma_{\bar{\theta}} = \rho_{\theta\bar{\theta}}\sigma_\theta$). Also, note that $\bar{q}$ is collinear with p, and hence $\rho_{\bar{q}\bar{\theta}} = \rho_{p\bar{\theta}}$ and $\rho_{q\bar{q}} = \rho_{qp}$. Hence, we get that:

$$\rho_{q\bar{\theta}} = \rho_{p\bar{\theta}} \cdot \rho_{qp}.$$

(Fourth Equation) The price is collinear with the price, and hence:

$$\sigma_p = \beta \cdot N \cdot \sigma_{\bar{q}}.$$

As before, we use that $\sigma_{\bar{q}} = \rho_{q\bar{q}}\sigma_q = \rho_{qp}\sigma_q$. Hence,

$$\sigma_p = \beta \cdot N \cdot \rho_{qp} \cdot \sigma_q.$$

■

Proof of Lemma polol **3.** Using Lemma lne 6, in any linear Nash equilibrium:

$$q_i = \frac{\mathbb{E}[\theta_i|\{s_{ij}\}_{j\in J}, p] - p}{1+\lambda}, \tag{44}$$ ffoc

for some $\lambda \geq -1/2$. Taking expectations and using the law of iterated expectations:

$$\mu_q = \frac{\mu_\theta - \mu_p}{1+\lambda}. \tag{45}$$ focf

In Lemma lemmendo 2 we proved that:

$$\mu_p = \alpha + \beta \cdot N \cdot \mu_q.$$

Replacing in (45):

$$\frac{\mu_p - \alpha}{\beta \cdot N} = \frac{\mu_\theta - \mu_p}{1+\lambda}. \tag{46}$$

Hence,

$$\mu_p = \frac{\beta \cdot N \cdot \mu_\theta + \alpha \cdot (1+\lambda)}{1+\lambda+\beta \cdot N}.$$

Rearranging terms we get the result. ■

Proof of Lemma 4. We begin with the restriction on the correlation coefficients given by (24). We explain each step subsequently:

$$\begin{aligned}
\rho_{\Delta q \Delta \theta} &= \frac{\mathrm{cov}(\Delta q_i, \Delta \theta_i)}{\sigma_{\Delta q}\sigma_{\Delta \theta}} && (47)\\
&= \frac{\mathrm{cov}(q_i, \theta_i) - \mathrm{cov}(\bar{q}, \bar{\theta})}{\sigma_{\Delta q}\sigma_{\Delta \theta}} && (48)\\
&= \frac{(\rho_{q\theta}\sigma_q\sigma_\theta - \rho_{p\bar{\theta}}\rho_{pq}\sigma_q\rho_{\theta\bar{\theta}}\sigma_\theta)}{\sigma_{\Delta q}\sigma_{\Delta \theta}} && (49)\\
&= \frac{(\rho_{q\theta}\sigma_q\sigma_\theta - \rho_{p\bar{\theta}}\rho_{pq}\sigma_q\rho_{\theta\bar{\theta}}\sigma_\theta)}{\sqrt{(1-\rho_{qp}^2)\sigma_q^2(1-\rho_{\theta\bar{\theta}}^2)\sigma_\theta^2}} && (50)\\
&= \frac{(\rho_{q\theta} - \rho_{p\bar{\theta}}\rho_{pq}\rho_{\theta\bar{\theta}})}{\sqrt{(1-\rho_{qp}^2)(1-\rho_{\theta\bar{\theta}}^2)}} && (51)
\end{aligned}$$

(47) is the definition of the covariance. (48) is as follows:

$$\mathrm{cov}(\Delta q_i, \Delta \theta_i) = \mathrm{cov}(q_i - \bar{q}, \theta_i - \bar{\theta}) = \mathrm{cov}(q_i, \theta_i) - \mathrm{cov}(\bar{q}, \bar{\theta}),$$

where the symmetry of the distribution is used to show that $\mathrm{cov}(\bar{q}, \bar{\theta}) = \mathrm{cov}(q_i, \bar{\theta}) = \mathrm{cov}(\bar{q}, \theta_i)$. The numerator of (49) is using the definition of the covariance and $\sigma_{\bar{\theta}}^2 = \rho_{\bar{\theta}\theta}\sigma_\theta^2$ and $\sigma_{\bar{q}}^2 = \rho_{pq}\sigma_q^2$ (see Lemma 1 and Lemma 2). (50) is as follows:

$$\sigma_{\Delta\theta}^2 = \mathrm{cov}(\theta_i - \bar{\theta}, \theta_i - \bar{\theta}) = \sigma_\theta^2 - \sigma_{\bar{\theta}}^2 = (1-\rho_{\theta\bar{\theta}}^2)\sigma_\theta^2.$$

where once again in the last equality we used Lemma 1. $\sigma_{\Delta q}^2$ is calculated in an analogous way (using Lemma 2). (51) is by simplifying the variances.

We use the results and equations used in the proof of Lemma 3. We begin by proving (22). Multiplying (44) by p, taking expectations, and using the law of iterated expectations, we get:

$$\mathbb{E}[q_i p] = \frac{\mathbb{E}[p \cdot \theta_i] - \mathbb{E}[p^2]}{1+\lambda} \tag{52}$$

Multiplying (45) by μ_p on both sides and subtracting it from (52):

$$\mathbb{E}[q_i p] - \mu_q \cdot \mu_p = \frac{\mathbb{E}[p\theta] - \mu_p \cdot \mu_\theta - (\mathbb{E}[p^2] - \mu_p^2)}{1+\lambda} \tag{53}$$

Hence, we have that:

$$\mathrm{cov}(q_i, p) = \frac{\mathrm{cov}(\theta_i, p) - \sigma_p^2}{1+\lambda}. \tag{54}$$

Note that:

$$\mathrm{cov}(q_i, p) = \mathrm{cov}(\bar{q}, p) = \frac{\sigma_p^2}{\beta \cdot N} \text{ and } \mathrm{cov}(\theta_i, p) = \mathrm{cov}(\bar{\theta}, p) = \rho_{\bar{\theta}p}\sigma_{\bar{\theta}}\sigma_p. \tag{55}$$

Hence, we have that:

$$\frac{\sigma_p^2}{\beta \cdot N} = \frac{\rho_{\bar{\theta}p}\sigma_{\bar{\theta}}\sigma_p - \sigma_p^2}{1+\lambda}. \tag{56}$$

Rearranging terms we get (22).

We now prove (23). We first note that:

$$\sigma_q^2 = \mathrm{var}(q_i) = \mathrm{var}(\bar{q} + \Delta q_i) = \sigma_{\bar{q}}^2 + \sigma_{\Delta q_i}^2,$$

where we use the $\mathrm{cov}(\bar{q}, \Delta q_i) = 0$.[8] Since $\bar{q}$ is collinear with p with a constant of proportionality of $\beta \cdot N$, we can use (22) to directly show that:

$$\sigma_{\bar{q}}^2 = \frac{\rho_{\bar{\theta}p}^2 \sigma_{\bar{\theta}}^2}{(\beta \cdot N + 1 + \lambda)^2}. \tag{57}$$

Multiplying (44) by q_i, and taking expectations, we get:

$$\mathbb{E}[q_i^2] = \frac{\mathbb{E}[\theta_i q_i] - \mathbb{E}[p \cdot q_i]}{1+\lambda} \tag{58}$$

Multiplying (45) by μ_q on both sides and subtracting it from (52):

$$\mathbb{E}[q_i^2] - \mu_q^2 = \frac{\mathbb{E}[\theta_i \cdot q_i] - \mu_q \cdot \mu_\theta - (\mathbb{E}[p \cdot q_i] - \mu_p \cdot q_i)}{1+\lambda} \tag{59}$$

Hence, we have that:

$$\sigma_q^2 = \frac{\mathrm{cov}(\theta_i, q_i) - \mathrm{cov}(p, q_i)}{1+\lambda}. \tag{60}$$

Dividing (54) by $\beta \cdot N$ and using that p and $\bar{q}$ are collinear with a constant of proportionality of $\beta \cdot N$, we have that:

$$\sigma_{\bar{q}}^2 = \frac{\mathrm{cov}(\theta_i, \bar{q}) - \mathrm{cov}(p, \bar{q})}{1+\lambda}. \tag{61}$$

[8] To check that $cov(\bar{q}, \Delta q_i) = 0$, simply note that $cov(\bar{q}, \Delta q_i) = cov(\bar{q}, q_i - \bar{q}) = cov(\bar{q}, q_i) - cov(\bar{q}, \bar{q}) = 0$.

Subtracting (61) from (60), and using (57) we get:

$$\sigma^2_{\Delta q} = \frac{\mathrm{cov}(\Delta\theta_i, \Delta q_i)}{1+\lambda}.$$

Hence, we have that:

$$\sigma_{\Delta q} = \frac{\rho_{\Delta q \Delta\theta} \cdot \sigma_{\Delta\theta_i}}{(1+\lambda)}. \tag{62}$$

Using (57) and (62), we have that:

$$\sigma^2_q = \sigma^2_{\bar{q}} + \sigma^2_{\Delta q} = \frac{\rho^2_{\bar{\theta}p}\sigma^2_{\bar{\theta}}}{(\beta \cdot N + 1 + \lambda)^2} + \frac{\rho^2_{\Delta q\Delta\theta} \cdot \sigma^2_{\Delta\theta_i}}{(1+\lambda)^2}$$

Hence, we prove the result. ■

Proof of Lemma 5. From (56) we have that σ_p is positive if and only if $\rho_{p\bar{\theta}}$ is positive. The standard deviation of any random variables is always positive, hence, we get (25). From (62) we have that $\sigma_{\Delta q}$ is positive if and only if $\rho_{\Delta\theta\Delta q}$ is positive. The standard deviation of any random variables is always positive, hence, $\rho_{\Delta\theta\Delta q} \in [0,1]$. Using (24) we get (26). ■

Proof of Lemma 3. The random variables $(\Delta\theta_i, \bar{\theta}, \Delta q_i, p,)$ is a linear combination of the random variables $(\theta_i, \bar{\theta}, q_i, p,)$. Hence, if the distribution of $(\Delta\theta_i, \bar{\theta}, \Delta q_i, p)$ is completely determined by $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q\Delta\theta}) \in (-1/2, \infty) \times [0,1] \times [0,1]$, then the distribution of $(\theta_i, \bar{\theta}, q_i, p)$ is completely determined by $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q\Delta\theta}) \in (-1/2, \infty) \times [0,1] \times [0,1]$.

We now show that the distribution of $(\Delta\theta_i, \bar{\theta}, \Delta q_i, p)$ is given by:

$$\begin{pmatrix} \Delta\theta_i \\ \bar{\theta} \\ \Delta q_i \\ p \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ \mu_{\bar{\theta}} \\ 0 \\ \mu_p \end{pmatrix}, \begin{pmatrix} \sigma^2_{\Delta\theta} & 0 & \rho_{\Delta q\Delta\theta}\sigma_{\Delta\theta}\sigma_{\Delta q} & 0 \\ 0 & \sigma^2_{\bar{\theta}} & 0 & \rho_{p\bar{\theta}}\sigma_{\bar{\theta}}\sigma_p \\ \rho_{\Delta q\Delta\theta}\sigma_{\Delta\theta}\sigma_{\Delta q} & 0 & \sigma^2_{\Delta q} & 0 \\ 0 & \rho_{p\bar{\theta}}\sigma_{\bar{\theta}}\sigma_p & 0 & \sigma^2_p \end{pmatrix} \right). \tag{63}$$

To check this, not that:

$$\mathbb{E}[\Delta\theta_i] = \mathbb{E}[\theta_i - \bar{\theta}] = \mathbb{E}[\theta_i] - \frac{1}{N}\sum_{j\in N}\mathbb{E}[\theta_j] = 0,$$

where in the last equality we used the symmetry of the distribution (that is, $\mathbb{E}[\theta_j] = \mathbb{E}[\theta_i]$). Hence, $\mu_{\Delta\theta} = 0$. Similarly, $\mu_{\Delta q} = 0$. For the terms in the variance covariance matrix, note that:

$$\mathrm{cov}(\Delta q_i, \bar{\theta}) = \mathrm{cov}(q_i - \frac{1}{N}\sum_{j\in N} q_j, \frac{1}{N}\sum_{j\in N}\theta_j) = \frac{1}{N}\sum_{j\in N}\mathrm{cov}(q_i, \theta_j) - \frac{1}{N^2}\sum_{j\in J}\sum_{\ell\in N}\mathrm{cov}(q_\ell, \theta_j) = 0.$$

where once again we just used the symmetry of the distribution and the colinearity of the covariance. Similarly,

$$\mathrm{cov}(\Delta q_i, \bar{q}) = \mathrm{cov}(\Delta\theta_i, \bar{\theta}) = \mathrm{cov}(\Delta\theta_i, \bar{q}) = 0.$$

For a given $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q \Delta\theta}) \in (-1/2, \infty) \times [0,1] \times [0,1]$, it is clear that μ_p and σ_p^2 are determined by (21) and (22). On the other hand, σ_q^2 is determined by (23) and we have that:

$$\sigma_{\Delta\theta}^2 = \sigma_q^2 - \sigma_{\bar{q}}^2 = \sigma_q^2 - \frac{\sigma_p^2}{(\beta N)^2}.$$

Hence, $\sigma_{\Delta\theta}^2$ is also determined by $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q \Delta\theta}) \in (-1/2, \infty) \times [0,1] \times [0,1]$. Looking at (63) it is clear that $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q \Delta\theta}) \in (-1/2, \infty) \times [0,1] \times [0,1]$ (plus (21), (22), (23)) determines the complete distribution of (63), and hence, also the distribution of $(\theta_i, \bar{\theta}, q_i, p)$. ■

Proof of Proposition 4. To be completed. ■

Proof of Proposition 5. To be completed. ■

Proof of Proposition 6. Using Lemma 6, in any linear Nash equilibrium agents submit demand functions:

$$x_i(p) = \frac{\mathbb{E}[\theta_i | s_1, ..., s_N, p] - p}{1+\lambda}$$

Let φ_i be defined as follows:

$$\varphi_i \triangleq \mathbb{E}[\theta_i | s_1, ..., s_N].$$

That is, $\bar{\varphi}_i$ is the best prediction of θ_i, given all the signals. Hence, we can rewrite agents' demand functions as follows:

$$x_i(p) = \frac{\mathbb{E}[\varphi_i | s_1, ..., s_N, p] - p}{1+\lambda}$$

Hence, we can replicate the argument in Proposition 1, but using φ_i instead of θ_i.

In any equilibrium with market power λ, the equilibrium price and the equilibrium quantity bought by agent i is given by:

$$p^* = \frac{\beta \cdot N \cdot \bar{\varphi} + (1+\lambda)\cdot\alpha}{(1+\lambda+\beta\cdot N)} \quad \text{and} \quad q_i = \frac{\bar{\varphi} - \alpha}{(1+\lambda+\beta\cdot N)} + \frac{\varphi_i - \bar{\varphi}}{1+\lambda} \tag{64}$$

We clearly have that:

$$\mathrm{corr}(p^*, \bar{\theta}) = \mathrm{corr}(\bar{\varphi}, \bar{\theta}) = \mathrm{corr}(\mathbb{E}[\bar{\theta}|s_1, ..., s_N], \bar{\theta}) = \frac{\sigma_{\bar{\theta}}^2}{\sigma_{\bar{\theta}}^2 + \sigma_{\bar{\varepsilon}}^2}.$$

$$\text{corr}(\Delta q_i, \Delta\theta_i) = \text{corr}(\Delta\varphi_i, \Delta\theta_i) = \text{corr}(\mathbb{E}[\Delta\theta_i|s_1, ..., s_N], \Delta\theta_i) = \frac{\sigma^2_{\Delta\theta}}{\sigma^2_{\Delta\theta} + \sigma^2_{\Delta\varepsilon}}.$$

Hence, the induced equilibrium outcome is determined by the market power $\lambda \in [-1/2, \infty)$ and the parameters:

$$(\rho_{p\bar{\theta}}, \rho_{\Delta q\Delta\theta}) = (\frac{\sigma^2_{\bar{\theta}}}{\sigma^2_{\bar{\theta}} + \sigma^2_{\bar{\varepsilon}}}, \frac{\sigma^2_{\Delta\theta}}{\sigma^2_{\Delta\theta} + \sigma^2_{\Delta\varepsilon}}). \tag{65}$$

We note that:

$$\sigma^2_{\bar{\theta}} = \frac{N\rho_{\theta\theta} + (1 - \rho_{\theta\theta})}{N} \text{ and } \sigma^2_{\Delta\theta} = \frac{(N-1)(1-\rho_{\theta\theta})}{N},$$

and similarly for $\sigma^2_{\bar{\varepsilon}}$ and $\sigma^2_{\Delta\varepsilon}$. Hence, it is easy to check that for any $(\rho_{p\bar{\theta}}, \rho_{\Delta q\Delta\theta}) \in [0,1]$, there exists $(\sigma^2_{\varepsilon}, \rho_{\varepsilon\varepsilon}) \in [0, \infty] \times [\frac{-1}{N-1}, 1]$ such that (65) is satisfied. Hence, for any $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q\Delta\theta}) \in [-1/2, \infty) \times [0,1] \times [0,1]$, there exists a set of public signals indexed by $(\sigma^2_{\varepsilon}, \rho_{\varepsilon\varepsilon}) \in [0, \infty] \times [\frac{-1}{N-1}, 1]$ and an equilibrium selection index by λ, such that the equilibrium outcome is given by the parameters $(\lambda, \rho_{p\bar{\theta}}, \rho_{\Delta q\Delta\theta})$. ■

Proof of Proposition 7 . Before we provide the proof, we note that:

$$s_i = \theta_i + \varepsilon_i + (\gamma - 1)(\bar{\theta} + \bar{\varepsilon}) = (\Delta\theta_i + \Delta\varepsilon_i) + \gamma(\bar{\theta} + \bar{\varepsilon}).$$

Consistent with the notation previously defined, we define:

$$\bar{s} \triangleq \gamma \cdot (\bar{\theta} + \bar{\varepsilon}) \text{ and } \Delta s_i \triangleq \Delta\theta_i + \Delta\varepsilon_i.$$

We note that the random variables $(\bar{\theta}, \bar{\varepsilon}, \bar{s})$ are orthogonal to $(\Delta\theta_i, \Delta\varepsilon_i, \Delta s_i)$ (see Lemma 3). Hence, the expectation can be written as follows:

$$\mathbb{E}[\theta_i|s_i, \bar{s}] = \frac{\sigma^2_{\Delta\theta}}{\sigma^2_{\Delta\theta} + \sigma^2_{\Delta\varepsilon}} \cdot \Delta s_i + \frac{\sigma^2_{\bar{\theta}}}{\sigma^2_{\bar{\theta}} + \sigma^2_{\bar{\varepsilon}}} \cdot \frac{1}{\gamma} \cdot \bar{s}$$

This can be rewritten as follows:

$$\mathbb{E}[\theta_i|s_i, \bar{s}] = \frac{\sigma^2_{\Delta\theta}}{\sigma^2_{\Delta\theta} + \sigma^2_{\Delta\varepsilon}} \cdot s_i + (1 - \tilde{\gamma})\left(\frac{\sigma_{\bar{\theta}}}{\sigma_{\bar{\theta}} + \sigma_{\bar{\varepsilon}}}\frac{1}{\gamma}\bar{s}\right), \tag{66}$$

where $\tilde{\gamma}$ is defined as in (31).

In any linear Nash equilibrium, the equilibrium price must be a linear function of the signals $\{s_i\}_{i \in N}$. Using the symmetry of the conjectured equilibrium, we have that in any symmetric linear Nash equilibrium, there exists constants $\hat{c}_0, \hat{c}_1$ such that the equilibrium price satisfies:

$$p^* = \hat{c}_0 + \hat{c}_1 \cdot (\frac{1}{N}\sum_{i \in N} s_i).$$

We note that:

$$\mathbb{E}[\theta_i|s_i, p^*] = \mathbb{E}[\theta_i|s_i, \bar{s}].$$

Hence, agent i in equilibrium buys a quantity:

$$q_i^* = \frac{\mathbb{E}[\theta_i|s_i, \bar{s}] - p^*}{1+\lambda}, \tag{67}$$

for some $\lambda \geq -1/2$. Using the market clearing condition, we must have that the equilibrium price is given by:

$$p^* = \frac{(1+\lambda)\alpha + \beta \cdot N \cdot (\frac{\sigma_{\bar{\theta}}^2}{\sigma_{\bar{\theta}}^2 + \sigma_{\bar{\varepsilon}}^2} \cdot \frac{1}{\gamma} \cdot \bar{s})}{1 + \lambda + \beta \cdot N}. \tag{68}$$

Given the equilibrium price in (68) (as a function of λ), we can find an expression for the expected value of θ_i conditional on the private information of agent i and the equilibrium price:

$$\mathbb{E}[\theta_i|p^*, s_i] = \left(\frac{\sigma_{\Delta\theta}^2}{\sigma_{\Delta\theta}^2 + \sigma_{\Delta\varepsilon}^2} \cdot s_i + (1-\tilde{\gamma})\right) \frac{p^*}{\beta \cdot N} \cdot (1 + \lambda + \beta \cdot N) - \frac{(1+\lambda)\alpha}{\beta \cdot N}\Bigg).$$

This corresponds to rewriting (66) in terms of p^* instead of $\bar{s}$. Remember that in equilibrium agent i submits demand function:

$$x_i(p) = \frac{\mathbb{E}[\theta_i|p^*, s_i] - 1}{1+\lambda}.$$

Hence, the slope of the demand submitted by agent i is equal to:

$$m = \frac{1 - (1-\tilde{\gamma})\frac{1}{\beta \cdot N} \cdot (1 + \lambda + \beta \cdot N)}{1+\lambda}.$$

Yet, λ is determined by (39). Hence, we have that λ is the root to a quadratic equation:

$$\lambda = \frac{1}{2}\Bigg(-1 - N \cdot \beta \cdot \frac{\tilde{\gamma}(N-1) - 1}{\tilde{\gamma}(N-1) + 1} \pm \sqrt{\left(N \cdot \beta \cdot \frac{\tilde{\gamma}(N-1) - 1}{\tilde{\gamma}(N-1) + 1}\right)^2 + 2 \cdot N \cdot \beta + 1}\Bigg). \tag{69}$$

Only the positive root is a valid solution as the negative root yields a $\lambda < -1/2$, and hence, this is not an equilibrium. Hence, for a fixed $\tilde{\gamma}$, there is a unique symmetric linear Nash equilibrium. Additionally, inverting (69) (using the positive solution), we have that $\tilde{\gamma}$ as a function of λ is given by (9). Hence, if $\tilde{\gamma}$ is given by (9) there is a unique linear Nash equilibrium in which the equilibrium market power is λ.

Note that the equilibrium outcome (67) and (68) is the same as the outcome of an equilibrium with public signals for any $\lambda \geq -1/2$. Hence, using Proposition 6 we have that all equilibrium outcomes can be decentralized as a unique linear Nash equilibrium with a one-dimensional signal as in (29). ∎

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